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Advanced Lab in Theoretical Physics PHYC30320

Simulation of a driven pendulum

The equation of motion of a driven pendulum is

$$\frac{\partial^2 x}{\partial t^2} = -\sin x + a_0 \sin(\omega_0 t)$$

where the second term describes the action of a periodic external force.

Use 4th order Runge-Kutta method for both the position and velocity to solve the equation of motion and investigate the dynamic regimes at different a_0 and ω_0 . Find regions of regular and chaotic motion, compare the results to the motion of a free pendulum ($a_0 = 0$).

The 4th order RK solves $\dot{\vec{x}} = f(\vec{x}(t), t)$ approximately by iterating

$$\vec{x}(t + \Delta t) = \vec{x}(t) + \frac{1}{6}(K_1 + 2K_2 + 2K_3 + K_4)$$

with

$$K_1 = \Delta t f(\vec{x}(t), t); \quad K_2 = \Delta t f\left(\vec{x}(t) + \frac{K_1}{2}, t + \frac{\Delta t}{2}\right)$$

$$K_3 = \Delta t f\left(\vec{x}(t) + \frac{K_2}{2}, t + \frac{\Delta t}{2}\right); \quad K_4 = \Delta t f(\vec{x}(t) + K_3, t + \Delta t)$$