Introduction

Computational Space Plasma Physics

Outline

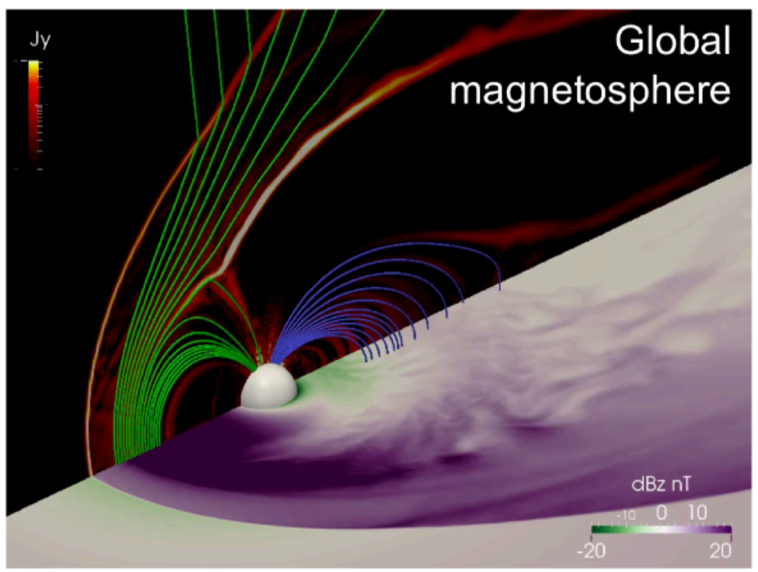
- Space Plasma Physics
- Computational
- Examples and Applications
- Finite difference approximation

What is Space Plasma Physics

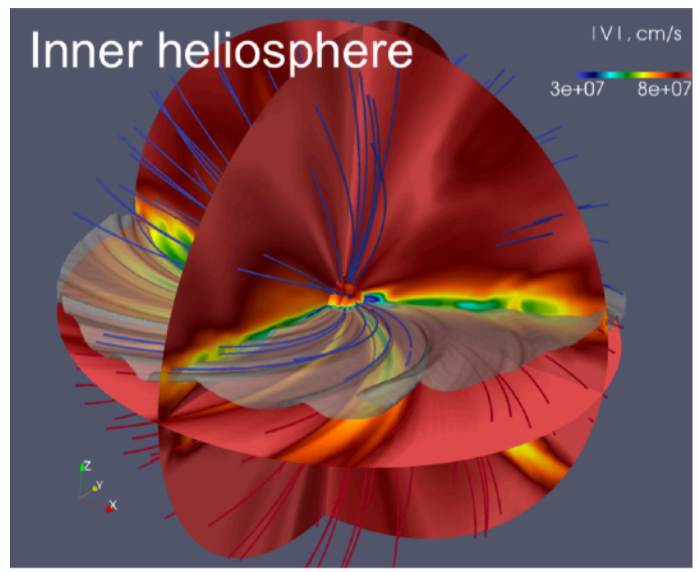
Space plasma physics is the study of the Sun's hot, ionized outer atmosphere and its interactions with the planets and other bodies of the solar system.

Why Study Space Plasma Physics?

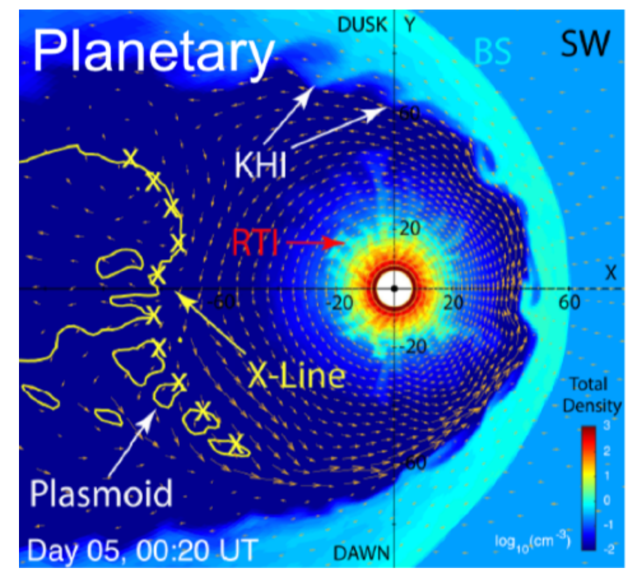
Geospace Dynamics



Solar and Solar Wind



Planetary Sciences



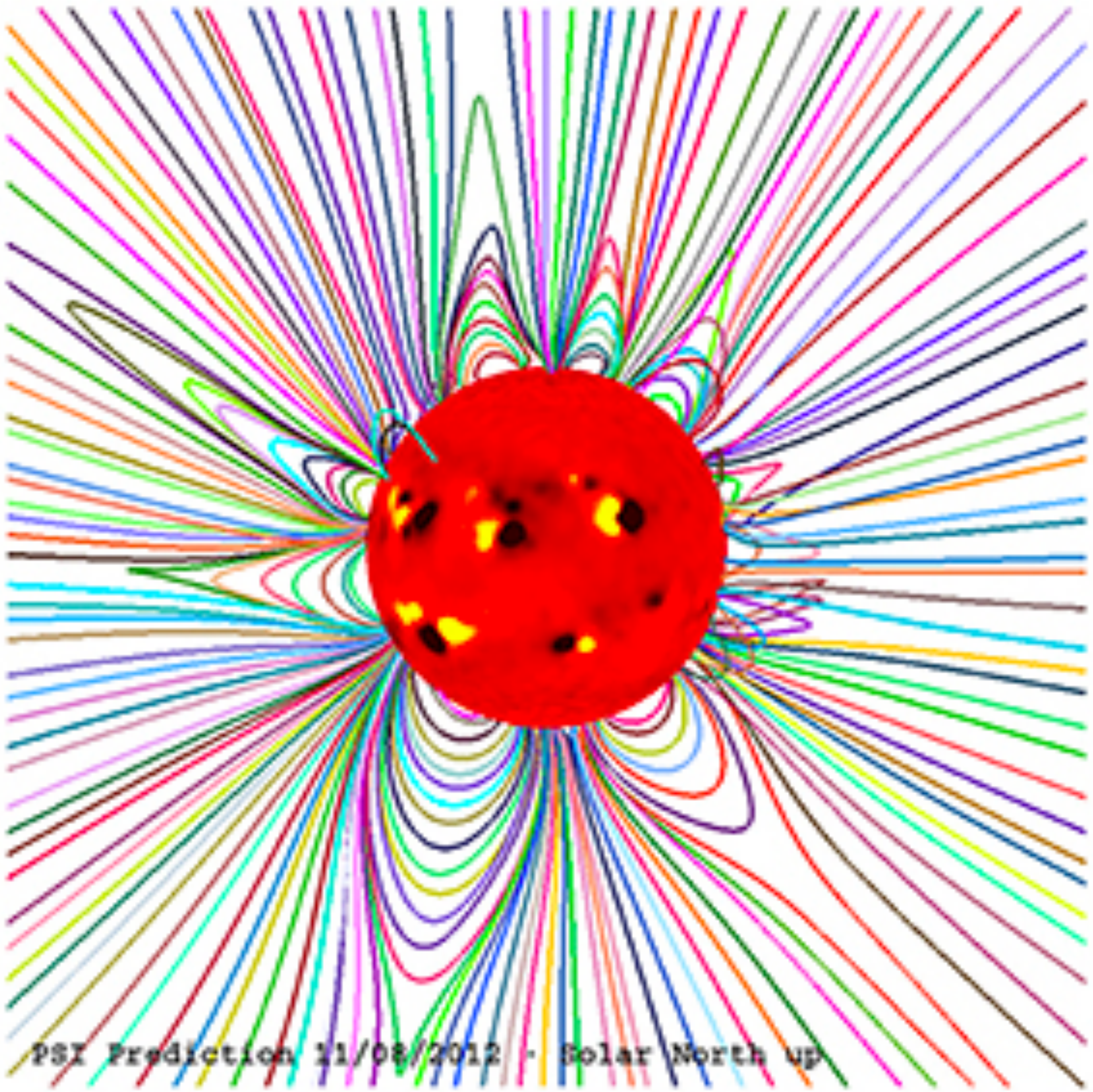
Basic Plasma

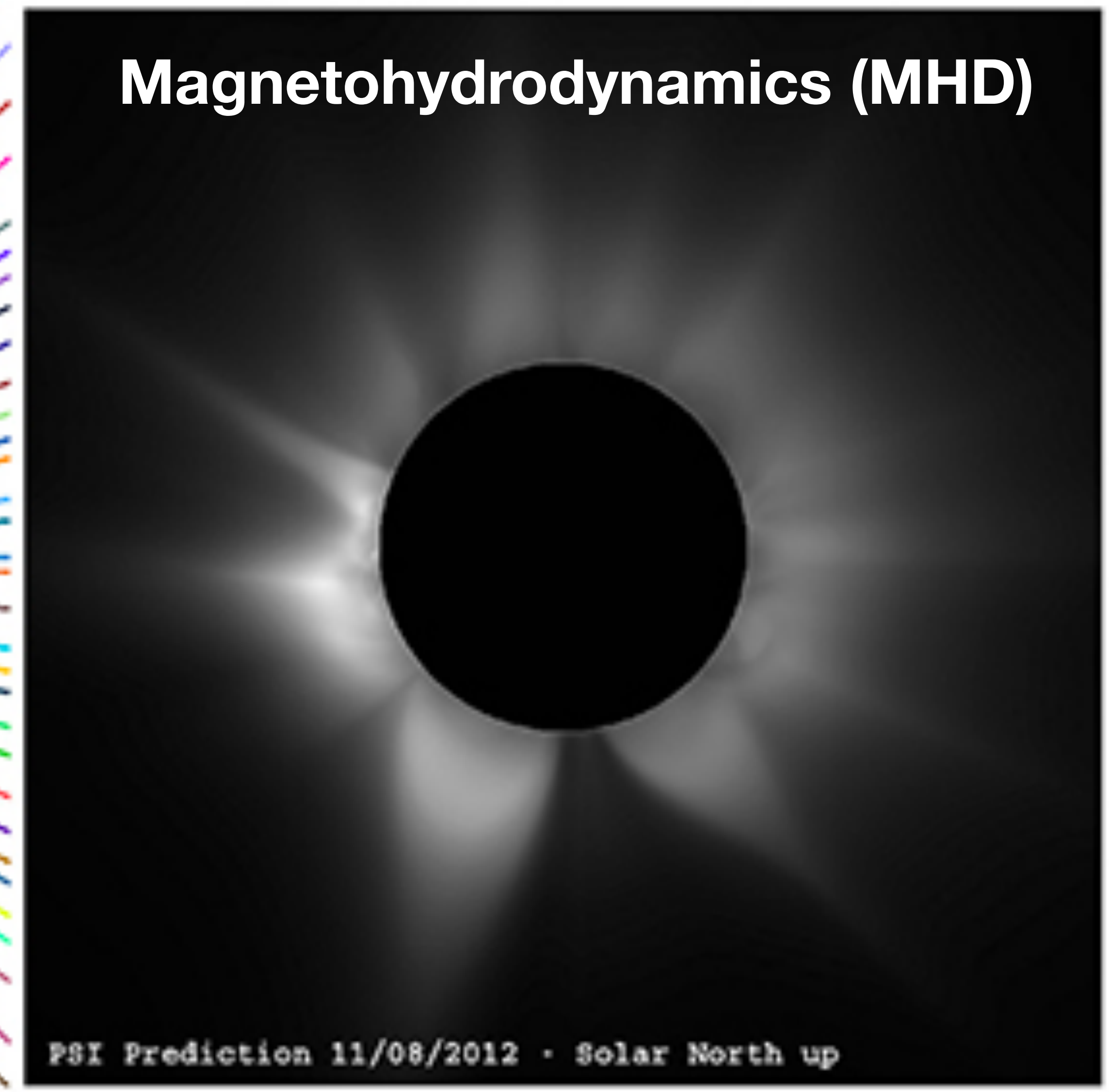
Who Cares About Space Plasma Physics?

