Homework 4: Phys 7310 (Fall 2021)

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***Problem* 4.1** (Potential in a box)**:** A hollow cube has conducting walls defined by six planes $x = 0, y = 0, z = 0$ and $x = a, y = a, z = a$. The walls $z = 0$ and $z = a$ are held at a constant potential V. The other four sides are at zero potential.

(a) Find the potential $\Phi(x, y, z)$ at any point inside the cube.

(b) Evaluate the potential at the center of the cube numerically, accurate to three significant figures. How many terms in the series is it necessary to keep in order to attain this accuracy? Compare your numerical result with the average value of the potential on the walls. See Problem 2.28.

(c) Find the surface charge density on the surface $z = a$.

Solution.

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***Problem* 4.2** (Almost a corner)**:** The two-dimensional region, $\rho \geq a, 0 \leq \phi \leq \beta$, is bounded by conducting surfaces at $\phi = 0, \rho = a$, and $\phi = \beta$ held at zero potential, as indicated in the sketch. At large ρ the potential is determined by some configuration of charges and/or conductors at fixed potentials.

(a) Write down a solution for the potential $\Phi(\rho, \phi)$ that satisfies the boundary conditions for finite ρ.

(b) Keeping only the lowest nonvanishing terms, calculate the electric field components E_ρ and E_ϕ and also the surface charge densities $\sigma(\rho, 0), \sigma(\rho, \beta)$, and $\sigma(a, \phi)$ on the three boundary surfaces.

(c) Consider $\beta = \pi$ (a plane conductor with a half-cylinder of radius a on it). Show that far from the half-cylinder the lowest order terms of part (b) give a uniform electric field normal to the plane. Sketch the charge density on and in the neighborhood of the half-cylinder. For fixed electric field strength far from the plane, show that the total charge on the half-cylinder (actually charge per unit length in the z direction) is twice as large as would reside on a strip of width $2a$ in its absence. Show that the extra portion is drawn from regions of the plane nearby, so that the total charge on a strip of width large compared to a is the same whether the half-cylinder is there or not.

Solution.

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***Problem* 4.3** (A sphere with a hole in the top)**:** A spherical surface of

Solution.

□

***Problem* 4.4** (Conducting disc)**:** *Solution.*

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