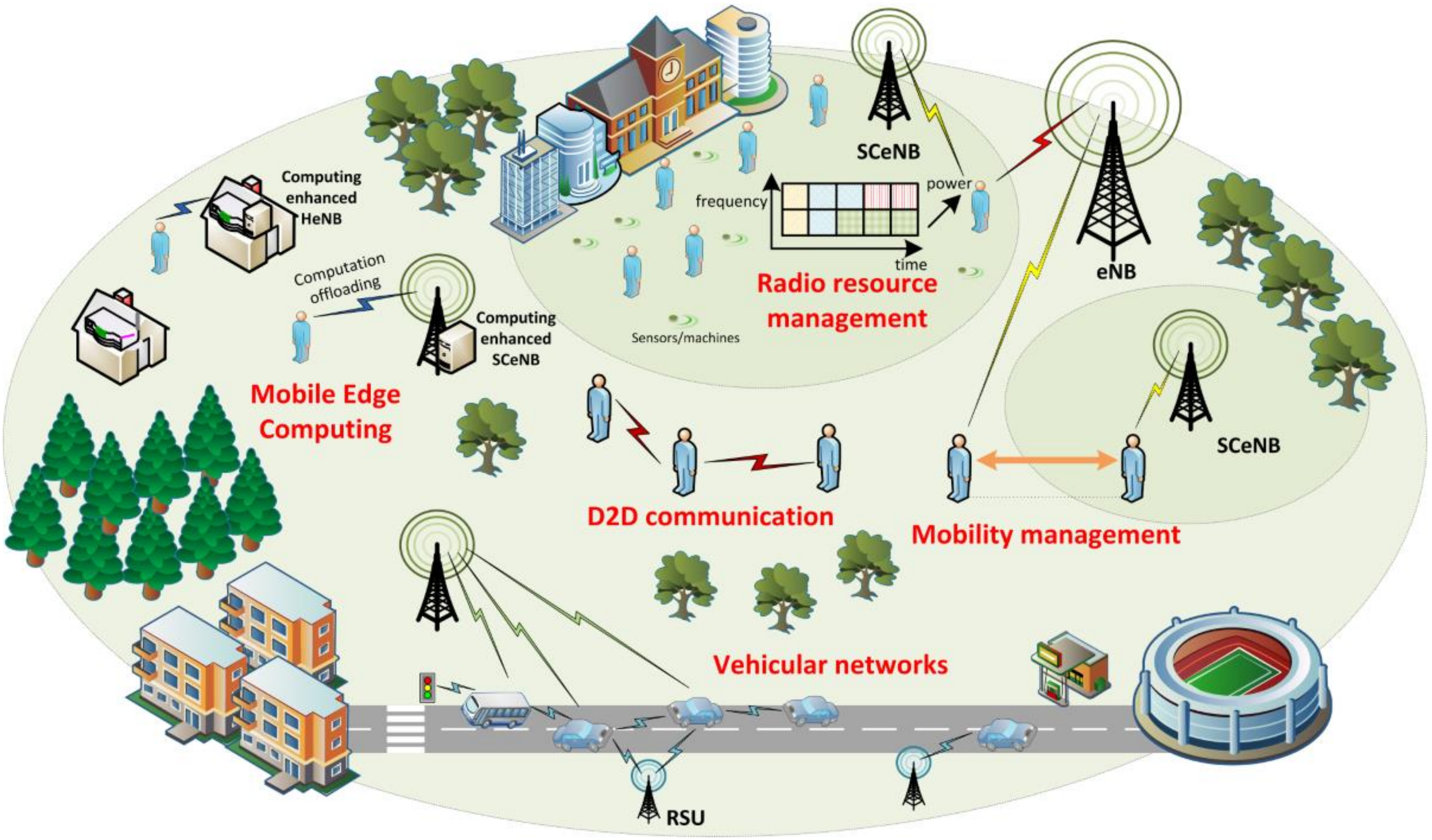
Matching Theory

ECE 697AA/597AA

Outline

- Motivation
- Matching Theory:
  - Definition
  - Classifications
  - Examples
- Wireless Matching Problems:
  - Modeling
  - Applications

5G and Beyond Resource Allocation



Cited from: http://5gmobile.fel.cvut.cz/activities/

5G and beyond challenges:

- Node/cell heterogeneous characteristics;
- Ultra-dense network;
- Mobility management;

Centralized solution:

- Global information;
- Significant overhead;
- Network management complexity;
- High latency.

Shift from centralized mechanism to self-organized distributed solutions.

Definitions

- ***Matching theory*** is a mathematical framework attempting to describe the formation of mutually beneficial relationships over time.
- Tractable solutions for the combinatorial problem of matching players in two different sets with preferences.
- Matching theory in graph:
  - Let G=(V,E) be an undirected graph, where $\boldsymbol{n}$=|$\boldsymbol{V}$ | and $\boldsymbol{m}$=|$\boldsymbol{E}$|;
  - A set $M \subseteq E$ is called a matching in G;

In wireless resource management

1) Tractable models for characterizing interactions

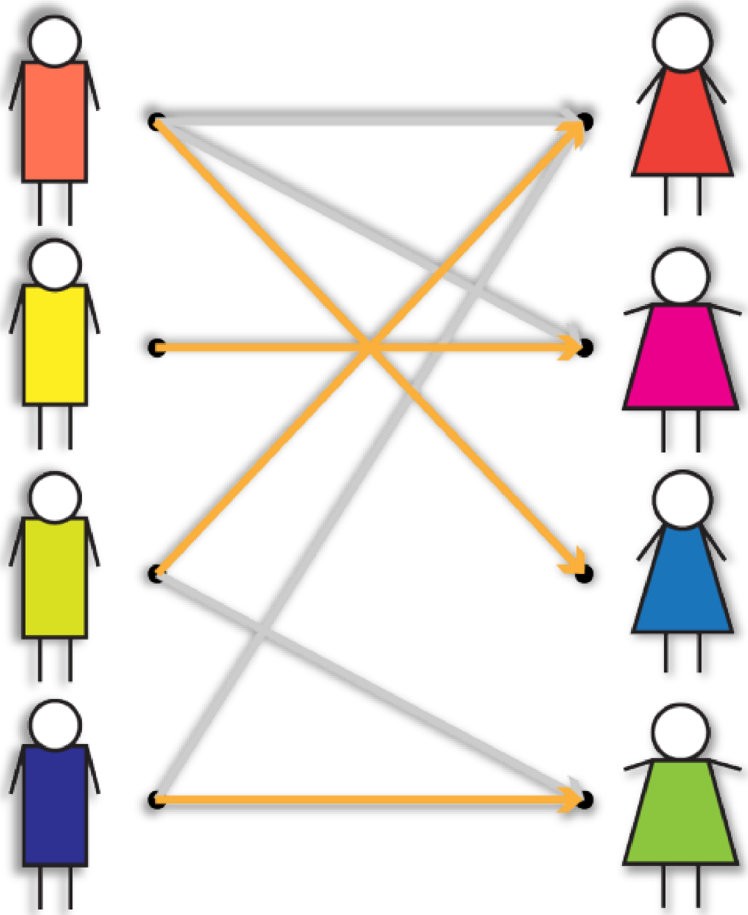
between heterogeneous nodes, each of which has its own type,

objective, and information.

