Temperature dependence of black-body radiation using a barium fluoride prism spectrometer

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(Dated: November 12, 2022)

Using a barium fluoride prism and thermopile detector, we measured the spectrum of a blackbody source at various temperatures in order to make comparisons with Planck's law and confirm the Stefan-Boltzmann law. We found that our spectra followed the Stefan-Boltzmann equation.

I. INTRODUCTION

The investigation of black-body radiation was fundamental in the establishment of the basis of quantum mechanics. The classical view of radiation predicted that a blackbody, via the equipartition theorem, would have an infinite heat capacity and infinitely growing spectrum of emitted radiation. This is known as the ultraviolent catastrophe, and was remedied by Planck's law. [1] [2]

Practical applications of black-body radiation include thermal imaging devices. Black-body radiation is prevalent in the fields of astronomy and cosmology, and has uses such as estimating the temperature relation between planets and stars and the study of cosmic microwave background radiation. [2]

To investigate black-body radiation, study Planck's law, and confirm the Stefan-Boltzmann law, we used a prism spectrometer to measure the spectrum of a black-body source at various temperatures.

II. THEORETICAL BACKGROUND

In 1858, Balfour Stewart used a thermopile and sensitive galvanometer read by a microscope to measure radiated power of plates of substances that radiated and absorbed for various qualities. He quantitatively confirmed that at thermal equilibrium, substances that emit and absorb selectively followed the principle of selective equality of emission and absorption, and he proposed that radiation was absorbed and emitted by particles of matter throughout the media it propagated within. He concluded that when leaving the surface of a substance, the reflected and emitted radiant heat within an enclosure in thermal equilibrium would be the same if the surface was composed of lamp-black (carbon black). [3]

In 1859, Gustav Kirchhoff, after noting the relation between absorption and emission of spectral wavelengths and the dependence on the temperature difference between emitter and absorber by bright or dark spectral lines, considered bodies emitted and absorbing thermal radiation in an opaque cavity in equilibrium. [4] Kirchhoff offered a new principal, although his qualitative theoretical proof is considered by some to be invalid, that the ratio of emitting power for a specific wavelength to absorptivity, denoted in equation 3, has the same value for all bodies, labeled i, that emit and absorb at that wavelength.

$$\frac{E(\lambda, T, i)}{\alpha(\lambda, T, i)}, \tag{1}$$

where both $E(\lambda, T, i)$, the emitting power of a body, and $\alpha(\lambda, T, i)$, the absorption ratio, are at temperature T and specified wavelength λ. This quantity has dimensions of emitting power. [4]

Unaware of Stewart's measurements of specific qualities of radiation, Kirchhoff sought to prove that the ratio in equation 1 did not depend on the nature i of the body by comparing an arbitrary non-ideal body with a postulated "perfectly black body" which corresponded to Stewart's observed bodies coated in lamp-black. Kirchhoff noted that a perfectly black body's ratio in equation 1 would just be $E(\lambda, T, BB)$, since by definition the absorptivity $\alpha(\lambda, T, BB)$ would be 1. Finally, he argued that

$$\frac{E(\lambda, T, i)}{\alpha(\lambda, T, i)} = E(\lambda, T, BB) = B_\lambda(\lambda, T), \tag{2}$$

however this function would not be determined by Planck for 40 years. Kirchhoff's law of thermal radiation can be stated:

"For any material at all, radiating and absorbing in thermodynamic equilibrium at any given temperature T, for every wavelength λ, the ratio of emissive power to absorptivity has one universal value, which is characteristic of a perfect black body, and is an emissive power which we represent by $B_\lambda(\lambda, T)$." [4]

A. Stefan-Boltzmann Equation

In 1879, following John Tyndall's measurements of infrared emission from a platinum filament and correspond-

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ing color in 1864, Josef Stefan determined the proportionality to the fourth power of absolute temperature. [5]In 1884, following Adolfo Bartoli's derivation of radiation pressure using thermodynamic principle's in 1876, Ludwig Boltzmann derived the relation theoretically by considering an ideal heat engine using electromagnetic radiation. [6]

The power given off by this ideal blackbody is given by the Stefan-Boltzmann law,

$$P = \sigma T^4 A, \tag{3}$$

where $\sigma = 5.670 \cdot 10^{-8} \frac{W}{m^2 K^4}$ is the Stefan-Boltzmann constant, T is the absolute temperature in Kelvin, and A is the area of the emitting surface.

