Vibrations of Springs

Tiffany Huff*
Physics Department, University of Texas at Austin

(Dated: November 17, 2022)

I. SPRING:

A. Introduction

Springs are typically used as a first example for simple harmonic motion since they easily depict wave phenomena. [1] The first part of this experiment was done using a spring to excite the first, second, and third harmonics. The data was recorded using a meter stick and a timer. The transverse modes were produced by holding the two ends of the spring and using arm movements to create a vibration. For the longitudinal vibration modes, the spring coils were compressed and then released to oscillate freely.

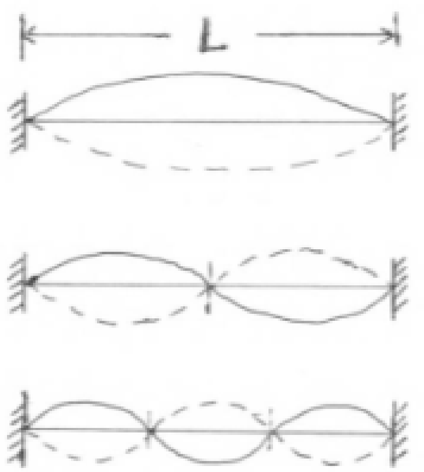

FIG. 1. Spring

FIG. 2. Spring

B. Data

Table 1 presents the data for the first three transverse modes found for the 9ft±1ft (3m±0.3m) spring.

TABLE I. Frequency of Spring's Transverse Modes

Mode	Nodes	ν [Hz]
1	0	2 ± 0.3
2	1	1 ± 0.3
3	2	0.5 ± 0.3

C. Analysis

Table 3 displays the wavelengths of the first three transverse modes for spring one which were calculated using

$$\lambda = 2L/N, \tag{1}$$

where N is the harmonic number of the mode and L is the length of the spring. The product of each mode's frequency and wavelength is also listed.

TABLE II. Wavelength of Spring's Transverse Modes

Mode	λ [m]	$\lambda \cdot \nu$ [m/s]
1	5.5 ± 0.6	11 ± 0.2
2	2.7 ± 0.3	2.7 ± 0.1
3	1.8 ± 0.2	0.9 ± 0.1

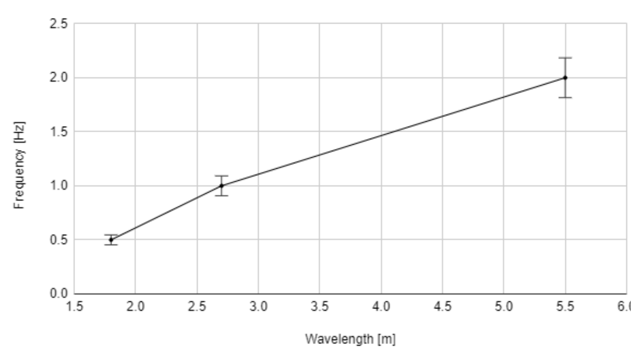

FIG. 3. Transverse mode Frequency [Hz] vs. Wavelength [m]

* tiffanynicolehuff@utexas.edu

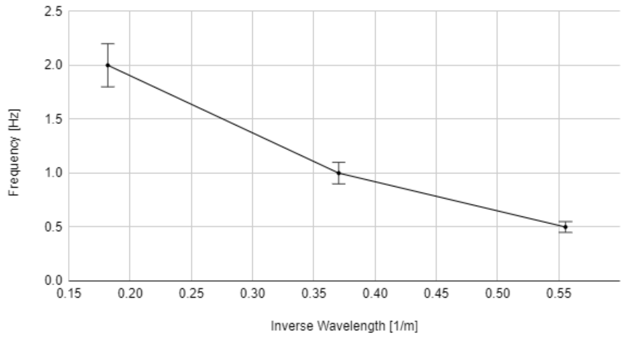

FIG. 4. Transverse Mode Frequency [Hz] vs. Inverse Wavelength [m^{-1}]

