Problem 1: Flint Water Crisis

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1 Non-technical Summary

In 2014, the city of Flint, Michigan made headlines as it came to light that a recent water source change had resulted in elevated lead levels in Flint tap water. The city worked to remedy this problem, and in 2017, it was announced that the lead levels in Flint tap water had returned to acceptable levels. However, citizens are still wary of the tap water and prefer to use bottled water instead. The city of Flint is now faced with the difficult task of convincing its citizens that the tap water is safe to drink.

We have created a model to predict how trust in Flint tap water will progress over time. Our model captures the idea that people can gain trust in something either through (1) their own experiences, or (2) word of mouth. By providing bottled water to citizens, the Flint government builds government trust, which gradually translates to trust in the government's statement that the tap water is safe. Once a citizen becomes trusting of the water, they have the potential to convince other citizens that the water is safe. We model this peer-to-peer dynamic using the SIR model for infectious diseases, because trust can be viewed as an infectious process.

While building trust in tap water is important, it is also imperative to consider the costs associated with building that trust. Since directly comparing citizen trust to monetary costs is impossible, our model incorporates utility functions relating citizen trust to the costs of distributing water bottles. The government, then, must balance this tradeoff via cost-benefit analysis, in order to effectively determine a stopping point for government water bottle distribution.

Our model shows that the amount of bottled water distributed at any given time can significantly impact how long the government will need to continue to distribute water. Another finding is that interaction between citizens has a huge effect on how long the government will need to continue to distribute water. For example, it will take over twice as long to build up trust in tap water if a Flint citizen who trusts the water supply can convince *one* other Flint citizen per year to begin trusting the water, compared to if a Flint citizen who trusts the water can convince *three* other Flint citizens per year to begin trusting the water.

Using our model, the mayor of Flint will be able to predict how long the government will have to distribute bottled water until trust in the tap water reaches a certain threshold. Additionally, the mayor can see how potential government policies can impact how quickly trust is built up.

We capture the complex dynamics of trust building in a model that is easily interpretable and flexible, which makes it useful in determining the optimal time for the city of Flint to stop bottled water distribution.

2 Introduction

In 2014, the residents of Flint, Michigan were exposed to elevated lead levels in their water after the city of Flint began using a new water source without proper water treatment. After the elevated lead levels were discovered, a federal state of emergency was declared. By 2017, the water lead levels had returned to acceptable levels. However, Flint citizens are still unwilling to trust Flint water.

The city of Flint is now faced with the problem of convincing their citizens that the water is safe. There are two channels through which citizens can become trusting of the tap water: they begin to trust the tap water on their own as they build up trust in the government and gradually begin to use tap water in their daily lives, or they are either convinced by peers. The main lever the Flint government has at their disposal is bottled water distribution. Distributing bottled water has a positive effect and a negative effect. The positive effect is that distributing bottled water strengthens the relationship between the government and the Flint citizens, causing citizens to become more likely to trust what the government says. One negative effect is that when citizens are consuming bottled water, they lose out on the opportunity to see for themselves that the tap water is harmless. Another negative effect is the cost.

Once a citizen becomes trusting of the water, they have the potential to convince other citizens that the water is safe, thus speeding up the process of building trust in the tap water. The more interactions that citizens have with each other, the bigger the effect of this speed up will be.

Our model captures the effects of the two channels through which citizens can become trusting of the tap water and allows us to determine an optimal distribution policy to help the city of Flint convince citizens of the safety of the tap water.

3 Model

The goal of our model is to predict the number of citizens who *trust* Flint tap water, as well as the number of citizens who *distrust* Flint tap water at any given time t (where $t \in (0, \infty)$). Using this model, the mayor of Flint can precisely determine at what time t to stop supplying water bottles. Additionally, the mayor can use this model to see how varying parameters (through policy decisions) affects the dynamics of citizen trust.

We can model the number of citizens who trust and distrust the water supply using a SIR model [1]. In this model:

1. S = Number of citizens who distrust the Flint water supply
2. I = Number of citizens who trust the Flint water supply
3. $S + I = N$ where N is the total population of Flint.

Over time, distrusting citizens in S will transition to state I at a rate of F by learning to trust the water supply.

$$\boxed{S} \xrightarrow[F]{} \boxed{I}$$

3.1 Assumption 1

The total Flint population N does not vary with time.

Remark: To simplify this model, we are only considering states S and I without taking birth and death states into account.

3.2 Assumption 2

A trusting citizen will not learn to distrust the tap water.

Remark: For this reason, we are not including the R state, which is a part of traditional SIR models, since we are assuming that I is an absorbing state. No citizens will be able to "recover" from trusting the tap water supply; once a citizen starts trusting the tap water, they will continue to trust the tap ware. This assumption is supported by the fact that the city tap water is indeed safe (as declared by the city), so citizens who start using the tap water will not experience any negative effects.

3.3 Assumption 3

Flint citizens who do not currently trust the tap water can learn to trust the tap water supply by one of two ways:

1. Untrusting citizens can be gradually "weaned off" of bottled water over time, in which they will learn trust the government's assessment that the tap water is safe.
2. Untrusting citizens will be influenced by their trusting neighbors and friends to try the tap water, which they will realize is safe.

