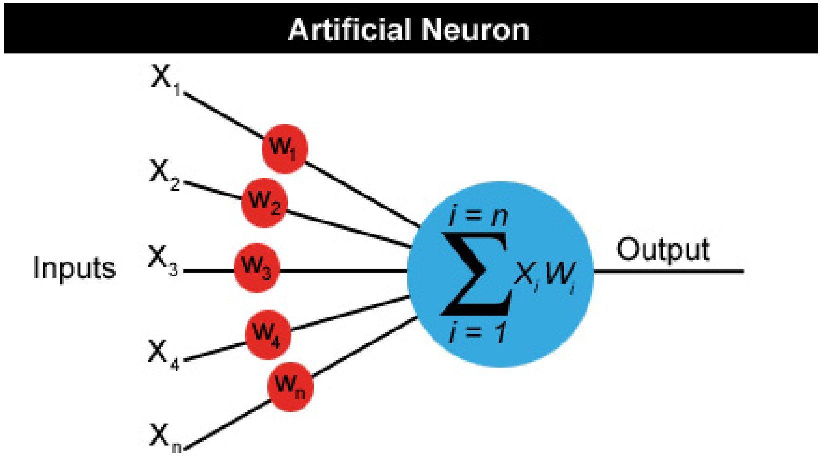
Lecture 04
Multi-Layer Perceptron (MLP)

Neural networks

- A neuron can have any number of inputs from one to n, where n is the total number of inputs.
- The inputs may be represented therefore as x_1, x_2, $x_3 \ldots x_n$.
- And the corresponding weights for the inputs as w_1, w_2, $w_3 \ldots w_n$.
- Output $a = x_1w_1 + x_2w_2 + x_3w_3 \ldots + x_nw_n$

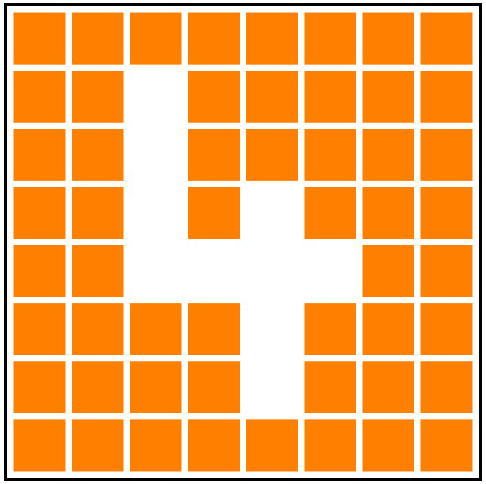


Feedforward Networks

- Feedforward network: The neurons in each layer feed their output forward to the next layer until we get the final output from the neural network.
- There can be any number of hidden layers within a feedforward network.
- The number of neurons can be completely arbitrary.

Example

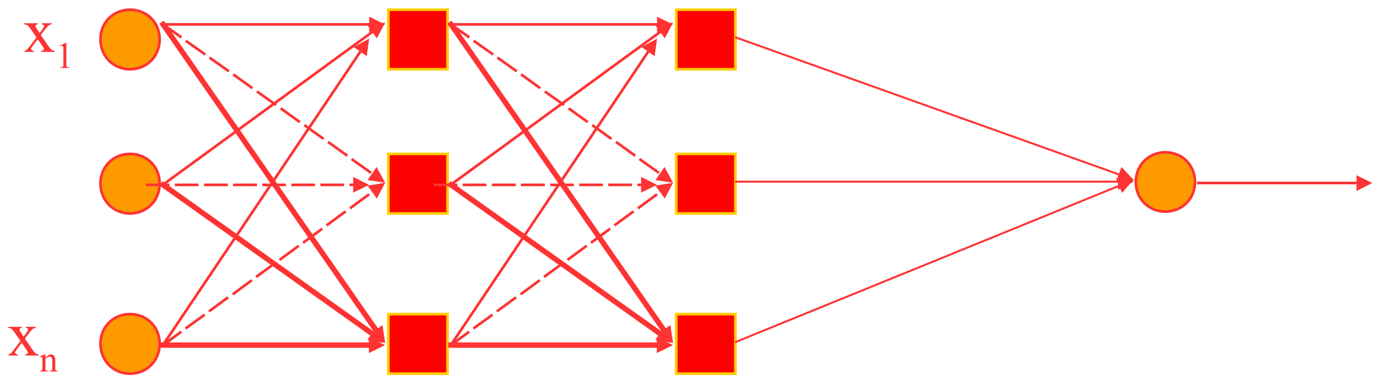
- Let's design a neural network that will detect the number '4'.
- Given a panel made up of a grid of lights which can be either on or off, we want our neural net to let us know whenever it thinks it sees the character '4'.
- The panel is eight cells square and looks like this:
- the neural net will have 64 inputs, each one representing a particular cell in the panel and a hidden layer consisting of a number of neurons (more on this later) all feeding their output into just one neuron in the output layer



Neural Networks by an Example

- Initialize the neural net with random weights
- Feed it a series of inputs which represent the different panel configurations
- For each configuration we check to see what its output is and adjust the weights accordingly so that whenever it sees something looking like a number 4 it outputs a 1 and for everything else it outputs a zero.

