Interference and Diffraction

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I. INTRODUCTION

A. Theoretical Background

Waves described by linear differential equations obey the superposition principle which means that a wave generated by two sources is the combination of the waves generated by each source individually. [1] The maximum ("crests") and minimum ("troughs") displacements can align resulting in twice the displacement of if the wave was generated by just one source and is referred to as constructive interference. Total destructive interference occurs when the maxima of one wave align with the minima of the other wave, yielding displacements at those points of zero. [1]

The intensity of this combined wave will be proportional to the squared time average (four times) of the intensity of the wave from an individual source. This is observable in the well known Young's double slit experiment where light is directed through a screen. [1] The resulting fringes are characterized by

$$X = L\frac{\lambda}{d}, \tag{1}$$

where X is the distance between the bright fringes, L is the distance between the resulting light and the screen, d is the slit width, and λ is the wavelength of the light source. [1]

B. Huygens's Principle

Huygens's principle states that the points on a wavefront act as sources of secondary waves which are identical to the waves generated by a point source, providing a way to understand wave propagation. [1]

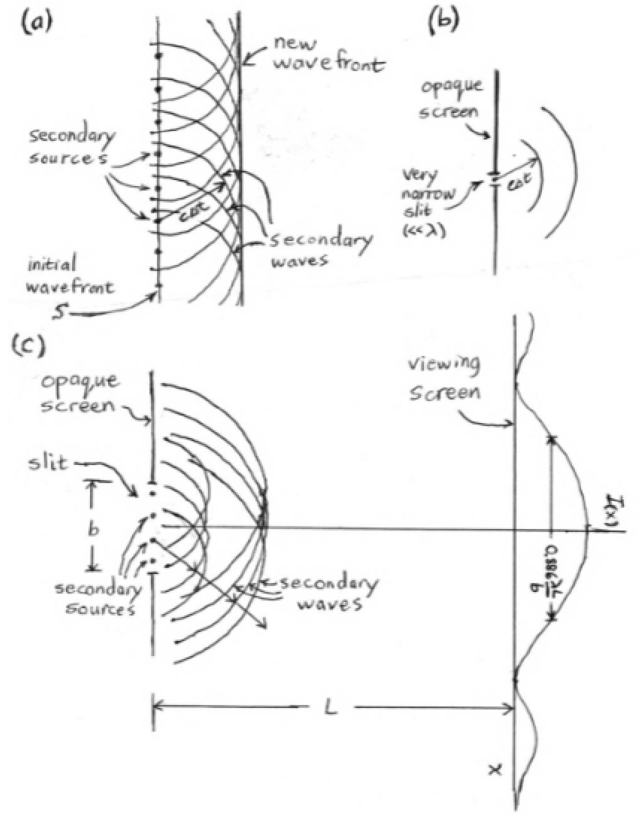

FIG. 1. (a) A plane wave viewed according to Huygens's principle (b) Application of Huygens's principle to light through a slit that is small in comparison to its wavelength (c) Light through a slit of finite width b according to Huygens's principle [1]

This principle can be applied to determine the limiting behavior of the propagation of light through a slit of narrow width in comparison to the wavelength. The resulting wave is an outgoing circular wave, as if from a single secondary source. [1] Single slit diffraction occurs when the slit width is not small compared to the light's wavelength λ. The resulting intensity of the light projected through the slit and onto a screen a distance L from the slit is

$$I(x) = I_0\frac{sin^2(\frac{b\pi}{\lambda L}x)}{(\frac{b\pi}{\lambda L}x)^2}. \tag{2}$$

Multiple slit interference yields a resulting wave with intensity

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$$I(\theta) = I_0 f(\theta) \frac{sin^2(\frac{\pi N d sin\theta}{\lambda})}{sin^2(\frac{\pi d sin\theta}{\lambda})}, \quad (3)$$

where $I_0 f(\theta)$ is the intensity from one slit and θ is the angle between the beam of light on the screen and the incident ray.

The solutions of Maxwell's equations consist of waves of vibrating electric and magnetic fields. The most general solution is given by

$$\overrightarrow{E} = E_{01}\hat{x}cos(\omega t - kz + \phi_{01}) + E_{02}\hat{y}cos(\omega t - kz + \phi_{02}) \quad (4)$$

which propagates in the $\hat{z}$ direction with frequency ω.

The polarization of this electromagnetic wave refers to the vibration of $\overrightarrow{E}$ in the $x-y$ plane, and linear polarization occurs when $\phi_{01} = \phi_{02} = \phi_0$. Then

$$\overrightarrow{E} = (E_0\hat{e})cos(\omega t - kz + \phi_0), \quad (5)$$

where $\hat{e}$ is the polarization vector and points in the direction of vibration of the electric field. [2]

Polarizers produce linearly polarized radiation. [2] A wire-grid polarizer drives an oscillating current through the component of the electromagnetic field that is parallel to the metal wires of the wire-grid polarizer. [2] The polarizer is constrained by its wire spacing, which must be small compared to the wavelength of the incident radiation.

