Chapter 2
Quantization

Ha Hoang Kha, Ph.D.
Ho Chi Minh City University of Technology
Email: hhkha@hcmut.edu.vn

1. Quantization process

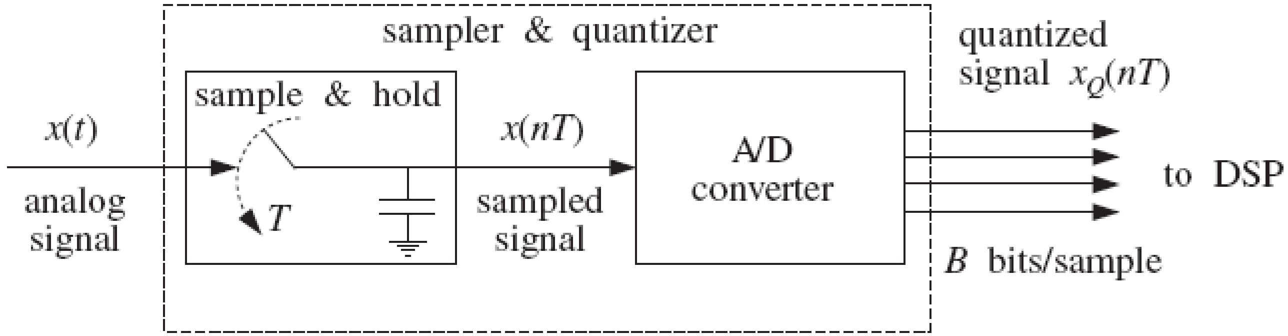

Fig: Analog to digital conversion

- The quantized sample $x_Q(nT)$ is represented by ***B*** **bit**, which can take 2^B possible values.

- An A/D is characterized by a **full-scale range R** which is divided into 2^B quantization levels. Typical values of R in practice are between 1-10 volts.

1. Quantization process

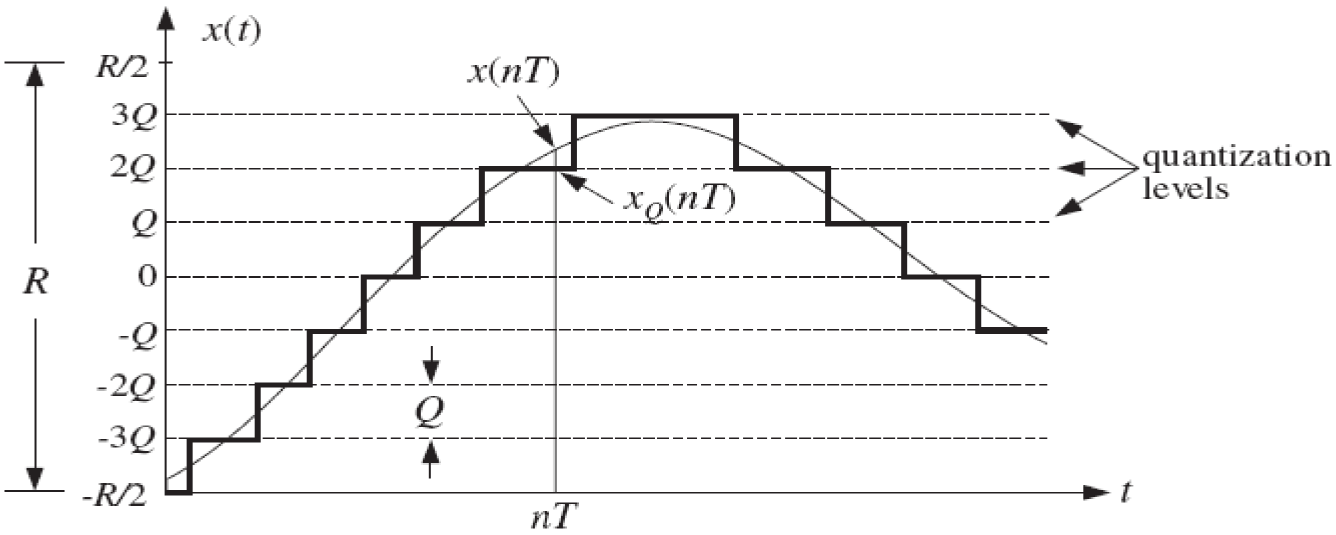

Fig: Signal quantization

- Quantizer resolution or quantization width $Q = \dfrac{R}{2^B}$
- A **bipolar** ADC $-\dfrac{R}{2} \leq x_Q(nT) < \dfrac{R}{2}$
- A **unipolar** ADC $0 \leq x_Q(nT) < R$