Multi-Layer Perceptron (MLP)



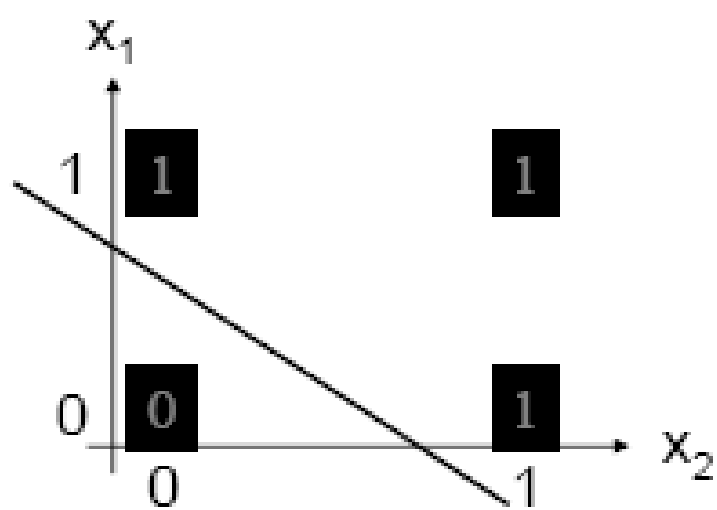
- We will introduce the MLP and the backpropagation algorithm which is used to train it
- MLP used to describe any general feedforward (no recurrent connections) network
- However, we will concentrate on nets with units arranged in layers
- Different books refer to the above as either 4 layer (no. of layers of neurons) or 3 layer (no. of layers of adaptive weights). We will follow the latter convention

Perceptron Learning Theorem

- *Recap*: A perceptron (threshold unit) can *learn* anything that it can *represent* (i.e. anything separable with a