Hence, we represent the flow of citizens from S to I by

$$F = \beta SI + cS$$

where β and c are positive constants. The number of citizens who are weaned off bottled water is proportional to the number of untrusting citizens and is represented by the cS term. The number of untrusting citizens that are converted to trusting citizens via influence from trusting peers is proportional to both the number of untrusting citizens (bigger $S =$ more potential converts) and the number of trusting citizens (bigger $I =$ more people spreading the word about how safe the water is).

3.4 Assumption 4

The government will continue to supply the appropriate number of water bottles to the citizens of Flint as long as the number of untrusting citizens in state S is above some small proportion ϵ of the total population N.

Remark: The above assumptions help detail the dynamics of the SIR model, but they neglect overall cost logistics. As time progresses, costs of the operation will rise, and more people begin to trust the tap water supply. How do we go about comparing aggregate citizen trust to monetary costs? From the above model, it is apparent that the rate of transition F from state S to I decreases over time since the number of citizens in S gets smaller over time. While F decreases over time, total costs will still be rising. Thus, we can say that there is some point in time in which the government should stop distributing water bottles, even though S may not be 0 (some stubborn citizens will always continue to distrust the tap water). In this sense, our goal is to find an ϵ that balances the benefits of citizens trusting the tap water with the total cost of the water distribution operation. Since comparing monetary costs to abstract trust in tap water is practically impossible, we can convert both to a common unit: utilities. We can express the tradeoff between trust in tap water and operation cost as some goal utility function G.

We can represent the goal utility function of the government as $G(\frac{I}{N}, C)$ where $\frac{I}{N}$ represents the proportion of Flint citizens who trust the water supply, and C represents the total cost of the operation. The government will choose $\frac{I}{N}, C$ to maximize its overall goal utility function G.

$$G\left(\frac{I}{N}, C\right) = U\left(\frac{I}{N}\right) + V(C)$$

$U(\frac{I}{N})$ can be seen as a increasing utility function, which returns larger utilities as $\frac{I}{N} \to 1$. This is the case since one of the government's goals is to get a large proportion of Flint citizens to trust the tap water supply. $V(C)$ can be seen as a negative utility function, which returns more negative utilities as $C \to \infty$ since high operation costs are not preferable.

Since the government stops the water bottle operation at time t_ϵ where $\epsilon = 1 - \frac{I}{N}$, the overall goal utility function G must have been maximized at that point in time. By setting ϵ as the maximum proportion of untrusting citizens, we are also equating $\frac{I}{N} = 1 - \epsilon$. Additionally, we are assuming that the government optimally allocates water bottles at an overall cost of $C = C_\epsilon$. As a result, we arrive at $\frac{I}{N} = 1 - \epsilon$ and $C = C_\epsilon$ as the arguments that maximize the overall goal utility function G.

Thus we arrive at the result

$$G\left(\frac{I}{N}, C\right) \leq G\left(1 - \epsilon, C_\epsilon\right) \quad \forall \frac{I}{N} \in (0,1), C \in (0, \infty)$$

Note that there are few clarifying restrictions on the individual utility functions themselves. Since comparing monetary costs to trust of their citizens is ultimately subjective, we leave it up to the mayor of Flint to decide the appropriate tradeoff between the two and compute the appropriate ϵ for stopping the water bottle operation. As such, we will not be specifying utility functions U and V any further than what is given.

4 Solving the Model

The overall model, as detailed in the section above, determines the optimal combination of citizen trust as a proportion $\frac{I}{N}$ and overall operation cost C. The subsections below detail how we evaluate citizen trust over time, as well as, how we can determine an optimal ϵ to balance citizen trust and operation cost (given utility functions U and V). Once $\frac{I}{N} \geq 1 - \epsilon$ (or, equivalently, $\frac{S}{N} \leq \epsilon$), our model will recommend that the Flint mayor stop the water bottle protocol since we will have already reached the maximum of the goal utility G. If the mayor continued distributing bottled water beyond this optimal point, the goal G would decrease because the marginal benefit of additional water distribution would be outweighed by the marginal costs of distributing water.

4.1 Evaluating Citizen Trust over Time

We can begin by evaluating the SIR model to find the number of citizens in state S at any given time t before we progress to finding the t_ϵ that maximizes goal utility G. We have established above that the dynamics of change in citizen trust in Flint tap water can be modeled by

$$\frac{dS}{dt} = -\beta SI - cS$$

We assume that each citizen either trusts the Flint tap water or does not trust the tap water, so $S + I = N$, where N is the total population of Flint. $S(t), I(t) \geq 0 \ \forall t$ because we cannot have a negative number of people in a group. We also assume that no citizens trust the water in the beginning, so $S(0) = N$.

