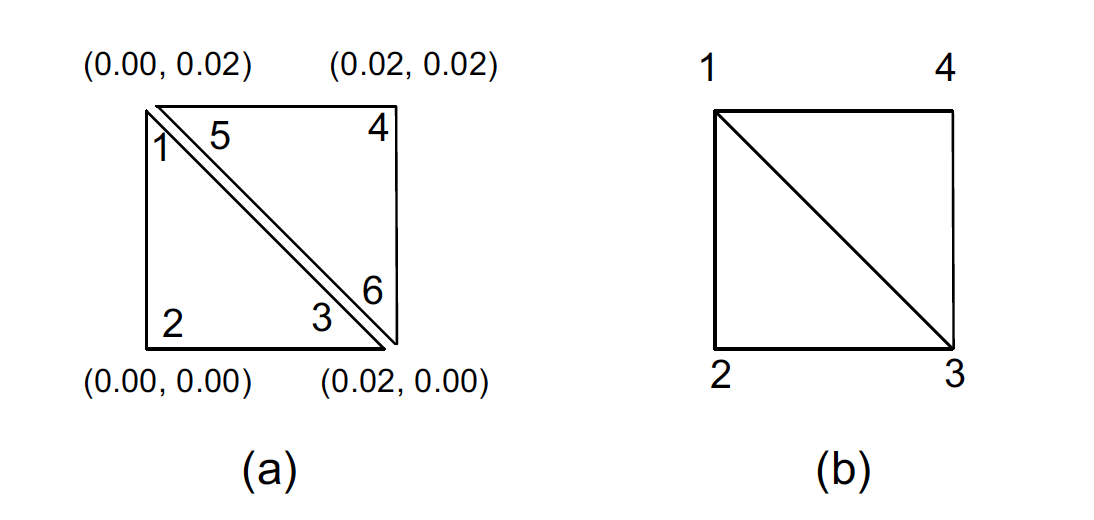
Tiffany Wang

260684152

ECSE 543 – Assignment 2

Question 1:



*Figure 1. (a) disjoint node numbering (b) global node numbering*

* calculations:
* S calculation:

Question 2:

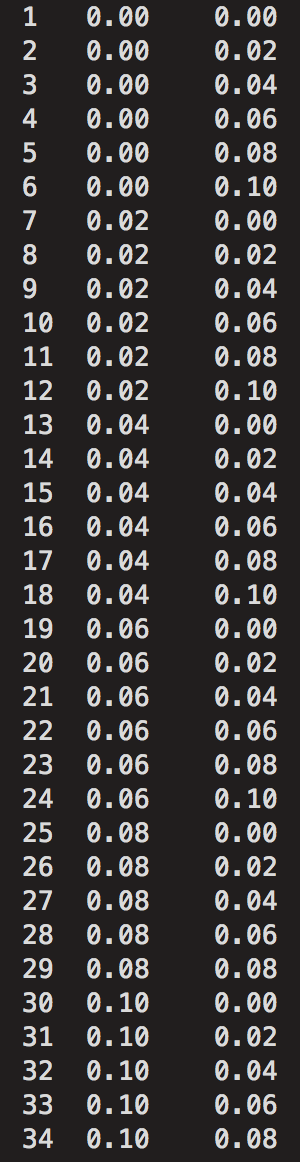
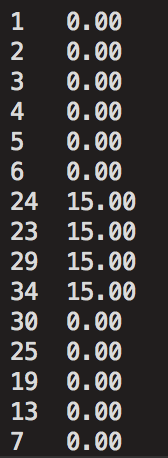
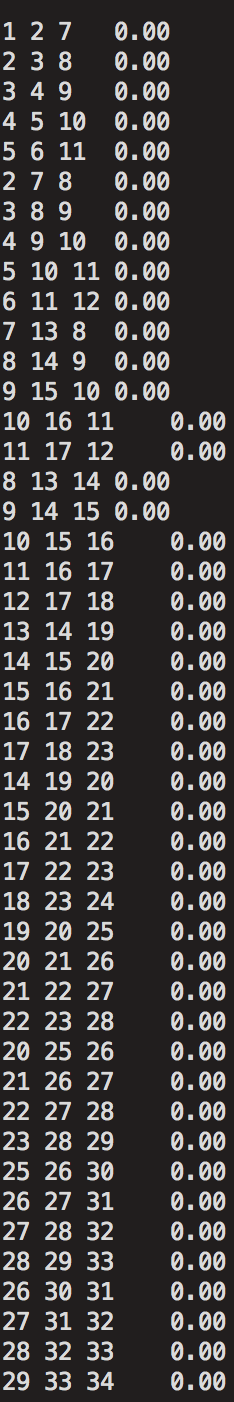
I used the following node numbering to solve the problem

