Statistics 510, Homework 2

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Directions: Do all the problems. The total grade is 40 points, with all problems being 5 points each. You may work with others in the class or use any sources you like to study these problems, but when you go to write the solutions you should be able to write it using only the textbook and course notes. If you obtain facts, advice, or inspiration from any source outside the textbook or course notes, you must indicate in your solution what the source is. In all cases, you must make it clear to the reader what logic/work was needed to reach your solution. Solutions must be posted to Canvas – any legible version, handwritten or typed, is acceptable.

#1

A box contains three coins with a head on each side, four coins with a tail on each side, and two fair coins. If one of these nine coins is selected (uniformly) at random and tossed once, what is the probability that a head will be obtained?

#2

A box contains three cards. One card is red on both sides, one is green on both sides, and one is red on one side and green on the other. One card is selected from the box at random, and one side is observed. If this side is green, what is the probability that the other side of the card is green?

#3

A box contains five coins and for each coin there is probability $p_i, i \in \{1,2,3,4,5\}$ that a head is obtained when the ith coin is tossed. Suppose $p_1 = 0$, $p_2 = 1/4$, $p_3 = 1/2$, $p_4 = 3/4$, and $p_5 = 1$.

(a) Suppose that one coin is selected at random from the box and when it is tossed once, a head is obtained. What is the conditional probability that the ith coin was selected?

(b) If the same coin is tossed again, what is the probability of obtaining a second head?

#4

Suppose an urn starts with one red ball and one white ball. At each stage of this process, you select one ball from the urn randomly. If you draw a red ball, then put the red ball back and add another red ball to the urn. Similarly, if you draw a white ball, put the white ball back and add another white ball to the urn. So after the nth draw, there are $n+2$ balls in the urn. Let R_n be the number of the $n+2$ that are red. Write the distribution of R_n.

#5

Recall X is a geometric random variable with parameter $p \in (0,1)$ if its pmf is

$$f_X(x) = p(1-p)^{x-1}, x \in \{1,2,3,...\}.$$

(a) Show that $\mathbf{P}(X = n+k | X > n) = \mathbf{P}(X = k)$ for any integers $k, n \geq 1$.

(b) Now, suppose that Y is a random variable satisfying

$$\mathbf{P}(Y = n+k | Y > n) = \mathbf{P}(Y = k)$$

for any integers $k, n \geq 1$. Show that Y has *some* geometric pmf. That is, find a value p such that $f_Y(y) = p(1-p)^{y-1}$ for all $y \in \{1,2,3,...\}$.

#6

An insurance agent sells a policy with a deductible of $d = 100$ and a cap of $c = 5000$. This means that when the policyholder files a claim, the policyholder must pay the first 100. After the first 100 is paid, the insurance company pays the rest of the claim up to a maximum payment of 5000. Any excess must be paid by the policyholder. Suppose that the dollar amount X of a claim has a continuous distribution with pdf $f(x) = 1/(1+x)^2$ for $x > 0$. Let Y be the amount that the insurance company will pay on the claim. Find the cdf of Y.

#7

An electronic device has lifetime denoted by T. The device has value $V = 5$ if it fails before time $t = 3$; otherwise, it has value $V = 2T$. Find the cdf of V, if T has pdf with $f_T(t) = \frac{2}{3}e^{-2t/3}$ for all $t > 0$ and 0 otherwise.

#8

Let X be a random variable with cdf $F_X(x)$ and $a < b$. Let

$$Y = \begin{cases} a & X < a \\ X & a \leq X \leq b \\ b & X > b \end{cases} \quad \text{and} \quad Z = \begin{cases} X & |X| \leq b \\ 0 & |X| > b \end{cases}.$$

Write the cdfs of Y and Z.