1. Quantization process –Quantization error

- Quantization by rounding: replace each value x(nT) by the nearest quantization level.

- Quantization by truncation: replace each value x(nT) by its below quantization level.

- Quantization error: $e(nT) = x_Q(nT) - x(nT)$

- Consider rounding quantization: $-\frac{Q}{2} \le e \le \frac{Q}{2}$

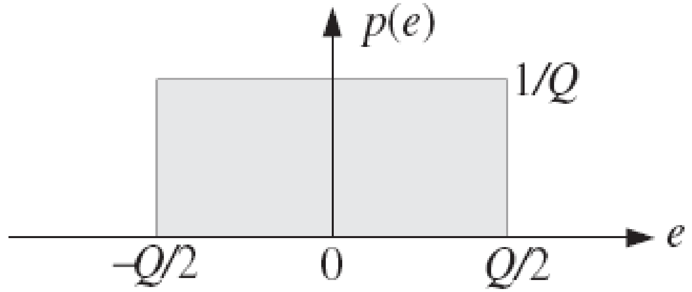

Fig: Uniform probability density of quantization error

1. Quantization process –Quantization error

- The mean value of quantization error $\bar{e} = \int_{-Q/2}^{Q/2} ep(e)de = \int_{-Q/2}^{Q/2} e\frac{1}{Q}de = 0$

- The mean-square error (power) $\sigma^2 = \overline{e^2} = \int_{-Q/2}^{Q/2} e^2 p(e)de = \int_{-Q/2}^{Q/2} e^2 \frac{1}{Q}de = \frac{Q^2}{12}$

- Root-mean-square (rms) error: $\boxed{e_{rms} = \sigma = \sqrt{\overline{e^2}} = \frac{Q}{\sqrt{12}}}$

- R and Q are the ranges of the signal and quantization noise, then the signal to noise ratio (SNR) or dynamic range of the quantizer is defined as

$$\boxed{SNR_{dB} = 20\log_{10}\left(\frac{R}{Q}\right) = 20\log_{10}(2^B) = B\log_{10}(2) = 6B\ dB}$$

which is referred to as 6 dB bit rule.

1. Quantization process –Example

- In a digital audio application, the signal is sampled at a rate of 44 KHz and each sample quantized using an A/D converter having a full-scale range of 10 volts. Determine the number of bits B if the rms quantinzation error mush be kept below 50 microvolts. Then, determine the actual rms error and the bit rate in bits per second.

2. Digital to Analog Converters (DACs)

❖ We begin with A/D converters, because they are used as the building blocks of successive approximation ADCs.