Of course scientists do, especially plasma physicists since the space is nature's lab for plasma experiments!
Engineers also do when applying space weather knowledge to satellite design and operation

Solar Corona

The Solar corona

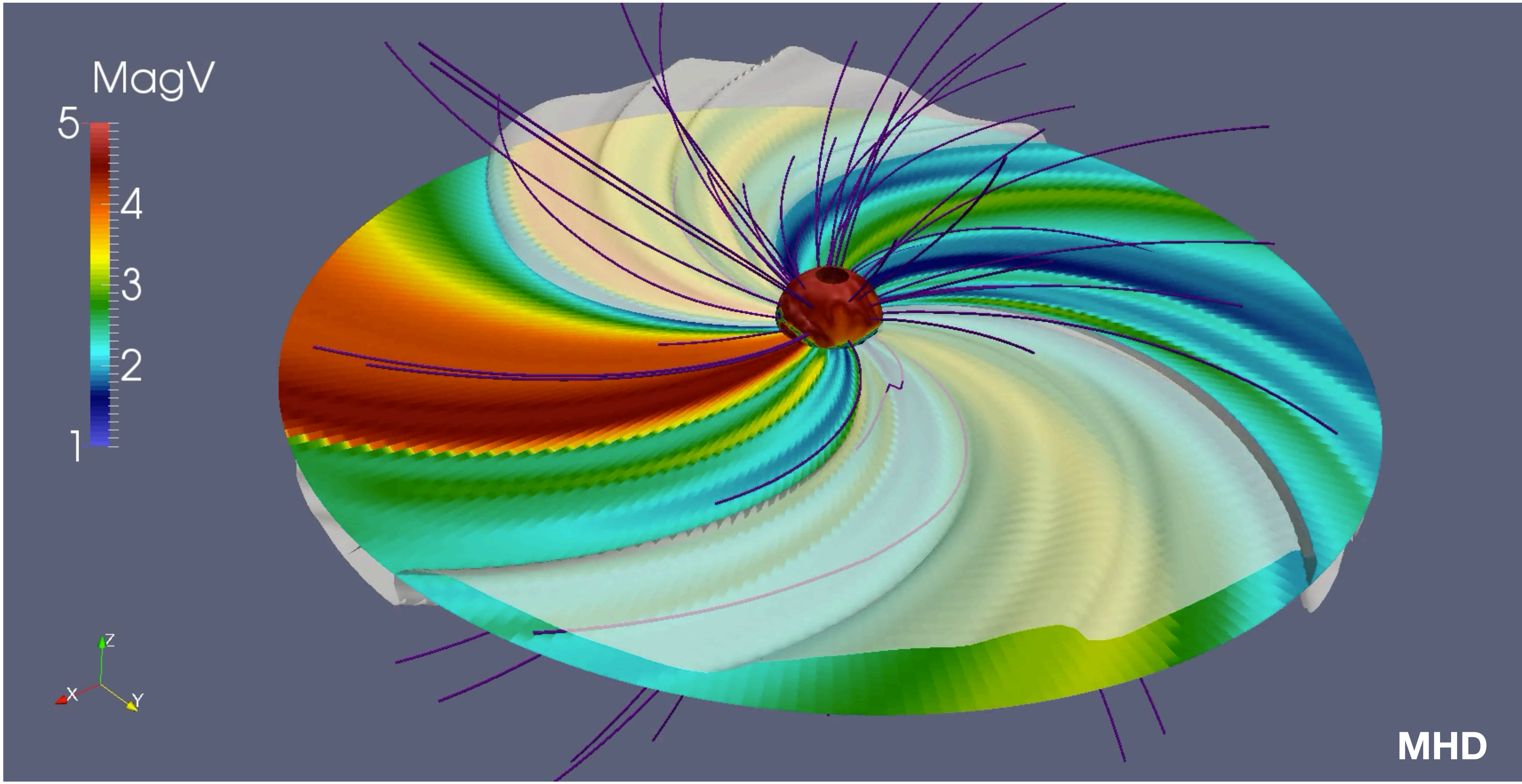




Domain: Surface of the Sun to 10-20 Solar Radius
Density: 10^9 particle/cm^3
Temperature: 10^7 K
Speed: ~100 km/s

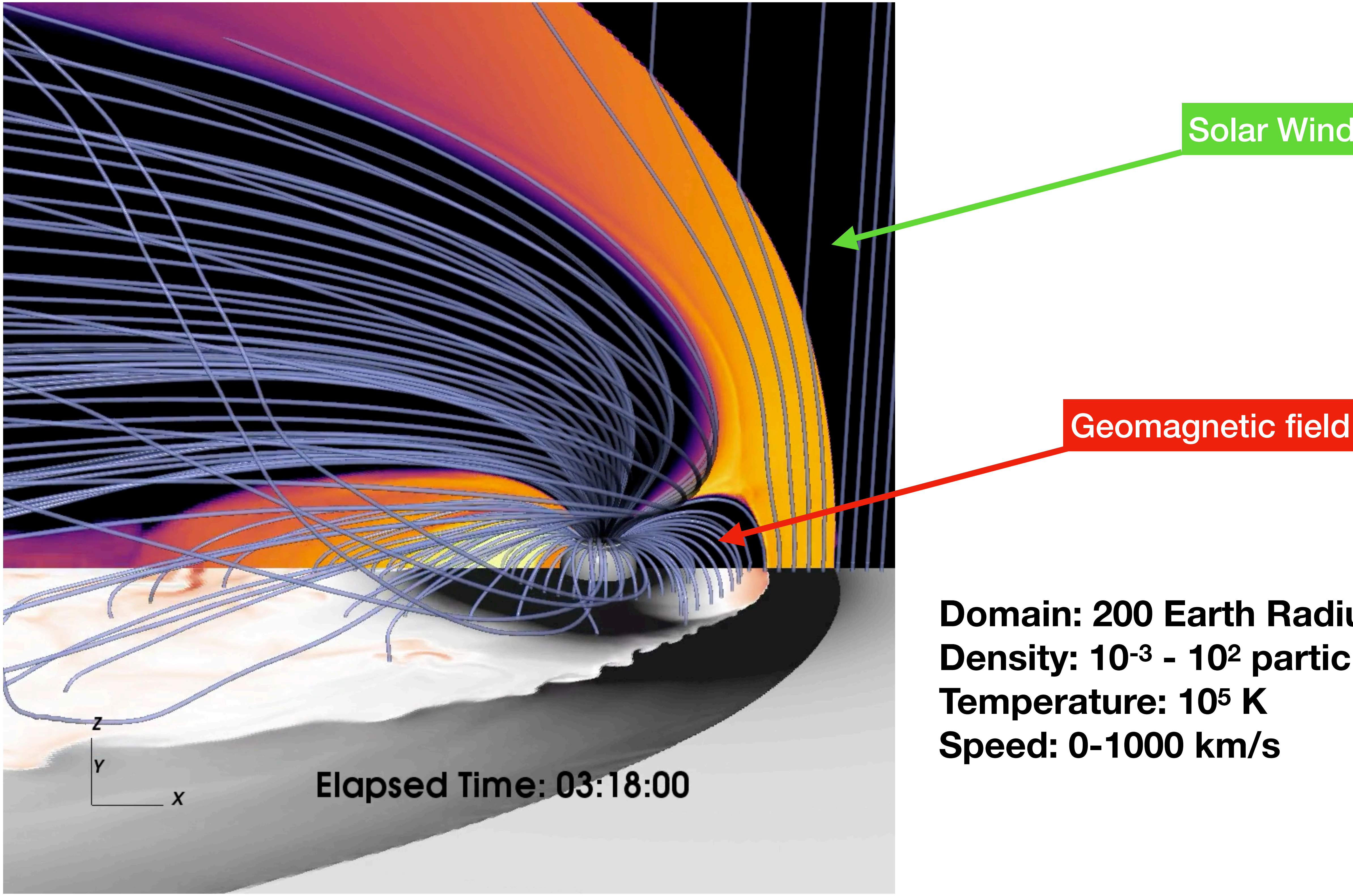
The Solar Wind

The Solar Wind (Parker Spiral)



Domain: 20 Solar Radius to several AU
Density: 10 particle/cm3
Temperature: 10^5 - 10^6 K
Speed: 400-1000 km/s

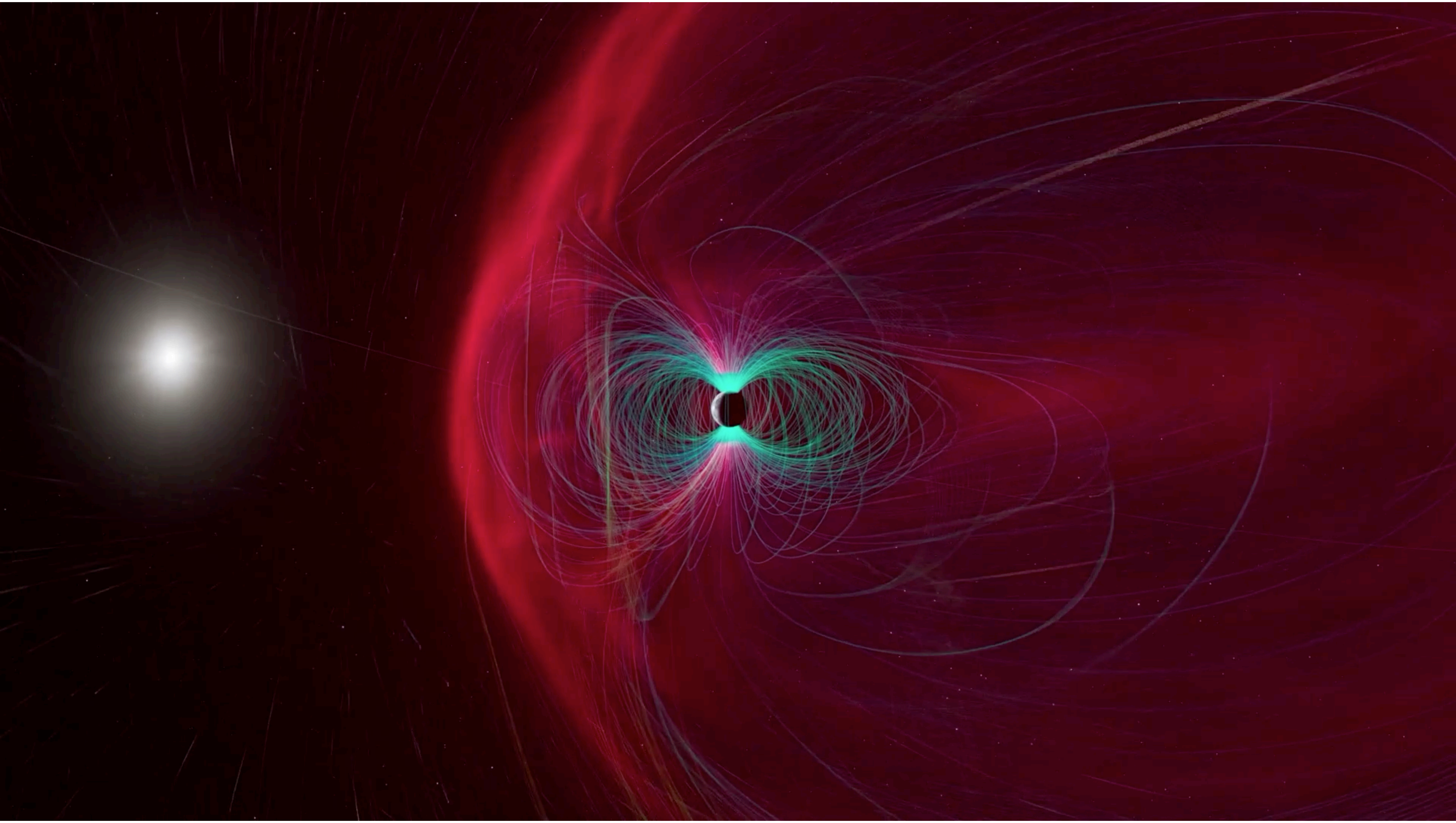
Earth's Magnetosphere



Domain: 200 Earth Radius
Density: 10^{-3} - 10^{2} particle/cm3
Temperature: 10^{5} K
Speed: 0-1000 km/s