B. Planck Radiation

Max Planck developed a theory for the radiance spectrum, referred to as the Planck spectrum, given by

$$B_{Planck}(\lambda, T) = \frac{2hc^2}{\lambda^5} \frac{1}{e^{\frac{hc}{\lambda k_b T}} - 1} \tag{4}$$

where $h = 6.62607015 \cdot 10^{-34} J \cdot s$ is Planck's constant, $c = 2.998 \cdot 10^8 m/s$ is the speed of light in a vacuum, λ is the wavelength, $k_B = 1.380649 \cdot 10^{-23} J/K$ is the Boltzmann constant. Emitted radiation is described by Planck's law when near thermodynamic equilibrium. Planck derived this formula by assuming a hypothetical electrically charged oscillator in a cavity containing black-body radiation could change its energy E in increments only proportional to the frequency ν of the electromagnetic wave. [1] The maximum intensity of this radiation is only dependent upon temperature.

C. Prism Spectroscopy

A prism spectrometer uses a prism to produce the spectrum of a source. Spectrometers first used to measure to spectrum of a blackbody were similar to prism spectrometers.

The spectra these spectrometers measured wavelengths depend on the focal length of the lenses used, mirror reflectivity, abosrption and reflection of light by the prism, and detector sensitivity. Minimizing the angular deviation is a common method used to calibrate these spectral measurements.

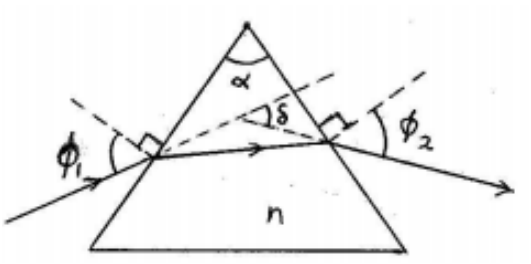

FIG. 1. Deflection of light rays incident on a prism [7]

The prism used in this experiment is made of barium fluoride and is equilateral so it has an apex angle of 60°. The angular deflection δ, the angle between the input and output ray of a prism, is given by

$$\delta = \phi_1 + sin^{-1}(nsin(\alpha - sin^{-1}(\frac{sin\phi_1}{n}))) - \alpha, \tag{5}$$

where the angles referenced are indicated in figure 1. Substituting in our values of $\alpha = 60°$, and our index of refraction n for barium fluoride at 635 nanometers, we found a minimum deflection angle δ_{min} of 34.894°. We adjusted our prism so that the angular deflection was at a minimum, and therefore the angles of incidence ϕ_1 and ϕ_2 were both equal to 47.447°. With the prism fixed at this minimum deflection angle, we determined the formula for angular deflection vs. index of refraction n, shown in figure 2.

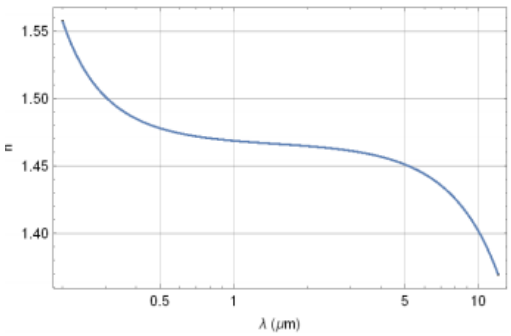

FIG. 2. Index of refraction of barium fluoride as a function of wavelength [7]

Over the wavelength range of 0.2 to 11 micrometers, the index of refraction of barium fluoride as a function of wavelength is given by

$$n = \sqrt{1.3 + \frac{0.81070\lambda^2}{\lambda^2 - 0.10065^2} + \frac{0.19652\lambda^2}{\lambda^2 - 29.87^2} + \frac{4.52469\lambda^2}{\lambda^2 - 53.82}}, \tag{6}$$

and is depicted in figure 3.

