7. Parallel lines are preserved in **affine geometry**. However, angles and distances are not. An affine transformation is defined as $\mathbf{x} \mapsto A\mathbf{x} + \mathbf{b}$, where A is an invertible matrix. Show that an affine transformation is not linear for $\mathbf{b} \neq \mathbf{0}$.

An affine transformation extends to $Ax+b$

Let $S(x) = Ax+b$, $b \neq 0$

Let $\alpha, \beta \in F$ & $x, y \in$ domain of S

$$S(\alpha x + \beta y) = A(\alpha x + \beta y) + b$$
$$= A\alpha x + A\beta y + b$$
$$= \alpha Ax + \beta Ay + b$$
$$= \alpha S(x) + \beta S(y) + b$$

$$S(\alpha x + \beta y) \neq \alpha S(x) + \beta S(y)$$

$\therefore S(x) = Ax+b$ is not linear transformation

4. Let $P = \begin{bmatrix} 4 & -9 & 5 \\ -3 & -1 & 6 \\ 9 & -2 & -6 \end{bmatrix}$, $\mathbf{b}_1 = \begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix}$, $\mathbf{b}_2 = \begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix}$, and $\mathbf{b}_3 = \begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$.

Find a basis $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ for $\mathbb{R}^3$ such that P is the change-of-coordinates matrix from $\{\mathbf{a}_1, \mathbf{a}_2, \mathbf{a}_3\}$ to the basis $\{\mathbf{b}_1, \mathbf{b}_2, \mathbf{b}_3\}$.

given P and b_1, b_2, b_3 and $\{b_1, b_2, b_3\}$

$P = [(a_1)_B, (a_2)_B, (a_3)_B]$ where $(a_1)_{B_i}$ is the coordinate vector with respect to the basis B

$a_1 = 4b_1 - 3b_2 + 9b_3$

$$a_1 = 4\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} - 3\begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix} + 9\begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} 15 \\ 8 \\ -30 \end{bmatrix}$$

$a_2 = -9b_1 - b_2 - 2b$

$$a_2 = -9\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} - 1\begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix} - 2\begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix} = \begin{bmatrix} -10 \\ -2 \\ -11 \end{bmatrix}$$

$$a3 = 5\begin{bmatrix} 0 \\ -1 \\ 3 \end{bmatrix} + 6\begin{bmatrix} 4 \\ 5 \\ -4 \end{bmatrix} - 6\begin{bmatrix} 3 \\ 3 \\ -6 \end{bmatrix}$$

$$= \begin{bmatrix} 6 \\ 7 \\ 27 \end{bmatrix}$$

$a_1 = \begin{bmatrix} 15 \\ 8 \\ -30 \end{bmatrix}$ $a_2 = \begin{bmatrix} -10 \\ -2 \\ -11 \end{bmatrix}$ $a_3 \begin{bmatrix} 6 \\ 7 \\ 27 \end{bmatrix}$ are the basis of R^3

8. Find the transformation matrix T that satisfies the following:

$$T(x_1, x_2, x_3) = (2x_1 + x_3, -x_2 + 4x_3, x_1 + 6x_2, 4x_1)$$

- given transformation is from $R^3 \to R^4$
- Standard Basis for R^3 $\{(1,0,0),(0,1,0),(0,0,1)\}$
- Standard Basis for R^4 is $\{(1,0,0,0),(0,1,0,0),(0,0,1,0),(0,0,0,1)\}$

$T(1,0,0) = (2,0,1,4)$

$T(0,1,0) = (0,-1,6,0)$

$T(0,0,1) = (1,4,0,0)$

so $(2,0,1,4) = 2a + 0b + 1c + 4d$

$(0,-1,6,0) = 0a - 1b + 6c + 0d$

$(1,4,0,0) = 1a + 4b + 0c + 0d$

$$\boxed{\text{Matrix } T = \text{Transformation} \begin{bmatrix} 2 & 0 & 1 \\ 0 & -1 & 4 \\ 1 & 6 & 0 \\ 4 & 0 & 0 \end{bmatrix}}$$