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CS 174A

ASSIGNMENT 2 - PART 1

1. Step 1: $a_i^t = \frac{F^t_{i,total}}{m_i}$ $\quad F_{total} = -\gamma_i \dot{x}_i + g_i + f_i$

γ - DAMPING COEFF

Step 2: $v_i^{t+1} = v_i^t + \Delta t\, a_i^t$ $\quad g$ - TOTAL INTERNAL FORCE

f - EXTERNAL FORCE

Step 3: $x_i^{t+1} = x_i^t + \Delta t\, v_i^{t+1}$

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2. $f_{ext} = (2, 14.7, -5)^T \quad m = 1 \quad dt = 1 \quad v(0) = 0 \quad a(0) = 0$

$\underline{t=0}$ $F_{total} = (2, 14.7, -5) + (0, -9.8, 0) = (2, 4.9, -5)$

$x(0) = (0,0,0)^T$

$a(0) = \frac{F_{total}}{m} = (2, 4.9, -5)$

$v(0+1) = v(0) + \Delta t\, a(0) = 0 + 1(2, 4.9, -5) = (2, 4.9, -5)^T$

$x(0+1) = x(0) + \Delta t\, v(t+1) = (0,0,0)^T + 1(2, 4.9, -5) = (2, 4.9, -5)^T$

$\underline{t=1}$ $a(1) = (0, -9.8, 0)^T$

$v(1+1) = (2, 4.9, -5) + 1(0, -9.8, 0) = (2, -4.9, -5)^T$

$x(1+1) = (2, 4.9, -5) + 1(2, -4.9, -5) = (4, 0, -10)^T$

$\underline{x(2) = (4, 0, -10)^T}$

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3. LAGRANGE EQUATION OF MOTION: $m_i \ddot{x}_i + \gamma_i \dot{x}_i - g_i - f_i = 0$

γ_i - DAMPING COEFF $\quad f_i$ - external force $\quad g_i$ - total force due to springs

a) HEATING & MELTING DEFORMABLE MODELS - MASS-SPRING MODEL

- DIFFUSION OF HEAT IN MATERIALS: $\frac{\partial}{\partial t}(\mu \sigma \theta) - \nabla \cdot (C \nabla \theta) = q$

q = rate heat gen/loss per vol $\quad \mu$ - kg/m^3 $\quad \sigma$ - SPECIFIC HEAT $\quad \theta$ - Temp, Kelvin

C - THERMAL CONDUCTIVITY MATRIX $\quad \nabla = \left[\frac{\partial}{\partial u}, \frac{\partial}{\partial v}, \frac{\partial}{\partial w}\right]$

- HOMOGENEOUS, ISOTROPIC MATERIAL: $\frac{\partial}{\partial t}(\mu \sigma \theta) - c \nabla^2 \theta = q$ $\quad [\cdots] = \star\theta$

DISCRETIZE HEAT EQ:
$$\mu\sigma \frac{(\theta^{t+\Delta t} - \theta^t)}{\Delta t} - c\left[\frac{\theta^t_{u+\Delta u,v,w} - 2\theta^t_{u,v,w} + \theta^t_{u-\Delta u,v,w}}{\Delta u^2} + \frac{\theta^t_{u,v+\Delta v,w} - 2\theta^t_{u,v,w} + \theta^t_{u,v-\Delta v,w}}{\Delta v^2} + \frac{\theta^t_{u,v,w+\Delta w} - 2\theta^t_{u,v,w} + \theta^t_{u,v,w-\Delta w}}{\Delta w^2}\right] = q$$

UPDATE θ
$$\theta^{t+\Delta t}_{u,v,w} = \theta^t_{u,v,w} + \frac{\Delta t}{\mu\sigma} c(\star\theta)$$

b) LIQUIDS – PARTICLE MODELS

TOTAL FORCE : $g_i(t) = \sum_{j \neq i} g_{ij}(t)$

$$g_{ij}(t) = m_i m_j (x_i - x_j)\left[-\frac{\alpha}{(r_{ij}+\xi)^a} + \frac{\beta}{(r_{ij})^b}\right] \qquad a=2,\; b=4$$

