2.1 We want to produce an estimate of the original signal that minimizes the mean square error, which may be expressed.

$$\epsilon(f) = E|X(f) - \hat{X}(f)|^2$$

where E denotes expectation.

If we substitute in the expression for $\hat{X}(f)$, the above can be ~~an~~ rearranged to

$$\epsilon(f) = E|X(f) - G(f)Y(f)|^2$$

$$= E|X(f) - G(f)[H(f)X(f) + V(f)]|^2$$

$$= E|[1 - G(f)H(f)X(f) - G(f)V(f)|^2$$

If we expand the quadratic, we get the following:

$$\epsilon(f) = [1 - G(f)H(f)][1 - G(f)H(f)]^* E|X(f)|^2$$

$$- [1 - G(f)H(f)]G^*(f)E\{X(f)V^*(f)\}$$

$$- G(f)[1 - G(f)H(f)]^* E\{V(f)X^*(f)\}$$

$$+ G(f)G^*(f)E|V(f)|^2$$

However, we are assuming that the noise is independent of the signal, therefore:

$$E\{X(f)V^*(f)\} = E\{V(f)X^*(f)\} = 0$$

Also, we are defining the power spectral densities as follows.

$$S(f) = E|X(f)|^2$$

$$N(f) = E|V(f)|^2$$

Therefore, we have:

$$\epsilon(f) = [1 - G(f) H(f)] [1 - G(f) H(f)]^* S(f) + G(f) G^*(f) N(f)$$

To find the minimum error value, we calculate the Wirtinger derivative with respect to $G(f)$ and set it equal to zero.

$$\frac{d\epsilon(f)}{dG(f)} = G^*(f) N(f) - H(f) [1 - G(f) H(f)]^* S(f) = 0$$

This final equality can be rearranged to give the Wiener filter.

<u>2.2</u>. A message $s[n]$ is transmitted over a multiplicative channel so that the received signal $r[n]$ is

$$r[n] = s[n] f[n].$$

Suppose $s[n]$ and $f[n]$ are zero mean and independent. We wish to estimate $s[n]$ from $r[n]$ using a Wiener filter.

Again, we have

$$R_{sr}[m] = R_{\hat{s}r}[m]$$
$$= h[m] * \underbrace{R_{rr}[m]}_{R_{ss}[m] R_{ff}[m]}.$$

But we also know that $R_{sr}[m] = 0$. Therefore $h[m] = 0$. This example emphasizes that the optimality of a filter satisfying certain constraints and minimizing some criterion does not necessarily make the filter a good one. The constraints on the filter and the criterior have to be relevant and appropriate for

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