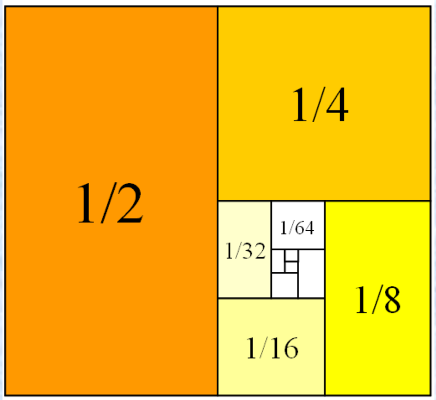


Chapter 1: Function and Graphs

Department of Mathematics, FPT University

Chapter 1: Function and Graphs

Objectives

- Four ways to represent a function
- Basis functions and the transformations of functions

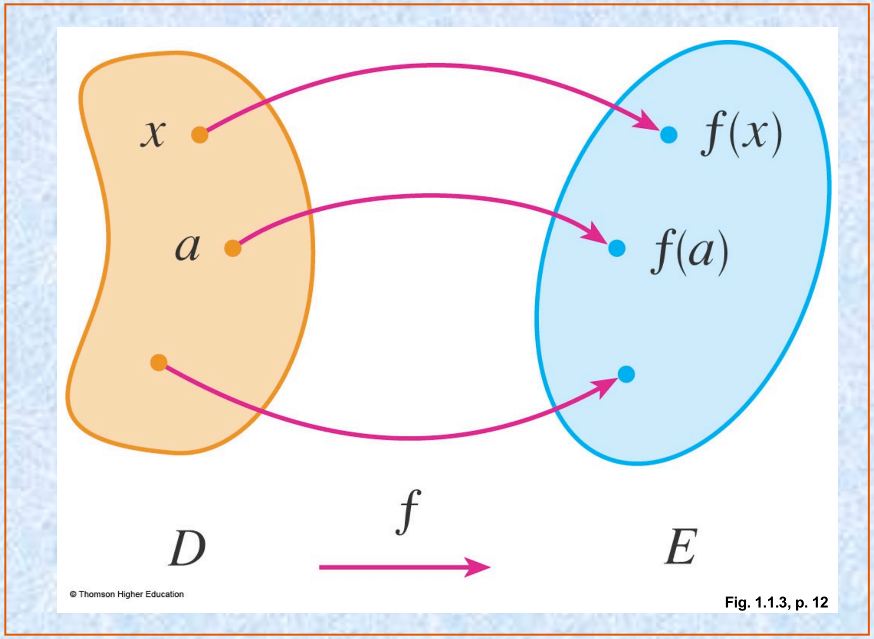
1.1
Review of Functions

FUNCTION

A **function** *f* is a rule that assigns to each element *x* in a set ***D*** *exactly one* element, called *f*(*x*), in a set ***E***.

The set *D* is called the **domain** of the function *f*.

The **range of *f*** is the set of all possible values of *f*(*x*) as *x* varies throughout the domain.

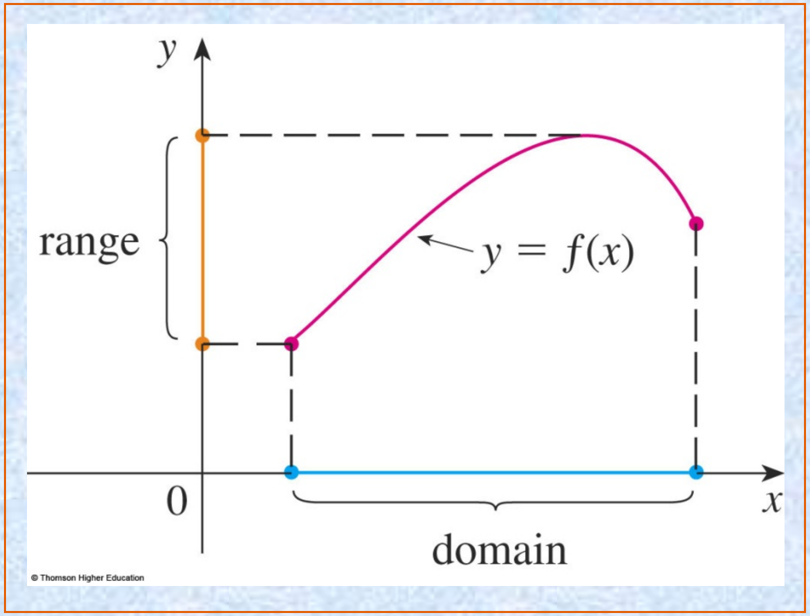


GRAPH

The **graph** of f is the **set** *of all points (x, y) in the coordinate plane such that y = f(x) and x is in the domain of f*.

The graph of *f* also allows us to picture:

- The **domain of *f*** on the *x*-axis
- **Its range** on the *y*-axis



GRAPH

Example 1

The graph of a function *f* is shown.

a. Find the values of *f*(1) and *f*(5).

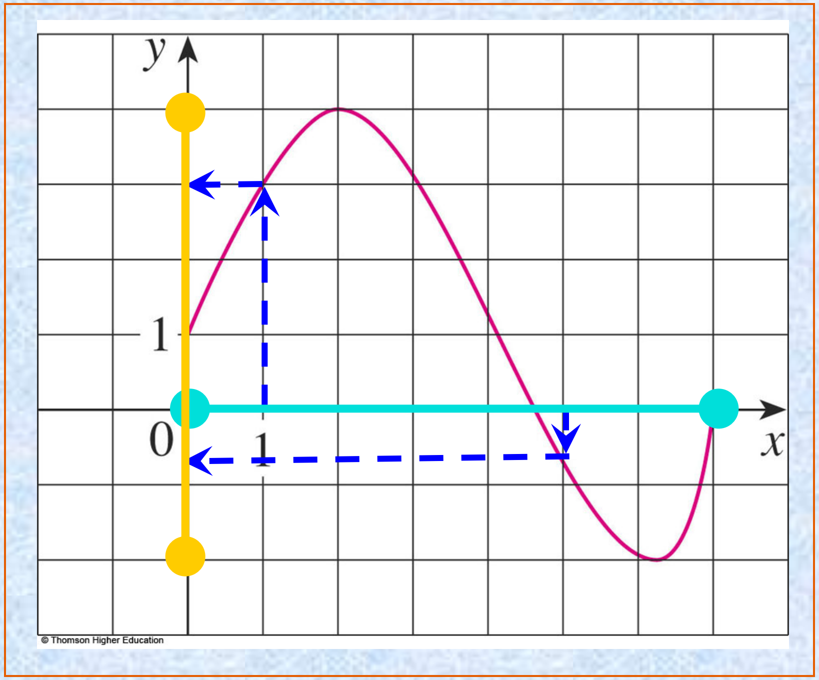
b. What is the domain and range of *f* ?

f(1) = 3

f(5) = - 0.7

D = [0, 7]

Range(f) = [-2, 4]



Find the domain and region of the functions (if it is a function).

a.

$f(n)=\sqrt{n}$ for all natural numbers n.

b.

$g(x)$ is any real number such that larger than x

REPRESENTATIONS OF FUNCTIONS

There are four possible ways to represent a function:

- Algebraically (by an explicit formula)
- Visually (by a graph)
- Numerically (by a table of values)
- Verbally (by a description in words)

EXAMPLE

The human population of the world *P* depends on the time *t*.

- The table gives estimates of the world population *P*(*t*) at time *t*, for certain years.
- However, for each value of the time *t*, there is a corresponding value of *P*, and we say that *P* is a function of *t*.

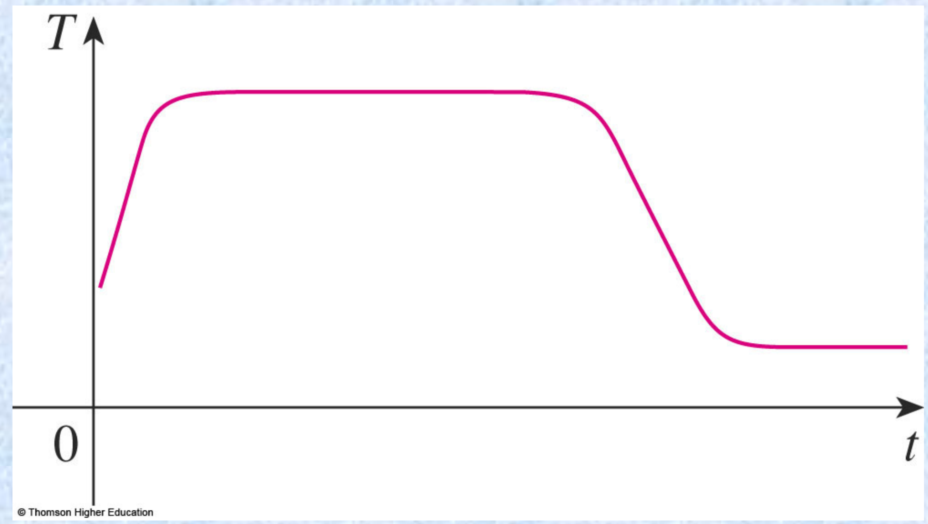
Year	Population (millions)
1900	1650
1910	1750
1920	1860
1930	2070
1940	2300
1950	2560
1960	3040
1970	3710
1980	4450
1990	5280
2000	6080

