MHD Modeling in Space Plasmas

Applications and Examples

Outline

- Forms of MHD equations
- History of global MHD simulations (Heliosphere and Magnetospheres)
- Applications
- Future directions

Forms of the MHD equations

- Up to now, we have been working with the primitive form of the MHD equations in one spatial dimension
- The “conservative form” of the MHD equations are more commonly used in space plasma physics and research applications
- Physically the two forms are equivalent, but mathematically they are not. The reason is due to the existence of “weak" solutions in wave equations
- Maxwell’s equation requires particular treatment when going to multi-dimensional flow

Conservative versus Primitive

Conservative (semi) form

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u}\mathbf{u} + \nabla p - \mathbf{J} \times \mathbf{B} = 0$$

$$\frac{\partial \mathscr{E}}{\partial t} + \nabla \cdot \mathbf{u}\left(\mathscr{E} + p\right) - \mathbf{u} \cdot \mathbf{J} \times \mathbf{B} = 0$$

Primitive form

$$\frac{\partial}{\partial t}\rho + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0$$

$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mathbf{J} \times \mathbf{B} = 0$$

$$\frac{\partial}{\partial t} p + \mathbf{u} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{u} = 0$$

Maxwell: $\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$ $\quad$ $\mathbf{J} = \nabla \times \mathbf{B}$ $\quad$ $\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$

Primary variables: $\rho, \mathbf{u}, p, \mathbf{B}$

Conserved variables: $\rho, \rho\mathbf{u}, \mathscr{E}, \Phi$ $\quad$ $\mathscr{E} = \frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1}$ Plasma energy

Secondary/Derived variables: $\mathbf{E}, \mathbf{J}$

Conservative versus Primitive

Maxwell: $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E} \qquad \mathbf{J} = \nabla \times \mathbf{B} \qquad \mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$$

Conservative (semi) form

$$\frac{\partial}{\partial t}\rho + \nabla \cdot (\rho \mathbf{u}) = 0$$

$$\frac{\partial \rho \mathbf{u}}{\partial t} + \nabla \cdot \rho \mathbf{u}\mathbf{u} + \nabla p - \mathbf{J} \times \mathbf{B} = 0$$

$$\frac{\partial \mathscr{E}}{\partial t} + \nabla \cdot \mathbf{u}\left(\mathscr{E} + p\right) - \mathbf{u} \cdot \mathbf{J} \times \mathbf{B} = 0$$

- Solve velocity, pressure indirectly
- Allows non-smooth solutions, i.e., profiles do not need to be differentiable
- Does satisfy the entropy conditions when shocks occur, i.e., correct shock solutions

Primitive form

$$\frac{\partial}{\partial t}\rho + \mathbf{u} \cdot \nabla \rho + \rho \nabla \cdot \mathbf{u} = 0$$

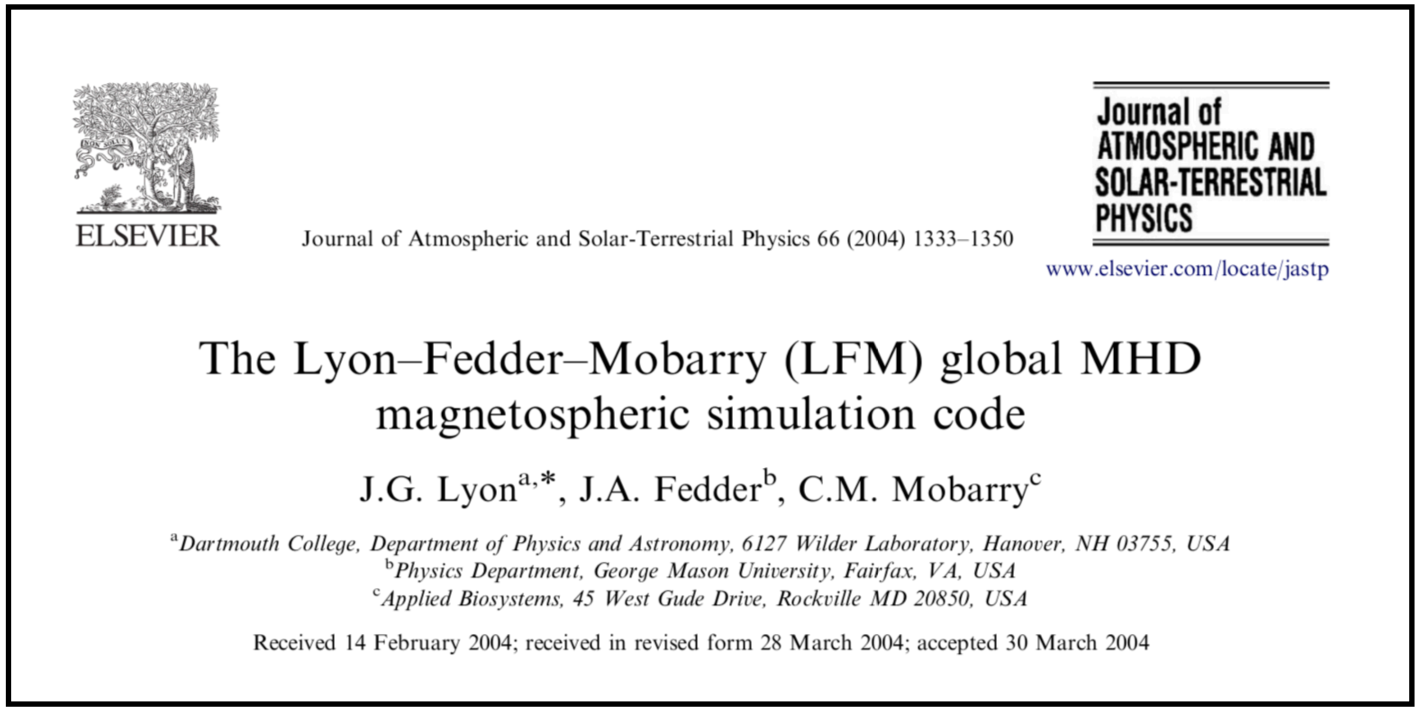
$$\rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \cdot \nabla \mathbf{u} + \nabla p - \mathbf{J} \times \mathbf{B} = 0$$

$$\frac{\partial}{\partial t} p + \mathbf{u} \cdot \nabla p + \gamma p \nabla \cdot \mathbf{u} = 0$$

- Solve velocity, pressure directly
- Only allows smooth solutions, i.e., profiles need to be differentiable
- Does not satisfy the entropy conditions when shocks occur, i.e., wrong solutions

The LFM MHD Code

LFM: the Lyon-Fedder-Mobarry global MHD code, developed mainly by John Lyon in the early 80s, the backbone of CISM geospace model series



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Journal of ATMOSPHERIC AND SOLAR-TERRESTRIAL PHYSICS

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The Lyon–Fedder–Mobarry (LFM) global MHD magnetospheric simulation code

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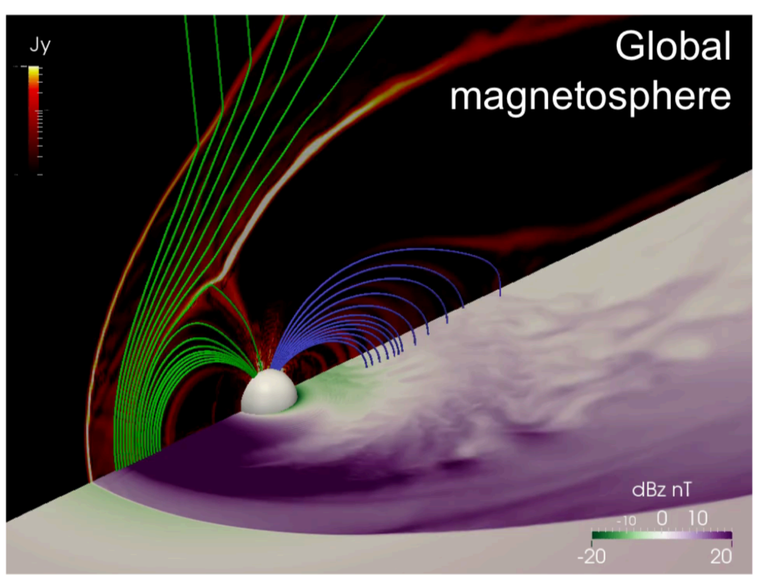
Lyon et al. 2004, JASTP

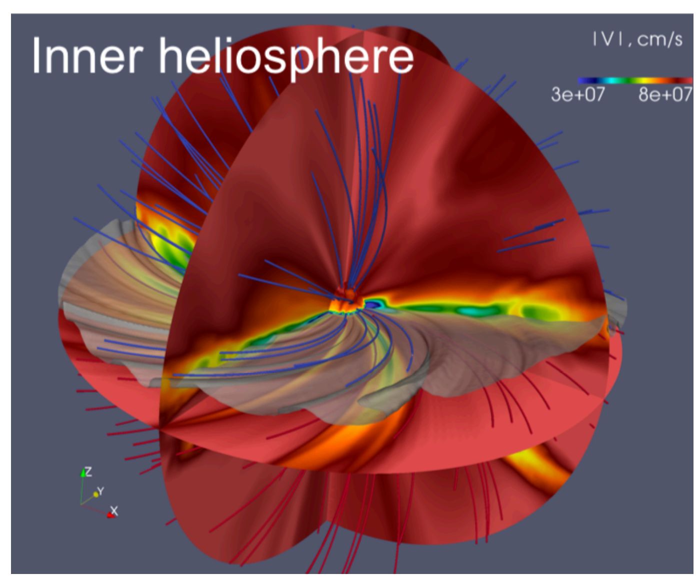
A couple of notes:

- The 3-D MHD solver was done near 1982
- The ionospheric solve was done near 1986
- The numerics paper was published in 2004
- TIEGCM coupled (CMIT) successfully in 2001
- CMIT paper published in 2004
- RCM coupled (LR) successfully in 2012
- LR paper published in 2013
- Multi-Fluid extension (MFLFM) done near 2009
- Multi-fluid paper paper coming soon…

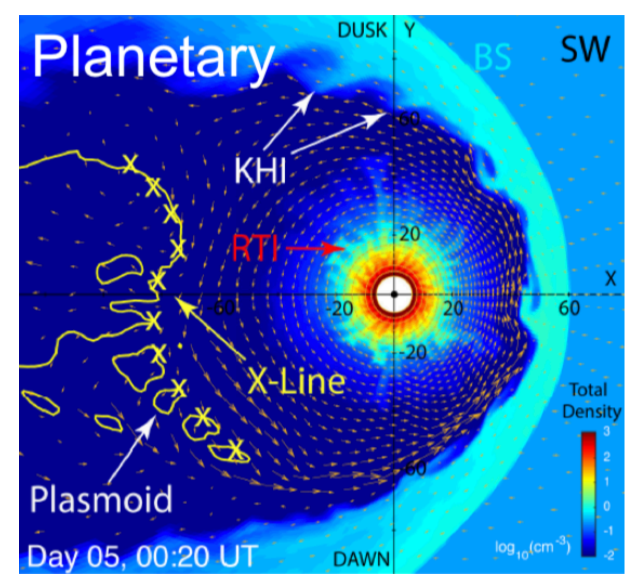
Earth (LFM)



Solar Wind (LFM-helio)



Jupiter (LFM-J)

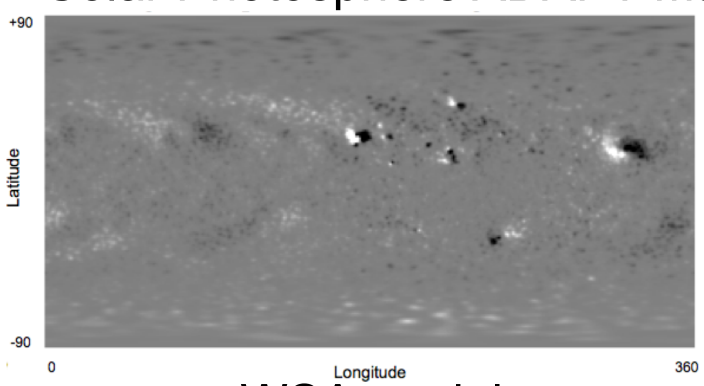


Basic (BOX)

And other applications such as: Venus (1998), Outer heliosphere (1998), Saturn/Neptune (unpublished)

