# MCA101 : COMPUTER GRAPHICS

## 2D GEOMETRY REPRESENTATION

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# OUTLINE

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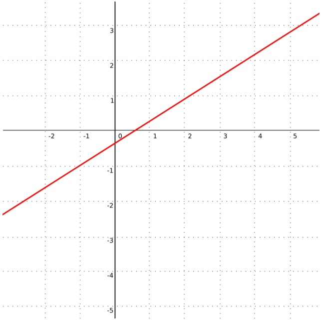
# $y = mx + c$

$$y = f(x) = mx + c$$



# PARAMETRIC FORM

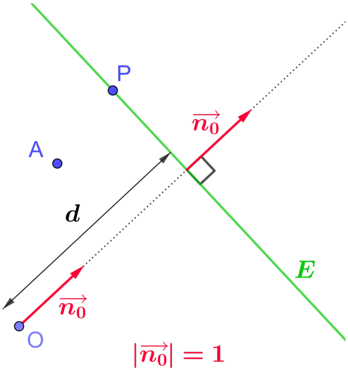
For any two vectors $\mathbf{u}, \mathbf{v} \in V$, a point on the line segment joining them is given parameterised by $t \in [0, 1]$, as

$$\mathbf{p} = f(t) = (1 - t)\mathbf{u} + t\mathbf{v}$$

# PARAMETRIC FORM

Any point on a line in the direction of unit vector $\mathbf{u} : \|\mathbf{u}\|_2^2 = 1$, and an incident point $\mathbf{p}_0$ may be given parameterised by $t \in \mathbb{R}$ as,

$$\mathbf{p} = f(t) = \mathbf{p}_0 + t\mathbf{u}$$




Distance from the origin $O$ to the line $E$ calculated with the Hesse normal form. Normal vector in red, line in green, point O shown in blue.

Given,
Normal to the line $\mathbf{n}_0 : \|\mathbf{n}_0\|_2^2 = 1$, and
its distance from origin, $d$;

The point on the line is given implicitly as the locus of all points $\mathbf{p}$ that satisfy,

$$\mathbf{n}_0 \cdot \mathbf{p} - d = 0$$

# OUTLINE

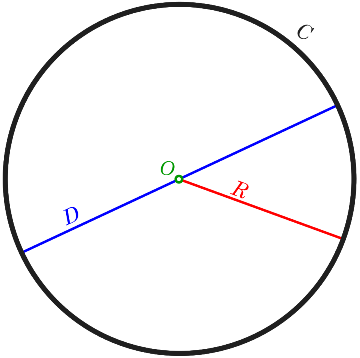



FIGURE: Image Courtesy: Wikipedia

Implicit Form:

$$f\begin{pmatrix} x \\ y \end{pmatrix} = x^2 + y^2 - r^2 = 0$$

Parametric Form:

$$f(r, t) = \begin{bmatrix} r\cos t \\ r\sin t \end{bmatrix}$$

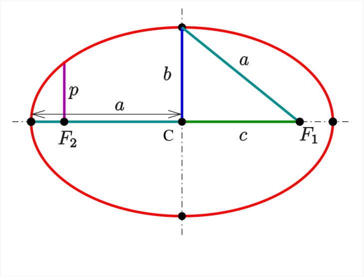



FIGURE: Image Courtesy: Wikipedia

Standard form

$$f\begin{pmatrix} x \\ y \end{pmatrix} = \frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0$$

Parametric Form

$$f(t; a, b) = \begin{bmatrix} a \cos t \\ b \sin t \end{bmatrix}$$

# OUTLINE

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# PROBLEM

In a quantised (pixelated or discrete) 2d plane, find the set of points that visually approximate a given curve, say a straight line or a conic.

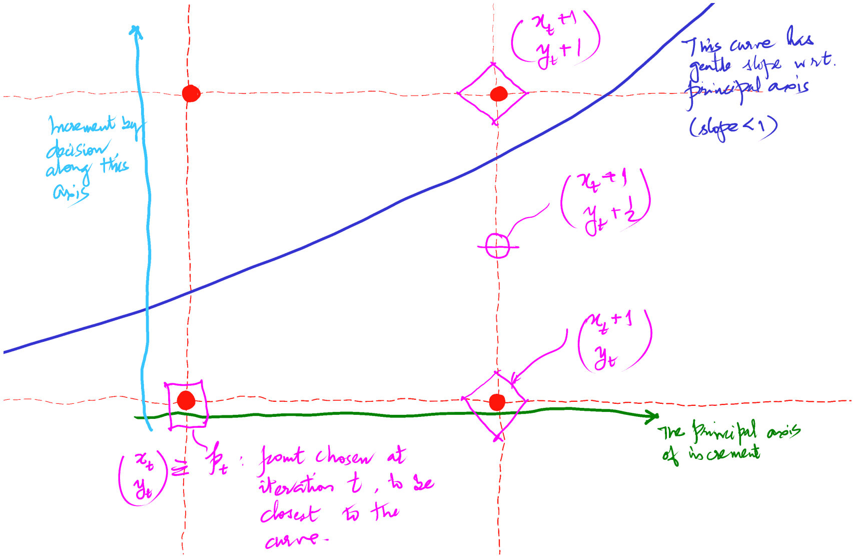
# METHOD

Iteratively, increment along one axes,
with respect to which, the slope of the curve
is gentle.