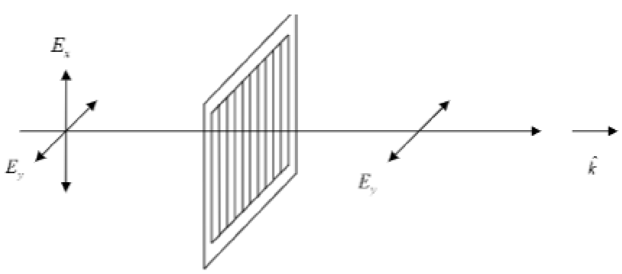

FIG. 2. Wire grid polarizer [1]

C. Malus' Law

When an electromagnetic wave propagates through two polarizers in sequence, the fraction of intensity transmitted through the second polarizer is given by

$$I_t = I_i cos^2(\theta) \quad (6)$$

known as Malus' law. [2] Therefore the intensity of a beam can be attenuated by adjusting the angle of the two polarizers. [2]

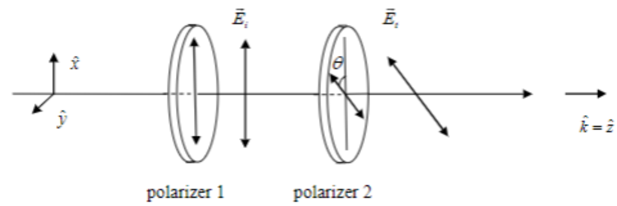

FIG. 3. Malus' law [1]

D. Photodiodes

When two types of semiconductors, p-type and n-type, are placed in junction with one another, current will flow in one direction but not the opposite.

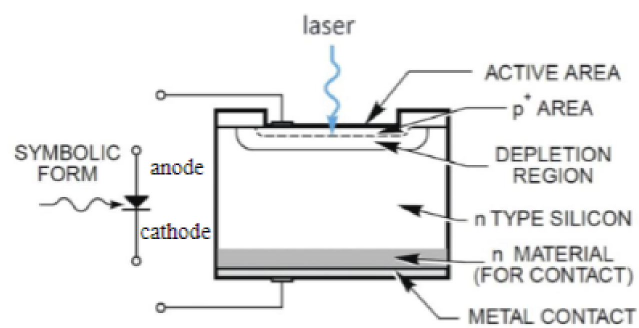

FIG. 4. Silicon photodiode [1]

II. EXPERIMENTAL PROCEDURE

A. Interference Patterns Using Cornell Slit-film Demonstrator

We used a Cornell slit-film interference and diffraction demonstrator and cell-phone flashlight beam directed through various slit patterns to observe characteristics of the diffraction patterns such as coloration, weak and strong fringes, and other trends.

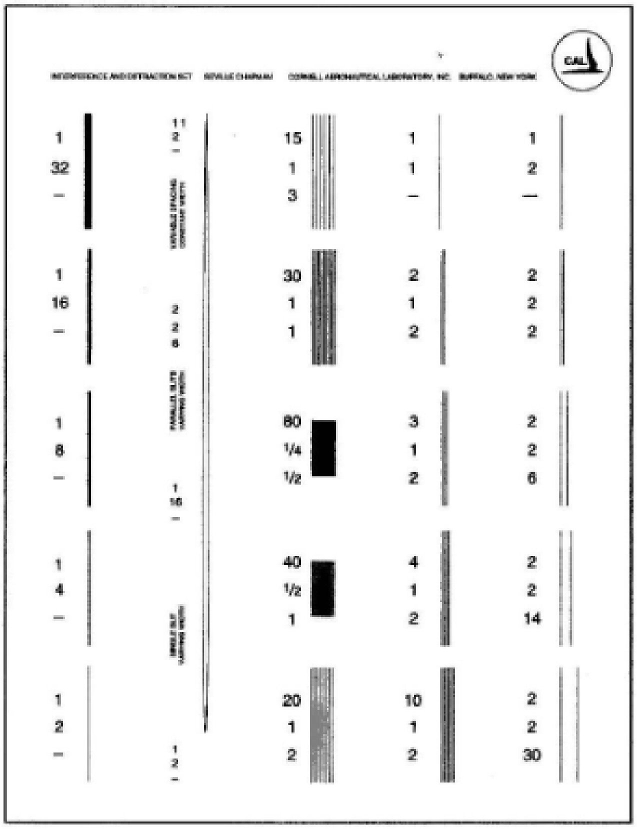

FIG. 5. Silicon photodiode [1]

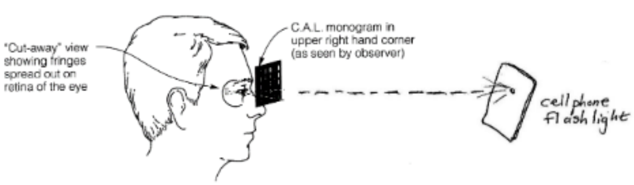

FIG. 6. Silicon photodiode [1]

Next we observed the flashlight beam through the single slit in the upper right corner of our slit-film to investigate single slit diffraction patterns.

