The Catholic University of America

CSC 527: Fundamental of Neural Networks

Project 1 – Report

Double-Moon Classification

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I – Introduction of Rosenblatt's Perceptron

- Rosenblatt's Perceptron is built around a non-linear neuron, or McCulloch-Pitts model of a neuron. The model consists of m inputs (x_1, x_2, …, x_m) and each input has its own weight (w_1, w_2, …, w_m). There is also a parameter called bias (b). And the output of the perceptron is calculated as in the following formula:

$$y = \sum_{1}^{m} x_i w_i + b$$

- The goal of Rosenblatt's Perceptron is to correctly classify the inputs (x_1, x_2, …, x_m), or the external applied stimuli, into one or two classes C1 and C2. The classification will assign the points represented by the inputs to C1 if the output of the perceptron is 1, and to C2 if the output of the perceptron is -1.

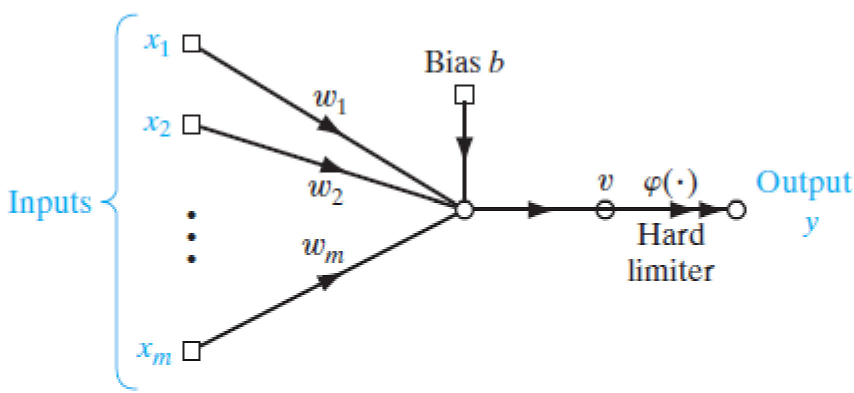


Fig 1. Representation of a Rosenblatt's Perceptron

II – Project Report

1. Introduction

- Knowing the general definition of Rosenblatt's Perceptron, I will apply it on a real pattern classification problem: The double-moon classification problem

