Effect Size: Comparing Cohen d and Pearson r

Notes:

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Effect size quantifies the strength of the relationship between variables or the magnitude of the difference between groups.

Effect size - Cohen's *d*

Cohen d is used to quantify the size of the difference between two groups, taking into account the variability within each group.

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}}$$

s_1 is the sample standard deviation for group 1, whereas s_2 is the sample standard deviation for group 2.

Effect Size - Pearson Product Moment *r*

The Pearson product-moment correlation coefficient, often referred to as the Pearson correlation coefficient, quantifies the strength of a linear relationship between two variables.

$$r_{xy} = \frac{cov(x, y)}{s_x s_y} = \frac{\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^{n}(x_i - \bar{x})^2}\sqrt{\sum_{i=1}^{n}(y_i - \bar{y})^2}}$$

Cohen's *d* and Pearson's *r*

The following effect size numbers are based on Jacob Cohen. (*reference is included at the end of this tutorial.*)

Effect Size	d	r
Small	0.2	0.1
Medium	0.5	0.3
Large	0.8	0.5

Comments

Cohen's d can assume values ranging from 0 to infinity (i.e., can be greater than 1), whereas Pearson's r is confined to a range between -1 and 1.

Note that, based on my best understanding (not sure), Cohen's d can be negative as well, depending on how to define m_1 and m_2 and which order to put them.

Convert r to d, and vice versa

While Cohen's d looks different from Pearson's r, there is internal connection between these two.

Group 1 and group 2 may be considered to be a dichotomy or a two point scale.

Thus, you can convert r to d, and vice versa.

Convert r to d

$$d = \frac{2r}{\sqrt{1-r^2}}$$

Convert d to r

$$d^2 = \frac{4r^2}{1-r^2}$$

$$d^2 = 4r^2 + r^2d^2 = r^2(4+d^2)$$

$$r^2 = \frac{d^2}{4+d^2}$$

$$r = \frac{d}{\sqrt{4+d^2}}$$

According to Cohen, sometimes, people might want to think of effect sizes for mean differences d in terms of r. Thus, you can convert d to r if needed.

For instance, group 1 and group 2 are two experimental manipulations. The effect size typically is d. But you can convert it into r.

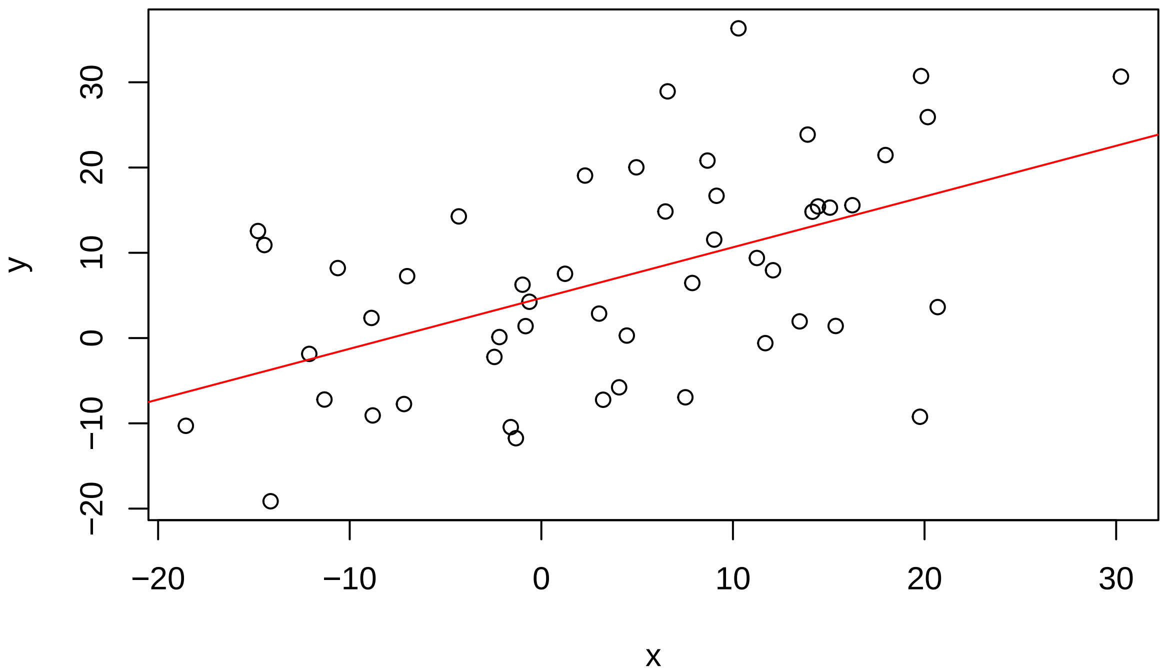
I will provide data to demonstrate the connection between d and r.

