

Assignment 2 - Part 1

1. Step 1: $a_i^t = \frac{F_{i, \text{total}}^t}{m_i}$

$$F_{\text{total}} = -\gamma \dot{x}_i + g_i + f_i$$

γ - DAMPING COEFF

Step 2: $v_i^{t+1} = v_i^t + \Delta t a_i^t$

g - TOTAL INTERNAL FORCE
 f - EXTERNAL FORCE

Step 3: $x_i^{t+1} = x_i^t + \Delta t v_i^{t+1}$

2. $f_{\text{ext}} = (2, 14.7, -5)^T$ $m = 1$ $\Delta t = 1$ $v(0) = 0$ $a(0) = 0$

$t=0$ $F_{\text{total}} = (2, 14.7, -5) + (0, -9.8, 0) = (2, 4.9, -5)$

$x(0) = (0, 0, 0)^T$

$a(0) = \frac{F_{\text{total}}}{m} = (2, 4.9, -5)$

$v(1) = v(0) + \Delta t a(0) = 0 + 1(2, 4.9, -5) = (2, 4.9, -5)^T$

$x(1) = x(0) + \Delta t v(1) = (0, 0, 0)^T + 1(2, 4.9, -5) = (2, 4.9, -5)^T$

$t=1$ $a(1) = (0, -9.8, 0)^T$

$v(2) = (2, 4.9, -5) + 1(0, -9.8, 0) = (2, -4.9, -5)^T$

$x(2) = (2, 4.9, -5) + 1(2, -4.9, -5) = (4, 0, -10)^T$

$x(3) = (4, 0, -10)^T$

3. LAGRANGE EQUATION OF MOTION: $m_i \ddot{x}_i + \gamma_i \dot{x}_i - g_i - f_i = 0$

γ_i - DAMPING COEFF f_i - external force g_i - total force due to springs

a) HEATING & MELTING DEFORMABLE MODELS - MRS-SPRING MODEL

- DIFFUSION OF HEAT IN MATERIALS: $\frac{\partial}{\partial t} (\mu \sigma \theta) - \nabla \cdot (C \nabla \theta) = q$

q = rate heat gain/loss per vol μ - kg/m³ σ - specific heat θ - Temp, kelvin

C - THERMAL CONDUCTIVITY MATRIX $\nabla = \left[\frac{\partial}{\partial u}, \frac{\partial}{\partial v}, \frac{\partial}{\partial w} \right]$

- HOMOGENEOUS, ISOTROPIC MATERIAL: $\frac{\partial}{\partial t} (\mu \sigma \theta) - c \nabla^2 \theta = q$ $[\dots] = \star \theta$

DISCRETE HEAT EQ: $\mu \sigma \frac{(\theta^{t+\Delta t} - \theta^t)}{\Delta t} - c \left[\frac{\theta_{u+\Delta u, v, w}^t - 2\theta_{u, v, w}^t + \theta_{u-\Delta u, v, w}^t}{\Delta u^2} + \frac{\theta_{u, v+\Delta v, w}^t - 2\theta_{u, v, w}^t + \theta_{u, v-\Delta v, w}^t}{\Delta v^2} + \frac{\theta_{u, v, w+\Delta w}^t - 2\theta_{u, v, w}^t + \theta_{u, v, w-\Delta w}^t}{\Delta w^2} \right] = q$

UPDATE θ $\theta_{u, v, w}^{t+\Delta t} = \theta_{u, v, w}^t + \frac{\Delta t}{\mu \sigma} c (\star \theta)$