*y*

*x*

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 6 | 12 | 18 | 24 |  | 15 V |
| 5 | 11 | 17 | 23 | 29 | 34 |
| 4 | 10 | 16 | 22 | 28 | 33 |
| 3 | 9 | 15 | 21 | 27 | 32 |
| 2 | 8 | 14 | 20 | 26 | 31 |
| 1 | 7 | 13 | 19 | 25 | 30 |

There is a total of 34 nodes and 46 elements (triangles).

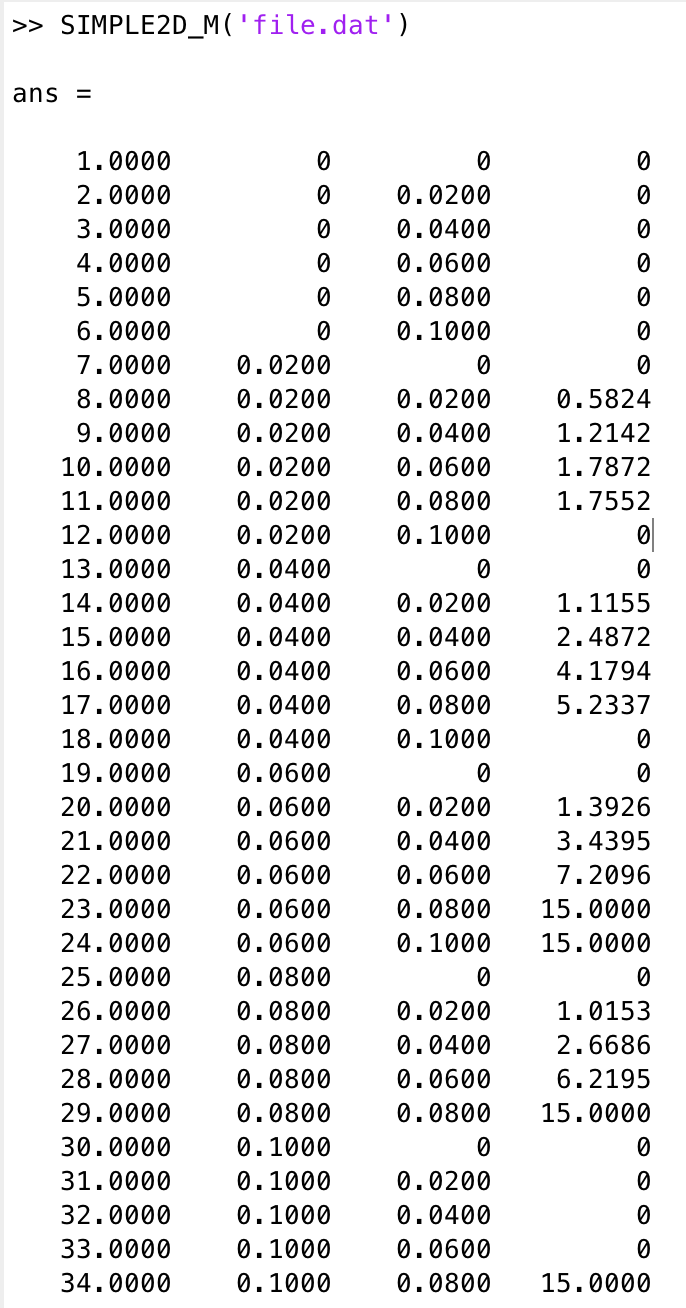


*Figure 4. Fixed node potentials*

*Figure 2. Node numbering and coordinates*

*Figure 3. Triangle definition*

b) Potential at each of the 34 nodes, computed by Matlab SIMPLE\_2D



*Figure 5. Potential approximation obtained with Matlab SIMPLE\_2D*

c) Since we have the nodes potential approximated in b, I will use the following formula to compute the energy inside the quarter of the coaxial cable:

I wrote a function which parse the file.dat file into the triangle coordinates, local and global S matrices. The code for the function can be found in the appendix.