REPRESENTATIONS

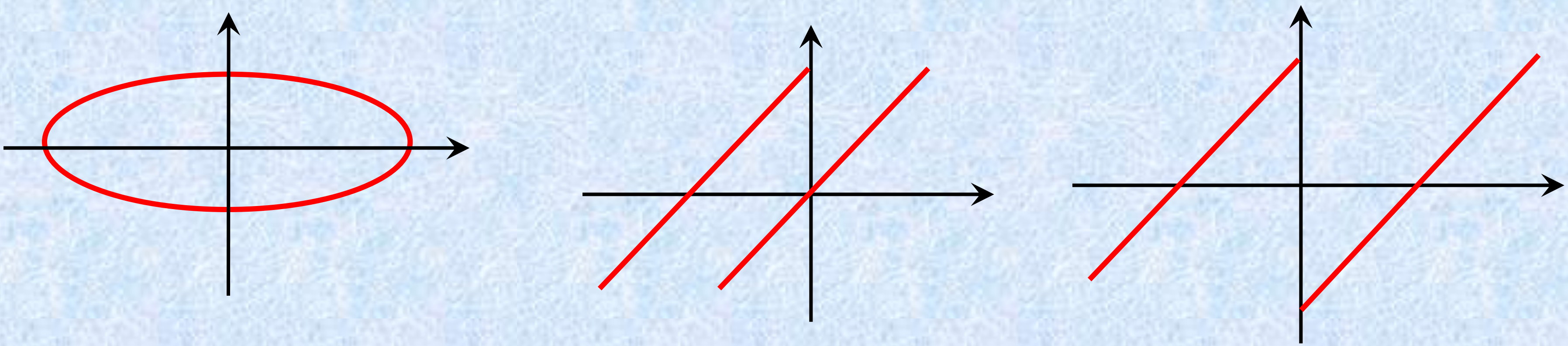
Example

"*When you turn on a hot-water faucet, the temperature T of the water depends on how long the water has been running*".

Draw a rough graph of T as a function of the time t that has elapsed since the faucet was turned on.



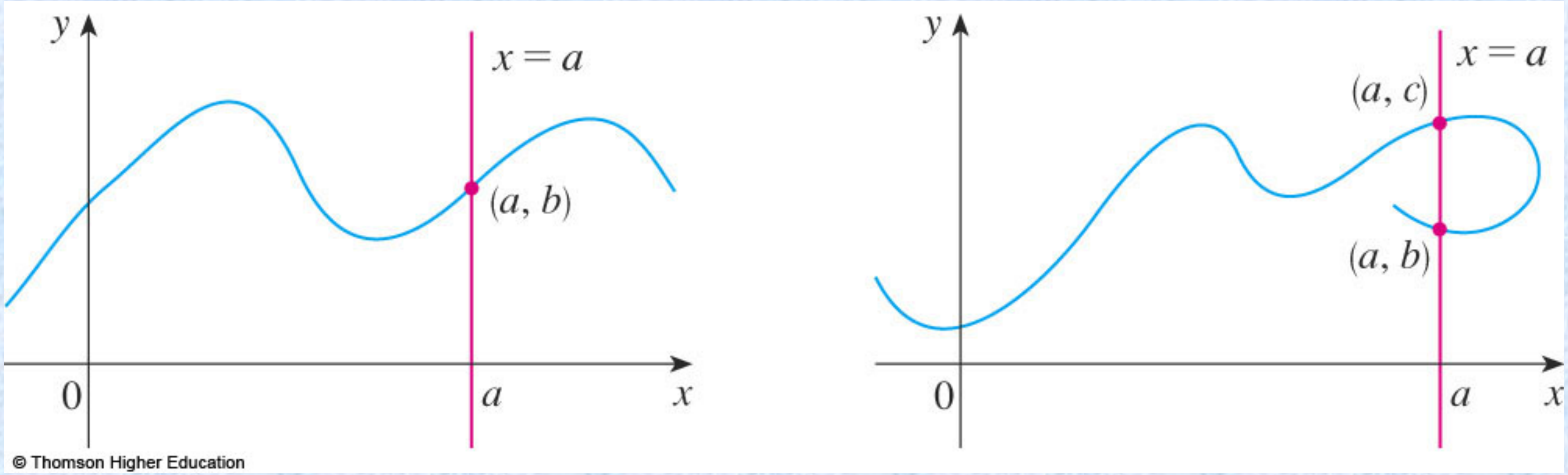
THE VERTICAL LINE TEST



A curve in the *xy*-plane is the graph of a function of x if and only if **no vertical line** intersects the curve **more than once**.

THE VERTICAL LINE TEST

The reason for the truth of the Vertical Line Test can be seen in the figure.



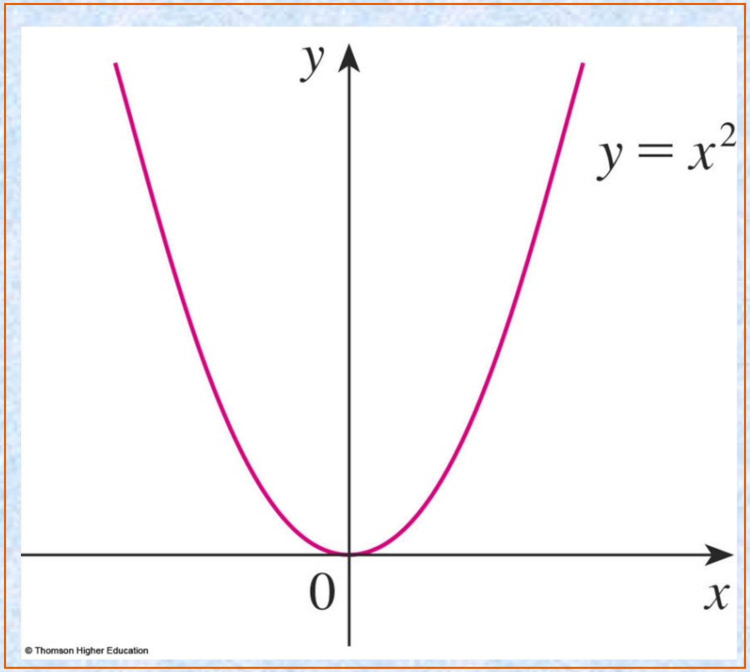
INCREASING AND DECREASING FUNCTIONS

A function f is called **increasing on an interval** $\boldsymbol{I}$ if:

$$f(x_1) < f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$

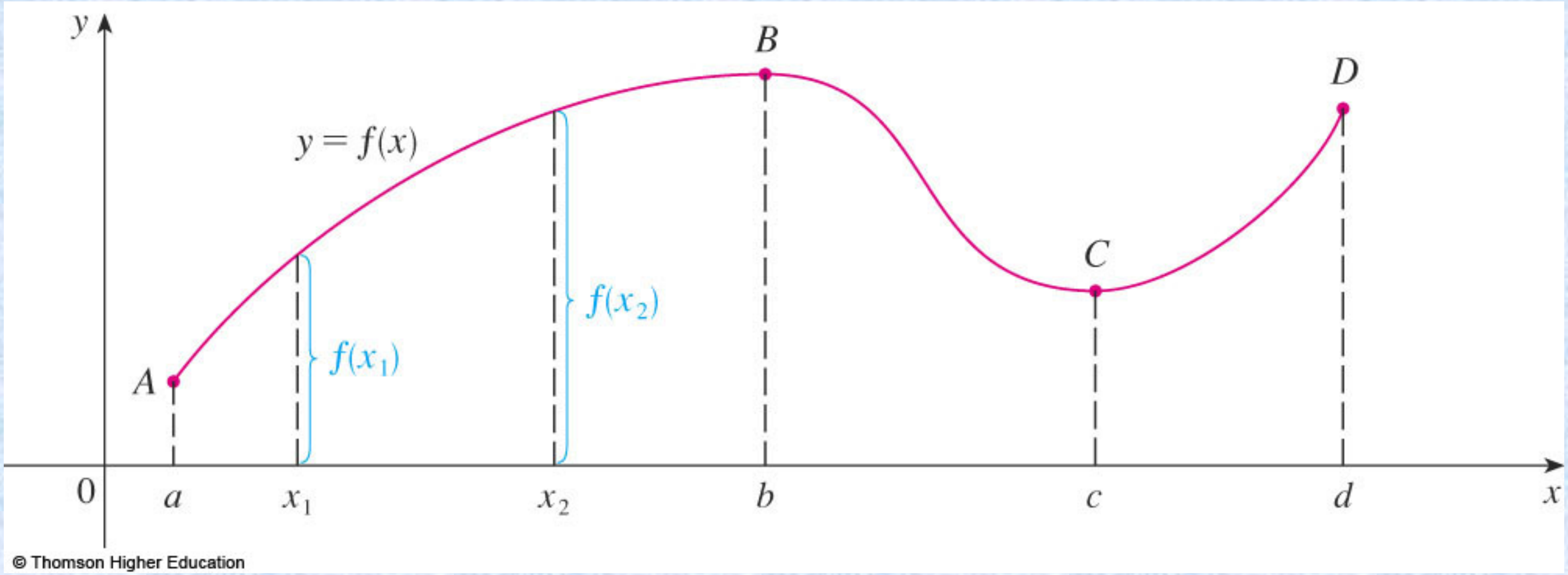
It is called **decreasing on** $\boldsymbol{I}$ if:

$$f(x_1) > f(x_2) \quad \text{whenever } x_1 < x_2 \text{ in } I$$



INCREASING AND DECREASING FUNCTIONS

The function *f* is said to be **increasing on the interval [*a*, *b*]**, decreasing on [*b*, *c*], and increasing again on [*c*, *d*].