Then we observed the two-slit interference fringes produced by the beam through two double slit patterns in the second row of the rightmost column. We observed how the produced pattern changed as we proceeded to higher slit spacing.

We then observed the variations in the diffraction pattern when viewing the beam through the variable double slit on the second column from the left.

Finally, we viewed the beam through the multiple slit patterns and noted the interference peaks and features of the pattern.

B. Laser Wavelength

To determine the wavelength of our laser, we began by creating a mount by cutting holes in pieces of foam board and placing them in slots in wooden blocks. We then created a similar holder using foam board and slotted wooden blocks to hold our slit-film in such a way that we could easily adjust which slit we directed our laser beam through. We then aligned the laser with our desired slit pattern, and created a "screen" using more foam board and wooden blocks in order to view our resulting diffraction pattern.

III. DATA AND ANALYSIS

A. Voltmeter

Table I lists the voltages recorded for each pair of resistors.

TABLE I. Potential Difference Between Resistor Pairs

R_1, R_2 ± 1%	Voltage [V] ± 0.01V
1.00 $k\Omega$, 523 Ω	3.12
2.00 $k\Omega$, 523 Ω	4.62
2.00 $k\Omega$, 1.00 $k\Omega$	4.50

B. Photodiode

We found that the photodiode did act as ordinary diode when no light was incident upon it. We also noted that the diode conducted current in forward bias but not reverse bias. We did measure that the voltage across the resistor was less than the battery voltage for forward bias. This could potentially be due to our incident beam being larger than the 1.245 millimeter by 1.245 millimeter collecting area of the photodiode.

C. Polarization

When rotating the polarizer sheet, we noted that the measured intensity varied, therefore we discovered that our laser was linearly polarized.

D. Malus' Law

A graph of our recorded power as a function of θ for two polarizers in series fit to a $cos^2(\theta)$ function is shown in figure 7. We confirm that within our bounds of uncertainty, Malus' law holds.

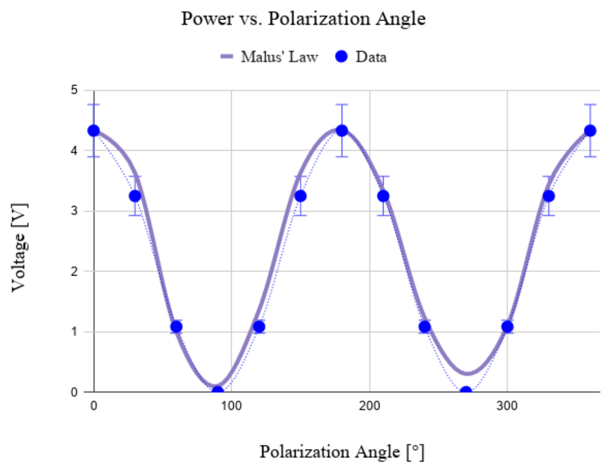

FIG. 7. Power of laser beam incident upon photodiode through two polarizers as a function of polarization angle

E. Three Polarizers

We found that by rotating the third polarizer 360°, the intensity decreased from 0° to 90°, increased from 91° to 180°, decreased from 181° to 270°, and again increased from 271° to 360°.

Our measured angle of maximum transmission was 180°± 0.02°, and minimum was 270°± 0.02°. In order to understand how light can pass through when the third polarizer is added even though it was previously blocked by the two polarizers, the quantum-mechanical view of the system must be adopted. The photons that previously passed through the two polarizers do not retain any information from past measurements. [3]

F. Brewster's Angle

We measured Brewster's angle to be 55°±0.02° and the polarization of the beam for minimum reflection to be 90°±0.02°. Our value of Brewster's angle does not agree for the theoretical value for the plexiglass block within our bounds of uncertainty. Our value was within 1.78%± 0.04% of the theoretical value of Brewster's angle.

When varying the polarization but leaving the plexiglass block at Brewster's angle, we noted that the reflected beam increased and decreased in intensity.

If the beam of light incident upon the plexiglass block at Brewster's angle was unpolarized, the polarization of the reflected light would be partially polarized.

IV. CONCLUSION

A systematic error in this experiment is that we neglected the index of refraction of air. We also did not make our measurements within a vacuum. [2] [4]

The values could be determined more accurately by repeating the measurements. [4] A systemic source of error was that the polarization and incidence angles could not be measured directly since we did not have a protractor and instead had to be calculated from length measurements which could only be measured with a precision of 1 millimeter. The data could also be made more precise by using a voltmeter with more values known than the 10 millivolt limited voltmeter used. Another issue in this experiment is that the voltmeter was constrained to values less than 10.00 Volts. [2]

To improve this experiment, more advanced equipment could be used such as a digital multimeter with various functions as opposed to the three wire voltmeter employed.

Issues in this experiment could have arisen due to measurement error. [4] The deviation values were also not decided precisely. [4]

[1] D. Heinzen, Interference and diffraction (v.2), (2020).
[2] I. Beskin, Lenses, (2020).
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[7] D. Heinzen, Photodiode and light beam polarization, (2020).
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