Section 2.1 Linear and Quadratic Functions with Modeling

The general polynomial function

$f(x) = a_n x^n + a_{n-1}x^{n-1} + a_{n-2}x^{n-2} + \ldots + a_1 x + a_0$

f is a *polynomial function of degree n*. The *leading coefficient* is a_n.

Low & No degree polynomial functions

Zero function $f(x) = 0$

Constant function $f(x) = a;\ a \neq 0$

Linear function $f(x) = ax + b;\ a \neq 0$

Quadratic function $f(x) = ax^2 + bx + c;\ a \neq 0$

Average rate of change

If $y = f(x)$ is a linear function, then the average rate of change between $x = a$ and $x = b$, $a \neq b$, is $\dfrac{f(b) - f(a)}{b - a}$.

Write an equation for a linear function f if $f(3) = 7$ and $f(-5) = 2$.

(3, 7) (-5, 2) $m = \dfrac{7-2}{3-(-5)} = \dfrac{5}{8}$

$y - 7 = \frac{5}{8}(x - 3)$ or $y - 2 = \frac{5}{8}(x + 5)$

Characteristics of a *Linear Function*:

Polynomial of degree 1.

$f(x) = mx + b \ (m \neq 0)$

Slant line with slope m and y-intercept b.

Function with a constant nonzero rate of change m; f is increasing if $m > 0$, decreasing if $m < 0$; the initial value of f is $f(0) = b$

The *correlation coefficient*, r, is a number between -1 and 1 that measures the strength and direction of the linear correlation of a data set.

If $|r| \approx 1$, then the data set has a strong linear correlation.

When $|r| \approx 0$, then the data set has a weak or no linear correlation.

$f(x) = a(x-h)^2 + k$ is the vertex form of a quadratic function.

Vertex: (h, k) Line of Symmetry: x = h

Find the vertex and line of symmetry of the graph of $g(x) = -3(x+2)^2 - 1$.

Vertex: $(-2, -1)$ Line of Symmetry: $x = -2$

Change $f(x) = ax^2 + bx + c$ to *vertex form.*

$$f(x) = a\left(x^2 + \frac{b}{a}x\right) + c \qquad \left(\frac{1}{2}\frac{b}{a}\right)^2 = \left(\frac{b}{2a}\right)^2$$

$$f(x) = a\left(x^2 + \frac{b}{a}x + \frac{b^2}{4a^2}\right) + c - \frac{b^2}{4a}$$

$$f(x) = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$$

$$\therefore h = \frac{-b}{2a}; \quad k = c - \frac{b^2}{4a}$$

$$\text{or } k = f\left(\frac{-b}{2a}\right)$$

$f(x) = ax^2 + bx + c$ is the general form of a quadratic function.

Vertex: $\left(\frac{-b}{2a}, c - \frac{b^2}{4a}\right)$ or $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

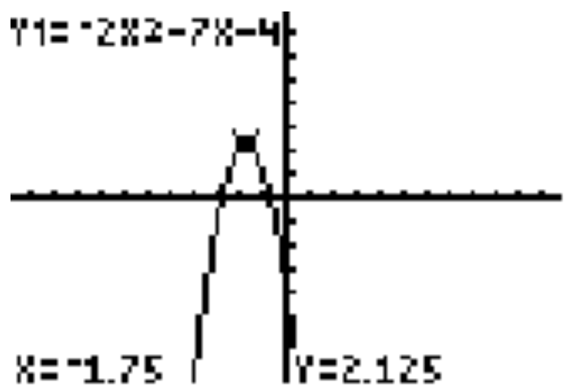
Line of Symmetry: $x = \frac{-b}{2a}$

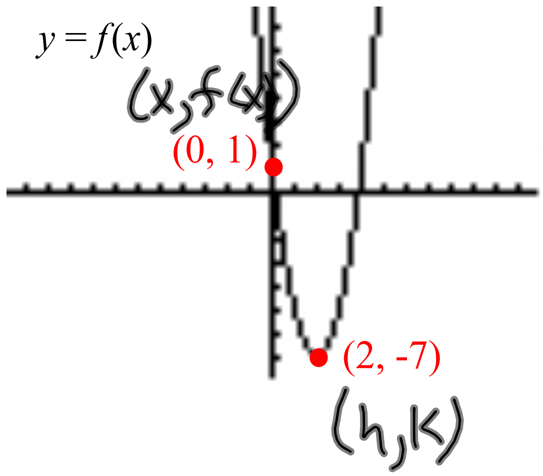
Find the vertex and line of symmetry of the graph of $f(x) = -2x^2 - 7x - 4$.