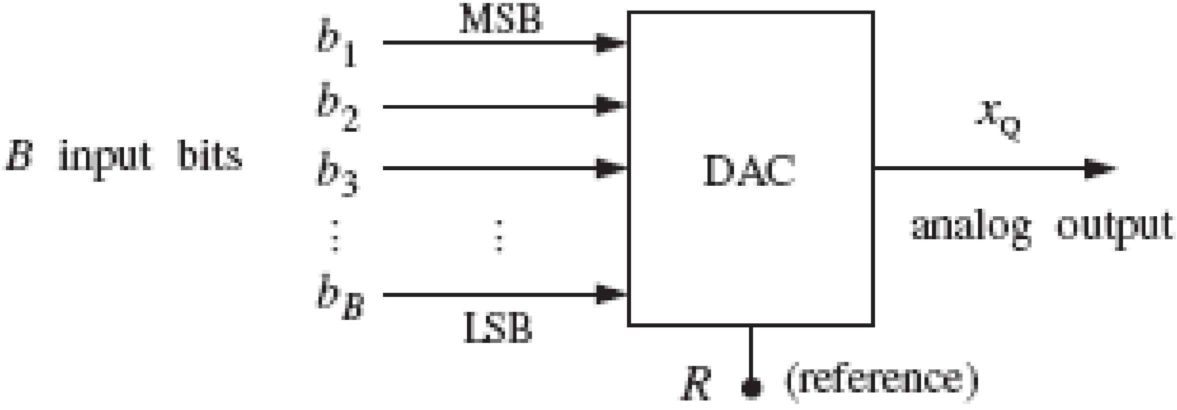

Fig: B-bit D/A converter

❖ Vector B input bits : $b=[b_1, b_2,\ldots,b_B]$. Note that b_B is the least significant bit (LSB) while b_1 is the most significant bit (MSB).

❖ For unipolar signal, $x_Q \in [0, R)$; for bipolar $x_Q \in [-R/2, R/2)$.

2. DAC-Example DAC Circuit

❖ Full scale $R=V_{REF}$, B=4 bit

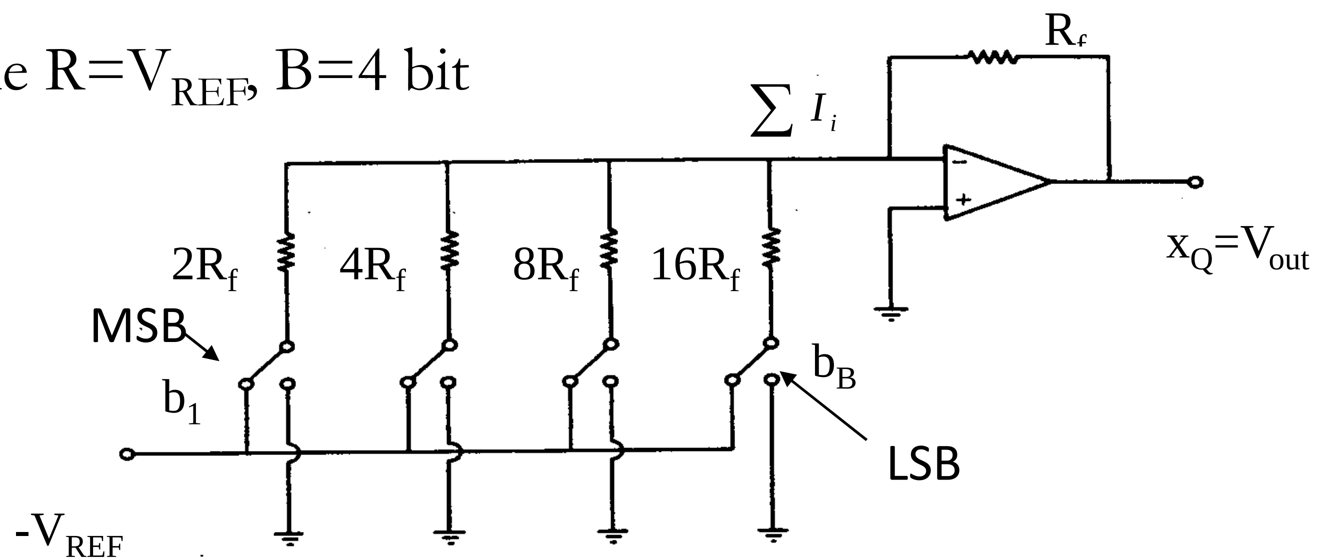

Fig: DAC using binary weighted resistor

$$\sum I = V_{REF}\left(\frac{b_1}{2R_f}+\frac{b_2}{4R_f}+\frac{b_3}{8R_f}+\frac{b_4}{16R_f}\right)$$

$$x_Q = V_{OUT} = \sum I \cdot R_f = V_{REF}\left(\frac{b_1}{2}+\frac{b_2}{4}+\frac{b_3}{8}+\frac{b_4}{16}\right)$$

$$x_Q = R2^{-4}\left(b_1 2^{-3}+b_2 2^{-2}+b_3 2^{-1}+b_4 2^{0}\right) = Q\left(b_1 2^{-3}+b_2 2^{-2}+b_3 2^{-1}+b_4 2^{0}\right)$$