Since since the coaxial cable is symmetrical, the total energy of the whole cross-section of the cable would be four times that of the quarter cable we studied in this problem.

Finally, the capacitance per meter of the cable is found using:

The capacitance per meter of the coaxial cable is found to be: ../../../../../../Desktop/Screen%20Shot%202017-11-18%20at%201.26.01

Question 3:

The potential of the quarter coaxial cable will be approximated using the Conjugate Gradient method.

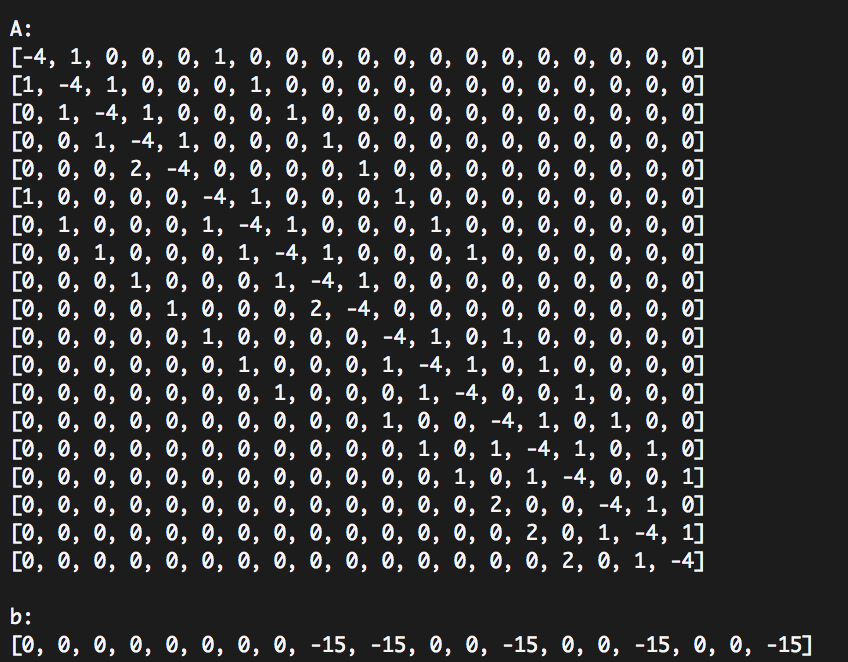
*A:=* five points difference matrix

*x:=* approximated potential at the nodes

blue:

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| 0V | 4 | 9 | 15 V | | |
| 3 | 8 |
| 2 | 7 | 12 | 15 | 18 |
| 1 | 6 | 11 | 14 | 17 |
| 0 | 5 | 10 | 13 | 16 |
|  | | | | |

The *b* value is equal to -1 x fixed potential directly neighboring the corresponding node. In this case, every value in *b* is *0*, except for index 8, 9, 12, 15 where *b[index] = -15*