Substituting $N - S$ for I, we can rewrite the equation above as

$$\frac{dS}{dt} = -\beta S(N-S) - cS = -\beta SN + \beta S^2 - cS$$

This is a separable differential equation, which we can solve by rearraging the terms and integrating

$$\frac{dS}{dt} = -\beta S(N-S) - cS = -\beta SN + \beta S^2 - cS$$

$$\int \frac{1}{-\beta SN + \beta S^2 - cS} ds = \int dt$$

$$\frac{1}{\beta N + c} \ln\left(\frac{|\beta S - \beta N - c|}{|S|}\right) + \text{constant} = t$$

To solve for the constant, we can plug in the initial condition $S(0) = N$.

$$\text{constant} = -\frac{\ln\left(\frac{c}{N}\right)}{\beta N + c}$$

Thus we have the model:

$$\frac{1}{\beta N + c}\left[\ln\left(\frac{|\beta S - \beta N - c|}{|S|}\right) - \ln\left(\frac{c}{N}\right)\right] = t$$

The above equation determines the amount of people in state S at time t given unknown constants β, c, and N. It is also important to note that S will never reach 0 at finite time t since stubborn citizens who refuse to believe the tap water is safe will always persist.

4.2 Detailing Unknown Constants

Here we will discuss the relevance of the above unknown constants and how they influence the number of people in S at time t.

4.2.1 Total Population

N represents the total population of Flint throughout the duration of the water bottle operation. Under Assumption 1, we have already specified that this value does not change over time. As a result, we can estimate this value by determining the population of Flint at time $t = 0$.

4.2.2 Trust Infection Rate

β represents the rate at which untrusting citizens are "infected" by their trusting friends and neighbors. This rate is incorporated in the overall flow from S to I:

$$\frac{dS}{dt} = -\beta SI - cS$$

Note that fluctuating β influences only the $-\beta SI$ term. To make sense of β, we can represent β as a rate $\frac{\gamma}{N}$ and substitute this into our original expression $-\beta SI$ to yield $\frac{-\gamma SI}{N}$.

This can be rewritten as $-\gamma I \frac{S}{N}$ where each trusting citizen in I is able to infect, on average, γ total citizens at random. However, some of those citizens may already trust the tap water supply, which won't change the number of Flint citizens in I. Therefore, we must weight γ citizens by the proportion of the population that doesn't trust the tap water supply $\left(\frac{S}{N}\right)$. Thus we arrive at our original rate $-\gamma I \frac{S}{N}$ where γ represents the total number of people infected by a trusting citizen in I.

Now that we have a general intuition about what γ represents, we can see that $\beta = \frac{\gamma}{N}$ is simply the average proportion of the entire population infected by a trusting citizen in I. Note that since β is a proportion of the total population N, $\beta \in (0, 1)$. Higher values of β will result in faster transitions from S to I since each trusting person will tend to influence more untrusting citizens on average.

4.2.3 Government's Constant

The government's constant c represents the rate at which untrusting citizens gradually "wean off" of bottled water over time by the government's efforts. This rate accounts for the other half of the overall flow from S to I.

$$\frac{dS}{dt} = -\beta SI - cS$$

We know that $c \in (0, 1)$ since $c > 1$ would result in a greater decrease in S than the number of people in S. It is in the government's best interest to maximize c by providing the appropriate number of water bottles to the citizens of Flint. As $c \to 1$, $S \to 0$ at a much faster rate and will converge to ϵ at a smaller value of t, which is desirable.

However, increasing c is not a trivial task for the government since supplying too many water bottles won't wean the citizens off of using water bottles. Supplying too few water bottles will force citizens to obtain water by other means (rather than using tap water) and will build distrust in the government.

We can model this scenario as a tradeoff with two opposing forces present. We consider c to be the sum of these two forces.

1. Trust in government (T_g) is a positive increasing function of number of water bottles b where $0 \leq b \leq \infty$.

2. Tap water consumed (W_c) is a positive (possibly parabolic) function of number of water bottles b where $0 \leq b \leq \infty$.

To keep things finite in terms of water bottles b, we can restrict the maximum number of bottles given to the citizens of Flint at d where d represents the aggregate demand for water. Meeting this demand will increase T_g, but also result in $W_c = 0$ since no citizen will try the tap water if all their demands are met by bottled water. Additionally, $b = 0$ will also result in $W_c = 0$ (as well as $T_g = 0$) since citizens will continue to distrust the government and will seek out other water sources instead of trying the tap water.