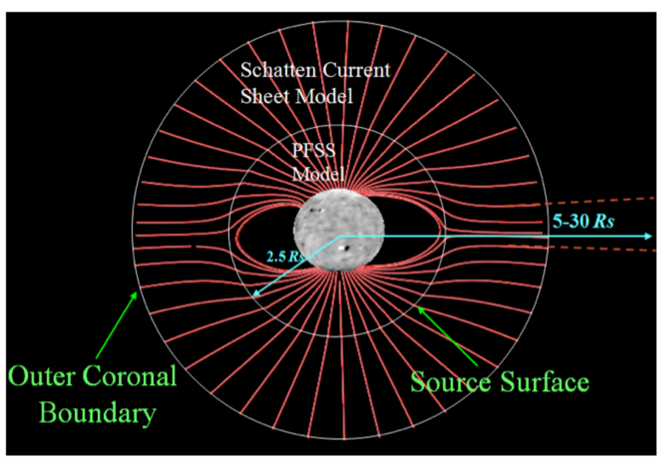
Inner Heliosphere (solar wind) Model

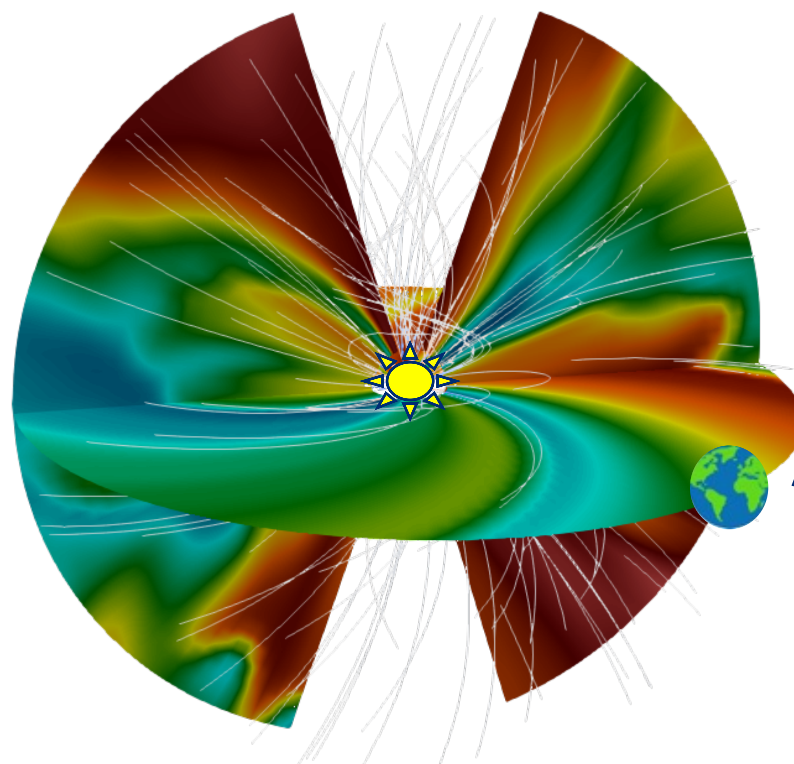
Solar Photosphere ADAPT Maps



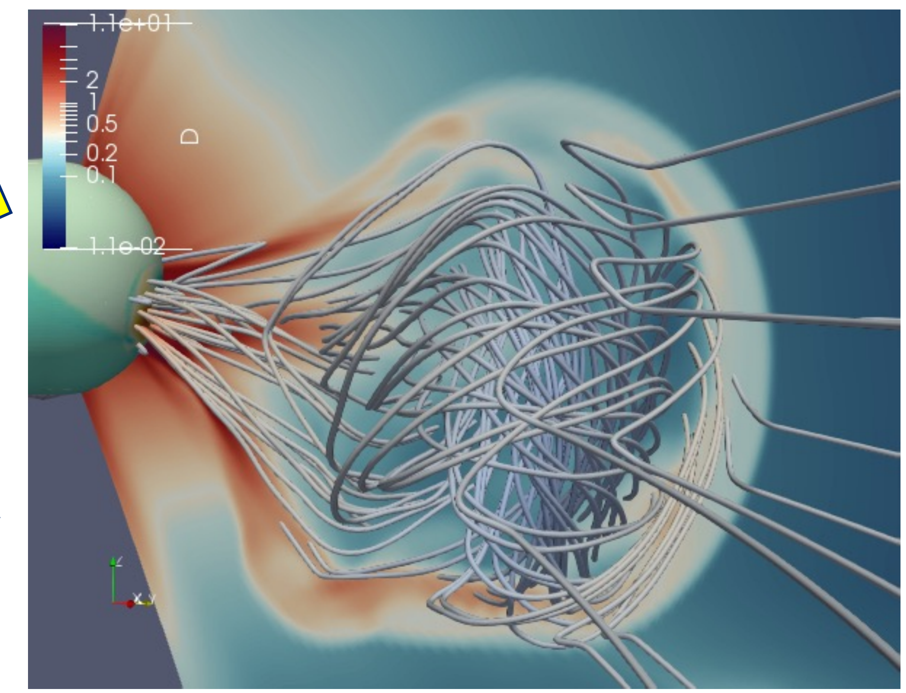
WSA model

Figure courtesy N. Arge

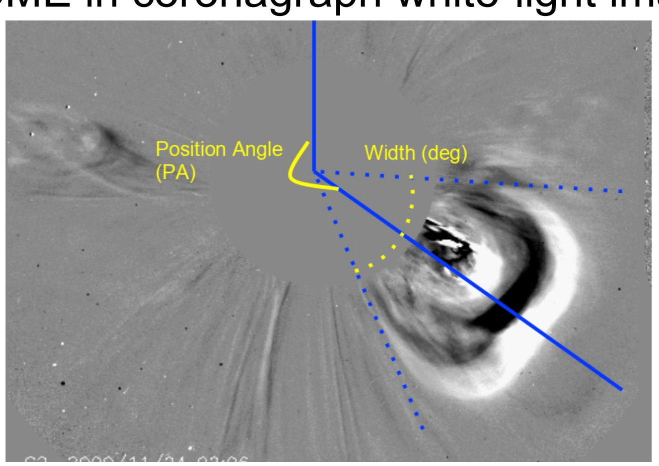




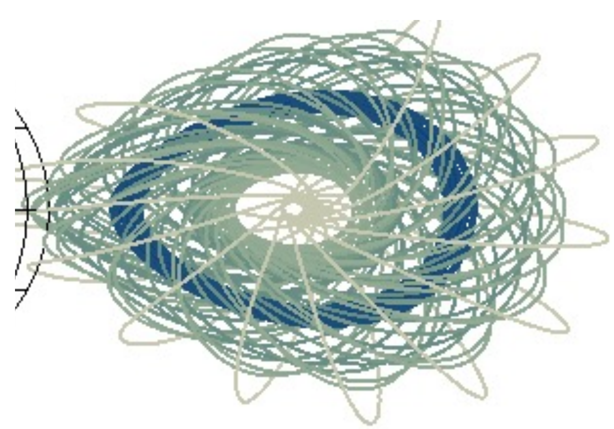
CME in the inner heliosphere



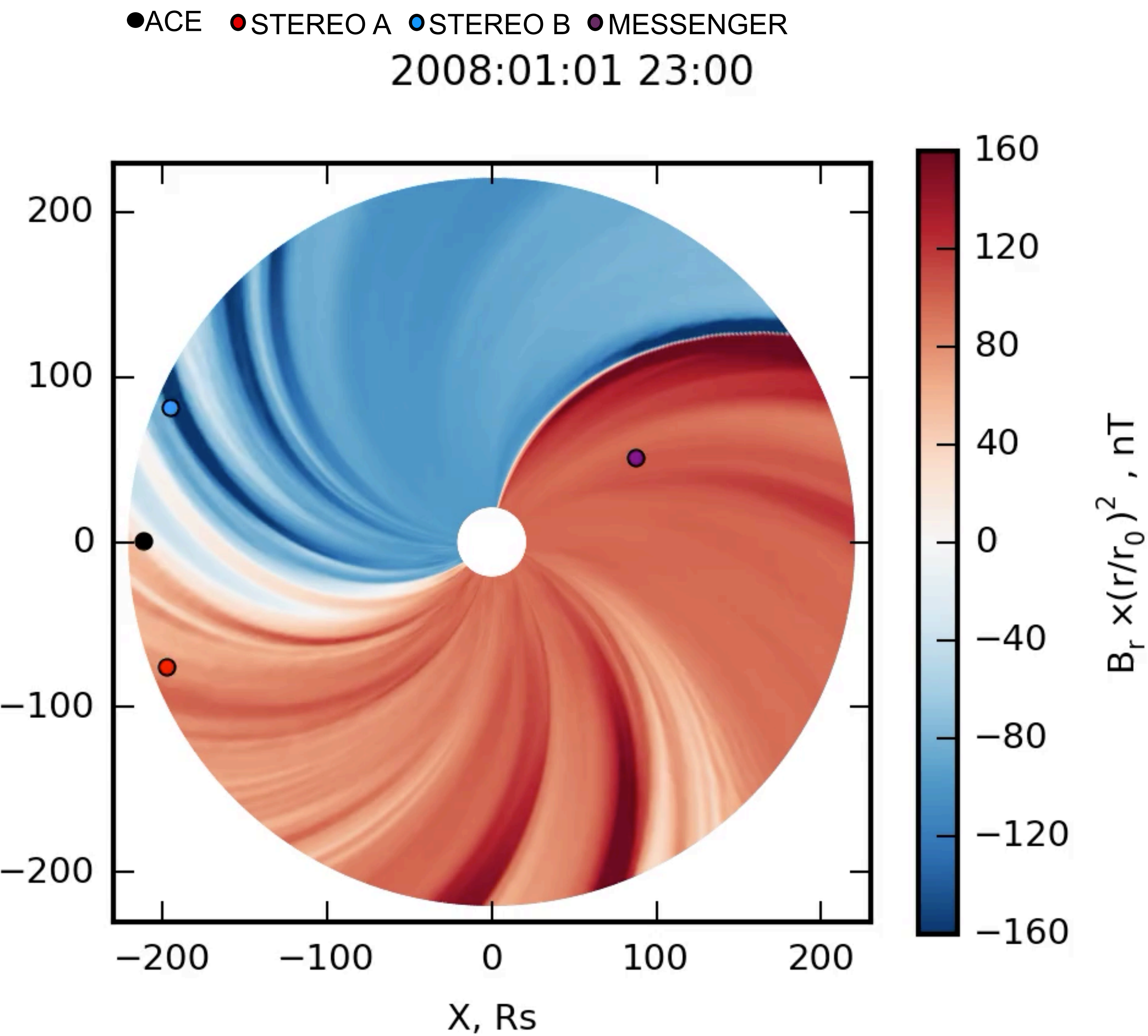
CME in coronagraph white-light image



GLOW flux rope model
(Gibson and Low 1998)

