JASON TIEU
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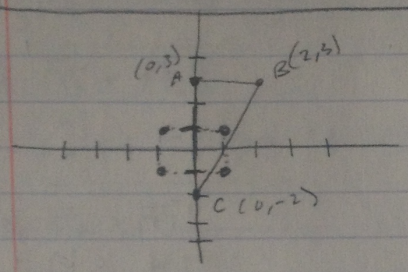
ASSIGNMENT 3

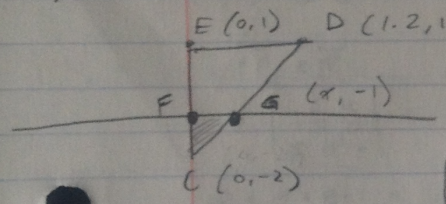
1) a) ACD

b) $B = (KLM)^{-1} = M^{-1}L^{-1}K^{-1}$

2)



b) BOTTOM EDGE



$y = -1$

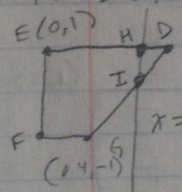
$G = (0.4, -1)$
$F = (0, -1)$
$E = (0, 1)$
$D = (1.2, 1)$

line $C \to D$: $\langle 0,-2\rangle + t\langle 1.2, 3\rangle$

$1.2t = x \qquad x = 0.4$

$-2 + 3t = -1 \qquad t = \frac{1}{3}$

c) RIGHT EDGE



$H = (1, 1)$

$I = (1, y)$

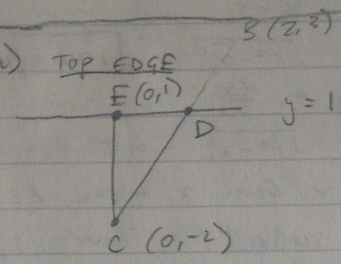
$x = 1$ line $D \to G$: $\langle 1.2, 1\rangle + t\langle -0.8, -2\rangle$

$1.2 - 0.8t = 1 \qquad t = \frac{1}{4}$

$1 - 2t = y = 0.5$

$H = (1, 1) \qquad E = (0, 1)$
$I = (1, 0.5) \qquad F = (0, -1)$
$G = (0.4, -1)$

a) TOP EDGE



$D = (x, 1)$

$C_x - B_x$, $C_y - B_y$

line $B \to C$: $\langle 2, 3\rangle + t\langle -2, -5\rangle$

$E = (0, 1)$
$D = (1.2, 1)$
$C = (0, -2)$

$-2t + 2 = x$

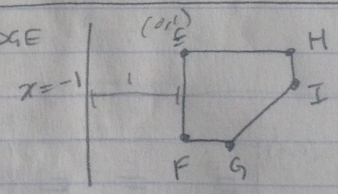
$-5t + 3 = 1 \qquad t = \frac{2}{5}$

$t = \frac{-3}{-2}$ / -5

$-2\left(\frac{2}{5}\right) + 2 = x$

$-\frac{4}{5} + \frac{10}{5} = x = \frac{6}{5} = 1.2$

D) LEFT EDGE



NO NEW POINTS FROM PART C

3) a) $x^2 + y^2 - r^2 = 0$

$x_0^2 + y_0^2 - r^2 = 0$

b) $dx = x_2 - x_1$

$dy = \sqrt{r^2 - x_2^2} - \sqrt{r^2 - x_1^2}$

$slope = \frac{dy}{dx}$

$y = \sqrt{r^2 - x_1^2} + 0.5$

```
for x from x1 to x2 do
    setPixel(x, Floor(y));
    y = y + slope * (x - x1)
```

c)

```
int x = 0, y = r, d = 3 - 2r;
while (x <= y)
    setPixel(x, y);
    if (d < 0)
        d = d + 4x + 6;
    else
        d = d + 4(x - y) + 10
        y--;
    x++;
```