2) The ability to define general “preferences” that can handle heterogeneous and complex considerations related to wireless quality-of-service (QoS).

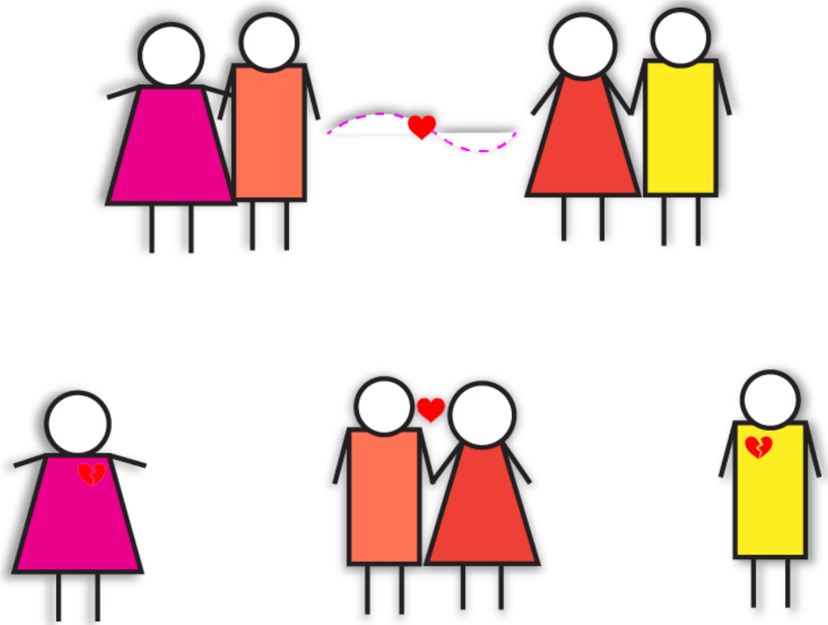
3) Tractable analysis of stability and optimality, that accurately reflect different system objectives.

4) Efficient implementations.



Everyone finds a date!

Unstable marriage!



How to design algorithm to prevent unstable results?

Definitions

- Basic elements (***Stable Marriage***):
  - ***Agents***: A set of men/women, and a set of women/men;
  - ***Preference list:*** A sorted list of men/women based on her/his preferences;
  - ***Blocking pair (BP)*** (m,w):
    - 1). m is unassigned or prefers w to his current partner;
    - 2). w is unassigned or prefers m to her current partner;
  - ***Stable matching***: A matching admit no BPs.
  - ***Gale-Shapley*** Algorithm (also known as the **Deferred Acceptance algorithm**): find a stable matching in SM.

```
function stableMatching {
    Initialize all m ∈ M and w ∈ W to free
    while ∃ free man m who still has a woman w to
propose to {
       w = first woman on m's list to whom m has not
yet proposed
       if w is free
         (m, w) become engaged
       else some pair (m', w) already exists
         if w prefers m to m'
            m' becomes free
           (m, w) become engaged
         else
           (m', w) remain engaged
    }
}
```

GS algorithm


Adam

Geeta, Heiki, Irina, Fran


Bob

Irina, Fran, Heiki, Geeta

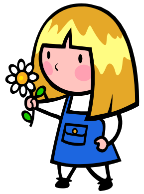

Carl

Geeta, Fran, Heiki, Irina


David

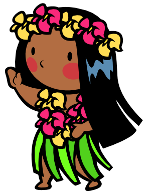
Irina, Heiki, Geeta, Fran

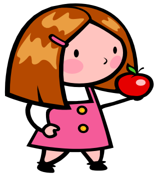
We reach a stable marriage!


Fran


Geeta

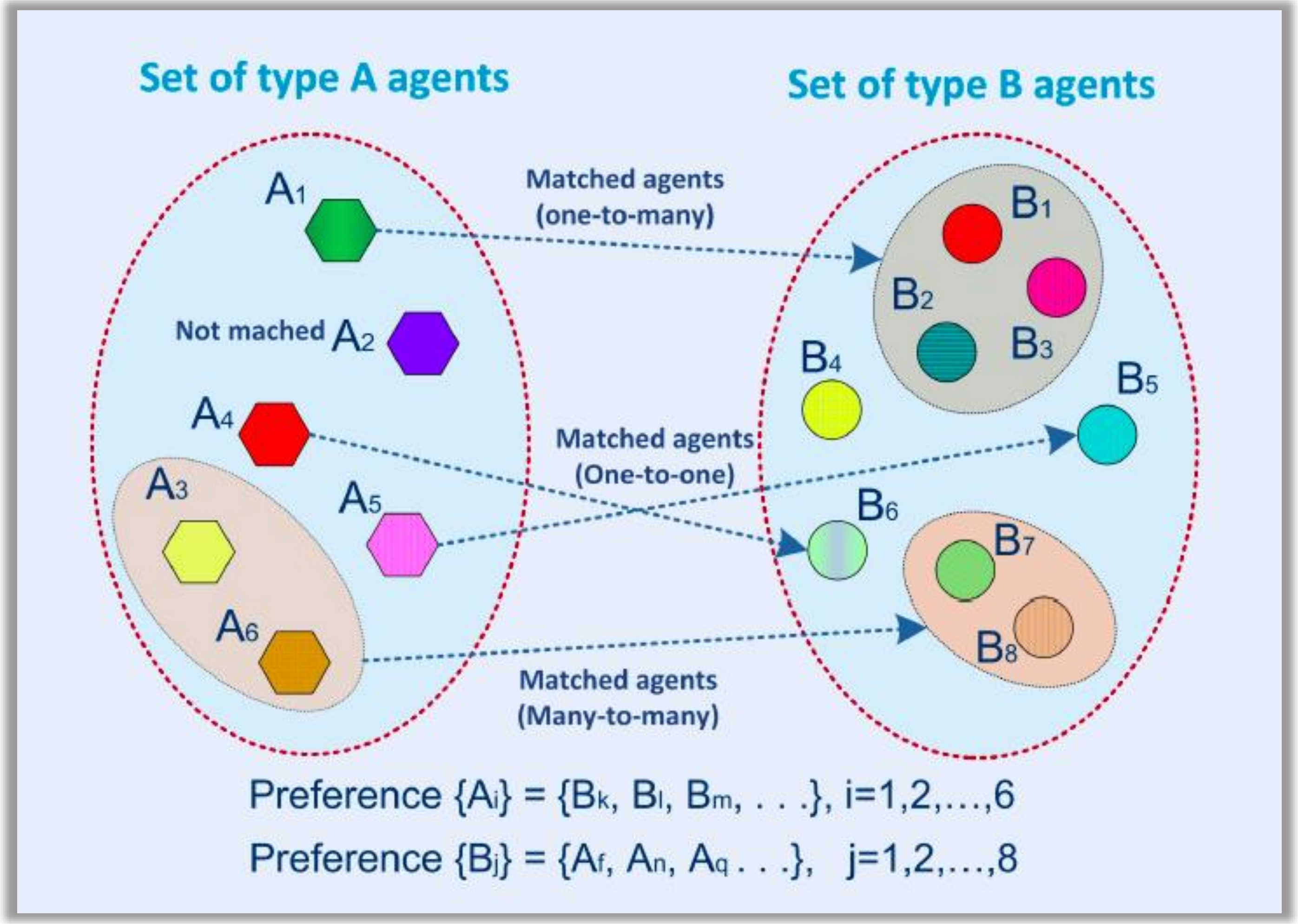
Carl > Adam


Heiki


Irina

David > Bob

Classification 1



- One-to-one:
  - Stable roommate (SR);
  - Stable marriage (SM);
- Many-to-one:
  - House allocation
  - Student admission
- Many-to-many
  - Worker-Firm (WF)
  - Assign paper to reviewer