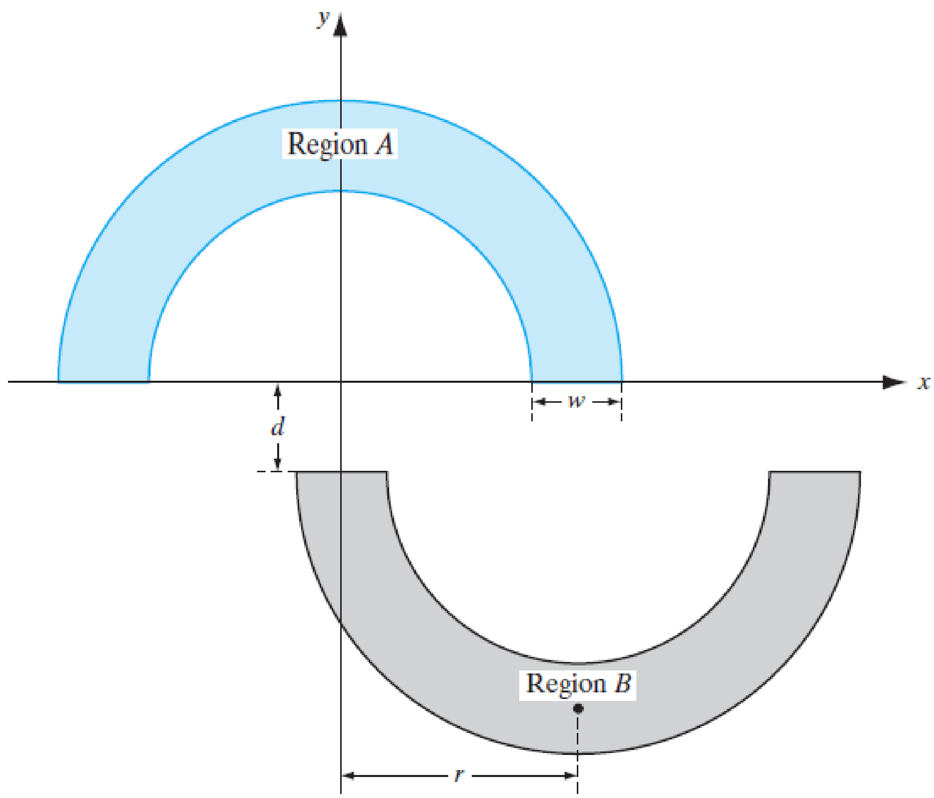


Fig 2. The double-moon classification problem

- Fig 2. shows a pair of "moon" facing each other. Moon A, whose center is at O(0,0) and is positioned symmetrically to the y-axis, whereas moon B is displaced to the right by an amount equal to the radius r of the moon. The width of each moon is w and the distance between 2 moon is d.
- The code to generate 2 moons are given. I will use Python for this project in order to do the classification and plot the result.

2. Implementation

- First, I initialize the data: Learning rate, Number of Epoch, the moons' dimensions

```
#initialize variables
LEARNING_RATE = 0.0001
N_EPOCH = 100

#create a data set
dataset = moon(2000, 1, 10, 6)
dataset = np.asarray(dataset)
```

- Since we want the dataset to be labelled either 1 or -1, which is in the form [x_i, y_i, label], we need to modify the dataset (because "moon" function returns the dataset as 4 lists of values of [$\mathbf{x}_1$, $\mathbf{y}_1$, $\mathbf{x}_2$, $\mathbf{y}_2$])

```
#process the dataset
dataset1 = np.resize(deepcopy(dataset[0]), (1, len(dataset[0])))
dataset1 = np.append(dataset1, np.resize(deepcopy(dataset[2]), (1, len(dataset[0]))), axis=0)
dataset1 = np.append(dataset1, np.ones((1, len(dataset1[0])))*-1, axis=0)
dataset1 = dataset1.transpose()

dataset2 = np.resize(deepcopy(dataset[1]), (1, len(dataset[1])))
dataset2 = np.append(dataset2, np.resize(deepcopy(dataset[3]), (1, len(dataset[1]))), axis=0)
dataset2 = np.append(dataset2, np.ones((1, len(dataset2[0]))), axis=0)
dataset2 = dataset2.transpose()

processedDataset = np.append(dataset1, dataset2, axis=0)

#shuffle the data set
np.random.shuffle(processedDataset)
```

- After preprocessing, I now train the dataset. The TrainWeights function that I used takes 3 parameters:
  + The "dataset" parameter: the dataset that has been preprocessed
  + The "learningRate" parameter: the learning rate of the neural network
  + The "nEpoch" parameter: number of epochs

  The returned value of this function is a mean square error list and weights of the neural network

```
def TrainWeights(dataset, learningRate, nEpoch):
    weights = [0.0 for i in range(len(dataset[0]))]
    MSE_list = []

    #for each epoch, calculate MSE and update the weights of the neural network
    for epoch in range(nEpoch):
        MSE = 0.0
        for row in dataset:
            prediction = Predict(row, weights)
            error = row[-1] - prediction
            MSE += error ** 2
            weights[0] = weights[0] + learningRate * error
            for i in range(len(row) - 1):
                weights[i+1] = weights[i+1] + learningRate * error * row[i]
        MSE = MSE / len(dataset)
        MSE_list.append(MSE)

        # if MSE is equal to 0, break the loop
        if(MSE == 0.0):
            break
    return MSE_list, weights
```

