

# Effect Size: Cohen d vs Pearson r

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## Effect size - Cohen's $d$

Cohen  $d$  is used to quantify the size of the difference between two groups, taking into account the variability within each group.

### Population

$$d = \frac{\mu_2 - \mu_1}{\sigma}$$

### Sample

We can use a sample to estimate the  $d$  by replacing with sample means and sample standard deviation.

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}}$$

$s_1$  is the sample standard deviation for group 1, whereas  $s_2$  is the sample standard deviation for group 2.

## Effect Size - Pearson Product Moment $r$

The Pearson product-moment correlation coefficient, often referred to as the Pearson correlation coefficient ( $\rho$ ), quantifies the strength of a linear relationship between two variables.

### Population

$$\rho = \frac{cov(X, Y)}{\sigma_X \sigma_Y}$$

### Sample

We can use a sample correlation coefficient, namely  $r$ , to estimate population correlation coefficient  $\rho$ .

$$r_{xy} = \frac{cov(x, y)}{s_x s_y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

# Comparing Cohen's $d$ and Pearson's $r$

## 1. Basic Idea

The following effect size numbers are based on Jacob Cohen's *Statistical Power Analysis for the Behavioral Sciences* (Second Edition, p. 82)

Effect Size	$d$	$r$
Small	0.2	0.1
Medium	0.5	0.3
Large	0.8	0.5

## 2. Additional comments

Cohen's  $d$  can assume values ranging from 0 to infinity (i.e., can be greater than 1), whereas Pearson's  $r$  is confined to a range between -1 and 1.

Note that, based on my best understanding (not sure), Cohen's  $d$  can be negative as well, depending on how to define  $m_1$  and  $m_2$  and which order to put them.

## 3. Convert $r$ to $d$ , and vice versa

While Cohen's  $d$  looks different from Pearson's  $r$ , there is internal connection between these two.

The basic idea is that, the membership for group 1 and group 2 may be considered to be a dichotomy or a two point scale.

Thus, you can convert  $r$  to  $d$ , and vice versa.

### Convert $r$ to $d$

$$d = \frac{2r}{\sqrt{1-r^2}}$$

### Convert $d$ to $r$

$$d^2 = \frac{4r^2}{1-r^2}$$

$$d^2 = 4r^2 + r^2d^2 = r^2(4 + d^2)$$

$$r^2 = \frac{d^2}{4 + d^2}$$

$$r = \sqrt{\frac{d^2}{4 + d^2}} = \frac{d}{\sqrt{4 + d^2}}$$

For the formula converting  $d$  to  $r$ , you can refer to Jacob Cohen's *Statistical Power Analysis for the Behavioral Sciences* (Second Edition, p.23).

According to Cohen, sometimes, people might want to think of effect sizes for mean differences  $d$  in terms of  $r$ . Thus, you can convert  $d$  to  $r$  if needed.

For instance, group 1 and group 2 are two experimental manipulations. The effect size typically is  $d$ . But you can convert it into  $r$ .

I will provide data to demonstrate the connection between  $d$  and  $r$ .

## Q1: What is the use case of effect size?

Effect size can be used to decide sample size.

As effect size decreases, the need for a larger sample size increases.

1. Effect size = 0.40

```
# Load the pwr package (if not already installed)  
# install.packages("pwr")
```

```
library(pwr)  
pwr.t.test(n=NULL,  
           d = 0.4,  
           sig.level = 0.05,  
           power = 0.80,  
           type = "two.sample",  
           alternative="two.sided")  
  
##  
##      Two-sample t test power calculation  
##  
##              n = 99.08032  
##              d = 0.4  
##      sig.level = 0.05  
##      power = 0.8  
##      alternative = two.sided  
##  
## NOTE: n is number in each group
```

2. Effect size = 0.30

```
pwr.t.test(n=NULL,  
           d = 0.3,  
           sig.level = 0.05,  
           power = 0.80,  
           type = "two.sample",  
           alternative="two.sided")  
  
##  
##      Two-sample t test power calculation  
##  
##              n = 175.3847  
##              d = 0.3  
##      sig.level = 0.05  
##      power = 0.8  
##      alternative = two.sided  
##  
## NOTE: n is number in each group
```

Changing effect size from 0.4 to 0.3, the required sample size increases from 99 to 135.

