Electron charge quantization from mineral oil drops between charged plates

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To measure the quantization of electron charge, we spray oil droplets between two charged plates and use a ThorCams camera to record their movement. After determining their rising and falling velocities, we calculate the charge and infer the number of electrons we have measured, thus determining an electron charge of $q_e = 1.59 \cdot 10^{-19}$. Our result agreed with the known value of $q_e = 1.602176634 \cdot 10^{-19}C$ within our margins of error, and we found 99.2% agreement.

I. INTRODUCTION

Millikan's oil-drop experiment is a fundamental experiment which established the assumption of charge quantization in the 20th century. [1]

Currently, the fundamental charge is defined as $q_e = 1.602176634 \cdot 10^{-19}C$ as determined in 2019 by the International System of Quantities. [2]

In order to confirm the quantization of electron charges, we used an atomizer to spray mineral oil between charged plates, and determined their rising and falling velocities using a ThorCams camera that recorded their behavior. We reversed the charges between the plates, switching the direction the droplets were moving, in order to measure one droplet multiple times. [3] We then used these velocities, the charged plate separation, and the density of the oil to determine the charge of our droplets. After inferring the number of electrons these charges corresponded to, we plotted a histogram of our results and determined the electron charge from the separation between the peaks of our histogram.

II. THEORETICAL BACKGROUND

Charge quantization refers to the charge of any object occurring in integer multiples of the elementary charge. This phenomena was first observed and measured by Robert A. Millikan and Harvey Fletcher in 1909. [1]

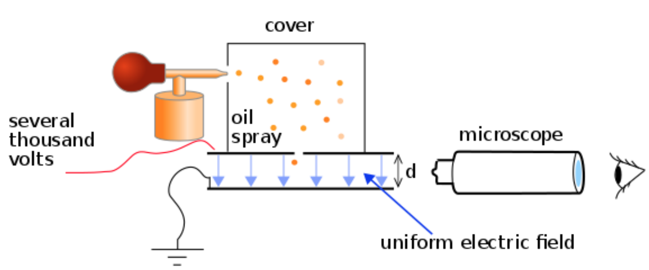

FIG. 1. Milikan's oil drop experiment diagram [4]

Millikan's fundamental oil drop experiment included observing minuscule droplets of mineral oil between two parallel, horizontal charged capacitor plates held apart by insulating material. [1] Between the plates, the forces acting on a drop were given by

$$F = F_E + F_g + F_f, \tag{1}$$

where F_E is the electrical force, F_g is the gravitational force, and F_f is the frictional force. The electrical force is given by

$$F_E = qE = q\frac{V}{d}, \tag{2}$$

while the gravitational force is given by

$$F_g = mg, \tag{3}$$

and the frictional force is given by

$$F_f = 6\pi\nu a v_f, \tag{4}$$

where q is the charge on the drop, d is the plate separation, V is the voltage between the plates, m is the mass of the drop, ν is the viscosity of air, a is the radius of the drop, and v_f is the terminal velocity of the drop falling in the absence of any electric field. [1]

Holes were cut into this insulating ring for viewing and to allow light to enter the chamber. Above the plates, an atomizer was used to spray oil which became electrically charged via friction with the nozzle or induced by an external ionising source. Since the droplets were then charged, when they moved between the capacitor plates their direction could be reversed by switching the charges on the plates. [1]

With no electric field present, Millikan then measured the terminal velocity of a falling droplet which allowed him to determine the drag force on the droplet that was equivalent to the gravitational force. Since both of these forces are dependent on the radius of the droplet, the mass and gravitational force were then known. Then an electric field was induced by a voltage applied between

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the plates until drops were suspended in the chamber, balancing the forces upon them. [1]

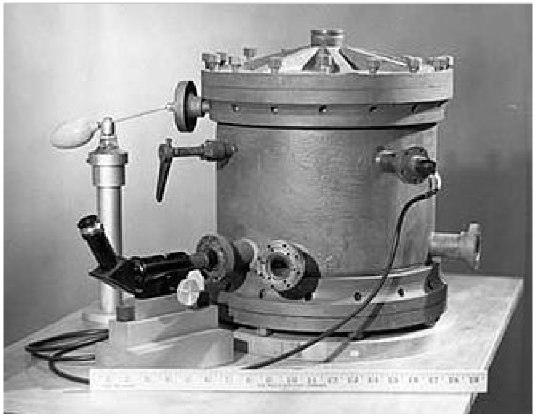

FIG. 2. Milikan's oil drop apparatus [4]

With this electric field known, the charges on the droplets could finally be determined. After repeating the procedure multiple times, Millikan and Fletcher confirmed that each measured charge on a droplet existed in an integer multiple of a specific value that they determined to be 0.06% different from the accepted value today of $q_e = 1.602176634 \cdot 10^{-19}C$. [2] [1]

Walter H. Shottky determined that the value of e would be known with a limited accuracy, and experimentally observed the first direct detection of Laughlin quasiparticles in the fractional quantum Hall effect.

