MDO Assignment

Part 1

by

Group 15

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Students:	Ties Nieuwenhuizen	4148495
	Nicolas Wahler	4446976

TU Delft

Contents

Nomenclature

Symbol	Explanation
$\hat{a}$	Copy of output variable "a" for use as extra design variable
a^*	Resultant value of variable "a" as output of the optimiser
a^0	Initial value of variable "a" as initiation for the optimiser
b	Total span
b_i	Span of inboard section
b_o	Span of outboard section
C_r	Root chord
C_k	Kink chord
C_t	Tip chord
$CST_{A,i,u}$	CST coefficients for the upper curve of the inner aerofoil (superscript indicates n-th element)
$CST_{A,i,l}$	CST coefficients for the lower curve of the inner aerofoil (superscript indicates n-th element)
$CST_{A,o,u}$	CST coefficients for the upper curve of the outer aerofoil (superscript indicates n-th element)
$CST_{A,o,l}$	CST coefficients for the lower curve of the outer aerofoil (superscript indicates n-th element)
CST_L	CST coefficients for the loading (superscript indicates n-th element)
c_0	Outputs of inequality constraints
C_c	Outputs of consistency constraints
C_T	Thrust specific fuel consumption
D	Drag
f_{tank}	Fuel tank volume correction factor
L	Lift
$MTOW$	Maximum takeoff weight
N_1	Shape factor for loading CST shape function
N_2	Shape factor for loading CST shape function
R	Range
S	Total surface area
S_i	Surface area of inboard section
S_o	Surface area of outboard section
S_1	Fuel tank root area
S_2	Fuel tank kink area
S_3	Fuel tank tip area
S_i, min	Minimum area of the inboard wing section
SF_L	Scaling factor for the loading CST-curve
V	Cruise velocity
V_{fuel}	Fuel volume
V_{tank}	Fuel tank volume
W_f	Fuel weight
$\left(\frac{W}{S}\right)_{max,ref}$	Maximum wing loading of the reference aircraft
$\mathbf{x}$	Design vector
$y_k ink$	Kink spanwise location
ζ_r	Root CST curve
ζ_k	Kink CST curve
ζ_t	Tip CST curve
η	Chordwise position of the CST-curve
ρ_f	Fuel density
Λ_{LE}	Leading edge sweep
Λ_{TE_i}	Trailing edge sweep
λ_o	Taper ratio of outboard trapezoidal element
ϕ_o	Twist angle at wing tip

Table 1: Nomenclature

1

Introduction

With the ever rising fuel prices and a more and more developing ecologic consciousness, airline demands for the most fuel efficient and optimised airframes is higher than ever. Their striving to minimise operational costs and use low fuel consumption as marketing tool challenges airframe manufacturers around the world to stay on top of their game and be innovative to stay competitive in their market. Thus, every airframe manufacturer must strive to develop and implement strategies that can utilise the scarce available resources in an optimum way to create the best possible product, that can satisfy the needs of the airlines best while keeping research, development and testing expenses at the minimum possible cost for the airframer.

One means to achieve this is to utilise the massively increased computational power available to automate many design processes that were previously done manually or experimentally, thus allowing a competent algorithm to sweep through a much larger range of possible design variables and thus to ideally find a more optimum solution to the design problem in a shorter period of time using less resources than a human design team ever could. The purpose of this study is to implement such an automated multi-discipline design optimisation for the wing of an AVRO RJ 115 aircraft with the design objective to reduce the required fuel weight for a design mission profile by altering and improving the wing design, and thus increasing aerodynamic and structural efficiency of said wing.

This report is part one in a series of two and presents the formal problem statement of the optimisation problem for the British Aerospace RJ115 wing, as well as its planned implementation using the Individual Discipline Feasible (IDF) architecture. Chapter 2 presents the problem specification, including the design vector, bounds, objective function, equality and inequality constraints. Chapter 3 contains the Extended Design Structure Matrix (XDSM) diagram detailing the planned implementation architecture.

2

Part 1.1: Problem Specification

Following from the problem statement in the introduction, this chapter describes the formal specifications, used equations and specified bounds in more detail, including their values and justifications for the chosen design variables and imposed bounds. Section 2.1 describes the components of the implemented design vector and the rationale behind choosing these parameters. Section 2.2 then build upon this vector by stating the defined bounds on each variable. Section 2.3 then details the defined objective function in order to fulfil the goal of minimising fuel weight. Lastly, sections 2.4 and 2.5 describe the necessary inequality and consistency constraints to fully characterise the problem and assure a working IDF architectural implementation.

2.1. Design Vector

The design vector combines all the required design variables that are necessary to fully define the wing planform, aerofoil shapes, wing loads and other parameters that re necessary to characterise the performance of the aircraft and thus allow to find a required fuel mass for the given design mission. The main challenge in this area, is that the complex problem should be broken down in as few and as basic parameters as possible, in order to simplify and accelerate the optimisation.

