Radio Frequency Transmission Lines

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I. INTRODUCTION

A. Theoretical Background

A key radio frequency, electromagnetic waves within the frequency range of 300kHz to 3GHz, component is the transmission line. Transmission lines are comprised of two parallel conductors, with a common example being a coaxial cable. Coaxial cables have an inner conducting wire surrounded by an outer cylindrical conductor which is encased by an insulating jacket.

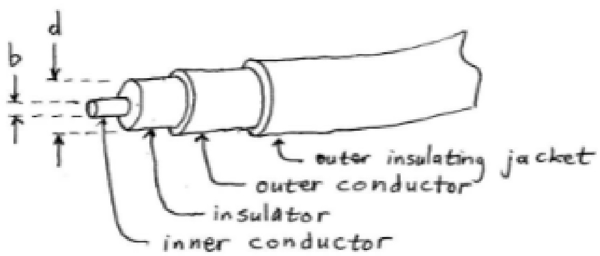

FIG. 1. Coaxial Cable

At radio frequencies, these two conductors have equal and opposite currents, therefore cancelling the fields they radiate. The potential difference $V(z,t)$ between the two conductors and the current $I(z,t)$ of the inner conductor are described by

$$\frac{\partial V}{\partial z} = -L\frac{\partial I}{\partial t} \tag{1}$$

$$\frac{\partial I}{\partial z} = -C\frac{\partial V}{\partial t} \tag{2}$$

where C is the capacitance per unit length of the transmission line and L is the inductance per unit length. The solutions are of the form of a wave equation and are given by

$$V(z,t) = V_0 cos(wt - kz) \tag{3}$$

$$I(z,t) = I_0 cos(wt - kz). \tag{4}$$

The impedance of the line is defined by

$$Z = \frac{V_0}{I_0} = L\frac{\omega}{k} = \frac{k}{C\omega} = \sqrt{\frac{L}{C}}. \tag{5}$$

So the voltage and current waves must oscillate in phase, and have phase velocity

$$v_\phi = \frac{\omega}{k} = \sqrt{\frac{1}{LC}} \tag{6}$$

For coaxial cables, the inductance and capacitance per unit length are given by

$$L = \frac{\mu_0}{2\pi} ln\frac{d}{b} \tag{7}$$

$$C = \frac{2\pi\epsilon_0\epsilon}{ln(d/b)} \tag{8}$$

where d is the inner diameter of the outer conductor, b is the diameter of the inner conductor, $\epsilon_0 = 8.854 \cdot 10^{-12}$ F/m is the electric constant, and $\mu_0 = 4\pi \cdot 10^{-7} N/A^2$ is the magnetic constant. We see that the speed of the wave is

$$v_\phi = \sqrt{\frac{1}{LC}} = \sqrt{\frac{1}{\epsilon_0\mu_0}\epsilon_r} = \frac{c}{\sqrt{\epsilon_r}} \tag{9}$$

where c is the speed of light in a vacuum. The impedance of the cable is

$$Z = \sqrt{\frac{L}{C}} = \frac{1}{2\pi}\sqrt{\frac{\mu_0}{\epsilon_0\epsilon_r}} ln(\frac{d}{b}). \tag{10}$$

B. Impedance Discontinuity

If a traveling voltage wave is moving through a cable of impedance Z_1 that is joined to a cable of impedance Z_2, the wave will partially reflect at this junction. The amplitude voltage reflection coefficient is given by

$$r = \frac{Z_2 - Z_1}{Z_1 + Z_2}, \tag{11}$$

and the amplitude voltage transmission coefficient is

$$t = 1 + r = \frac{2Z_2}{Z_1 + Z_2}. \tag{12}$$

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C. Radio Frequency Power

If a voltage source is driven through one end of a cable, then the cable will draw a current

$$I(t) = V(t)/Z, \tag{13}$$

and the instantaneous power transmitted past a given point is

$$P(t) = VI = I^2Z = I_0^2Zcos^2(wt - kz) \tag{14}$$

with a time-average of

$$P_{avg} = \frac{1}{2}I_0^2Z = \frac{V_0^2}{2Z} = \frac{V_{rms}^2}{Z}, \tag{15}$$

where

$$V_{rms} = \frac{1}{\sqrt{2}}V_0 \tag{16}$$

is the root-mean-square (rms) voltage.