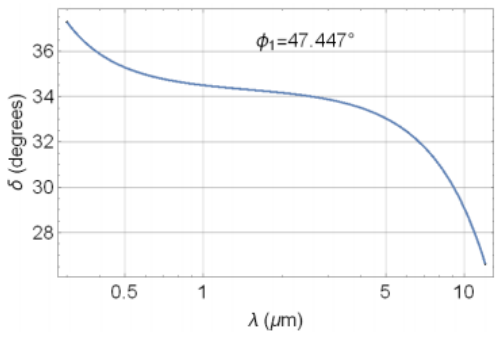

FIG. 3. Angular deflection as a function of wavelength for a prism at minimum deviation for a wavelength of 635 nm [7]

From these relations, we now have the angular deflection as a function of wavelength. [7]

Our recorded spectra had known absorption lines from the modes of the carbon dioxide molecules in the atmosphere which we corrected for.

III. EXPERIMENTAL PROCEDURE

A. Spectrometer

To measure the blackbody spectrum for temperatures within the range of 500 to 1500 Kelvin, and wavelengths between 0.35 micrometers and 10 micrometers, we used a broadband prism spectrometer similar to the spectrometers first used to precisely measure the blackbody spectrum.

A radiation source is placed in front of an entrance slit S1. The radiation travels through S1 and then completely illuminates a Thorlabs MPD169-P01 1 inch diameter off-axis parabolic mirror M1, which has a reflected focal length of 6 inches designed for right angle reflections. This mirror is placed 6 inches from S1.

A light chopper is placed between M1 and the prism in order to reduce noise in the signal.

The parallel beam of light from M1 passes through the chopper wheel and reaches the input surface of a Crystan Optics part number BAFPRISM25.4-60 barium fluoride (BaF_2) equilateral prism with faces 25.4 millimeters wide by 25.4 millimeters high. The light is then again refracted into a parallel output beam, which is then intercepted by another mirror M2 identical to M1.

This beam is directed down towards another slit S2, which reaches the Dexter Research model 2M Thin Film Based Thermopile Detector with internal thermistor, barium fluoride window, and xenon gas fill that we selected for its ability to respond over our desired wavelength range of 0.35 to 10 micrometers. [7]

The configuration of this detector is a 2 millimeter by 2 millimeter sensitive area consisting of a series array of semiconductor thermocouple junctions mounted on a thin membrane of low thermal conductance to serve as the "hot junctions", and a second series array of thermocouple junctions mounted to a ceramic substrate in contact with the device case to serve as the "cold junctions". When radiation is transmitted through the BaF_2 window and heats the sensitive area of the detector, the two thermocouple arrays make a differential measurement of the difference between the hot and cold junctions' temperatures. Therefore, the device yields a voltage output reading proportional to the radiation's incident power. [7]

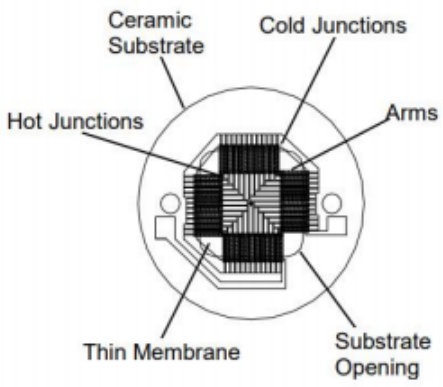

FIG. 4. Thermopile Detector Design [7]

The sensitive area of this detector features a high absorptivity coating to maximize the conversion to heat from optical power, and this absorptivity has a negligible dependence on wavelength. [7]

Since the signal from this thermopile detector was too small to measure with our computer, we used a BNC cable to connect its output to a Dexter model 1010 low-noise amplifier, providing a voltage gain of 1,000 to our signal.

The amplifier was then connected using a BNC cable to our National Intruments DAQ interface A01 input port, which was connected to our computer via USB cable.

We mounted M2, S2, and the thermopile detector onto a platform on a Newport Model URM100 PE motorized rotation stage. With a source of a specific wavelength, rotating the platform yielded a change in wavelength and differing deviation angle that passed through S2 and onto the detector, allowing a measurement of the detected power as a function of wavelength and therefore the spectrum of our source. We controlled this rotation stage using the buttons on our Newport model ESP300 controller, which was connected via USB to our computer.