Thus we arrive at the following graphs for T_g and W_c below:

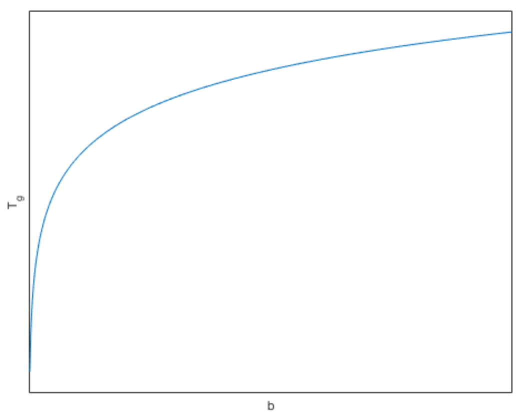

Figure 1: Plot of T_g vs. b

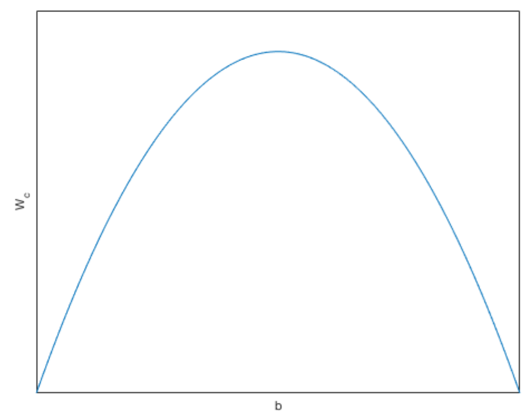

Figure 2: Plot of W_c vs. b

Since $c = W_c + T_g$, we can add the two graphs above to generate a graph for c in terms of the number of distributed bottles b. As shown in Figure 3 below, there exists an optimal $b^* \in (0, d)$ that maximizes c.

4.3 Evaluating Overall Utility given Cost and Citizen Trust

Our choice for the government constant c above may not rely solely on the maximum achievable value for c given b^*. While larger values of c result in a faster convergence of Flint citizens to state I, larger c also may increase operation costs C and result in a larger negative utility $V(C)$. Thus it is possible for $G = U(\frac{I}{N}) + V(C)$ to be less than the maximum attainable value for G, even though we are maximizing c given b^*.

We can evaluate the total cost of the operation as the total number of water bottles purchased over the entire time period. The number of bottles required to build trust will decrease over time since more citizens will start to trust the tap water (as citizens move from S to I). To provide a conservative estimate of how cost influences our overall goal utility G, we will overestimate our cost C by taking the number of bottles b^* that maximizes $c(b)$ at time $t = 0$, and we will compute overall cost C assuming that $b = b^*$ across all t. Thus, total operation cost can be represented as $C = (p)(b^*)(T)$ where b^* is number of bottles distributed across the first timestep $t = 0$, p is the unchanging cost of a water bottle, and T is the total number of timesteps t until ϵ-convergence.

By our fourth assumption, the government has utility functions $U\left(\frac{I}{N}\right)$ and $V(C)$. Summing these two functions results in the overall goal utility function: $G(\frac{I}{N}, C)$. Since making further assumptions regarding these utility functions may restrict the generalizability of the model, we will refrain from further characterizing these utility functions. However, upon obtaining the specific utility functions, G can be minimized at

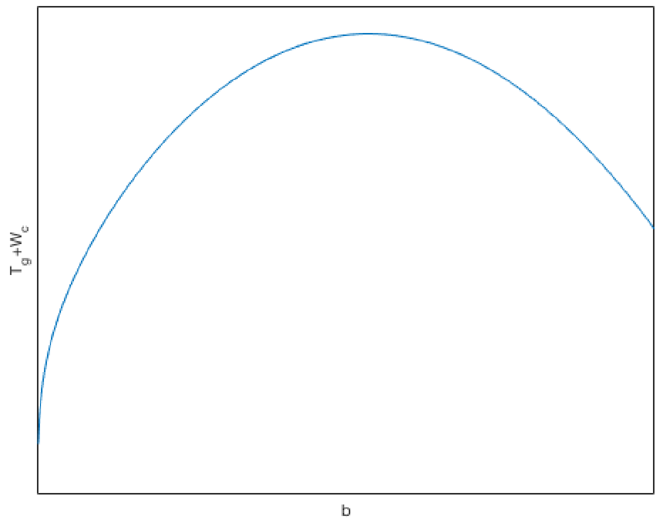

Figure 3: Plot of $T_g + W_c$ vs. b

some t_ϵ where $\frac{I}{N} = 1 - \epsilon$. To reach this point, it is possible to not have chosen b^* to maximize c at every timestep.

5 Model Simulation

In this section, we investigate how our model changes as we change the model parameters. Specifically, we look at how varying c and β affects the rate of S convergence (i.e., the amount of time required for S to decrease to below the ϵ threshold)

5.1 Varying c

The city of Flint can influence the values of c by varying the amount of bottled water that it distributes. For now, we take as given that we can vary the value of c and do not focus on the mechanism through which bottled water distribution influences c.

In Figure 4, we plot the change in S (the number of people who distrust the water) over time for $c = 0.2, 0.4, 0.6, 0.8$. Recall that the government's constant c represents the proportion of distrusting citizens that spontaneously become trusting of the water during one unit of time. For convenience, we use years as our unit of time. Despite not having information regarding the goal utility function G, we can arbitrarily set an ϵ threshold to 0.05, which means that when only 5% of citizens remain distrustful of Flint tap water, the government will stop distributing bottled water.