## Q2: For effect size, any difference between correlation and linear regression?

### 1. Theoretical part

Correlation:

$$r_{xy} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Estimated slope in simple linear regression:

$$\hat{b} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$$

Standardized regression coefficient:

$$\hat{\beta} = \hat{b} \frac{s_x}{s_y} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2} \frac{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2}}{\sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}} = \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sqrt{\sum_{i=1}^n (x_i - \bar{x})^2} \sqrt{\sum_{i=1}^n (y_i - \bar{y})^2}}$$

Thus, correlation coefficient is equal to the standardized regression coefficient for the same X and Y in simple linear regressions (i.e., a regression model with one X and one Y).

### 2. Simulate a data for correlation / simple linear regression

```
# set the seed for reproducibility
set.seed(123)

# specify marginal means and standard deviations
mean_x <- 5
mean_y <- 6
sd_x <- 12
sd_y <- 14

## specify coefficient, which must be between -1 and 1
corcoef <- 0.5

## covariance between two variables
covariance <- corcoef * sd_x * sd_y

## variance-covariance matrix
sigma <- matrix(c(sd_x^2, covariance, covariance, sd_y^2), nrow = 2)

xy <- MASS::mvrnorm(n = 50, mu = c(mean_x, mean_y), Sigma = sigma)
colnames(xy) <- c("x", "y")
df <- data.frame(xy)
head(df)

##           x           y
## 1 -2.188776  0.1167428
```

```
## 2  3.014416  2.8711033
## 3 20.168055 25.9184510
## 4 -4.307292 14.2689957
## 5  7.878421  6.4575243
## 6 10.288163 36.3225845
```

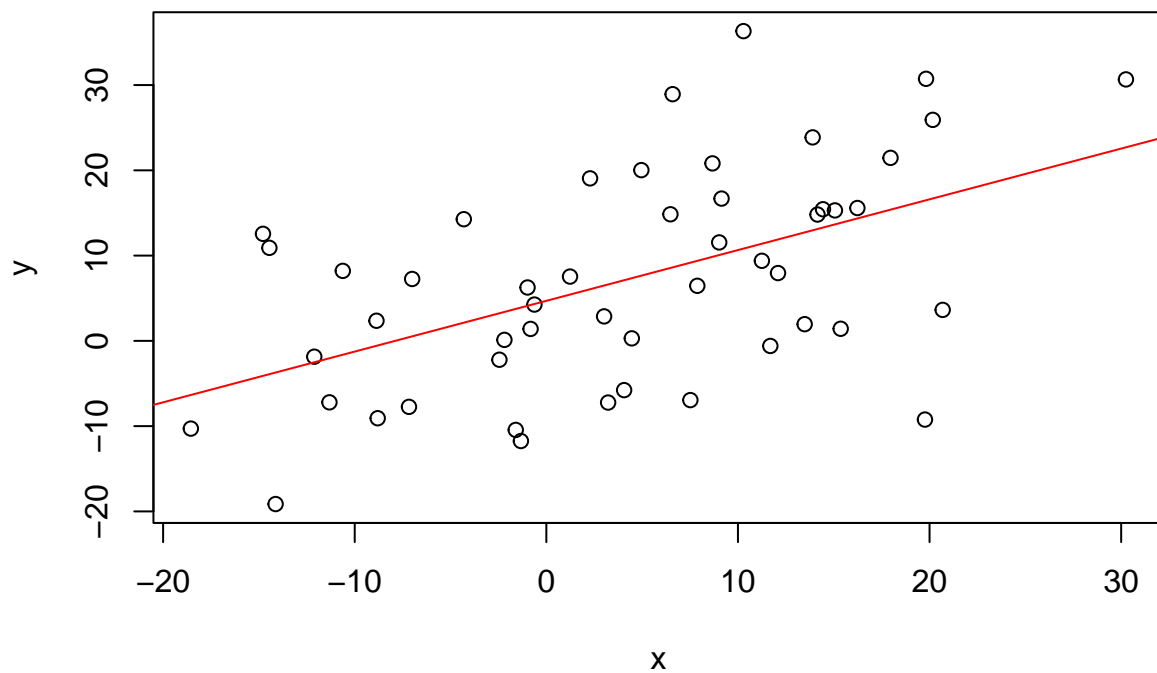
```
# check the means
sapply(df, mean)
```

```
##          x          y
## 4.260339  7.231809
```

```
# check the standard deviation
sapply(df, sd)
```

```
##          x          y
## 11.20860  12.75766
```

```
# plot it
plot(df)
abline(lm(y ~ x,data=df), col = "red")
```



