Homework 4: Phys 5210 (Fall 2021)

Tien Vo

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***Problem* 1:** Two particles move about each other in circular orbits under the influence of gravitational forces (so that the potential energy is $U = -\alpha/r$), with a period τ.

Their motion is suddenly stopped at a given instant of time, and they are then released and allowed to fall into each other.

Prove that they collide after a time $\tau/(4\sqrt{2})$.

Solution.

□

***Problem* 2:** A central force potential is given by

$$U(r) = \begin{cases} 0, & r > a \\ -U_0, & r < a \end{cases} \tag{2.1}$$

Here $U_0 > 0$. We would like to study the scattering in this potential.

(a) If a particle approaches the potential with the kinetic energy E and the impact parameter s, find the angle φ and $\tilde{\varphi}$ as shown in the Figure above. The figure represents the potential as a circle of radius a, with the trajectory of a particle shown in blue. Use conservation of energy and conservation of angular momentum to find $\tilde{\varphi}$.

(b) Determine the scattering angle θ in terms of φ and $\tilde{\varphi}$.

(c) Express s as a function of θ. This step is tricky as it is easy to find θ in terms of s, harder the other way around. To do that, take the ratio $\sin\frac{\tilde{\varphi}}{\sin\varphi}$. On the one hand, use the results of part (a) to figure out what it is equal to. On the other hand, express $\tilde{\varphi}$ in terms of φ and θ, and work to solve the resulting equation for φ. Finally, use the previous result relating φ to s.

(d) Finally, find the differential cross section $d\sigma/d\Omega$ where $d\Omega = 2\pi\sin(\theta)d\theta$ for a particle scattering in this potential.

Solution.

□