OR function

x_1	x_2	y
0	0	0
0	1	1
1	0	1
1	1	1

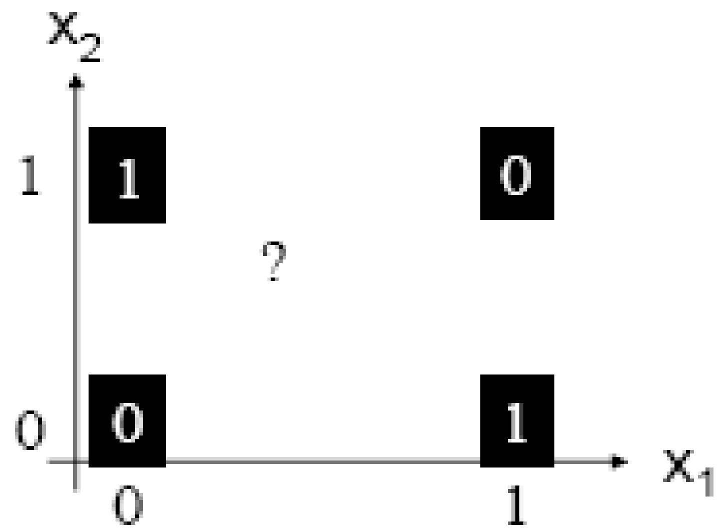


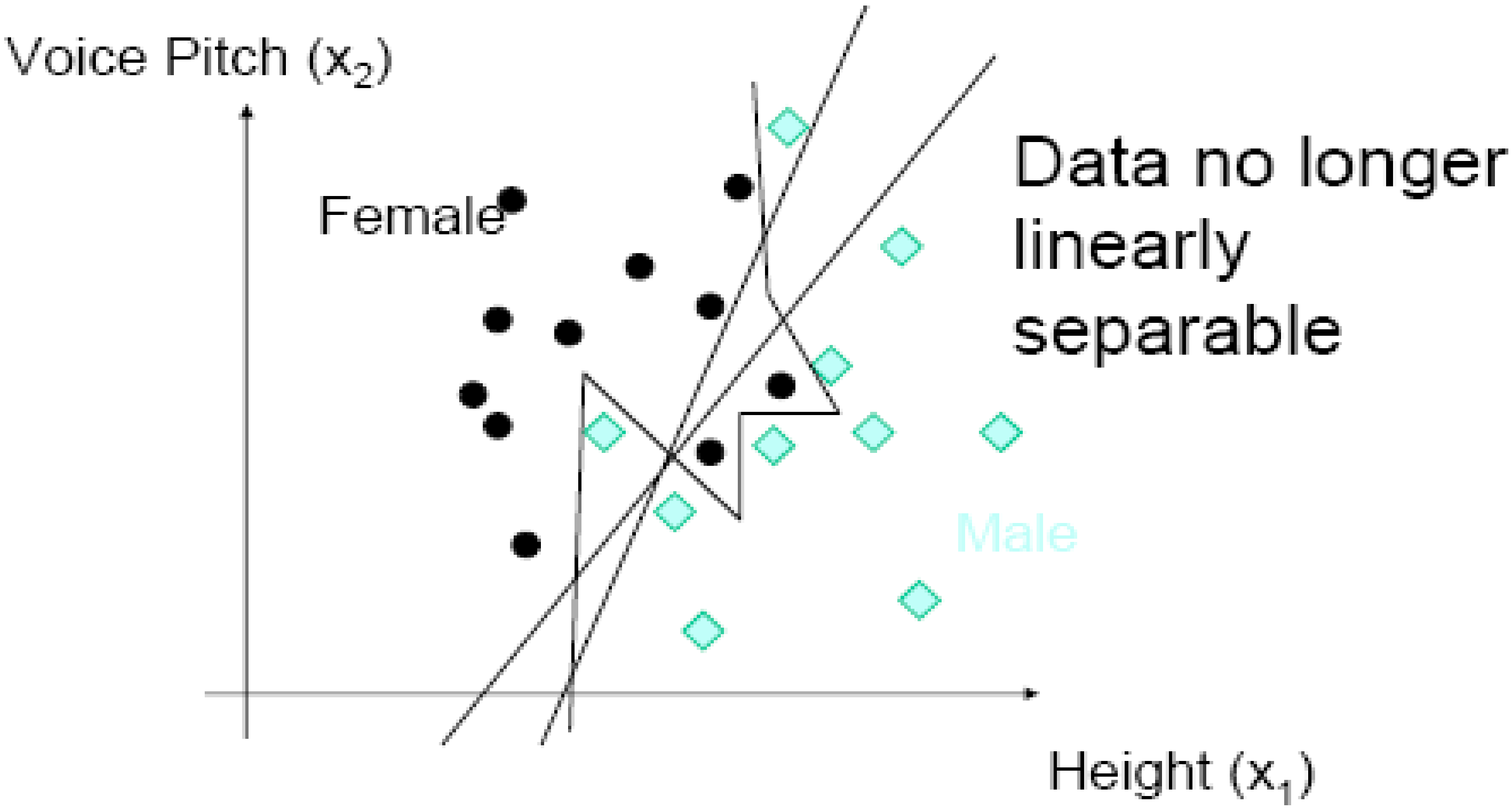
The Exclusive OR problem

A Perceptron cannot represent Exclusive OR since it is not linearly separable.