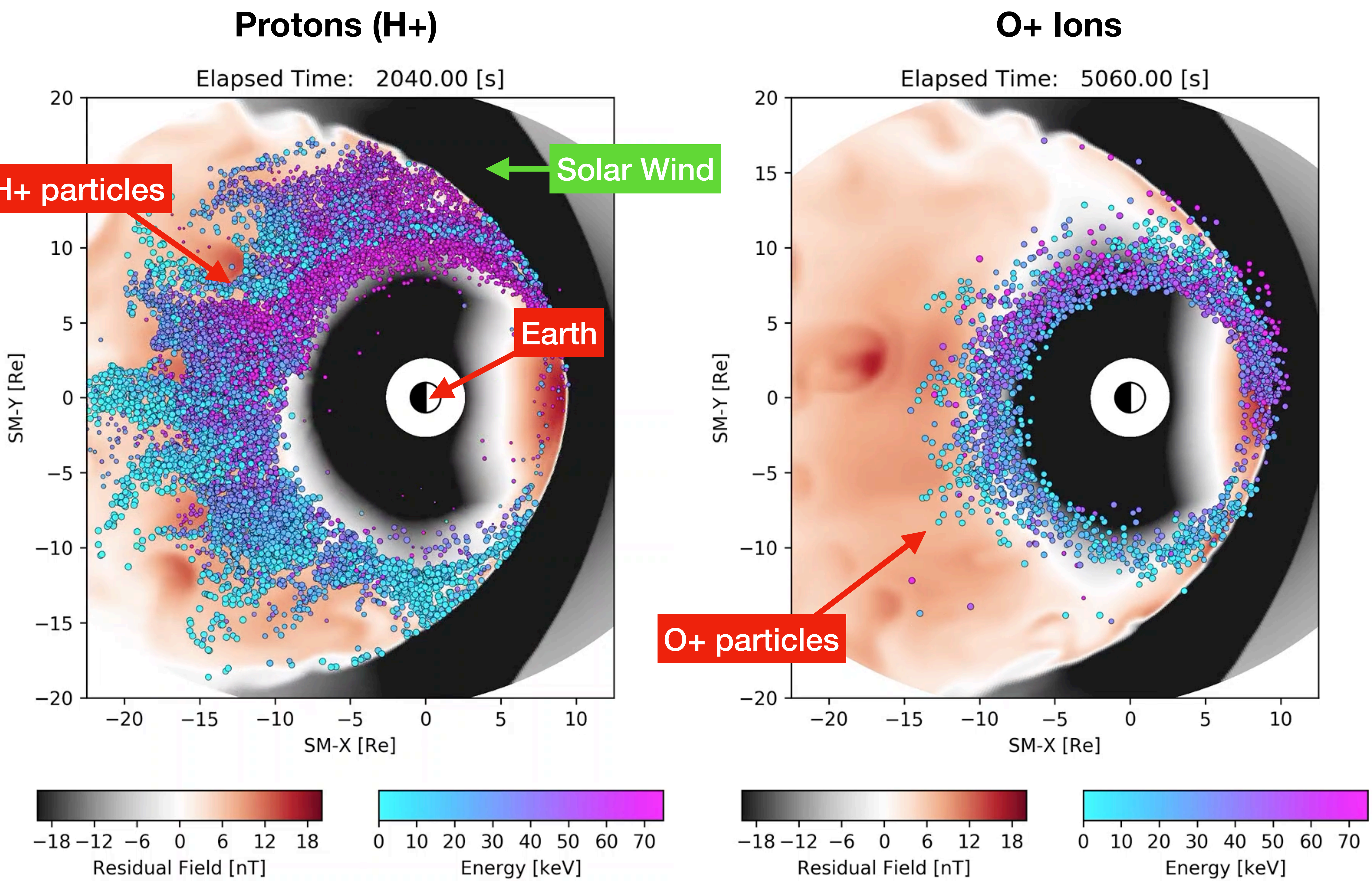
MHD + Ionospheric electrodynamics

Earth's Magnetosphere



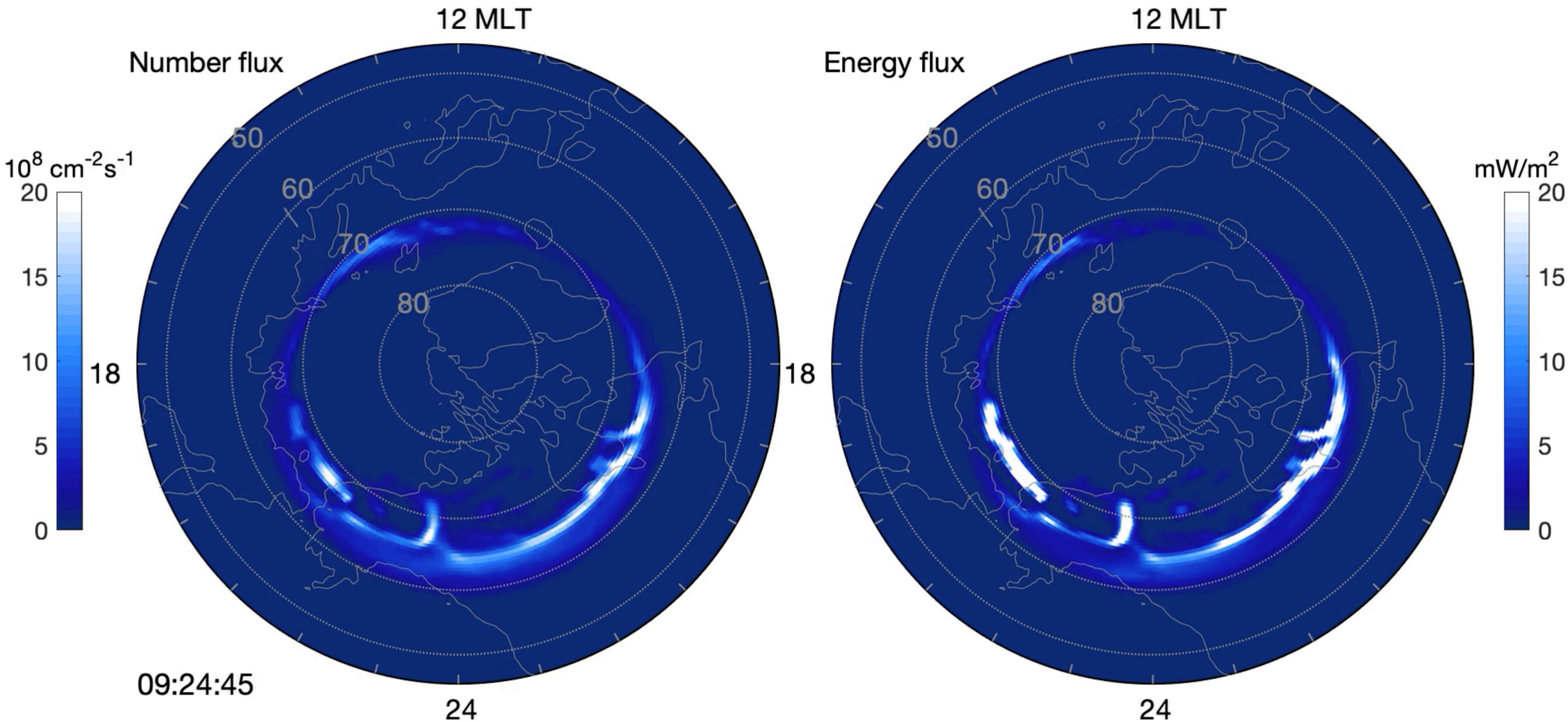
MHD + visualization

Earth's Radiation Belt Particles



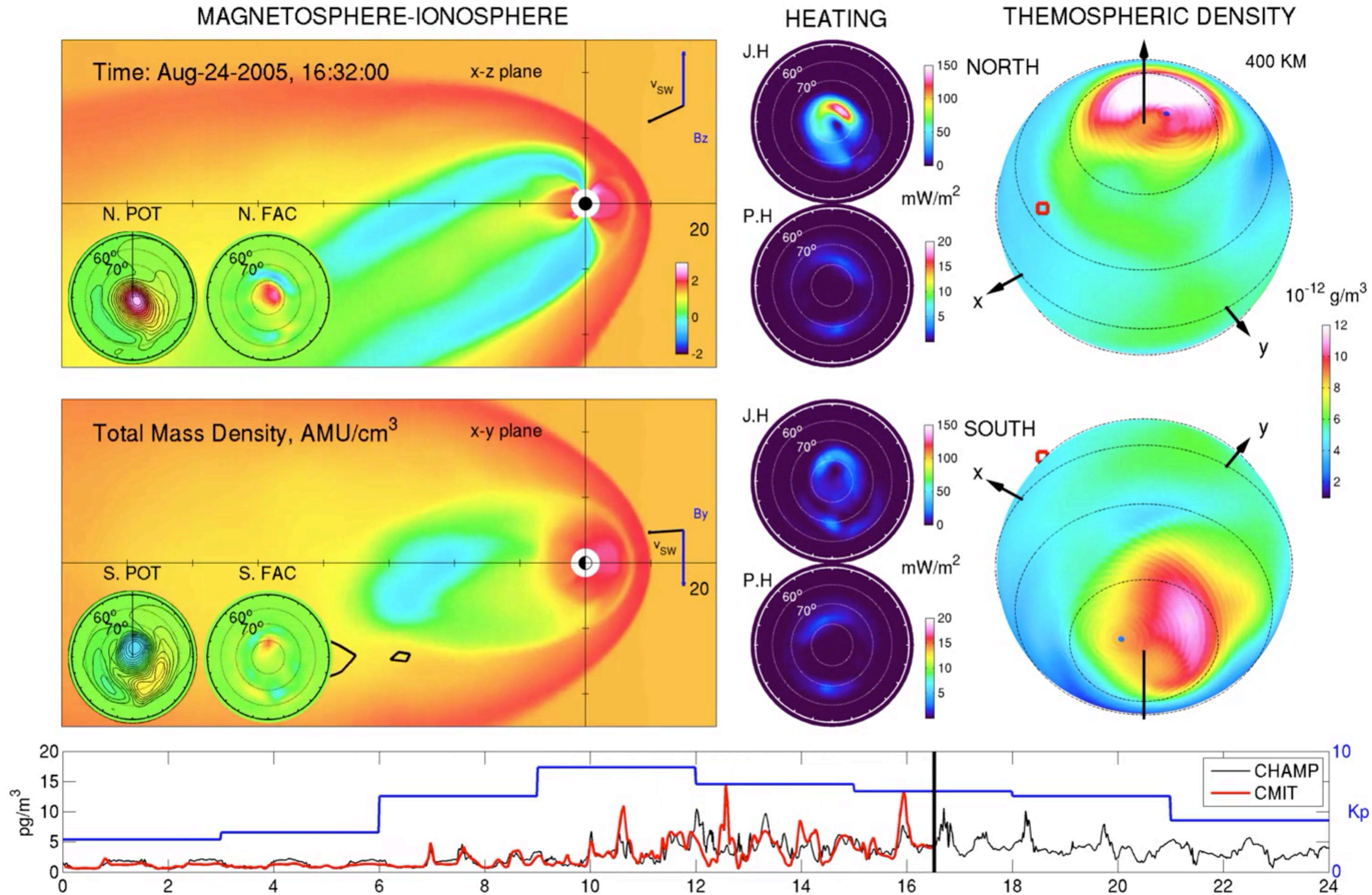
Particle Dynamics

Earth's Aurora (Northern Light)