Decide whether it is required to increment
along the perpendicular axis or not.

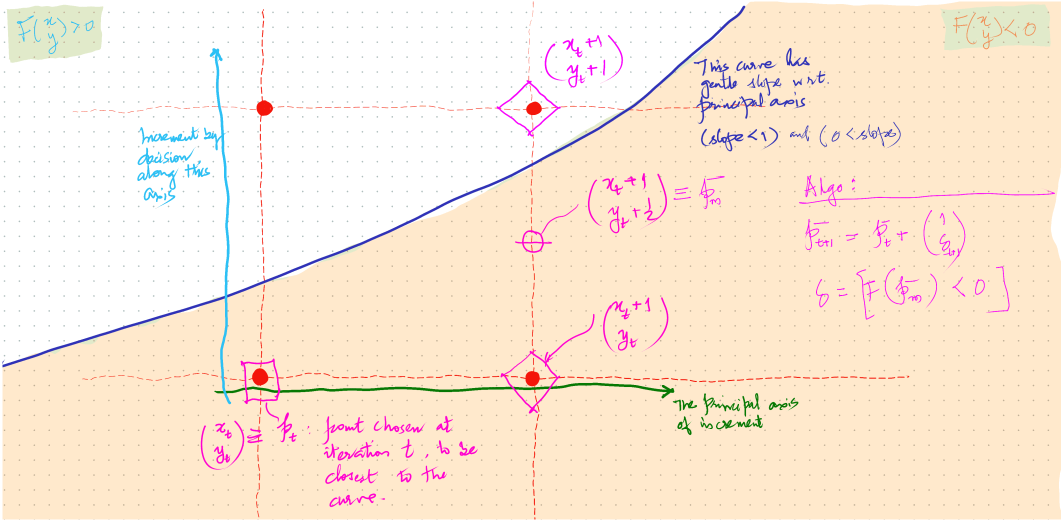
Increment if required.

# EXAMPLE



Increment by decision along this axis
$\begin{pmatrix} x_t+1 \\ y_t+1 \end{pmatrix}$
This curve has gentle slope w.r.t. principal axis (slope < 1)
$\begin{pmatrix} x_t+1 \\ y_t+\frac{1}{2} \end{pmatrix}$
$\begin{pmatrix} x_t+1 \\ y_t \end{pmatrix}$
$\begin{pmatrix} x_t \\ y_t \end{pmatrix} \equiv p_t$ : point chosen at iteration $t$, to be closest to the curve.
The principal axis of increment

# EXAMPLE


$F\binom{x}{y} > 0$
$F\binom{x}{y} < 0$
$\binom{x_t+1}{y_t+1}$
This curve has gentle slope w.r.t. principal axis
(slope < 1) and (0 < slope)
Increment by decision along this axis
$\binom{x_t+1}{y_t+\frac{1}{2}} \equiv p_m$
Algo:
$p_{t+1} = p_t + \binom{1}{\delta_{t+1}}$
$\delta = [F(p_m) < 0]$
$\binom{x_t+1}{y_t}$
$\binom{x_t}{y_t} \equiv p_t$ : point chosen at iteration $t$, to be closest to the curve.
The principal axis of increment

Mid-point algorithm is applicable to a curve within a given finite interval, iff

1. The curve increases monotonically;
2. The curve increases gradually.

In other words,

$$0 \leqslant \mathrm{d}y \leqslant \mathrm{d}x$$

# GENERIC ALGO

**Algorithm 1:** Generic Mid-point Algorithm

**Input:** $x_0, x_{\max} \in \mathbb{Z}$ — Start and end x-coordinates.

**Input:** $F : \mathbb{R}^2 \to \mathbb{R}$ — Signed Distance Function from the curve.

**Output:** $C \equiv \{\mathbf{p}_0, \ldots, \mathbf{p}_{\max}\} \subset \mathbb{Z}^2$ — Curve in discrete 2D space.

$\mathbf{p}_0 \leftarrow \begin{bmatrix} x_0 \\ \lceil y_0 \rceil \end{bmatrix} \vdash F\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = 0$

**for** $t \in \{1, \ldots, \max\}$ **do**

- $\mathbf{p}_{\mathrm{mid}} \leftarrow \mathbf{p}_{t-1} + \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$
- $\delta_t \leftarrow I[F(\mathbf{p}_{\mathrm{mid}}) < 0]$
- $\mathbf{p}_t \leftarrow \mathbf{p}_{t-1} + \begin{pmatrix} 1 \\ \delta_t \end{pmatrix}$