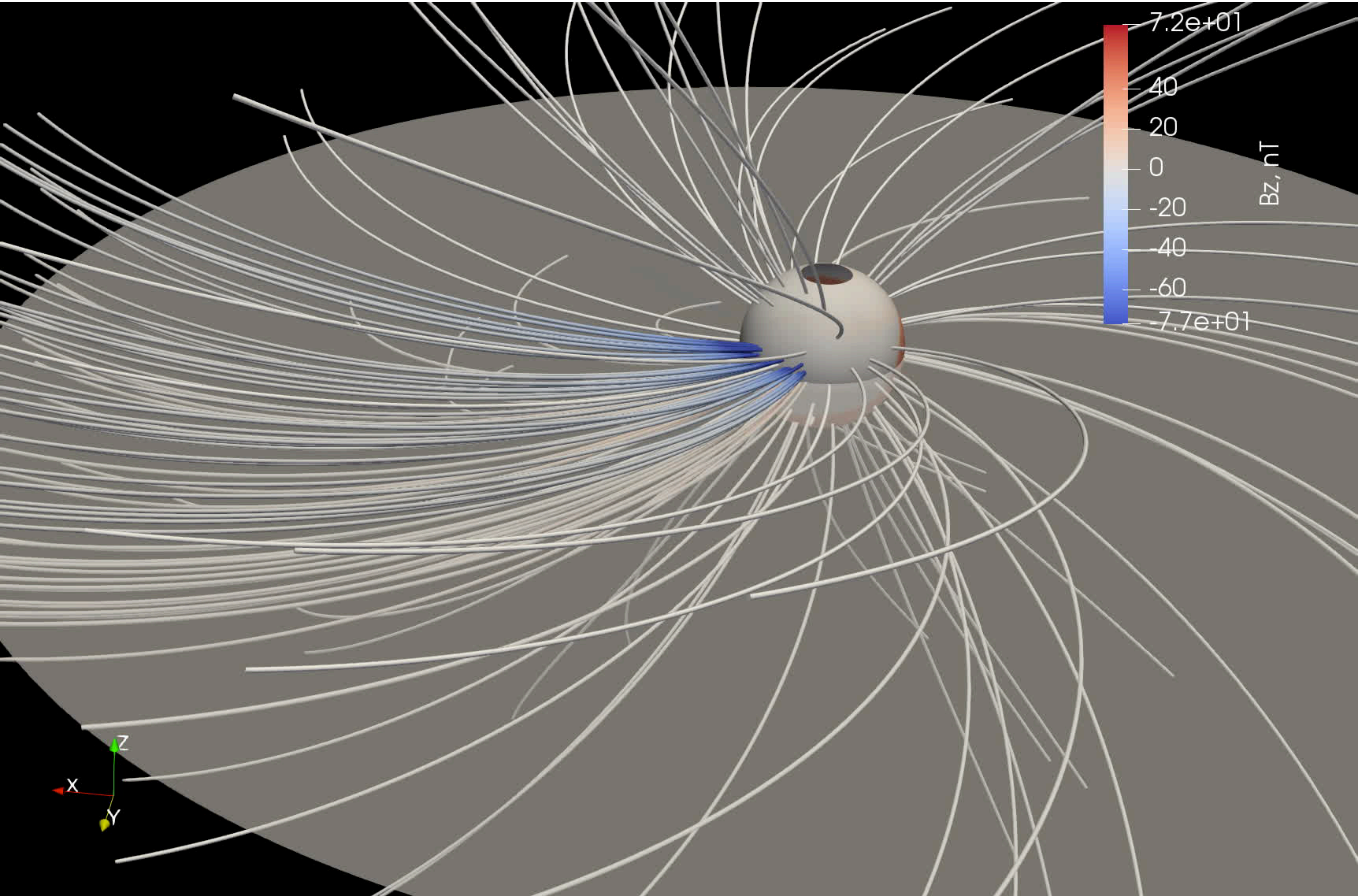


Inner Heliosphere (solar wind) Model

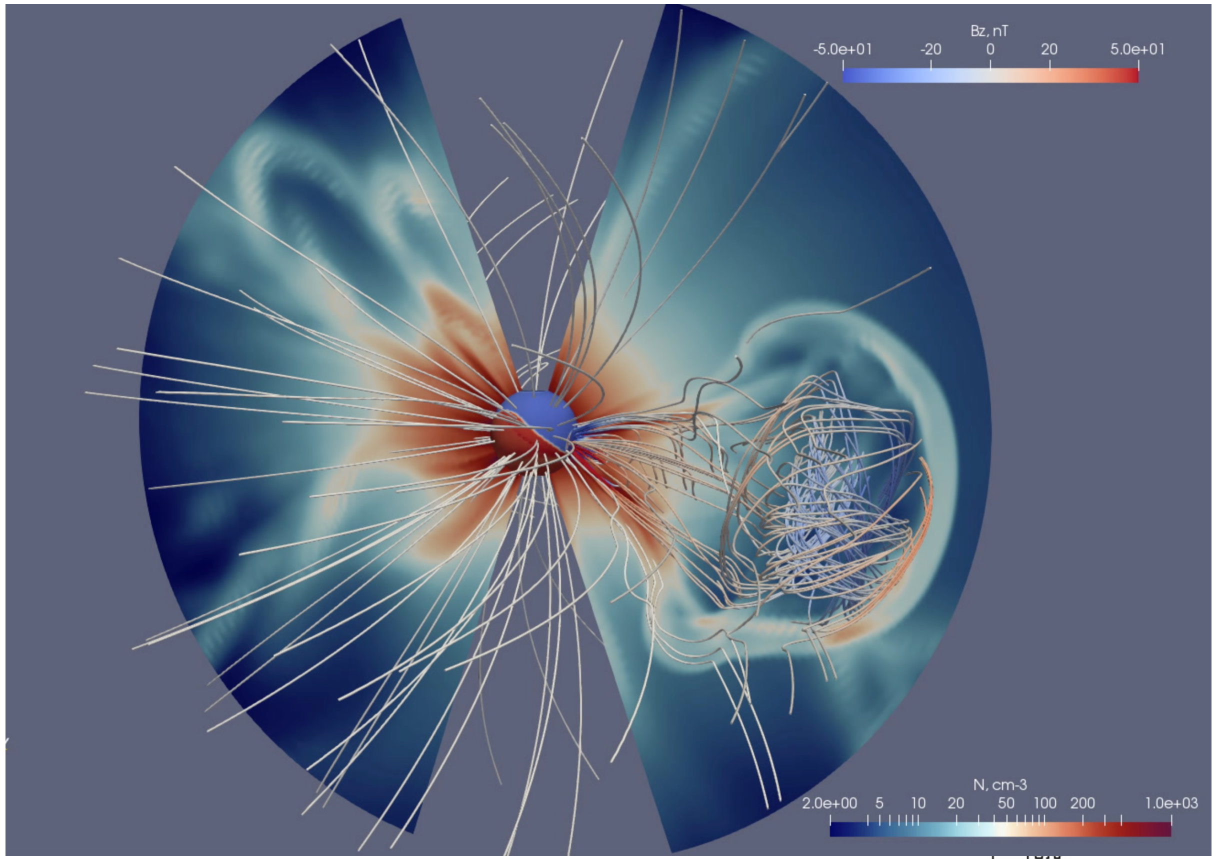


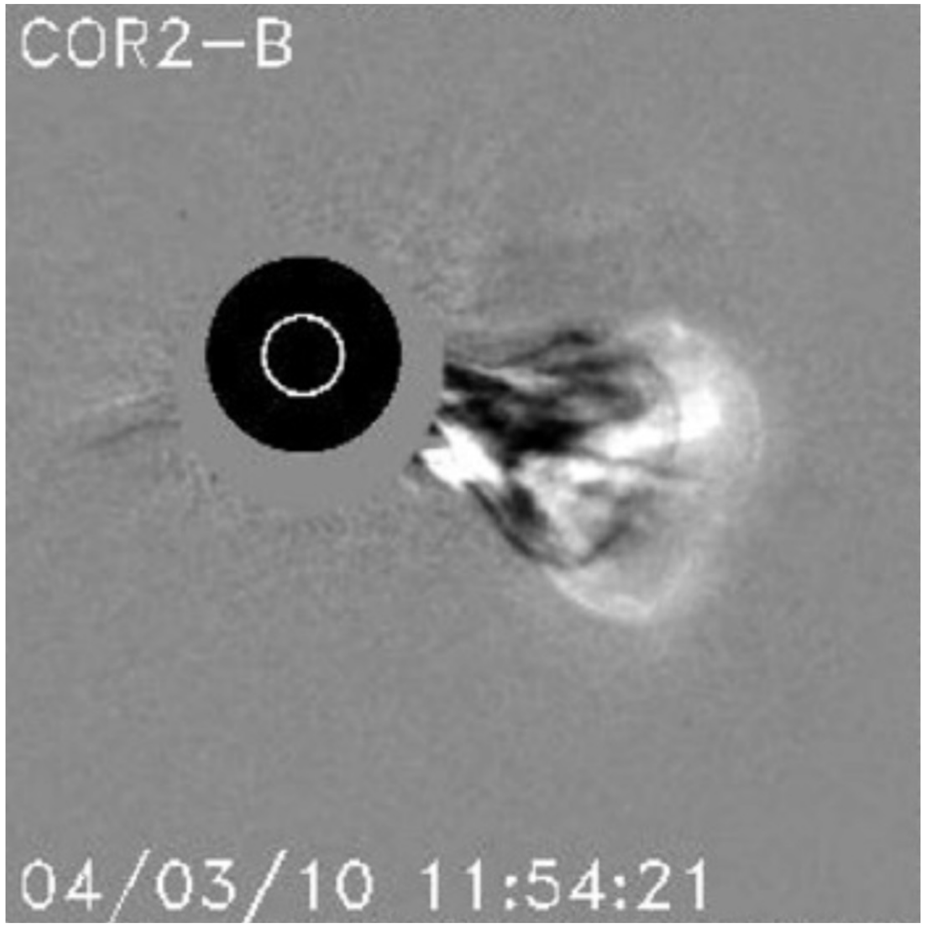
- HCS moves around
- Transient SW velocity streams
- Complex HCS crossings/ transitions

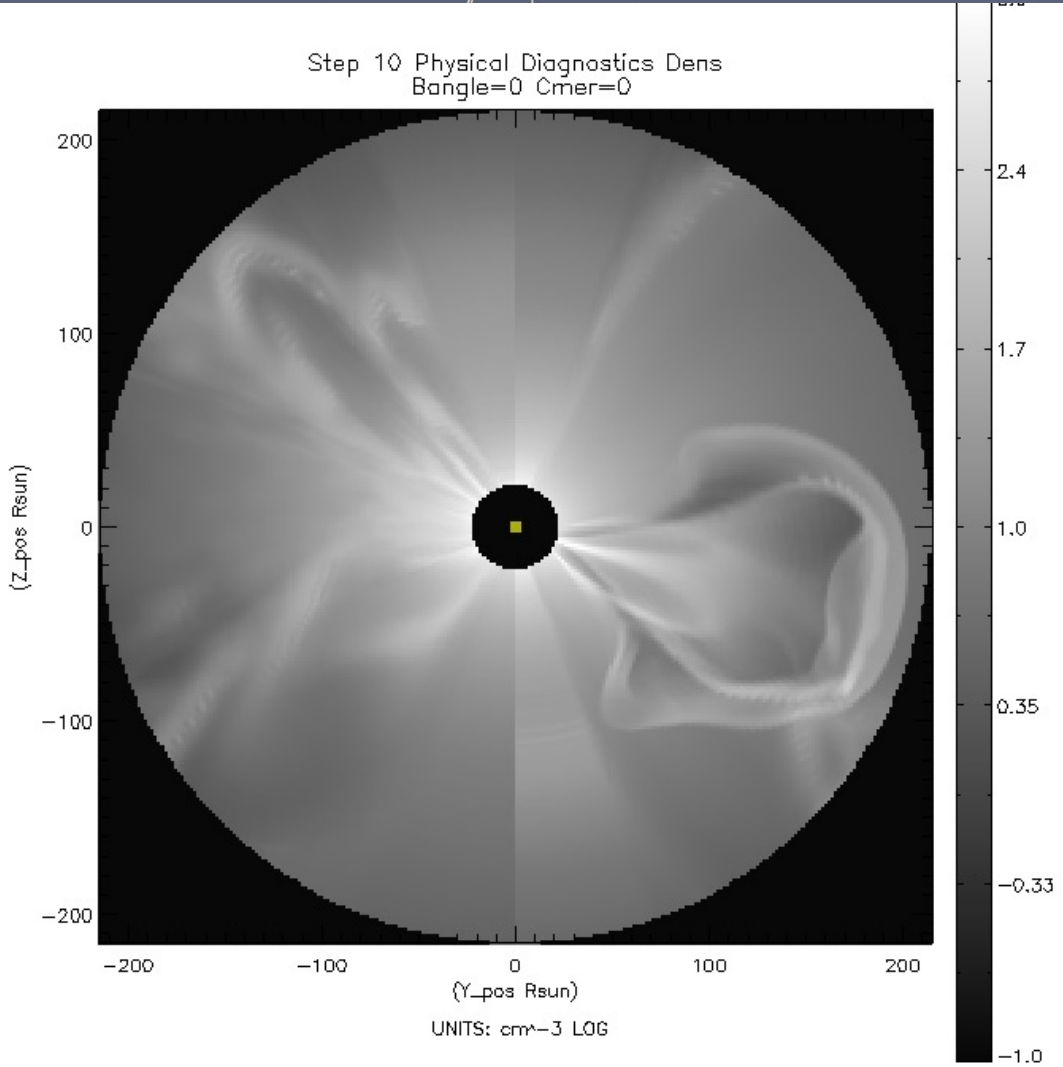
Coronal Mass Ejection (Eruption) Model



Compare with observations

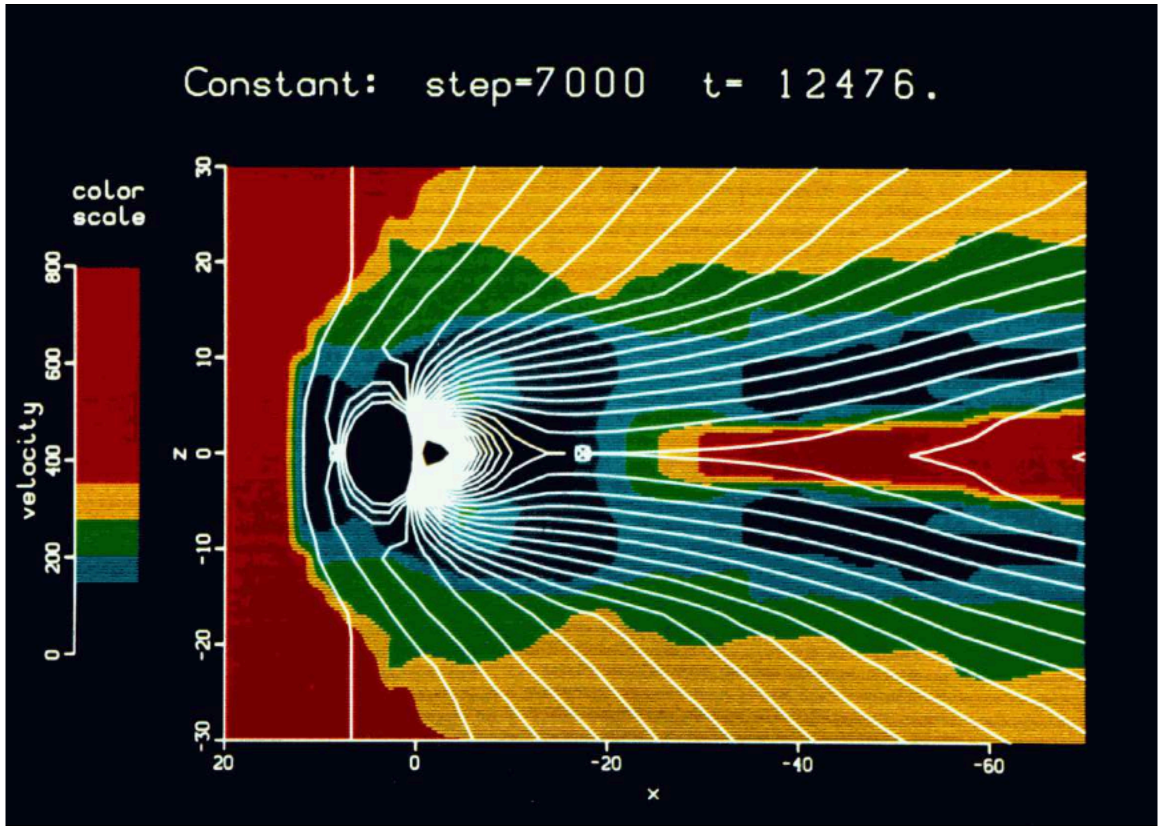






The Magnetosphere Model (LFM)

Lyon et al., [1986], 60x20 cells, 20th-order



The code uses a leapfrog time-integration scheme and a 20th-order finite-difference approximation to the spatial derivatives. Flux-corrected transport[12] is used to provide the minimum possible numerical dissipation consistent with a solution showing no spurious extreme.

Lyon et al. 1981, PRL

What to model in 1986?

Magnetosheath and Bowshock

Modeling Capability of the LFM Geospace Model

Physical phenomenon	Model	Physical phenomenon	Model
Bowshock/Magnetopause	LFM	Electron Precipitation	LFM
Kelvin-Helmholtz Instability	LFM	Ionosphere convection	LFM
Flux Transfer Events	LFM	Field-aligned currents	LFM
Bursty-Bulk Flows	LFM	Proton Precipitation	LFM
ULF waves	LFM	Drift-kinetics Ring Current	LFM-RCM
Alfvénic activities	LFM	Radiation Belt Dynamics	LFM+particle pushing
Cusp dynamics	LFM	Theremosphere dynamics	LFM-TIEGCM
Plasmasphere	multi-fluid (MF) LFM	Tongue of Ionization	LFM-TIEGCM (high-res)
Ionospheric outflow	MFLFM-IPWM	Topside ionosphere	MFLFM-IPWM-TIEGCM

Substorms	Not really
Ion Physics	No
Anisotropic Pressure	No
Hall Physics	Yes and No

Super geomagnetic storms	MFLFM+RCM+TIEGCM-IPWM+WACCMX = NO

What makes the LFM model unique?

1. **High-resolving power numerics**
2. **Problem-adapted grid design**
3. **Coupling to other physics codes**

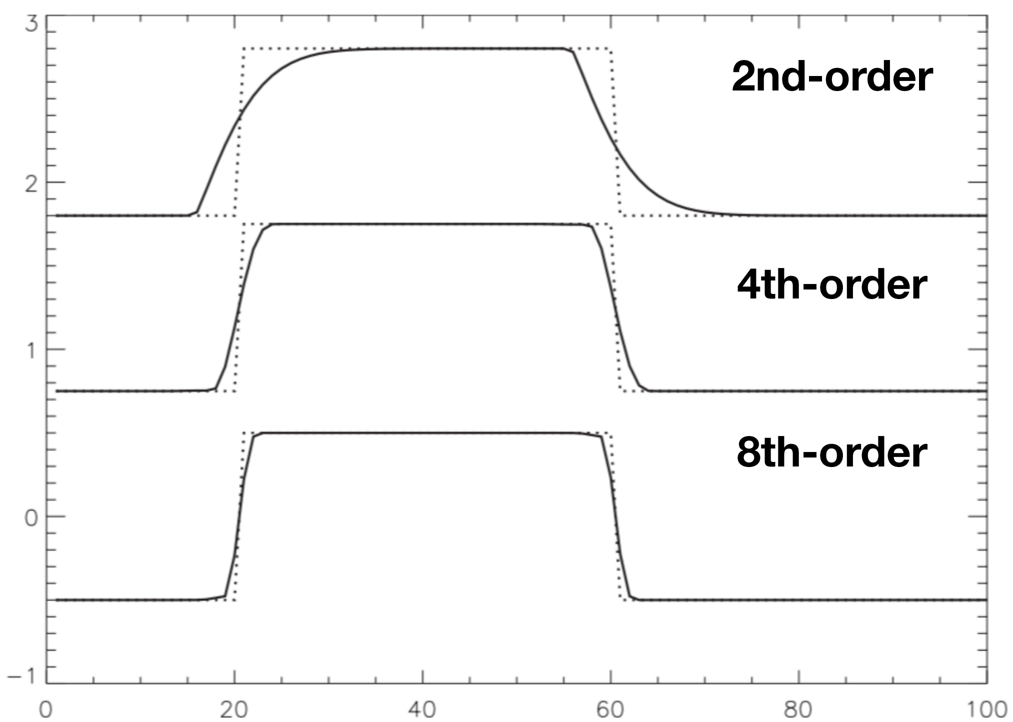
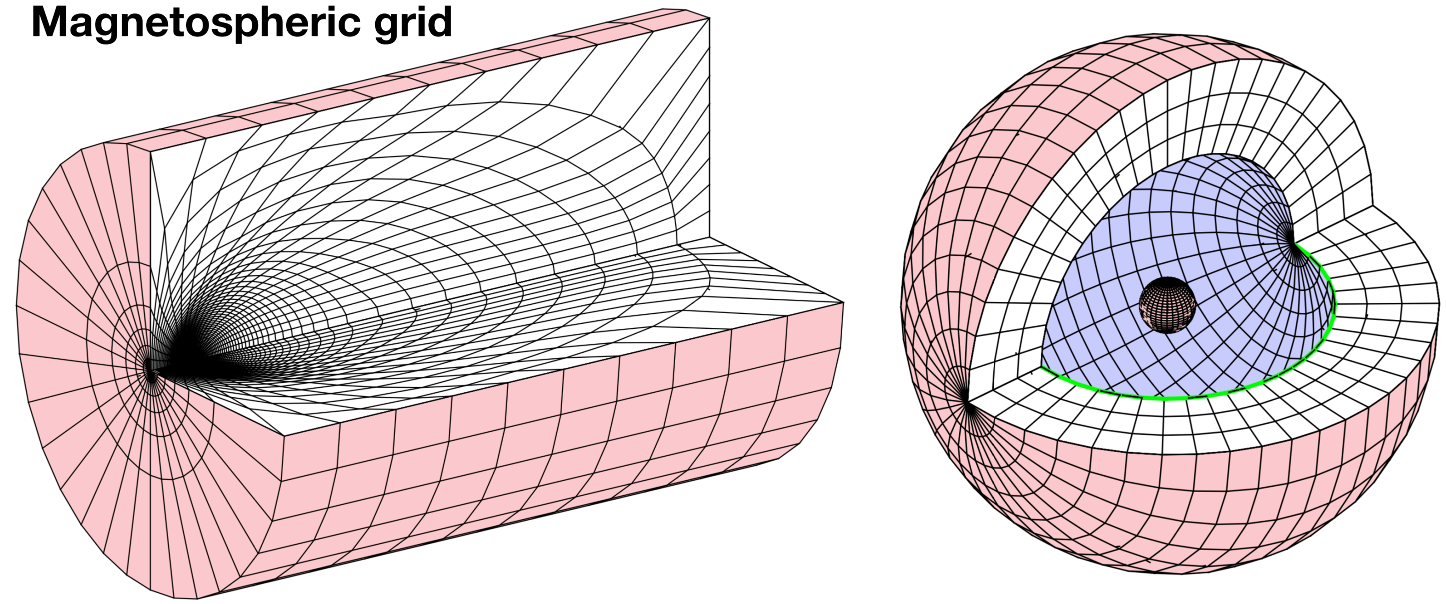