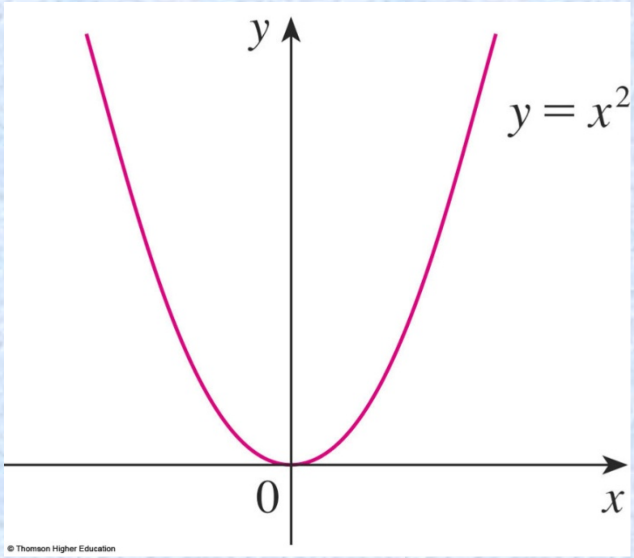
SYMMETRY: EVEN FUNCTION

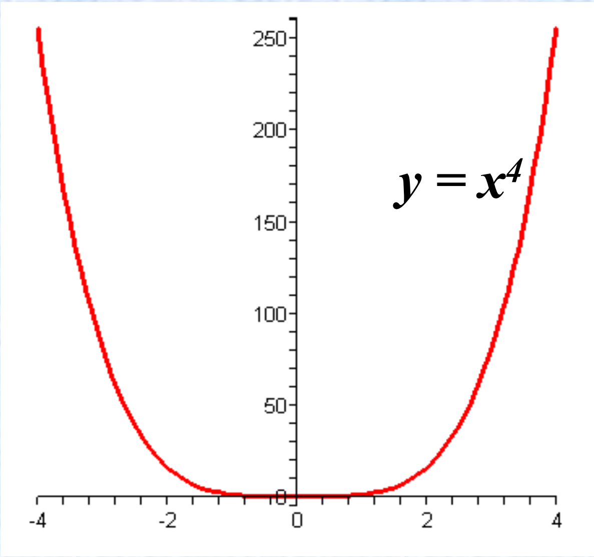
If a function *f* satisfies:

$$\boxed{f(-x) = f(x), \text{ for all } x \text{ in } D}$$

then *f* is called an **even function.**

- The geometric significance of an even function is that its graph is **symmetric with respect to the *y*-axis**.





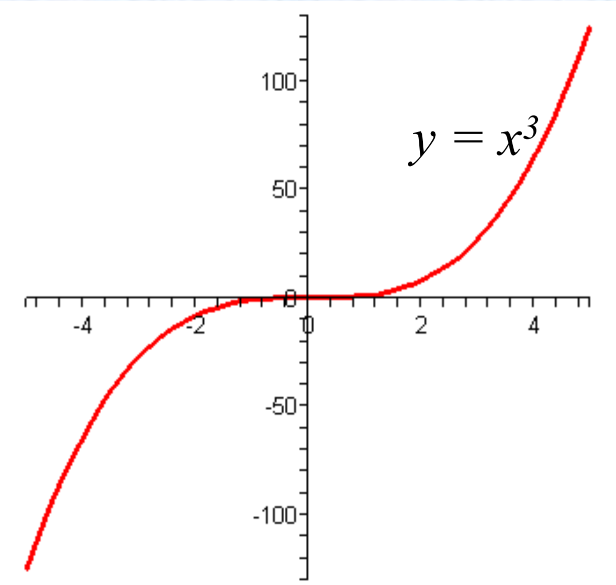
SYMMETRY: ODD FUNCTION

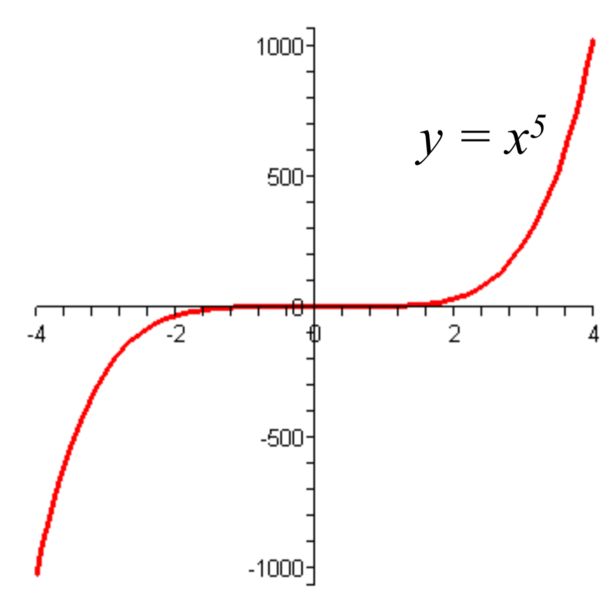
If f satisfies:

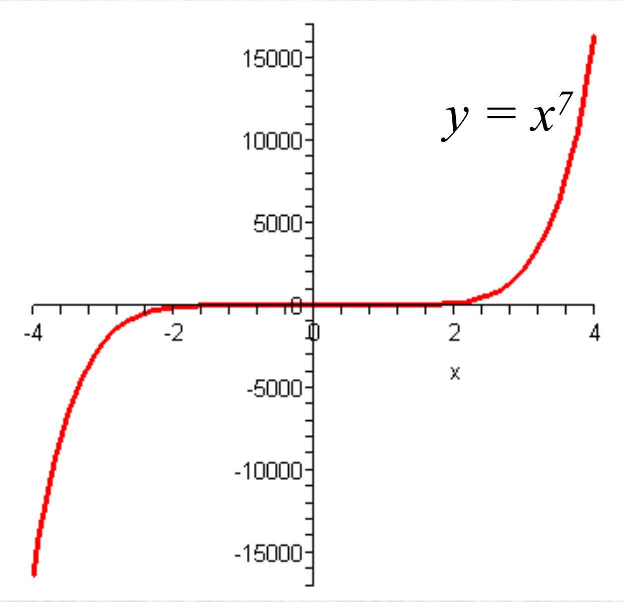
$$\boxed{f(-x) = -f(x), \text{ for all } x \text{ in } D}$$

then f is called an **odd function**.

- The graph of an odd function is **symmetric about the origin**.







Example

Let f is an **odd function**. If (-3,5) is in the graph of f then which point is also in the graph of f?

a. (3,5) b. (-3,-5) c. (3,-5) d. All of the others

Answer: c

Example

Suppose f is an odd function and g is an even function.

What can we say about the function f.g defined by $(f.g)(x)=f(x)g(x)$?

Prove your result.

QUIZ QUESTIONS

1) If f is a function then $f(x+2)=f(x)+f(2)$

a. True

b. False

2) If $f(s)=f(t)$ then $s= t$

a. True

b. False

3) Let f be a function.
We can find s and t such that $s=t$ and $f(s)$ is not equal to $f(t)$

a. True

b. False

COMBINATIONS OF FUNCTIONS

- Two functions *f* and *g* can be combined to form new functions:
  - $(f + g)x = f(x) + g(x)$
  - $(f - g)x = f(x) - g(x)$

$$(fg)(x) = f(x)g(x) \qquad \left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}$$

  - $(f \circ g)(x) = f(g(x))$

Let $h(x)=f(g(x))$.