The first aspect to be considered is the wing planform, as most disciplines directly relate to the wing shape. The wing is modelled as two trapezoidal elements, the inboard one having a constant span and trailing edge sweep. Another assumption that was made to simplify the analysis was, that although trailing edge sweep might differ for the two elements, the leading edge sweep shall be the same for both wing planforms. Using a set of nonlinear equations (eqs. (2.1) to (2.3)), this allows to fully define the whole planform with only four design variables: total wing area (S), wing span (b), leading edge sweep (Λ_{LE}) and the taper ratio for the outboard wing (λ_o). All other required geometric parameters directly follow from geometric relations.

$$b = b_i + b_o; \qquad S = S_i + S_o \tag{2.1}$$

$$S_i = b_i \left(\frac{C_r + C_k}{2} \right); \qquad S_o = b_o \left(\frac{C_k + C_t}{2} \right) \tag{2.2}$$

$$C_k = C_r - b_i tan(\Lambda_{LE}) - b_i tan(\Lambda_{TE_i}); \qquad \lambda_o = \frac{C_t}{C_k} \tag{2.3}$$

The aerodynamics and load disciplines use the planform geometry to define the position of the leading edge for the specified wing sections, as well as the resulting chord length at those points. Furthermore, leading edge twist (ϕ_o) is introduced as design variable for the wing shape, which will be applied linearly over the whole span, starting at zero value for the root. Furthermore, the aerofoil shapes for both root and tip are required design variables for these disciplines. The original aerofoil was parameterised using 5^{th} order CST-functions for upper and lower side for both root and tip aerofoil respectively. Lastly, the required lift coefficient at the design point is computed using the weight of the aircraft as computed from the last iteration of the optimiser. For the loads discipline, this is corrected for the maximum load factor the aircraft shall be able to tolerate.

The structures discipline takes the same design variables as specified before, however, now also the parameterised loading is taken into account. For this, a 4^{th} order CST-function was deemed satisfactory, in conjunction with one variable shape factor of the shape function and a scaling factor to scale the resulting CST shape to the magnitude of the loads computed in the loads discipline. The shape factor N_1 is chosen as zero since the curve will have a finite value at the root. N_2 however is included in the parameterisation to obtain a better fitting curve. This curve fit will be performed by an `fmincon` optimisation within the loading block.

Lastly, the performance discipline does not require any new design variables, it only takes the results from the aerodynamics and structures discipline from the previous iteration to determine an updated fuel weight, which is then passed on to the objective function.

All other inputs required by the different disciplines, such as flight condition, spar locations or engine locations are considered constants throughout the whole analysis and thus do not warrant any additional design variables.

Summarising this description, the total design vector can be assembled and is presented in eq. (2.4).

$$\mathbf{x} = \left[S, b, \Lambda_{LE}, \lambda_o, CST_{A,i,u}, CST_{A,i,l}, CST_{A,o,u}, CST_{A,o,l}, \phi_o, \widehat{MTOW}, \widehat{W_f}, \widehat{CST_L}, \widehat{SF_L}, \widehat{N_2}, \widehat{\frac{L}{D}} \right] \tag{2.4}$$

Where:

$$CST_L = [CST_L^1, CST_L^2, CST_L^3, CST_L^4, CST_L^5] \tag{2.5}$$

$$CST_{A,i,u} = \left[CST^1_{A,i,u}, CST^2_{A,i,u}, CST^3_{A,i,u}, CST^4_{A,i,u}, CST^5_{A,i,u}, CST^6_{A,i,u}\right] \tag{2.6}$$

$$CST_{A,i,l} = \left[CST^1_{A,i,l}, CST^2_{A,i,l}, CST^3_{A,i,l}, CST^4_{A,i,l}, CST^5_{A,i,l}, CST^6_{A,i,l}\right] \tag{2.7}$$

$$CST_{A,o,u} = \left[CST^1_{A,o,u}, CST^2_{A,o,u}, CST^3_{A,o,u}, CST^4_{A,o,u}, CST^5_{A,o,u}, CST^6_{A,o,u}\right] \tag{2.8}$$

$$CST_{A,o,l} = \left[CST^1_{A,o,l}, CST^2_{A,o,l}, CST^3_{A,o,l}, CST^4_{A,o,l}, CST^5_{A,o,l}, CST^6_{A,o,l}\right] \tag{2.9}$$

2.2. Bounds

To further accelerate the optimiser, it is necessary to specify bounds on the design variables whenever possible.