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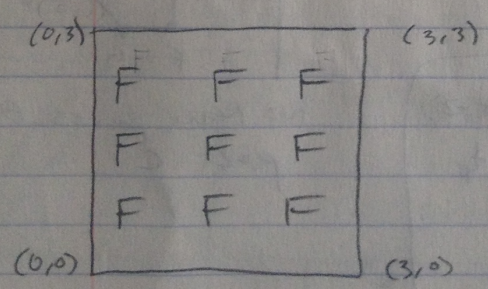
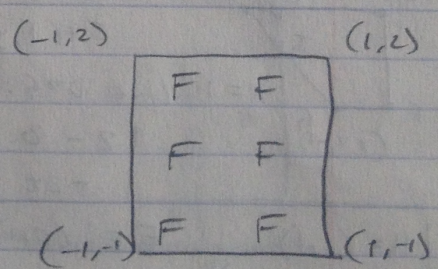
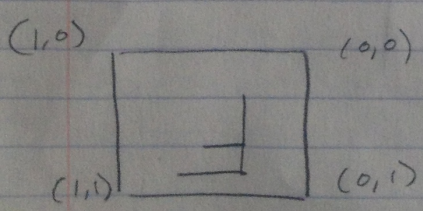
4) a)



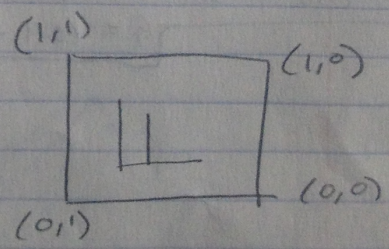
b)



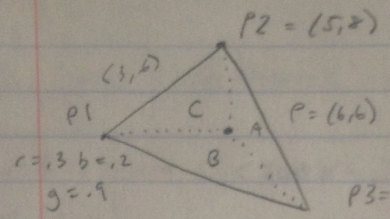
c)



d)



5)



$\vec{PP_1} = P_1 - P = \langle -3, 0 \rangle$

$\vec{PP_2} = P_2 - P = \langle -1, 2 \rangle$

$\vec{PP_3} = P_3 - P = \langle 2, -3 \rangle$

$A = \frac{1}{2} \| \vec{PP_3} \times \vec{PP_2} \|$

$$\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ -1 & 2 & 0 \end{vmatrix} = i(0-0) - j(0-0) + k(4-+3) = \frac{1}{2}\|\langle 0,0,1 \rangle\| = \frac{1}{2}$$

$B = \frac{1}{2} \| \vec{PP_3} \times \vec{PP_1} \|$

$$\begin{vmatrix} i & j & k \\ 2 & -3 & 0 \\ -3 & 0 & 0 \end{vmatrix} = i(0-0) - j(0-0) + k(0-+9) = \frac{1}{2}\|\langle 0,0,-9 \rangle\| = \frac{9}{2}$$

$C = \frac{1}{2} \| \vec{PP_2} \times \vec{PP_1} \|$

$$\begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -3 & 0 & 0 \end{vmatrix} = i(0) - j(0) + k(0 - -6) = \frac{1}{2}\|\langle 0,0,6 \rangle\| = 3$$

$D = A + B + C = 8$

$a = \frac{A}{D} = \frac{1}{16}$

$b = \frac{B}{D} = 9/16$

$c = C/D = 3/8$

P's $\langle r,g,b \rangle = a\langle .3, .9, .2 \rangle + b\langle .5, .5, .9 \rangle + c\langle .7, .2, .7 \rangle$