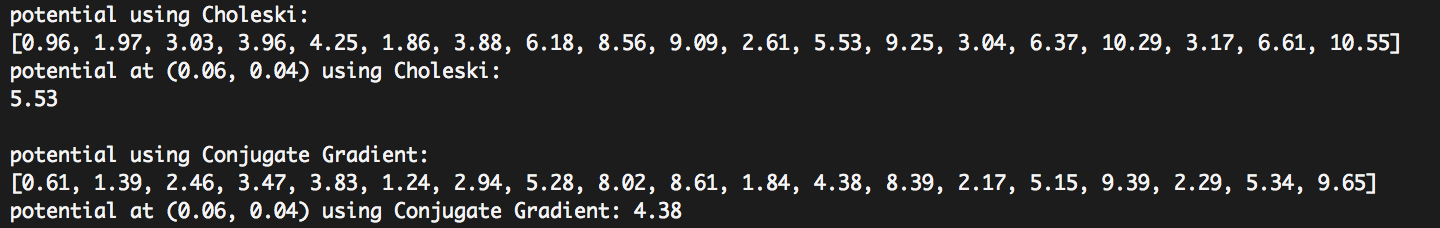
*Figure 6. A and b generated for the conjugate gradient*

1. A is not a symmetric matrix: A[4][3] ≠ A[3][4]

In order to obtain a singular symmetric positive definite matrix, we will multiple both side of the equation by *AT*

The potential is approximated using this equation.

1. The potential at each load with

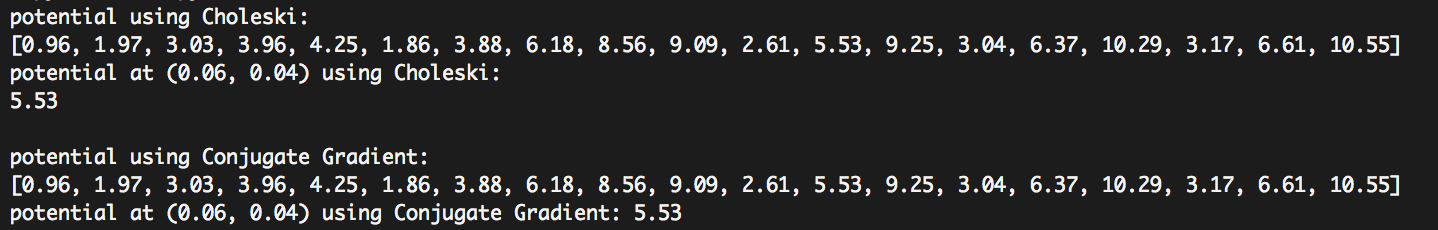


*Figure 7. Potential approximation obtained using Choleski and 19 iterations for Conjugate Gradient*

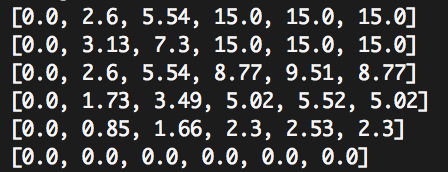
The potential difference between the approximated potentials are too high. It can be deduced that the Conjugate Gradient cannot be completed in *O(n)* complexity.

Therefore, I continued changed the Conjugate Gradient to iterate through the approximation until the residual is less than 0.005.

Then, the accuracy of the approximation is a lot higher.



*Figure 8. Potential approximation obtained using Choleski and Conjugate Gradient with residuals*



*Figure 9. Potential approximation results using SOR (from Assignment 1)*

The results were really different compared to the results obtained with both Choleski and Conjugate Gradient.

