Standing Waves

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I. INTRODUCTION

For our experiment exploring standing waves, we used sound waves to find nodes of an open ended tube as well as a half-open tube. Next, we observed radio frequency cavities in microwaves by microwaving marshmallows in a lattice configuration and observing the node and antinode pattern created.

II. THEORETICAL BACKGROUND

Standing waves can be written as the superposition of two travelling waves. The general equation for a standing wave is

$$d(x,t) = Asin(kx - \omega t), \tag{1}$$

where d is the displacement from equilibrium, A is the amplitude, k is the wave vector, and ω [rad/s] is the angular frequency. The superposition of two travelling waves that are identical except in their direction yields a wave with displacement

$$d(x,t) = 2Acos(\omega t)sin(kx). \tag{2}$$

This equation represents a standing wave with oscillations of amplitude $2Asin(kx)$ that varies with position, and $cos(\omega t)$ that is time variant.

A standing wave has points called nodes that are characterized by a displacement that is always zero. Similarly, points of maximum amplitude are referred to as antinodes. [1] These antinodes have a separation of half of the wavelength.

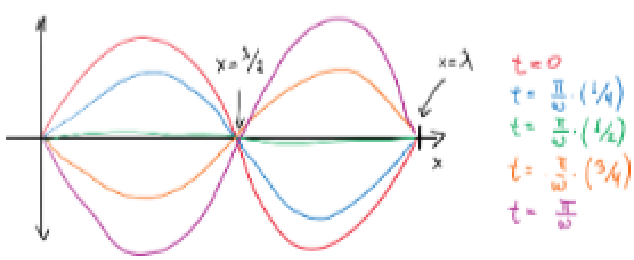

FIG. 1. Standing Waves

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A. Microwaves

The charges within the wall of a microwave oven orient to oppose the electric field parallel to the conducting walls that encase the inside of the microwave, mandating the component of the electric field parallel to the wall's surface to be zero. The transverse electromagnetic wave that is reflected from the walls is a standing wave with a displacement variable of the electric field E inside the microwave. This example of a Radio Frequency Cavity has a standing wave solution that depends on the position of the walls and antenna that is used to excite the wave.

The points in the microwave near the nodes do not experience a changing electric field and therefore do not heat up. We were able to observe these points and determine the wavelength of our microwave by microwaving a uniform setup so the nodes were easily discernible. This one-dimensional scalar interpretation of the wave is an oversimplified view of the true three-dimensional wave vector displacement, although we are still able to approximate the wavelength from the antinode separation.

B. Soundwaves

Sound waves are disturbances in the pressure of the air which is described by

$$P(x,t) = P_0 + p(x,t), \tag{3}$$

where $P_0 = 1$ atm is the pressure corresponding to zero disturbance in the atmosphere, and p is the displacement of the pressure.

As a travelling wave pulse of sound propagates through a pipe with speed v, the leading end of the pulse has a higher pressure than the ambient pressure, and the trailing end has a much lower pressure. The higher and lower pressure regions have corresponding higher and lower regions of density of gas molecules, which are produced by longitudinal displacements of the gas $s(x,t)$. The pressure and displacement are related by

$$p(x,t) = -B\frac{\partial s}{\partial x} \tag{4}$$

where B is the bulk modulus of the gas.

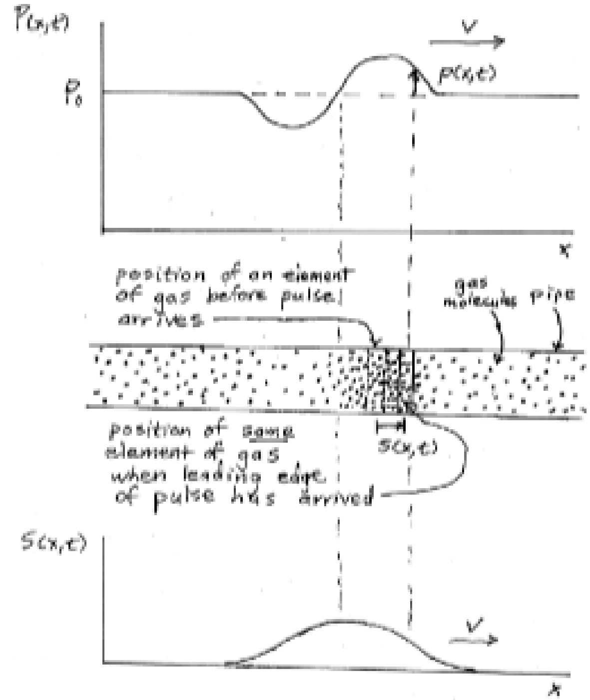

FIG. 2. Sound Wave Pulse in Pipe

If sound waves are excited in a tube of gas, standing waves of sound are produced due to the strong reflection of the sound waves from both ends of the tube. [2]

If a sound wave is excited in a tube with one end closed and the other opened, the closed end must correspond to a node, since the amplitude of oscillation must be zero. At the open end, the gas volume suddenly increases just outside the end of the tube. [1] This corresponds to a node in the pressure at a distance L_E past the tube end which can be approximated by

$$L_E = 0.3D, \tag{5}$$

where D is the inner diameter of the tube.