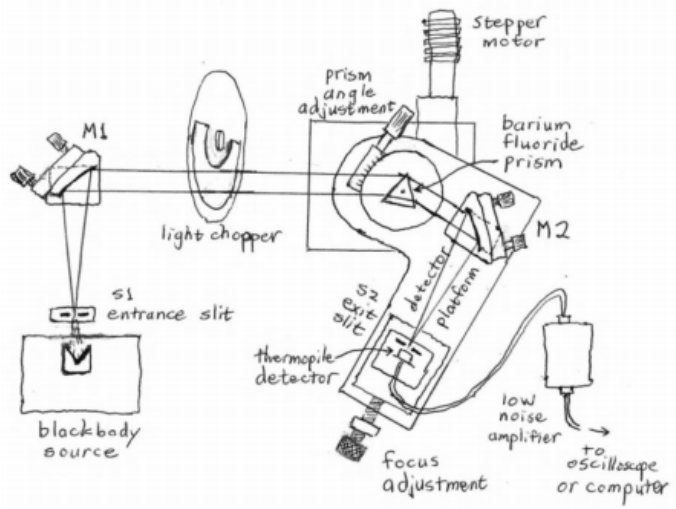

FIG. 5. Top View of Spectrometer Configuration [7]

Our LabView program was configured to read the input signal from the detector and amplifier connected to the A01 input port. The program also controlled the rotating stage and moved through our range of specified angles and angle increments. The program recorded with a time interval in multiples of the 2 second period of the chopper wheel, since the signal recorded for the closed portion of its period was subtracted from the signal for the open portion to reduce the recorded noise. Finally, the program produced a plot of angle versus voltage, which we then saved.

B. Procedure

To properly align our components and determine the relationship between prism index of refraction and wavelength, we first used a 635 nanometer Thorlabs LED as a visible radiation source in place of our blackbody source. We plugged in our LED to a power source, and connected it to a 50 millimeter lens. After screwing the LED into our spectrometer mounting board, we directly aligned the lens with our S1 entrance slit using an adjustable knob.

We used our visible light beam to ensure that the radiation was uniformly illuminating the entrance aperture of M1, with an angle of reflection near 90°, and that the beam of light leaving the prism also uniformly illuminated M2 with a reflection angle of 90°.

We then adjusted the height of S2 so that the image of the entrance slit, which was 6 inches from the center of M2, was at the same height. To maximize the signal and resolution, we used the focus adjustment on the rotation stage platform to ensure S2 coincided with the focused image of S1.

In order to calibrate the wavelength and angle of our spectrometer, we adjusted our prism for minimum deflection angle by turning the prism angle adjustment knob and rotating the stage until our measured signal was at a maximum. Then, we manually reprogrammed this measured angle on our controller to the angular deflection for 635 nanometers in barium fluoride of 34.894°. [7]

Finally, we used our LabView program to record a spectrum of our LED to ensure that the 635 nanometer wavelength was measured correctly and infer our wavelength resolution from the width of this spectrum.

C. Blackbody Source

We replaced our LED with an Infrared Systems Development IR-564 Blackbody Reference Source, aligning its largest aperture with S1 to fully illuminate M1.

Our blackbody source was controlled using an Infrared Systems Development IR-301 Digital Temperature Controller, which we used to vary the temperature.

We used our LabView program to take spectral recordings at temperatures of 800°C, 1000°C, and 1200°C, beginning with the lowest temperature.

IV. DATA

The spectrum we recorded for the 635 nanometer LED source for calibration and spectrometer alignment purposes is depicted in figure 6.

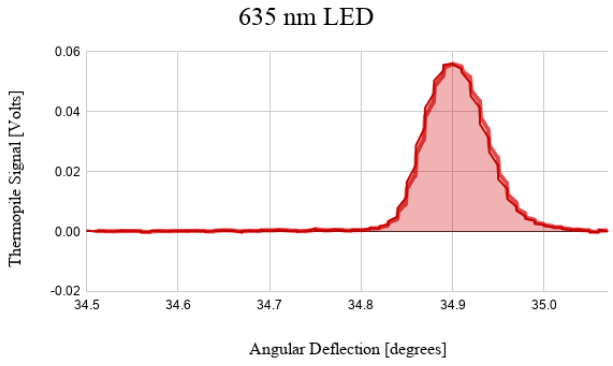

FIG. 6. Spectrum recorded for 635 nm LED

Our recorded blackbody spectra for the temperatures of 800°C, 1000°C, and 1200°C are depicted in figures 7, 8, and 9 respectively.

