Homework 1: PHYS 7310 (Fall 2021)

Tien Vo

August 26, 2021

Problem 1.1 (Properties of conductors). Use Gauss' theorem [and (1.21) if necessary] to prove the following:

(a) Any excess charge placed on a conductor must lie entirely on its surface. (A conductor by definition contains charges capable of moving freely under the action of applied electric fields.)

(b) A closed, hollow conductor shields its interior from fields due to charges outside, but does not shield its exterior from the fields due to charges placed inside it.

(c) The electric field at the surface of a conductor is normal to the surface and has a magnitude σ/ϵ_0, where σ is the charge density per unit area on the surface.

Problem 1.2 (A neutral hydrogen atom). The time-averaged potential of a neutral hydrogen atom is given by

$$\Phi = \frac{q}{4\pi\epsilon_0}\frac{e^{-\alpha r}}{r}\left(1 + \frac{\alpha r}{2}\right) \tag{1}$$

where q is the magnitude of the electronic charge, and $\alpha^{-1} = a_0/2$, a_0 being the Bohr radius. Find the distribution of charge (both continuous and discrete) that will give this potential and interpret your results physically.

Problem 1.3 (Capacitors). A simple capacitor is a device formed by two insulated conductors adjacent to each other. If equal and opposite charges are placed on the conductors, there will be a certain difference of potential between them. The ratio of the magnitude of the charge on one conductor to the magnitude of the potential difference is called the capacitance (in SI units it is measured in farads). Using Gauss Law, calculate the capacitance of

(a) two large, flat, conducting sheets of area A, separated by a small distance d;

(b) two concentric conducting spheres with radii a, b $(b > a)$;

(c) two concentric conducting cylinders of length L, large compared to their radii a, b $(b > a)$.

Problem 1.4 (Green's reciprocation theorem and grounded parallel plates). (a) Prove *Green's reciprocation theorem*: If Φ is the potential due to a volume-charge density ρ within a volume V and a surface-charge σ on the conducting surface S bounding the volume V, while Φ' is the potential due to another charge distribution ρ' and σ', then

$$\int_V \rho\Phi' d^3x + \int_S \sigma\Phi' da = \int_V \rho'\Phi d^3x + \int_S \sigma'\Phi da \tag{2}$$

(b) Two infinite grounded parallel conducting planes are separated by a distance d. A point charge q is placed between the planes. Use the reciprocation theorem of Green to prove that the total induced charge on one of the plane is equal to $(-q)$ times the fractional perpendicular distance of the point charge from the other plane. (*Hint*: As your comparison electrostatic problem with the same surfaces choose one whose charge densities and potential are known and simple.)