When $c = 0.2$, the government has to distribute water for 1.788 years until the proportion of the population that is distrusting of the water falls below 0.05. 1.788 was determined by finding the t value at which the blue line intersects the green line. When $c = 0.4$, the government has to distribute water for 1.497 years. When $c = 0.6$, the government has to distribute water for 1.318 years. When $c = 0.8$, the government can stop distributing distributing water after only 1.188 years. Increasing c can be beneficial since this increases the rate at which distrusting citizens become trusting of the tap water. However, we see that there are diminishing returns to increasing c. Increasing c from 0.2 to 0.4 decreases the duration of water distribution by $1.788 - 1.497 = 0.2910$ years, whereas increasing c from 0.6 to 0.8 decreases the duration by only

$1.318 - 1.188 = 0.1300$ years

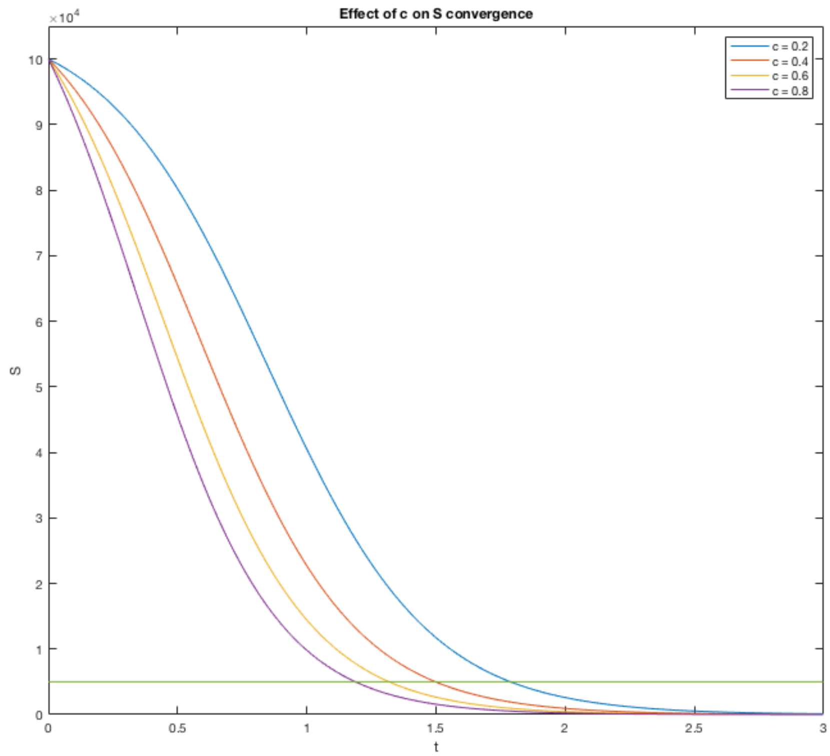

Figure 4: Plots of S over time for $c = 0.2, 0.4, 0.6, 0.8$. N is fixed at 100,000 and β is fixed at $3/N$. The horizontal line represents an ϵ threshold of 0.05.

5.2 Varying β

In the equation $\frac{dS}{dt} = -\beta SI - cS$, which describes the transition of citizens from distrusting (S) to trusting (I), β plays a crucial role. As described in Section 4.2.2, β is equivalent to $\frac{\gamma}{N}$, where γ represents the total number of people infected by a trusting citizen in I. More concretely, tap water safety is likely a frequent topic of conversation among Flint citizens. When a Flint citizen who trusts the water interacts with other Flint citizens, they either reinforce the beliefs of citizens who already trust the water, or they may be able to convince a distrusting citizen that the water is, in fact, safe.

We can interpret γ as proportional to the number of other Flint citizens that each trusting citizen talks to about water safety. We assume that the government can vary γ by spending money on campaigns to encourage discourse about tap water safety (e.g., by distributing pamphlets that encourage citizens to talk to each other about water safety). By varying γ, we also vary β, since $\beta = \frac{\gamma}{N}$.

Figure 5 shows the effect of varying β by varying γ. When $\gamma = 1$, the government will have to distribute water for 3.954 years before the percentage of citizens who distrust the tap water falls below 5%.When $\gamma = 2$, the distribution period is reduced to 2.431. When $\gamma = 3$, the distribution period is 1.766. When $\gamma = 4$, the distribution period is 1.427. When $\gamma = 5$, the distribution period is 1.193. Clearly, increasing γ (i.e., increasing the interactions between citizens who trust the Flint tap water and those who do not trust the tap water) helps increase the rate at which the Flint citizenry as a whole moves towards trusting the tap water.