- The TrainWeights function will process the data based on the number of pre-defined epochs and learning rate. For each epoch, the program calculates the Mean-Squared Error based on the following formula:

$$\text{MSE} = \frac{1}{n} \sum_{i=1}^{n} (y_i - \tilde{y}_i)^2$$

- The Predict function is called inside the TrainWeights function and is served as the prediction step. The Predict function takes 2 parameters:
  + The "row" parameter: the row of the current data
  + The "weights" parameter: the weights of the perceptron

  The returned value of this function is the label of the data (either 1 or -1)

```
def Predict(row, weights):
    activation = weights[0]
    for i in range(len(row) - 1):
        activation += weights[i+1] * row[i]
    if activation >= 0.0:
        return 1.0
    else:
        return -1.0
```

- The Activation function used in the Predict function is Signum function

$$\text{sgn}(v) = \begin{cases} +1 & \text{if } v > 0 \\ -1 & \text{if } v < 0 \end{cases}$$

  Using the Signum function is suitable in this project because to determine the output, it calculates the weighted sum of the inputs, add a bias and gives the result based on the final summation: output is 1 if the summation is greater than or equal to 0, and output is -1 if the summation is less than 0

3. Result

- I first plot the learning curve based on the MSE calculated above. In the plot, the x-axis represents the number of epochs and the y-axis represents MSE
- Then I plot the decision boundary line. The formula applied here is: weight[0] + weight[1]*x + weight[2]*y = 0
  where:

+ x is in range [-20, 35]

+ y is calculated as: y = -(weight[0] + weight[1]*x)/weight[2]

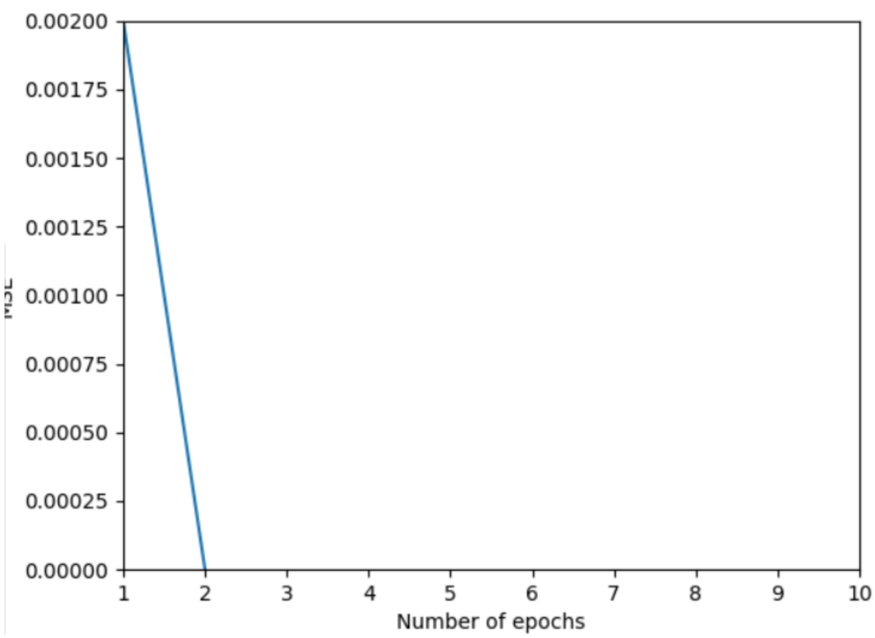
```
#plot classification line
x = np.asarray([-20,35])
y = -(weights[0] + weights[1]*x)/weights[2]
plt.plot(x, y, c="g")
```