## Correlation

```
# check the correlation coefficient
cor.test(df$x,df$y)

##
## Pearson's product-moment correlation
##
## data: df$x and df$y
## t = 4.2497, df = 48, p-value = 9.8e-05
## alternative hypothesis: true correlation is not equal to 0
## 95 percent confidence interval:
## 0.2861663 0.6994209
## sample estimates:
## cor
## 0.5228658
```

## Simple linear regression

```
# simple linear regression
#raw x and y
lm(y ~ x,data=df)
```

```
##
## Call:
## lm(formula = y ~ x, data = df)
##
## Coefficients:
## (Intercept)          x
## 4.6964      0.5951
```

```
# simple linear regression
#standardized both x and y
lm(scale(y) ~ scale(x),data=df)
```

```
##
## Call:
## lm(formula = scale(y) ~ scale(x), data = df)
##
## Coefficients:
## (Intercept)    scale(x)
## -2.463e-17    5.229e-01
```

## Observation

Thus, correlation coefficient is the same as the standardized regression coefficient for the same X and Y.

### Q3: Why is $r$ different from $d$ ?

Effect Size	$d$	$r$
Small	0.2	0.1
Medium	0.5	0.3
Large	0.8	0.5

```
# dichotomize x
df$dichotomized_x<- ifelse(df$x >mean(df$x), 1, 0)
head(df)
```

```
##           x           y dichotomized_x
## 1 -2.188776  0.1167428             0
## 2  3.014416  2.8711033             0
## 3 20.168055 25.9184510             1
## 4 -4.307292 14.2689957             0
## 5  7.878421  6.4575243             1
## 6 10.288163 36.3225845             1
```

Since we got a column of dichotomized  $x$ , we can actually use the formula mentioned to calculate Cohen's  $d$ . That is, we can calculate the means for groups 1 and 2. Then, we can calculate the sample standard deviations.

$$d = \frac{m_2 - m_1}{s_{pooled}} = \frac{m_2 - m_1}{\sqrt{\frac{s_1^2 + s_2^2}{2}}}$$

```
# calculate group means
means<-aggregate(df$y, list(df$dichotomized_x), FUN=mean)

# calculate sample standard deviation - pooled
variances<- aggregate(df$y, list(df$dichotomized_x), FUN=var)
s_pooled=sqrt((variances$x[1]+variances$x[2])/2)

d_calculated<-(means$x[2]-means$x[1])/s_pooled
print(d_calculated)
```

```
## [1] 1.252203
```

$$d = \frac{2r}{\sqrt{1 - r^2}}$$

```
# check the correlation coefficient
cor_result<-cor.test(df$x,df$y)
correlation_coefficient<-cor_result$estimate
d_converted<-2*correlation_coefficient/sqrt(1-correlation_coefficient^2)
print(d_converted)
```

```
##           cor
## 1.226787
```

## References

1. Jacob Cohen, 1988, Statistical Power Analysis for the Behavioral Sciences, Second Edition
2. What is the relation between the effect size and correlation?

<https://stats.stackexchange.com/questions/412590/what-is-the-relation-between-the-effect-size-and-correlation>

3. What is Effect Size and Why Does It Matter? (Examples)

<https://www.scribbr.com/statistics/effect-size/>

4. How to simulate a strong correlation of data with R

<https://stackoverflow.com/questions/72894192/how-to-simulate-a-strong-correlation-of-data-with-r>

5. FAQ How is effect size used in power analysis?

<https://stats.oarc.ucla.edu/other/mult-pkg/faq/general/effect-size-power/faqhow-is-effect-size-used-in-power-analysis/>