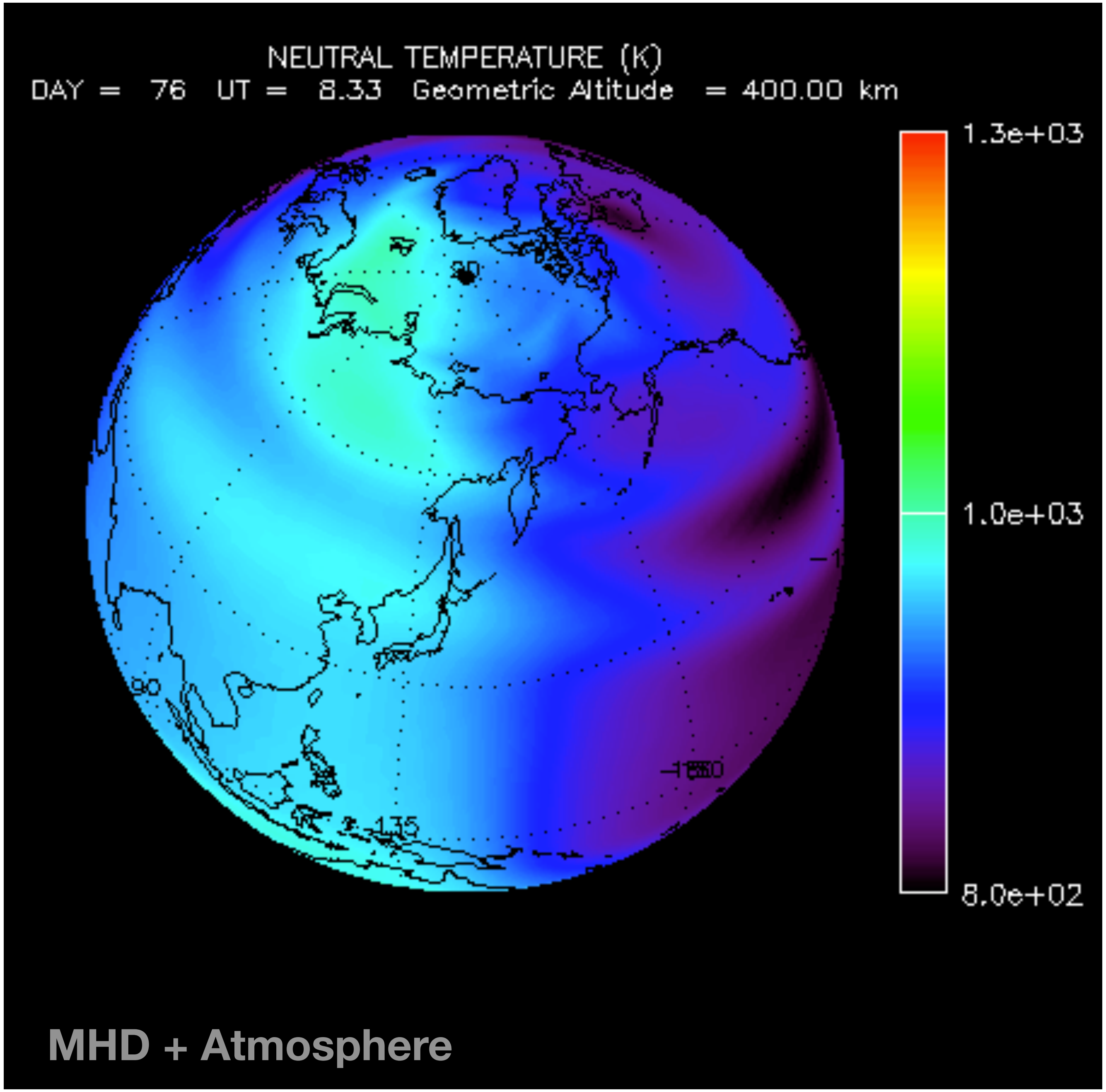
MHD + Kinetic parameterization

Geomagnetic Storms (Space Weather)



MHD + Drift Kinetics + Atmosphere

Ionosphere/Upper Atmosphere



Ionosphere

Domain: 90-500 km altitude

Density: 10^{-1} - 10^{6} particle/cm^{3}

Temperature: 1 - 10^{2} K

Speed: 0-1000 m/s

Upper Atmosphere

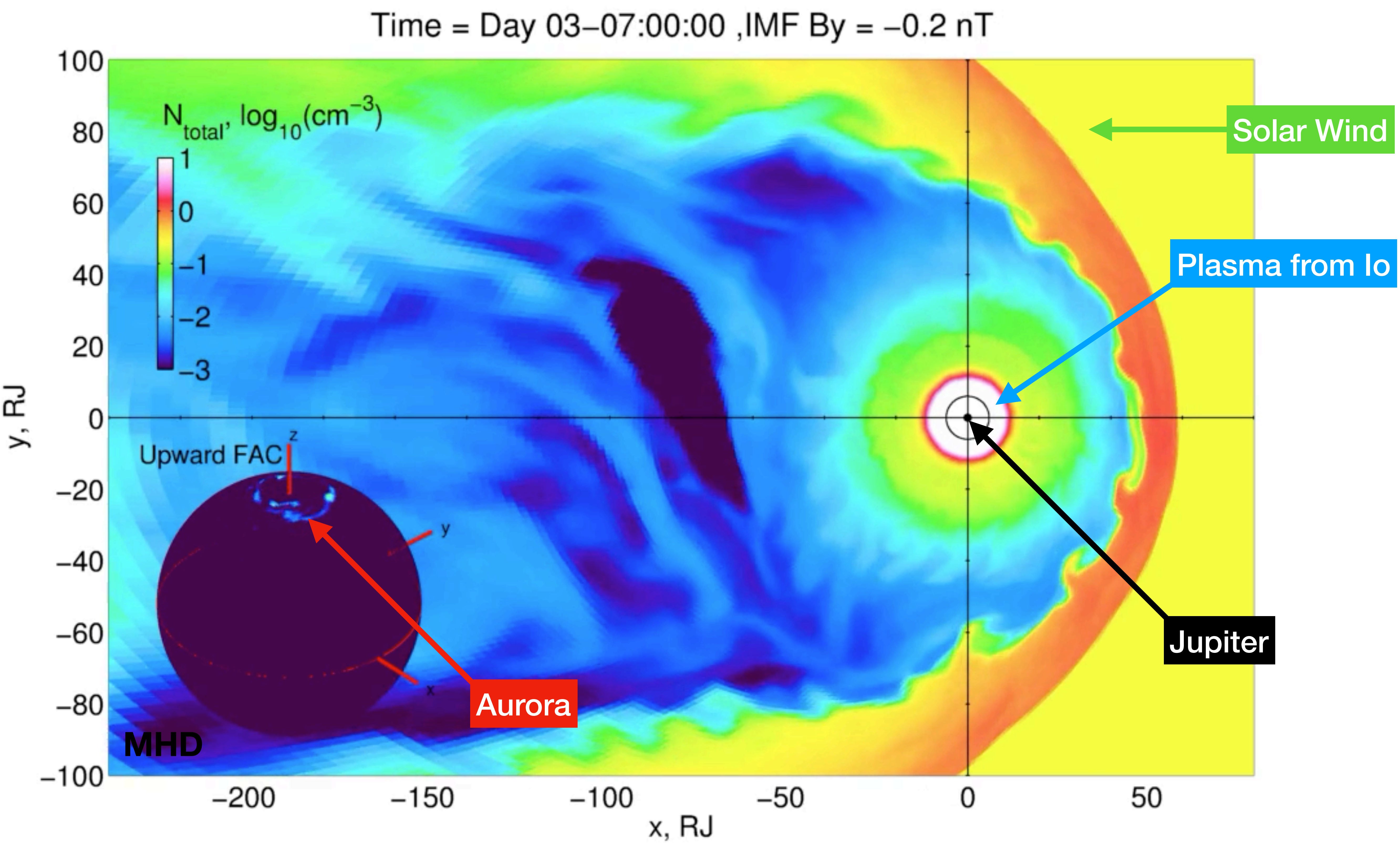
Domain: 50-700 km altitude

Density: 10^{-14} - 10^{-5} kg/cm^{3}

Temperature: 10 - 10^{3} K

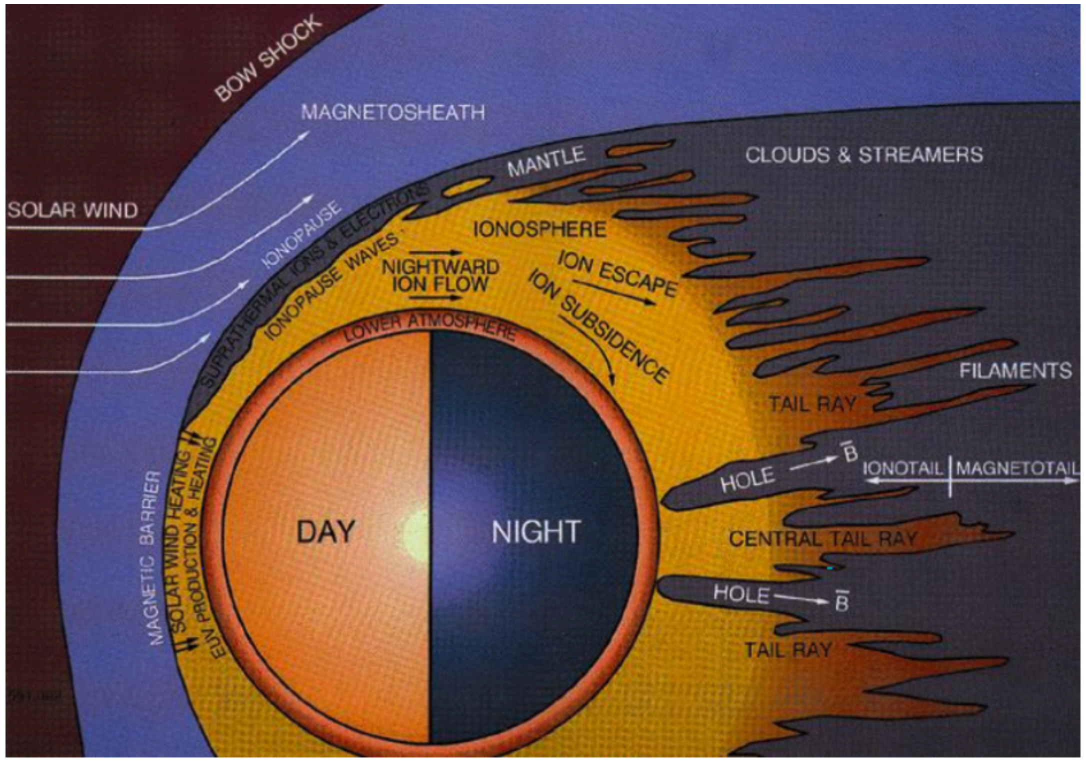
Speed: 0-100 m/s

Giant Magnetospheres (Jupiter)



