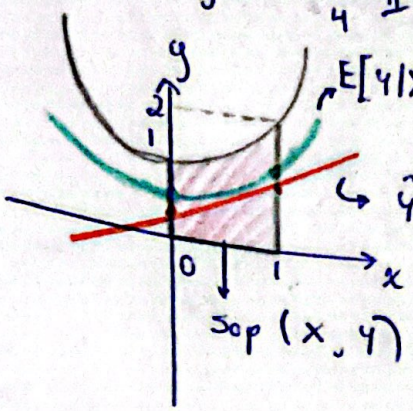
$$P_{XY}(x,y) = \frac{3}{4}\mathbb{1}\{0<y<1+x^2,\ 0<x<1\}$$



$$E[Y|X=x] = \frac{1}{2}(x^2+1)$$

$$f_X(x) = \frac{3}{4}(1+x^2)\mathbb{1}\{0<x<1\}$$

$$f_{Y|X=x}(y) = \frac{1}{1+x^2}\mathbb{1}\{0<y<1+x^2\}$$

para $x \in (0,1)$

$$E[Y|X=x] = \int_0^{1+x^2} y\,\frac{1}{1+x^2}\,dy = \frac{1}{2}(x^2+1)$$

$$E[X] = \frac{3}{4}\int_0^1 (x+x^3)\,dx = \frac{9}{16} \qquad E[Y] = E[E[Y|X]] = \frac{1}{2}E[X^2] + \frac{1}{2} = \frac{7}{10}$$

$$Var(X) = E[X^2] - E^2[X] = \frac{3}{4}\int_0^1 (x^2+x^4)\,dx - \left(\frac{9}{16}\right)^2 = \frac{107}{1280}$$

$$Cov(X,Y) = \underbrace{E(XY)}_{\int_0^1\int_0^{1+x^2} \frac{3}{4}xy\,dy\,dx} - E(X)E(Y) = \frac{7}{16} - \frac{7}{10}\cdot\frac{9}{16} = \frac{7}{160}$$

$$\hat{Y} = \frac{Cov(X,Y)}{Var(X)}(X - E[X]) + E[Y] = \frac{7/160}{107/1280}\left(X - \frac{9}{16}\right) + \frac{7}{10}$$

$$= \frac{56}{107}X + \frac{217}{535}$$

Error Bayesiano

$$E[Var(Y|X)] = E\left[E[Y^2|X] - E^2[Y|X]\right]$$

$$= E\left[\frac{(x^2+1)^2}{3}\right] - E^2\left[\frac{1}{2}(x^2+1)\right]$$

$$= \frac{1}{3}\left(E[X^4] + 2E[X^2] + 1\right) - \frac{1}{4}\left(E[X^2]+1\right)^2$$

$$= \frac{1}{3}\left(\frac{9}{35} + \frac{4}{5} + 1\right) - \frac{1}{4}\left(\frac{2}{5}+1\right)^2 = \frac{137}{700}$$