GAME THEORY
Cooperative Game

ECE 697AA/597AA

Cooperation in Wireless Networks

- Cooperation in wireless networks
  - Cooperation among network nodes
    - Gains in terms of capacity, energy conservation or improved Bit Error Rate (BER)
  - Ubiquitous in many networks
    - Cognitive radio, sensor networks, WiMAX, ….
- Levels of cooperation
  - Network Layer Cooperation
    - Routing and Packet forwarding
  - Physical Layer Cooperation
    - Traditional Relay channel
    - Virtual MIMO

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Cooperative Game Theory

- Players have mutual benefit to cooperate
  - Startup company: everybody wants IPO, while competing for more stock shares.
  - Coalition in Parlement
- Namely two types
  - Bargaining problems
  - Coalitional game
- For coalitional game
  - Definition and key concepts
  - New classification
  - Applications in wireless networks

Walid Saad, Zhu Han, Merouane Debbah, Are Hjorungnes, and Tamer Basar, ``Coalitional Game Theory for Communication Networks'', IEEE Signal Processing Magazine, Special Issue on Game Theory, p.p. 77-97, September 2009.

Introduction to Bargaining

- Bargaining situation
  - A number of individuals have a **common** interest to cooperate but a conflicting interest on **how to cooperate**
- Key tradeoff
  - Players wish to reach an agreement rather than disagree but…
  - Each player is self interested
- What is bargaining?
  - Process through which the players on their own attempt to reach an agreement
  - Can be tedious, involving offers and counter-offers, negotiations, etc.
- Bargaining theory studies these situations, their outcome, and the bargaining process

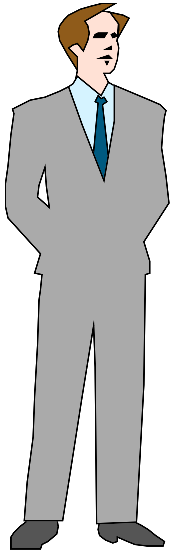
Introduction

- Key issues in bargaining
  1. The players must inspect efficiency and the effect of delay and disagreement on it
     - They seek a jointly efficient mutual agreement
  2. Distribution of the gains from the agreement
     - Which element from the efficient set must the players elect?
  3. What are the joint strategies that the players must choose to get the desired outcome?
  4. How to finally enforce the agreement?
- Link to game theory
  - Issues 1 and 2 are tackled traditionally by cooperative game theory
  - Issues 3 and 4 are strongly linked to non-cooperative game theory

Motivating Example

I can give you 100$ if and only if you agree on how to share it

Bargaining theory and the Nash bargaining solution!



Can be deemed unsatistifactory
Given each Person's wealth!!!

Rich Man

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The Nash Bargaining Solution

- **John Nash**'s approach
  - When presented with a bargaining problem such as the rich person – poor person case, how can we pick a reasonable outcome?
  - Interested in the **outcome** rather than the **process**
- **The Nash Bargaining Solution** was proposed in 1950 using an axiomatic approach and is considered as one of the key foundations of bargaining problems

The Nash Bargaining Solution

- Given a bargaining problem between **two** players
- Consider a **utility** region *S* that is compact and convex
  - A utility is a function that assigns a value to every player, given the strategy choices of **both** players
- Define the **disagreement or threat point** *d in S* which corresponds to the minimum utilities that the players want to achieve
- A Nash bargaining problem is defined by the pair *(S,d)*

The Nash Bargaining Solution

- Can we find a *bargaining solution*, i.e., a function f that specifies a **unique** outcome $f(S,d) \in S$?
- Axiomatic approach proposed by Nash
  - Axiom 1: Feasibility
  - Axiom 2: Pareto efficiency
  - Axiom 3: Symmetry
  - Axiom 4: Invariance to linear transformation
  - Axiom 5: Independence of irrelevant alternatives

The Nash Bargaining Solution

- Axiom 1: Feasibility
  - Can be sometimes put as part of the definition of the space *S*
- Feasibility implies that
  - The outcome of the bargaining process, denoted (u^*,v^*) cannot be worse than the disagreement point $d = (d_1,d_2)$, i.e., $(u^*,v^*) \geq (d_1,d_2)$
  - Strict inequality is sometimes defined
- Trivial requirement but important: the disagreement point is a benchmark and its selection is very important in a problem!