1) If $g(x)=x-1$ and $h(x)=3x+2$ then $f(x)$ is:

a. $3x+3$ b. $3x+4$ c. $3x+1$ d. None of them

2) If $h(x)=3x+2$ and $f(x)=x-1$ then $g(x)$ is:

a. $3x+3$ b. $3x+4$ c. $3x+1$ d. None of them

Answer: 1) d 2) a

QUIZ QUESTIONS

1) If f and g are functions, then $f \circ g = g \circ f$

a. True

b. False

2)

x	1	2	3	4	5	6
$f(x)$	3	2	1	0	1	2
$g(x)$	6	5	2	3	4	6

$(f \circ g)(2)$ is

a. 5

b. 1

c. 2

d. None of the others

1.2

BASIC CLASSES OF FUNCTIONS

ALGEBRAIC FUNCTIONS

LINEAR MODELS

When we say that *y* is a **linear function** of *x*, we mean that the graph of the function is a line.

- So, we can use the slope-intercept form of the equation of a line to write a formula for the function as

$$y = f(x) = mx + b$$

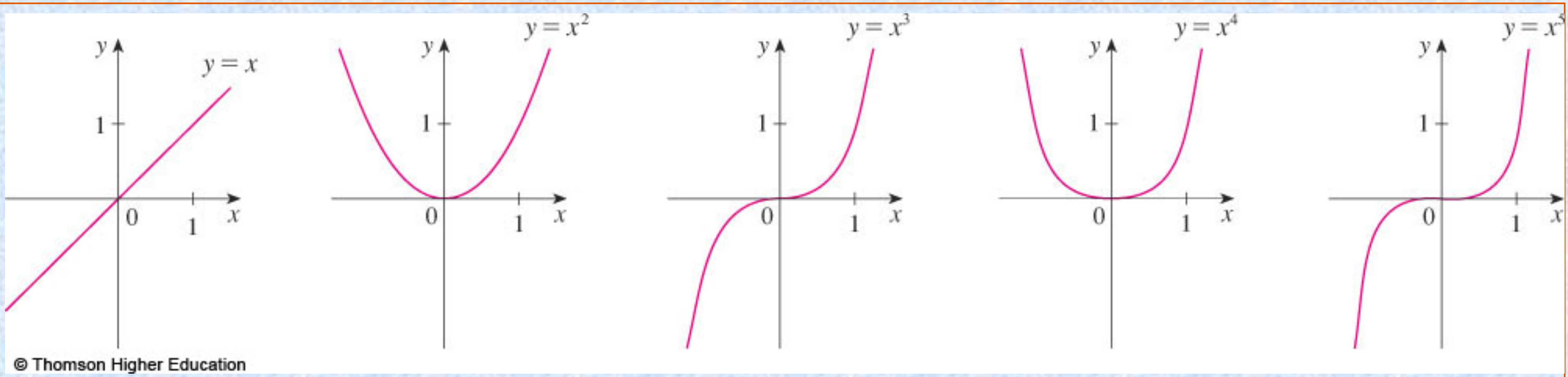
where *m* is the slope of the line and *b* is the *y*-intercept.

ALGEBRAIC FUNCTIONS

POWER FUNCTIONS

A function of the form $f(x) = x^{a}$, where *a* is constant,

is called **a power function**.



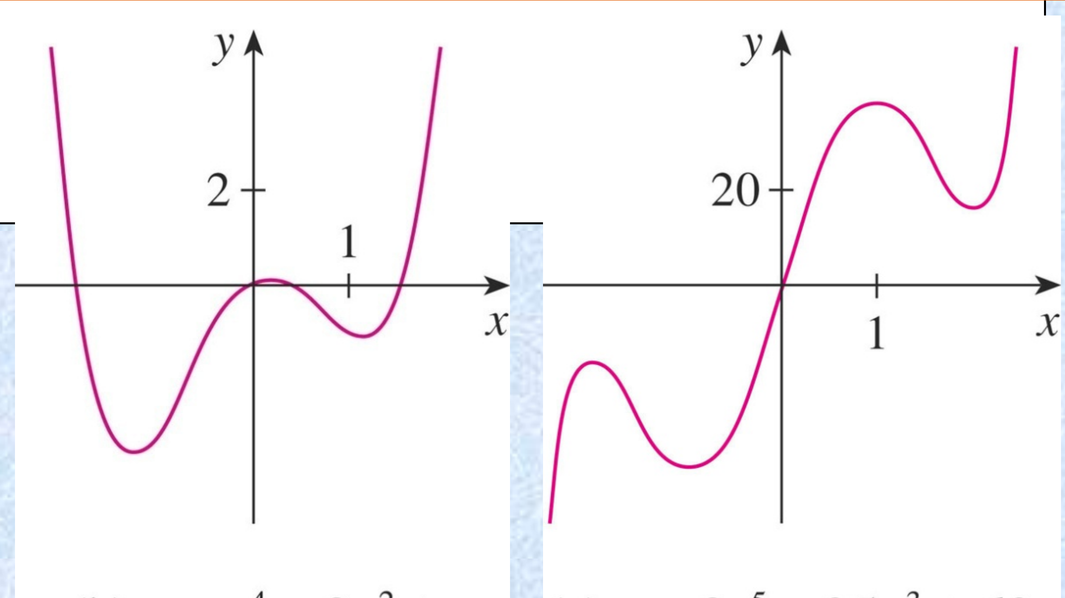
ALGEBRAIC FUNCTIONS

POLYNOMIALS

A function *P* is called a polynomial if

$$P(x) = a_n x^n + a_{n-1} x^{n-1} + \ldots + a_2 x^2 + a_1 x + a_0$$

where n is a nonnegative integer and the numbers $a_0, a_1, a_2, \ldots, a_n$ are constants called the coefficients of the polynomial.



(b) $y = x^4 - 3x^2 + x$

(c) $y = 3x^5 - 25x^3 + 60x$

ALGEBRAIC FUNCTIONS

RATIONAL FUNCTIONS

A rational function *f* is a ratio of two polynomials

$$f(x) = \frac{P(x)}{Q(x)}$$

where *P* and *Q* are polynomials.

- The domain consists of all values of *x* such that $Q(x) \neq 0$.