Classification 2

- ***Bipartite*** matching problem with ***two-sided*** preferences.
  - Assign junior doctors to hospitals
  - Assign pupils to schools
- ***Bipartite*** matching problem with ***one-sided*** preferences.
  - Campus housing allocation
  - DVD rental markets
- ***Non-bipartite*** matching problem with preferences.
  - Form pairs of agents for chess tournaments
  - Create partnership in P2P network
  - Kidney exchange market

Classification 3

- Matching ***without external effect***:
  - The preferences only depend on the identity of their partners
  - E.g. Hospital resident problem
- Matching ***with external effect***:
  - The preferences depend on other agents' actions
  - E.g. multiple users share the same resource band
  - Solution: post-matching swap

Classification 4

- Matching ***without transfer***:
    - No transactions between any two agents
    - E.g. stable marriage problem
- Matching ***with transfer***:
    - Transactions between the matched agents
    - Transfer: money, credit, service, resources and so on…
    - E.g. spectrum trading between PU and SU
    - Solution: the assignment game

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- Motivation
  - Future Networks
  - Distributive Solutions
- Matching Theory Basics:
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  - Classifications
  - Examples
- Wireless Oriented Matching:
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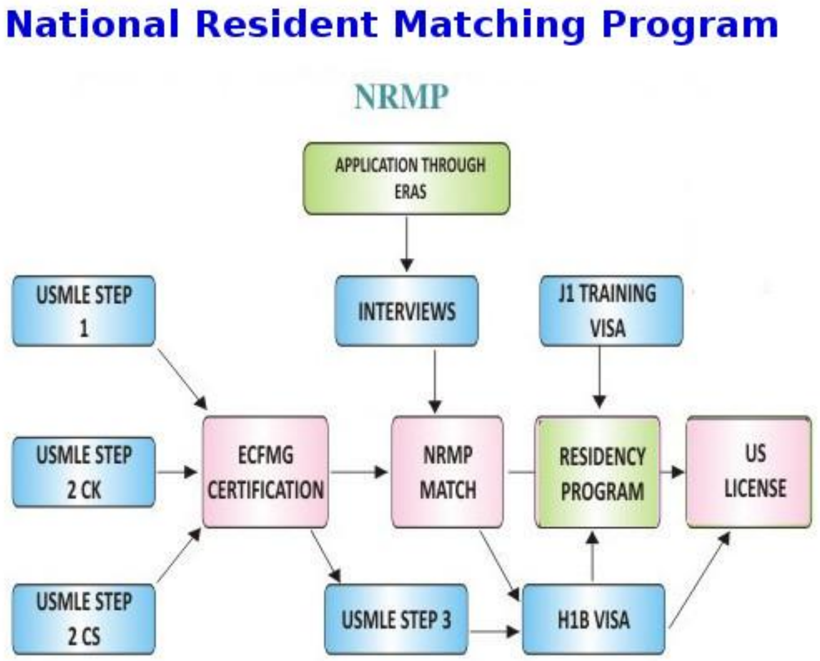
  - Applications

The Hospital Resident problem (Bipartite two-sided);
The House Allocation problem (Bipartite one-sided);
The Stable Roommate problem (Non-bipartite);
The Assignment problem (Matching with transfers).

1. The Hospital Resident (HR) problem



- Basics:
  - A set of hospitals, a set of residents;
  - Hospitals have capacity restriction;
- Generalized GS algorithm
- Rural Hospital Theorem:
  - The same residents are assigned in all stable matchings;
  - Each hospital is assigned the same number of residents in all stable matchings;
  - Any hospital that is undersubscribed in one stable matching is assigned exactly the same set of residents in all stable matchings.

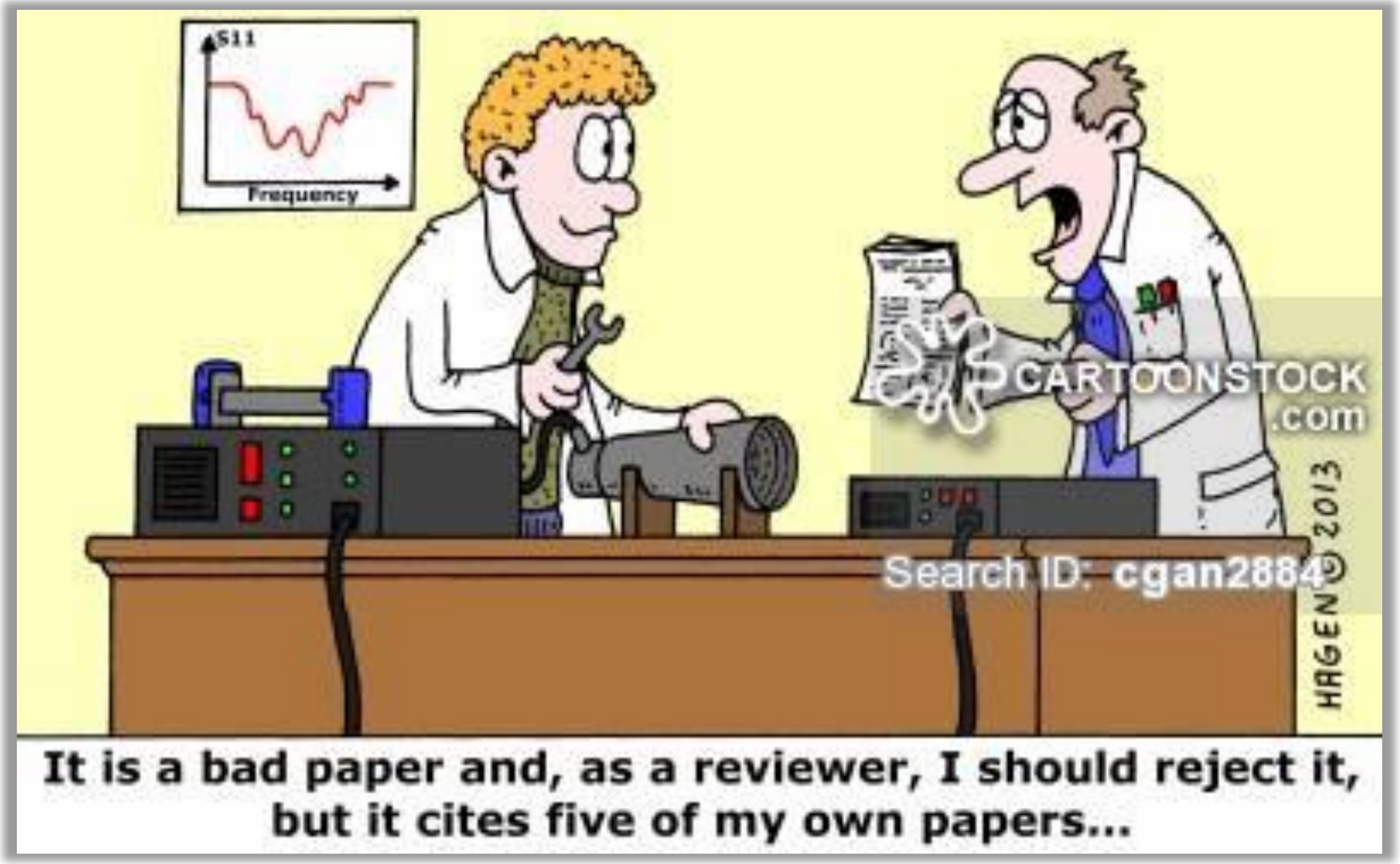
HR & its variants

- ***Three gender SM problem (3GSM)*:**
  - Men, women and dogs
  - Strictly-ordered preferences over pairs
  - Preferences over individual agents
- ***HR with Couples (HRC)*:**
  - Couples want to be allocated to the same hospital
- ***Mapping workers to firms (WF)*:**
  - many-to-many matching
  - Individual/group preferences