2. D/A Converters

❖ Unipolar natural binary $\boxed{x_Q = R(b_1 2^{-1} + b_2 2^{-2} + \ldots + b_B 2^{-B}) = Qm}$

where m is the integer whose binary representation is b=[b_1, b_2,…,b_B].

$$\boxed{m = b_1 2^{B-1} + b_2 2^{B-2} + \ldots + b_B 2^0}$$

❖ Bipolar offset binary: obtained by shifting the x_Q of unipolar natural binary converter by half-scale R/2:

$$\boxed{x_Q = R(b_1 2^{-1} + b_2 2^{-2} + \ldots + b_B 2^{-B}) - \frac{R}{2} = Qm - \frac{R}{2}}$$

❖ Two's complement code: obtained from the offset binary code by complementing the most significant bit, i.e., replacing b_1 by $\overline{b_1} = 1 - b_1$.

$$\boxed{x_Q = R(\overline{b_1} 2^{-1} + b_2 2^{-2} + \ldots + b_B 2^{-B}) - \frac{R}{2}}$$

2. D/A Converters-Example

❖ A 4-bit D/A converter has a full-scale R=10 volts. Find the quantized analog values for the following cases ?

a) Natural binary with the input bits b=[1001] ?

b) Offset binary with the input bits b=[1011] ?

c) Two's complement binary with the input bits b=[1101] ?

3. A/D converter

- A/D converters quantize an analog value x so that is is represented by B bits $b=[b_1, b_2, \ldots, b_B]$.

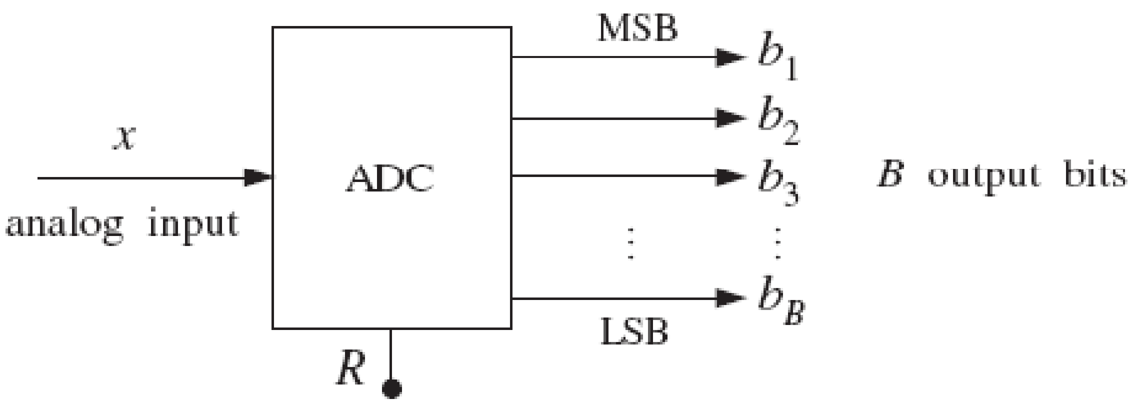

Fig: B-bit A/D converter

3. A/D converter

- One of the most popular converters is the successive approximation A/D converter