α & β DETERMINE

ξ – SEPARATION STRENGTH OF ATTRACTION & REPULSION FORCES $\quad r_{ij} = \|x_j - x_i\|$

c) CLOTH – VISCOELASTICITY – MASS-SPRINGS MODEL

$$m_i \ddot{x}_i + \gamma_i \dot{x}_i + c_{ij} r_{ij} = f_i$$
$$m_j \ddot{x}_j + \gamma_j \dot{x}_j - c_{ij} r_{ij} = f_j$$
$$c_{ij}(x_i, x_j) = \frac{k_{ij} e_{ij} + \gamma_{ij} \dot{e}_{ij}}{\|r_{ij}\|}$$

$$\begin{bmatrix} m_i & 0 \\ 0 & m_j \end{bmatrix}\begin{bmatrix} \ddot{x}_i \\ \ddot{x}_j \end{bmatrix} + \begin{bmatrix} \gamma_i & 0 \\ 0 & \gamma_j \end{bmatrix}\begin{bmatrix} \dot{x}_i \\ \dot{x}_j \end{bmatrix} + \begin{bmatrix} -c_{ij} & c_{ij} \\ c_{ij} & -c_{ij} \end{bmatrix}\begin{bmatrix} x_i \\ x_j \end{bmatrix}$$

$$M\ddot{x} + G\dot{x} + K(x)x = f$$

4. EYE POSITION: $(2,10,3)^T$, LOOK AT PT $(-2,2,0)^T$ UP VECTOR $(-1,-1,0)^T$

$$k = \frac{P_{eye} - P_{ref}}{|P_{eye} - P_{ref}|} = \frac{(2,10,3)^T - (-2,2,0)^T}{|(2,10,3)^T - (-2,2,0)^T|} = \frac{(4,8,3)^T}{|(4,8,3)^T|}$$

$$\boxed{k = \frac{1}{\sqrt{89}}(4,8,3)^T}$$

$$i = \frac{v_{up} \times k}{|v_{up} \times k|} \qquad v_{up} \times k \begin{vmatrix} i & j & k \\ -1 & -1 & 0 \\ \frac{1}{\sqrt{89}} \to 4 & 8 & 3 \end{vmatrix} = (-3-0)i - (-3-0)j + (-8+4)k$$

$$v_{up} \times k = \frac{1}{\sqrt{89}}(-3, +3, -4)^T$$

$$|v_{up} \times k| = \frac{9}{89} + \frac{9}{89} + \frac{16}{89} = \sqrt{\frac{34}{89}}$$

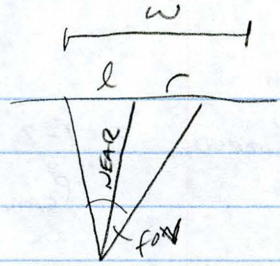
$$\boxed{i = \frac{(-3,+3,-4)^T}{\sqrt{34}}}$$

$$j = k \times i = \begin{vmatrix} i & j & k \\ \frac{1}{\sqrt{89}}\; 4 & 8 & 3 \\ \frac{1}{\sqrt{34}}\; -3 & +3 & -4 \end{vmatrix} = \frac{1}{\sqrt{89}}\frac{1}{\sqrt{34}}\left[(-32-9)i - (-16+9)j + (+12+24)k\right]$$

$$\boxed{j = \frac{(-41, 7, 36)^T}{\sqrt{89} \cdot \sqrt{34}}}$$

$$M_{CAM}^{-1} = \begin{bmatrix} i_x & i_y & i_z & 0 \\ J_x & J_y & J_z & 0 \\ k_x & k_y & k_z & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 & 0 & -P_{eye\,x} \\ 0 & 1 & 0 & -P_{eye\,y} \\ 0 & 0 & 1 & -P_{eye\,z} \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{-3}{\sqrt{34}} & \frac{3}{\sqrt{34}} & \frac{-4}{\sqrt{34}} & 0 \\ -41/\sqrt{3026} & \frac{7}{\sqrt{3026}} & \frac{36}{\sqrt{3026}} & 0 \\ 4/\sqrt{89} & 8/\sqrt{89} & 3/\sqrt{89} & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$X \begin{bmatrix} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 10 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$