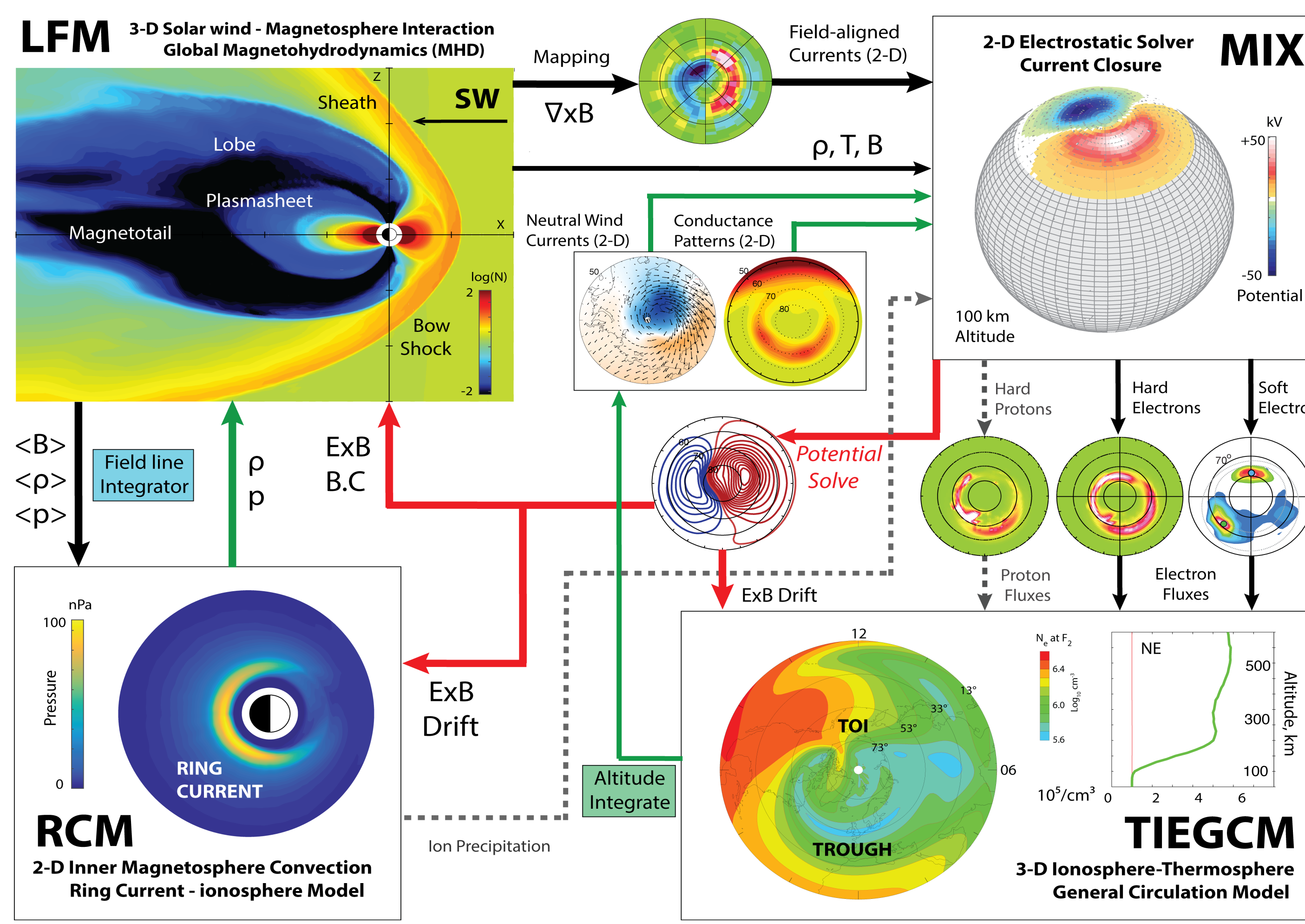
Table 1
Comparison of different advection schemes

Algorithm	Width	Relative *R*
Donor Cell	42	1
2nd Order	11	15
4th Order	4	100
8th Order	2	400

LFM advection scheme only needs 4 cells to resolve contact discontinuity, with a relative grid Reynolds number > 400 in 1D

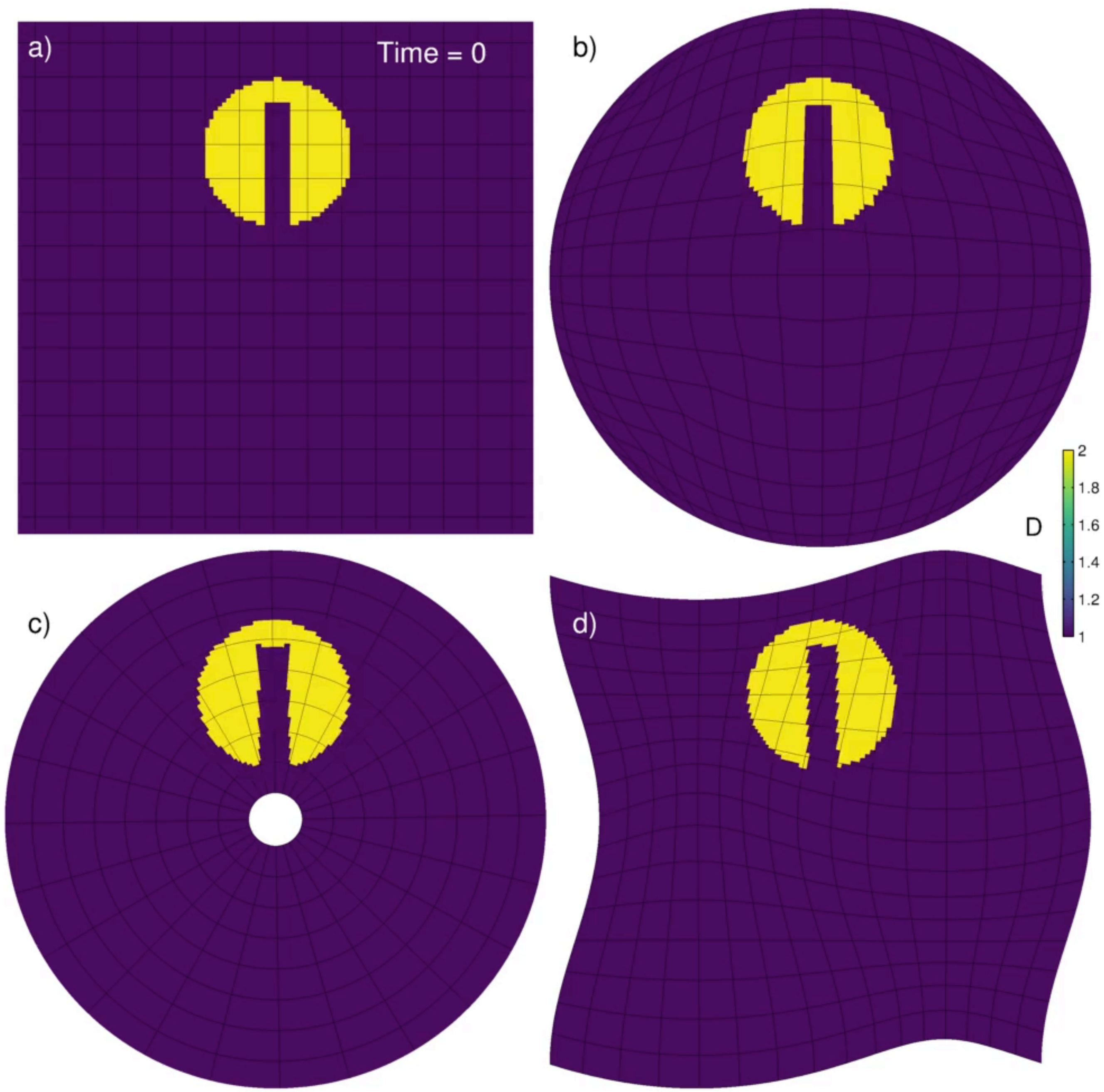
Magnetospheric grid



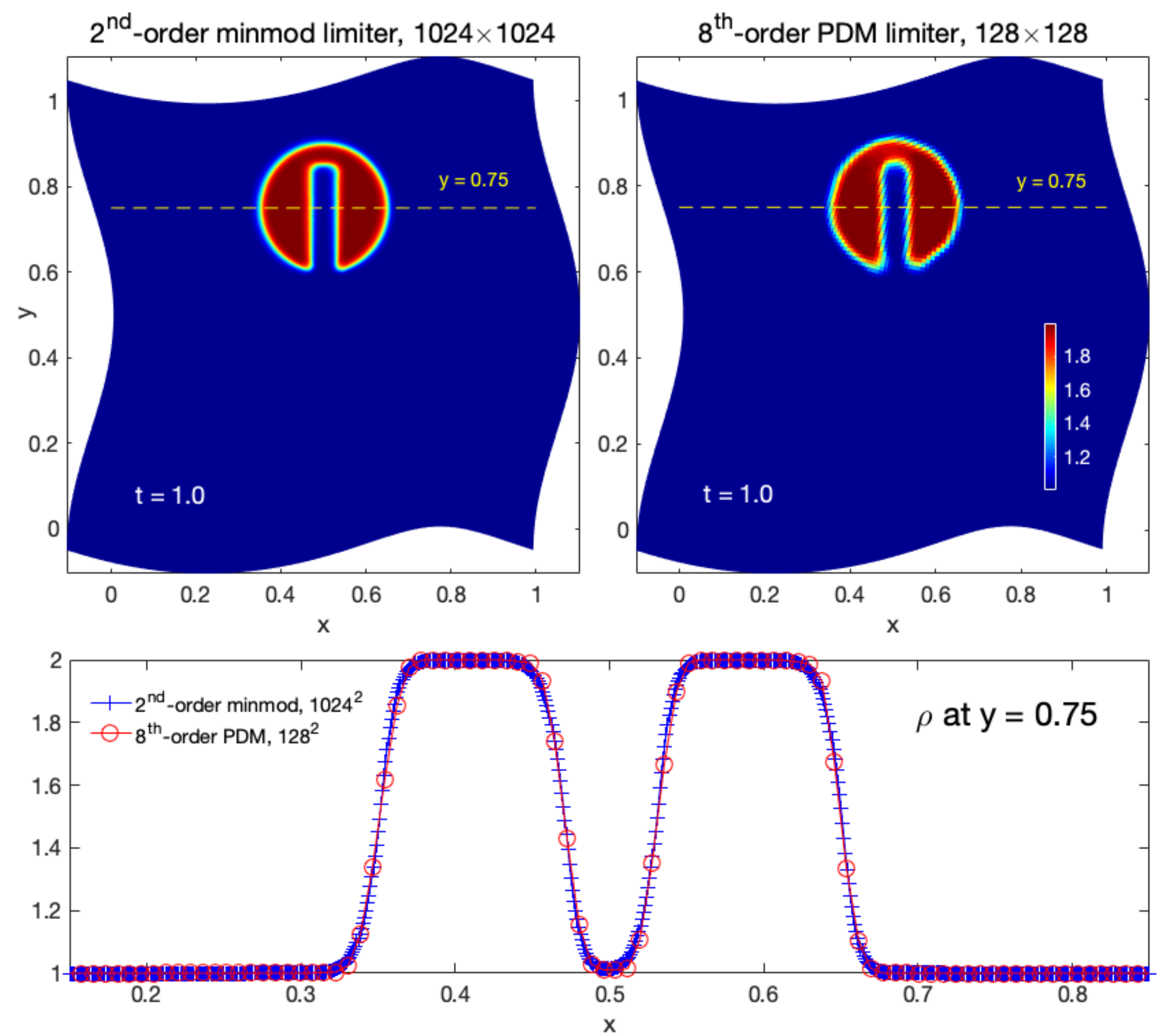


Quality of the Advection Schemes

2-D circular advection



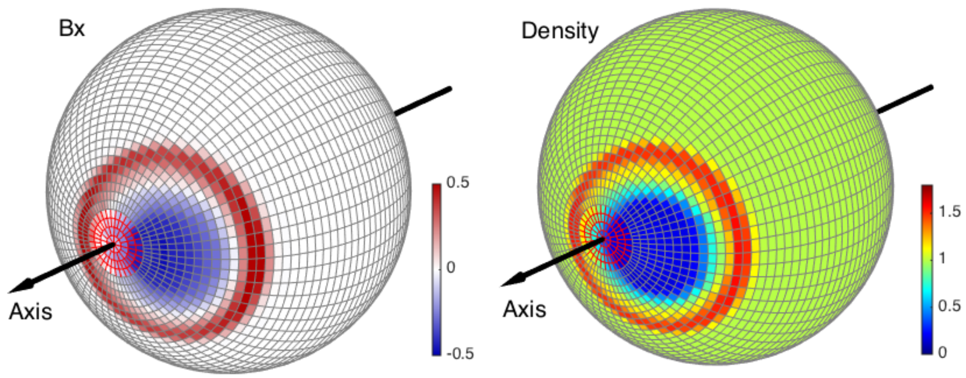
128x128 cells



- The 7th-order reconstruction scheme preserves the slotted cylinder well with 128x128 cells on non-orthogonal grid
- The 2nd-order one needs 1024x1024 to resolve the same density gradient

Singularity Treatment

Ring average for axis singularity



Zhang et al. (2018)