2. The House Allocation (HA) problem

- Basics:
  - A set of applicants and a set of houses
  - Houses have capacity limitations
- Optimality:
  - ***Pareto optimal***
    - No applicant can be better off without requiring another one to be worse off
  - ***Popular***
    - A matching is popular if there's no other matching that is preferred by the majority of the applicants
  - ***Profile-based optimal***
    - Satisfy certain condition of agents' rankings of their partners

HA variant: Reviewer assignment problem



HA & its variants

- ***Reviewer assignment problem (RA)*：**
  - Assignment considerations
    - Reviewers’ interest and expertise
    - Conflict of interest
    - Coverage
    - Load balancing
  - Solution Techniques
    - Heuristics, approximation algorithms, integer programming, polynomial-time algorithms and strategy-proof mechanisms;
  - Leximin optimal matching
    - Maximize the profile of the worst-off reviewer;
    - Fairness.

3. The stable roommate (SR) problem

- Basics:
  - A set of students looking for roommates;
  - One-to-one pair;
- Stability:
  - Stable matching may not exist;
- Algorithm:
  - Irving's algorithm;
  - Tan-Hsueh algorithm (conceptional simpler)

SR & its variants

- *Stable crews (SC)*

- *Stable fixtures (SF)*

- *Stable activity (SA)*
- *Stable multiple activity (SMA)*

4. The Assignment Game

The Assignment Game

- A two-sided ***buyer-seller market***:
  - Seller owes one product, and buyer seeks for one product;
  - Product (indivisible) exchange for money;
- Formulation: binary linear programming
- Distributed algorithm
  - Feasible solution
  - Making sequential price offer
  - Reaching ***competitive equilibrium*** (stability)

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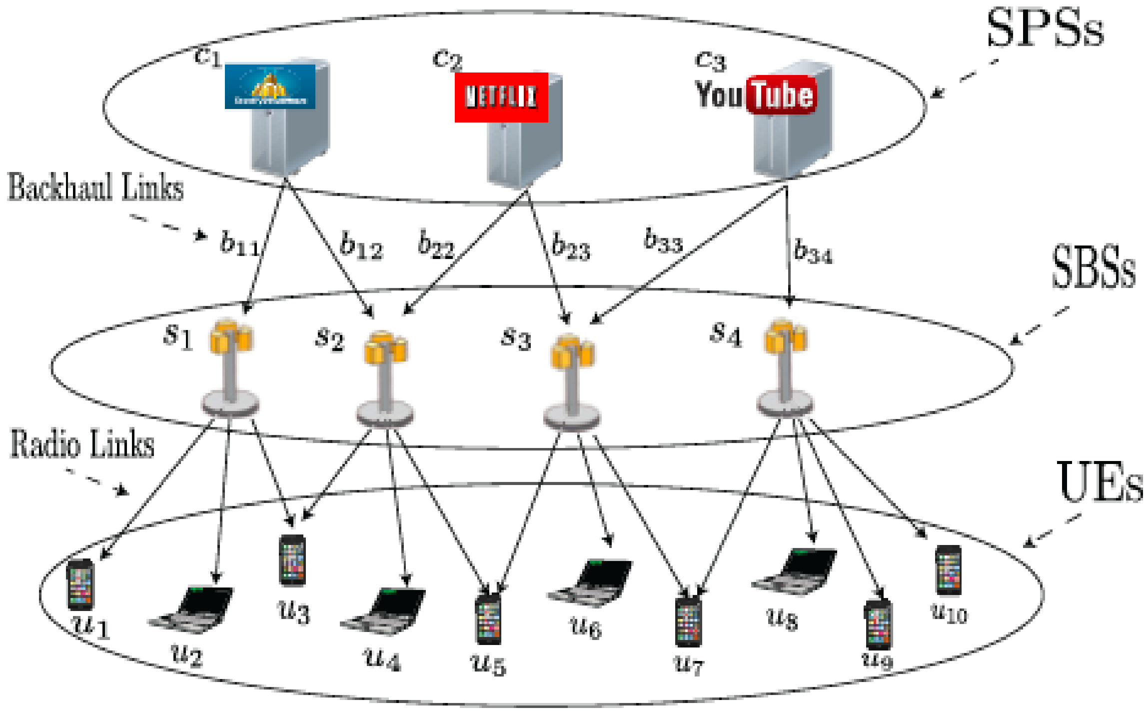
Wireless-Oriented Matching

- Modeling:
  - Users, resources: agents;
  - System QoS requirements: preference lists;
  - System objectives: stability, popularity, Pareto optimality…
  - Solutions: distributed, low-complexity algorithms.
- Advantages:
  - Game theory:
    - don’t require other players’ actions;
    - two-sided stability;
    - Not necessarily utility function needed;
  - Centralized optimization: Distributive and fast deployment.

Applications

- LTE-Unlicensed: static and dynamic stability
- D2D communication
- Physical player security
- Social caching in small cell
- LTE assisted V2X communication