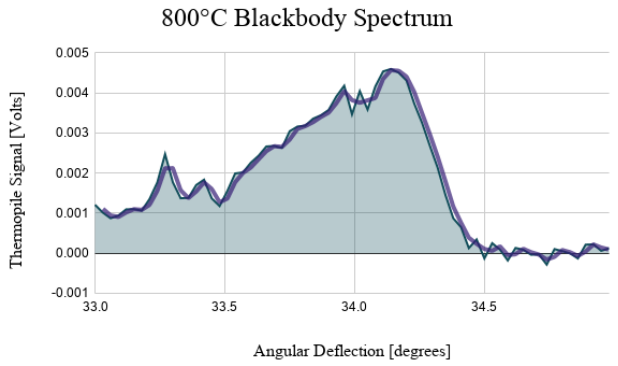

FIG. 7. Radiation spectrum recorded for a blackbody source at 800°C

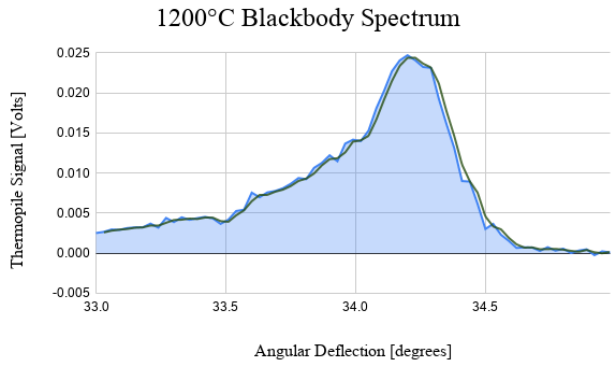

FIG. 9. Radiation spectrum recorded for a blackbody source at 1200°C

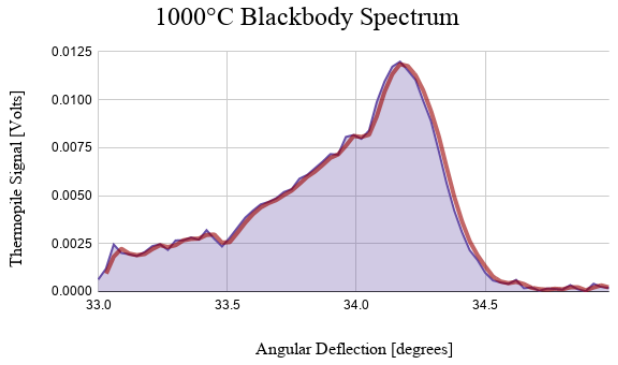

FIG. 8. Radiation spectrum recorded for a blackbody source at 1000°C

V. CONCLUSION

We found that our measured spectra indeed depended on temperature as described by the Stefan-Boltzmann law and verified by Planck's law.

A systematic source of error is that the index of refraction of air was neglected in our experiment. Our measurements were also not taken in a vacuum, which contributes to error since the known indices of refraction for our barium fluoride prism are specified for a vacuum. We also neglected the index of refraction of air, which had minimal impact on our measurements.

Other sources of systematic error could potentially be from our thermopile detector. The semiconductor island's differential temperature measurements would be affected by the temperature in our laboratory on a given day. [7]

We also would have systematic error from atmospheric interference on our radiation. We corrected for the known aborption lines from carbon dioxide, but there could be interference from dust particles or other unknown sources. [8]

A. Acknowledgements

We would like to acknowledge the University of Texas at Austin's physics department for allowing us the laboratory space to perform this experiment. We would like to thank the University for providing us with the equipment to carry out our experiment. We are thankful to William Debenham, Connor Davis, and Dr. Daniel Heinzen for the efforts and insight they provided us with during this experiment.

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