MHD + rotation

Unmagnetized Magnetosphere (Venus)



<u>Venus Ionosphere-Magnetosphere</u>

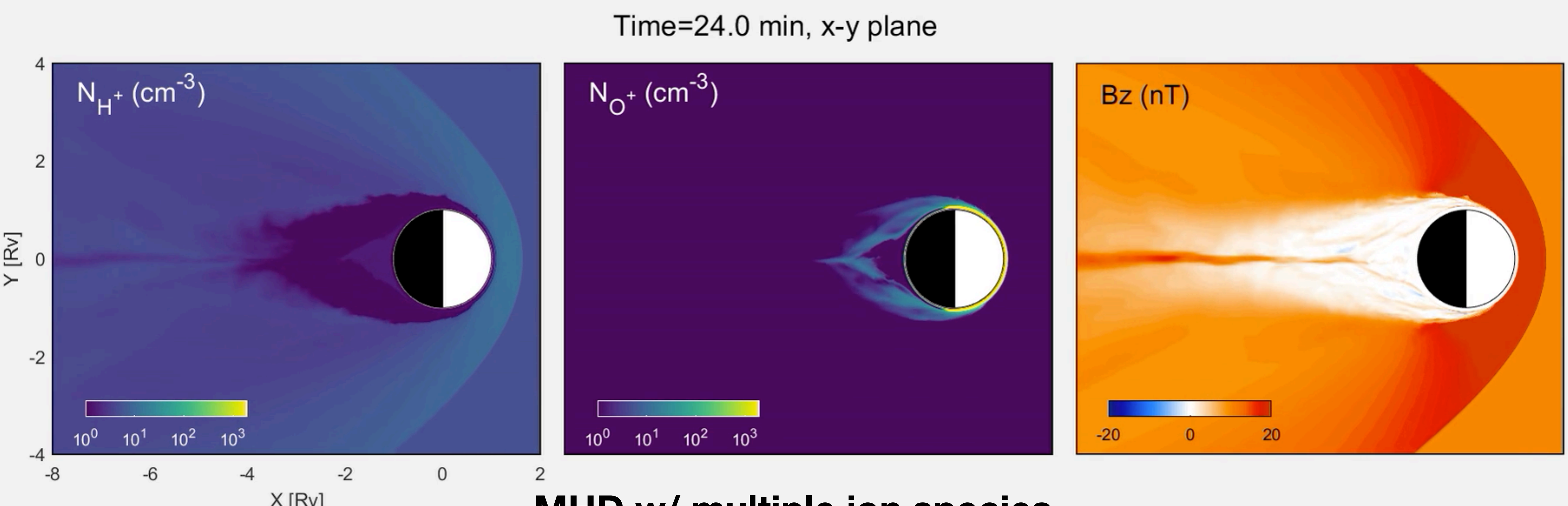
Domain: 90 km altitude to several Rv

Density: 10^{-2} - 10^{8} particle/cm^{3}

Plasma Species: H^{+}, O^{+}, O_2^{+}, CO_2^{+}

Temperature: 1 - 10^{5} K

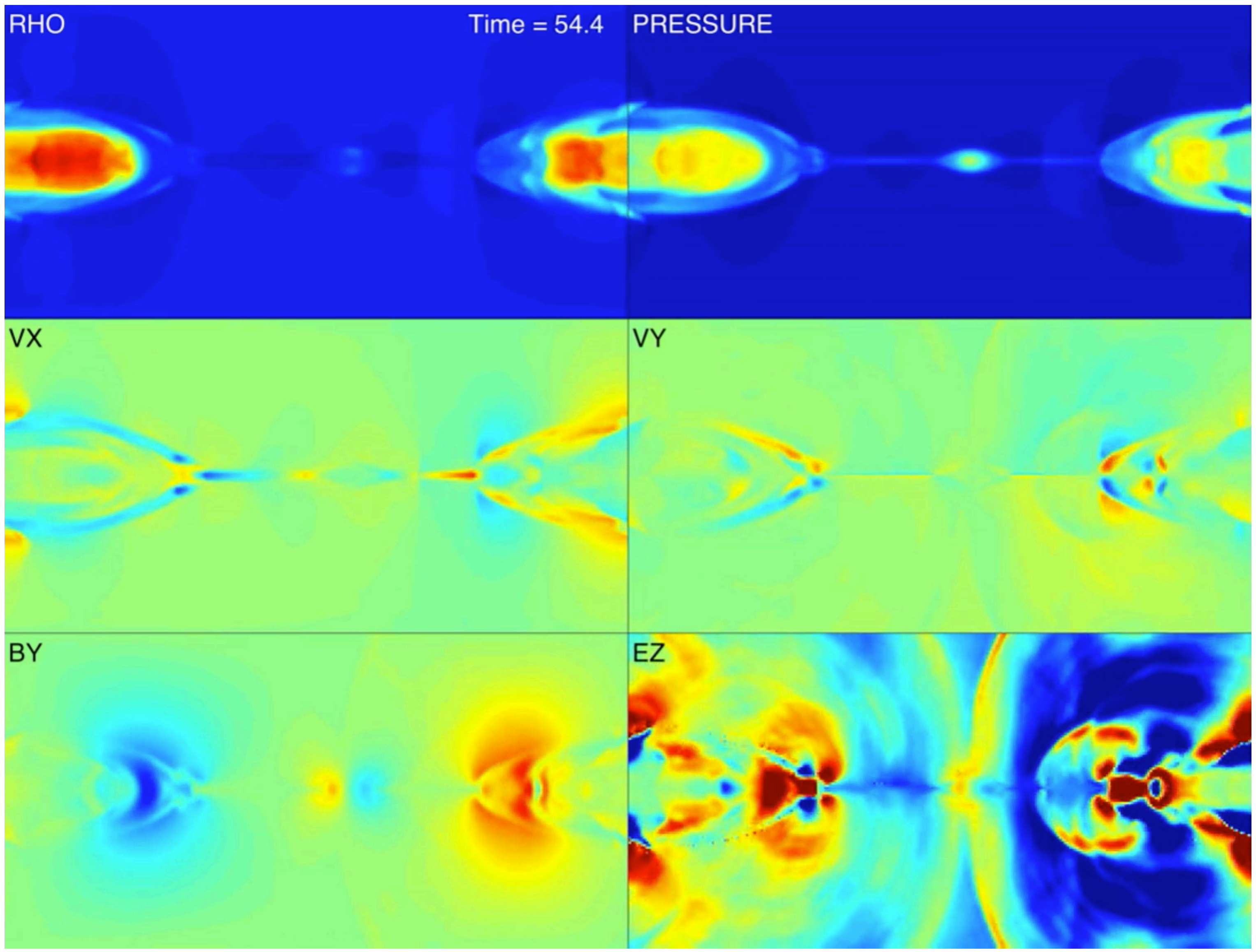
Speed: 0-500 m/s



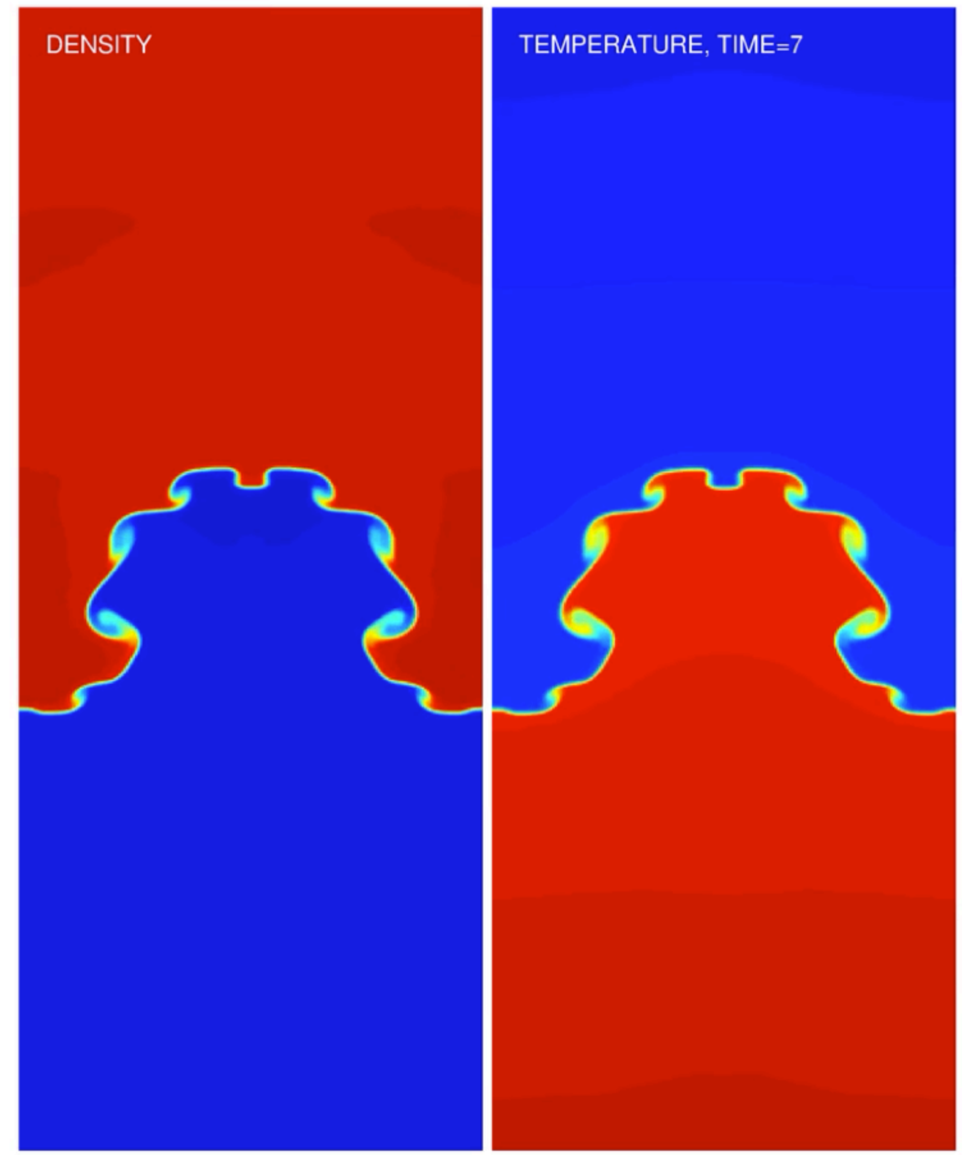
MHD w/ multiple ion species

Basic Plasma Physics

Magnetic Reconnection

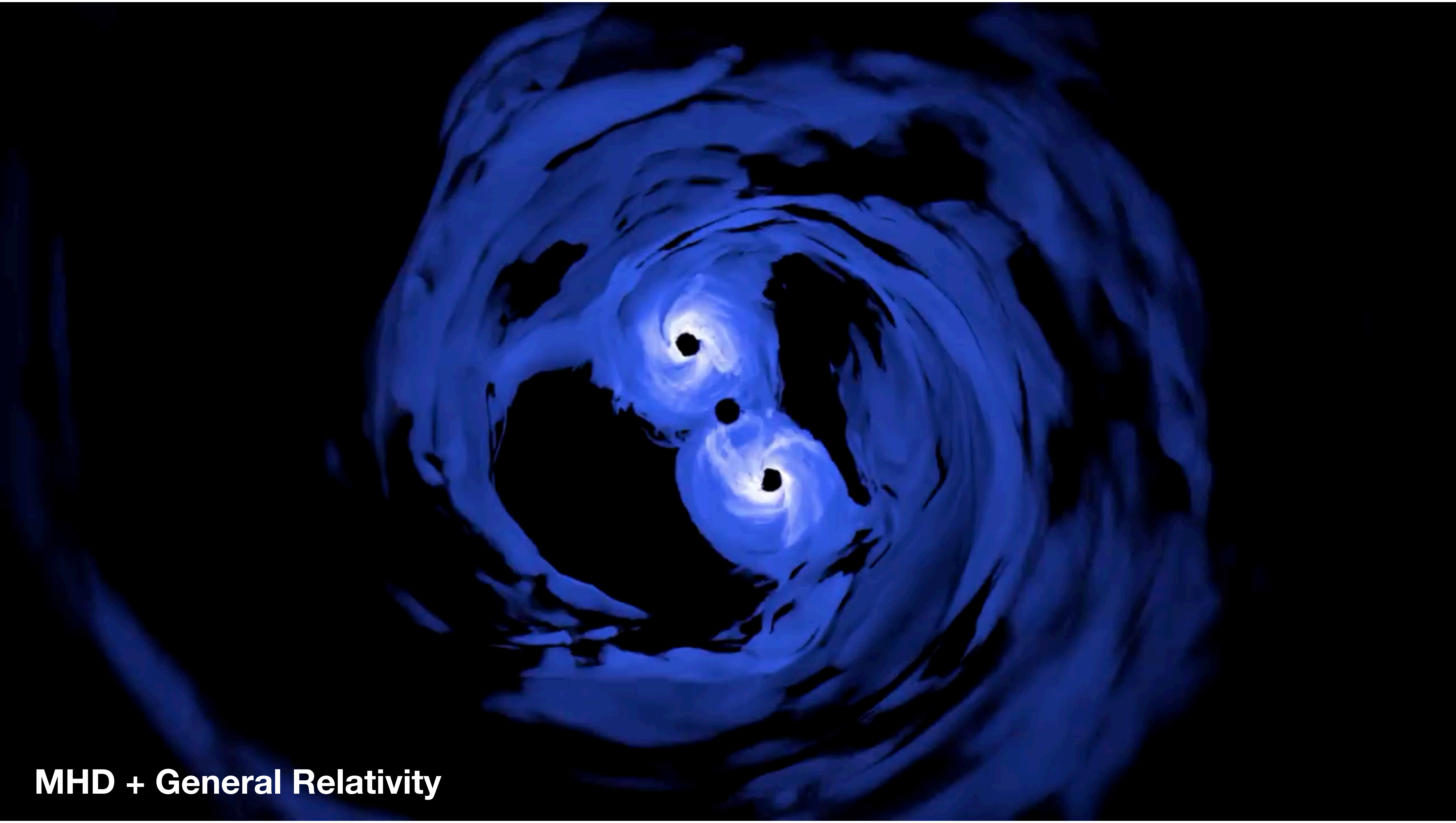


Rayleigh-Taylor Instability



Fundamental plasma physics problems - depending on the physics

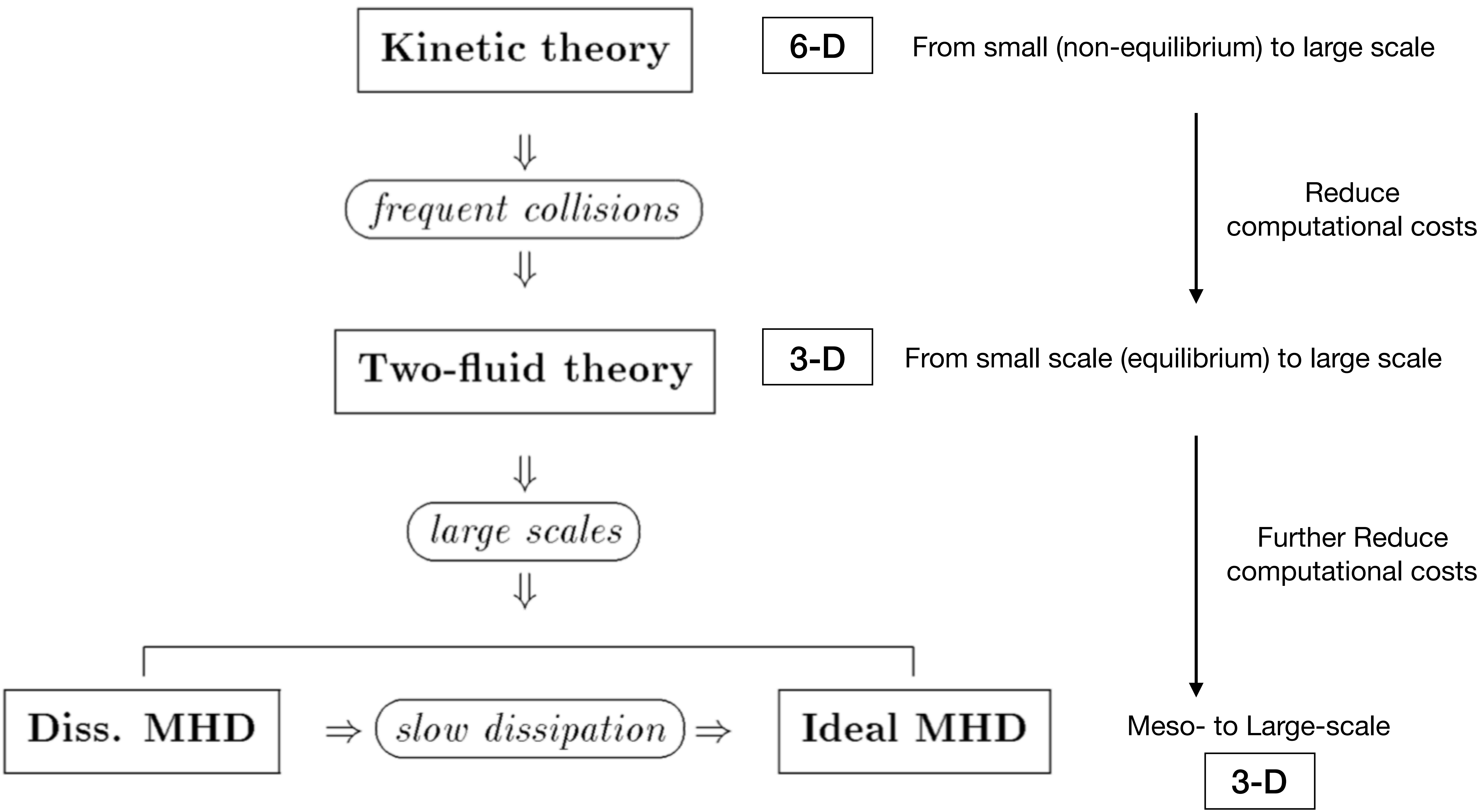
Binary Black Holes (Astrophysics)



Movie courteous: S. Noble

Descriptions of Space Plasmas

General picture



Descriptions of Space Plasmas

Fluid equations

Fluid Dynamics

