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CS/MATH111 ASSIGNMENT 4

due 8AM, Tuesday, November 22

Individual assignment: Problems 1 and 2.
Group assignment: Problems 1,2 and 3.

Problem 1: Give the asymptotic value (using the Θ-notation) for the number of letters that will be printed by the algorithms below. In each algorithm the argument n is a positive integer. Your solution needs to consist of an appropriate recurrence equation and its solution, including a justification for both.

(a) **Algorithm** PrintAs (n : **integer**)
    if $n < 3$
        print("A")
    else
        PrintAs($\lceil n/3 \rceil$)
        for $i \leftarrow 1$ **to** 9 **do** print("A")

(b) **Algorithm** PrintBs (n : **integer**)
    if $n < 3$
        print("B")
    else
        PrintBs($\lfloor n/3 \rfloor$)
        PrintBs($\lceil n/3 \rceil$)
        for $i \leftarrow 1$ **to** $9n$ **do** print("B")

(c) **Algorithm** PrintCs (n : **integer**)
    if $n < 3$
        print("C")
    else
        PrintCs($\lceil n/3 \rceil$)
        PrintCs($\lceil n/3 \rceil$)
        PrintCs($\lceil n/3 \rceil$)
        for $i \leftarrow 1$ **to** $9n$ **do** print("C")

(d) **Algorithm** PrintDs (n : **integer**)
    if $n < 3$
        print("D")
    else
        for $j \leftarrow 1$ **to** 4 **do** PrintDs($\lceil n/3 \rceil$)
        for $i \leftarrow 1$ **to** $9n$ **do** print("D")

(e) **Algorithm** PrintEs (n : **integer**)
    if $n < 3$
        print("E")
    else
        for $j \leftarrow 1$ **to** 4 **do** PrintEs($\lceil n/3 \rceil$)
        for $i \leftarrow 1$ **to** $9n^2$ **do** print("E")

Solution 1:

a)

Because ther is the statement: if $n < 3$ followed by a print("A") we know that for $n \geq 3$

The Recurrence Equation:

$A(n) = A(n/3)$ +9

Solving the recurrance equation:

Using the Master Theorem where $a = 1$, $b = 3$, $c = 9$, $d = 0$ we see that:

$1 = 3^0$

$1 = 1$

By the Master Theorem:

$A(n) = \Theta(logn)$

b)

for $n > 3$:

The recurrence equation is:

$B(n) = 2B(n/3) + 9n$

The $2B(n/3)$ comes from the two recurrance calls, and the 9n comes from the parameters in the for loop.

Solving the recurrence equation using the Master Theorem where $a = 2$, $b = 3$, $c = 9$, $d = 1$ we see that:

$a > b^d$

$2 > 3^1$

By the Master Theorem:

for $a < b^d$; $f_n = \Theta(n^d)$

$B(n) = \Theta(n^1)$

$B(n) = \Theta(n)$

c)

for $n > 3$:

The recurrence equation is:

$C(n) = 3C(n/3) + 9n$

This is because there are 3 recursive calls and a forloop that prints("C") 9n times.
Solving the recurrnace equation using the Master Theorem where: $a = 2$, $b = 3$, $c = 9$, $d = 1$ we see that:

$a = b^d$

$3 = 3^1$

By the Master Theorem:

for $a = b^d$; $f_n = \Theta(n^d logn)$

So:

$C(n) = \Theta(n^1 logn)$

$C(n) = \Theta(nlogn)$

d)

for $n > 3$:

The recurrence equation is:

$D(n) = 4D(n/3) + 9n$

This is because there are 4 recursive calls and a forloop that prints("D") 9n times.

Solving the recurrnace equation using the Master Theorem where: $a = 4$, $b = 3$, $c = 9$, $d = 1$ we see that:

$4 > 3^1$

$a > b^d$

By the Master Theorem:

for $a > b^d$; $f_n = \Theta(n^{log_b a})$

So:

$D(n) = \Theta(n^{log_3 4})$

e)

for $n > 3$:

The recurrence equation is:

$E(n) = 4E(n/3) + 9n^2$

This is because there are 4 recursive calls and a forloop that prints("D") $9n^2$ times.