XOR function

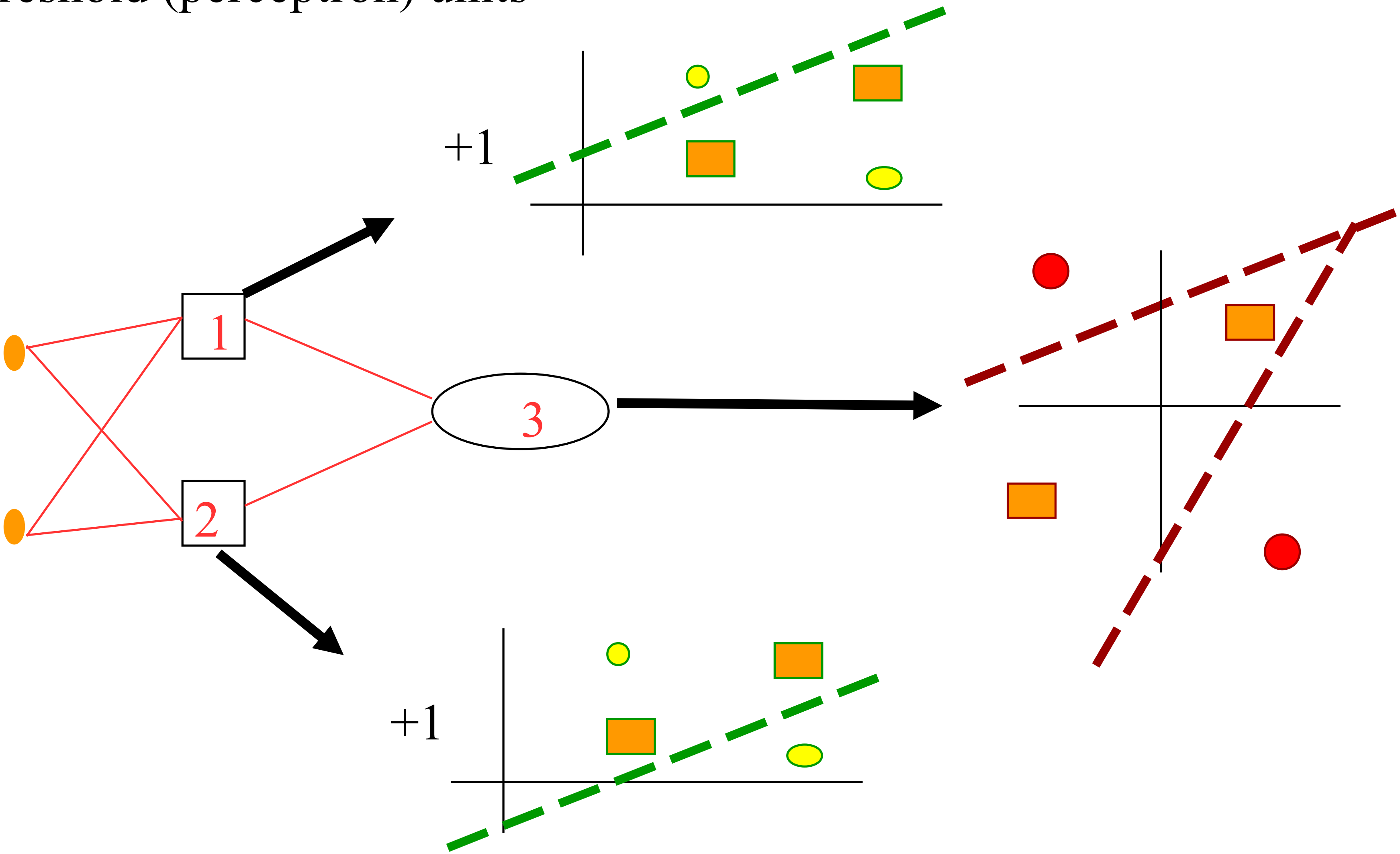
x_1	x_2	**y**
0	0	0
0	1	1
1	0	1
1	1	0



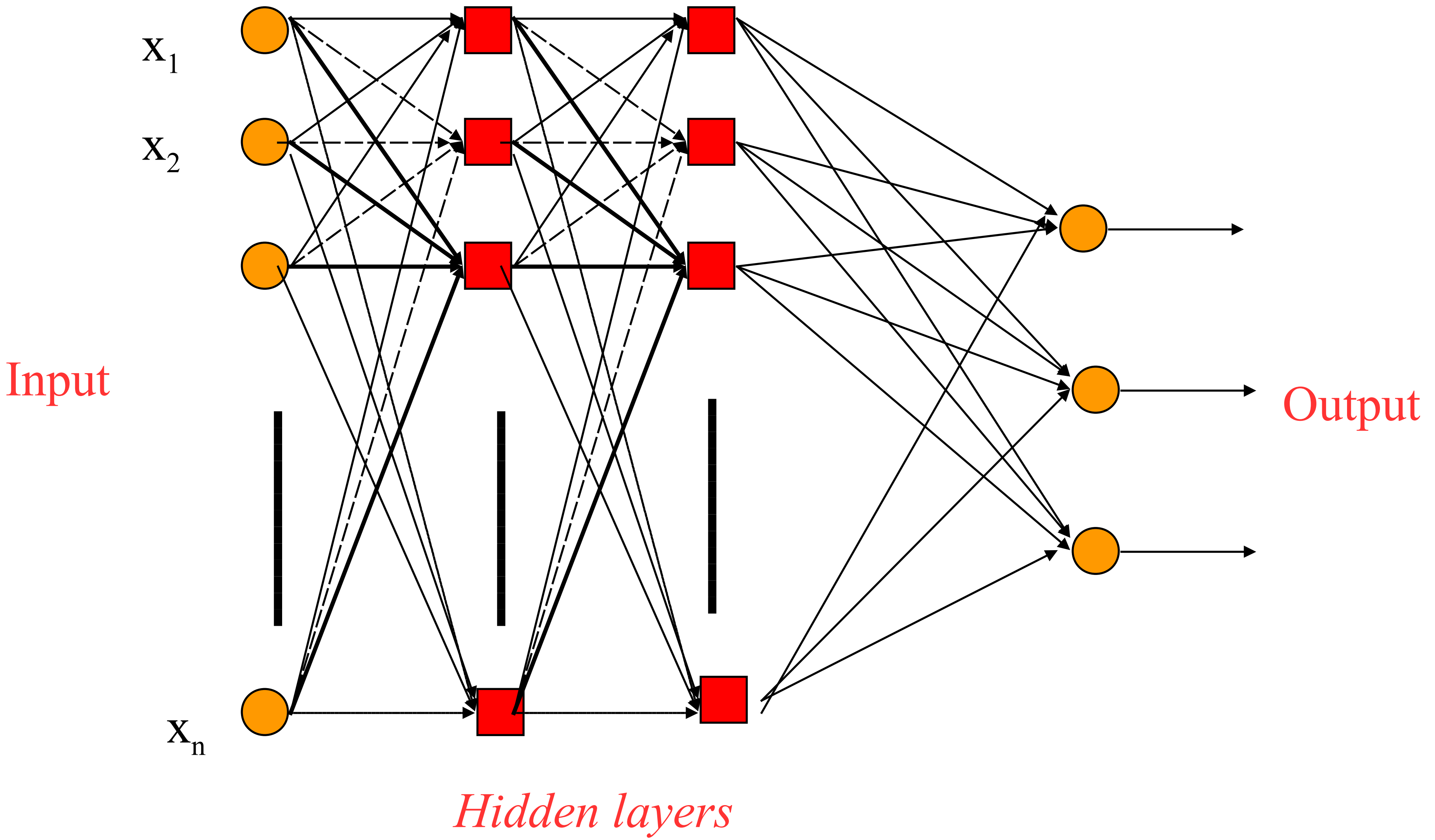


What is a good decision boundary ?

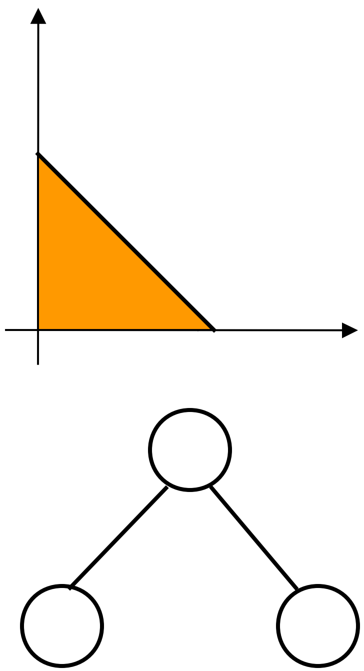
Minsky & Papert (1969) offered solution to XOR problem by combining perceptron unit responses using a second layer of Units. Piecewise linear classification using an MLP with threshold (perceptron) units



