1 Problem 1

Consider the game with the following bimatrix

	A	B	C
a	1, 1	3, x	2, 0
b	2x, 3	2, 2	3, 1
c	2, 1	1, x	xˆ2, 4

(a) Find x so that the game has no pure Nash equilibrium.

(b) Find x so that the game has (c, C) as a pure Nash equilibrium.

(c) What is the Pareto optimal solution when x = 0?

2 Problem 2

Consider the battle of the sexes game from class where player 1 prefers to do activity B with their partner and player 2 prefers to do activity F with their partner. For this payoff matrix (shown below) we calculated in class a Nash Equilibrium where player 1 chooses B 2/3 of the time and F 1/3 of the time; whereas player 2 chooses B 1/3 of the time and F 2/3 of the time: (2/3, 1/3), (1/3, 2/3). Show your calculations in your answers.

	B	F
B	2, 1	0,0
F	0,0	1,2

(a) If the players are rational and are correct in assuming the other player is also rational, the players will choose the activities with the above frequencies. In this case, in what proportion of their outings is the couple together (doing the same activity)?

(b) In what proportion of their outings is player 1 doing B by himself/herself while player 2 does F?

(c) In what proportion of their outings is player 2 doing B by himself/herself while player 1 does F?

(d) Sometimes for convenience, we refer to units of utility as “utils”. Assuming the same mixed strategies above, when they go out, what is the average expected payoff for player 1 in utils during each outing? For player 2?

(e) Consider the case where player 2 knows that player 1 is selfish and stubborn and will always choose activity B. what is the average expected payoff for player 1 in utils during each outing? For player 2?

3 Problem 3

Consider the following extensive form game:

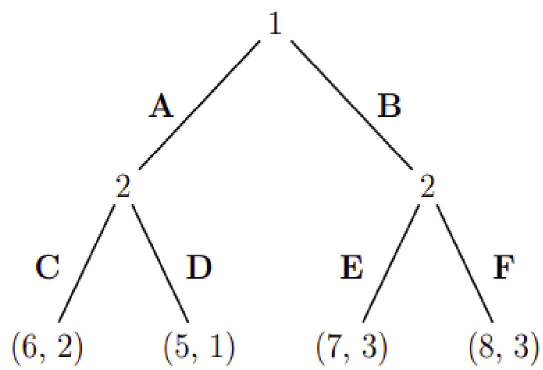

Figure 1

(a) Transform this game into a strategic game.

(b) Use backward induction to find a pure Nash equilibrium.

(c) Identify all subgame perfect Nash equilibrium.

4 Problem 4

Let us assume that there are n nodes (players) sending data on the network simultaneously. Let $x_i \geq 0$ be the size of data sent by node i. Each node i chooses x_i independently. The speed of network is inversely proportional to the total size of the data, so that it takes $x_i\tau(x_1, ..., x_n)$ minutes to send the message where

$\tau(x_1, ..., x_n) = x_1 + ... + x_n$

The payoff of node i is

$\theta_i x_i - x_i\tau(x_1, ..., x_n)$,

where $\theta_i \in 1, 2$ is a payoff parameter of node i, privately known by the node itself. For each $j \neq i$, independent of θ_j , node j assigns probability 1/2 to $\theta_i = 1$ and probability 1/2 to $\theta_i = 2$.

(a) Write this game formally as a Bayesian game.

(b) Compute the symmetric Bayesian Nash equilibrium of this game. Here symmetric means that $x_i(\theta_i) = x_j(\theta_j)$ when $\theta_i = \theta_j$.