III – Task 1

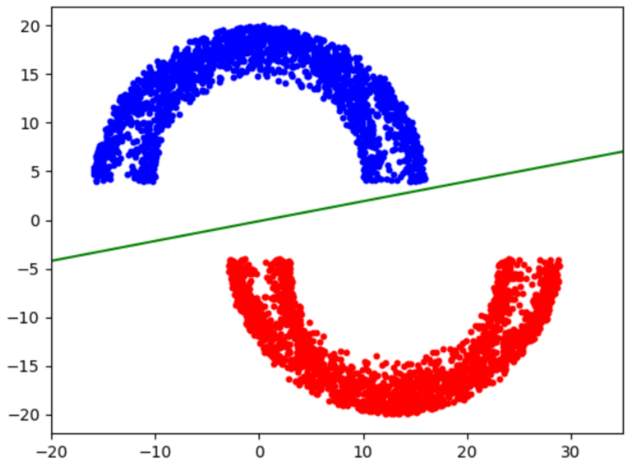
1. Part 1

Learning rate	0.0001
Number of epochs	100
Number of points	2000
Distance (d)	4
Radius (r)	10
Width(w)	6

The Learning curve plot:



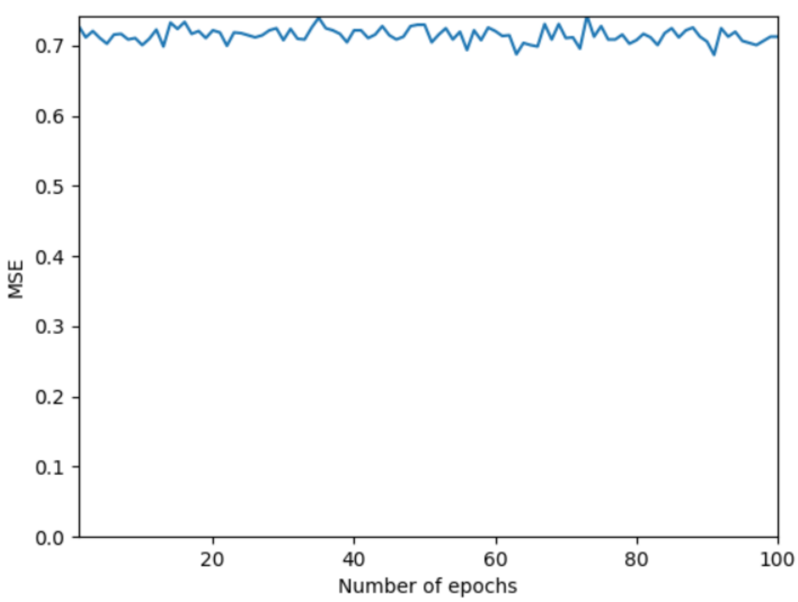
The Classification line plot:



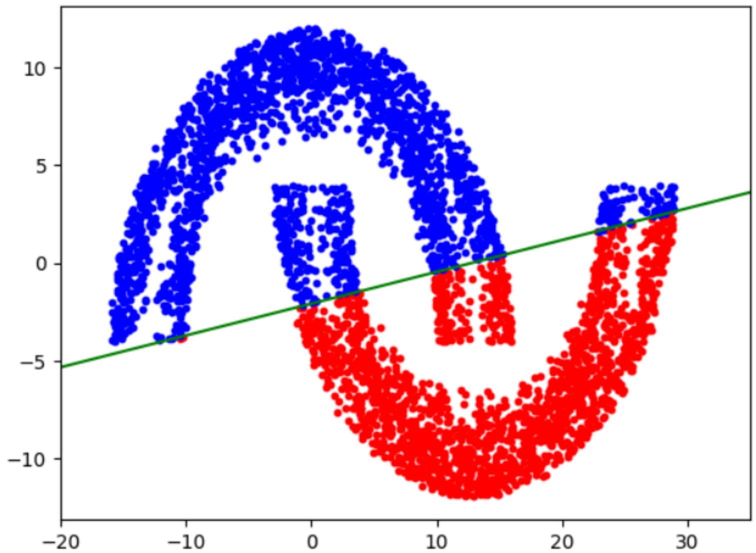
2. Part 2

Learning rate	0.0001
Number of epochs	100
Number of points	2000
Distance (d)	-4
Radius (r)	10
Width(w)	6