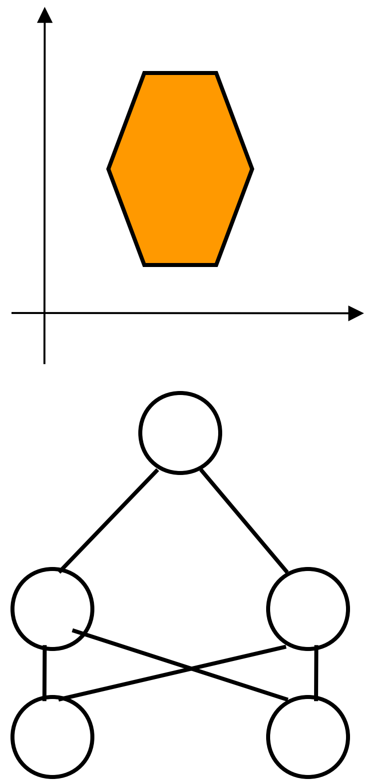
Three-layer networks



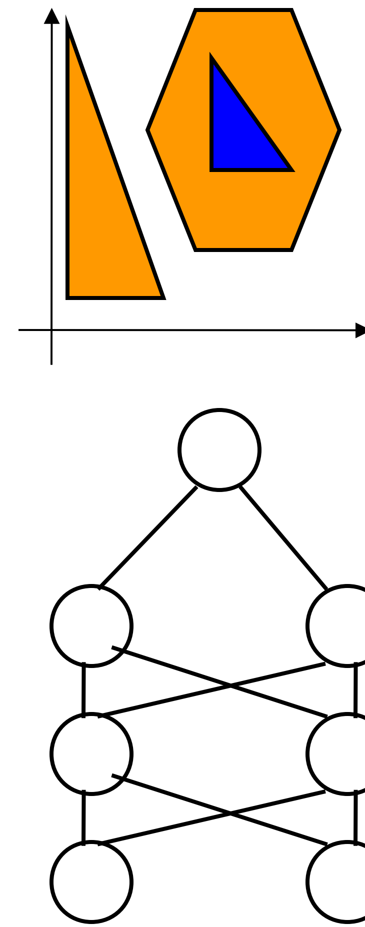
What do each of the layers do?



1st layer draws linear boundaries



2nd layer combines the boundaries



3rd layer can generate arbitrarily complex boundaries

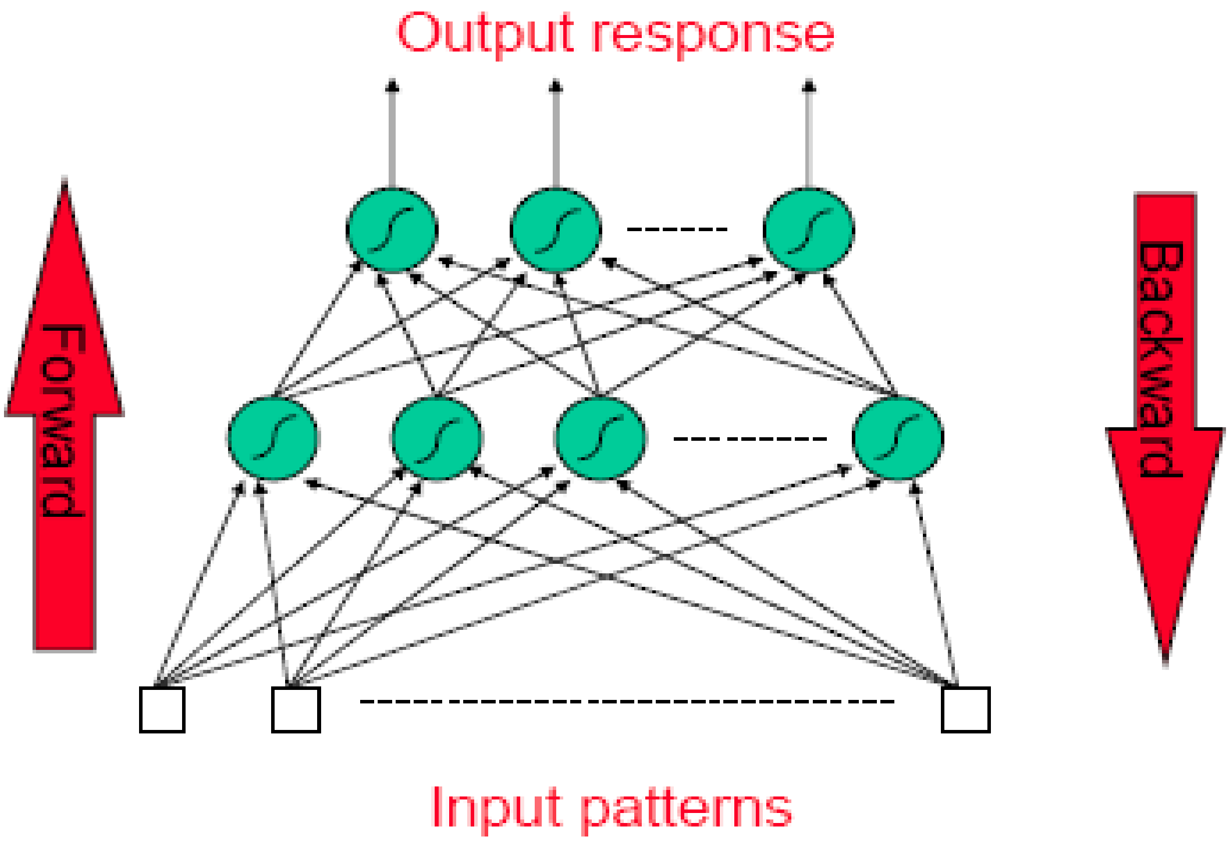
Backpropagation learning algorithm

Backpropagation has two phases:

Forward pass phase: computes 'functional signal', feed forward propagation of input pattern signals through network

Backward pass phase: computes 'error signal', *propagates* the error *backwards* through network starting at output units (where the error is the difference between actual and desired output values)

