Mechanical Resonance

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I. INTRODUCTION

When studying oscillations in classical mechanics, damped and driven systems are typically among the first topics introduced. This is due to the importance of harmonic oscillators in many areas of physics since they describe the behavior of any mass in stable equilibrium that is subjected to a force for small vibrations. [1]

For our experiment, we measured the free decay of a cantilever oscillator using a laser and Pasco light sensor to determine its natural frequency, decay rate, and quality factor. We then used a driving coil to record a driven and damped oscillator's damping constant and frequencies above, below, and near resonance. Finally we compared the quality factor and frequency of the free decay with the driven oscillations.

II. THEORETICAL BACKGROUND

A harmonically oscillating system is damped if a frictional force proportional to the velocity is present, and driven if a time dependent external force is in effect. [1]

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 = 0 \tag{1}$$

where $\omega_0 = \sqrt{k/m}$ is the oscillator's natural vibration frequency, and $\gamma = b/m$ is the damping constant.

Depending on the frictional coefficient, a system experiencing damping can be underdamped, oscillating at a lower frequency than the undamped system with an amplitude that decays with time, overdamped, exhibiting no oscillations and decaying to an equilibrium state, or critically damped, returning to steady state as quickly as possible without oscillating. [1] The case of over damping occurs when

$$x(t) = Ae^{-\frac{\gamma}{2}}cos(\omega_0' t + \phi), \tag{2}$$

where $\omega_0' = \sqrt{\omega_0^2 - (\gamma/2)^2}$, $\phi = tan^{-1}(\gamma/(2\omega_0'))$, and $A = x_0/cos(\phi)$.

For our experimental purposes, we considered only the strongly underdamped case, where $\omega_0' \approx \omega_0, \phi \approx 0$, $A \approx x_0$, and

$$x(t) \approx x_0 e^{-\gamma/2} cos(\omega_0 t). \tag{3}$$

Similarly, an oscillating system can be driven and damped, and is then described by

$$\frac{d^2x}{dt^2} + \gamma\frac{dx}{dt} + \omega_0^2 = \frac{F_0}{m}cos(\omega t). \tag{4}$$

If an applied oscillating force's frequency is close to the system's resonant frequency ($\omega \approx \omega_0$), the frequency at which the corresponding amplitude reaches a maximum, the system will oscillate at a higher amplitude than if the same force is applied at non-resonant frequencies. [2] A useful parameter called the quality factor or Q factor, defined in equation 1, characterizes an oscillator's rate of energy loss relative to the overall stored energy of the oscillator. [1]

$$Q = \omega_0/\gamma \tag{5}$$

III. EXPERIMENTAL PROCEDURE

The experimental setup consisted of a breadboard, foam pad, laser, Pasco light sensor, mounts, lens, polarizer, and cantilever. A cantilever is a thin metal bar with one end supported and the other free with a bent piece of metal acting as a "detection flap". The free end is also weighted with a magnet and holder allowing us to drive the motion of the cantilever using a coil.

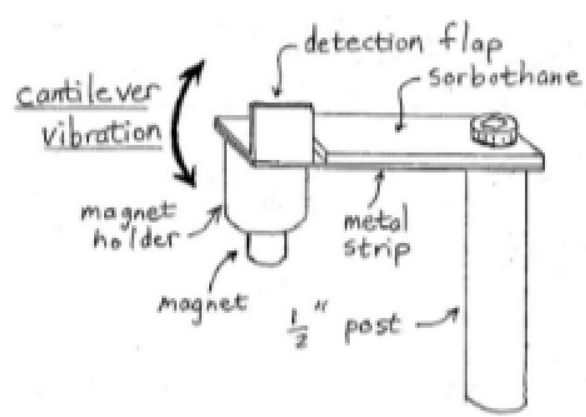

FIG. 1. Cantilever

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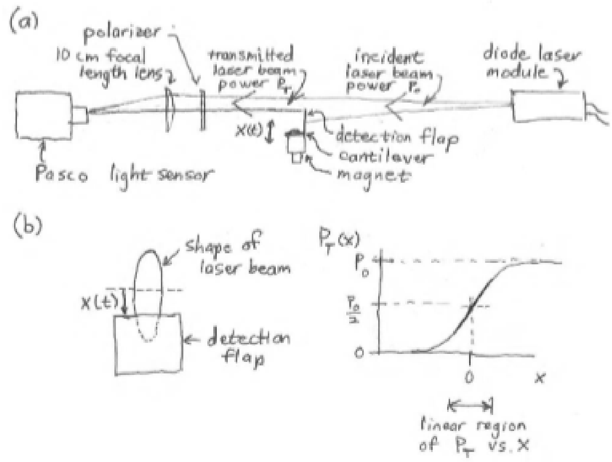

