# Homework 2: Astr 5140 (Fall 2021)

Tien Vo

September 4, 2021

***Problem* 1** (**$\mathbf{J} \times \mathbf{B}$** force)**:** Using the two-fluid equations, calculate the Lorentz force per unit volume on a quasi-neutral (MHD) plasma using the definition of $\mathbf{J}$. Separate the electric force ($\mathbf{F}_E$) from the magnetic force ($\mathbf{F}_B$). Show that if the plasma is quasi-neutral, then $\mathbf{F}$ reduces to the standard MHD result.

***Problem* 2** (EM review: Waves in a plasma)**:** Using Maxwell's equations and setting the current to be the electron motion only

$$\frac{\partial \mathbf{J}}{\partial t} = \frac{ne^2 E}{m_e} \tag{2.1}$$

Show that the solution of a transverse light wave becomes:

$$\omega^2 = \omega_{pe}^2 + k^2 c^2 \tag{2.2}$$

where $\omega_{pe}^2 = ne^2/\epsilon_0 m_e$.

***Problem* 3** (Pick-up Ion at Mars)**:** Suppose an $O$ atom that escaped from Mars is at rest (in our frame) at $x = 0, y = 0$. It is photo-ionized (charge $e$) at $t = 0$ in the solar wind ($v_{sw} = 350\,\mathrm{km/s}$ in the $x$ direction) with a magnetic field $\mathbf{B} = 10\,\mathrm{nT}$ in the $z$ direction (see diagram). Assume that the photo-ionization does not move the $O^+$ ion.

(1) Describe the subsequent motion of the $O^+$ ion, $x(t), y(t), v_x(t), v_y(t)$, in our rest frame (not the plasma frame). *Hint*: What is the solar wind electric field? Calculate $v_x(t)$ and $v_y(t)$ then integrate. Apply boundary conditions, $x(t = 0) = 0, y(t = 0) = 0, v_x(t = 0) = 0, v_y(t = 0) = 0$ to get an exact solution.

(2) Sketch the $O^+$ path.

(3) The drift and gyration cause the $O^+$ ion to slow down and speed up in our rest frame. What is the drift speed? What is the gyration speed? What is the maximum velocity (km/s) and energy (keV) that the $O^+$ ion reaches?

(4) Using the gyration speed only, what is the perpendicular (to $\mathbf{B}$) temperature of that ion in $^\circ$K? (In 2D, temperature and energy are equal.)

***Problem* 4** (Current sheet)**:** A current sheet is such that $\mathbf{J}$ is in the $y$ direction and $\mathbf{B}$ is in the $z$ direction. $\mathbf{B}, \mathbf{J}$, and $P$ vary only with $x$ (see diagram). Derive a solution for $\mathbf{B}$, and $\mathbf{J}$ under the condition $\mathbf{J} \sim P$ that is valid for $-L < x < L$. Make sure that your solution satisfies the boundary conditions of $\mathbf{B}(x = L) = -B_0\hat{\mathbf{z}}$, and $\mathbf{B}(x = 0) = \mathbf{0}$. $L$ is a characteristic length. Sketch your results.

*Hint*: There is more than one possible solution – just give any valid solution. Be careful, the condition $\mathbf{J}^2 \sim P$ is NOT the same as in the Harris solution.

***Problem*** **5** (Magnetic diffusion)**:** Consider a magnetic field $\mathbf{B} = B_z(x, t)\hat{\mathbf{z}}$ where $B_z(x, t = 0) = B_0 \cos(k_1 x) + B_0 \cos(k_2 x)$ in a resistive plasma with $k_2 \gg k_1$.

(a) Find the solution for $B_z(x, t)$.

(b) Sketch (accurate plot not needed) the solution for $t = 0$ and $t > 0$. What happens to the high-$k$ wave?