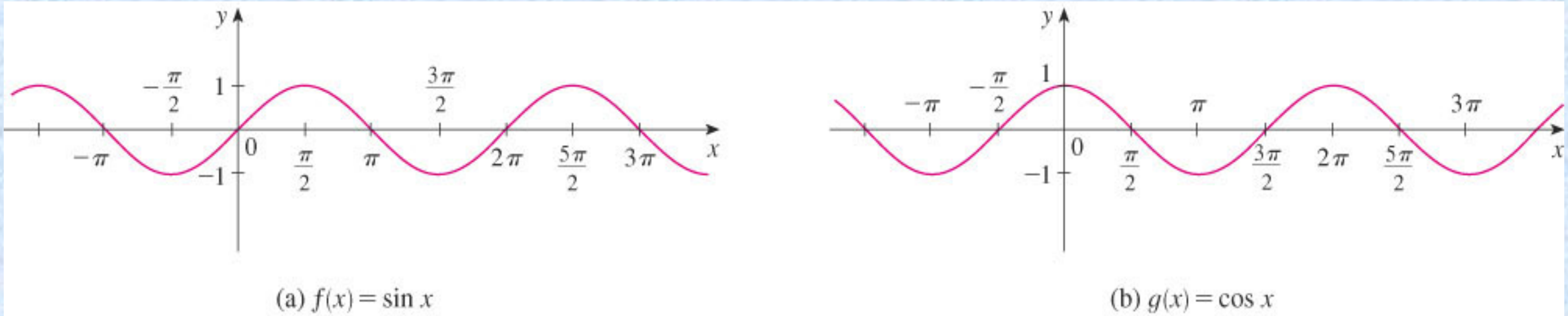
TRANSCENDENTAL FUNCTIONS

TRIGONOMETRIC FUNCTIONS

$f(x) = \sin x$ $\qquad D = (-\infty, \infty)$

$g(x) = \cos x$ $\qquad R = [-1, 1]$

$$\sin(x + k2\pi) = \sin x \qquad \cos(x + k2\pi) = \cos x; \quad k \in Z$$



(a) $f(x) = \sin x$ (b) $g(x) = \cos x$

TRANSCENDENTAL FUNCTIONS

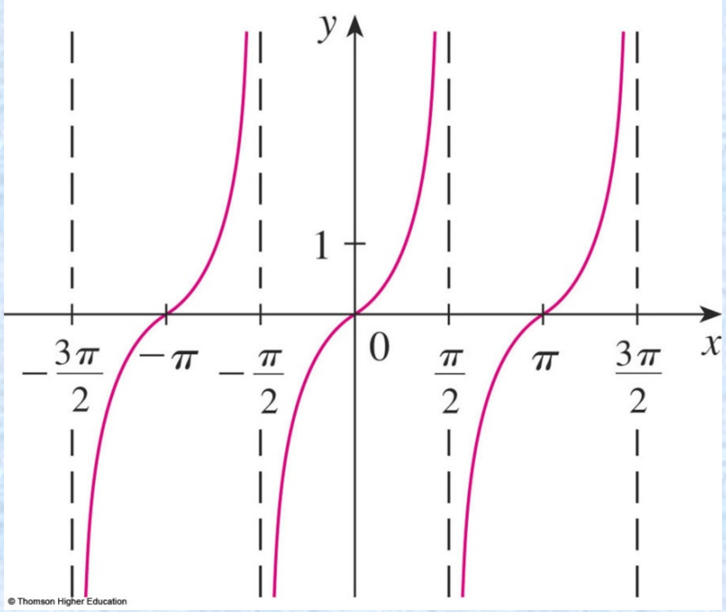
TRIGONOMETRIC FUNCTIONS

$$\tan x = \frac{\sin x}{\cos x}$$

$$x \neq \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \ldots$$

$$R = (-\infty, \infty)$$

$$\tan(x + k\pi) = \tan x; \quad k \in Z$$



TRANSCENDENTAL FUNCTIONS

TRIGONOMETRIC FUNCTIONS

The reciprocals of the sine, cosine, and tangent functions are

$$\cos ecx = \frac{1}{\sin x}$$

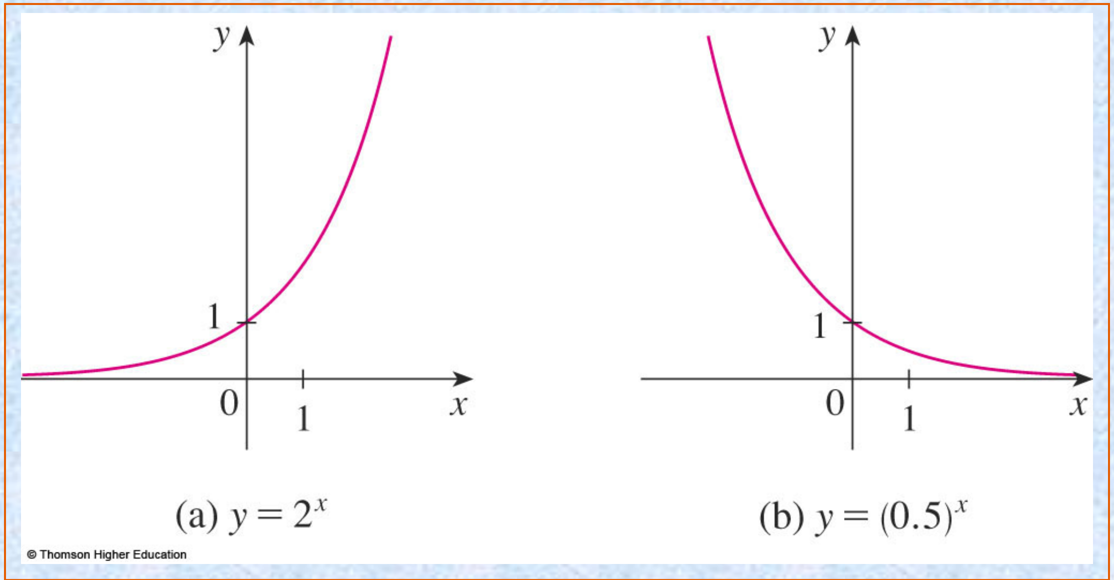
$$\sec x = \frac{1}{\cos x}$$

$$\cot anx = \frac{1}{\tan x}$$

EXPONENTIAL FUNCTIONS

The **exponential functions** are the functions of the form $\boxed{f(x) = a^x}$, where the base *a* is a positive constant.

- The graphs of $y = 2^x$ and $y = (0.5)^x$ are shown.
- In both cases, the domain is $(-\infty, \infty)$ and the range is $(0, \infty)$.



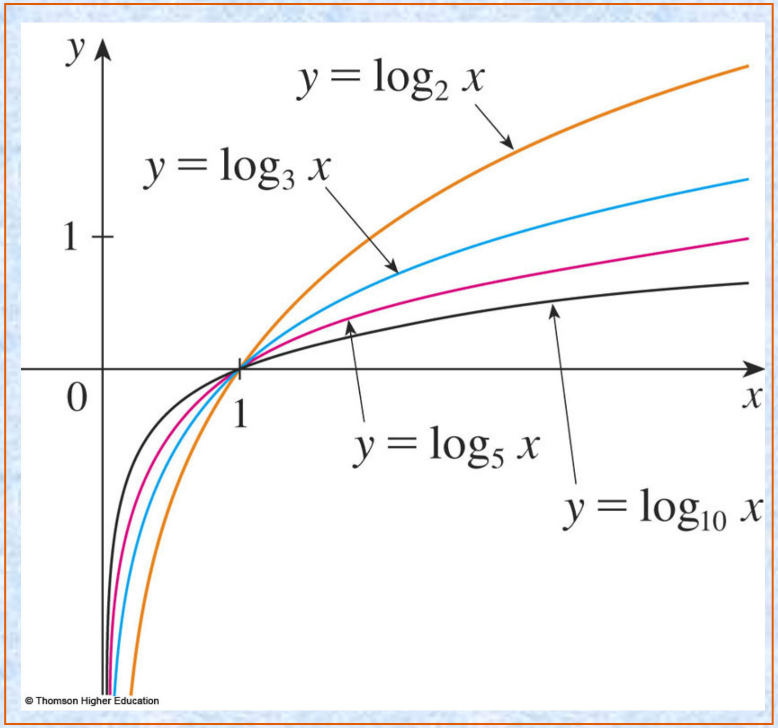
TRANSCENDENTAL FUNCTIONS

LOGARITHMIC FUNCTIONS

The logarithmic functions $\boxed{f(x) = \log_a x,}$

where the base *a* is a positive constant, are the inverse functions of the exponential functions.

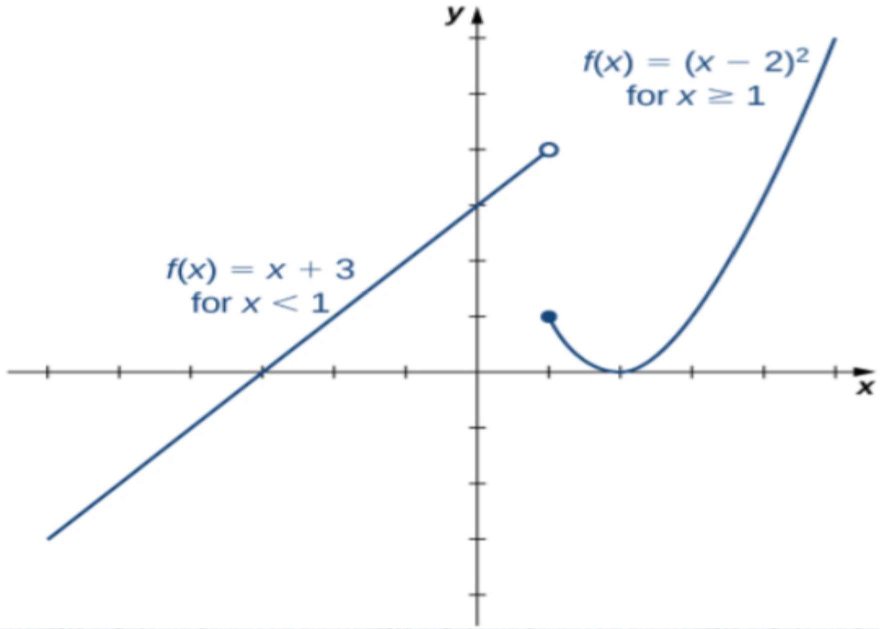
The figure shows the graphs of four logarithmic functions with various bases.