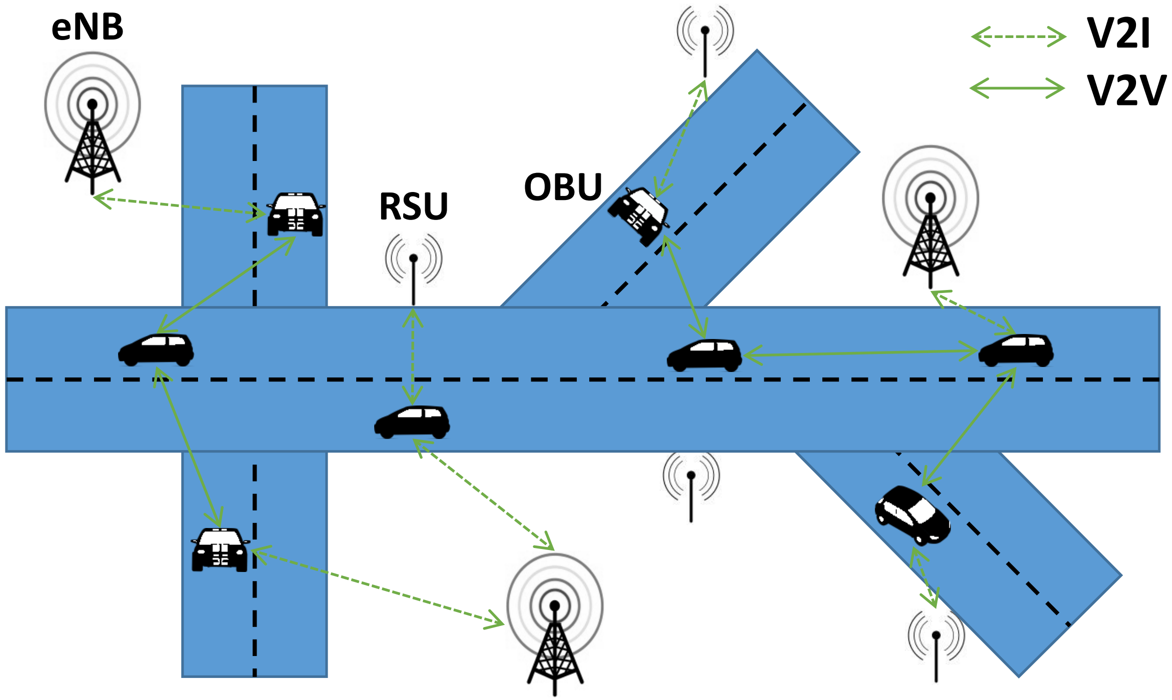
Social Caching in Small Cells



- ***Many-to-many matching*** between videos and SBSs
- Preference: Delay, Popularity
- Modified GS
- Pairwise stability (not blocked by individual nor pairs)

Hamidouche, K.; Saad, W.; Debbah, M., "Many-to-many matching games for proactive social-caching in wireless small cell networks," in Modeling and Optimization in Mobile, Ad Hoc, and Wireless Networks (WiOpt), 2014 12th International Symposium on , vol., no., pp.569-574, 12-16 May 2014

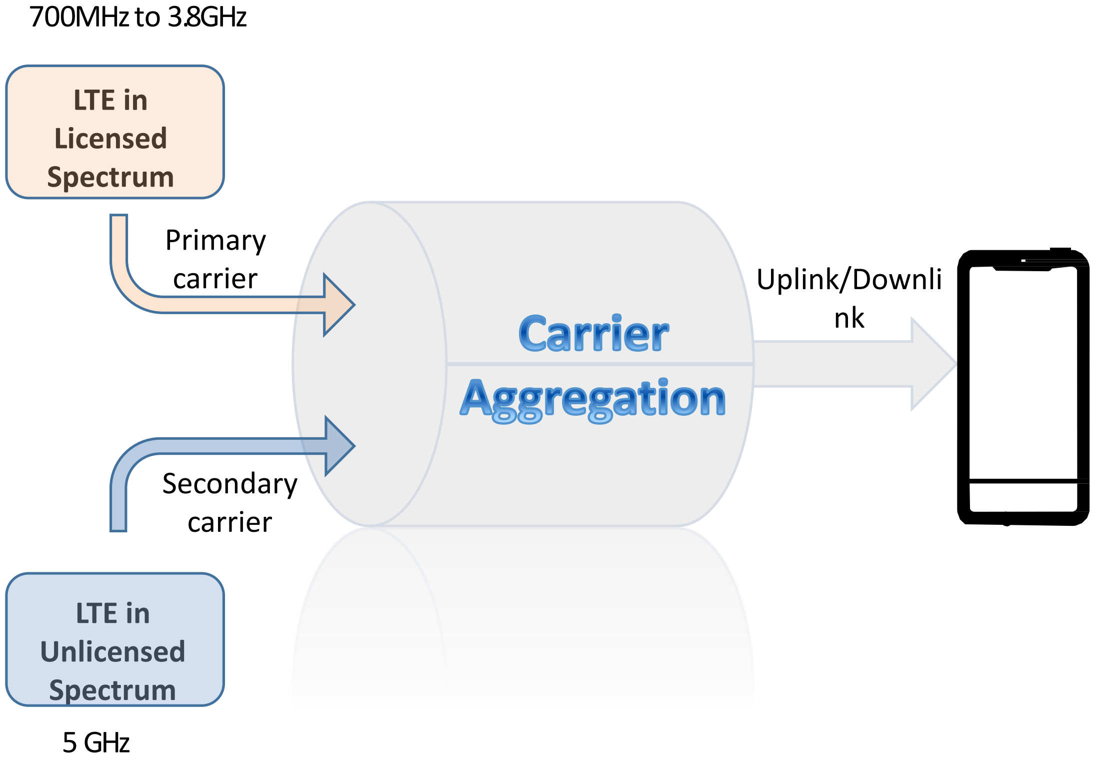
LTE Assisted V2X communications

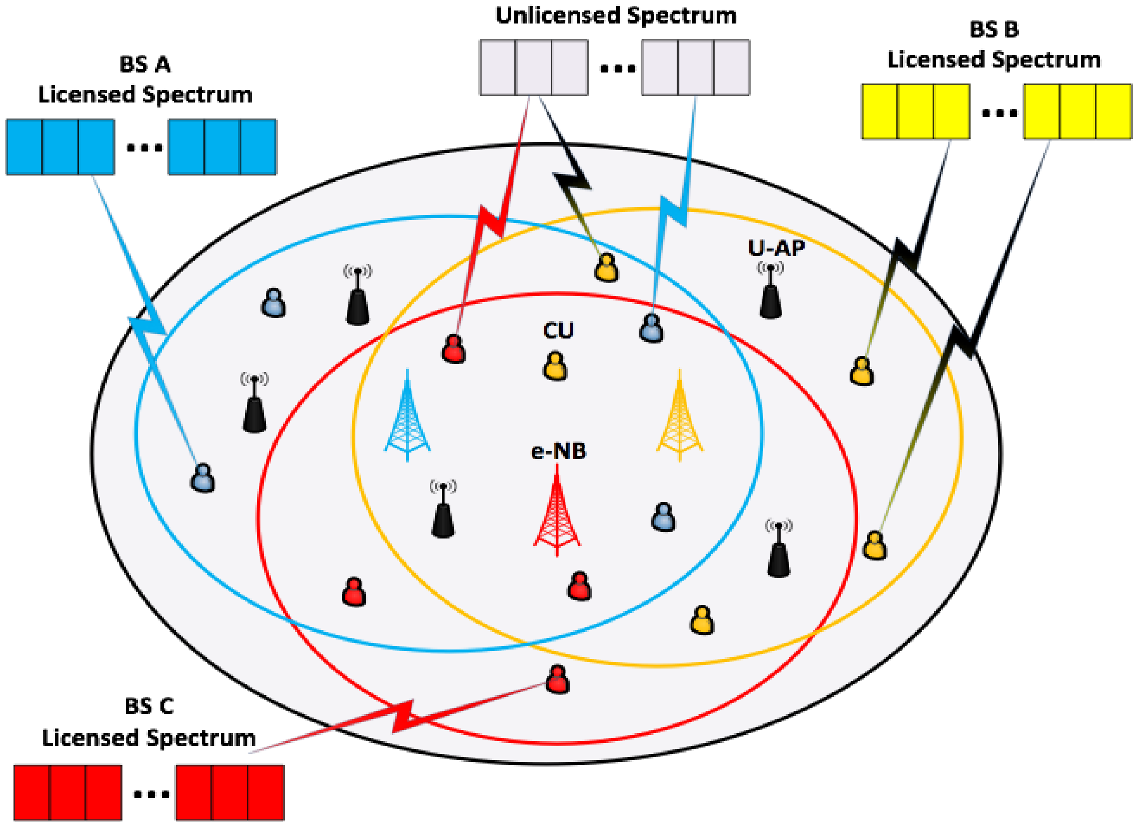


- LTE assisted vs 802.11p-based V2X:
  - Safety critical applications;
  - Better reliability;
  - Lower latency;
  - More efficient content sharing;
- Optimize content sharing;
  - Data class diversity;
  - Communication link quality;
- ***Stable fixture (SF)*** model;
  - Coalition formation/clustering;
  - Flexible many-to-many matching.

