Homework 3: Phys 7320 (Spring 2022)

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***Problem* 1** (Scattering for a perfectly conducting sphere)**:** In class we discussed scattering off a dielectric sphere; now we consider the case of a perfectly conducting sphere of radius a. Jackson discusses this in Sec. 10.1.C so we will fill in the details.

When the radiation hits the target, it induces both an electric dipole moment $\mathbf{p}$ and a magnetic dipole moment $\mathbf{m}$, which in Gaussian units take the form,

$$\mathbf{p} = a^3\mathbf{E}_{\text{inc}}, \qquad \mathbf{m} = -\frac{a^3}{2}\mathbf{B}_{\text{inc}} \tag{1.1}$$

or for SI units, see Jackson (10.12)–(10.13).

(a) Show why this leads to the differential scattering cross section given in Jackson (10.14) (in either unit system).

(b) Calculate the results for scattering for polarization both parallel and perpendicular to the plane of scattering, as in Jackson (10.15). Include the factor 1/2 so that when they are added together in part (c), you are calculating the total cross section averaged over initial polarizations.

(c) Caculate the total differential cross-section averaged over polarizations (10.16) and the polarization $\Pi(\theta)$ (10.17). Is radiation more likely to continue in a forward direction, or to be scattered backwards? Is radiation more likely to be scattered parallel to, or perpendicular to, the plane of scattering? Compare these behaviors to the case of scattering from a dielectric sphere.

Solution.

□

Problem 2 (Power scattered and power absorbed)**:** An unpolarized wave of frequency $\omega = ck$ is scattered by a *slightly* lossy uniform isotropic dielectric sphere of radius R much smaller than a wavelength. The sphere is characterized by an ordinary real dielectric constant ϵ_r and a real conductivity σ. The parameters are such that the skin depth δ is very *large* compared to the radius R.

(a) Calculate the differential and total *scattering* cross sections.

(b) Show that the absorption cross section is

$$\sigma_{\text{abs}} = 12\pi R^2 \frac{R Z_0 \sigma}{(\epsilon_r + 2)^2 + (Z_0\sigma/k)^2} \tag{2.1}$$

(c) From part (a) write down the forward scattering amplitude and use the optical theorem to evaluate the total cross section. Compare your answer with the sum of the scattering and absorption cross sections from parts (a) and (b). Comment.

Solution.

□

***Problem* 3** (An atom in a spring)**:** Suppose you model the atom as an electron on a spring of natural frequency ω_0. Include a damping force $\mathbf{F} = -\Gamma d\mathbf{x}/dt$. Put the spring in an external electric field $\mathbf{E}e^{-i\omega t}$, solve for the steady state motion, compute the resulting dipole moment, and drop the result into the appropriate formula for the differential cross section. Now do physics: note how you recover Rayleigh scattering for $\omega \ll \omega_0$. At high frequency, the cross section becomes independent of frequency. This is called the Thomson cross section. (It is the same formula for scattering off a free electron). There is a dimensionful constant

$$r_0 = \frac{e^2}{m_e c^2} \tag{3.1}$$

where m_e is the electron mass, which characterizes the scale of elastic light scattering on electrons and hence on atoms. Find a number for this in cm. This says that the natural scale for light scattering on the electrons in atoms is $\sigma \sim r_0^2$ (or maybe more correctly, $r_0^2(\omega/\omega_0)^4$, which is even smaller). Notice how all the interesting polarization behavior is frequency-independent. This isn't true when you work in complete generality, but it is true in dipole approximation. There are other ways to derive this result which we'll encounter later on in the course.

Solution.

$\square$