Midterm: Phys 7320 (Spring 2022)

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Problem **M.1** (Radiation from two dipoles)**:** Consider a pair of identical dipoles, with dipole moments $p = qd$ where d is the separation of the charges in the dipole. The dipoles sit on the z-axis centered at locations $z = \pm L$. Assume that $d \ll L$, so that we can treat each dipole as essentially point-like. The dipoles are made to oscillate at frequency $e^{-i\omega t}$.

(a) First, assume both dipoles are pointing in the positive z-direction. Go to the radiation zone $r \gg L$ and $r \gg \lambda$, but do not assume anything about λ vs. L, and find an expression for the antenna pattern $dP/d\Omega$ for the whole system.

(b) Find the limit of the result of part (a) for large wavelengths $\lambda \gg L$. Show this matches the leading multipole moment of the system, and give the total power P in this limit.

(c) Now assume the dipole at $z = L$ is pointing in the positive z-direction, but the dipole at $z = -L$ is pointing in the negative z-direction (all times the common factor $e^{-i\omega t}$). Again without assuming anything about λ vs. L, find the antenna pattern $dP/d\Omega$ for the whole system in the radiation zone.

(d) Again take the limit of the result of part (c) for large wavelengths $\lambda \gg L$ and find the leading nonzero contribution. Independently calculate the leading multipole moment of the system and show this matches what you just found. You don't have to calculate the total power.

Solution.

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Problem M.2 (Diffraction from rectangular slits)**:** A perfectly conducting screen sits in the xy-plane at $z = 0$. Planar electromagnetic radiation is incident on the screen, moving in the positive z-direction. We will approximate the EM wave with a scalar field ψ.

(a) First, let the screen have one aperture in it, a rectangle centered on the origin from $x = -a$ to $x = a$ and from $y = -b$ to $y = b$. In the Fraunhofer limit, use the scalar Dirichlet Kirchhoff formula to calculate the diffracted field ψ and the intensity $|\psi|^2$. Describe what will appear on a screen far away for $ka \sim kb \sim 20$.

(b) Reduce your result for $|\psi|^2$ from part (a) to the case of a slit: the limit $b \to 0$ with k and a fixed, and E_0 scaled large so that the quantity $\hat{E}_0 \equiv 2bE_0$ stays finite in the limit; write the result in terms of $\hat{E}_0$. Describe what happens to the y-dependence, and why this is now effectively a 1-dimensional diffraction pattern. Sketch the intensity in the x direction for $ka \sim 20$.

(c) Now change the screen so that it consists of two rectangular slits, each of width $2a$ in the x direction, centered at $x = \pm d$, with $d > a$. Again using the Dirichlet scalar Kirchhoff formula in the Fraunhofer limit, calculate the intensity for this case. Sketch the intensity as a function of x, assuming that d is bigger than a by a factor of approximately 3.

Solution.

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Problem M.3 (A moving car)**:** Consider a car with length l, and choose the origin in its rest frame such that the front of the car is at $x_F = 0$, and the rear of the car is at $x_R = l$.

(a) Move to an inertial frame with coordinates x', t' where the car is moving to the left with constant velocity v. Find an equation relating x'_F and t'_F (the worldline of the front of the car in this frame), and another equation relating x'_R and t'_R (the world line of the back of the car in this frame).

(b) Use your results from the previous part to obtain the length of the car in the primed frame in terms of l (that is, rederive length contraction from your results). Also, what is the slope of the two worldlines in the spacetime diagram with ct' the vertical axis and x' the horizontal axis?

(c) The car has a resting length of $l = 5\,\text{m}$, a rest mass of $2000\,\text{kg}$, and in the primed frame is moving at $3c/5$. Find the energy and momentum of the car in its rest frame, and the energy, momentum and length of the car in the moving frame. (The speed of light is $3 \times 10^8\,\text{m/s}$.)

Solution.

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