For an impedance discontinuity, the power reflectionn coefficient is

$$R_P = r^2 = \frac{(Z_1 - Z_2)^2}{(Z_1 + Z_2)^2}, \tag{17}$$

and the power transmission coefficient is

$$T_P = t^2\frac{Z_2}{Z_1} = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2} \tag{18}$$

II. EXPERIMENTAL PROCEDURE

For simplicity, the various configurations and measurements are separated into numbered subsections, and these numbers are referred to in the Data (III) and Analysis (IV) sections. First we placed a BNC tee and 50 Ohm terminator on the oscilloscope's channel 1 and 2 inputs to impedance match the cables to the inputs. After plugging in and turning on the oscilloscope, we restored the settings to default and configured our display for the experiment and well as exploring the general functions of the instrument. We then connected our voltage controlled oscillator to the open end of the BNC tee on oscilloscope's channel 1 input.

We then changed the vertical sensitivity of Volts per division by turning the knob under the yellow "1" button to 5 Volts per division. Next we set the horizontal sensitivity by turning the knob under the "acquire" button to 25 nanoseconds per division, yielding a display of individual cycles of sine waves. We next changed the voltage attenuation setting to $1x$ and set the signal to DC coupling for channel's 1 and 2. We then configured the triggering settings, the feature that synchronizes waveforms from one sweep to the next, to "Type Edge" triggering and set the source to channel 1. We ensured that the signal was set to "Slope Rising". We finally set the trigger mode to "Auto" and the "Level" knob to "Triggered".

We set our oscillator frequency to 60 MHz by adjusting the knob on the VCO supply and checking the trigger rate on the oscilloscope.

A. Measurements

To take measurements of this signal, we utilized the oscilloscope's automatic measurement function as well as cross checking using the oscilloscope's measurement cursors, although these measurements can also be taken by reading the oscilloscope's display grid manually and calculating values based on the horizontal and vertical scales.

When utilizing cursors to make measurements, we first pressed the "Cursor" button, then changed "Type" to "Amplitude" with the "Source" set to Channel 1. Then we could adjust the multipurpose knob to move the cursor line to our selected points and record the values shown. To measure the amplitude, we positioned this cursor to the top of our sine wave, and pushed the "Cursor 2" button and moved it to the bottom of the wave. After changing the "Type" to "Time", we measured the period of oscillation and noted the displayed voltage level at specific points.

1. Output Power of Voltage Controlled Oscillator (VCO)

To measure using the automatic function on the oscilloscope, we pressed the "Measure" button, selected "Channel 1", and used the multipurpose knob to choose Period, Frequency, Peak-Peak, and RMS. We determined the output power of the VCO from the root-mean-square of the voltage signal given by the RMS value from this automatic measurement function.

2. Capacitance of RG-58 Coaxial Cable

To measure the capacitance of our RG-58 cable, we turned the VCO off and disconnected it from the scope along with the BNC tee and 50 Ohm terminator from the channel 1 input. We then plugged in the avalanche pulser power supply and connected a 6 foot long 50 Ohm cable to the "cable" port, leaving the other end of the cable open. We connected the pulser output to channel 1 of the oscilloscope via a 10 foot cable without using a terminator. We then increased the horizontal time per division scale to 2.5 microseconds, yielding an exponential decay after the sharp increase in voltage at the trigger point. We utilized the oscilloscope's time cursors to measure the time constant of this decay. We substituted a 40 foot cable for the 10 foot cable and observed how the early time signal changed.

3. Propagation Speed of Signal in RG-58 Coaxial Cable

We then placed a BNC tee and 50 Ohm terminator back onto channel 1 of the oscilloscope, and plugged the pulser into the scope using a 10 foot cable, adjusting the scope settings until we could observe the "transmission line" limit.

Next, we reconnected the pulser to the oscilloscope with a BNC tee and 50 Ohm terminator on the pulser output in order to suppress pulse reflections travelling back to the pulser. In addition, this reduces the amplitude of the voltage pulse by a factor of two.

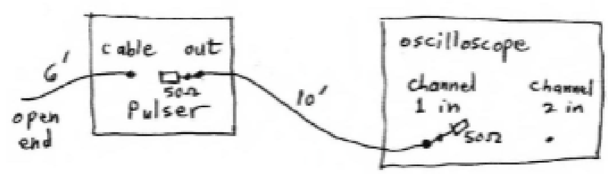

FIG. 2. Setup for determining propagation of speed of signal in an RG-58

We then connected the output of the directional coupler using a 3 foot BNC to the channel 1 input of the oscilloscope, the CPL using a 3 foot BNC to the channel 2 oscilloscope input, and the directional coupler's IN to the pulser out using a 10 foot BNC, leaving the 'CABLE' output open ended with a 6 foot BNC. This is configured so that we simultaneously recorded the the pulse transmitted through our directional coupler, and the pulse from the coupler's "CPL" port. We made sure in this stage to check whether the directional coupler was inverting our measured output by observing whether our channel 2 pulse was reading as negative. We measured the heights of these pulses to determine the attenuation of the signal transmitted through the output port of the coupler, and the ratio of the coupled port signal to the input port signal to compare this with the specified -20 dB coupling.