Mass conservation: $$\frac{\partial}{\partial t}(n_\alpha m_\alpha) + \nabla \cdot (n_\alpha \mathbf{u}_\alpha) = S_\alpha$$

Momentum conservation: $$\frac{\partial}{\partial t}(n_\alpha \mathbf{u}_\alpha) + \nabla \cdot (n_\alpha \mathbf{u}_\alpha \mathbf{u}_\alpha + \mathbf{P}_\alpha) - \frac{n_\alpha q_\alpha}{m_\alpha}(\mathbf{E} + \mathbf{u}_\alpha \times \mathbf{B}) = \mathbf{R}_\alpha$$

Energy conservation: $$\frac{\partial \mathscr{E}_\alpha}{\partial t} + \nabla \cdot \mathbf{u}_\alpha \left(\mathscr{E}_\alpha + p_\alpha\right) - \mathbf{u}_\alpha \cdot \mathbf{J} \times \mathbf{B} = 0$$

Electrodynamics

Faraday's law: $$\frac{\partial \mathbf{B}}{\partial t} = -\nabla \times \mathbf{E}$$

Ampere's law: $$\mathbf{J} = \nabla \times \mathbf{B}$$

Ohm's law: $$\mathbf{E} + \mathbf{u} \times \mathbf{B} = 0$$

This is known as the Magentohydrodynamics (MHD)