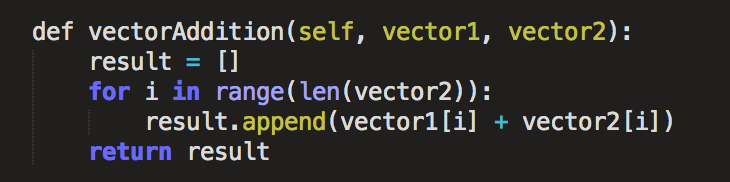
|  |  |  |  |
| --- | --- | --- | --- |
| Potential approximation | Choleski | Conjugate Gradient | SOR |
| (0.02, 0.02) | 0.96 | 0.96 | 0.85 |
| (0.02, 0.04) | 1.97 | 1.97 | 1.73 |
| (0.02, 0.06) | 3.03 | 3.03 | 2.60 |
| (0.02, 0.08) | 3.96 | 3.96 | 3.13 |
| (0.02, 0.10) | 4.25 | 4.25 | 2.60 |
| (0.04, 0.02) | 1.86 | 1.86 | 1.66 |
| (0.04, 0.04) | 3.88 | 3.88 | 3.49 |
| (0.04, 0.06) | 6.18 | 6.18 | 5.54 |
| (0.04, 0.08) | 8.56 | 8.56 | 7.30 |
| (0.04, 0.10) | 9.09 | 9.09 | 5.54 |
| (0.06, 0.02) | 2.61 | 2.61 | 2.30 |
| (0.06, 0.04) | 5.53 | 5.53 | 5.02 |
| (0.06, 0.06) | 9.25 | 9.25 | 8.77 |
| (0.06, 0.08) | 15.0 | 15.0 | 15.0 |
| (0.06, 0.10) | 15.0 | 15.0 | 15.0 |
| (0.08, 0.02) | 3.04 | 3.04 | 2.53 |
| (0.08, 0.04) | 6.37 | 6.37 | 5.52 |
| (0.08, 0.06) | 10.29 | 10.29 | 9.51 |
| (0.08, 0.08) | 15.0 | 15.0 | 15.0 |
| (0.08, 0.10) | 15.0 | 15.0 | 15.0 |
| (0.10, 0.02) | 3.17 | 3.17 | 2.30 |
| (0.10, 0.04) | 6.61 | 6.61 | 5.02 |
| (0.10, 0.06) | 10.55 | 10.55 | 8.77 |
| (0.10, 0.08) | 15.0 | 15.0 | 15.0 |
| (0.10, 0.10) | 15.0 | 15.0 | 15.0 |

1. df

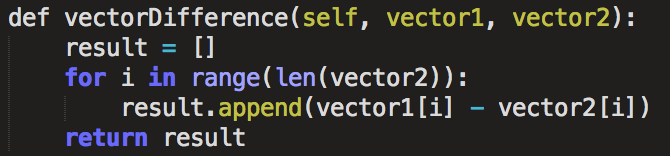
*Appendix*

Matrix function:

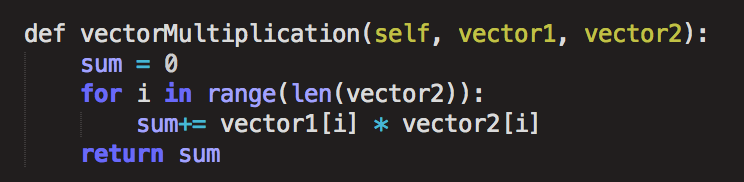
* Vector addition

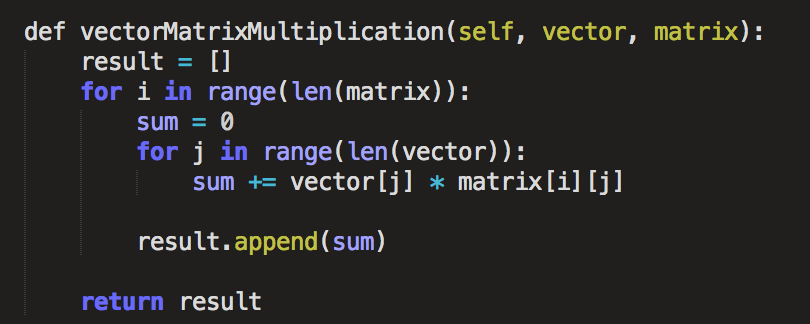


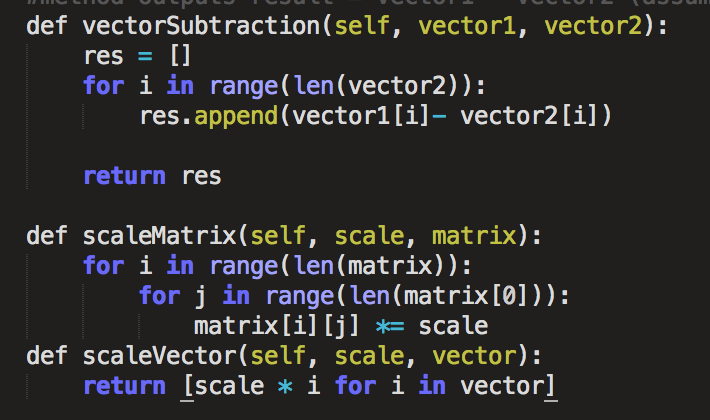
* Vector subtraction (Vector1 – Vector2)