The Nash Bargaining Solution

- Axiom 2: Pareto efficiency
  - Players need to do as well as they can without hurting one another
- At the bargaining outcome, no player can improve without decreasing the other player's utility
  - Pareto boundary of the utility region
- Formally, no point (u,v) ϵ S exists such that $u > u^*$ **and** $v \geq v^*$ or $u \geq u^*$ **and** $v > v^*$

The Nash Bargaining Solution

- Axiom 3: Symmetry
  - If the utility region is symmetric around a line with slope 45 degrees then the outcome will lie on the line of symmetry
  - Formally, if $d_1 = d_2$ and S is symmetric around $u = v$, then $u^* = v^*$
- Axiom 4: Invariance to linear transformation
  - Simple axiom stating that the bargaining outcome varies linearly if the utilities are scaled using an affine transformation

The Nash Bargaining Solution

- Axiom 5: Independence of irrelevant alternatives
  - If the solution of the bargaining problem lies in a subset *U* of *S*, then the outcome does not vary if bargaining is performed on *U* instead of *S*
- Controversial axiom
  - If we increase the maximum utility achievable by a player, the outcome does not change!
  - It is shown that although the bargaining power of one player might improve in the bigger set, the other would not
  - We will explore an alternative in later slides

The Nash Bargaining Solution

- Nash showed that there exists a unique solution *f* satisfying the axioms, and it takes the following form (Nash product)

$$(u^*, v^*) = f(S, d) = \max_{(u,v)\in S} (u - d_1)(v - d_2)$$

Example: Splitting a cake

2 people are bargaining over splitting a cake:

- u= S (share of the cake)   d1 = 1/3
- v = 1 – S   d2 = 1/8

- What is the Nash Bargaining solution?

Rich person – poor person problem revisited

- Considering logarithmic utilities and considering that what the person's current wealth is as the disagreement point
  - The Nash solution dictates that the rich person receives **a larger** share of the 100$
- Is it fair?
  - Fairness is subjective here, the rich person has more bargaining power so he can threaten more to stop the deal
    - The poor person also values little money big as he is already poor!
  - Variant of the problem considers the 100$ as a debt, and, in that case, the NBS becomes fair, the richer you are the more you pay!

The Nash Bargaining Solution

- The NBS is easily extended to the N-person case
  - The utility space becomes N-dimensional and the disagreement point as well
  - Computational complexity definitely increases and coordination on a larger scale is required
- Solution to the following maximization problem

$$(u_1^*, \ldots, u_N^*) = f(S, d) = \max_{(u_1, \ldots, u_N) \in S} \prod_{i=1}^{N} (u_i - d_i)$$

The Nash Bargaining Solution

- If we drop the Symmetry axiom we define the <u>Generalize Nash Bargaining Solution</u>
- Solution to the following maximization problem

$$(u_1^*, \ldots, u_N^*) = f(S, d) = \max_{(u_1,\ldots,u_N)\in S} \prod_{i=1}^{N} (u_i - d_i)^{\alpha_i}$$

Value between 0 and 1 representing the bargaining power of player *i*
If equal bargaining powers are used, this is equivalent to the NBS

Nash Bargaining Solution – Summary

- The NBS/GNBS are a very interesting concept for allocating utilities in a bargaining problem
  - Provide Pareto optimality
  - Account for the bargaining power of the players but..
  - Can be unfair, e.g., the rich person – poor person problem
  - Require convexity of the utility region
  - Independence of irrelevant alternatives axiom
  - Provide only a static solution for the problem, i.e., no discussion of the bargaining process
- Alternatives?
  - Dynamic bargaining and the Rubinstein process

Dynamic Bargaining

- Dynamic bargaining
  - Interested in the players interactions to reach an agreement
  - Broader than static bargaining, although linked to it
  - Unlimited offers
  - Alternating offers
  - Delays are costly
  - In this trial lecture, we cover the Rubinstein process although many others exists

Rubinstein Bargaining Process

- Two players *A* and *B* bargain over the division of a cake of size 1
- Alternating-offers process
  - At time 0, *A* makes an offer to *B*
  - If *B* accepts, the game ends, otherwise
  - *B* rejects and makes a counter offer at time $\Delta > 0$
  - The process continues infinitely until an agreement is reached
- The payoff of a player *i* at any time is $x_i\delta^t$
  - x_i is the share of the cake for player *i*, $0 < \delta < 1$ a discount factor and *t* is the number of time intervals Δ elapsed
  - The discount factor is also function of a discount rate

Rubinstein Bargaining Process

- Rubinstein (1982) modeled this process as an extensive form non-cooperative game
  - When making an offer the player's strategy is the value of the share he requests
  - When responding to an offer the strategy is either accept or reject the offer (as a function of the history of the game so far)
- Rubinstein showed that the game admits a **unique subgame perfect equilibrium**
  - The equilibrium is also Pareto efficient

Rubinstein Bargaining Process

- At the equilibrium the shares of the players are

$$x_A^* = \frac{1}{1+\delta}, \quad x_B^* = \frac{\delta}{1+\delta}$$

  - At date t = 0, player *A* offers x_A, player *B* accepts and the game ends
  - First-mover advantage, player *A* gets more than player *B* but the result is Pareto efficient
  - As δ increases (the interval Δ decreases) this advantage starts to disappear
  - The rations depend highly on the relative discount rates
    - The winner is the strongest

Rubinstein Bargaining Process

- The Rubinstein model shows that being more patient increases your bargaining power!
  - The smaller the cost of “haggling”, the more waiting time you can sustain, the higher is your bargaining power
- If the process is frictionless then it becomes indeterminate!
- We reach NBS/GNBS as *Δ* goes to *0*