If one end of the tube is open, the effective length of the tube becomes

$$L_{EFF} = L + 0.3D, \tag{6}$$

where L is the length of the tube.

If both ends of the tube are open, the effective length becomes

$$L_{EFF} = L + 0.6D. \tag{7}$$

Since the wave is constrained by these boundary conditions, it can only vibrate at specific normal mode frequencies. For an open ended tube, the wavelength of the nth mode is given by

$$\lambda_n^o = \frac{2L_{EFF}}{n}, \tag{8}$$

where n = 1, 2, 3,...

The frequency of the nth mode of an open tube is

$$f_n^o = \frac{v_s}{\lambda_n^o}, \tag{9}$$

where v_s is the speed of sound which is 343 m/s in dry air.

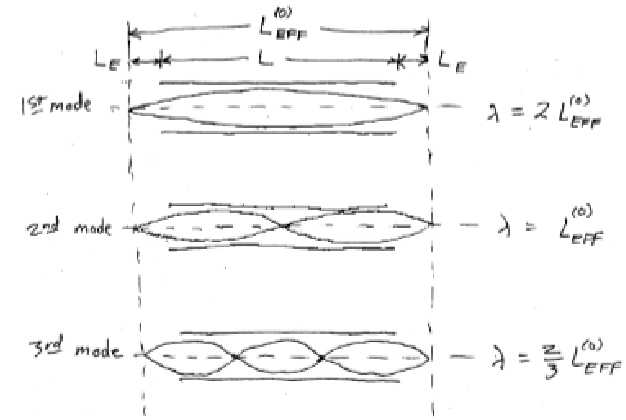

FIG. 3. Wavelength of modes for open ended tube

For a half-open ended tube, the wavelength of the nth mode is

$$\lambda_n^{ho} = \frac{4L_{EFF}}{2n-1}, \tag{10}$$

where n = 1, 2, 3,...

The frequency of the nth mode of an open tube is

$$f_n^o = \frac{v_s}{\lambda_n^{ho}}, \tag{11}$$

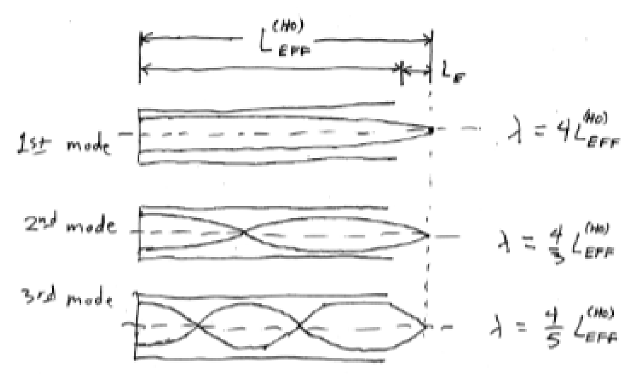

FIG. 4. Wavelength of modes for half-open ended tube

III. EXPERIMENTAL PROCEDURE

We removed the rotating plate from our microwave so the nodes and antinode points would stay fixed and easily observable. We then placed a piece of paper on a large plate and set up our marshmallows on the paper in a lattice pattern with spaces no larger than 8 mm. Next, we placed the plate in the center of the microwave and ran the microwave for about 1 to 2 seconds at a time, checking the progression of the melting of the marshmallows between runs. When the node pattern was visible and some "cold" spots were still remaining in the marshmallow lattice, we removed the plate from the microwave and measured the distance between the antinodes to determine the wavelength of the microwave.

For the next part of the experiment observing standing waves, we used an open ended 76.2 cm PVC tube and iOS frequency function generator app on our iPhone. The app produced a range of frequencies from specified starting and ending points for 10 second "sweeps." Beginning with a frequency range of 80 Hz to 1000 Hz, we placed the phone speaker centered at a distance $\approx D$, the inner diameter of the tube, from one end of the tube and kept our ear $\approx D$ at the other end to listen for the sharp increases in volume that corresponded to the nodes of the tube. After determining the number of nodes and approximate frequency values, we repeated this process using smaller ranges to determine the nodes with greater precision.