Solving the recurrnace equation using the Master Theorem where: $a = 4$, $b = 3$, $c = 9$, $d = 2$ we see that:

$4 < 3^2$

$a < b^d$

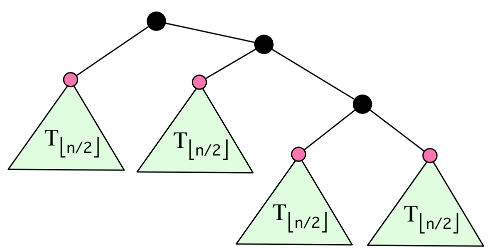
By the Master Theorem:

for $a < b^d$; $f_n = \Theta(n^d)$

So:

$E(n) = \Theta(n^2)$

Problem 2: For each integer $n \geq 1$ we define a tree T_n, recursively, as follows. T_1 consists of only a single node. For $n \geq 2$, T_n is obtained from four copies of $T_{\lfloor n/2 \rfloor}$ and three additional nodes, by connecting them as follows:



Let $s(n)$ be the number of nodes in T_n.

(a) Give a recurrence equation for $s(n)$ and justify it.

(b) Determine the exact values of $s(n)$ for $n = 1, 2, 3, 4, 5, 6, 7, 8, 9$. (Use the table formatting, as show at the end of the homework.)

(c) Draw T_5. (You can use a drawing software or draw it by hand, and include a pdf file in the latex source.)

(d) Give the asymptotic formula for $s(n)$, by solving the recurrence from part (a). Justify your solution.

Solution 2:

a)

The inital condition is:

for $n \geq 1$:

$s(n) = 4s_{\lfloor n/2 \rfloor}$

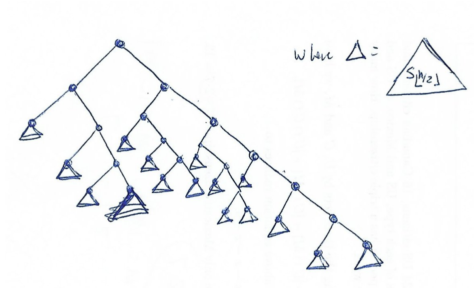
This is the recurrance eqaution because there are 4 recursive calls to $T_{\lfloor n/2 \rfloor}$ (which is rounded down) plus an additional 3 nodes.

b)

n	1	2	3	4	5	6	7	8	9
$s(n)$	1	7	7	31	31	31	31	127	127

c)

This is T_5 :



d)

Given the recurance equation $s(n) = 4s_{\lfloor n/2 \rfloor} + 3$

Using the Master Theorem where $a = 4$, $b = 3$, $c = 3$, and $d = 0$:

$a > b^d$

$4 > 2^0$

By the Master theorem when $a > b^d$, $\Theta(n^{log_b a})$

$s(n) = \Theta(n^{log_2 4})$

So the asymptotic formula for the equation is:

$s(n) = \Theta(n^2)$

Problem 3: RockCyc is a mountain bicycle that is available with three upgrade options: for the fork, brakes, or wheels. 105 customers bought a RockCyc bike. Among them:

- 45 upgraded fork
- 36 upgraded brakes
- 25 upgraded wheels
- 17 upgraded fork and brakes
- 13 upgraded fork and wheels

- 10 upgraded brakes and wheels
- 4 upgraded all three components

(To clarify, these categories are not exclusive. For example, the set of 45 customers that upgraded the fork includes those that upgraded the fork and the brakes.)

Use the inclusion-exclusion principle to determine how many customers bought the basic model of Rock-Cyc, without any upgrades. Show your work.

Submission. To submit the homework, you need to upload the pdf file into ilearn by 8AM, Tuesday, November 22, and turn-in a paper copy in class.

Here is how you can format the table for the solution of Problem 2b:

n	1	2	3	4	5	6	7	8	9
$s(n)$	a	b	c	d	e	f	g	h	i