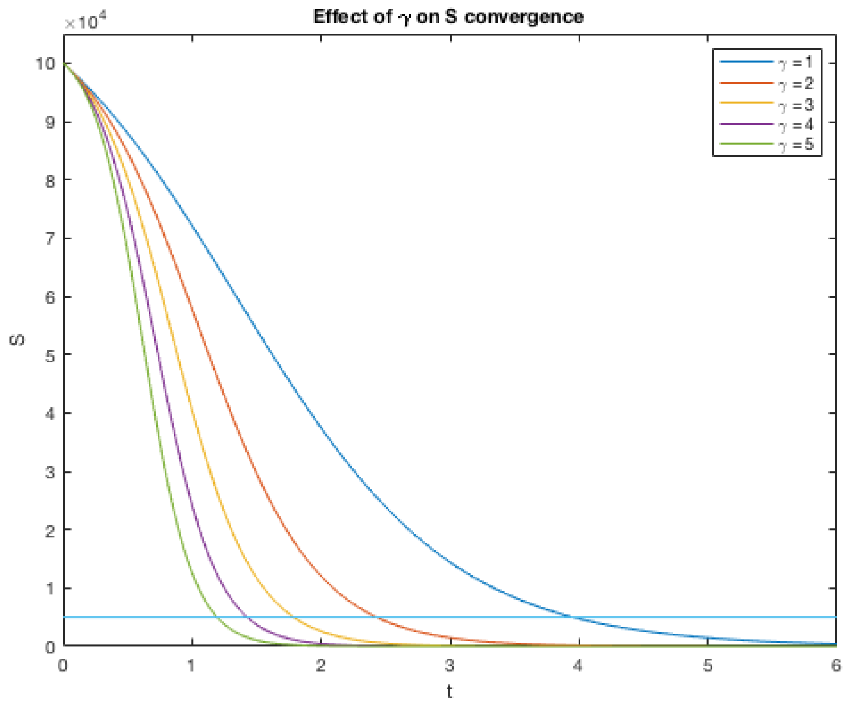

Figure 5: Plots of S over time for $\gamma = 1, 2, 3, 4, 5$. N is fixed at 100,000 and c is fixed at 0.2. The horizontal line represents an ϵ threshold of 0.05.

6 Application to Flint Water Crisis

We have the following notable dates regarding the Flint Water Crisis:

Dates	Time	Event	Average Citizen Reaction
January 5th, 2016	N/A	Declaration of Lead in Water	Very Negative
January 24th, 2017	0	Declaration of Water being Safe	Distrustful
February 20th, 2017	1/12	Considering Stopping Bottled Water	Distrustful
April 6th, 2018	4/3	Stopping Bottled Water	Less Distrustful

We can let January 24th, 2017 be time $t = 0$ because this was the day when Flint's tap water was declared safe. We will consider February 20th, 2017 as $t = 1/12$ since February 20th is roughly one month after January 24th ($t = 0$), and t is given in years. Finally, we will consider April 6th, 2018 as $t_\epsilon = 4/3$ since the government announced that it will be stopping the supply of bottled water 15 months after the Flint water was first declared to be safe.

We know that the population of Flint at time $t = 0$ is roughly 100,000 people [2] By Assumption 1, we assume that this total population doesn't change over time. On Feburary 20th, 2017, the Flint government considered stopping the bottled water supply [3]. This means that the government was originally considering assigning $t_\epsilon = 1/12$. Working backwards, we can use $t_\epsilon = 1/12$ to estimate ϵ given different values of c and β.

Figure 6 and Figure 7 show how the value of S/N at $t = 1/12$ changes as we vary c and γ respectively.

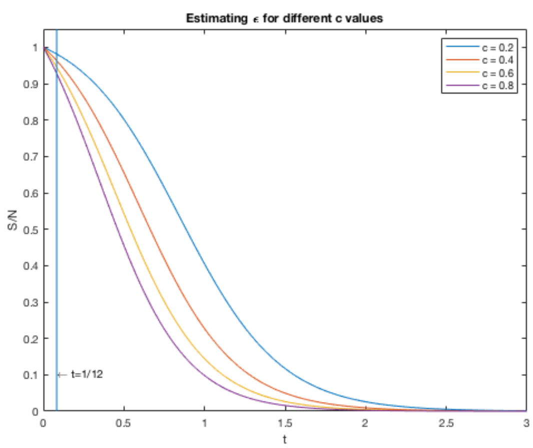

Figure 6: Plots of S/N (the fraction of Flint citizens that don't trust the tap water) over time for $=0.2, 0.4, 0.6, 0.8$. N is fixed at 100,000 and β is fixed at $3/N$. The vertical line on the left corresponds to the first time that the government publicly announced that they were considering the discontinuation of bottled water distribution (February 20th, 2017).

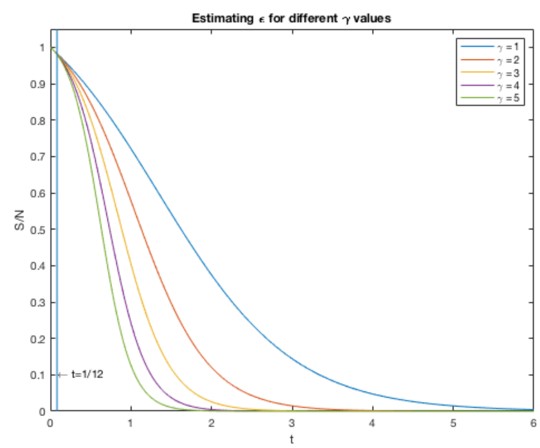

Figure 7: Plots of S/N (the fraction of Flint citizens that don't trust the tap water) for $\gamma = 1, 2, 3, 4, 5$. N is fixed at 100,000 and c is fixed at 0.2. The vertical line on the left corresponds to the first time that the government publicly announced that they were considering the discontinuation of bottled water distribution (February 20th, 2017).