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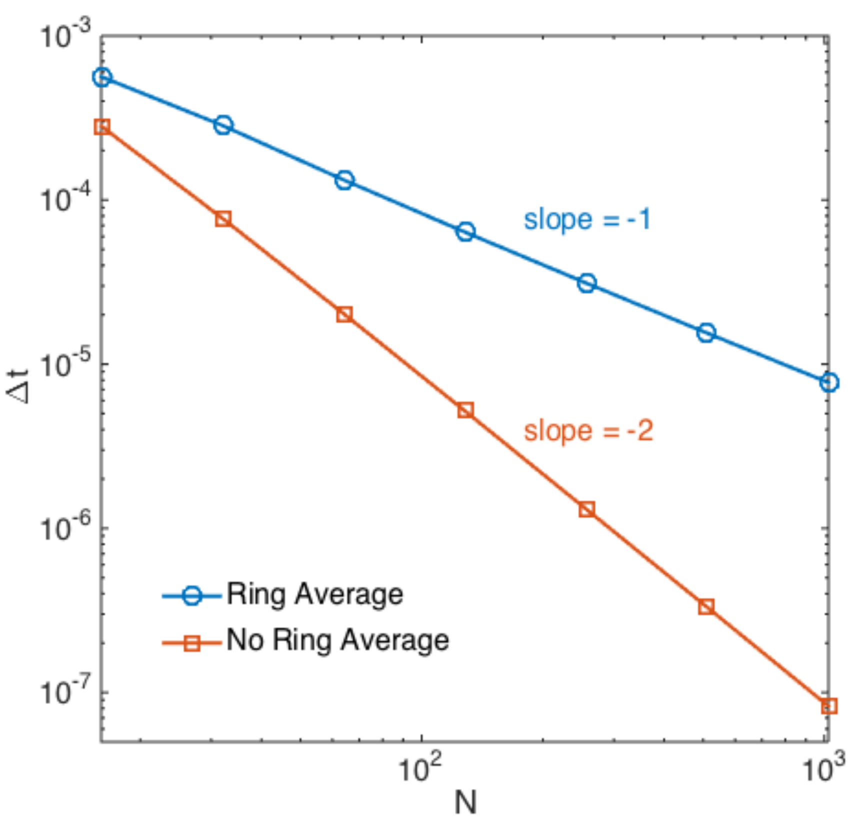
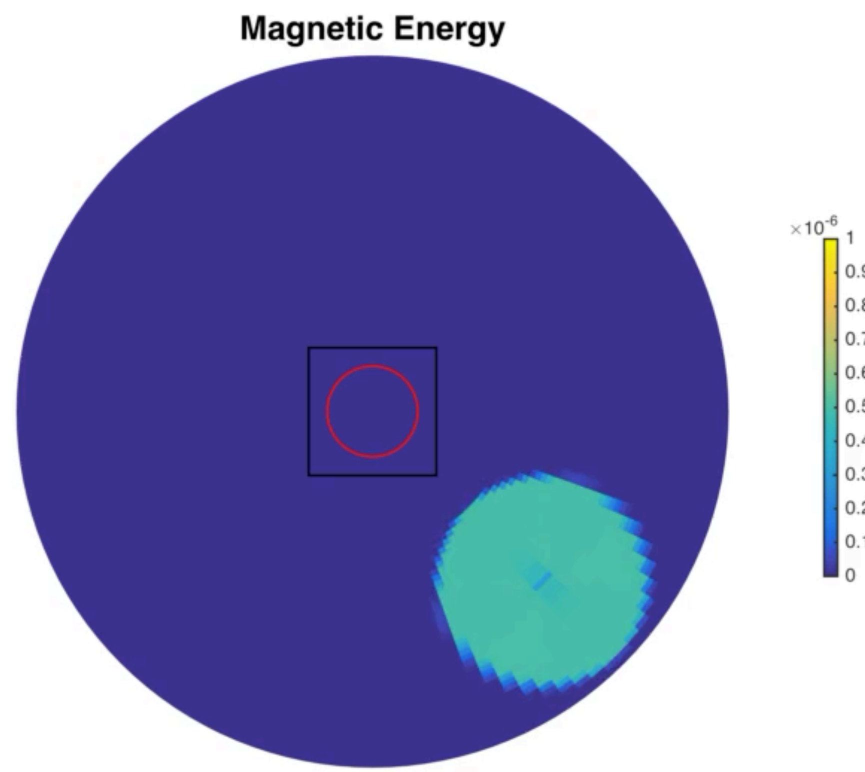
Journal of Computational Physics
Volume 376, 1 January 2019, Pages 276-294

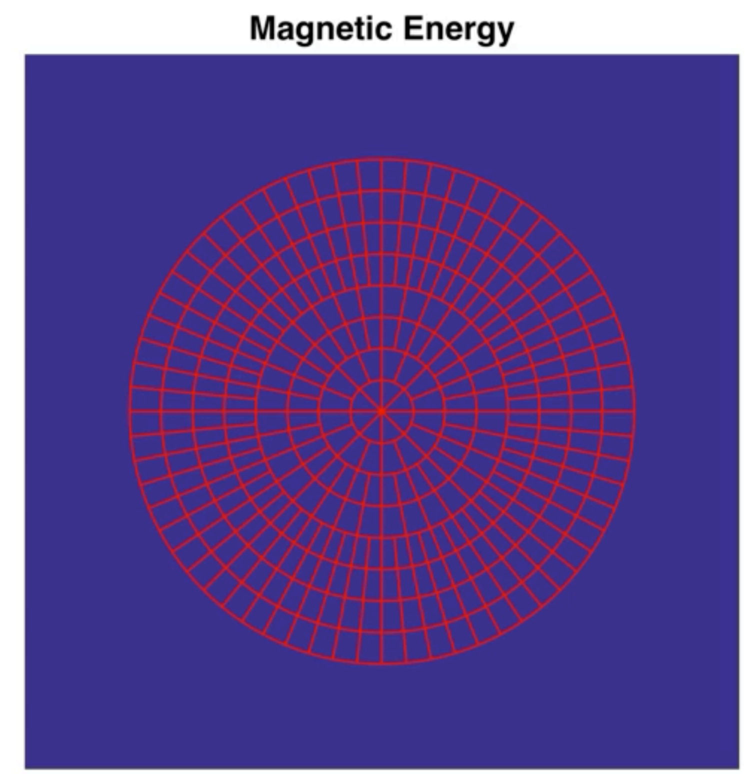
Conservative averaging-reconstruction techniques (Ring Average) for 3-D finite-volume MHD solvers with axis singularity

Binzheng Zhang [a, b, c], Kareem A. Sorathia [d], John G. Lyon [e], Viacheslav G. Merkin [d], Michael Wiltberger [c]



dt drops linearly with Ring Average





Zhang et al. (2018)

Gameraspheres

From an MHD solver to global Magnetosphere Models

Physical considerations

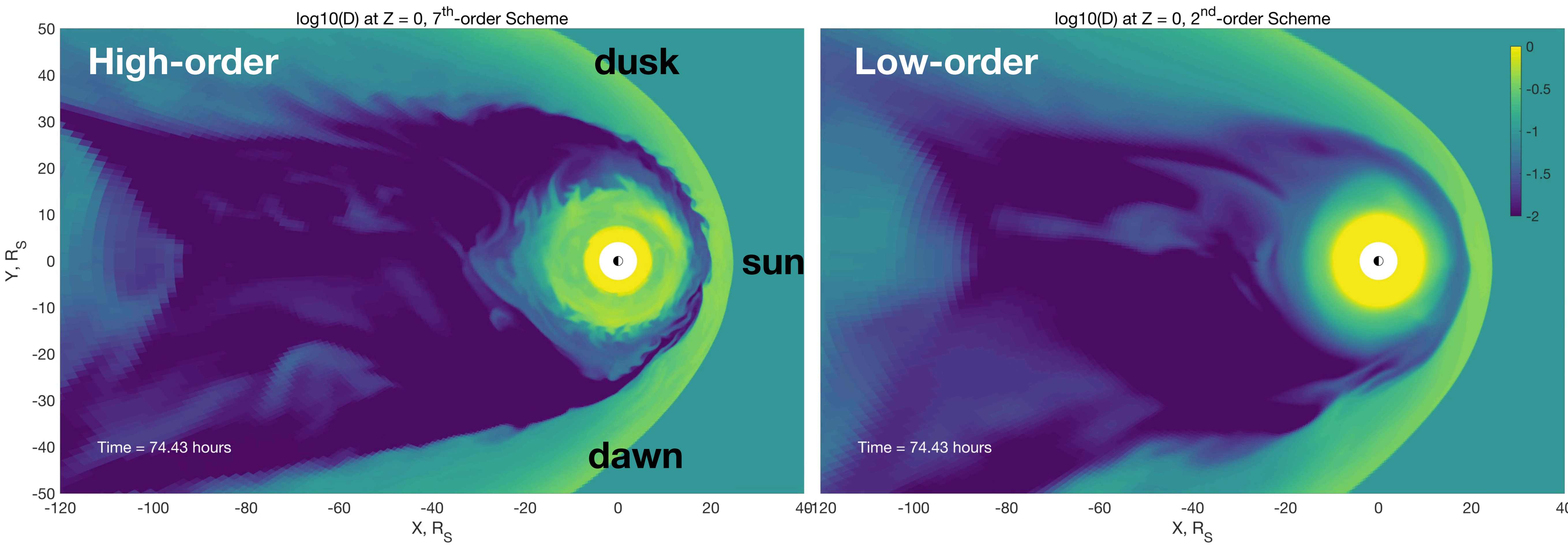
- Solar wind
- Dipole field
- M-I Coupling
- Non-MHD processes

Numerical considerations in a global magnetosphere model

Difficulties	Solutions
Large computational box (400x200x200 RE^3)	**non-orthogonal curvilinear grid**
Spherical Axis Singularity	**Ring Average technique**
Boundary layer resolving	**high-order reconstruction scheme**
Extremely low beta plasma (<1e-5)	**Background splitting technique**
Solar wind Boundary	**ballistic propagation in ghost zones**
low-altitude boundary	**electrostatic current closure**
Ring current drift kinetics	**Rice Convection Model (RCM)**
Ionosphere-thermosphere dynamics	**TIEGCM/WACCMX coupling**
Rotating dipole?	good luck…

Numerical schemes matter in Magnetosphere Simulations

Effects of Order of reconstruction on Rotating Giant magnetospheres

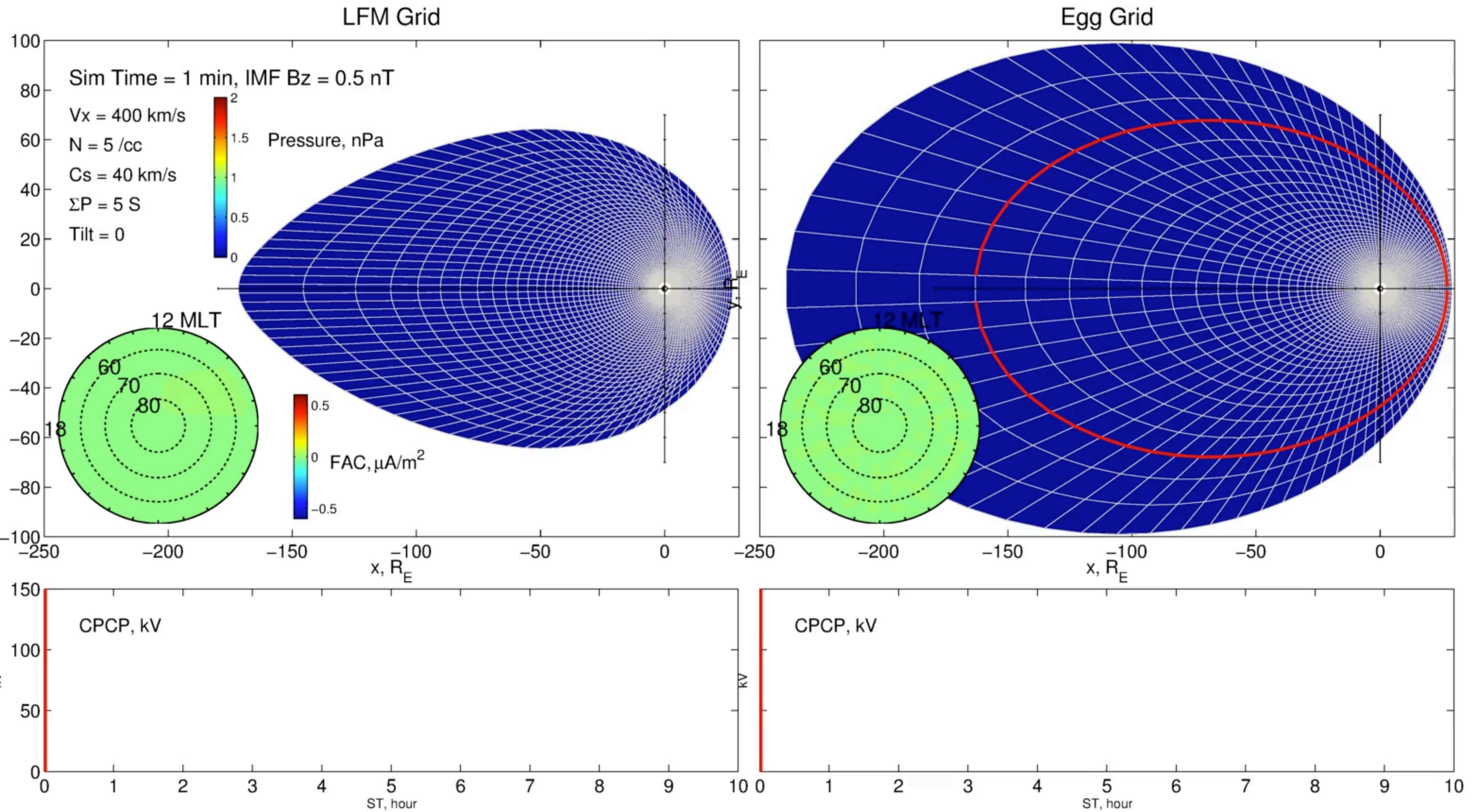


- 288x288x256 cells, dr = 0.2 Rs near the magnetopause boundary
- Steady solar wind (Mach 10) and IMF (dawn-dusk, 0.5 nT)
- Impose co-rotation potential through M-I coupling

- Asymmetric KH at the boundary in 7th-order run
- Little KHI in the 2nd order run
- Consistent with Cassini observations

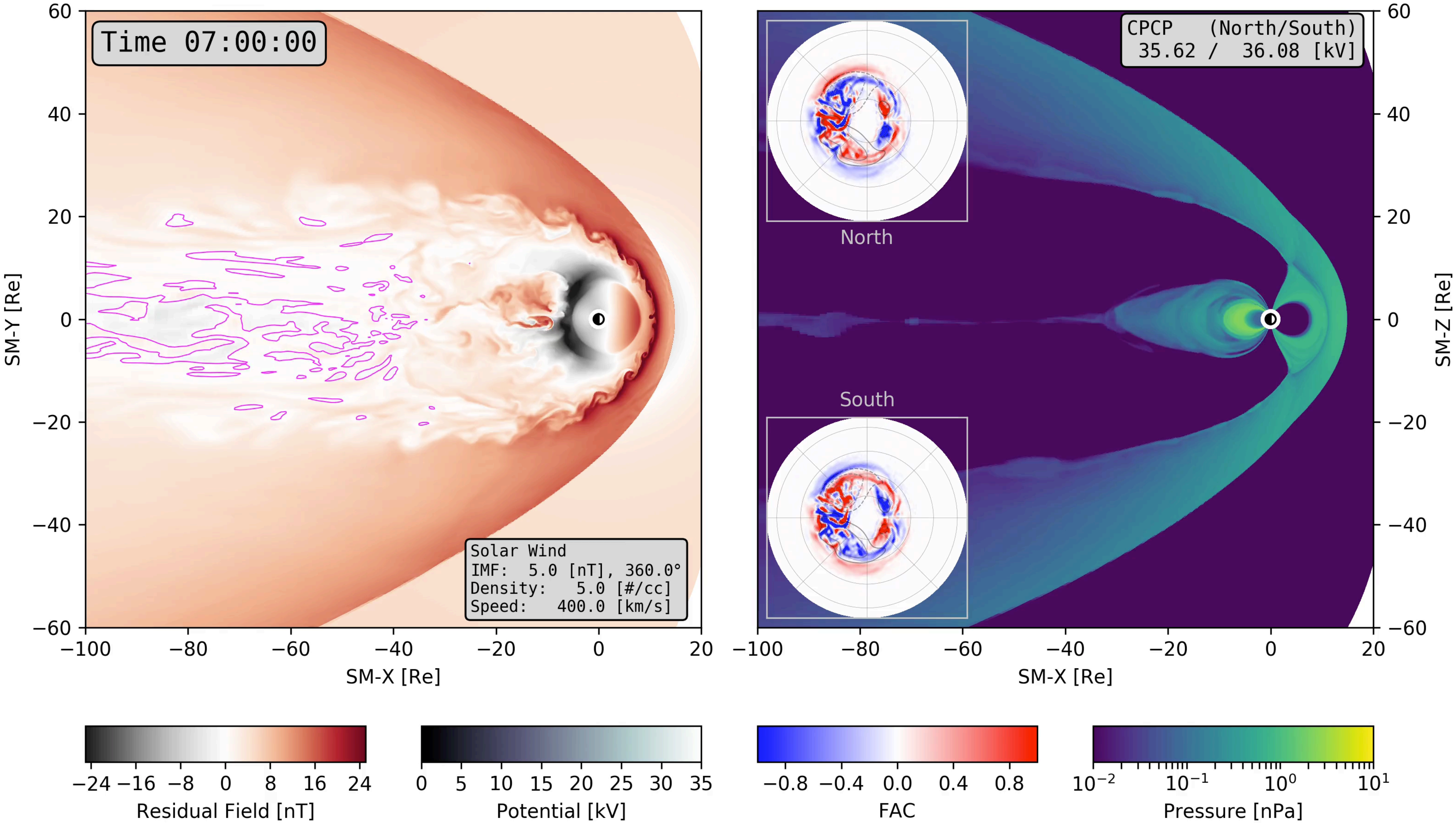
Earth Magnetosphere (Grid Independent)

