A Brief Introduction to Independent Set Reconfiguration and Related Problems

Duc A. Hoang

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anhduc.hoang1990@gmail.com

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Outline

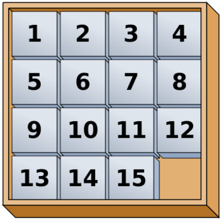
General Framework: Combinatorial Reconfiguration

Independent Set Reconfiguration

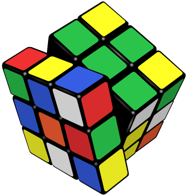
My PhD Journey: The SLIDING TOKEN problem

General Framework: Combinatorial Reconfiguration

Reconfiguration: An Overview



15-PUZZLE

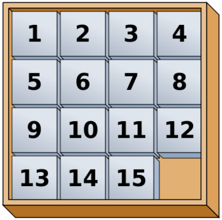


RUBIK'S CUBE

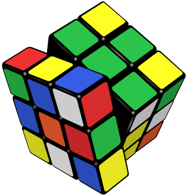
RUSH-HOUR

The figures were originally downloaded from various online sources, especially Wikipedia

Reconfiguration: An Overview



15-PUZZLE



RUBIK'S CUBE

RUSH-HOUR

They are all examples of Reconfiguration Problems:

Given two configurations, and a specific rule describing how a configuration can be transformed into a (slightly) different one

Ask whether one can transform one configuration into another by applying the given rule repeatedly

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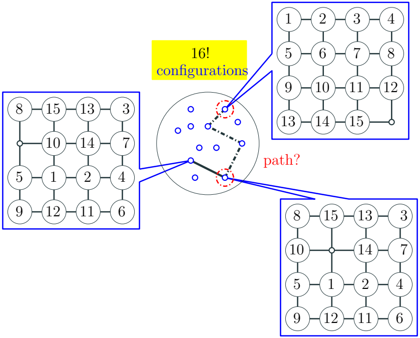
Reconfiguration: An Overview

Reconfiguration Graph

Vertex Set: Set of all configurations

Edge Set: Two vertices (configurations) are adjacent if one can be obtained from the other by applying the given rule once



The reconfiguration graph of 15-PUZZLE

Reconfiguration: An Overview

New insights into the computational complexity theory

Given	Configurations A, B, and a transformation rule
Decision	Decide if A can be transformed into B
Find	A transformation sequence?
Shortest	A shortest transformation sequence?

See also the “Masterclass Talk: Algorithms and Complexity for Japanese Puzzles” by R. Uehara at ICALP 2015

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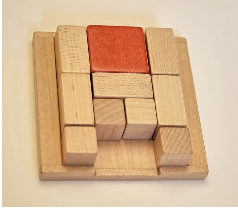
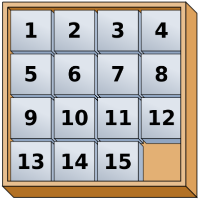
Reconfiguration: An Overview

New insights into the computational complexity theory

	Sliding-block Puzzle	15-puzzle
Decision	PSPACE-complete	Linear
Find	PSPACE-complete	Poly-time
Shortest	PSPACE-complete	NP-complete



Sliding-block Puzzle



15-puzzle

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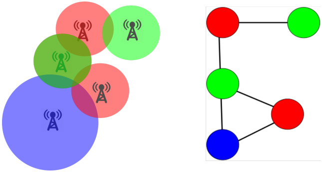
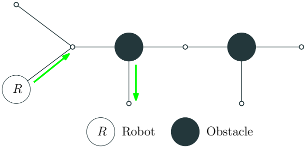
Reconfiguration: An Overview

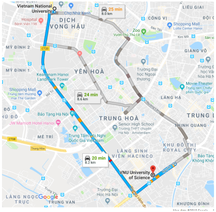
Real-world situations involving movement and change



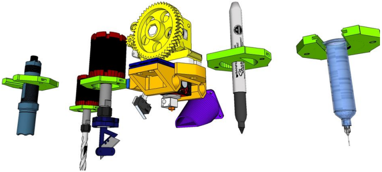
Frequency Re-Assignment



Robot Motion



Path Planning



Printer's Multi-heads Motion

The figures were originally downloaded from various online sources, especially Wikipedia

Reconfiguration: An Overview

Surveys on Reconfiguration

Jan van den Heuvel (2013). "The Complexity of Change." In: *Surveys in Combinatorics*. Vol. 409. London Mathematical Society Lecture Note Series. Cambridge University Press, pp. 127–160. DOI: 10.1017/CBO9781139506748.005