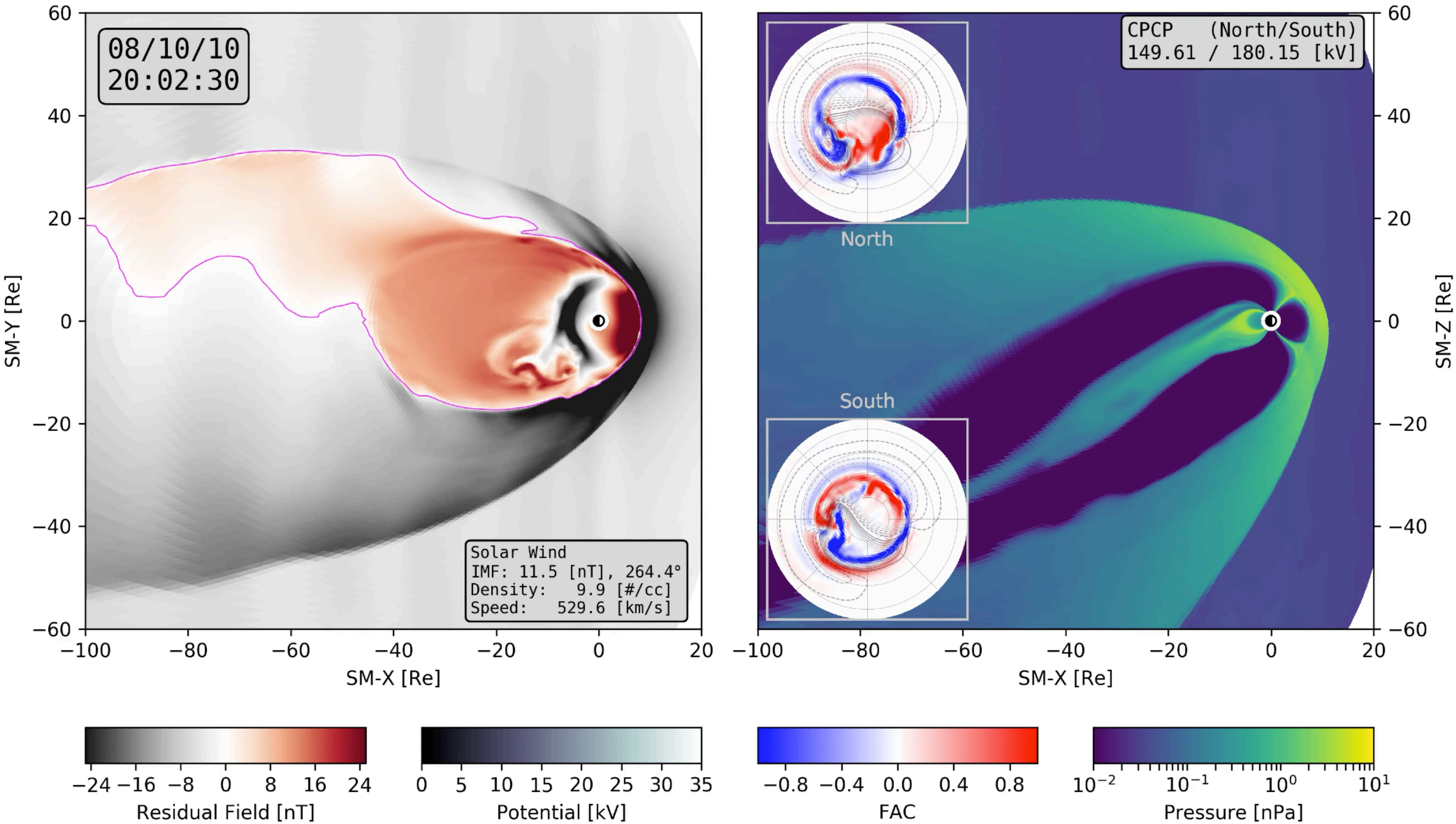
Conserved variables:

$$\rho, \rho\mathbf{u}, \mathscr{E}, \mathbf{B}$$

$$\mathscr{E} = \frac{1}{2}\rho u^2 + \frac{p}{\gamma - 1}$$ Plasma energy

Application of the Fluid equations

The Solar Wind - Magnetosphere Interactions



Descriptions of Space Plasmas

Kinetic equations

Vast majority of plasma physics in contained in the Vlasov-Maxwell equations that describe self-consistent evolution of ***distribution function*** f(**x**, **v**, t) and electromagnetic fields:

0 -> Vlasov Equation

$$\frac{\partial f_s}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f_s) + \nabla_{\mathbf{v}} \cdot (\mathbf{F}_s f_s) = \cancel{\left(\frac{\delta f_s}{\partial t}\right)_c}$$

Where Fs is the force

$$\mathbf{F}_s = q_s / m_s (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Boltzmann Equation

Together with the Maxwell's equation

$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} + \nabla \times \mathbf{E} = 0$$

$$\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_0 \sum_s q_s \int_{-\infty}^{+\infty} v f_s d^3 v$$

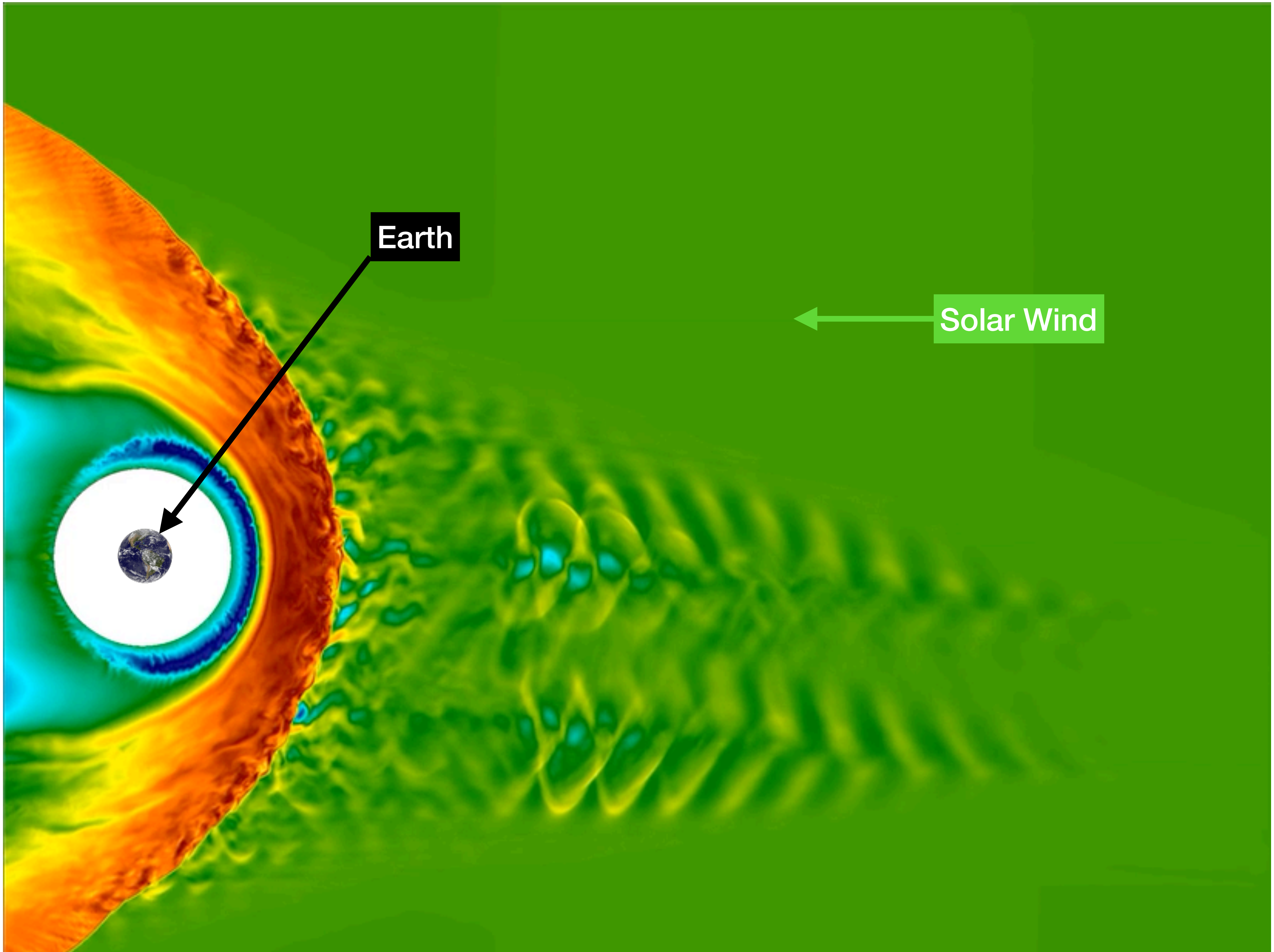
Maxwell's Equations

Highly nonlinear: fields tell particles how to move. Particle motion generates fields. This is a very difficult system of equations to solve! Theoretical and computational plasma physics consists of making approximations and solving these equations in specific situations.

Extremely expensive to solve numerically!

$f_s(\mathbf{x}, \mathbf{v}, t)$: 6-D + time

Application of the Kinetic equations



Why is solving the Vlasov equation so hard?

Despite being the fundamental equation in plasma physics the Boltzmann-Maxwell equations remain highly challenging to solve.

- **Highly nonlinear** with the coupling between fields and particles via currents and Lorentz force. Collisions can further complicate things due to long-range forces in a plasma; dominated by small-angle collisions

- **High dimensionality** and multiple species with large mass ratios: 6D phase-space, me/mp = 1/1836 and possibly dozens of species

- **Enormous scales** in the system: light speed and electron plasma oscillations; cyclotron motion of electrons and ions; fluid-like evolution on intermediate scales; resistive slow evolution of near-equilibrium states; transport scale evolution in tokamak discharges. 14 orders of magnitude of physics in these equations!

Plus it's also unnecessary to have all these information for a specific problem!

A Special Descriptions of Space Plasmas

Hybrid equations

Ions

Particles/distribution function

$$\frac{\partial f_s}{\partial t} + \nabla_{\mathbf{x}} \cdot (\mathbf{v} f_s) + \nabla_{\mathbf{v}} \cdot (\mathbf{F}_s f_s) = \left(\frac{\delta f_s}{\partial t}\right)_c$$

$$\mathbf{F}_s = q_s / m_s (\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

Maxwell's Equations

$$\frac{\partial \mathbf{B}}{\partial \mathbf{t}} + \nabla \times \mathbf{E} = 0$$

$$\mu_0 \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} - \nabla \times \mathbf{B} = -\mu_0 \sum_s q_s \int_{-\infty}^{+\infty} v f_s d^3 v$$

Electrons

Treated as Fluid

$$\frac{\partial}{\partial t}(n_e) + \nabla \cdot (n_e \mathbf{u}_e) = S_e$$

$$\frac{\partial}{\partial t}(n_e \mathbf{u}_e) + \nabla \cdot (n_e \mathbf{u}_e \mathbf{u}_e + \mathbf{P}_e) + \frac{n_e e}{m_\alpha}(\mathbf{E} + \mathbf{u_e} \times \mathbf{B}) = \mathbf{R}_e$$

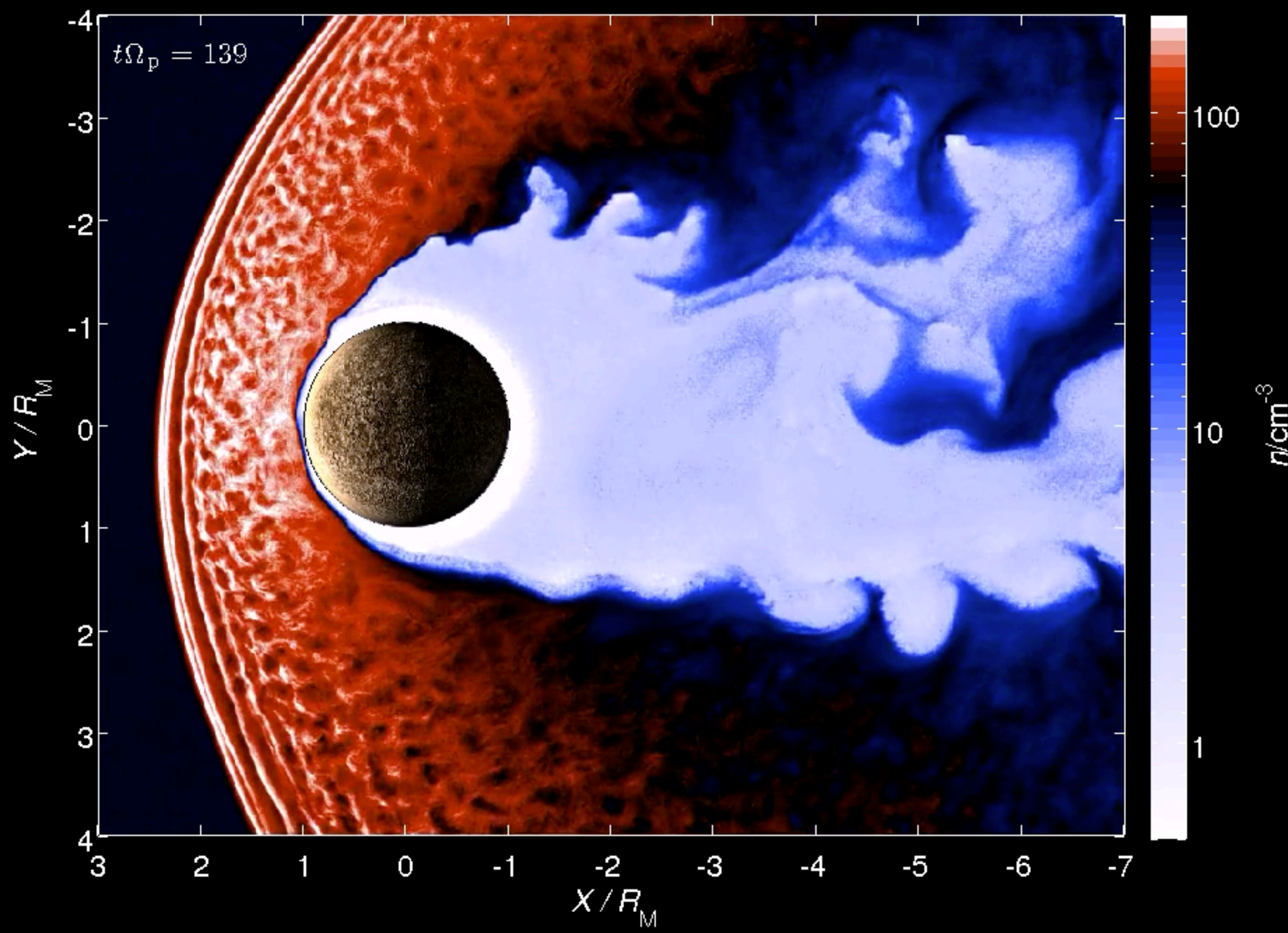
$$\frac{\partial p_e}{\partial t} + \mathbf{u}_e \cdot \nabla p_e + \gamma p_e \nabla \cdot \mathbf{u}_e = Q_e$$