Put everything together: low-res magnetosphere simulations using different grids



Extremely high resolution

Put everything together: extra high-res magnetosphere simulations using MHD



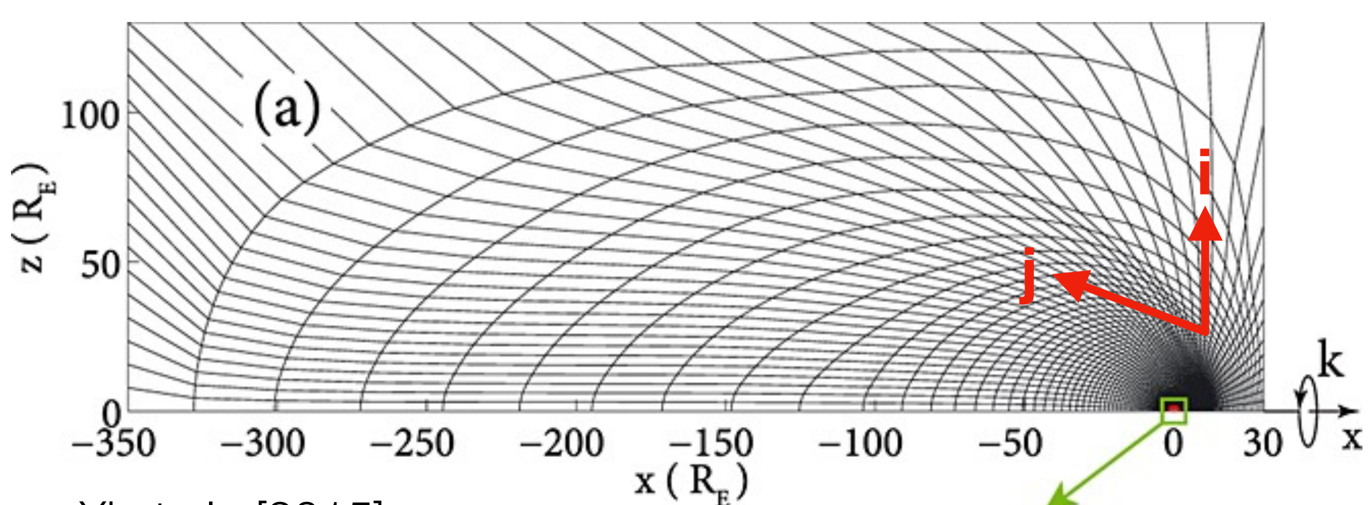
Comparisons between resolutions

- Boundary layer - Kelvin Helmholtz instabilities
- Tail dynamics - busty bulk flows
- Inner magnetospheric shielding - diamagnetic ring current
- Electron precipitation - hemispheric power

Fluid Limit? Above | Marginal | Below

	Single-Res	Double-Res	Quad-Res	Oct-Res	Hex-Res
Cells	64x32x32 (**ixjxk**)	64x64x64	128x128x128	256x256x256	512x512x512
MP Resolution (RE)	0.4	0.4	0.2	0.1	0.05
IM Resolution (RE)	0.5	0.5	0.25	0.126	0.0625
Tail Resolution (RE)	0.6	0.6	0.3	0.15	0.075
deltax/di	>4	>4	>2	~1	<1?

Computational Grid



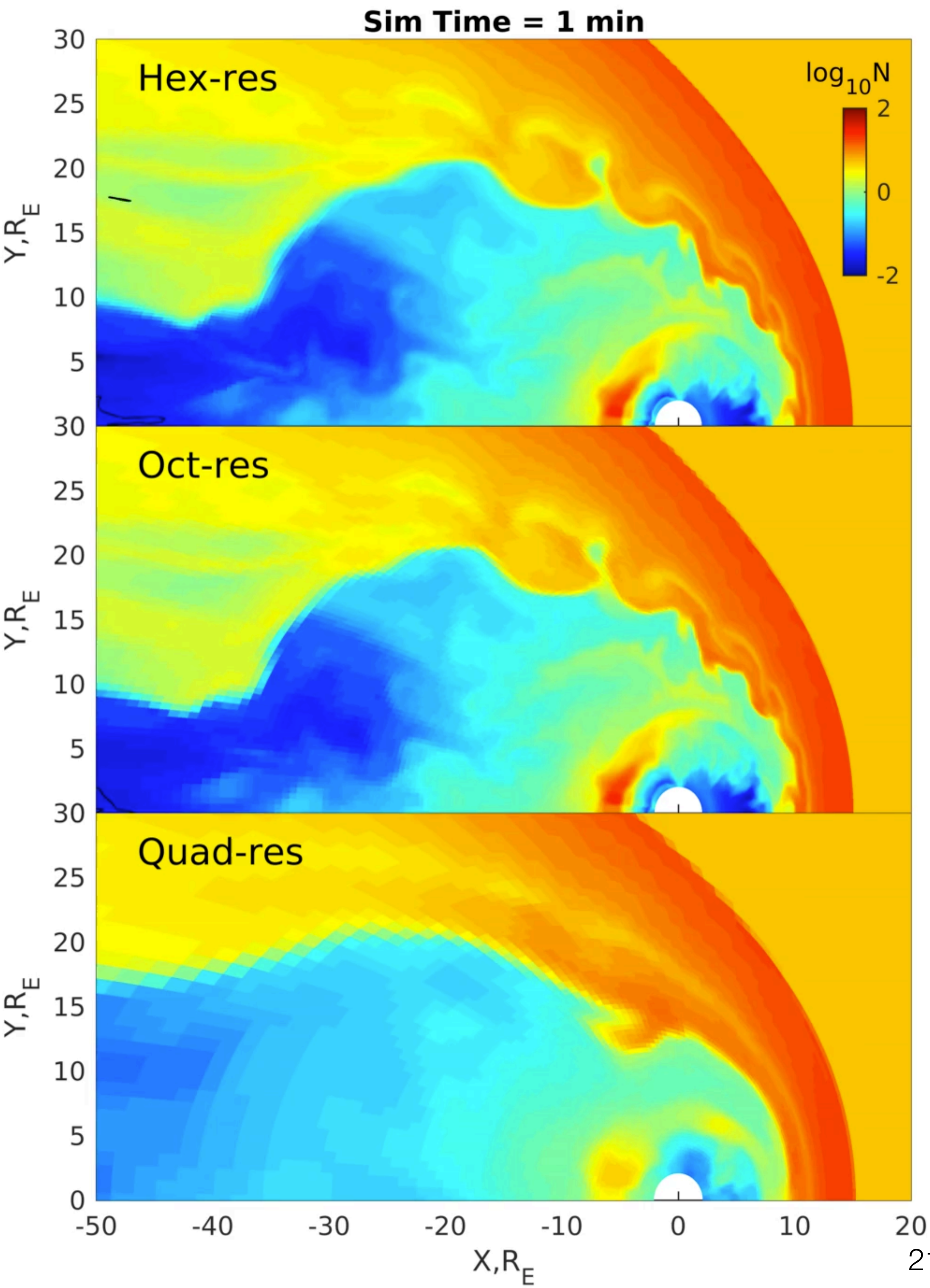
Xi et al., [2015]

Computational cost

	Quad	Oct	Hex
CPU-hour per sim hr	0.2-0.3k	4-5k	90-100k

e.g., an 10-hour hex-res event sim would cost 1M CPU hour - very expensive

Kelvin-Helmholtz Instabilities



SW/IMF conditions

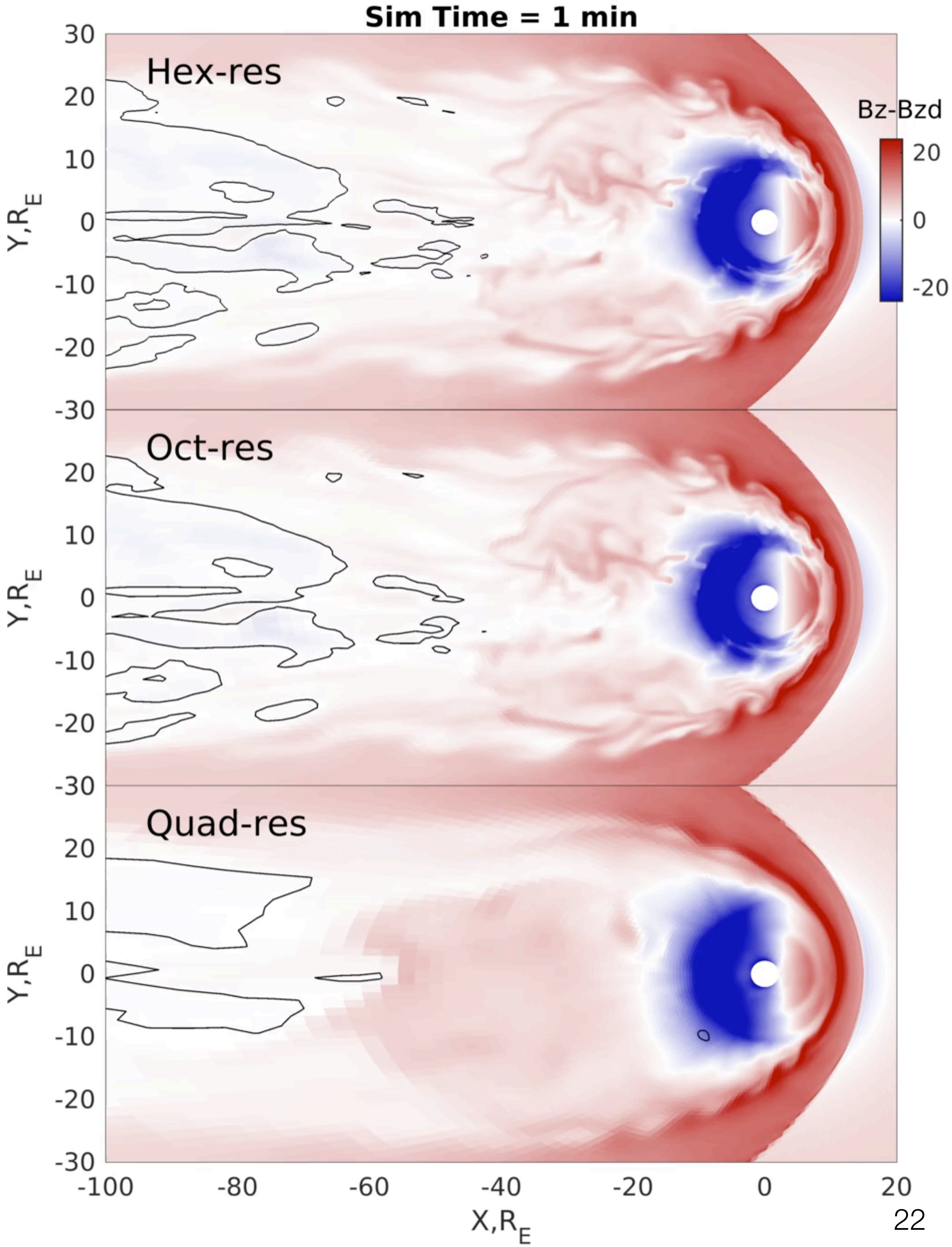
- **N = 5/cc**
- **V = 400 km/s**
- **Mach = 10**
- **Bz = +/-5 nT**

Results

- KH activities occur in all three resolutions
- Quad-res seems to under-resolve KHI
- The difference in boundary layer between oct-res and hex-res seems small - does not mean it's converging
- KHI occurs much less frequently during southward IMF driving - consistent with observations?

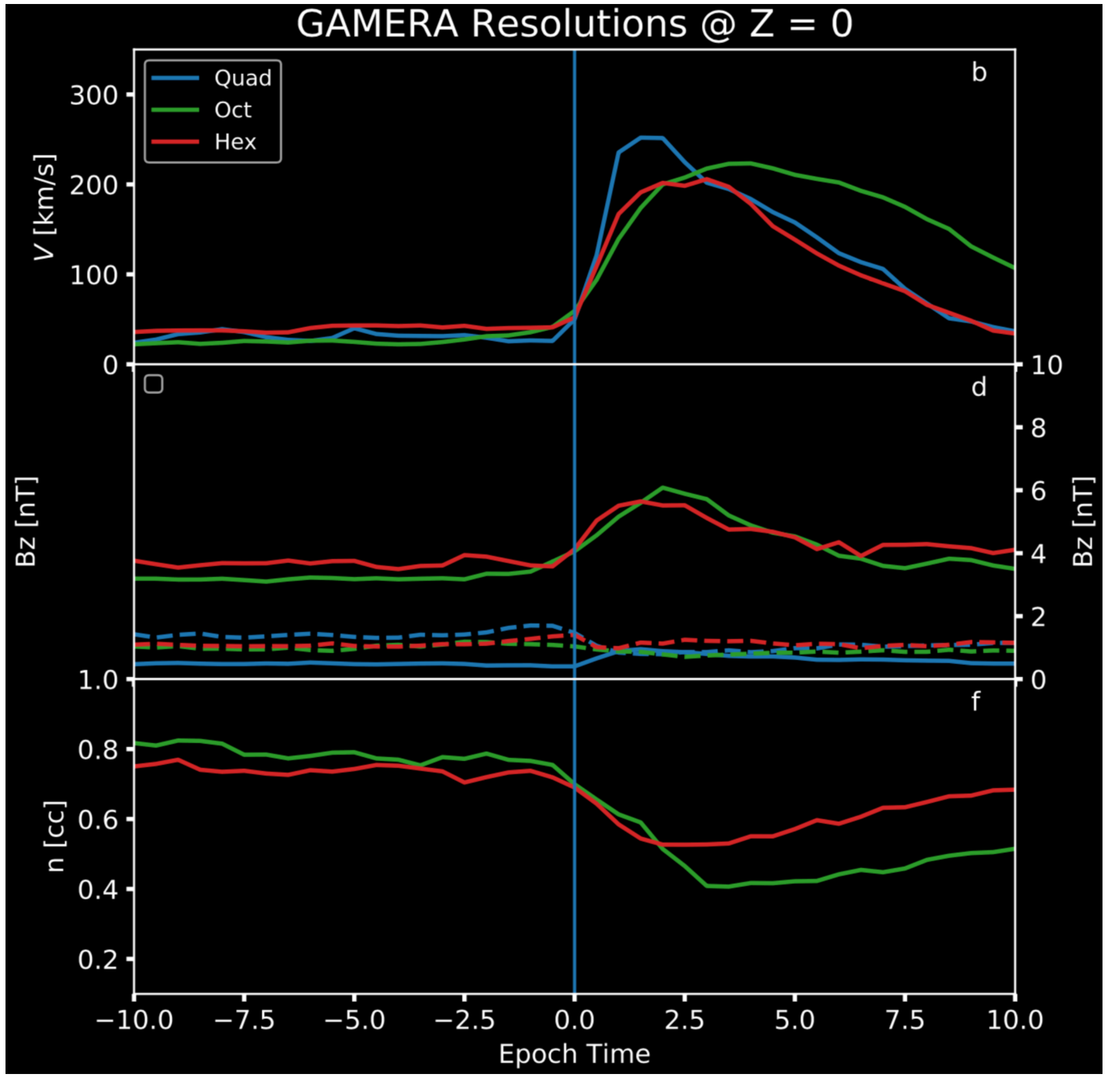
Quantitive studies like *Merkin et al.*, [2013] is needed

