Question 2:

for scaling, we multiply by 3 matrices:

$$P_{final} = point * T_{1(-Tx,-Ty)} * S_{(sx,sy)} * T_{2(Tx,Ty)}$$

$$P = \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} \; ; \; T = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix} \; ; \; S = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Generalized Scaling matrix: combining 3 matrices T_1, S, T_2 (M)

$$\Rightarrow T_1 * S * T_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -T_x & -T_y & 1 \end{bmatrix} * \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -T_x S_x & -T_y S_y & 1 \end{bmatrix} * \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ T_x & T_y & 1 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ -T_x S_x + T_x & -T_y S_y + T_y & 1 \end{bmatrix}$$

$$\Rightarrow M = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ T_x(1-S_x) & T_y(1-S_y) & 1 \end{bmatrix}$$

$$\Rightarrow P_{final} = P * M$$

$$= \begin{bmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{bmatrix} * \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ T_x(1-S_x) & T_y(1-S_y) & 1 \end{bmatrix}$$