MERRIMACK COLLEGE

Simulating Missile Trajectory with DEs

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Modeling and Simulation Final

GitHub

Introduction

In a ballistic missile trajectory simulation, the system of differential equations used to describe the ballistic model is a highly complex system. In particular, the six-degree of freedom model used most frequently, solves for the missile's components of acceleration, velocity, and position at discrete time intervals. The usual approach for simulation is the 4th Order Runge Kutta method. This poster will be diving into a different, and potentially more efficient algorithm, called the Parker-Sochacki Method (PSM for short).

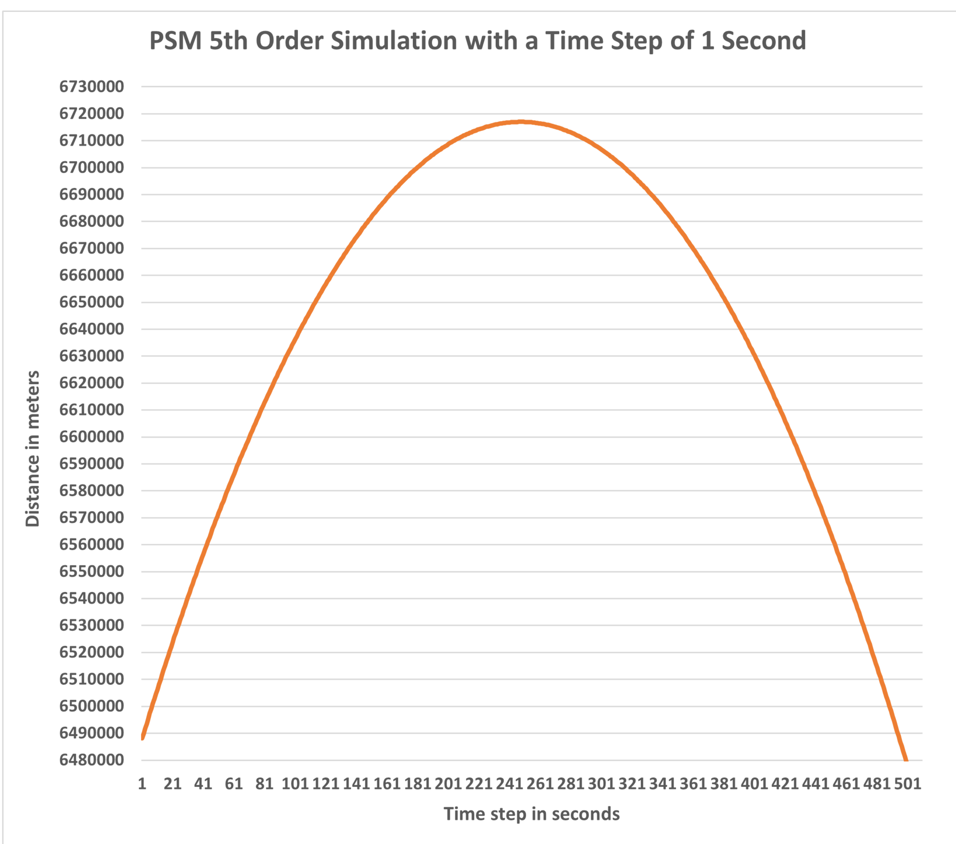
Variables and Assumptions

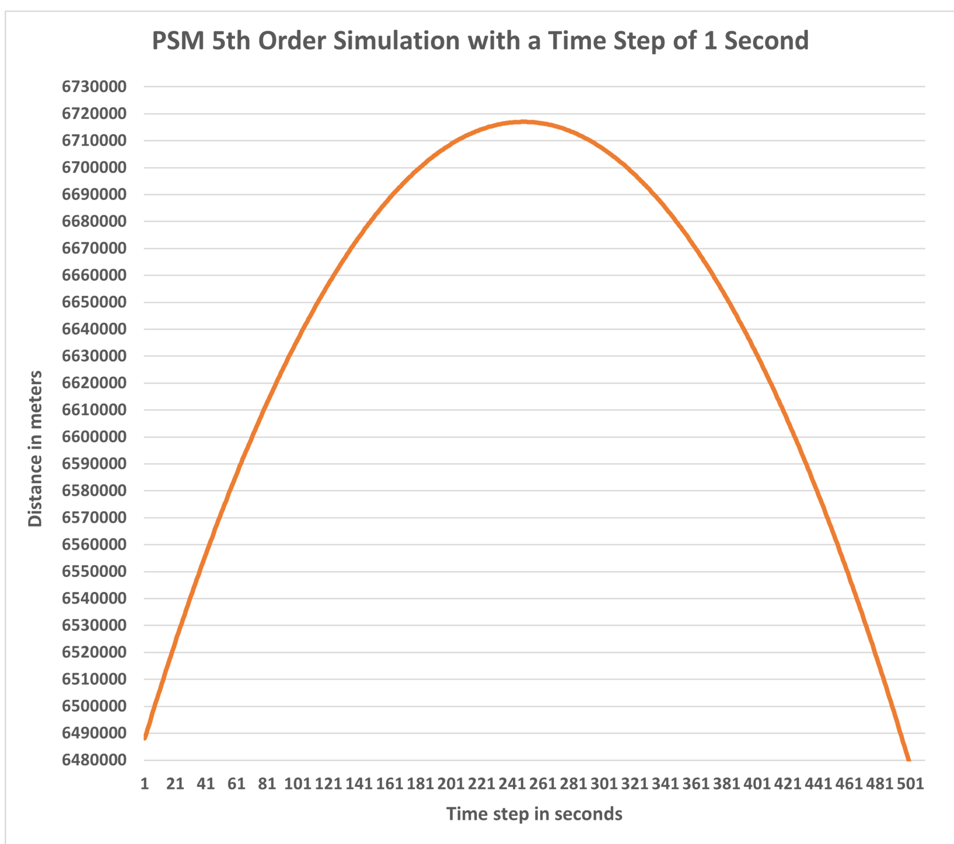
- Initial position coordinates, velocity, and acceleration
- PSM Order, time step, and tolerance
- Gravity coefficients for x, y, and z components
- Assuming that mass and acceleration are constants to be ignored

Problem Identification

The Parker-Sochacki method