In The Josephson effect, voltage oscillations are caused by the quantum effect electrons exhibit at low temperatures, two dimensional confinement, and strong magnetic fields. The Josephson constant is given by

$$K_J = \frac{2e}{h}, \tag{5}$$

and the von Klitzing constant

$$R_K = \frac{h}{e^2}. \tag{6}$$

The elementary charge can then be found by

$$e = \frac{2}{R_K K_J} \tag{7}$$

$$e^2 = \frac{2h\alpha}{\mu_0 c} = 2h\alpha\epsilon_0 c, \tag{8}$$

where α is the fine-structure constant, μ_0 is the magnetic constant, ϵ_0 is the electric constant, c is the speed of light, and h is Planck's constant.

A. 2019 Redefinition of SI Units

In 2019, the International system for standards in measurement redefined four of the seven base SI units. The constants that were redefined were those of kilogram, ampere, kelvin, and mole. This was due to the changing of the values for Planck's constant h, the elementary electric charge e, the Boltzmann constant k_B, and the Avogadro constant N_A. The changes are listed in table I. [2]

TABLE I. 2019 Redefinition of SI Base Units [2]

SI Base Unit	Previous Value	2019 Definition
h	-0.80 ± 0.10	$6.62607015 \cdot 10^{-34} J \cdot s$
k_B	-0.80 ± 0.10	$1.80649 \cdot 10^{-23} \frac{J}{K}$
N_A	-0.80 ± 0.10	$6.02214076 \cdot 10^{23} mol^{-1}$
e	-0.80 ± 0.10	$1.602176634 \cdot 10^{-19} C$

III. EXPERIMENTAL PROCEDURE

First we removed the AP-8210A PASCO Milikan Oil Drop Apparatus' chamber housing, lid, upper capacitor plate, and plate separator and washed them thoroughly with soap. After drying the components completely, we replaced them on the apparatus.

Next we adjusted the legs of the apparatus until the level bubble showed that the base was level. Then we connected the LED to an AC power source and switched it on.

Next we connected the ThorCams camera which had been attached to the pasco apparatus to our computer using a USB cable. Then we inserted the "focusing wire" between the capacitor plates and used the video feed on the ThorCams software to adjust the reticle focus, particle focus, LED brightness, and the frame rate and resolution settings on the ThorCams interface so that the wire appeared clearly. We then removed the wire and placed a ruler marked to the millimeter in the cameras view. We used the ThorCams measuring tool to draw a box on our screen that was a millimeter in height. This was done so that measuring the velocities of the droplets would be easier.

Next we connected the positive and negative terminals of our thermistor voltage terminals on the pasco apparatus to a digital multimeter using banana plugs. We turned the digital multimeter on so that it was set to measure voltage. We then recorded the temperature on the thermistor temperature table on the pasco apparatus that corresponded to the measured voltage. We recorded this temperature every 15 minutes throughout the experiment.

We connected our pasco power source's positive and negative terminals to the capacitor plate's cathode and anode on the pasco apparatus. We then connected the power source to power and set it to 500 Volts.

We made sure our atomizer was filled with mineral oil and held it to the chamber so that the nozzle was ninety degrees to the droplet hole. Using quick, hard squeezes we sprayed oil into the chamber and observed the ThorCams feed to determine when an adequate amount of oil droplets were in the chamber. We then placed a cover over the entrance hole, and used the plate voltage switch to determine which droplets were charged and isolate the most visible and adequately sized droplets. The manual advised us to choose droplets which were moving the most slowly. We also made sure to record droplets that we were able to measure the rising and falling velocities of multiple times.

Once we had isolated a suitable droplet, we used the plate voltage switch and a stopwatch to record the time necessary for the drop to move upwards and downwards between the measured area we had drawn. Making sure we maintained a constant voltage and housing chamber temperature, we repeated this process for multiple droplets.

IV. DATA ANALYSIS

We used each droplets measured rising and falling velocities to create a histogram of our data. Then we determined where our histogram bins should lie due to the grouping distribution of our data to determine how many charges each data point represented. We then determined the separation of our bins, yielding our value for the fundamental charge. [3]

V. CONCLUSION

Our determined value for the fundamental charge was $1.59 \cdot 10^{-19}C$ which is in 99.2% agreement with the accepted value of $1.602176634 \cdot 10^{-19}C$. [2]

A large amount of error is present in this experiment due to the variation of temperature in the housing chamber while we were recording data. [3] The voltage output by the power source was also not constant during our experiment, varying between $\pm$ 1.0 Volts. Another issue within our experiment is that our recorded velocities have some systematic error since our distance scale was set only for the center of the chamber. Some of the droplets could have been further or closer to the camera than the scale corresponded to and thus their velocities have not been measured exactly. The accuracy of these measurements could also be improved by timing the droplets using a laser or other means than the manual stop watch method we employed. [5]

Another source of systematic error is a result of our usage of the value for the viscosity of air. We could also improve our accuracy by performing this experiment in a vacuum.

A source of error which we minimized as much as possible during this experiment was the issue of drops evaporating during measurement, either partially or entirely. [5]

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