Naomi Nishimura (2018). "Introduction to Reconfiguration." In: *Algorithms* 11.4. (article 52). DOI: 10.3390/a11040052

Online Web Portal (maintained by Takehiro Ito)

http://www.ecei.tohoku.ac.jp/alg/core/

Independent Set Reconfiguration

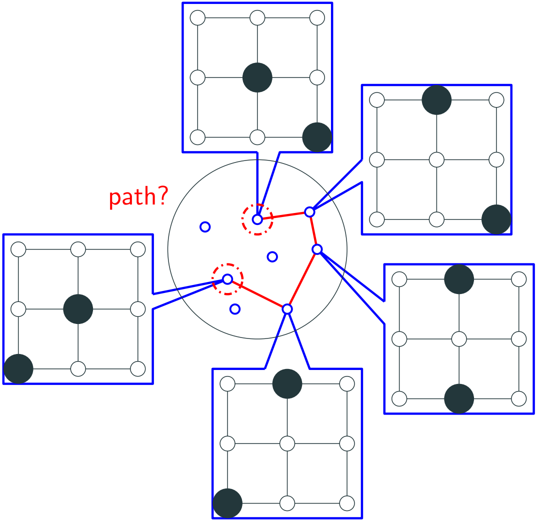
Independent Set Reconfiguration: Definition

Given two independent sets of a graph; and one of the following rules

- **Token Sliding** (TS) [Hearn and Demaine 2005]
- **Token Addition and Removal** (TAR(k)) [Ito et al. 2011]
- **Token Jumping** (TJ) [Kamiński et al. 2012]

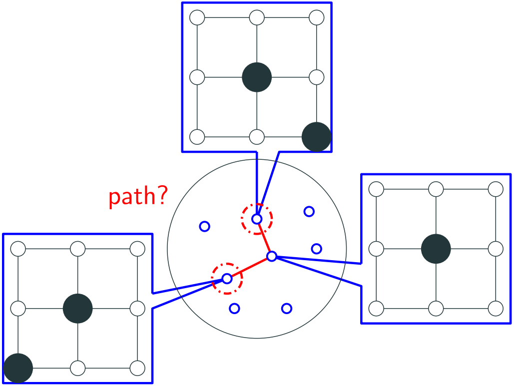
Ask whether one can transform one independent set into another by applying the given rule repeatedly

Independent Set Reconfiguration: Transformation Rule



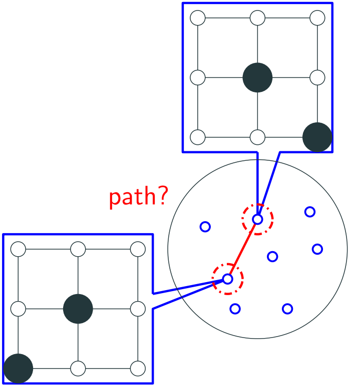
Token Sliding (TS): Swap one member for one non-member that is adjacent to the member (see [Hearn and Demaine 2005])

Independent Set Reconfiguration: Transformation Rule



Token Addition and Removal (TAR(k)): Either add or remove one vertex, while keeping the size of the independent sets at least some given threshold k (see [Ito et al. 2011])

Independent Set Reconfiguration: Transformation Rule



Token Jumping (TJ): Swap one member for one non-member
(see [Kamiński et al. 2012])

Independent Set Reconfiguration: Complexity

Graph	Adjacency			Complexity	Reference
	TS	TAR	TJ		
planar ∩ maximum degree 3	○	○	○	PSPACE-complete	[Hearn and Demaine 2005]
general	○	○	○	PSPACE-complete	[Ito et al. 2011]
line ⇐ MATCHING RECONF.		○	○	P	
perfect	○	○	○	PSPACE-complete	[Kamiński et al. 2012]
even-hole-free		○	○	P	
cograph (P_4-free)	○			P	
cograph (P_4-free)		○	○	P	[Bonsma 2014]
bounded bandwidth	○	○	○	PSPACE-complete	[Wrochna 2014]
claw-free (⊃ line)	○	○	○	P	[Bonsma et al. 2014]
tree (⊂ even-hole-free)	○	○	○	P	[Demaine et al. 2014]
bipartite permutation	○			P	[Fox-Epstein et al. 2015]
bipartite distance-hereditary	○				
planar ∩ maximum degree 3 ∩ bounded bandwidth	○	○	○	PSPACE-complete	[van der Zanden 2015]
cactus		○	○	P	[Mouawad et al. 2014]
cactus	○			P	[Hoang and Uehara 2016]
interval (⊂ even-hole-free)	○	○	○	P	[Bonamy and Bousquet 2017]
bipartite	○			PSPACE-complete	[Lokshtanov and Mouawad 2018]
		○	○	NP-complete	
split	○			PSPACE-complete	[Belmonte et al. 2018]
split (⊂ even-hole-free)		○	○	P	