We then replicated this procedure after placing a cap tightly on the PVC tube and placing our ear and the speaker both centered at the open end at a distance $\approx D$ from the opening.

FIG. 5. Marshmallow Setup Before Microwaving

IV. DATA

Figure 2 depicts the marshmallow lattice after microwaving in a 1000 watt microwave for 12 seconds.

FIG. 6. Marshmallow Setup After Microwaving

The distances between the antinodes are given in Table I.

TABLE I. Distance Between Antinodes in Marshmallow Lattice

Antinode Pair	Antinode Distance [cm]
1	7.3025 ± 0.1588
2	5.3975 ± 0.1588
3	7.4613 ± 0.1588
4	6.6675 ± 0.1588
5	7.7788 ± 0.1588
6	7.9375 ± 0.1588
7	8.4138 ± 0.1588

Table II lists the frequency values recorded for the four nodes found within the range of 80 Hz to 1000 Hz for a 76.2 cm open ended PVC tube.

TABLE II. Node Frequency of Open Tube

Node 1 [Hz]	Node 2 [Hz]	Node 3 [Hz]	Node 4 [Hz]
220.80 ± 0.01	436.40 ± 0.01	657.40 ± 0.01	876.40 ± 0.01
219.44 ± 0.01	434.00 ± 0.01	657.20 ± 0.01	874.00 ± 0.01
221.12 ± 0.01	428.40 ± 0.01	655.00 ± 0.01	876.20 ± 0.01
227.00 ± 0.01	426.00 ± 0.01	653.00 ± 0.01	879.60 ± 0.01
226.16 ± 0.01	430.00 ± 0.01	657.60 ± 0.01	871.00 ± 0.01
215.24 ± 0.01	433.60 ± 0.01	656.60 ± 0.01	872.80 ± 0.01
226.00 ± 0.01	436.00 ± 0.01	657.00 ± 0.01	874.20 ± 0.01
220.00 ± 0.01	434.40 ± 0.01	657.80 ± 0.01	873.60 ± 0.01
221.20 ± 0.01	435.60 ± 0.01	654.20 ± 0.01	882.00 ± 0.01
223.60 ± 0.01	436.60 ± 0.01	657.40 ± 0.01	873.00 ± 0.01

Table III presents the frequencies of the four nodes found for the same tube but only half-open.

TABLE III. Node Frequency of Half-Open Tube

Node 1 [Hz]	Node 2 [Hz]	Node 3 [Hz]	Node 4 [Hz]
333.30 ± 0.01	533.28 ± 0.01	765.26 ± 0.01	980.40 ± 0.01
331.20 ± 0.01	533.44 ± 0.01	768.90 ± 0.01	981.60 ± 0.01
329.70 ± 0.01	531.04 ± 0.01	759.54 ± 0.01	978.60 ± 0.01
329.10 ± 0.01	531.84 ± 0.01	766.40 ± 0.01	976.71 ± 0.01
327.00 ± 0.01	534.88 ± 0.01	767.52 ± 0.01	983.80 ± 0.01
332.40 ± 0.01	535.36 ± 0.01	765.60 ± 0.01	981.20 ± 0.01
327.30 ± 0.01	530.88 ± 0.01	763.00 ± 0.01	980.10 ± 0.01
330.00 ± 0.01	532.80 ± 0.01	760.34 ± 0.01	982.92 ± 0.01
330.00 ± 0.01	533.76 ± 0.01	765.88 ± 0.01	979.60 ± 0.01
327.00 ± 0.01	530.24 ± 0.01	764.90 ± 0.01	984.45 ± 0.01

V. ANALYSIS

The average distance between the microwave antinodes was found to be 7.2798 ± 0.3754 cm which corresponds to a wavelength of 14.560 ± 0.7508 cm.

The average values and standard deviations for the modes of the open ended tube are listed in Table IV, and the modes of the half-open tube are listed in table V. The open ended tube was found to have an effective length of 78.581 ± 0.2541 cm following equation 7. The half-open tube had an effective length of 77.391 ± 0.2064 cm following equation 6. The values and standard deviations were found using Mathematica.