# OUTLINE

# CHARACTERISING STRAIGHT LINES

$$F(x, y) = Ax - By + C$$

$$
\begin{array}{rcll}
0 \leqslant \mathrm{d}y/\mathrm{d}x \leqslant 1 & \mapsto & 0 \leqslant A \leqslant B & \quad \ldots\textsc{case } 1 \\
-1 \leqslant \mathrm{d}y/\mathrm{d}x \leqslant 0 & \mapsto & 0 \leqslant A \leqslant -B & \quad \ldots\textsc{case } 2 \\
0 \leqslant \mathrm{d}x/\mathrm{d}y \leqslant 1 & \mapsto & 0 \leqslant B \leqslant A & \quad \ldots\textsc{case } 3 \\
-1 \leqslant \mathrm{d}x/\mathrm{d}y \leqslant 0 & \mapsto & 0 \leqslant -B \leqslant A & \quad \ldots\textsc{case } 4
\end{array}
$$

# MID-POINT ALGO FOR ST. LINE

**Algorithm 2:** Mid-Point Algorithm for Straight Line (intermediate attempt)

**Input:** $x_0, x_{max} \in \mathbb{Z}$ — Start and end x-coordinates.

**Input:** $a, b, c \in \mathbb{Z} \vdash 0 \leqslant a < b$; $b$ even — Coefficients: $F(x, y) = ax - by + c$. **Rem:** $-by$.

**Output:** $C \equiv \{(x_0, y_0), \ldots, (x_{max}, y_{max})\} \subset \mathbb{Z}^2$

An ordered sequence; a curve in discrete 2D space.

$y_0 \leftarrow \left\lceil \frac{ax_0 + c}{b} \right\rceil$

$\delta_0 \leftarrow ax_0 - by_0 - \frac{b}{2} + c$ — **Rem:** $-by_0 - \frac{b}{2}$; $\frac{b}{2} \in \mathbb{Z}$ because $b$ even.

**for** $t \in \{1, \ldots, \max\}$ **do**

- $x_t \leftarrow x_{t-1} + 1$
- $\delta_t \leftarrow \delta_{t-1} + a - b \cdot I[\delta_{t-1} < 0]$
- $y_t \leftarrow y_{t-1} + I[\delta_t < 0]$

# MID-POINT ALGO FOR ST. LINE

**Algorithm 3:** Mid-Point Algorithm for Straight Line

**Input:** $x_0, x_{\max} \in \mathbb{Z}$ — Start and end x-coordinates.

**Input:** $a, b, c \in \mathbb{Z} \vdash 0 \leqslant a < b$; $b$ even — Coefficients: $F(x, y) = ax - by + c$. **Rem:** $-by$.

**Output:** $C \equiv \{(x_0, y_0), \ldots, (x_{\max}, y_{\max})\} \subset \mathbb{Z}^2$

An ordered sequence; a curve in discrete 2D space.

**Init:** $C$ as a new Array.

**Init:** $x, y, \delta$ as integers.

$x \leftarrow x_0$

$y \leftarrow \left\lceil \frac{ax+c}{b} \right\rceil$

$\delta \leftarrow ax - by - \frac{b}{2} + c$ — **Rem:** $-by - \frac{b}{2}$; $\frac{b}{2} \in \mathbb{Z}$ because $b$ even.

$C\cdot$ Push$((x, y))$

**for** $t \in \{1, \ldots, \max\}$ **do**

- $x \leftarrow x + 1$
- $\delta \leftarrow \delta + a - b \cdot I[\delta < 0]$
- $y \leftarrow y + I[\delta < 0]$
- $C\cdot$ Push$((x, y))$

# OUTLINE

# GENERIC ALGO

**Algorithm 4:** Generic Mid-point Algorithm

**Input:** $x_0, x_{\max} \in \mathbb{Z}$ — Start and end x-coordinates.

**Input:** $F : \mathbb{R}^2 \to \mathbb{R}$ — Signed Distance Function from the curve.

**Output:** $C \equiv \{\mathbf{p}_0, \ldots, \mathbf{p}_{\max}\} \subset \mathbb{Z}^2$ — Curve in discrete 2D space.

$\mathbf{p}_0 \leftarrow \begin{bmatrix} x_0 \\ \lceil y_0 \rceil \end{bmatrix} \vdash F\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = 0$

**for** $t \in \{1, \ldots, \max\}$ **do**

- $\mathbf{p}_{\mathrm{mid}} \leftarrow \mathbf{p}_{t-1} + \begin{pmatrix} 1 \\ \frac{1}{2} \end{pmatrix}$
- $\delta_t \leftarrow I[F(\mathbf{p}_{\mathrm{mid}}) < 0]$
- $\mathbf{p}_t \leftarrow \mathbf{p}_{t-1} + \begin{pmatrix} 1 \\ \delta_t \end{pmatrix}$