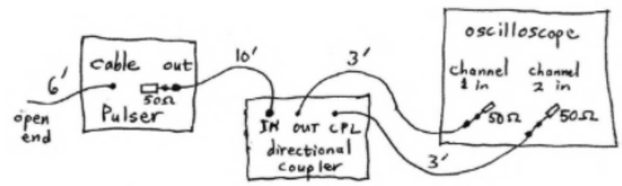

FIG. 3. Setup to record the pulse transmitted through the directional coupler, and from the coupler's "CPL" port of a forward propagating pulse

4. Bi-directional Coupler

Next, we added another directional coupler to our setup to create a bi-directional coupler, by connecting the "OUT" ports of both couplers via a 3 foot BNC. The second coupler's "CPL" port was connected to the oscilloscope's channel 2 port with a 3 foot BNC, and its "IN" port was left open circuited using a 40 foot BNC. The first directional coupler's "CPL" port was connected via a 3 foot BNC to the oscilloscope's channel 1 port, and its "IN" port was connected to the pulser with a 10 foot BNC. We then measured the delay between the leading edges of the channel 1 and channel 2 pulses. After removing the 40 foot cable, we repeated these measurements.

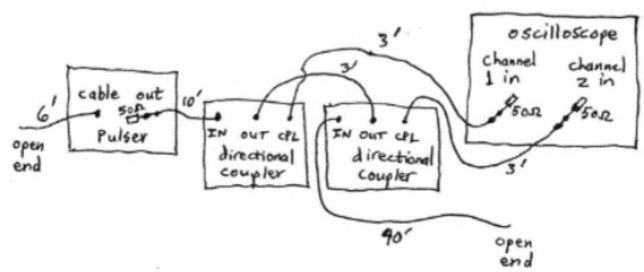

FIG. 4. Configuration to record both the forward and backward propagating pulse

5. Reflection Coefficients from Cable Terminations

We changed the 40 foot BNC from our previous configuration to a 6 foot, 50 Ohm cable. We then measured the reflected pulses when the end of the 6 foot cable (i) was terminated with a 50 Ohm terminator, (ii) left open circuited, (iii) shorted with the shorting cap, and (vi) terminated with a 93 Ohm terminator.

6. Reflection and Transmission Coefficients From Impedance Discontinuity

We then connected the 6 foot long, 50 Ohm cable to a 30 foot long, 93 Ohm cable using a BNC union yielding two reflected pulses. Once we connected the 93 Ohm terminator to the end of the 30 foot cable with a BNC union, the reflection from that cable end was eliminated.

Finally, we connected a single directional coupler's "CPL" port to the oscilloscope's channel 1 port using a 3 foot, 50 Ohm BNC, the "IN" port to the pulser's out port with a 10 foot, 50 Ohm cable, and the "CABLE" port of the pulser open ended with a 6 foot, 50 Ohm BNC. The "OUT" port of the directional coupler was connected to a 6 foot, 50 Ohm BNC which was connected to a 5 foot, 93 Ohm BNC using a BNC union, and finally connected to the oscilloscope's channel 2 input. This setup allowed us to measure the reflection coefficient r of the impedance discontinuity.

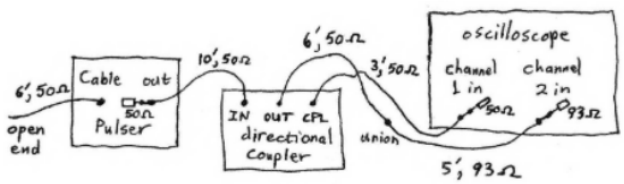

FIG. 5. Configuration to record both the forward and backward propagating pulse

III. DATA

1. Output Power of VCO

The VCO's pulse amplitude, frequency, period, and V_{rms} are given in table I. These values were found using the automatic measurement function on the oscilloscope.