PIECEWISE-DEFINED FUNCTIONS

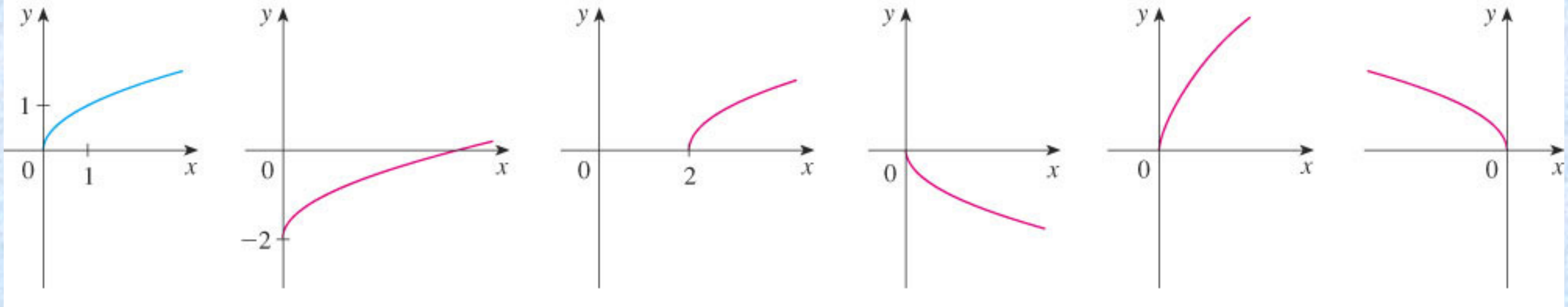
Example:

$$f(x) = \begin{cases} x+3, & x<1 \\ (x-2)^2, & x \geq 1 \end{cases}.$$



TRANSFORMATIONS OF FUNCTION

- Label the following graph from the graph of the function y=f(x) shown in the part (a) y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



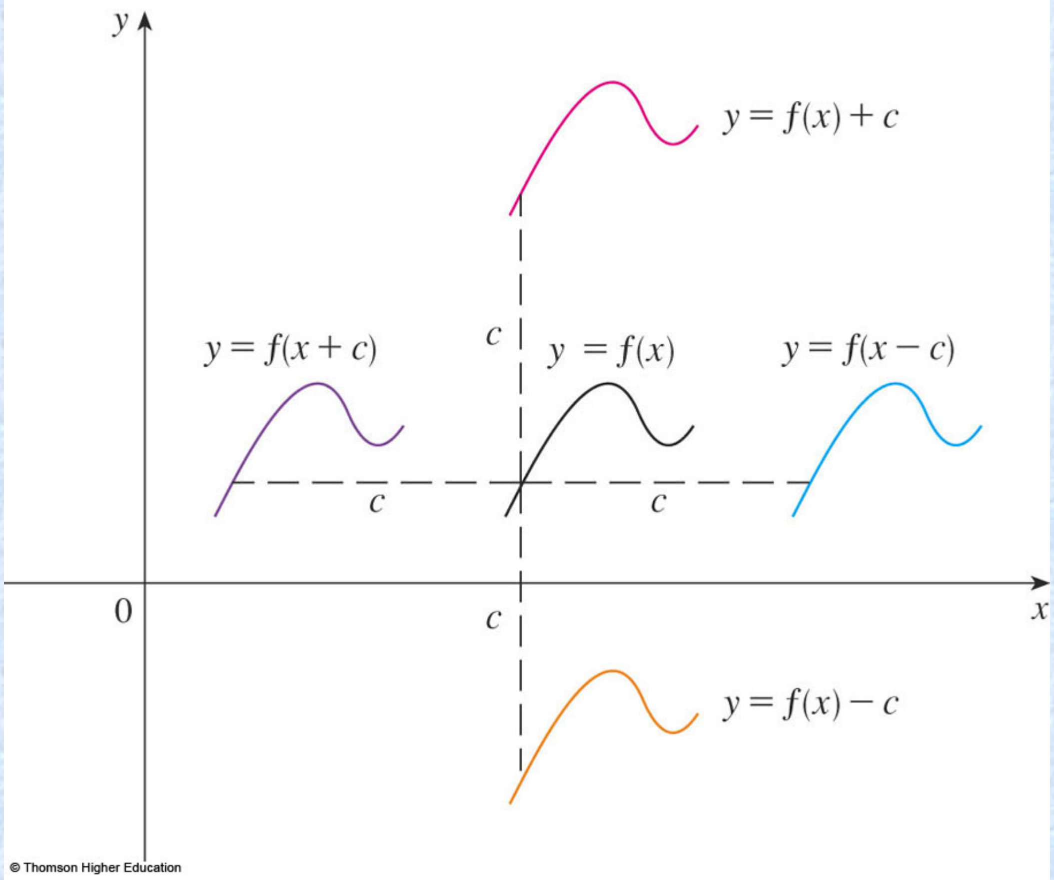
(a) $y=\sqrt{x}$

SHIFTING

- Suppose $c > 0$.

Why don't we consider the case c<0?

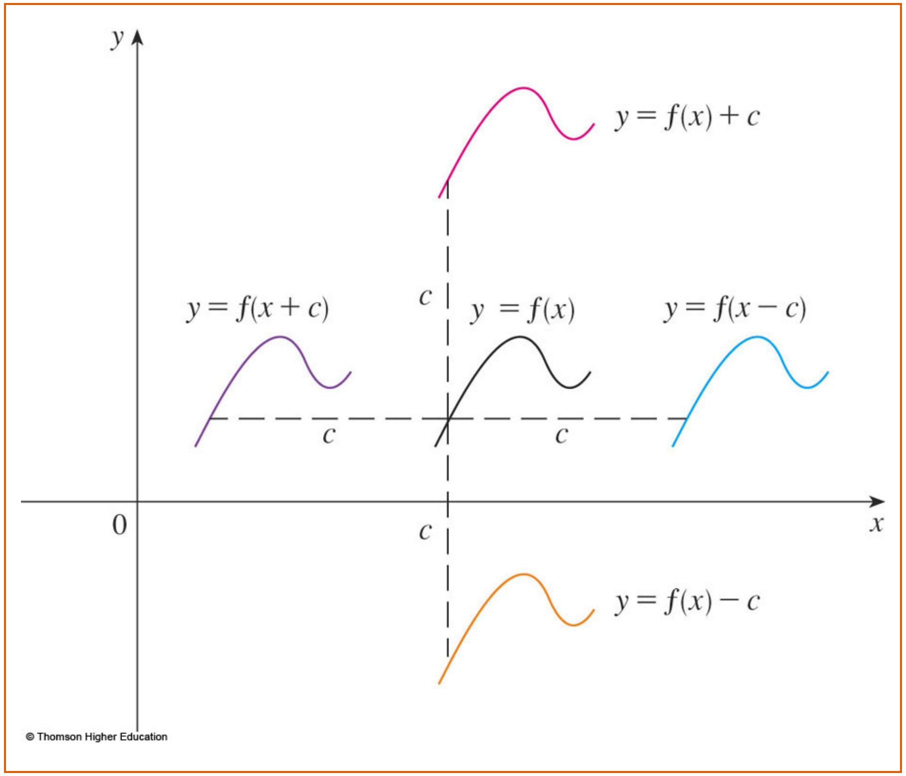
  - To obtain the graph of $y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward.
  - To obtain the graph of $y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward.



SHIFTING

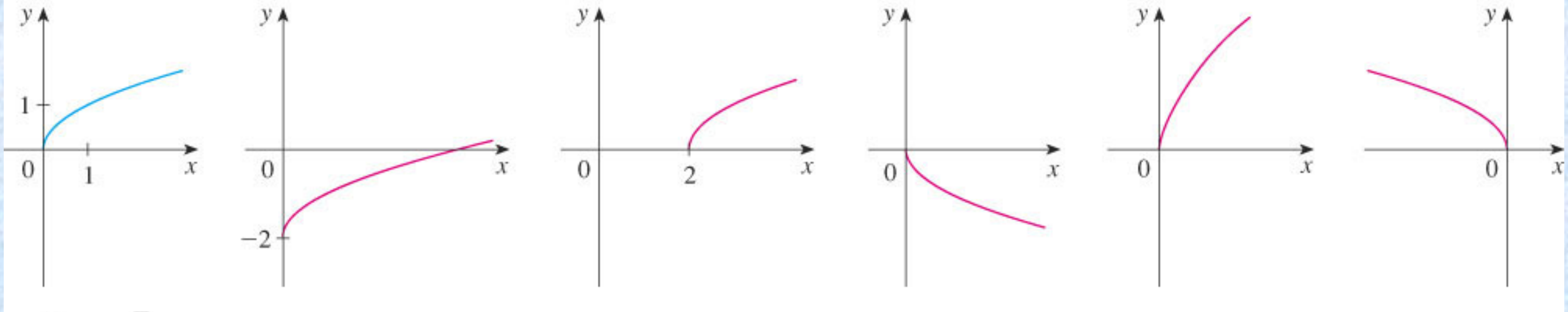
- To obtain the graph of $y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right.

- To obtain the graph of $y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left.



NEW FUNCTIONS FROM OLD FUNCTIONS

- Label the following graph from the graph of the function y=f(x) shown in the part (a) y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?



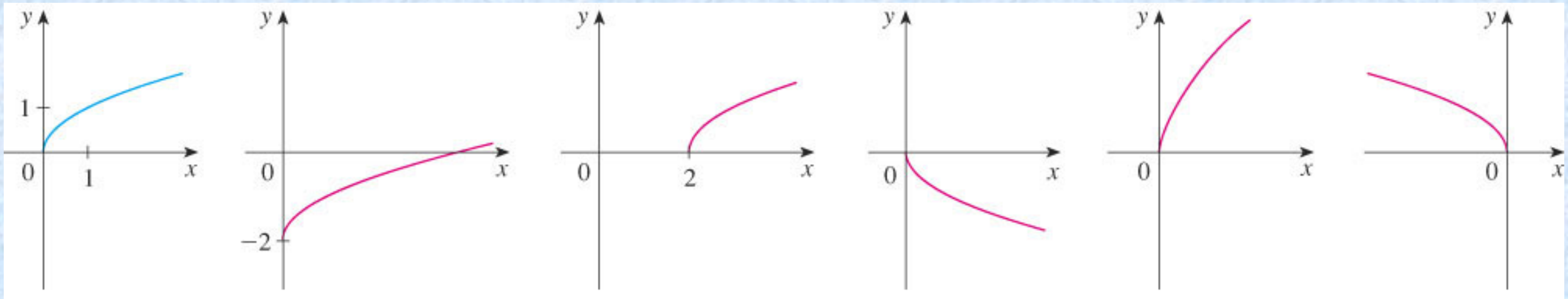
(a) $y = \sqrt{x}$

NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

- $y = \sqrt{x} - 2$ by shifting 2 units downward.
- $y = \sqrt{x-2}$ by shifting 2 units to the right.