Other dynamic bargaining solutions

- The Rubinstein model is given for 2-player case, recent bargaining literature looks into variants for N-player games
- Varying preferences over time (non-stationary models)
- Emphasis on tools from non-cooperative games, e.g., repeated games, extensive form games, or stochastic games
- Dynamic bargaining is **hot** in bargaining literature nowadays

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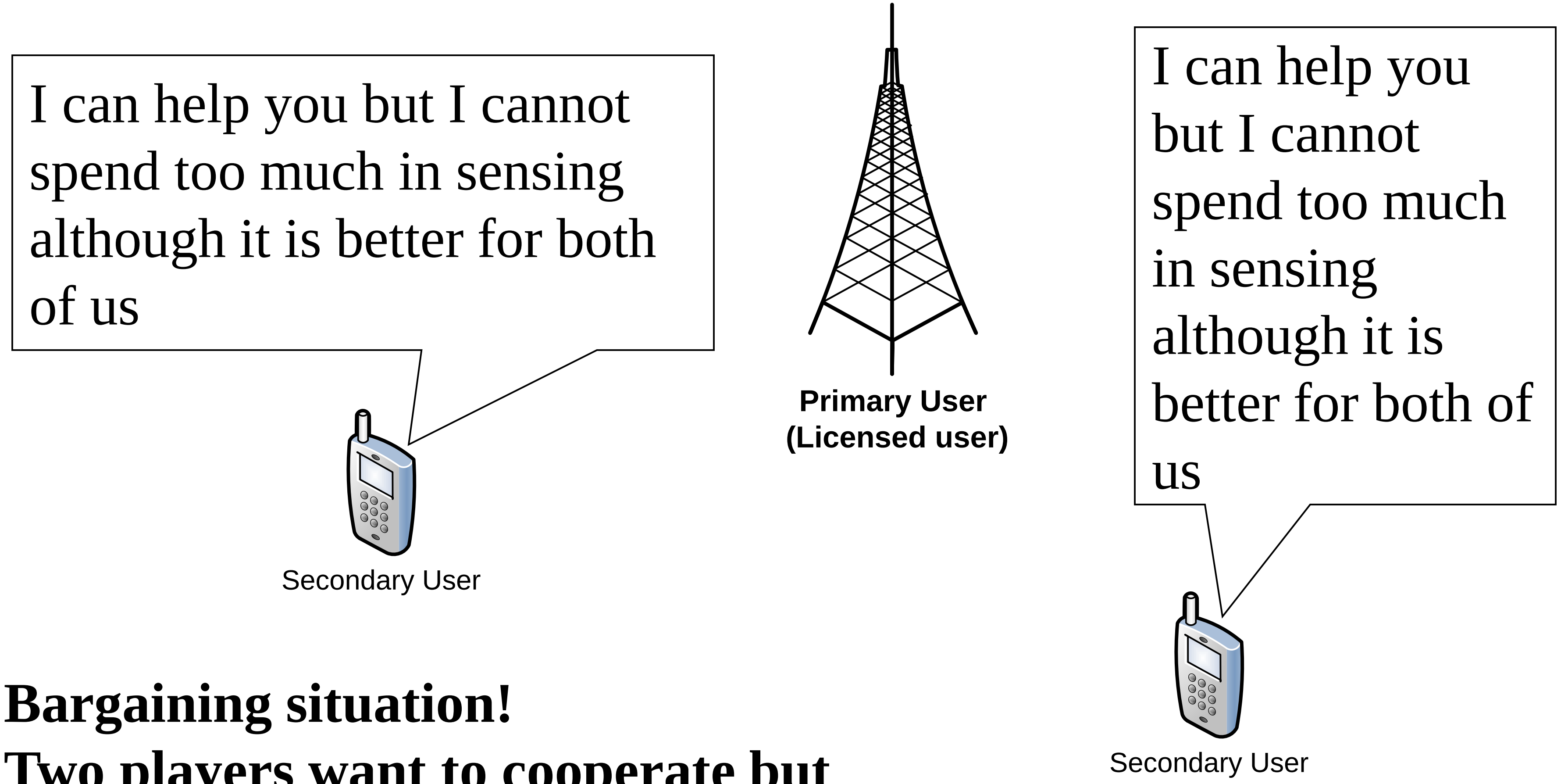
Bargaining in Cognitive Radio Networks – Spectrum Sensing



Bargaining situation!
Two players want to cooperate but they are self-interested

Bargaining in Cognitive Radio Networks – Spectrum Sensing

- Two secondary users bargain over sharing a time period
  - When and how long does each player spend in sensing given the loss in transmission time?
- Rubinstein problem of alternating offers
  - Two players
  - The share of the cake is the share of the time period each player uses for sensing
  - The utilities are a function of the time share, the rate, and the probability of detection
  - Discount rate is function of the PU – SU distance

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