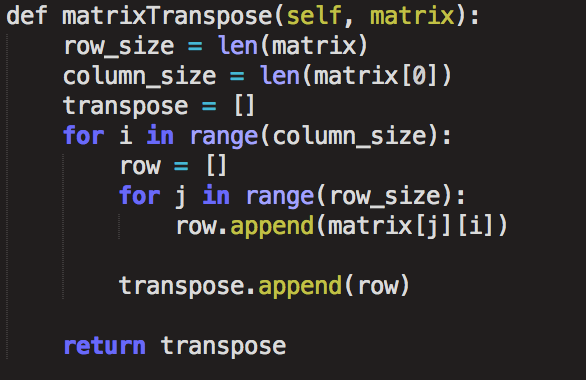
* Vector Dot Product (A · B)



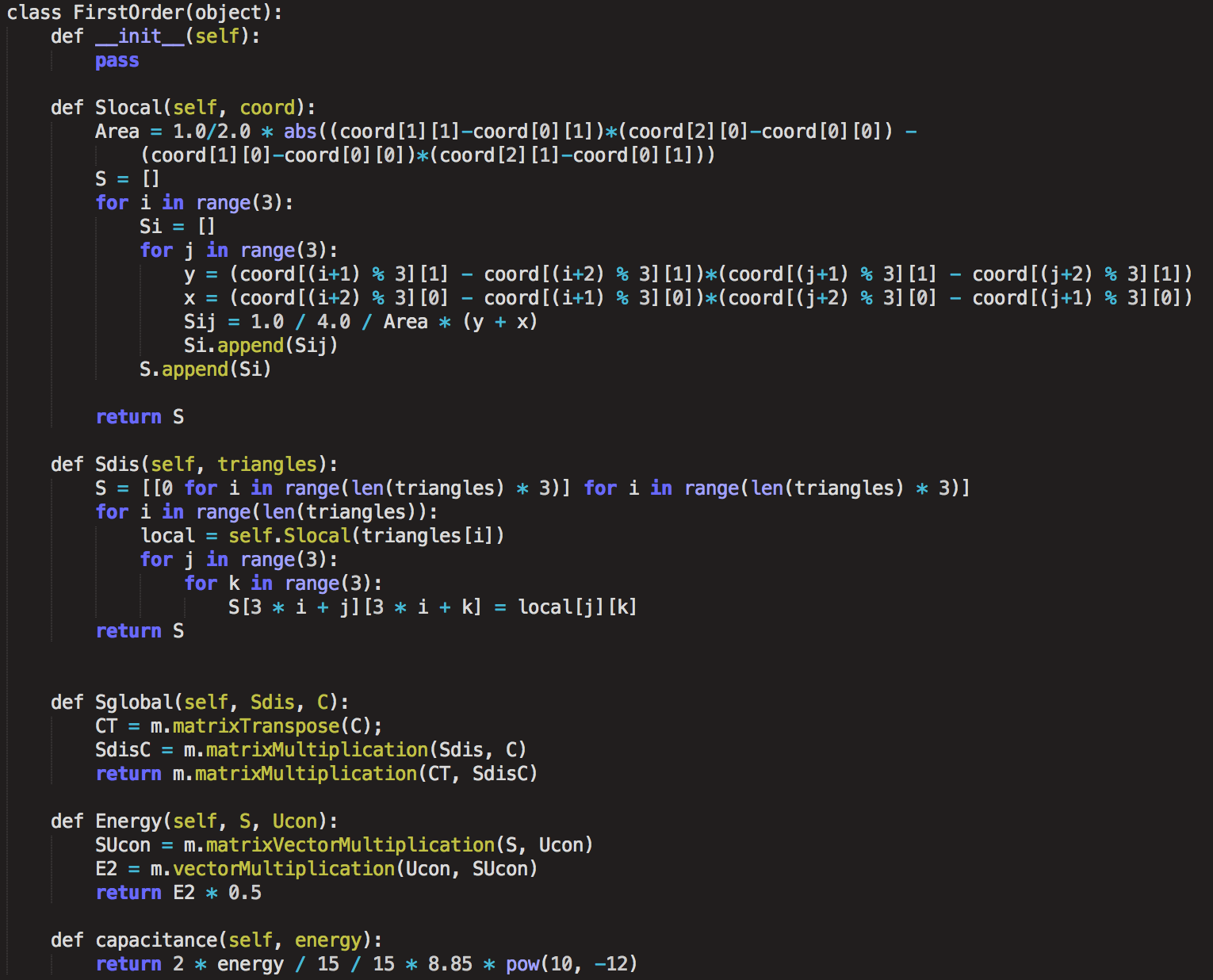
* Vector · Matrix
* Scale Matrix (a · Matrix)



