Statistics 510, Homework 5

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Directions: Do all the problems. The total grade is 40 points, with all problems being worth 5 points each. You may work with others in the class or use any sources you like to study these problems, but when you go to write the solutions you should be able to write it using only the textbook and course notes. If you obtain facts, advice, or inspiration from any source outside the textbook or course notes, you must indicate in your solution what the source is. In all cases, you must make it clear to the reader what logic/work was needed to reach your solution. Solutions must be posted to Canvas – any legible version, handwritten or typed, is acceptable.

#1

A random point (X, Y) is distributed uniformly on the square $[-1, 1]^2$ where $(x, y) \in [-1, 1]^2$ means $x \in [-1, 1]$ and $y \in [-1, 1]$. That is, the joint pdf is $f_{X,Y}(x, y) = 1/4$ on the square. Determine the probabilities of the following events.

(a) $X^2 + Y^2 < 1$
(b) $2X - Y > 0$
(c) $|X + Y| < 2$
(d) $|X - Y| > 1/2$
(e) $\max(X, Y) > 1/2$

#2

Let X and Y be random variables for which the joint pdf is

$$f_{X,Y}(x,y) = 2(x+y), 0 \leq x \leq y \leq 1.$$

Find the pdf of $X+Y$.

#3

Let X and Y be independent random variables with distribution $Exp(\lambda)$ for some $\lambda > 0$. Find the pdfs of $X-Y$ and $X+Y$ and prove your answer.

#4

Let $X \sim Gamma(\alpha_1, \beta)$ and $Y \sim Gamma(\alpha_2, \beta)$ be independent. Let $U = X/(X+Y)$ and $V = X+Y$. (a) Find the cdfs of U and V. (b) Show that U and V are independent random variables.

#5

Let X_1 and Y be independent random variables with distribution $Exp(1)$. Let $D = X - Y$ and $Q = X/Y$.

(a) Write the joint pdf of X and D and the conditional pdf of X given $D = 0$.
(b) Write the joint pdf of X and Q and the conditional pdf of X given $Q = 1$.

#6

Suppose the distribution of Y conditioned on $X = x$ is a normal distribution with mean x and variance x^2 and that the marginal pdf of X is uniform on $(0,1)$.

(a) Compute $\mathbf{E}[Y], \mathrm{Var}[Y]$, and $Cov(X,Y)$.
(b) Prove that Y/X and X are independent.

#7

Suppose that X and Y are random variables such that $\mathbf{E}[Y|X] = 10 - X$ and $\mathbf{E}[X|Y] = 7 - \frac{Y}{4}$. Find the correlation coefficient between X and Y.

#8

Let X and Y be random variables such that X has the standard normal distribution and the conditional distribution of Y given X is the normal distribution with mean $2X - 3$ and variance 12. Find the marginal pdf of Y and the correlation coefficient between X and Y.