$h = \frac{7}{2(-2)} = -\frac{7}{4}$; $k = f\left(-\frac{7}{4}\right) = \frac{17}{8}$

$\left(-\frac{7}{4}, \frac{17}{8}\right)$

$x = -\frac{7}{4}$





Write an equation for the parabola shown.

$f(x) = a(x-h)^2 + k$

$1 = a(0-2)^2 - 7$

$1 \overset{+7}{=} a(4) - 7^{+7}$

$8 = 4a$

$2 = a$

$f(x) = 2(x-2)^2 - 7$

Plot $g(x) = 5x^2 - 25x + 12$ using an algebraic method. $h = \frac{-b}{2a}$, $k = c - \frac{b^2}{4a}$

What is the vertex?

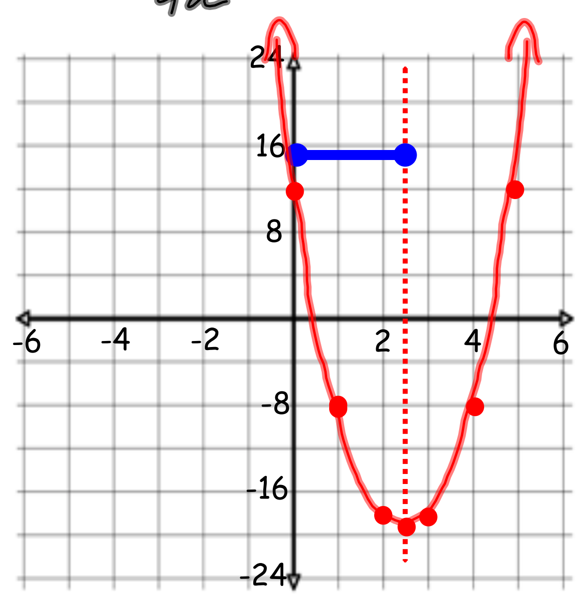
$h = \frac{25}{10} = \frac{5}{2}$

$k = 12 - \frac{625}{20} =$

$12 - 31\frac{1}{4} = -19\frac{1}{4}$

Draw the line of symmetry.

What is the y-int?



$31\frac{1}{4}$
12
$-19\frac{1}{4}$

x	$\frac{5}{2}$	0	5	1	4	2	3
y	$-19\frac{1}{4}$	12	12	-8	-8	-18	-18

33 $f(x) = x^2 - 4x + 6$

Vertex form: complete the sq $\left(\frac{b}{2}\right)^2$

$f(x) = (x^2 - 4x + 4) + 6 - 4$

$\left(-\frac{4}{2}\right)^2 = 4$ $f(x) = (x-2)^2 + 2$

Vertex: $(2, 2)$

line of symmetry: $x = 2$

37 $f(x) = 2x^2 + 6x + 7$

$f(x) = 2\left(x^2 + 3x + \frac{9}{4}\right) + 7 - \frac{9}{2}$

$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$

$f(x) = 2\left(x + \frac{3}{2}\right)^2 + \frac{5}{2}$

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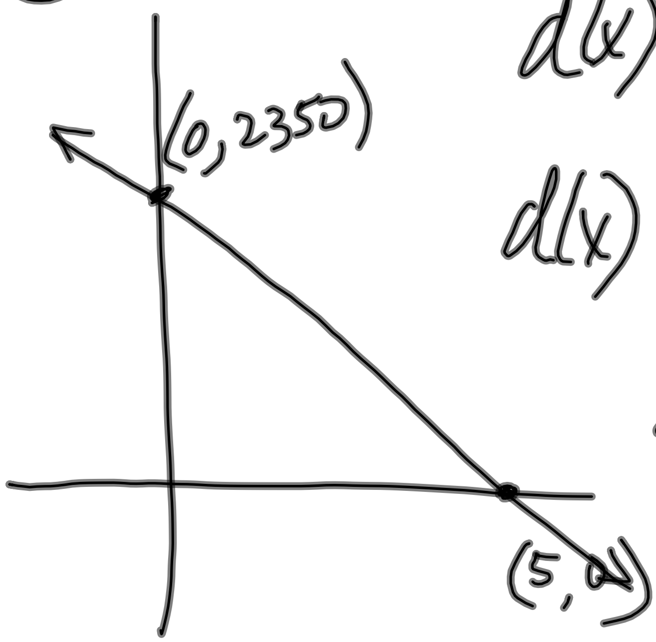

$d(x) = \frac{-2350}{5} x + 2350$

$d(x) = -470x + 2350$

$d(3) = 940$