For the wingspan, the minimum is the span of the inboard wing, to prevent "negative wing" problems. Similarly, the lower bound for S is chosen to prevent inconsistent wing parameterisation; the upper bound based on a value that is assumed to exceed the feasible design space. The upper bound for the span is chosen to stay within the same airport category [1] as the original aircraft. Sweep is limited to prevent excessive (45°is already high for such an aircraft) or reverse sweep, taper to prevent inverse taper and negative areas. Twist is bound not to exceed a value of 5 °, which is already well exceeding typical values of 2-3°. Aerofoil CST coefficients are bound to not deviate more than 15% from the original aerofoil coefficients. This is based on consultations with an instructor. If the optimum solution seems to converge towards a value close to these bounds, an increase will be considered to prevent artificial limitations on a more optimal solution. The consistency variables are all bound to prevent values that are physically unattainable on the lower side. The upper bounds are set based on values assumed to exceed the feasible design space. The exception here is the upper bound of the fuel mass, where an increase beyond the value for the current design point would defy the purpose of the optimiser. If the optimum solution for any design variable converges very close to an assumed limit, the same procedure as with the CST coefficient bounds will be considered.

These bounds are presented again in Table 2.1.

Variable	Lower Bound	Upper Bound	Unit
S	$S_{i,min}$	$2S^0$	$[m^2]$
b	y_{kink}	36	$[m]$
Λ_{LE}	0	45	[°]
λ_o	0	1	[-]
CST_A	$0.85 \cdot CST_A^0$	$1.15CST_A^0$	[-]
ϕ_o	-5	5	[°]
$\widehat{MTOW}$	0	$1.5\widehat{MTOW}^0$	[kg]
$\widehat{W_f}$	0	$\widehat{W_f}^0$	[kg]
$\widehat{CST_L}$	-10	10	[-]
$\widehat{SF_L}$	0	$3\widehat{SF_L}$	[N]
$\widehat{N_2}$	0	1	[-]
$\widehat{\frac{L}{D}}$	0	30	[-]

Table 2.1: Bounds

2.3. Objective Function

The purpose of the objective function is to relate design variables and outputs of disciplines to the parameter that is to be minimised in the optimiser. However, as the mission fuel weight is already a direct output of the performance discipline, in this analysis the objective function is a mere feedthough of the fuel weight. As a clarification, the (actual) objective function that is contained in the performance discipline is presented here in eq. (2.10). This is a rewritten form of the two Breguet range equations provided in the assignment specification [2].

Minimise:

$$f(\mathbf{x}) = \left[1 - 0.938 \cdot \exp\left(\frac{R \cdot C_T}{V \cdot \frac{L}{D}}\right)\right] \cdot MTOW \tag{2.10}$$

Where:

- $\frac{L}{D}$ is determined by the Q3D aerodynamic analysis.
- $MTOW$ is determined by the EMWET structural analysis.

2.4. Inequality Constraints

Inequality constraints have a similar function as bounds, however their implementation in the solver differs and they allow to bound certain relationships between different design variables. For this problem, two inequality constraints are used, for wing loading and fuel volume.

2.4.1. Wing Loading

The maximum wing loading is not to exceed the reference aircraft maximum wing loading to prevent deteriorated landing and takeoff performance.

$$\frac{MTOW}{S} \leq \left(\frac{W}{S}\right)_{max,ref} \tag{2.11}$$

2.4.2. Fuel Volume

This constraint ensures the fuel tanks located within the wings have sufficient space to accommodate at least the mission fuel. The fuel tank is split in two sections, similarly to the wing trapezoidal elements. The available fuel volume is found using eq. (2.13), where S_1,S_2 and S_3 describe the cross sectional area between front and rear spar at the root, kink and tip respectively. These areas are found by integration of the CST-functions of the airfoil between the front and rear spar locations of 15% and 60% chord length.

$$V_{fuel} = \frac{W_f}{\rho_f} \leq V_{tank} f_{tank} \tag{2.12}$$

with:

$$V_{tank} = \frac{y_{kink}}{3} \cdot \left(S_1 + S_2 + \sqrt{S_1 \cdot S_2}\right) + \frac{b - y_{kink}}{3} \cdot \left(S_2 + S_3 + \sqrt{S_2 \cdot S_3}\right) \tag{2.13}$$

$$S_1 = c_r \int_{0.15}^{0.6} \zeta_r(\eta) d\eta \tag{2.14}$$

$$S_2 = c_k \int_{0.15}^{0.6} \zeta_k(\eta) d\eta \tag{2.15}$$

$$S_3 = c_t \int_{0.15}^{0.6} \zeta_t(\eta) d\eta \tag{2.16}$$

2.5. Equality & Consistency Constraints

The consistency constraints are required to check for convergence not only in fuel weight, but also for all other outputs of the different disciplines. Only when they converge is the whole optimiser consistent and the results have a physical meaning.

$$\widehat{MTOW} = MTOW \tag{2.17}$$

$$\widehat{W_f} = W_f \tag{2.18}$$

$$\widehat{CST_L} = CST_L \tag{2.19}$$

$$\widehat{SF_L} = SF_L \tag{2.20}$$

$$\widehat{N_2} = N_2 \tag{2.21}$$

$$\widehat{\frac{L}{D}} = \frac{L}{D} \tag{2.22}$$

These equality constraints will be subject to a tolerance to improve convergence and performance.