Y. Gu, L. X. Cai, M. Pan, L. Song and Z. Han, "Exploiting the Stable Fixture Matching Game for Content Sharing in D2D-based LTE-V2X Communications," IEEE Global Communications Conference (GLOBECOM), Washington DC, MA, 2016.

LTE-Unlicensed: Static





- Better network performance
- Unified network management
- Enhanced user experience

Coexistence issue?

Y. Gu, Y. Zhang, L. X. Cai, M. Pan, L. Song and Z. Han, "Exploiting Student-Project Allocation Matching for Spectrum Sharing in LTE-Unlicensed," 2015 IEEE Global Communications Conference (GLOBECOM), San Diego, CA, 2015, pp. 1-6.

LTE-Unlicensed Coexistence Issues

- Impact of cellular users (CUs) on unlicensed users (UUs):
  - Listen before talk for UUs;
- Impact of UUs on CUs:
  - Minimum SINR requirement for CUs;
- Impact of CUs on other CUs:
  - TDMA to avoid CU-CU interference;
  - The more CUs, the smaller share of resource;
  - ***External effect in matching!***

Student Project Allocation Model

- CU → student, UU → Project, LTE-U BS→ Lecturer;
- Acceptable lists: coexistence constraints;
- Preference of CU over UU:

$$PL_{value}^{cu} = w_j \log\left(1 + r_{i,j,k}^{cu}\right)$$

- Preference of LTE-U BS over (CU,UU):

$$PL_{value}^{ap} = w_j \log\left(1 + r_{i,j,k}^{cu}\right) + w_j \log\left(1 + r_{i,j,k}^{ap}\right)$$

- SPA-(S,P) algorithm to find stable matching.

Initialize:
Build PLs;

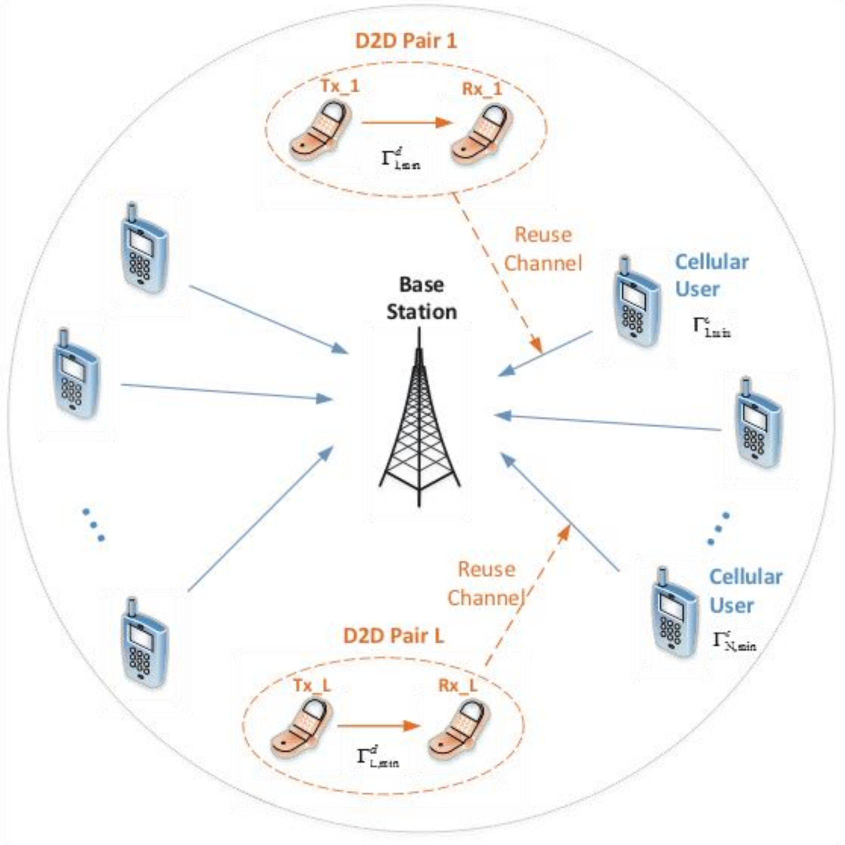
Stability is only guaranteed in conventional case.
TDMA brings external effect!
How to transform the matching into stable again?

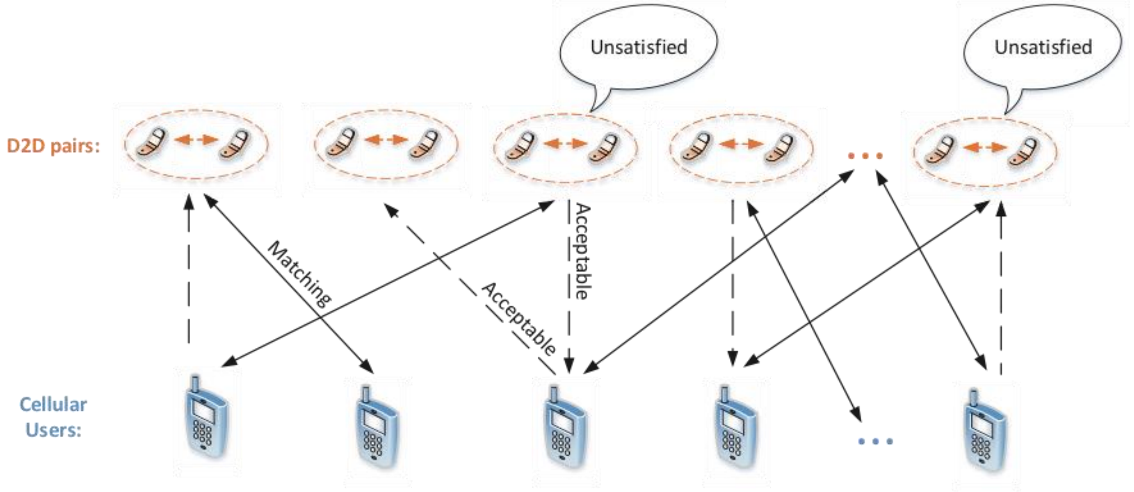
Yes

Yes

Terminate:
A stable matching.

D2D Communications





- D2D user pairs (DUs) search for cellular users (CUs) to share spectrum;
- ***Stable marriage*** model;
- Preference lists: channel condition, QoS;
- GS: stable one-one matching (DU,CU).

Some DUs are not matched to the best choice, and have the incentive improve by ***cheating*!**

Yunan Gu, Yanru Zhang, Miao Pan, and Zhu Han, "Matching and Cheating in Device to Device Communications Underlaying Cellular Networks," IEEE Journal on Selected Areas on Communications, Special Issue on Recent Advances in Heterogeneous Cellular Networks, vol. 33, no. 10, pp. 2156-2166, October 2015.

