Question 5: My student ID: 00054383**31**

x	y
-5	31
-3	10
0	1
3	10
5	31

the equation of the form: $y = a_0 + a_1 x + a_2 x^2$

a) - Number of unknowns: 3 (a_0, a_1, a_2)

- Least squares for a quadratic gives 3 normal equations

$$\begin{cases} na_0 + a_1 \sum x + a_2 \sum x^2 = \sum y \\ a_0 \sum x + a_1 \sum x^2 + a_2 \sum x^3 = \sum xy \\ a_0 \sum x^2 + a_1 \sum x^3 + a_2 \sum x^4 = \sum x^2 y \end{cases}$$

- using sums: $n, \sum x, \sum x^2, \sum x^3, \sum x^4, \sum y, \sum xy, \sum x^2 y$

b)

	x	y	x^2	x^3	x^4	xy	x^2y
	-5	31	25	-125	625	-155	775
	-3	10	9	-27	81	-30	90
	0	1	0	0	0	0	0
	3	10	9	27	81	30	90
	5	31	25	125	625	155	775
Σ	0	83	68	0	1412	0	1730

c)

$$\begin{cases} 5a_0 + 0a_1 + 68a_2 = 83 \\ 0a_0 + 68a_1 + 0a_2 = 0 \\ 68a_0 + 0a_1 + 1412a_2 = 1730 \end{cases}$$

$$\Rightarrow \left[\begin{array}{ccc|c} 5 & 0 & 68 & 83 \\ 0 & 68 & 0 & 0 \\ 68 & 0 & 1412 & 1730 \end{array}\right]$$

d) Solve matrix

$$\frac{1}{68}R_2 \left[\begin{array}{ccc|c} 5 & 0 & 68 & 83 \\ 0 & 1 & 0 & 0 \\ 68 & 0 & 1412 & 1730 \end{array}\right]$$

$$\Rightarrow \frac{1}{5}R_1 \left[\begin{array}{ccc|c} 1 & 0 & \frac{68}{5} & \frac{83}{5} \\ 0 & 1 & 0 & 0 \\ 68 & 0 & 1412 & 1730 \end{array}\right] \Rightarrow R_3 - 68R_1 \left[\begin{array}{ccc|c} 1 & 0 & \frac{68}{5} & \frac{83}{5} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \frac{2436}{5} & \frac{3006}{5} \end{array}\right]$$

$$\Rightarrow \frac{5}{2436}R_3 \left[\begin{array}{ccc|c} 1 & 0 & \frac{68}{5} & \frac{83}{5} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{501}{406} \end{array}\right] \Rightarrow R_1 - \frac{68}{5}R_3 \left[\begin{array}{ccc|c} 1 & 0 & 0 & \frac{-37}{203} \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \frac{501}{406} \end{array}\right]$$

$$\Rightarrow a_0 = -\frac{37}{203}\ ;\ a_1 = 0\ ;\ a_2 = \frac{501}{406}$$

e) $y = a_0 + a_1 x + a_2 x^2$

$$\Rightarrow y = -\frac{37}{203} + \frac{501}{406}x^2$$

$$\Rightarrow y \approx -0{,}1827 + 1{,}2340x^2$$