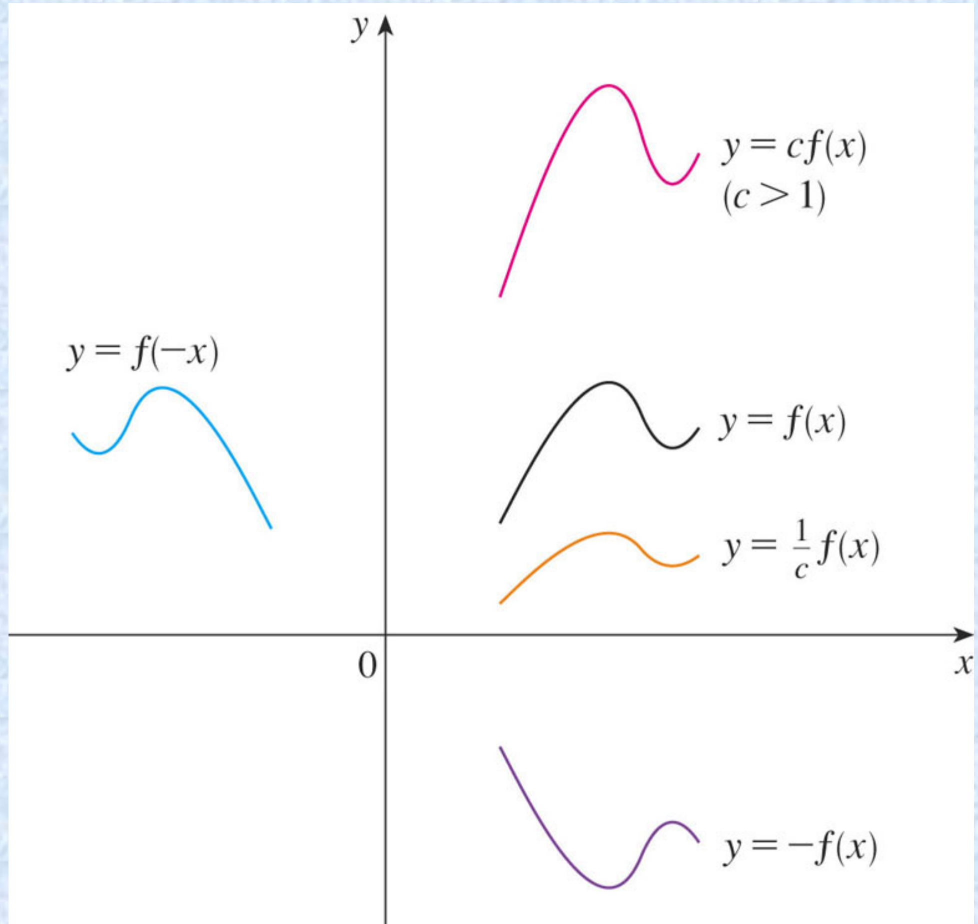


(a) $y = \sqrt{x}$ (b) $y = \sqrt{x} - 2$ (c) $y = \sqrt{x-2}$

TRANSFORMATIONS

- Suppose $c > 1$.
  - To obtain the graph of $y = cf(x)$, stretch the graph of $y = f(x)$ vertically by a factor of c.
  - To obtain the graph of $y = (1/c)f(x)$, compress the graph of $y = f(x)$ vertically by a factor of c.

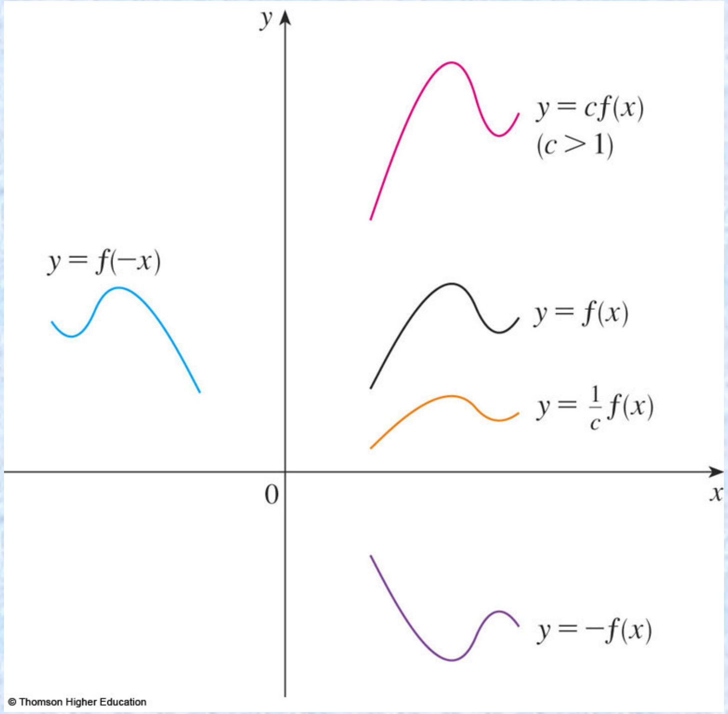
How about the case c<1?



TRANSFORMATIONS

- In order to obtain the graph of $y = f(cx)$, compress the graph of $y = f(x)$ horizontally by a factor of c.

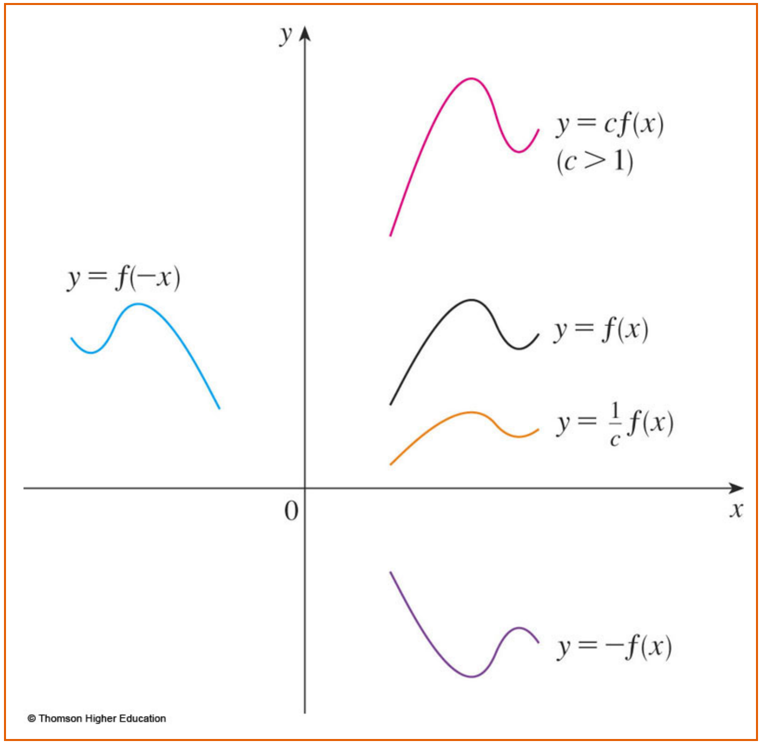
- To obtain the graph of $y = f(x/c)$, stretch the graph of $y = f(x)$ horizontally by a factor of c.



TRANSFORMATIONS

- In order to obtain the graph of $y = -f(x)$, reflect the graph of $y = f(x)$ about the x-axis.