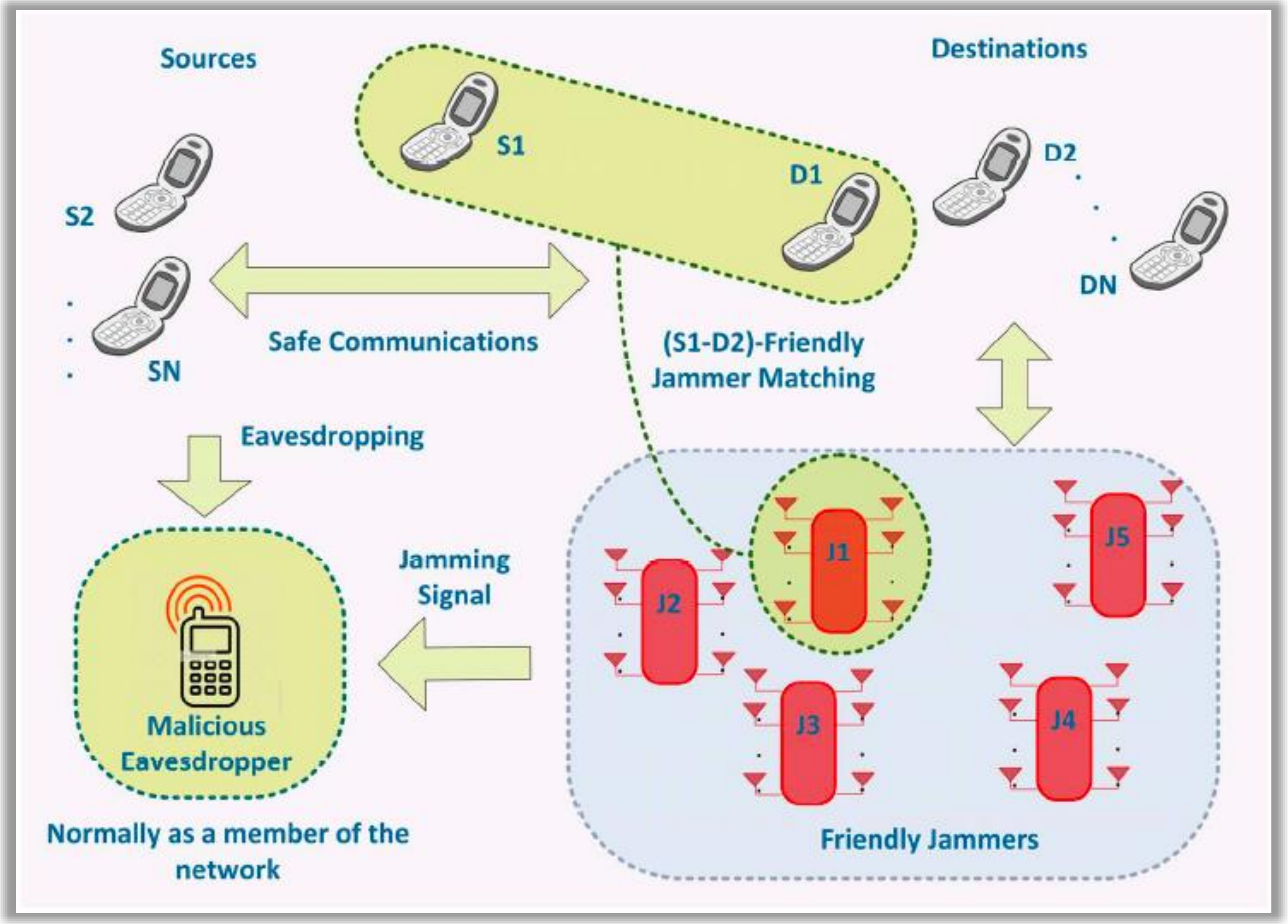
D2D Communications

- ***Strategic Issue (cheating)***
  - Achieve better partners by falsifying the preference lists;
- ***Coalition Strategy***
  - Find the ***cabal*** set
  - Find the ***accomplices*** of the cabal
  - Falsify the accomplices' preference lists
- If a cabal exist, then
  - Each man in the cabal is strictly better off;
  - Each man outside the cabal keeps their partners.

A group of men who prefer each other's partner to its own.

A group of men who would otherwise prevent the cabal to get their desired partners.

Physical Layer Security



- Form the (source, jammer) pair
- Decide the monetary compensation

The assignment game!

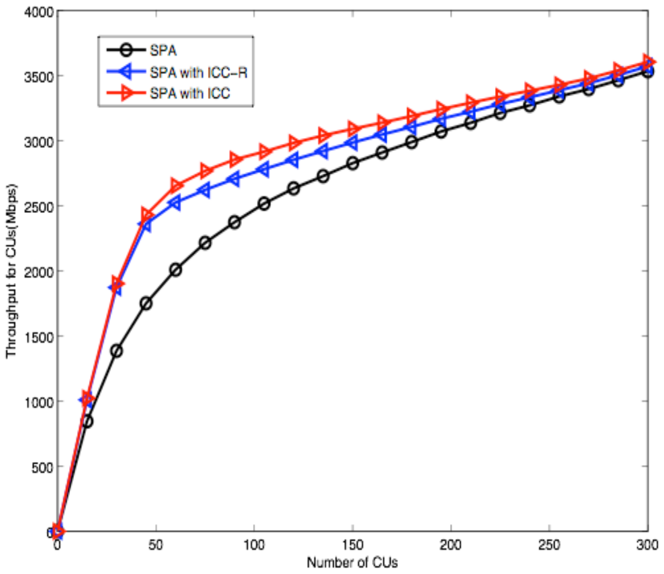
- Each jammer makes a price offer to source
- Source propose the the best jammer
- Jammer decides accept/reject, and increase the price by a certain amount
- Terminate with no new offer
- Competitive equilibrium (stability)

Bayat, S.; Louie, R.H.Y.; Zhu Han; Vucetic, B.; Yonghui Li, "Physical-Layer Security in Distributed Wireless Networks Using Matching Theory," in Information Forensics and Security, IEEE Transactions on , vol.8, no.5, pp.717-732, May 2013

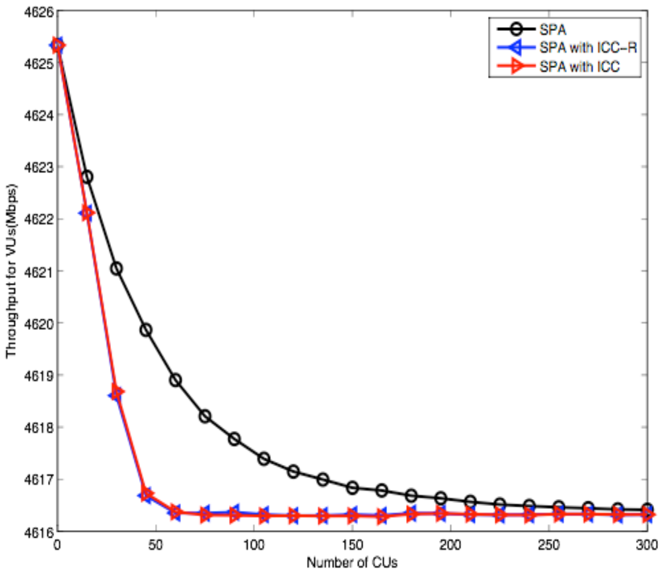
Inter Channel Cooperation (ICC)

- Stability
  - Only CUs can make changes;
  - Two-sided stability becomes one-sided: Pareto optimal
  - No player is better off without any other player(s) being worse off;
- ICC steps:
  - Firstly, search "unstable" CU-CU pairs regarding the current matching;
  - Secondly, check if the exchange is allowed (beneficial to all CUs);
  - Thirdly, find the allowed pair with the greatest throughput improvement, switch their partners, and update the current matching;
  - Keep searching "unstable" CU-CU pairs, until a trade-in-free environment.
- The convergence is guaranteed by the ***unreversibility*** of each switch.