Simulate both X and Y that are numerical

```r
# set the seed for reproducibility
set.seed(123)

# specify marginal means and standard deviations
mean_x <- 5
mean_y <- 6
sd_x <- 12
sd_y<- 14

## specify coefficient, which must be between -1 and 1
corcoef <- 0.5

## covariance between two variables
covariance <- corcoef * sd_x * sd_y

## variance-covariance matrix
sigma <- matrix(c(sd_x^2, covariance, covariance, sd_y^2), nrow = 2)

xy <- MASS::mvrnorm(n = 50, mu = c(mean_x, mean_y), Sigma = sigma)
colnames(xy) <- c("x", "y")
df <- data.frame(xy)
head(df)
```

```
##           x          y
## 1 -2.188776  0.1167428
## 2  3.014416  2.8711033
## 3 20.168055 25.9184510
## 4 -4.307292 14.2689957
## 5  7.878421  6.4575243
## 6 10.288163 36.3225845
```

```r
# plot it
plot(df)
abline(lm(y ~ x,data=df), col = "red")
```




```
cor.test(df$x,df$y)
```

```
##
##  Pearson's product-moment correlation
##
## data:  df$x and df$y
## t = 4.2497, df = 48, p-value = 9.8e-05
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
##  0.2861663 0.6994209
## sample estimates:
##       cor
## 0.5228658
```

r vs. d: Difference and Connection

Effect Size	d	r
Small	0.2	0.1
Medium	0.5	0.3
Large	0.8	0.5

```r
# dichotomize x
df$dichotomized_x<- ifelse(df$x >mean(df$x), 1, 0)
head(df)
```

```
##           x          y dichotomized_x
## 1 -2.188776  0.1167428              0
## 2  3.014416  2.8711033              0
## 3 20.168055 25.9184510              1
## 4 -4.307292 14.2689957              0
## 5  7.878421  6.4575243              1
## 6 10.288163 36.3225845              1
```

Method 1: Direct Calculating d

Since we got a column of dichotomized x, we can actually use the formula mentioned to calculate Cohen's d. That is, calculate the means for groups 1 and 2. Then, calculate the sample standard deviations.

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}}$$

```r
# calculate group means
means<-aggregate(df$y, list(df$dichotomized_x), FUN=mean)

# calculate sample standard deviation - pooled
variances<- aggregate(df$y, list(df$dichotomized_x),FUN=var)
s_pooled=sqrt((variances$x[1]+variances$x[2])/2)

d_calculated<-(means$x[2]-means$x[1])/s_pooled
print(d_calculated)
```

```
## [1] 1.252203
```

Method 2: Converting r to d

$$d = \frac{2r}{\sqrt{1 - r^2}} = \frac{2 * 0.52}{\sqrt{1 - 0.52^2}} = 1.2$$

```r
# check the correlation coefficient
cor_result<-cor.test(df$x,df$y)
correlation_coefficient<-cor_result$estimate
d_converted<-2*correlation_coefficient/sqrt(1-(correlation_coefficient^2))
print(d_converted)
```

```
##      cor
## 1.226787
```

References

1. Jacob Cohen, 1988, Statistical Power Analysis for the Behavioral Sciences, Second Edition
2. What is the relation between the effect size and correlation?

https://stats.stackexchange.com/questions/412590/what-is-the-relation-between-the-effect-size-and-correlation

3. What is Effect Size and Why Does It Matter? (Examples)

https://www.scribbr.com/statistics/effect-size/

4. How to simulate a strong correlation of data with R

https://stackoverflow.com/questions/72894192/how-to-simulate-a-strong-correlation-of-data-with-r

5. FAQ How is effect size used in power analysis?

https://stats.oarc.ucla.edu/other/mult-pkg/faq/general/effect-size-power/faqhow-is-effect-size-used-in-power-analysis/