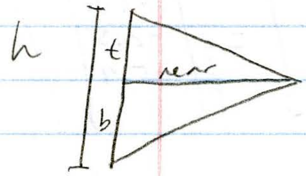
5. $n = 1 \quad f = 100 \quad fov_h = 30° \quad$ ASPECT RATIO = 1:2

$\tan\left(\frac{fov}{2}\right) = \frac{r}{near}$ $\quad r = \tan(15) = 0.268$

$l = -r = -0.268$

WIDTH OF IM. PLANE @ NEAR $\quad w = 2 \cdot r = 0.5356$

HEIGHT @ NEAR $\quad \frac{w}{h} = \text{RATIO} = \frac{1}{2} \quad h = 2 \cdot 0.5356 = 1.0712$



$t = \frac{h}{2} = 0.5356$

$b = -t = -0.5356$

$$M_{proj} = \begin{bmatrix} \frac{2 \cdot n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2 \cdot n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{n+f}{n-f} & \frac{-2fn}{n-f} \\ 0 & 0 & 1 & 0 \end{bmatrix} = \boxed{\begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & 2.02 \\ 0 & 0 & 1 & 0 \end{bmatrix}}$$

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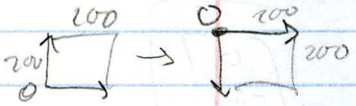
6. 200 pixels wide / 200 pixels high $\quad$ ORIGIN UPPER LEFT



$n_x = 200$

$n_y = 200$

TRANSLATION $\quad\quad$ SCALING

$$M_{VP} = \begin{bmatrix} 1 & 0 & 0 & \frac{n_x - 1}{2} \\ 0 & 1 & 0 & \frac{n_y - 1}{2} \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{n_x}{2} & 0 & 0 & 0 \\ 0 & \frac{n_y}{2} & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} * \rightarrow$$

$$\rightarrow * \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \boxed{\begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}$$

REFLECTION

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7. a) $P_{WORLD,a} = (3, 2, 1, 1)^T \quad\quad P_{cam} = M_{cam}^{-1} P_{WORLD}$

$$P_{cam,a} = \begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 3 \\ 2 \\ 1 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} -3.3 \\ -3.1 \\ -7.0 \\ 1 \end{bmatrix}}$$

b) $P_{WORLD,b} = (0, 0, -3, 1)^T$

$$P_{cam,b} = \begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ -3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -3.7 \\ -11.2 \\ 1 \end{bmatrix}}$$

c) $P_{WORLD,c} = (-2,-1,2,1)^T$

$$P_{CAM,c} = \begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -2 \\ -1 \\ 2 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} -2.9 \\ 0.9 \\ -11.3 \\ 1 \end{bmatrix}}$$

d) $P_{world,d} = (1,5,-1,1)^T$

$$P_{CAM,d} = \begin{bmatrix} -0.51 & 0.51 & -0.69 & -2.1 \\ -0.75 & 0.13 & 0.65 & -1.75 \\ 0.42 & 0.85 & 0.32 & -10.28 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ -1 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0.7 \\ -2.5 \\ -5.9 \\ 1 \end{bmatrix}}$$

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8)

a) $P_{CAM,a} = \begin{bmatrix} -3.3 \\ -3.1 \\ -7.0 \\ 1 \end{bmatrix}$

$$P_{clip,a} = M_{proj} \cdot P_{cam,a} = \begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & +2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -3.3 \\ -3.1 \\ -7.0 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} -12.3 \\ -5.8 \\ 9.16 \\ 7.0 \end{bmatrix}}$$

b) $P_{cam,b} = \begin{bmatrix} 0 \\ -3.7 \\ -11.2 \\ 1 \end{bmatrix}$

$$P_{clip,b} = M_{proj} \cdot P_{cam,b} = \begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & +2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0 \\ -3.7 \\ -11.2 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -6.9 \\ 13.44 \\ 11.2 \end{bmatrix}}$$

c) $P_{cam,c} = \begin{bmatrix} -2.9 \\ 0.9 \\ -11.3 \\ 1 \end{bmatrix}$

$$P_{clip,c} = M_{proj} \cdot P_{cam,c} = \begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & +2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} -2.9 \\ 0.9 \\ -11.3 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} -10.8 \\ 1.7 \\ 13.55 \\ 11.3 \end{bmatrix}}$$