Using $t_\epsilon = 1/12$ and assuming $\gamma = 3$, we get the following estimates for ϵ as we vary c. These estimates correspond to the value of S/N at the point of intersection of each curve with the vertical line at $t = 1/12$

in Figure 6.

c	ϵ
0.2	0.9813
0.4	0.9629
0.6	0.9449
0.8	0.9273

Using $t_\epsilon = 1/12$ and assuming $c = 0.2$, we get the following estimates for ϵ as we vary γ These estimates correspond to the value of S/N at the point of intersection of each curve with the vertical line at $t = 1/12$ in Figure 7.

γ	ϵ
1	0.9828
2	0.9820
3	0.9813
4	0.9804
5	0.9796

We can see in the above tables that ϵ was at least 0.92 for all combinations of parameters tested, which implies that at least 92% of people were still distrustful of the government at $t = 1/12$. For more robust results, we could test more combinations of c and γ values and create a matrix, but our preliminary analysis is enough to suggest that at $t = 1/12$, the proportion of people who are distrustful of the water is still very high.

We now turn our attention to $t = 4/3$, which corresponds to April 6th, 2018 the time at which the government actually went through with stopping bottled water distribution [4].

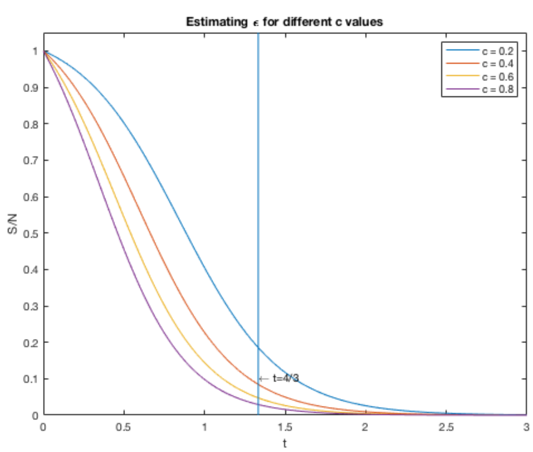

Figure 8: Plots of S/N (the fraction of Flint citizens that don't trust the tap water) over time for $=0.2, 0.4, 0.6, 0.8$. N is fixed at 100,000 and β is fixed at $3/N$. The vertical line in the middle of the plot corresponds to the time that the government actually discontinued bottled water distribution (April 6th, 2018).

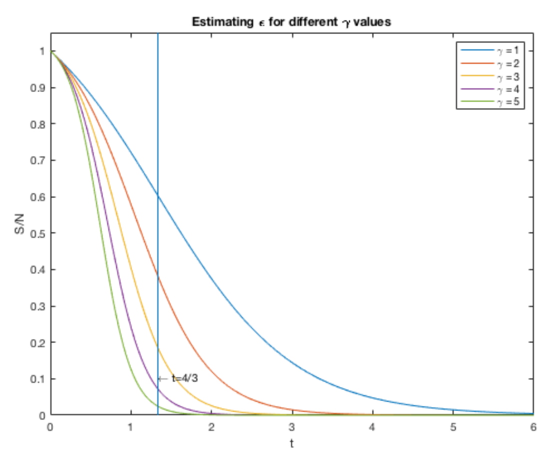

Figure 9: Plots of S/N (the fraction of Flint citizens that don't trust the tap water) over time for $\gamma = 1, 2, 3, 4, 5$. N is fixed at 100,000 and c is fixed at 0.2. The vertical line in the middle of the plot corresponds to the time that the government actually discontinued bottled water distribution (April 6th, 2018).

We can compute estimates of ϵ at various values of c and γ as we did with $t = 1/12$. These values are recorded below:

c	ϵ
0.2	0.1854
0.4	0.0845
0.6	0.0474
0.8	0.0292

γ	ϵ
1	0.6028
2	0.3821
3	0.1854
4	0.0723
5	0.0247

We can see in the above tables that ϵ was at least 0.0292, which implies that at least 2.9% of people were still distrustful of the government at $t = 4/3$. Since this value is under 0.05, this would explain why there was only partial public resentment with only some people going for water runs at the time of announcement [5].

As shown in the above results, the government of Flint ultimately set $t_\epsilon = 4/3$ since it may have believed that $t_\epsilon > 4/3$ would result in lower overall goal utility $G = U\left(\frac{I}{N}\right) + V(C)$. This may be the case since $V(C)$ (a negative utility function) grew too large with large operation costs C. At $t_\epsilon = 4/3$, it is possible that the marginal benefit of additional water distribution was outweighed by the marginal costs of distributing water. As a result, the government announced that they were no longer distributing bottled water.