* Transpose Matrix (result = *AT*)

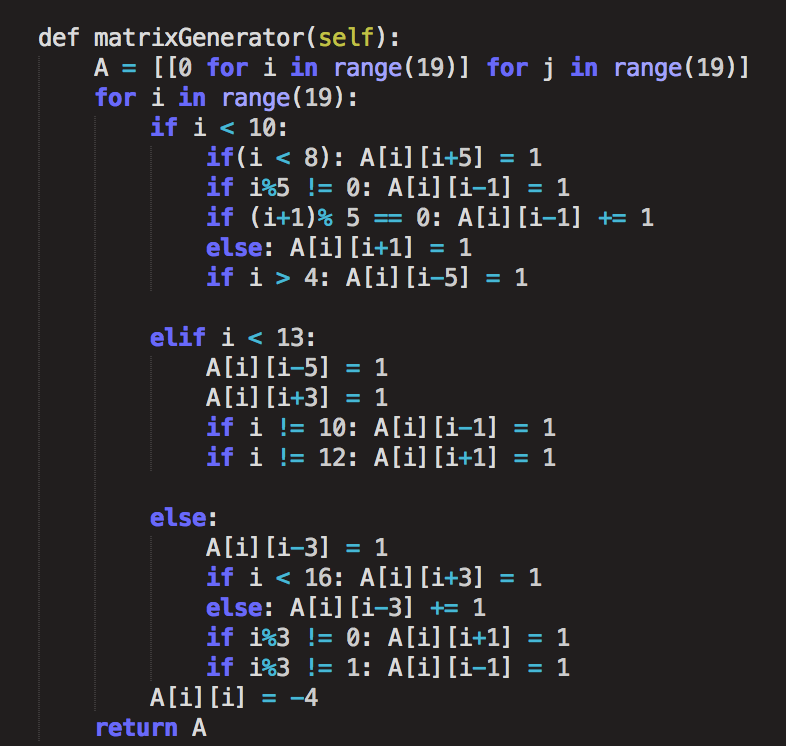


* First Order Class

Includes: Local S matrix generator, Disjoint S matrix generator, conjoint S matrix generator, Energy calculator, Capacitance calculator

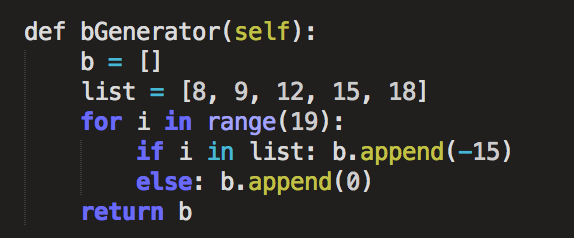
* Five points difference matrix generator

(for the specific quarter coaxial cable)