$\boxed{= \langle 0.5625, 0.4125, 0.7813 \rangle}$

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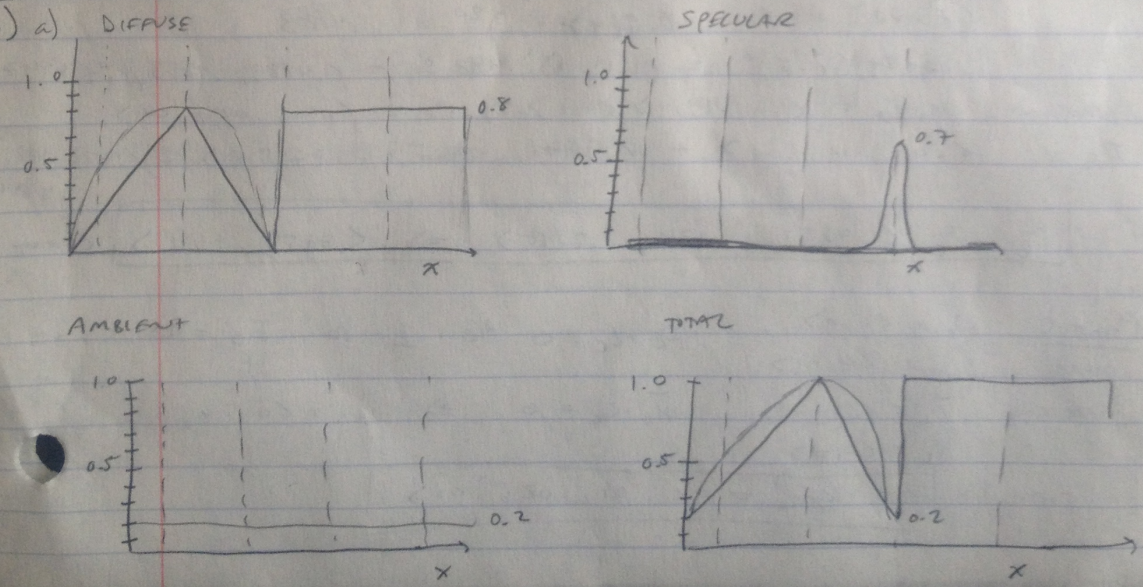
6) a)



6b) LOCAL ILLUMINATION IS DONE IN BETWEEN VIEWING TRANSFORMATION AND PROJECTION TRANSFORMATION IN THE GRAPHICS PIPELINE. THIS IS B/C THERE IS NO OBJECT-OBJECT INTERACTION & BASED IN THE VCS.

7) CALC NORMS IN AB + BC & INTERPOLATE

a) $\vec{AB} = B - A = \langle 5,1 \rangle$

$$\vec{N}_{AB} = \frac{\langle 1,-5 \rangle}{\|\langle 1,-5 \rangle\|} = \frac{1}{\sqrt{26}}\langle 1,-5 \rangle = \langle 0.1961, -0.9806 \rangle$$

$\vec{BC} = C - B = \langle 2, 2 \rangle$

$$N_{BC} = \frac{\langle 2,-2 \rangle}{\|\langle 2,-2 \rangle\|} = \frac{1}{\sqrt{2}}\langle 1,-1 \rangle = \langle .7071, -0.7071 \rangle$$

INTERPOLATE $\therefore$ $$N_B = \frac{\langle 0.1961, -0.9806 \rangle + \langle .7071, -.7071 \rangle}{\|\langle .1961, -0.9806 \rangle + \langle .7071, -.7071 \rangle\|} = \langle 0.4718, -.8817 \rangle$$

b) BLINN-PHONG MODEL: $I = k_A I_A + k_D I_D (\vec{N} \cdot \vec{L}) + k_S I_S (\vec{r} \cdot \vec{v})^n$

POINT B: $\vec{N}_B = \langle 0.4718, -.8817 \rangle$

DIFFUSE: $$\vec{L}_B = \frac{\langle 8,1 \rangle - \langle 6,3 \rangle}{\|\langle 2,2 \rangle\|} = \frac{1}{\sqrt{2}}\langle 1,-1 \rangle = \langle .7071, -.7071 \rangle$$

$\vec{N}_B \cdot \vec{L}_B = .9571$ $\quad k_D I_D (\vec{N} \cdot \vec{L}) = \langle .3, .8, .81 \rangle \cdot (.9571)$
$= \langle .2871, .7657, .7753 \rangle$

SPECULAR $$\vec{v} = \frac{E - B}{\|E - B\|} = \frac{\langle 2,-4 \rangle}{\|\langle 2,-4 \rangle\|} = \frac{1}{\sqrt{20}}\langle 2,-4 \rangle = \langle .4472, -.8944 \rangle$$