TABLE I. VCO Values Using an Oscilloscope

Amplitude [V]	Frequency [MHz]	Period [ns]	V_{rms} [mV]
1.96 ± 0.02	60.00 ± 0.02	16.64 ± 0.02	710 ± 0.01

2. Capacitance of RG-58 Coaxial Cable

TABLE II. Time Decay of RG-58 Coaxial Cable Pulse

Cable Length [ft]	Voltage [V]	Frequency [kHz]	τ [μs]
10	5.00 ± 0.02	322.5 ± 0.02	3.10 ± 0.02
40	3.40 ± 0.02	76.92 ± 0.02	13.0 ± 0.02

3. Pulse Height With and Without Directional Coupler

TABLE III. Pulse Height

Channel	Pulse Height [V]
1	10.20 ± 0.02
2	1.60 ± 0.02

4. Bi-directional Coupler

TABLE IV. Bi-Directional Coupler

Cable Length [ft]	Voltage [V]	Frequency [MHz]	Time Delay [ns]
40	6.00 ± 0.02	7.692 ± 0.02	130.00 ± 0.02
0	5.60 ± 0.02	200.00 ± 0.02	5.00 ± 0.02

5. Reflection Coefficients from Cable Terminations

(i) With the 50 Ω termination, only a signal from channel 1 was shown.

(ii) When left open circuited, the time delay was $24.00 \pm 0.02ns$, the frequency difference was $41.66 \pm 0.02MHz$, and the voltage difference was $3.60 \pm 0.02V$.

(iii) When shorted, the time delay was $25.00 \pm 0.02ns$, and the frequency difference was $40.00 \pm 0.02MHz$, and the voltage difference was $3.60 \pm 0.02V$.

(iv) With the 93 Ω terminator, the channel 1 voltage was $11.60 \pm 0.02V$, and the channel 2 voltage amplitude was $380 \pm 0.02mV$.

6. Reflection and Transmission Coefficients from Impedance Discontinuity

TABLE V. Impedance Discontinuity

Channel	Voltage [V]
1	10.8 ± 0.2
2	13.6 ± 0.2

The time delay between these pulses was also recorded at $11.2 \pm 0.02ns$

IV. ANALYSIS

1. Output Power of VCO

From equations 16 and 15, the VCO output power is 14.2 mW, or 41.52 dBm, which is comparable to the specified value.

2. *Capacitance of RG-58 Coaxial Cable*

For the 10 foot cable, the capacitance was 31 pF or 3.1 pF per foot.

For the 40 foot cable, the capacitance was 1.3 pF or 0.03 pF per foot.

The early time signal for the 40 foot cable was noiser than the 10 foot cable's early time signal.

3. *Propagation Speed of Signal in RG-58 Coaxial Cable*

The pulse height from channel 1 was $10.2 \pm 0.02V$, and the pulse height from channel 2 was $1.6 \pm 0.02V$. The ratio of the "CPL" port signal to the input port signal was 6.38 $\pm 0.04V$

4. *Bi-directional Coupler*

The difference in the time delay with and without the 40 foot cable was 125 $\pm 0.02ns$. This corresponds to a velocity of 97.54 $\pm$0.04 m/ μs. The dielectric constant of the insulator in the cable was found to be 3.07 from equation 9, meaning the cable insulator could be made of solid polyethylene.

5. *Reflection Coefficients from Cable Termination*

(i) Only channel 1 read a signal.
(ii) The reflection coefficient was 12.10 $\pm$0.02
(iii) The reflection coefficient was 10.94 $\pm$0.02
(iv) The reflection coefficient was 30.52 $\pm$0.02

6. *Reflection and Transmission Coefficients from Impedance Discontinuity*

The reflection coefficient was 0.79 ± 0.04 and the transmission coefficient was 1.79 ± 0.04.

V. CONCLUSION

Issues in this experiment could have arisen due to measurement error. The deviation values were also not decided precisely.

More accurate values for the node frequencies could be found by performing more repetitions of this experiment. [1]

The cable lengths and terminations could also not have been accurate. The BNC tees, unions, and jacks could possibly have been damaged which would lead to the recorded signals varying. The oscilloscope could also have been damaged, or recorded data differently due to the temperature in the lab on that particular day.

[1] P. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for Physical Sciences*, 3rd ed., Vol. 2 (McGraw-Hill Education, 2002).

[2] R. Fitzpatrick, *Oscillations and Waves: An Introduction*, 1st ed. (CRC Press, 2017).

[3] R. P. Feynman, Phys. Rev. **94**, 262 (1954).

[4] G. C. King, *Vibrations and Waves*, 1st ed. (Wiley, 2009).