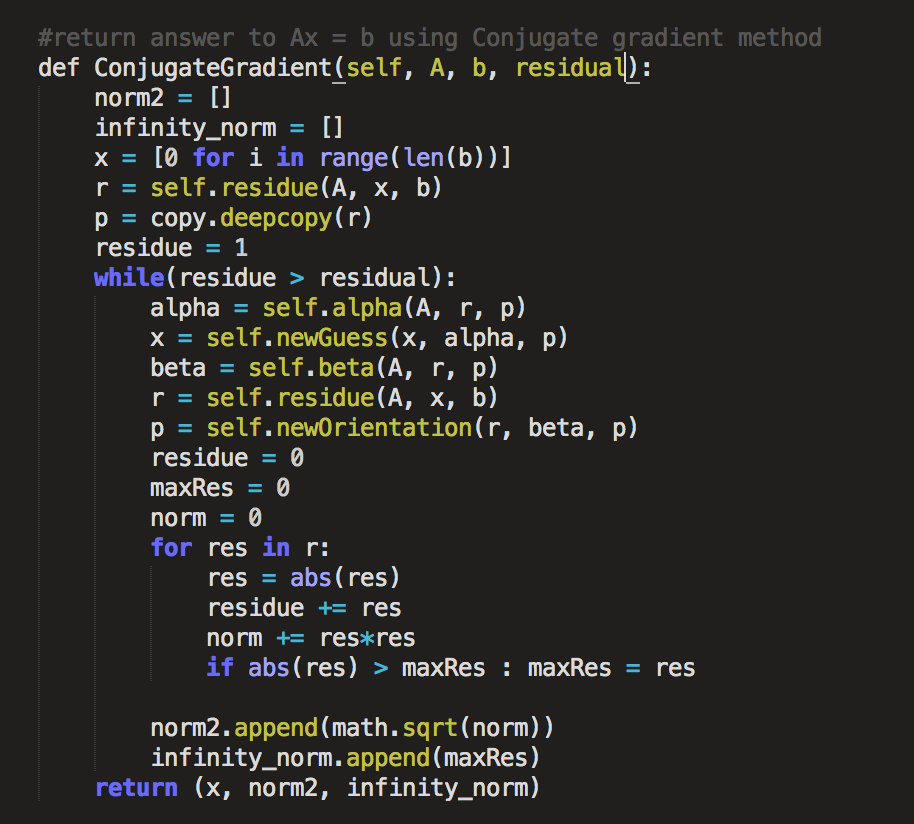


* resulting b vector generator

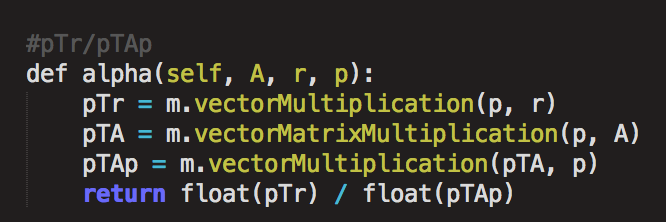
(for the specific quarter coaxial cable)



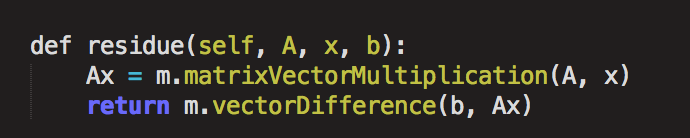
* Conjugate Gradient approximation method



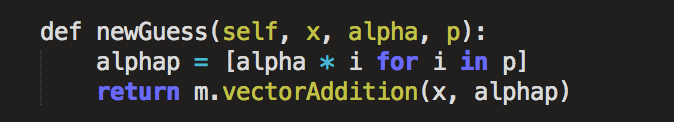
* first arrangement coefficient (alpha)



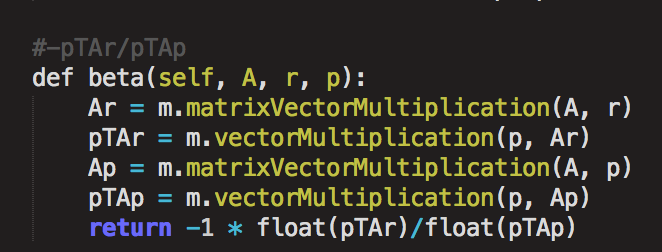
* Residue vector r calculator



* Next approximation (new guess) calculator



* New rearrangement coefficient calculator



* new orientation vector generator