$\vec{r} = 2(N(N \cdot L)) - L = 2\langle 0.4718, -.8817 \rangle (.9571) - \langle .7071, -.7071 \rangle$
$= \langle .1960, -.9807 \rangle$

$(\vec{r} \cdot \vec{v})^n = (\quad 0.9648 \quad)^{20} = .4883$

$k_S I_S (\vec{r} \cdot \vec{v})^n = (1,1,.9) .4883 = \langle .4883, .4883, .43947 \rangle$

AMBIENT $k_A I_A = \langle .1, .1, .1 \rangle \langle .1, .2, .1 \rangle = \langle .01, .02, .01 \rangle$

$I_B = \langle .01, .02, .01 \rangle + \langle .2871, .7657, .7753 \rangle + \langle .4883, .4883, .43947 \rangle$

$$\boxed{I_B = \langle .7854, 1.274, 1.2248 \rangle \Rightarrow \langle .7854, 1, 1 \rangle}$$ CLAMP

POINT C $N_C = \langle 1, 0 \rangle$ $\quad N_C \cdot L_C = 0$ B/C $\perp$ SO $I_D = \langle 0, 0, 0 \rangle$

DIFFUSE $L_C = \langle 0, -1 \rangle$

SPECULAR $\vec{V}_C = \langle 0, -1 \rangle$ $\quad \vec{H}_C \cdot \vec{N}_C = 0 \Rightarrow I_S = \langle 0, 0, 0 \rangle$

$H_C = \langle 0, -1 \rangle$

TOTAL $= \boxed{I_C = k_A I_A = \langle .01, .02, .01 \rangle}$

POINT D B/C FLAT SHADING MODEL $I_D = I_B$ OR $I_D = I_C$

C) POINT B & POINT C - SEE PART 7b

POINT D

$$I_D = \tfrac{1}{2} I_B + \tfrac{1}{2} I_C$$

$$K_A I_A + \tfrac{1}{2} k_D I_D (\vec{N_B} \cdot \vec{L_B}) + \tfrac{1}{2} k_S I_S (\vec{H_B} \cdot \vec{N_B})^n$$

$$I_D = \langle .01, .02, .01 \rangle + \tfrac{1}{2} \langle .2871, .7657, .7753 \rangle + \tfrac{1}{2} \langle .7254, 1.274, 1.2248 \rangle = \langle .5721, .8214, .7743 \rangle$$

D) POINT B + POINT C - SEE PART 7c

$$\vec{N_D} = \frac{\vec{N_B} + \vec{N_C}}{\|\vec{N_B} + \vec{N_C}\|} = \frac{\langle 0.4718, -.8817 \rangle + \langle 1, 0 \rangle}{\| \langle 1.4718, -.8817 \rangle \|} = \langle .8578, -.5139 \rangle$$

$$\vec{V_D} = \frac{E - D}{\|E - D\|} = \frac{1, -5}{\sqrt{26}} = \langle 0.1961, -0.9806 \rangle$$

$$\vec{L_D} = \frac{L - D}{\|L - D\|} = \frac{1, -3}{\sqrt{10}} = \langle .3162, -0.9487 \rangle$$

$$\vec{H_D} = \frac{L_D + V_D}{\|L_D + V_D\|} = \frac{\langle .5123, -1.9293 \rangle}{1.9962} = \langle 0.2567, 0.9665 \rangle$$

DIFFUSE $k_D I_D (\vec{N_D} \cdot \vec{L_D}) = \langle .3, .8, .9 \rangle \langle 1, 1, .9 \rangle (.75877329)$

$= \langle .2276, .6070, .5146 \rangle$

SPECULAR $k_S I_S (\vec{H_D} \cdot \vec{L_D})^n = \langle 1, 1, 1 \rangle \langle 1, 1, .9 \rangle (\langle 0.2567, .9665 \rangle \cdot \langle .3162, -0.9487 \rangle)^{25}$

$= \langle .0013, .0013, .0012 \rangle$

TOTAL : $I_D = \langle .2389, .6203, .6258 \rangle$