While the above perspective may have been true, it is also possible that the government of Flint may have incorrectly assessed that $t_\epsilon > 4/3$ would result in lower overall goal utility $G = U\left(\frac{I}{N}\right) + V(C)$. As shown at $t = 1/12$ (February 20th, 2017), the government considered setting $t_\epsilon = 1/12$, which it later deemed an incorrect stopping point since a higher value of G could be achieved at $t_\epsilon > 1/12$. While it is unclear from the data why the government revised its original estimate, the government may have come to the conclusion that it misvalued either one (or both) of the utility functions $U\left(\frac{I}{N}\right)$ and $V(C)$. This would have lead to an incorrect assessment of the maximum value of G and stopping point t_ϵ. This may also be the case at the current stopping point $t_\epsilon = 4/3$.

Additionally since c and γ are not known fixed values, it is possible that Flint was operating at a less than optimal curve. This could potentially lead to upward estimates of ϵ (greater than 2.9%), which implies that a larger proportion of people were still distrustful of the tap water at $t_\epsilon = 4/3$. In this case, the government of Flint may benefit from improving c and γ to reach higher values of goal utility G without taking on too much cost C.

7 Conclusion

In summary, we recommend the mayor of Flint to use our model to help determine the optimal time to stop bottled water distribution. Additionally, we recommend that the mayor estimate the impact of potential government policies by adjusting the model parameters accordingly. This will allow the mayor to perform a cost-benefit analysis (i.e., Is the benefit of increasing the rate of trust-building worth the extra cost of a given policy?).

7.1 Model Strengths

1. Our model factors in the effects that people have on each other. As one person learns to trust the tap water, they will notify their friends and family. As a result, we can expect the knowledge of tap water safety to propagate through friendship networks, which we model on average as constant γ.
2. Our model also factors in the effect of government intervention in regaining the trust of Flint's citizens. It expects the government to select intermediate values of b to avoid oversupplying or undersupplying its citizens.
3. Despite not delving within the utility tradeoff between cost C and proportion of $\frac{I}{N}$, our model still accounts for selecting some t_ϵ that maximizes the overall goal utility tradeoff G, and how one may select t_ϵ given utility function U for citizen trust and V for overall operation costs.

7.2 Model Weaknesses

1. Our model's current notion of stopping at t_ϵ does not account for the additional flow ($\frac{dS}{dt} = -\beta SI$) that would occur in the absence of water bottle distribution ($c = 0$). This would lead to an overestimate of ϵ at t_ϵ and an underestimate of $U\left(\frac{I}{N}\right)$ as a result.
2. Our model lacks a stochastic element, which we would hope to implement if more time was available. Instead of deterministically setting values for c and γ, they could be stochastically distributed to account for differences across time and across various groups of people within Flint.
3. In practice, it may be hard to determine how c varies with the number of bottles distributed. We take as given that c can be varied but in order to determine the exact relationship between c and the amount of bottled water the city distributes, the city of Flint would need to regularly conduct public opinion surveys.

7.3 Further Improvements

Our model could be improved by adding the birth and death dynamics of population models. The "death" rate would be used to refer citizens who moved away in addition to citizens lost to natural death. This "death" rate has had a nonneglible effect on Flint's population in the past decade. In fact, Flint's population decreased by 6.1% from April 2010 to July 2018 [2]. Adding in the effects of the "death" rate will help make our model a more accurate reflection of reality.

Additionally, we could make our results more robust by running our model with different initial conditions. We took as given that at $t = 0$, no one trusts the water, but perhaps this is not an accurate reflection of reality. For the purposes of expanding the generalizability of our model, we should run our model with various values of $S(0)$ and see how t_ϵ varies as a result.

Furthermore, our model could be expanded to recommend utility functions U and V to Flint's government and determine the optimal t_ϵ based on those recommendations. Unfortunately due to time constraints, we were unable to generate feasible utility functions to reflect our own assessment of the tradeoff between citizen trust and operation costs.

8 Bibliography

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9 Appendix

Letter to Mayor of Flint

To the Mayor of Flint:

You are faced with the difficult task of rebuilding citizen trust in Flint's tap water. We recognize that you have to balance the benefits of distributing bottled water with the cost to the taxpayers. Armed with an understanding of mathematics, we have created a model to help you better understand the dynamics of trust in the tap water and to help you determine an optimal policy for water distribution.

Our recommendations are as follows:

- Don't provide too much bottled water, as citizens will be unlikely to try the tap water if they can meet their water demands using the bottled water that you provide.
- Don't provide too little bottled water, as this will have a negative effect on trust in the government. Citizens will be less likely to trust your word that the water is safe if they feel like you do not care about them. Additionally, if citizens have a huge water demand that is unmet using the bottled water that you provide, they will find it worth it to seek out alternative bottled water sources and will be unlikely to try the tap water.
- Encourage Flint citizens who trust the tap water to talk to their neighbors about how safe the water is.
- Don't waste resources trying to convince stubborn people that the water is safe. Once around 95% of citizens trust the tap water, stop distributing bottled water.

It is our hope that you will take these recommendations into consideration as you plan out Flint's bottled water distribution for the coming months.

Sincerely,
Soryan Kumar
Tiffany Ding