Forward Activity – Backward Error



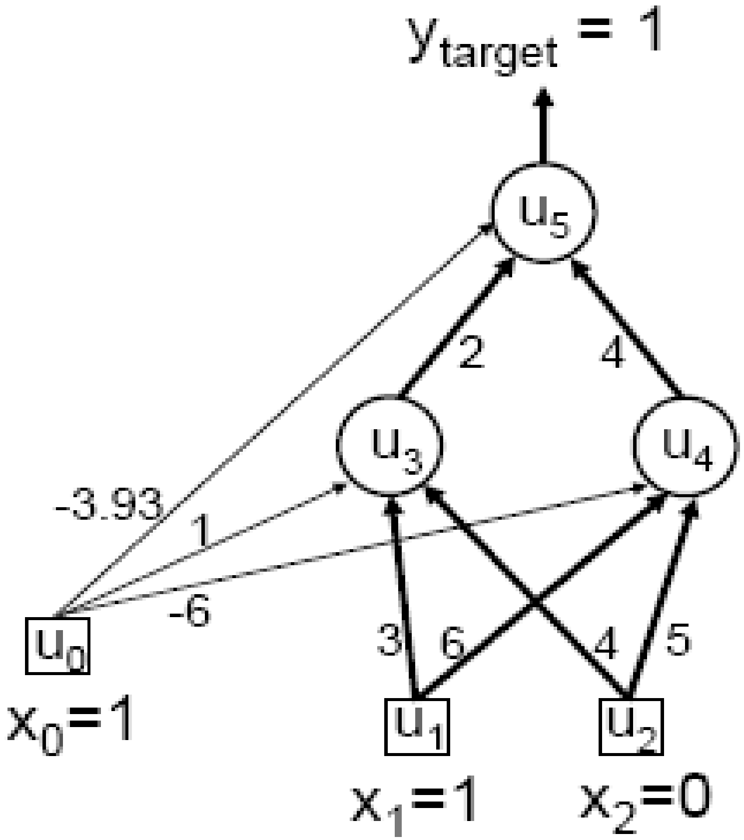
Forward Propagation of Activity

- **Step 1**: Initialise weights at random, choose a learning rate η
- Until network is trained:
- For each training example i.e. input pattern and target output(s):
- **Step 2**: Do forward pass through net (with fixed weights) to produce output(s)
  - i.e., in Forward Direction, layer by layer:
    - Inputs applied
    - Multiplied by weights
    - Summed
    - ‘Squashed’ by sigmoid activation function
    - Output passed to each neuron in next layer
  - Repeat above until network output(s) produced

Step 3. Back-propagation of error

- Compute error (delta or local gradient) for each output unit $\delta\ k$
- Layer-by-layer, compute error (delta or localgradient) for each hidden unit $\delta\ j$ by backpropagating errors
- **Step 4**: Next, update all the weights Δwij by gradient descent, and go back to Step 2
  - The overall MLP learning algorithm, involving forward pass and backpropagation of error (until the network training completion), is known as the Back Propagation (BP) algorithm

MLP/BP Example



Current state:

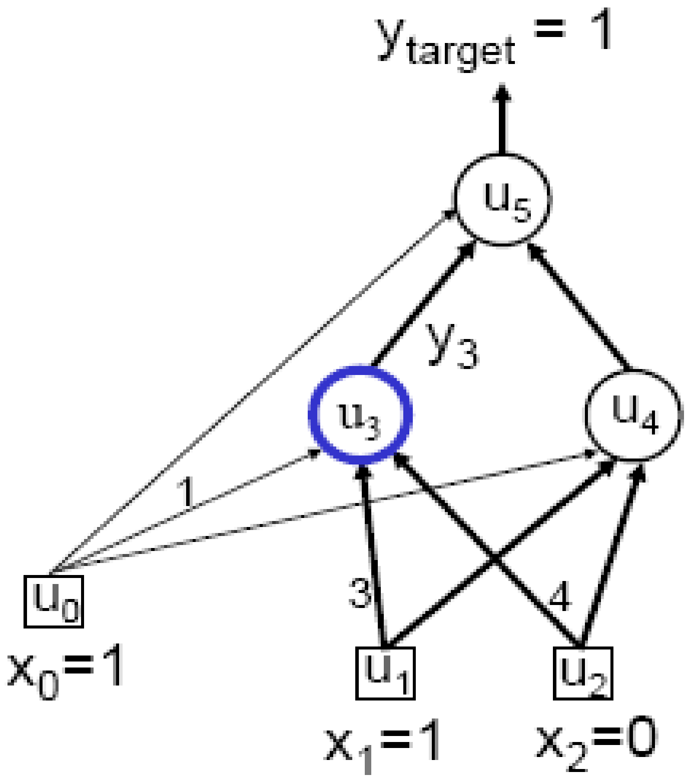
- Weights on arrows e.g. $w_{13} = 3$, $w_{35} = 2$, $w_{24} = 5$
- Bias weights, e.g.

bias for unit 4 (u_4) is w_{04}= -6

Training example (e.g. for logical OR problem):

- Input pattern is x_1=1, x_2=0
- Target output is y_{target}=1

Example: Forward Pass



Output for any neuron/unit j can be calculated from:

$$a_j = \sum_i w_{ij} x_i$$

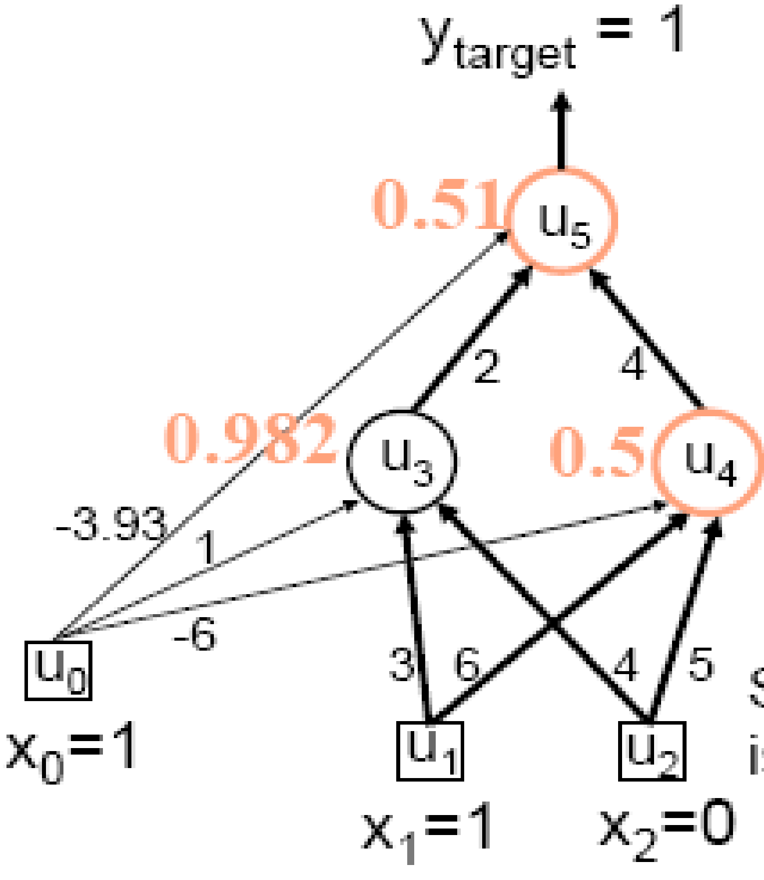
$$y_j = f(a_j) = \frac{1}{1+e^{-a_j}}$$

e.g Calculating output for Neuron/unit 3 in hidden layer:

$$a_3 = 1*1 + 3*1 + 4*0 = 4$$

$$y_3 = f(4) = \frac{1}{1+e^{-4}} = 0.982$$

Example: Forward Pass

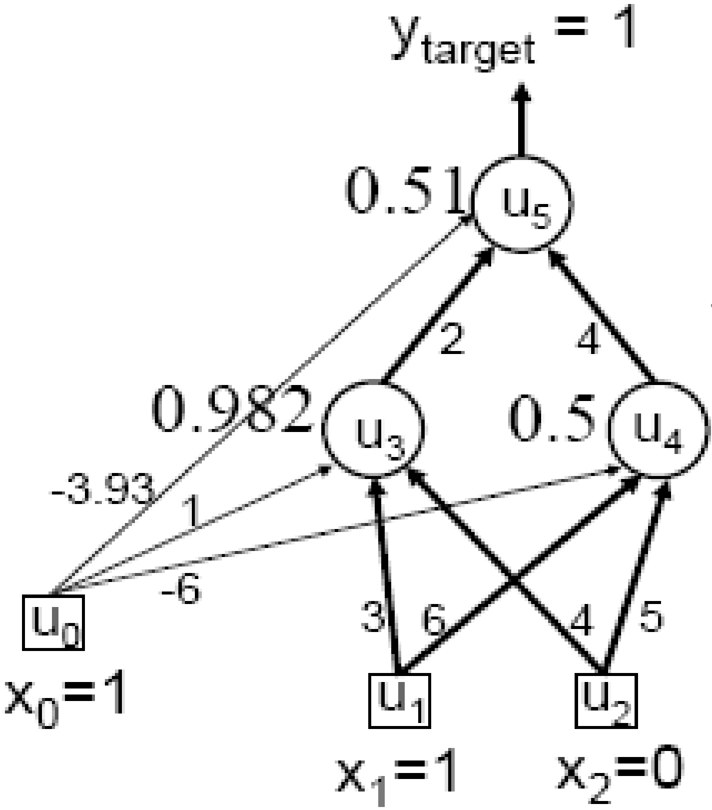


Unit	activation	output
	a_j	y_j
u_3	4.00	0.982
u_4	0.00	0.500
u_5	0.04	**0.510**

(network output)

So the error for this training example is: $(y_{target} - y_5) = (1 - 0.510) = 0.490$

Example: Backward Pass



Now compute delta values starting at the output:

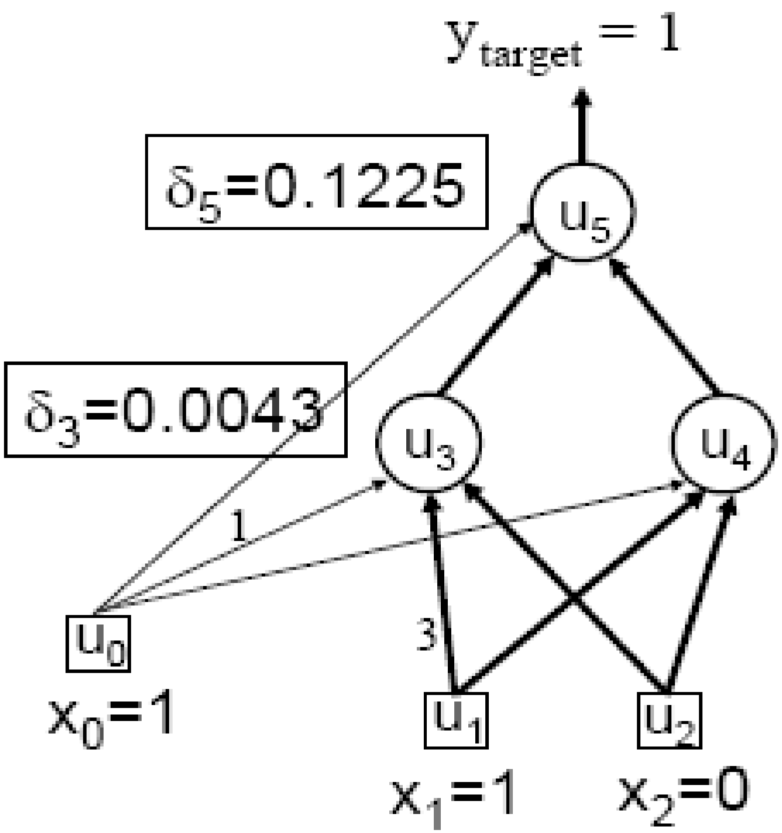
$$\delta_5 = y_5(1 - y_5)\,(y_{target} - y_5)$$
$$= 0.51(1 - 0.51) \times 0.49$$
$$= \mathbf{0.1225}$$

