The Meaning of Cohen's d

Note and Disclaimer

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Data Example

Group	*1*	*2*
	10	15
	20	25
	30	35
	40	45
	50	55
Mean	30	35
SD	15.81	15.81

Mean Difference

$$m_2 - m_1 = 35 - 30 = 5$$

Cohen's d

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}} = \frac{5}{15.81} = 0.32$$

Thus, the sole distinction between Cohen d and mean difference lies in the consideration of standard deviation.

Different Sizes of Effect Size

The following effect size numbers are from Jacob Cohen's *Statistical Power Analysis for the Behavioral Sciences*.

Effect Size	d
Small	0.2
Medium	0.5
Large	0.8

Cohen's d > 1

Cohen d can be greater than 1, when the pooled standard deviation is smaller than the mean difference.

Group	*1*	*2*
	1	6
	2	7
	3	8
	4	9
	5	10
Mean	3	8
SD	1.58	1.58

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}} = \frac{5}{1.58} = 3.16$$

Data Examples

Mean difference = 5 and SD = 1.58:

$$d = \frac{5}{1.58} = 3.16$$

```r
# Set the seed for reproducibility
set.seed(123)

# Specify parameters
n1 <- 100  # Sample size for group 1
n2 <- 100  # Sample size for group 2
mean_diff <- 5  # Desired mean difference between groups
sd <- 1.58  # Common standard deviation for both groups

# Generate random samples for two groups
group1 <- rnorm(n1, mean = 0, sd = sd)
group2 <- rnorm(n2, mean = mean_diff, sd = sd)

# Perform a two-sample t-test
t.test(group1, group2)
```

```
##
##  Welch Two Sample t-test
##
## data:  group1 and group2
## t = -22.309, df = 197.35, p-value < 2.2e-16
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -5.101572 -4.272897
## sample estimates:
## mean of x mean of y
## 0.1428413 4.8300761
```

Mean difference = 5 and SD = 10:

$$d = \frac{5}{10} = 0.5$$

```r
# Set the seed for reproducibility
set.seed(123)

# Specify parameters
n1 <- 100  # Sample size for group 1
n2 <- 100  # Sample size for group 2
mean_diff <- 5  # Desired mean difference between groups
sd <- 10  # Common standard deviation for both groups

# Generate random samples for two groups
group1 <- rnorm(n1, mean = 0, sd = sd)
group2 <- rnorm(n2, mean = mean_diff, sd = sd)

# Perform a two-sample t-test
t.test(group1, group2)
```

```
##
##  Welch Two Sample t-test
##
## data:  group1 and group2
## t = -2.2714, df = 197.35, p-value = 0.0242
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -5.6428618 -0.3980841
## sample estimates:
## mean of x mean of y
## 0.9040591 3.9245320
```

Mean difference = 5 and SD = 20:

$$d = \frac{5}{20} = 0.25$$

```r
# Set the seed for reproducibility
set.seed(123)

# Specify parameters
n1 <- 100  # Sample size for group 1
n2 <- 100  # Sample size for group 2
mean_diff <- 5  # Desired mean difference between groups
sd <- 20  # Common standard deviation for both groups

# Generate random samples for two groups
group1 <- rnorm(n1, mean = 0, sd = sd)
group2 <- rnorm(n2, mean = mean_diff, sd = sd)

# Perform a two-sample t-test
t.test(group1, group2)
```

```
##
##  Welch Two Sample t-test
##
## data:  group1 and group2
## t = -0.3914, df = 197.35, p-value = 0.6959
## alternative hypothesis: true difference in means is not equal to 0
## 95 percent confidence interval:
##  -6.285724  4.203832
## sample estimates:
## mean of x mean of y
##  1.808118  2.849064
```

Observations:

(1) While the mean difference is kept the same, different standard deviations to different effect sizes, which lead to different p-values.

(2) For the same sample size, larger effect sizes will lead to smaller p-values.