Verification of Model





Conclusion

conclusion stuff

Cauchy Product Example Problem

Use the power series method to solve $y' = y^2$; $y(0) = -1$

$$y = \sum_{k=0}^{\infty} c_k t^k \Rightarrow y' = \sum_{k=1}^{\infty} k c_k t^{k-1}$$

Plug into $y' = y^2$

$$y' = \sum_{k=1}^{\infty} k c_k t^{k-1} = \left(\sum_{k=0}^{\infty} c_k t^k\right)\left(\sum_{k=0}^{\infty} c_k t^k\right)$$

Rewrite y^2 to match the form of the cauchy product

$$y' = \sum_{k=1}^{\infty} k c_k t^{k-1} = \sum_{k=0}^{\infty} \underbrace{\left(\sum_{j=0}^{k} c_j c_{k-j}\right)}_{\text{Cauchy product}} t^k \tag{1}$$

Set $k = 1$ on both sides of the equation to solve for c_k

$$y' = \sum_{k=1}^{\infty} k c_k t^{k-1} = \sum_{k=1}^{\infty} \left(\sum_{j=0}^{k-1} c_j c_{k-j-1}\right) t^{k-1}$$

$$c_k = \frac{\sum_{j=0}^{k-1} c_j c_{k-j-1} t^{k-1}}{k} \tag{2}$$

Solving for c_k, we can see each coefficient is a function of previous coefficients

$$\therefore c_1 = c_0 c_0, c_2 = \frac{c_0 c_1 + c_1 c_0}{2}, c_3 = \frac{c_0 c_2 + c_1 c_1 + c_2 c_0}{2}, \ldots$$

References

Hello