TABLE IV. Modes of Open Tube

Mode	Experimental [Hz]	Theoretical [Hz]
1	220.06 ± 1.1544	218.25
2	433.10 ± 1.1674	436.49
3	656.32 ± 0.5243	654.74
4	875.28 ± 1.0605	872.97

TABLE V. Modes of Half-Open Tube

Mode	Experimental [Hz]	Theoretical [Hz]
1	329.70 ± 0.6943	332.62
2	532.75 ± 0.5444	554.00
3	764.73 ± 0.9402	775.61
4	980.94 ± 0.7534	997.20

The plot of the mode number versus frequency for an open ended tube is shown in figure 7, and a half-open tube in figure 8.

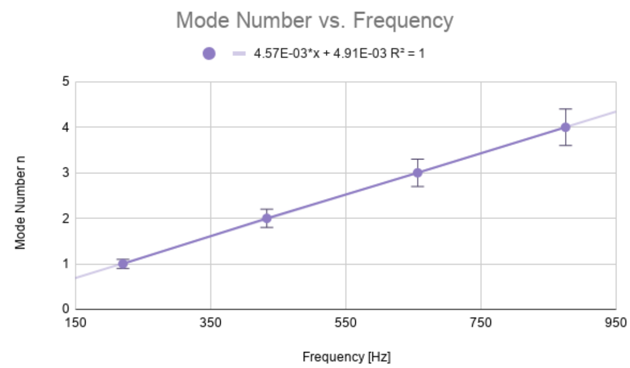

FIG. 7. Open Ended Tube

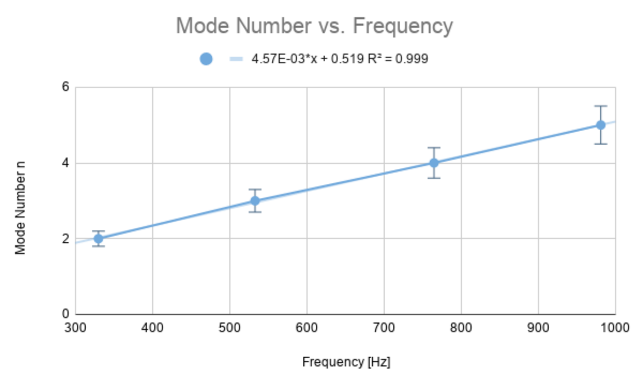

FIG. 8. Half-Open Tube

VI. CONCLUSION

Issues in this experiment could occur due to the large plate the marshmallows were placed on which was not completely flat. This could correspond to disruptions in the node pattern that would not be present otherwise. A large error in this part of the experiment is a result of each marshmallow not being uniform in size, shape, or consistency. This would result in uneven heating and a deformed node pattern. The marshmallow lattice was also not accurately space. This part of the experiment could be improved by using smaller marshmallows placed at precise distances, or using a more uniform material. Another issue in this part of our procedure was that the estimate of the wavelength was calculated using the distance between the antinodes. First, this is an oversimplification of the real three-dimensional wave which is a function of x, y, z, and t due to its dependence on the position of the microwave walls, as well as the type and position of the antenna which excites the mode. Second, the locations of these antinodes were determined by making the best estimate by eye.

Generally, more accurate values for the node frequencies of the waves could be found by performing more repetitions of this experiment. [3] The microwave wavelength could also be calculated more accurately by performing additional runs to take repeated measurements of the distance between the antinodes. [3] Length measurements were taken using a ruler with precision of a sixteenth of an inch, so more precise values could be taken using more profound instruments. Despite these issues, we were able to accomplish the goal of the experiment of observing the node pattern of a microwave and investigating standing wave phenomena.

Errors in the second portion of our experiment occurred due to estimating the nodes of the PVC pipe using the observed increase in volume as indication of a node frequency. This volume change would be different for each person who performs the experiment. The exact point of the volume increase was also not explicitly defined or recorded with accuracy each time, due to normal delay in stopping the sweep of the frequency function generator on the phone. This part of the experiment could be improved by using more sophisticated sonar software to record the point of this volume increase.

A large amount of error arose in this section of the experiment due to interference from sound waves in the environment that could not be eliminated or measured and corrected for. [3] A recommendation for improving this aspect of the experiment would be to perform future procedures in a soundproof environment.

Other energy was lost in the system from air resistance, although this effect was minimal. Energy was also lost by the system to the environment in the form of heat energy. [3] Noise occurred from air flow although this effect was negligible. [3]

[1] R. Fitzpatrick, *Oscillations and Waves: An Introduction*, 1st ed. (CRC Press, 2017).

[2] G. C. King, *Vibrations and Waves*, 1st ed. (Wiley, 2009).

[3] P. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for Physical Sciences*, 3rd ed., Vol. 2 (McGraw-Hill Education, 2002).

[4] R. P. Feynman, Phys. Rev. **94**, 262 (1954).