FIG. 2. Optical Detection of Cantilever Displacement

First, we placed the breadboard on a foam pad to reduce the vibrations of the breadboard since they can couple with the cantilever's vibrations. Then the laser was mounted at one end of the breadboard with the beam running down the center of the board and positioned 6 inches above the surface to allow room for the coil. We then placed the Pasco light sensor at the opposite end of the board so that the beam's focus was on its center. The light sensor was then connected to the Pasco 850 channel which was set to record the light sensor signal.

We next mounted the lens 10 cm in front of the light sensor. The elliptical beam produced by the laser was oriented so the longer axis was vertical. The polarizer was placed in front of the lens to reduce the laser power since at this point it was above the saturation power. We then adjusted the polarizer and set-screw on the polarizer's mount until the power was at 80. The cantilever was then mounted in the middle of the breadboard and adjusted so that it blocked 50 percent of the beam and the laser's signal was 40.

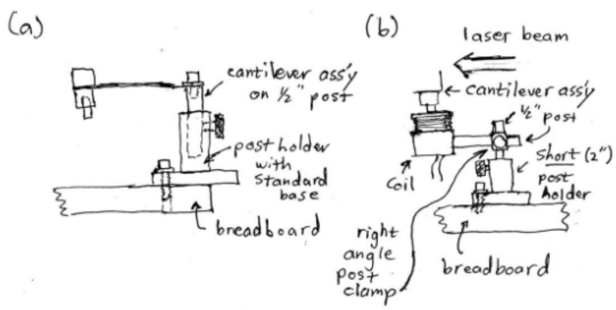

FIG. 3. Mounts for Cantilever

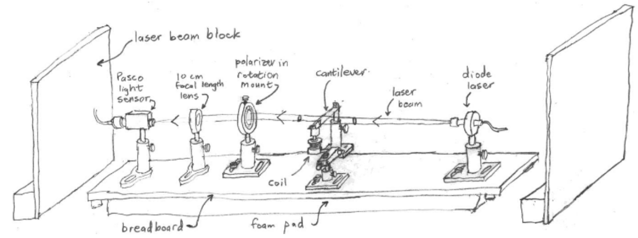

FIG. 4. Setup for Experiment

In order to measure the free decay of our damped oscillator, we manually displaced the cantilever and recorded the time dependent signal in the Pasco interface.

Finally, to measure the driven oscillations of a damped cantilever, we positioned the coil below the cantilever using two post holders so that the magnet was protruding into the bore as well as centered without touching the coil.

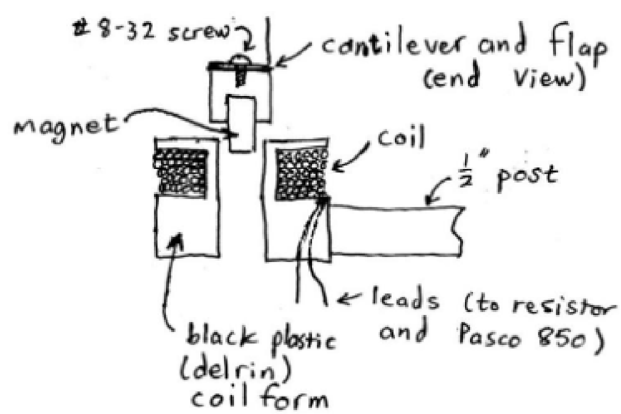

FIG. 5. Magnet and Coil

The coil was then connected to the Pasco 850 interfaces input 1 channel using a dual banana plug. We also connected 4.7 Ohm resistor in series with the coil to prevent overloading the Pasco analog output. The DIN connector of the coil assembly was connected to the Pasco 850's analog input B so we could measure the force applied to the coil which is proportional to the current through the coil.