d) $P_{cam,d} = \begin{bmatrix} 0.7 \\ -2.5 \\ -5.9 \\ 1 \end{bmatrix}$

$$P_{clip,c} = M_{proj} \cdot P_{cam,d} = \begin{bmatrix} 3.734 & 0 & 0 & 0 \\ 0 & 1.867 & 0 & 0 \\ 0 & 0 & -1.02 & +2.02 \\ 0 & 0 & -1 & 0 \end{bmatrix} \begin{bmatrix} 0.7 \\ -2.5 \\ -5.9 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 2.6 \\ -4.7 \\ 8.04 \\ 5.9 \end{bmatrix}}$$

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9) a) $P_{NDC,a} = P_{clip,a} / P_{clip,a}[3]$ — LAST ELEMENT OF $P_{clip,a}$

$$P_{NDCS,a} = \frac{1}{7.0} \begin{bmatrix} -12.3 \\ -5.8 \\ 9.16 \\ 7.0 \end{bmatrix} = \boxed{\begin{bmatrix} -1.8 \\ -0.8 \\ 1.31 \\ 1 \end{bmatrix}}$$

b) $P_{NDC,b} = P_{clip,b} / P_{clip,b}[3]$ - LAST ELEMENT of $P_{clip,b}$

$$P_{NDCS,b} = \frac{1}{11.2}\begin{bmatrix} 0 \\ -6.9 \\ 13.44 \\ 11.2 \end{bmatrix} = \boxed{\begin{bmatrix} 0 \\ -0.6 \\ 1.2 \\ 1 \end{bmatrix}}$$

c) $P_{NDC,c} = P_{clip,c} / P_{clip,c}[3]$ - LAST ELEMENT of $P_{clip,c}$

$$P_{NDCS,c} = \frac{1}{11.3}\begin{bmatrix} -10.8 \\ 1.7 \\ 13.55 \\ 11.3 \end{bmatrix} = \boxed{\begin{bmatrix} -0.96 \\ 0.15 \\ 1.2 \\ 1 \end{bmatrix}}$$

d) $P_{NDC,d} = P_{clip,d} / P_{clip,D}[3]$ - LAST ELEMENT of $P_{clip,d}$

$$P_{NDCS,d} = \frac{1}{5.9}\begin{pmatrix} 2.6 \\ -4.7 \\ 8.04 \\ 5.9 \end{pmatrix} = \boxed{\begin{pmatrix} 0.4 \\ -0.8 \\ 1.36 \\ 1 \end{pmatrix}}$$

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10) a) $P_{DCS,a} = M_{vp} * P_{NDCS,a}$

$$P_{DCS,a} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} -1.8 \\ -0.8 \\ 1.31 \\ 1 \end{bmatrix} = \boxed{\begin{pmatrix} 80.5 \\ 179.5 \\ 1.31 \\ 1 \end{pmatrix}}$$

b) $P_{DCS,b} = M_{vp} * P_{NDCS,b}$

$$P_{DCS,b} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0 \\ -0.6 \\ 1.2 \\ 1 \end{bmatrix} = \boxed{\begin{pmatrix} 99.5 \\ 159.5 \\ 1.2 \\ 1 \end{pmatrix}}$$

c) $P_{DCS,c} = M_{vp} * P_{NDCS,c}$

$$P_{DCS,c} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{pmatrix} -0.96 \\ 0.15 \\ 1.2 \\ 1 \end{pmatrix} = \boxed{\begin{bmatrix} 3.5 \\ 84.5 \\ 1.2 \\ 1 \end{bmatrix}}$$

d) $P_{DCS,d} = M_{vp} * P_{NDCS,d}$

$$P_{DCS,d} = \begin{bmatrix} 100 & 0 & 0 & 99.5 \\ 0 & -100 & 0 & 99.5 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} 0.4 \\ -0.8 \\ 1.36 \\ 1 \end{bmatrix} = \boxed{\begin{bmatrix} 139.5 \\ 179.5 \\ 1.36 \\ 1 \end{bmatrix}}$$