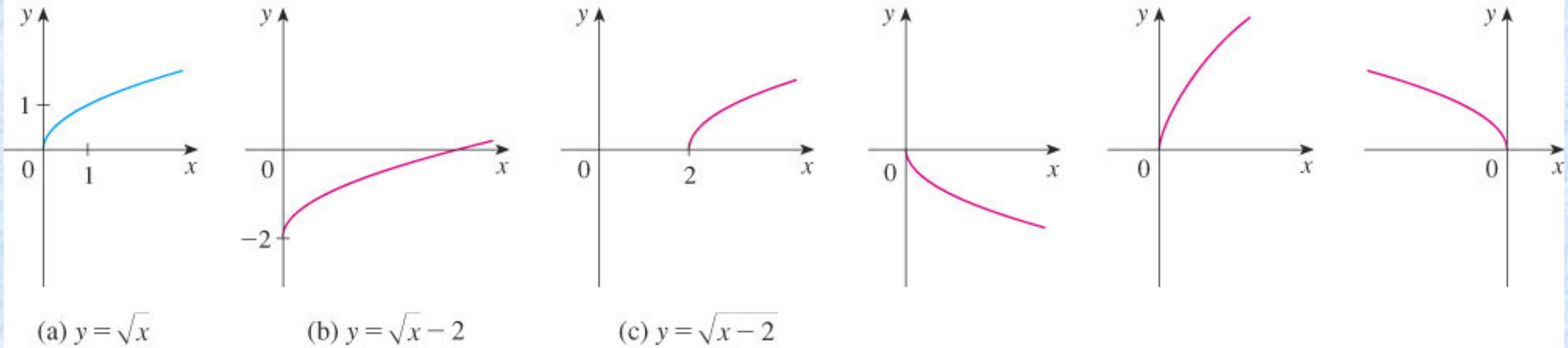
- To obtain the graph of $y = f(-x)$, reflect the graph of $y = f(x)$ about the y-axis.



NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

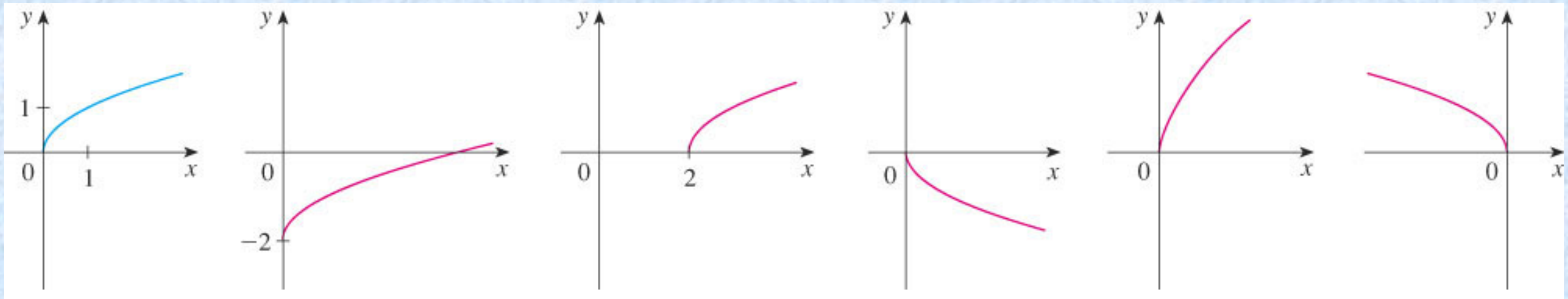


NEW FUNCTIONS FROM OLD FUNCTIONS

Label the following graph from the graph of the function $y = \sqrt{x}$ shown in the part (a):

y=f(x)-2, y=f(x-2), y=-f(x), y=2f(x), y=f(-x)?

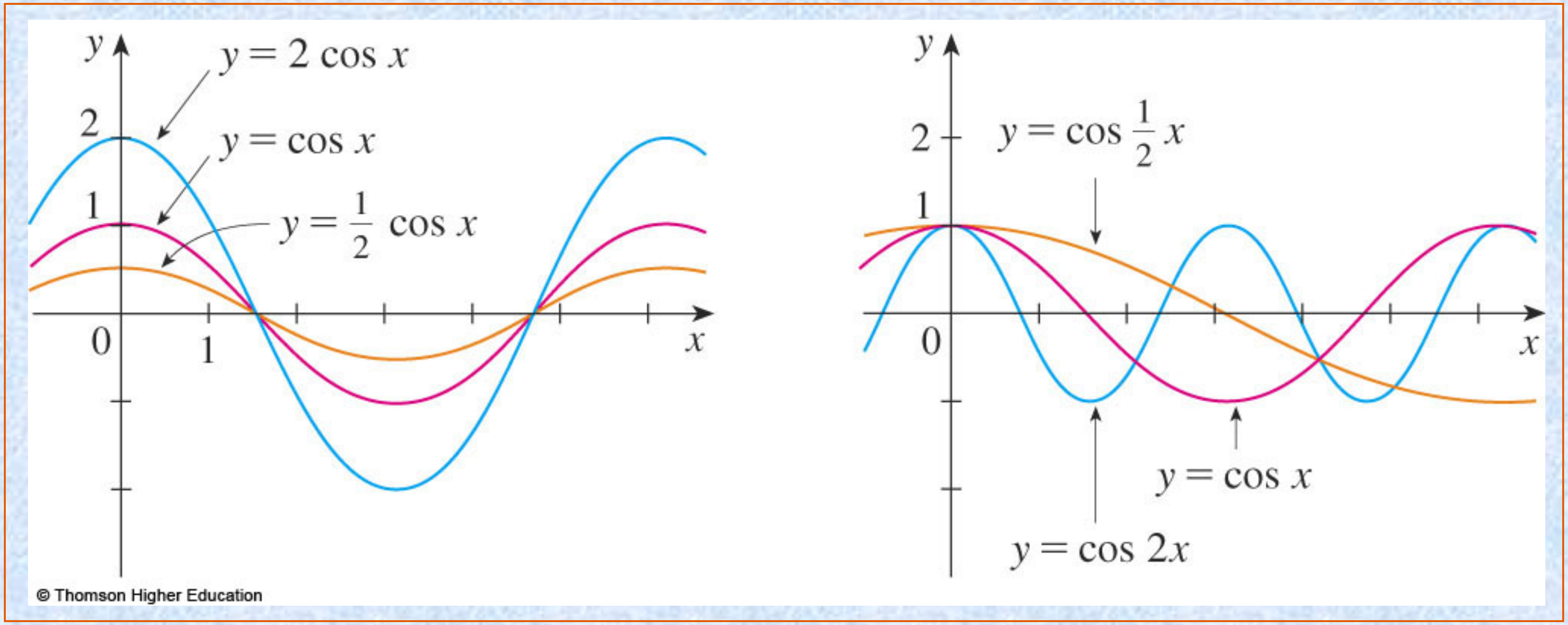
- $y = -\sqrt{x}$ by reflecting about the *x*-axis.
- $y = 2\sqrt{x}$ by stretching vertically by a factor of 2.
- $y = \sqrt{-x}$ by reflecting about the *y*-axis



(a) $y = \sqrt{x}$ (b) $y = \sqrt{x} - 2$ (c) $y = \sqrt{x-2}$ (d) $y = -\sqrt{x}$ (e) $y = 2\sqrt{x}$ (f) $y = \sqrt{-x}$

TRANSFORMATIONS

- The figure illustrates these stretching
- transformations when applied to the cosine
- function with c = 2.



Example

Suppose that the graph of f is given.
Describe how the graph of the function $f(x-2)+2$
can be obtained from the graph of f.
Select the correct answer.

a. Shift the graph 2 units to the left and 2 units down.

b. Shift the graph 2 units to the right and 2 units down.

c. Shift the graph 2 units to the right and 2 units up.

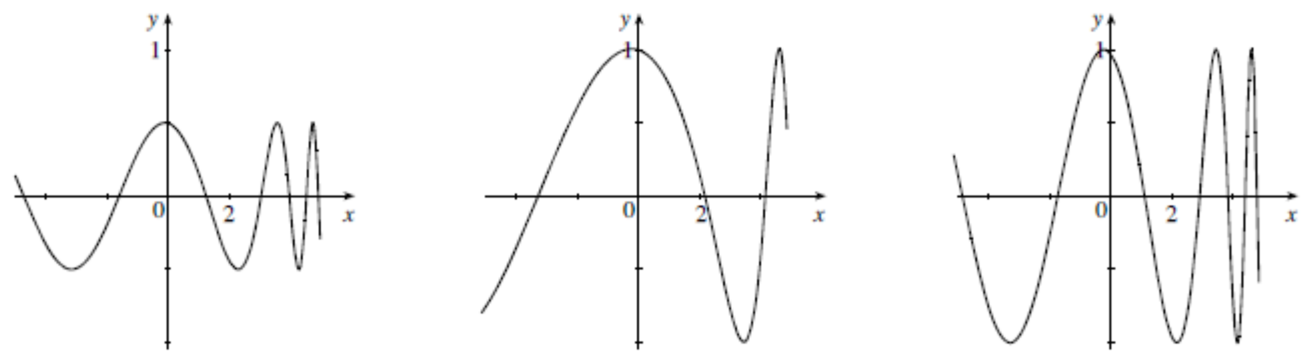
d. Shift the graph 2 units to the left and 2 units up.

e. none of these

Answer: c

QUIZ QUESTIONS

Label the following graphs: $f(x)$, $\frac{1}{2}f(x)$, $f\left(\frac{1}{2}x\right)$.



Answer: ½ f(x), f(½ x), f(x)

Thanks