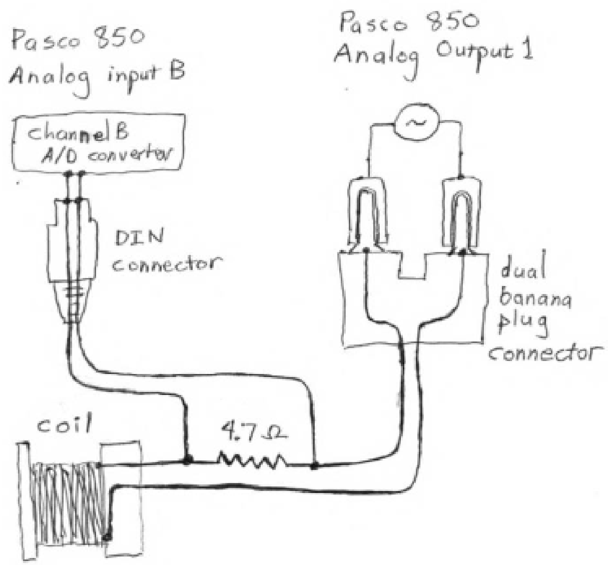

FIG. 6. Electrical Circuit and Pasco 850 Connection

IV. DATA

The cantilever's metal strip material was stainless steel with thickness 0.01 inches and a length of 3 inches. The sampling rate was approximately 10 measurements per cycle.

TABLE I. Free Decay of Cantilever

$\gamma/2$	ω [rad/s]
0.616 ± 0.004	101 ± 0.004
0.575 ± 0.003	101 ± 0.003
0.600 ± 0.004	100 ± 0.004
0.544 ± 0.003	101 ± 0.003
0.539 ± 0.005	101 ± 0.005
0.514 ± 0.004	101 ± 0.004
0.524 ± 0.005	101 ± 0.005
0.522 ± 0.004	101 ± 0.004

For the driven oscillations of a damped cantilever, we recorded a number of parameters for varying frequencies as listed in table II.

V. ANALYSIS

The parameters of resonant frequency ω_0, $\gamma/2$, the damping coefficient γ and the quality factor Q for the cantilever are listed in table 1. These were taken from the fit used when plotting the data in Capstone and further analysis using the equations introduced in the experimental section. Following the comments on error analysis in the manual, we used Mathematica to determine the error of these parameters.

TABLE II. Driven Oscillation of a Damped Cantilever

Frequency [Hz]	Force [N]	Displacement
2	0.837 ± 0.005	2.46 ± 0.001
5	0.840 ± 0.003	0.157 ± 0.003
8	0.839 ± 0.005	3.39 ± 0.003
15.5	0.829 ± 0.003	1.75 ± 0.004
15.7	0.830 ± 0.003	2.65 ± 0.001
15	0.832 ± 0.004	3.00 ± 0.004
16.2	0.786 ± 0.003	0.157 ± 0.003
16.5	0.829 ± 0.004	1.37 ± 0.001
16.7	0.831 ± 0.005	0.632 ± 0.001
16	0.817 ± 0.005	3.74 ± 0.003
32.5	0.808 ± 0.003	0.522 ± 0.002
49.6	0.761 ± 0.005	0.243 ± 0.001

TABLE III. Parameters of Free Decay of Cantilever

$\gamma/2$	γ	ω_0 [rad/s]	Q
0.55 ± 0.01	1.10 ± 0.02	100.9 ± 0.100	

TABLE IV. Parameters of Driven Oscillation of Cantilever

$\gamma/2$	γ	ω_0 [rad/s]	Q
0.55 ± 0.01	1.10 ± 0.02	100.9 ± 0.100	1

VI. CONCLUSION

The values measured could be made more accurate by recording more repetitions of the experiment.

Errors could have arisen due to the detection by the Pasco software which is affected by the temperature. [3] As stated in the experimental procedure, coupling can also occur between the cantilever and breadboard. The breadboard's oscillations can couple with the surroundings, including the building, although this effect would be minimal and has been dampened by the use of the foam pad. [3]

[1] R. Fitzpatrick, *Oscillations and Waves: An Introduction*, 1st ed. (CRC Press, 2017).

[2] G. C. King, *Vibrations and Waves*, 1st ed. (Wiley, 2009).

[3] P. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for Physical Sciences*, 3rd ed., Vol. 2 (McGraw-Hill Education, 2002).

[4] R. P. Feynman, Phys. Rev. **94**, 262 (1954).