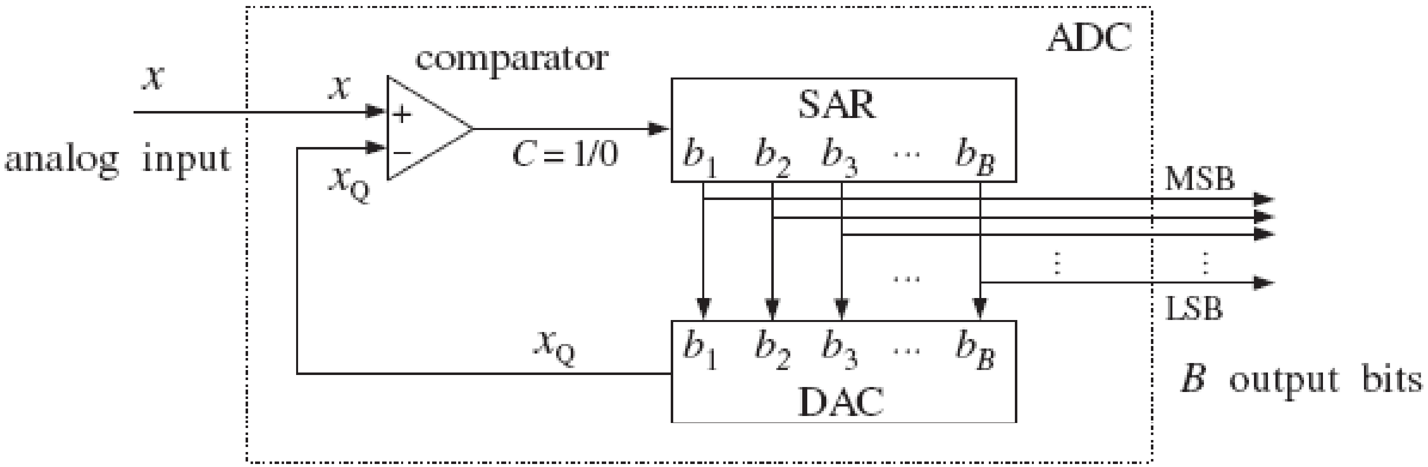

Fig: Successive approximation A/D converter

- After B tests, the successive approximation register (SAR) will hold the correct bit vector b.

3. A/D converter

❖ Successive approximation algorithm

for each x *to be converted, do:*
    initialize $\mathbf{b} = [0, 0, \ldots, 0]$
    for $i = 1, 2, \ldots, B$ *do:*
        $b_i = 1$
        $x_Q = \mathrm{dac}(\mathbf{b}, B, R)$
        $b_i = u(x - x_Q)$

where the unit-step function is defined by $u(x) = \begin{cases} 1 & if \;\; x \ge 0 \\ 0 & if \;\; x < 0 \end{cases}$

This algorithm is applied for the **natural and offset binary with truncation quantization.**

3. A/D converter-Example

- Consider a 4-bit ADC with the full-scale R=10 volts. Using the successive approximation algorithm to find offset binary of truncation quantization for the analog values x=3.5 volts and x=-1.5 volts.

3. A/D converter

- For rounding quantization, we shift x by Q/2:

$$
\begin{aligned}
&\textit{for each } x \textit{ to be converted, do:}\\
&\quad y = x + Q/2\\
&\quad \textit{initialize } \mathbf{b} = [0,0,\ldots,0]\\
&\quad \textit{for } i = 1,2,\ldots,B \textit{ do:}\\
&\qquad b_i = 1\\
&\qquad x_Q = \mathrm{dac}(\mathbf{b}, B, R)\\
&\qquad b_i = u(y - x_Q)
\end{aligned}
$$

- For the two's complement code, the sign bit b_1 is treated separately.

$$
\begin{aligned}
&\textit{for each } x \textit{ to be converted, do:}\\
&\quad y = x + Q/2\\
&\quad \textit{initialize } \mathbf{b} = [0,0,\ldots,0]\\
&\quad b_1 = 1 - u(y)\\
&\quad \textit{for } i = 2,3,\ldots,B \textit{ do:}\\
&\qquad b_i = 1\\
&\qquad x_Q = \mathrm{dac}(\mathbf{b}, B, R)\\
&\qquad b_i = u(y - x_Q)
\end{aligned}
$$

3. A/D converter-Example

❖ Consider a 4-bit ADC with the full-scale R=10 volts. Using the successive approximation algorithm to find offset and two's complement of rounding quantization for the analog values x=3.5 volts .

Homework

- Problems 2.1, 2.2, 2.3, 2.5, 2.6