Bursty Bulk Flows



Results

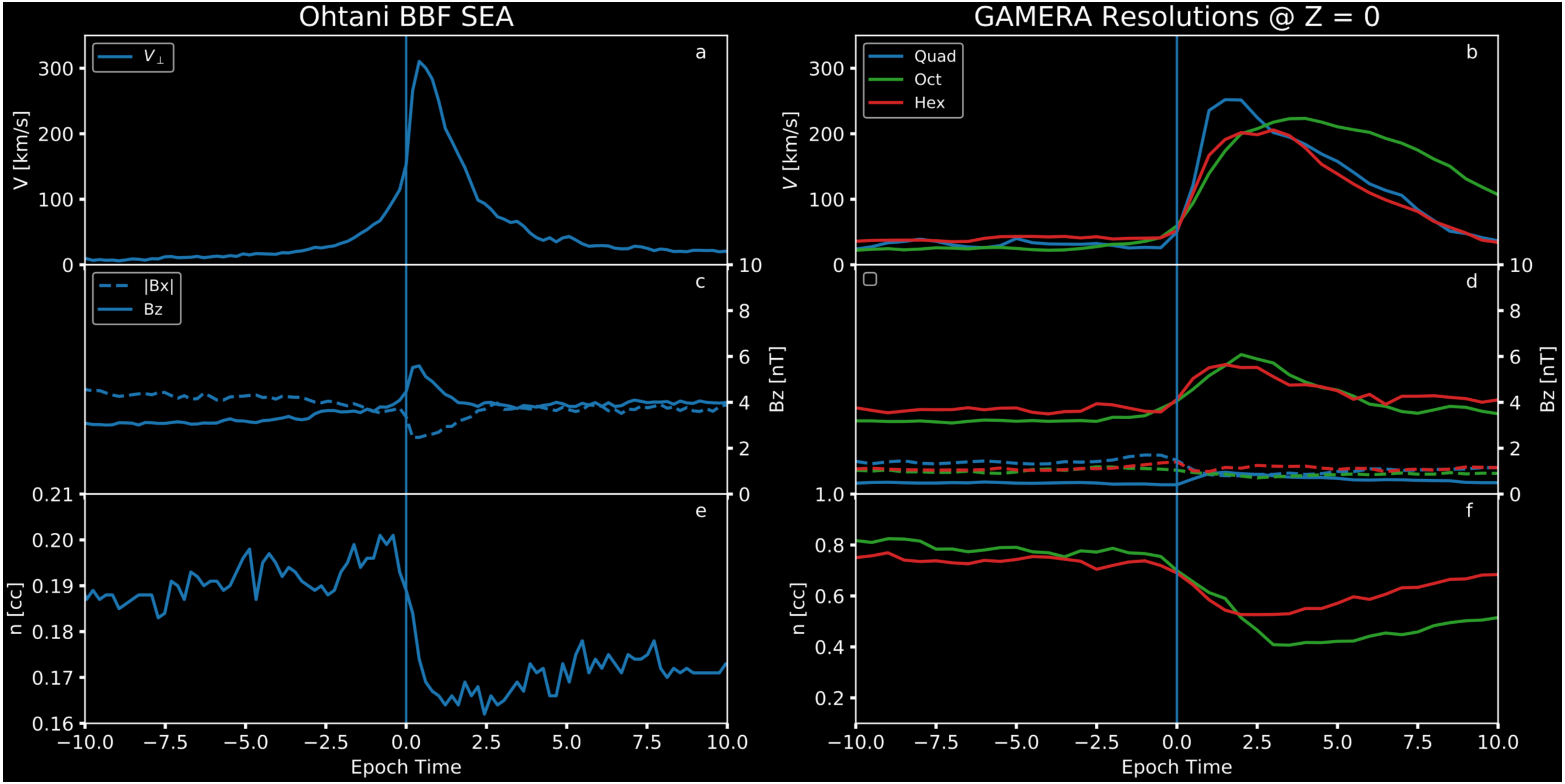
- BBFs occur in all three resolutions
- Quad-res seems to produce less BBF compared to the other two runs
- The difference in BBFs between oct-res and hex-res seems small - not a factor of 2, converging?
- BBF interacting with boundary layer - any observational evidence?



Analysis based on *Wiltberger et al.*, [2016]

Bursty Bulk Flows

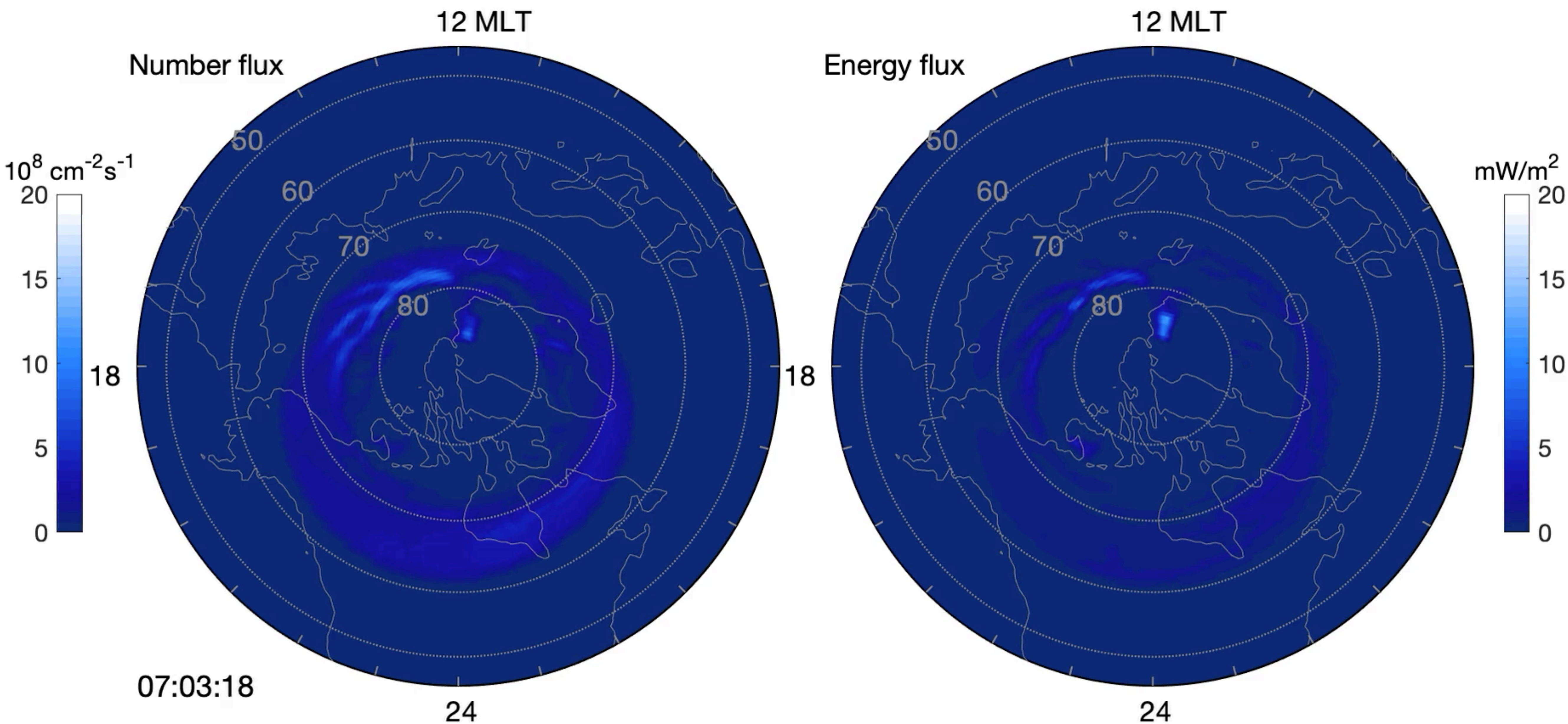
Compare with Ohtani BBF observations



Quick finding: the oct-res and hex-res are approximately in the same regime, not quad

Electron Precipitation (Aurora)

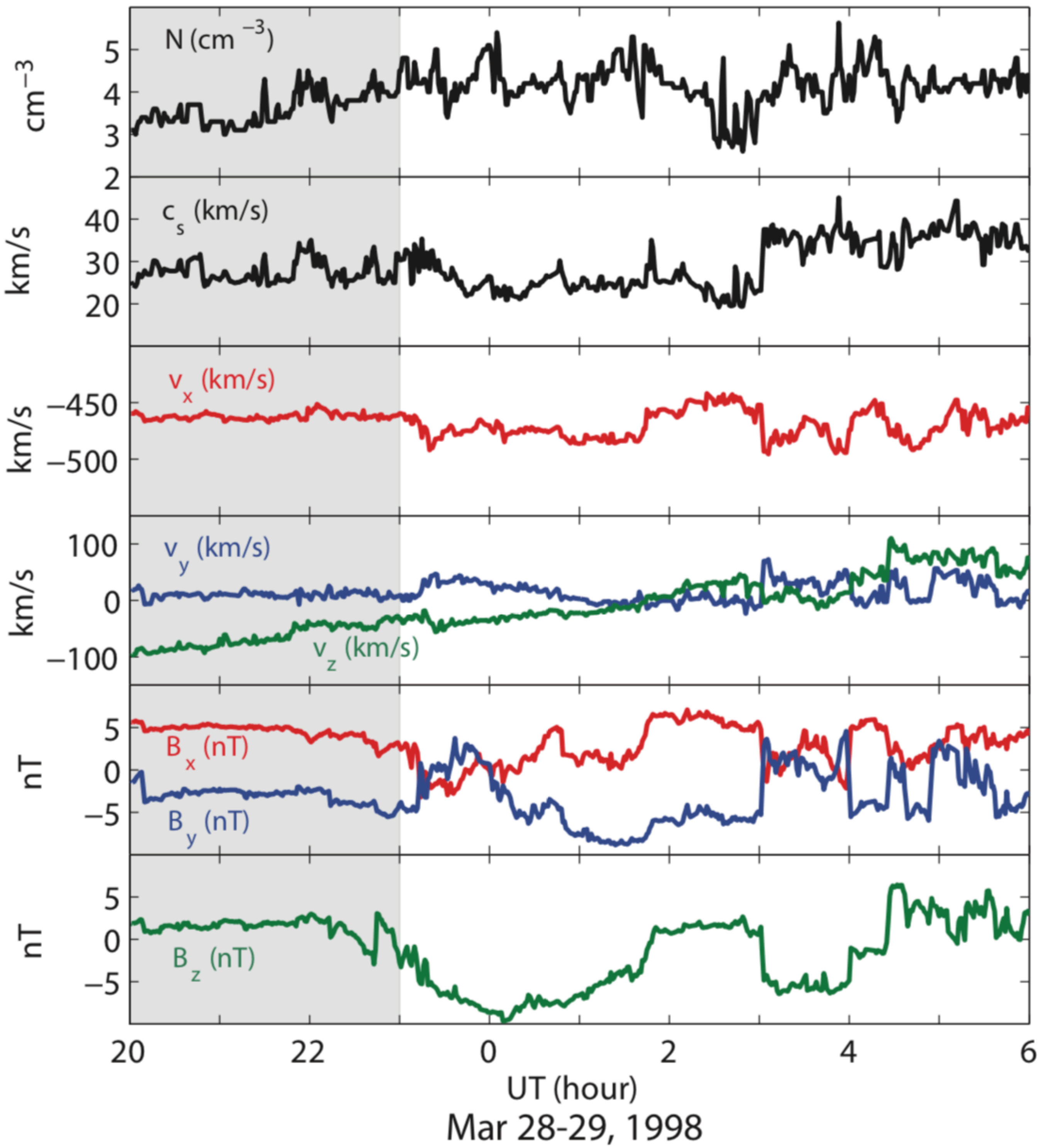
Idealized Simulations



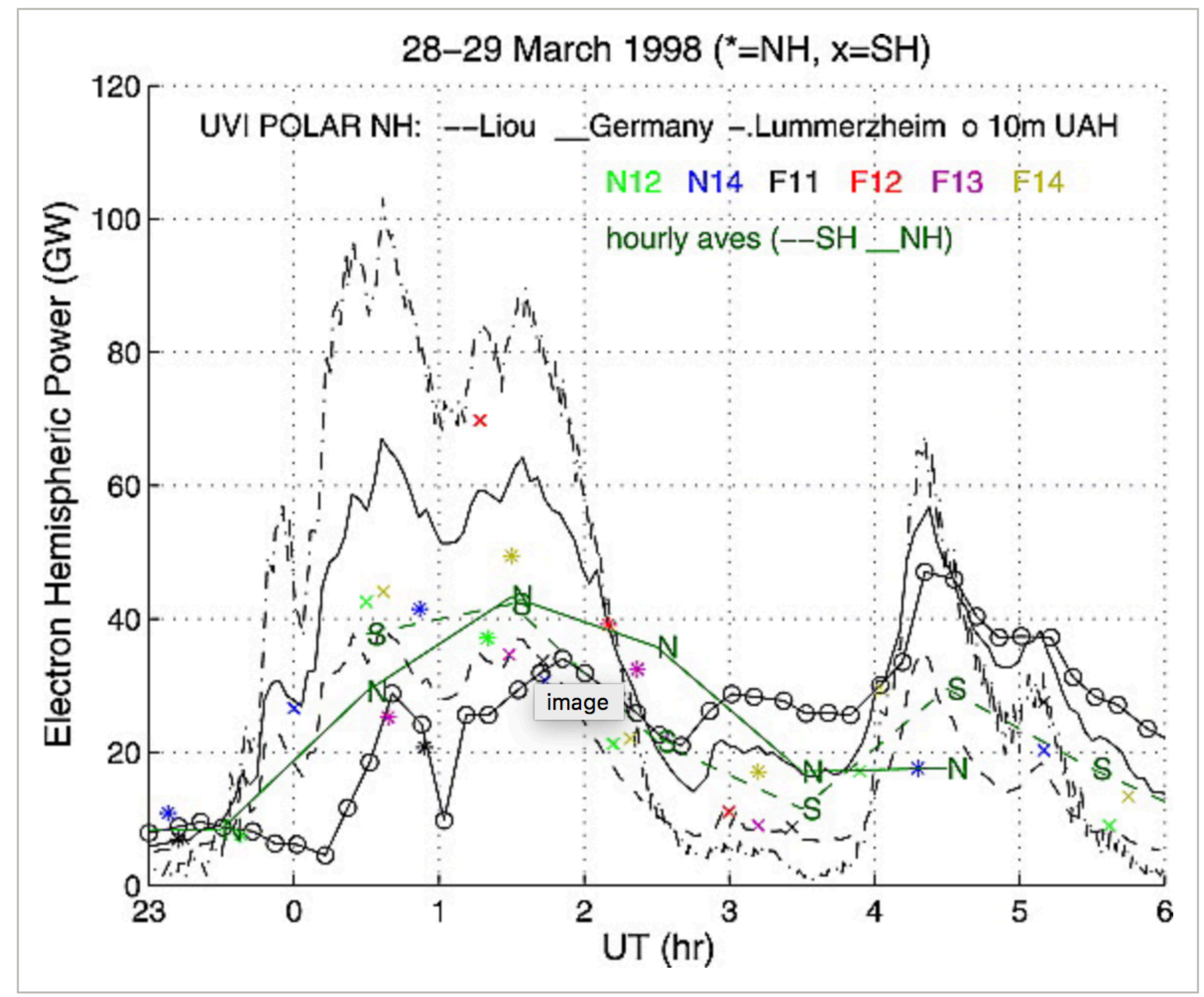
Electron Precipitation

The Mar 28-29, 1998 Event: UVI observations

IMF/SW conditions



Emery et al. [2008]