Then for hidden units:

$$\delta_4 = y_4(1 - y_4)\,w_{45}\,\delta_5$$
$$= 0.5(1 - 0.5) \times 4 \times 0.1225$$
$$= \mathbf{0.1225}$$

$$\delta_3 = y_3(1 - y_3)\,w_{35}\,\delta_5$$
$$= 0.982(1-0.982) \times 2 \times 0.1225$$
$$= \mathbf{0.0043}$$

Example: Update Weights Using Generalized Delta Rule (BP)



- Set learning rate η = 0.1
  Change weights by:
  $$\Delta w_{ij} = \eta \delta_j y_i$$
- e.g.bias weight on u_3:

Δw_{03} = $\eta \delta_3 x_0$
=0.1*0.0043*1
= 0.0004

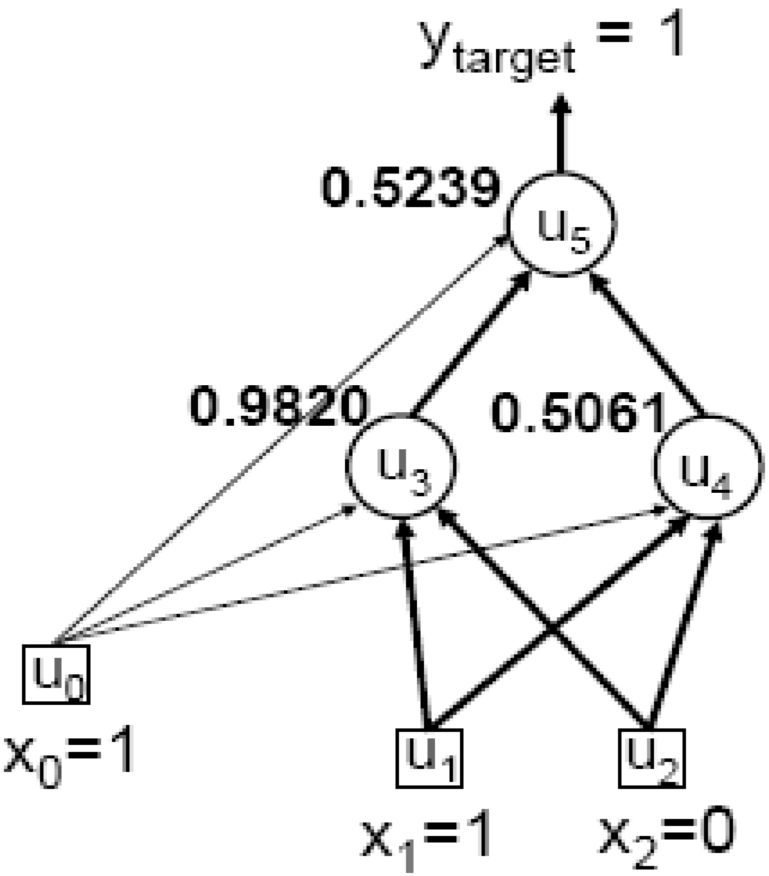
So, new w_{03} ⌫
w_{03}(old)+Δw_{03}
=1+0.0004=1.0004

- and likewise:
  w_{13} ⌫ 3 + 0.0004
  = 3.0004

Similarly for the all weights wij:

i	j	w_{ij}	δ_j	y_i	Updated w_{ij}
0	3	**1**	0.0043	1.0	**1.0004**
1	3	**3**	0.0043	1.0	**3.0004**
2	3	**4**	0.0043	0.0	**4.0000**
0	4	**-6**	0.1225	1.0	**-5.9878**
1	4	**6**	0.1225	1.0	**6.0123**
2	4	**5**	0.1225	0.0	**5.0000**
0	5	**-3.92**	0.1225	1.0	**-3.9078**
3	5	**2**	0.1225	0.9820	**2.0120**
4	5	**4**	0.1225	0.5	**4.0061**

Verification that it works



On next forward pass:

The new activations are:

y_3 = f (4.0008) = 0.9820

y_4 = f (0.0245) = 0.5061

y_5 = f (0.0955) = **0.5239**

Thus the new error

$(y_{target} - y_5)=(1 - 0.5239)=0.476$

has been reduced by **0.014**

(from **0.490** to **0.476**)

Ref: "Neural Network Learning & Expert Systems" by Stephen Gallant

Training

- This was a single iteration of back-prop
- Training requires many iterations with many training examples or *epochs* (one epoch is entire presentation of complete training set)
- It can be slow !
- Disjoint training and testing data sets
- learn from training data but evaluate performance (generalization ability) on unseen test data
    - **Aim**: minimize error on *test* data