Homework 1: Phys 7320 (Spring 2022)

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Due: January 19, 2022

***Problem* 1.1** (Radiating spherical metallic shell)**:** Two halves of a spherical metallic shell of radius R and infinite conductivity are separated by a very small insulating gap. An alternating potential is applied between the two halves of the sphere so that the potentias are $\pm V\cos\omega t$. In the long-wavelength limit, find the radiation fields, the angular distribution of radiated power, and the total radiated power from the sphere.

Solution.

$\square$

***Problem* 1.2** (Array of dipoles)**:** Consider an array of $2N+1$ dipoles. Each has a time-dependent dipole moment

$$\mathbf{p}(t) = p\hat{\mathbf{z}}e^{-i\omega t}, \tag{1.2.1}$$

and they are spaced each a distance D apart along the z-axis, centered on the origin. Find the resulting antenna pattern $dP/d\Omega$ at $r \gg D$. You may assume that the wavelength is large compared to each individual dipole, but do not assume it is large compared to the spacing between dipoles (that is, do not assume $\lambda \gg D$.) Explicitly work out what happens at $\theta \approx \pi/2$ where θ is the usual angle from the z-axis in spherica coordinates.

This is an iconic probem: many similar radiators distributed in space, so the antenna pattern tells you things about the spatial distribution of radiators.

Solution.

□

***Problem* 1.3** (Almost a sphere)**:** (a) The surface of a charge distribution of uniform density is almost a sphere: the radius as a function of the polar angle is

$$R(\theta) = R_0(1 + \gamma \cos\theta). \tag{1.3.1}$$

The quantity γ oscillates in time with frequency ω, so we are describing something like surface waves on a water balloon. Working to lowest nontrivial order in the small parameter γ, and in the long wavelength limit, first find an expression for the total charge on the surface Q, and then find an expression for the antenna pattern of emitted radiation $dP/d\Omega$ and the total power radiated P with these answers written in terms of Q.

(b) The previous part is a variation of Jackson 9.12, except in that problem $R(\theta) = R_0(1 + \beta P_2(\cos\theta))$. What is the leading mutipole behavior of the radiation in that case? Why? (You don't have to calculate it – just say which kind of multipole it is and why.)

Solution.

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