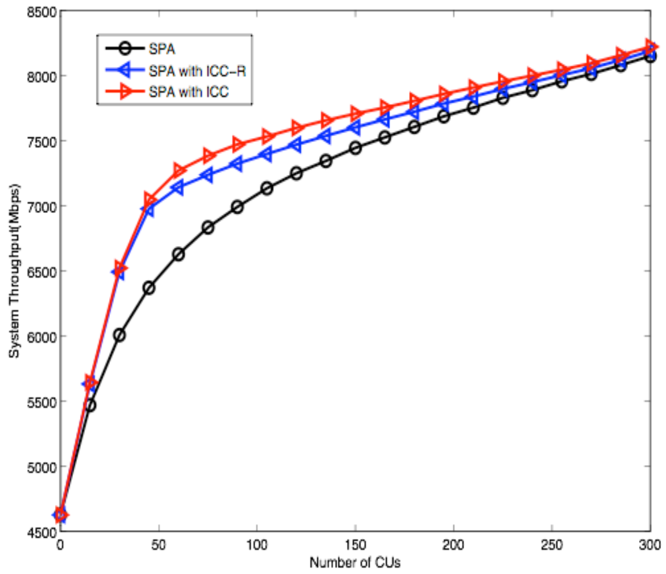
Performance Evaluation



(a) CUs' throughput



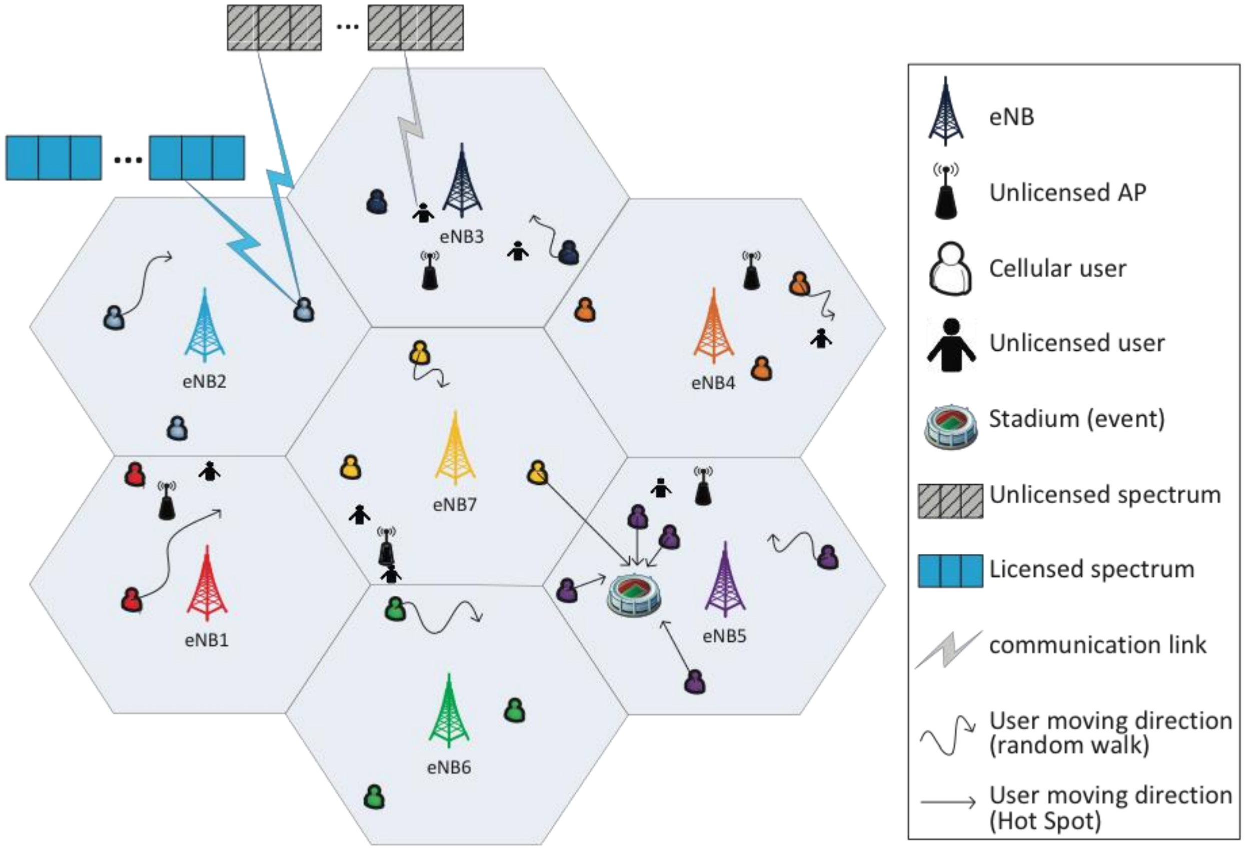
(b) UUs' throughput



(c) System throughput

- Throughput comparison: SPA, SPA with ICC-R, SPA with ICC.
  - Both ICC-R/ICC further improve CU/system throughput than SPA;
  - Both ICC-R/ICC slightly decrease UU throughput than SPA;
  - ICC outperforms ICC-R w.r.t. CU/system throughput.

Dynamic Stability in LTE-Unlicensed



User Mobility

Network dynamic

Dynamic Stability?

Yunan Gu, Lin X. Cai, Chunxiao Jiang, Lingyang Song, Miao Pan, Zhu Han, submitted to" Dynamic Path to Stability in LTE-Unlicensed with User Mobility: a matching theory framework", submitted to IEEE Transactions on Wireless Communications.

Random Path to Stability

- One-to-one:
  - ***Roth-Vande Vate (RVV)*** algorithm
  - Divorcing and remarrying operations
- Many-to-many (many-to-one):
  - Pairwise stability
  - ***RPTS*** algorithm (random path to stability)
  - Increase internal stable set by satisfying a blocking agent/pair
- Compared with repeated GS
  - ***Make use of the previous matching*** instead of staring from empty;
  - ***Reduce complexity.***

References

- *[1] http://www2.egr.uh.edu/~zhan2/*
- *[2] Li Huang; Guangxi Zhu; Xiaojiang Du; Kaigui Bian, "Stable multiuser channel allocations in opportunistic spectrum access," in Wireless Communications and Networking Conference (WCNC), 2013 IEEE , vol., no., pp.1715-1720, 7-10 April 2013*
- *[3]Siavash Bayat*, Raymond H. Y. Louie, Branka Vucetic and Yonghui Li, "Dynamic decentralized algorithms for cognitive radio relay networks with multiple primary and secondary users utilizing matching theory"*
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- *[9] Bayat, S.; Louie, R.H.Y.; Zhu Han; Vucetic, B.; Yonghui Li, "Distributed User Association and Femtocell Allocation in Heterogeneous Wireless Networks," in Communications, IEEE Transactions on , vol.62, no.8, pp.3027-3043, Aug. 2014*