1.1

Mel Frequency Cepstral Coefficient (MFCC):-

Automatic speech recognition system identifies the component of audio signal, which are good for identifying Linguistic content and discarding background noise and emotion. The shape that determines the sound of the vocal tract including tongue, teeth mainfests itself in the envelope of the short time power spectrum. Mel Frequency Cepstral Coefficient (MFCC) represents this envelope.

The steps that need to be followed to calculate MFCCs are - Frame the signal into short frames, for each frame the periodogram estimation of the power spectrum needs to be calculated, the mel filterbank is applied to the power spectra and the energy is summed in each filter, the logarithm of all filterband energies are need to be taken, and also the DCT of the log filterband energies, DCT coefficients 2-13 needs to be kept and should discard the rest.

The signal into 20-40ms frames are used because the audio signal is constantly changing. To calculate power spectrum, we need to estimate periodogram. In periodogram spectral, not ~~entre~~ all information are required for Automatic speech Recognition. So, we take clumps of periodogram bins and sum them up to get ~~an~~ the idea of

1.2

It's a high-pass filter

1.3

K / shift frames can be produced in total

2.1

2.1

(1,1), (1,3), (2,3), (4,4), (2,4)

$$A = \begin{array}{c} \begin{matrix} x & y \end{matrix} \\ \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \\ 4 & 4 \\ 2 & 4 \end{bmatrix} \end{array}$$

$$\therefore \bar{A} = \begin{bmatrix} \frac{1+1+2+4+2}{5} & \frac{2+3+3+4+4}{5} \end{bmatrix} = \begin{array}{c} \begin{matrix} \bar{x} & \bar{y} \end{matrix} \\ \begin{bmatrix} 2 & 3.20 \end{bmatrix} \end{array}$$

Now, $$cov(x,y) = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})(y_i - \bar{y})$$

$$= \frac{1}{n-1} a' \cdot a \quad \text{where } a$$

$$a = \begin{bmatrix} 1 & 2 \\ 1 & 3 \\ 2 & 3 \\ 4 & 4 \\ 2 & 4 \end{bmatrix} - \begin{bmatrix} 2 & 3.20 \\ 2 & 3.20 \\ 2 & 3.20 \\ 2 & 3.20 \\ 2 & 3.20 \end{bmatrix} = \begin{bmatrix} -1 & -1.33 \\ -1 & -0.33 \\ 0 & -0.33 \\ 2 & 0.67 \\ 0 & 0.67 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1.2 \\ -1 & -0.2 \\ 0 & -0.2 \\ 2 & 0.8 \\ 0 & 0.8 \end{bmatrix}$$

$$\therefore \text{cov}(X, Y) = \frac{1}{4} \begin{bmatrix} -1 & -1 & 0 & 2 & 0 \\ -1.2 & -.2 & -.2 & .8 & .8 \end{bmatrix} \begin{bmatrix} -1 & -1.2 \\ -1 & -0.2 \\ 0 & -0.2 \\ 2 & 0.8 \\ 0 & 0.8 \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 1+1+0+4+0 & 1.2+0.2+0+1.6+0 \\ 1.2+0.2+0+1.6+0 & 1.44+.04+.04+1.6+1. \end{bmatrix}$$

$$= \frac{1}{4} \begin{bmatrix} 8 & 3 \\ 3 & 3.64 \end{bmatrix} = \begin{bmatrix} 2 & 1.33 \\ 1.33 & 0.91 \end{bmatrix}$$

Now, det $\left(\text{cov}(X,Y)\right)$

$$\Rightarrow \det \begin{bmatrix} 2-\lambda & 1.33 \\ 1.33 & 0.91-\lambda \end{bmatrix}$$

$$\Rightarrow 1.82 - 2\lambda - 0.91\lambda + \lambda^2 - 1.76 = 0$$

$$\Rightarrow \lambda^2 - 2.91\lambda + 0.06 = 0$$

By solving, $\lambda_1 = 2.89$ $\quad \lambda_2 = 0.0176$

Now, for 1st principal component, we choose, $\lambda_1 = 2.89$

$$\therefore B = \begin{bmatrix} 2 - 2.89 & 1.33 \\ 1.33 & 0.91 - 2.89 \end{bmatrix}$$

And we need to solve,

$$B\vec{X} = 0$$

$$\begin{bmatrix} -.89 & 1.33 \\ 1.33 & -1.98 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 0$$

using row reduction we can solve,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 1.49 \\ 1 \end{bmatrix}.$$

$\therefore$ 1st principal component is $\begin{bmatrix} 1.49 \\ 1 \end{bmatrix}$

Ans