MCA101 : COMPUTER GRAPHICS

2D GEOMETRY REPRESENTATION

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OUTLINE

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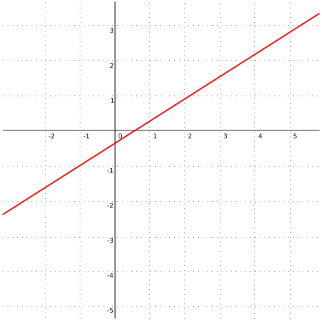
1. 2D GEOMETRY — INTRODUCTION
   - Straight Lines
   - Conics

2. MID-POINT ALGORITHM

$y = mx + c$

$$y = f(x) = mx + c$$



PARAMETRIC FORM

For any two vectors $\mathbf{u}, \mathbf{v} \in V$, a point on the line segment joining them is given parameterised by $t \in [0, 1]$, as

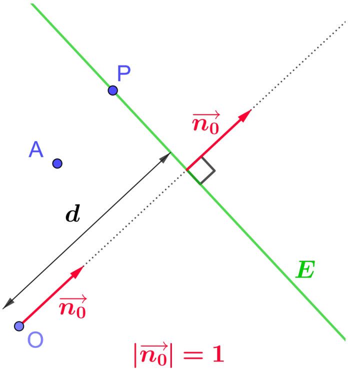
$$\mathbf{p} = f(t) = (1 - t)\mathbf{u} + t\mathbf{v}$$

PARAMETRIC FORM

Any point on a line in the direction of unit vector $\mathbf{u} : \|\mathbf{u}\|_2^2 = 1$, and an incident point $\mathbf{p}_0$ may be given parameterised by $t \in \mathbb{R}$ as,

$$\mathbf{p} = f(t) = \mathbf{p}_0 + t\mathbf{u}$$

HESSE NORMAL FORM



Distance from the origin O to the line E calculated with the Hesse normal form. Normal vector in red, line in green, point O shown in blue.

Given,
Normal to the line $\mathbf{n}_0 : \|\mathbf{n}_0\|_2^2 = 1$, and
its distance from origin, d;

The point on the line is given implicitly as the locus of all points $\mathbf{p}$ that satisfy,

$$\mathbf{n}_0 \cdot \mathbf{p} - d = 0$$

OUTLINE

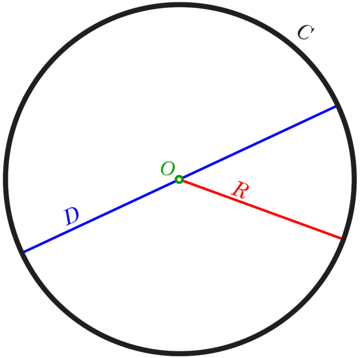


FIGURE: Image Courtesy: Wikipedia

Implicit Form:

$$f\begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2 - r^2 = 0$$

Parametric Form:

$$f(r, t) = \begin{bmatrix} r\cos t \\ r\sin t \end{bmatrix}$$

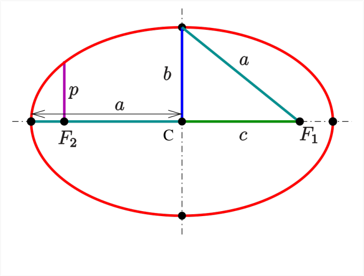


FIGURE: Image Courtesy: Wikipedia

Standard form

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

Parametric Form

$$f(t; a, b) = \begin{bmatrix} a \cos t \\ b \sin t \end{bmatrix}$$

OUTLINE

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PROBLEM

In a quantised (pixelated or discrete) 2d plane, find the set of points that visually approximate a given curve, say a straight line or a conic.

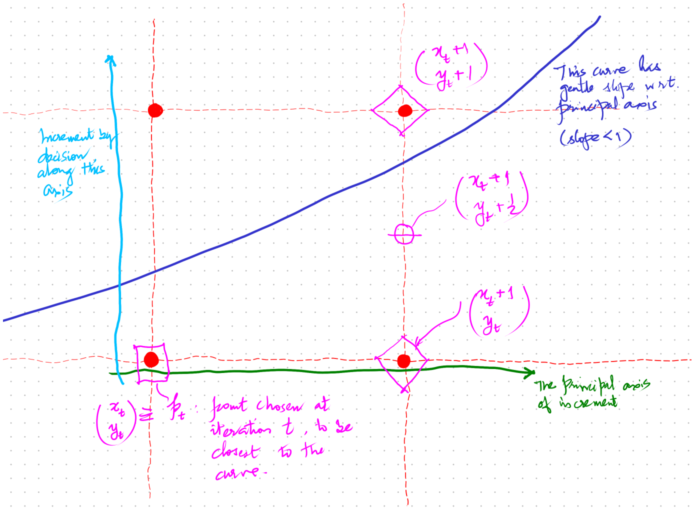
METHOD

Iteratively, increment along one axes,
with respect to which, the slope of the curve
is gentle.

Decide whether it is required to increment
along the perpendicular axis or not.

Increment if required.

EXAMPLE



Increment by decision along this axis
$\begin{pmatrix} x_t+1 \\ y_t+1 \end{pmatrix}$
This curve has gentle slope w.r.t. principal axis (slope < 1)
$\begin{pmatrix} x_t+1 \\ y_t+\frac{1}{2} \end{pmatrix}$
$\begin{pmatrix} x_t+1 \\ y_t \end{pmatrix}$
The principal axis of increment
$\begin{pmatrix} x_t \\ y_t \end{pmatrix} \equiv p_t$: point chosen at iteration t, to be closest to the curve.