Electrons and Ions are coupled through electrodynamics and collisions

Relatively expensive to solve numerically

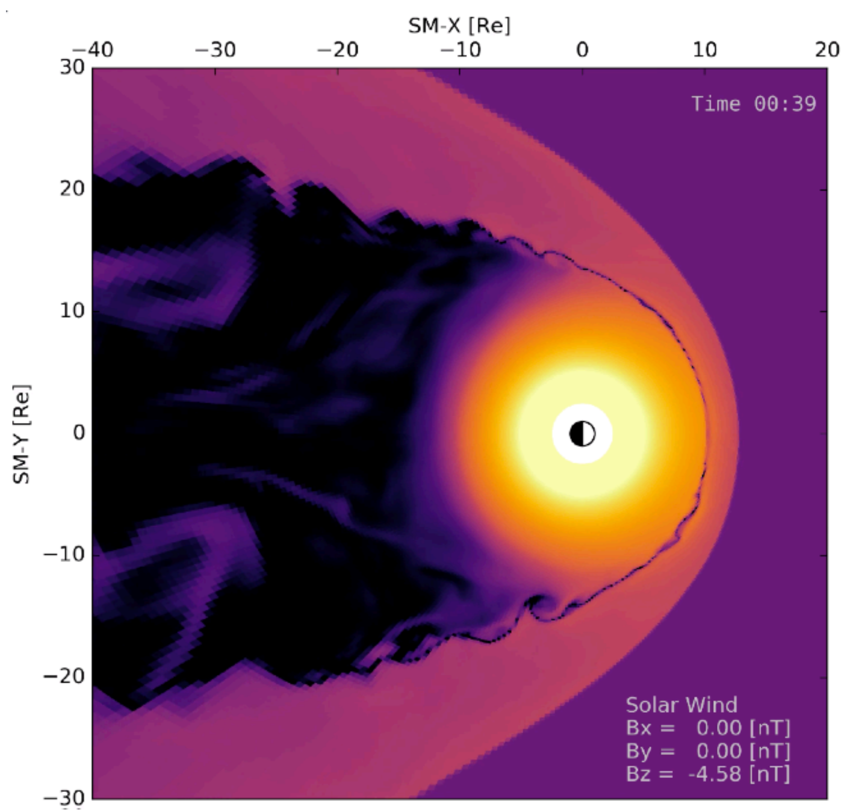
Application of the Hybrid equations

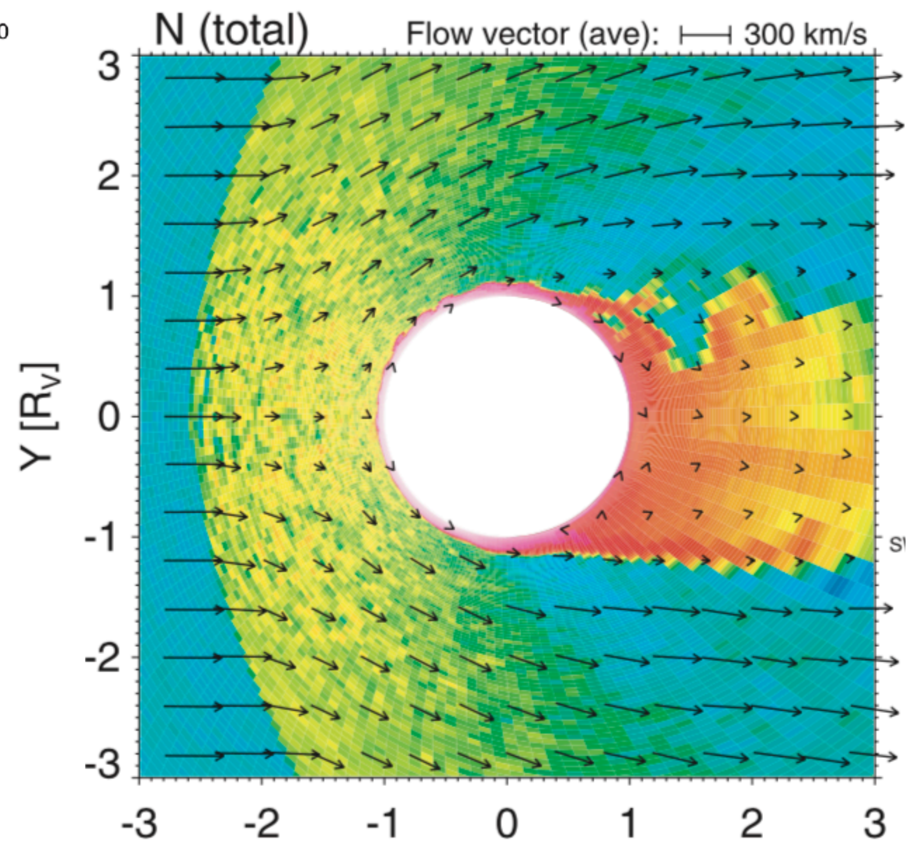
Mercury Magnetosphere



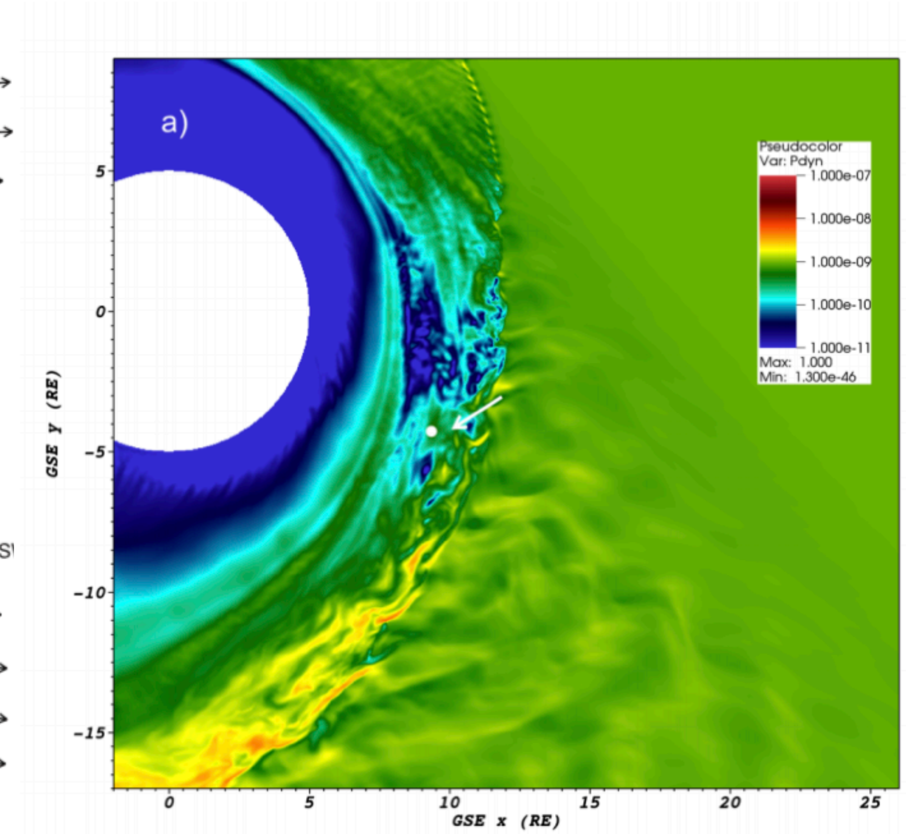
Comparing the Three Methods

Magnetohydrodynamics	Hybrid	Kinetic/Particle





Tareda et al. [2002]

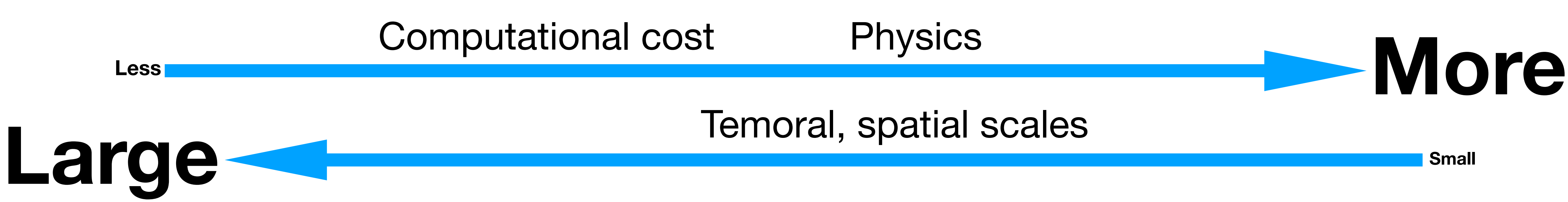


Palmroth et al. [2018]

- **Low computational cost**
- **Fluid physics, large scale**

- **High computation cost**
- **Ion physics, ion-scale**

- **Extremely high cost**
- **Particle physics, electron-scale**



This Course

What to expect and what you will learn

Goal:

- Hands-on experience with simulation codes, mostly 1-D plus other computational techniques
- Reasonable amount of programing, using Matlab

You can learn Matlab from: Codes, Help file and Other online documents

Codes:

- 1-D Advection code
- 1-D/2-D MHD code
- 1-D electrostatic particle code
- 1-D Hybrid code

Grades:

- 40% HW + 20% Midterm (Project) + 40% Final (Project)

Office hours:

- Open (BZ) + arrangement with TA

Honor code:

- Copying codes and figures not allowed
- Discussions of HW/projects allowed

Course Website: https://github.com/tiegcm/CPD