3

Part 1.2: XDSM

This chapter shows the XDSM for the wing shape optimisation described in the previous chapter. On the diagonal, the different disciplines, objective and constraints are shown. Outputs for each are shown in the respective row, inputs for each on the respective column. The white boxes show initial values to initialise the optimiser, as well as the final resultant values after convergence. The grey boxes show which data is passed on to which discipline within the optimiser's run.

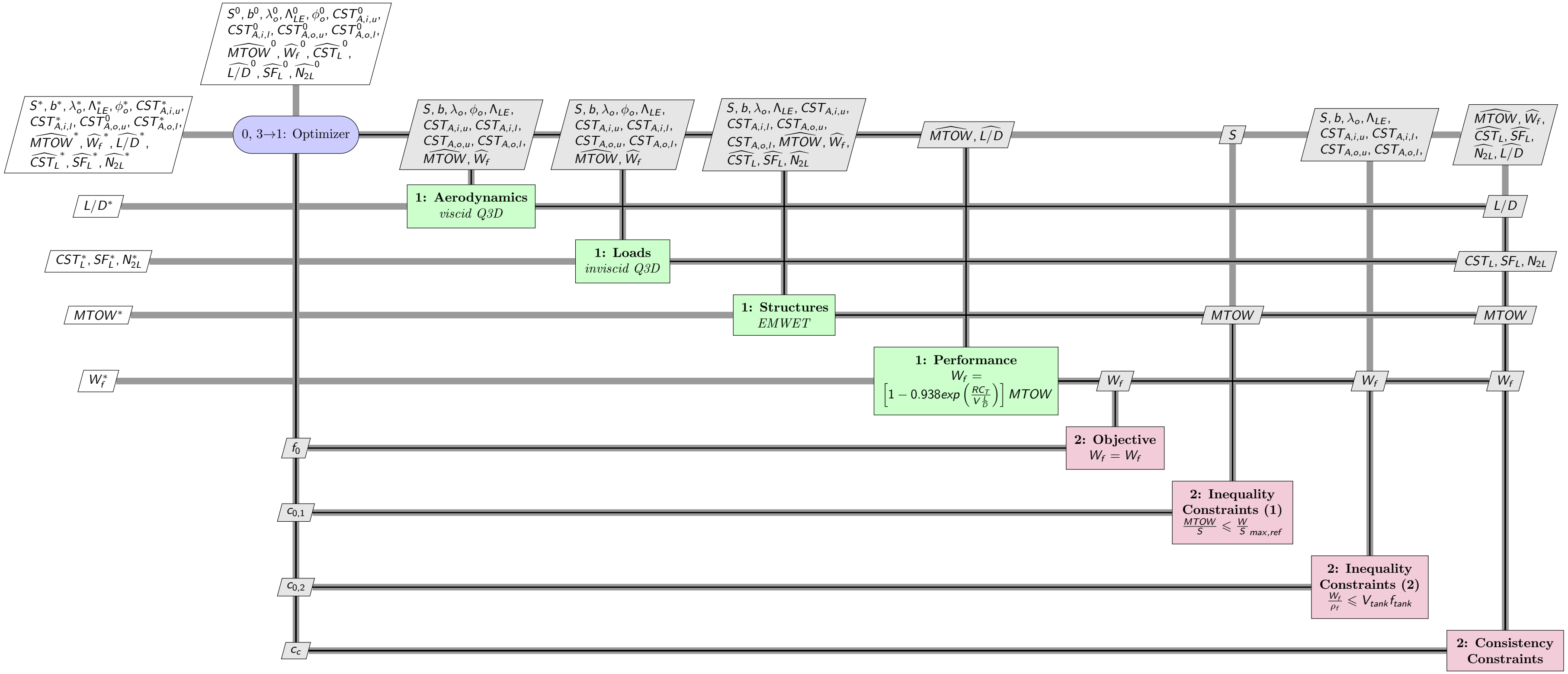

Figure 3.1: Extended Design Structure Matrix for Fuel Weight Optimisation of AVRO RJ 115

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[1] ICAO. Icao aerodrome reference code. URL https://www.skybrary.aero/index.php/ICAO_Aerodrome_Reference_Code.

[2] Gianfranco La Rocca. Ae4-205 mdo for aerospace applications homework assignment, 2018.