b.

1. Let's suppose $f(x) = 8x^4$. $g(x) = \frac{89}{9}x^2$.

So. $f'(x) = 32x^3$ $g'(x) = \frac{178}{9}x^2$

Thus the increasing speed of $f(x)$ will be greater than $g(x)$'s when $x \geqslant 1$.

Which means the value of $f(x) - g(x)$ will increase starting from $x=1$

This tells us. $f(x) - g(x)$ is strictly increasing when $x \geqslant 1$

We know that constants will not change the monotonicity of functions.

So $f(x) - g(x) - 1$ is strictly increasing when $x \geqslant 1$.

and we may find. $p(2) = 8 \times 2^4 - \frac{89}{9} \times 2^2 - 1$

$= 32 \times 4 - \frac{89}{9} \times 4 - 1$

$= (32 - 9\frac{8}{9}) \times 4 - 1$

$32 - 9\frac{8}{9} > 20$

$20 \times 4 > 1 > 0$

So $p(2) > 0$.

$$p(1) = 8 - \frac{89}{9} - 1$$
$$= 8 - 9\frac{8}{9} - 1$$
$$= -1\frac{8}{9} - 1 < 0.$$

Now we have the first conclusion.

① $p(x)$ has a root between 1 and 2 and $p(x)$ has no root when $x \geq 2$.

on the other hand. when x belongs to $[0, 1]$

x^4 will less than x^2

and the coefficient $8 < \frac{89}{9}$

So we can know $8x^4 < \frac{89}{9}x^2$

Thus we have another conclusion

$p(x) < 0$ when x belongs to $[0, 1]$.

Moreover $p(0) = -1$

We have the second conclusion.

② $p(x)$ has no root between $[0, 1]$.

Next, we may find that $p(-x) = p(x)$

This means $p(x)$ is an even function.

Even functions are Y-axis symmetric.

Now, we know the third conclusion.

$p(x)$ is Y-axis symmetric

Based on those 3 conclusions. we may say.

$p(x)$ has 2 roots. and one is belong to $(1, 2)$.

another is belong to $(-2, -1)$.

a. $p(x) = 8x^4 - \frac{89}{9}x^2 - 1$

let's say $a = x^2$. so. we can have.

$$y = 8a^2 - \frac{89}{9}a - 1$$

This means the max number of roots is 2.

one is positive, another is negative.

however, $a = x^2$. so a will not be negative.

so we have at most one possible value of a

which must be positive.

so for x, we have 2 possiblities, one is

positive, one is negative.

Now we have the conclusion:

for $p(x) = 8x^4 - \frac{89}{9}x^2 - 1$. the maximum number of

roots is 2.