The Learning curve plot:



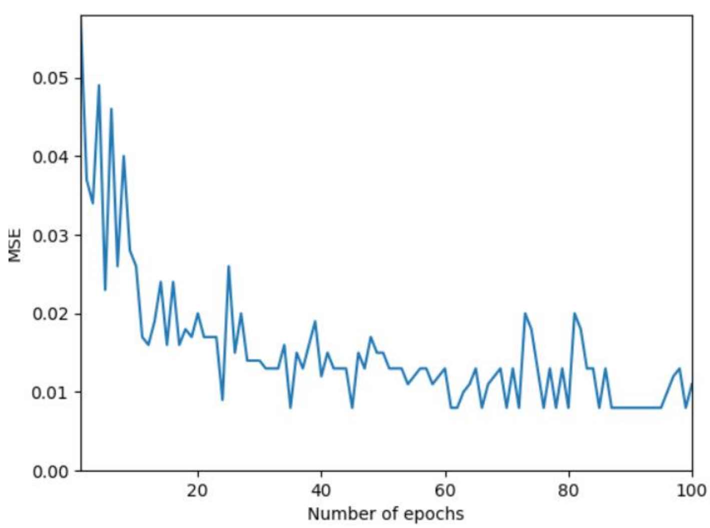
The Classification line plot:



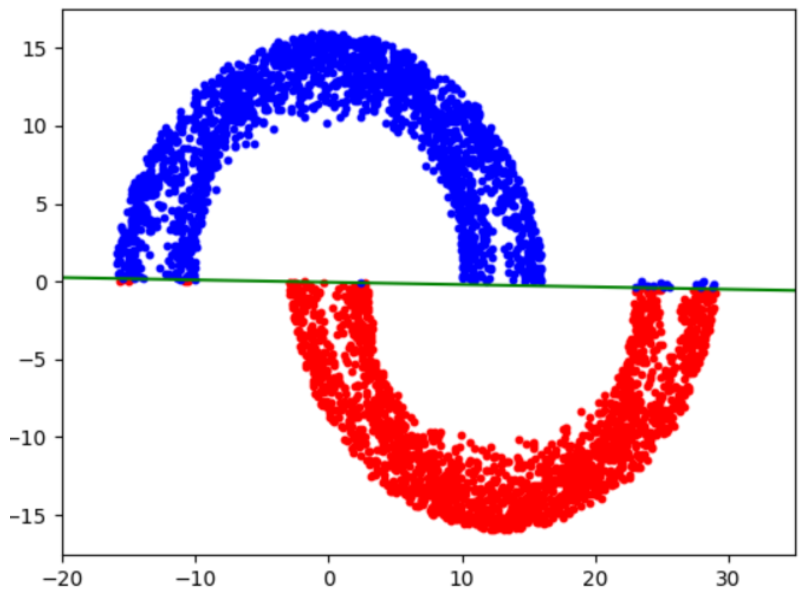
IV – Task 2

Learning rate	0.0001
Number of epochs	100
Number of points	2000
Distance (d)	0
Radius (r)	10
Width(w)	6

The Learning curve plot:



The Classification line plot:



V – Conclusion

- As can be seen from Task 1, where d = 4, the classification works perfectly
- However, when d = -4, which means 2 moons are very close to each other, the result shows a significant error in classification
- In Task 2, when d = 0, there are still some mis-classification from the program because the perceptron cannot converge.
- In general, this project is a great opportunity to apply what we have covered in class.
- GitHub link: https://github.com/tienpham2103/csc527/tree/master/Project1

VI – References

Simon, H. (2009). Neural Network and Learning Machine

Stephanie, G. (2013). Mean Squared Error: Definition and Example. Retrieved from: https://www.statisticshowto.com/mean-squared-error