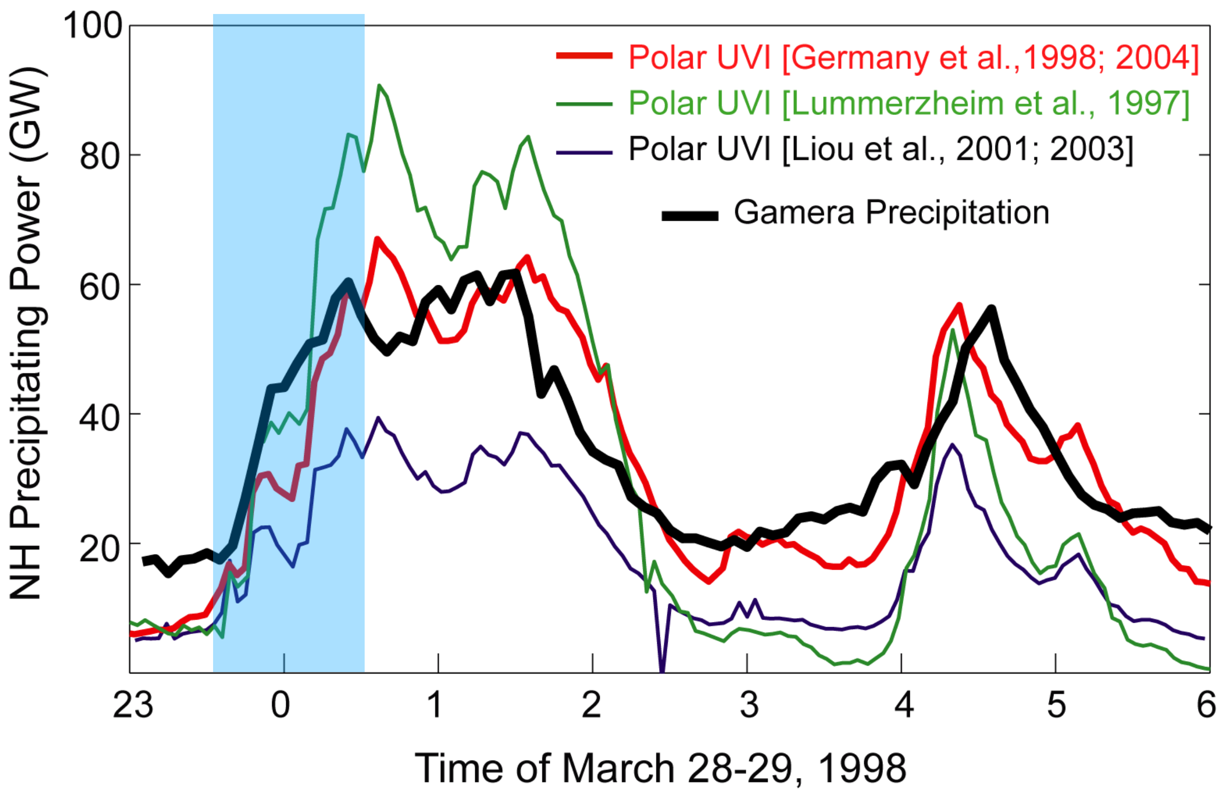


NH Polar UVI estimates of hemispheric power on 28–29 March 1998 from three separate scientific analyses [after *Palmroth et al.*, 2006] compared to adjusted Hpe estimates from NOAA and DMSP in the NH (asterisks) and SH (crosses) and their hourly averages. The 10-min automated routine estimates from Polar at UAH are also included as connected circles.

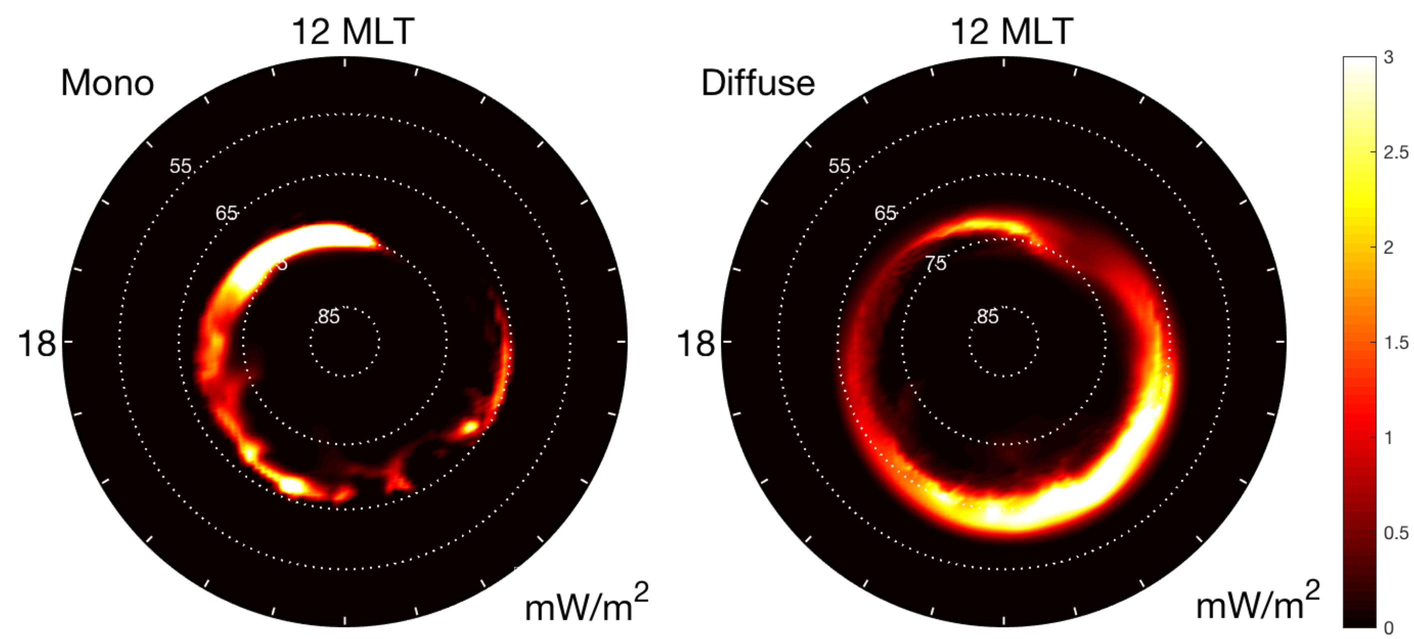
Electron Precipitation

The Mar 28-29, 1998 Event - MHD Simulations Versus Polar UVI

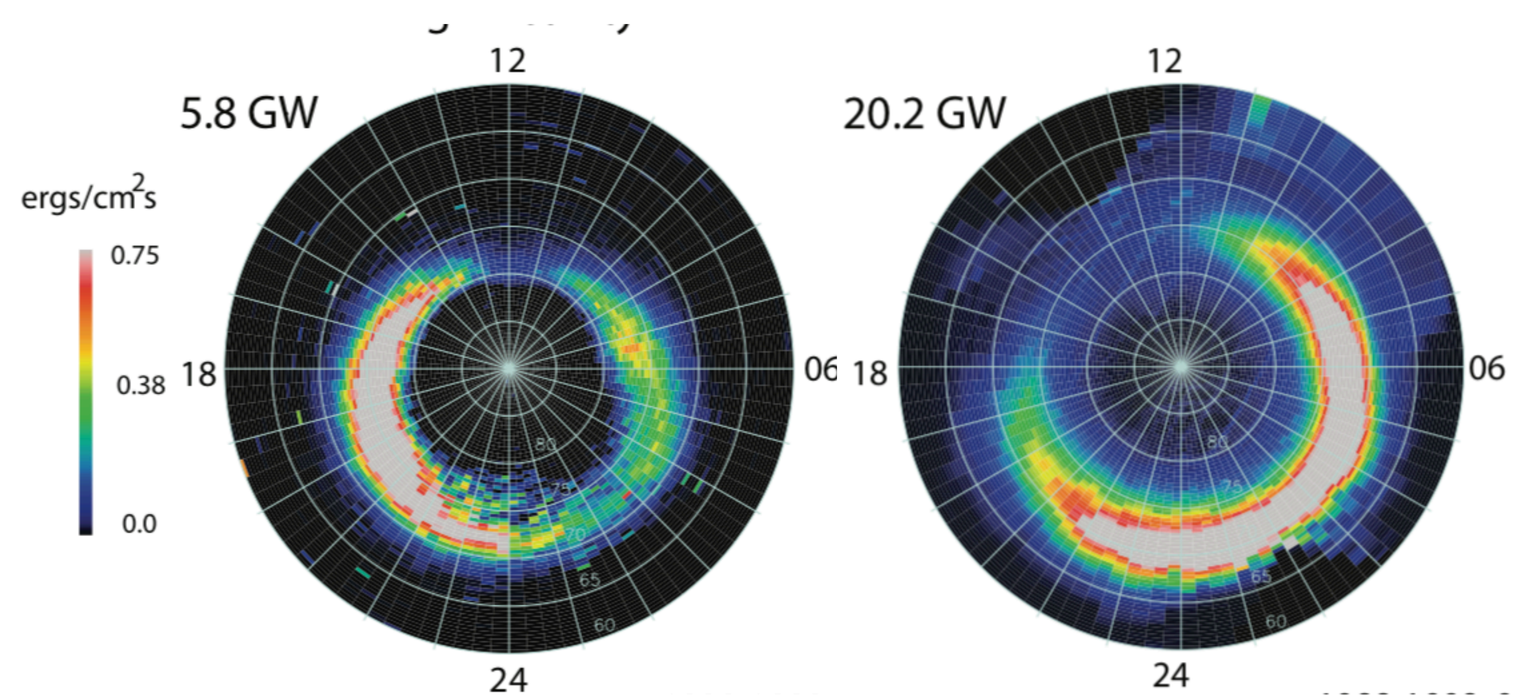
Power compared w/ Emery et al. [2008]



Diffuse and Monoenergetic Precipitation



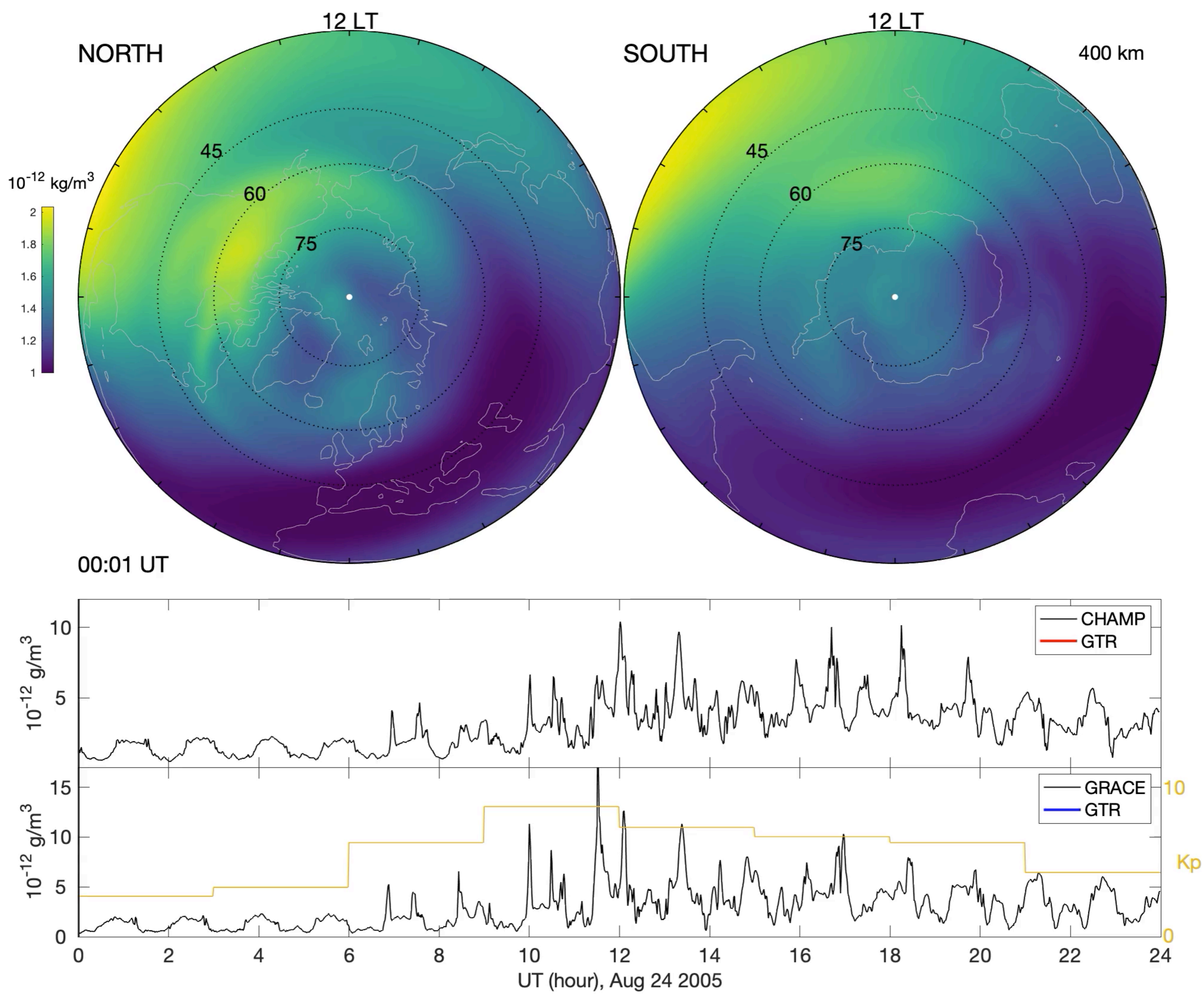
Mono/Diffuse = ~25%



Results

- Observations may give different estimations of auroral power based on Polar UVI - which one shall I use?
- The Fedder 95 model seems to work with proper temperature ratio and loss cone filling
- Both monoenergetic and diffuse precipitation resembles OVATION results

Upper Atmosphere



Take-home Messages

- **MHD simulations are used extensively in all kinds of space plasma/astrophysics problems**
- **3-D MHD simulations requires supercomputing techniques**
- **We can always increase the resolution (not yet enough for solar wind), but more physics is required resolving small scale structures**
- **The future direction will be merging different physics-based space plasma models with MHD**