Independent Set Reconfiguration: Related Problems

- Reconfiguration Graph
  - Find a (shortest) path between two vertices (configurations)
  - Determine whether the reconfiguration graph is connected
  - Determine whether its diameter is bounded
  - Determine whether the original graph and the corresponding reconfiguration graph are isomorphic/belong to the same graph class
- Optimization
  - Allow p tokens to move simultaneously (under TJ). Find smallest p such that there is a transformation sequence between any two independent sets of size k.
  - Find an independent set of maximum size that is reachable from an initial independent set I_0 of size at least k using TAR(k).

Independent Set Reconfiguration: Related Problems

- Independent Set as a Vertex-Subset
  - Reconfiguration of Cliques
  - Reconfiguration of Vertex Covers
  - Reconfiguration of (Labeled/Unlabeled) Tokens
- Independent Set as an Edgeless Induced Subgraph
  - Reconfiguration of (Induced/Spanning) Subgraphs
- Independent Set as a 1-colorable set
  - Reconfiguration of c-colorable sets ($c \geq 1$)

My PhD Journey: The SLIDING TOKEN problem

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- Erik D. Demaine, Martin L. Demaine, Takehiro Ito, Hirotaka Ono, Yota Otachi, Ryuhei Uehara, and Takeshi Yamada: Around 2014, they designed a $O(n^2)$-time algorithm for Sliding Token for caterpillars (a subclass of trees). At that time, Sliding Token for trees remains open.

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- With me joining the project, we designed a $O(n^2)$-time algorithm for Sliding Token for trees. This result was accepted to ISAAC 2014.

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- With Eli Fox-Epstein joining the project, we improved the running time of our algorithm to $O(n)$. This result was then published in the journal "Theoretical Computer Science" (600, 132–142, 2015).

My PhD Journey: The SLIDING TOKEN problem

- The ideas of Eli for improving our algorithm for trees are quite useful. We (Eli Fox-Epstein, Yota Otachi, Ryuhei Uehara, and I) thought (at that time) that our algorithm can be extended to the case for bipartite graphs using similar ideas.

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- Though we has not been successful with bipartite graphs, we designed tools and techniques for showing that Sliding Token for bipartite permutation graphs and bipartite distance-hereditary graphs can be solved in $O(n^3)$ time. This result was accepted to ISAAC 2015 (presented by Eli).

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- Very recently, Daniel Lokshtanov and Amer E. Mouawad showed that SLIDING TOKEN for bipartite graphs is PSPACE-complete. Their result (presented at SODA 2018) implied that our conjecture is indeed wrong.

My PhD Journey: The Sliding Token problem

- From April to July 2016, I visited Zhou&Ito Lab, Tohoku University. At that time, I had some rough ideas for designing a poly-time algorithm for Sliding Token on cactus graphs (generalizing the techniques for trees).

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- After several useful discussions with Takehiro Ito and the members of his laboratory, including Tatsuhiko Hatanaka, Haruka Mizuta, and Yuhei Moriya, I have finally been able to complete the algorithm. The result (co-authored with my supervisor Ryuhei Uehara) was then accepted to ISAAC 2016.

My PhD Journey: The Sliding Token problem

- From the algorithm for cacti, we (Eli Fox-Epstein, Ryuhei Uehara, and I) designed a $O(n^3)$-time algorithm for Sliding Token for block graphs. This result was presented at WALCOM 2017.

My PhD Journey: The Sliding Token problem

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- Very recently (around March 2018), Mariana Teatini Ribeiro and Vinícius Fernandes dos Santos showed us a counter-example of a proposition in our algorithm (for block graphs). As a result, the complexity of Sliding Token for block graphs has not yet been settled.

My PhD Journey: The SLIDING TOKEN problem

- Even though SLIDING TOKEN can be solved in linear time for trees, finding a shortest TS-sequence between two independent sets of a tree remains open. We (my supervisor and I) conjectured that the problem can be solved in poly-time for trees, and becomes NP-hard for graphs containing cycles.

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- Sugimori's $O(poly(n))$-time algorithm seems to have $poly(n)$ with high degree. With Amanj Khorramian joining the project, we designed a $O(n^2)$-time algorithm for finding shortest TS-sequences in spider graphs (a subclass of trees). We hope that our result provides a framework for improving Sugimori's algorithm. This result was recently presented at WAAC 2018 (August 27, 2018) by my supervisor Ryuhei Uehara.

Thank you very much for your attention!

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