D. Conclusion

Using a spring, we were able to excite the first, second, and third fundamental transverse modes. We used the measured frequencies of those modes to determine the wavelengths. We found a nearly linear relationship between transverse mode frequency and wavelength. Repetitions of this experiment would yield a more precise calculation of these values. Future reproductions of this experiment would benefit from using a laser or similar method to more accurately measure the frequency. Error could have arisen from undesired motion such as rotational, translational, or longitudinal. [2]

II. SONOMETER

A. Introduction

For the second section of this experiment, the primary instrument used was a sonometer along with a data acquisition system on a computer. Sonometers are typically used to measure tension, frequency, or density of vibrations. We also used a movable bridge for the sonometer, along with a weight hanger and slotted weights. Data was taken using a Pasco sound sensor and cable.

Our sonometer consisted of one string with one end held with a fixed tension using slotted weights and one end held by the movable bridge. The length of string between these points was plucked, producing vertical vibrations through the fixed bridge and to the sound box. This setup is depicted in figure 3.

The software uses a fast Fourier transform (FFT) algorithm to calculate and plot discrete Fourier transforms. This data then represents the amount of oscillations that occur at a specific frequency.

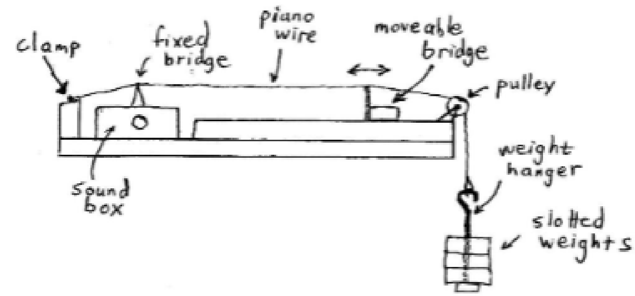

FIG. 5. Sonometer

B. Data

We recorded the frequencies of the fundamental modes first with a constant 50 centimeter length and varying tension, and then with a constant tension from a 5 kilogram mass and varying length.

TABLE III. Fundamental Modes of Constant Length Sonometer

Tension [N]	Frequency [Hz]
19.6	185.862 ± 0.6
29.4	221.163 ± 0.3
39.2	258.789 ± 0.2
49	278.32 ± 0.2
58.8	306.286 ± 0.2
68.6	181.996 ± 0.2

TABLE IV. Fundamental Modes of Constant Tension Sonometer

Length [cm]	Frequency [Hz]
65	201.546 ± 0.6
60	215.971 ± 0.3
50	257.287 ± 0.2
40	324.895 ± 0.2

C. Analysis

When plucking the sonometer's spring in different ways such as using a fingernail or fingertip, the waveforms and frequencies produced remained the same, while the strength of the harmonic increased when using a fingernail.

The fit curve found for the relationship between length and frequency was

$$L[cm] = 13896x^{-1.01}, \tag{2}$$

with an R^2 value of 0.999.

The relationship between tension and frequency determined from the data

$$Frequency[Hz] = -53.1 + 14.5x - 0.156x^2 \qquad (3)$$

with an R^2 value of 0.721.

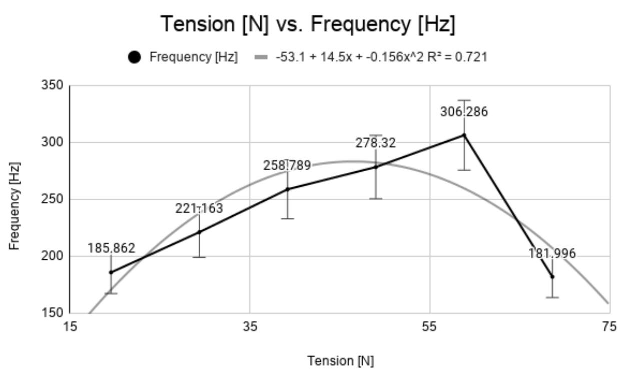

FIG. 6. Sonometer

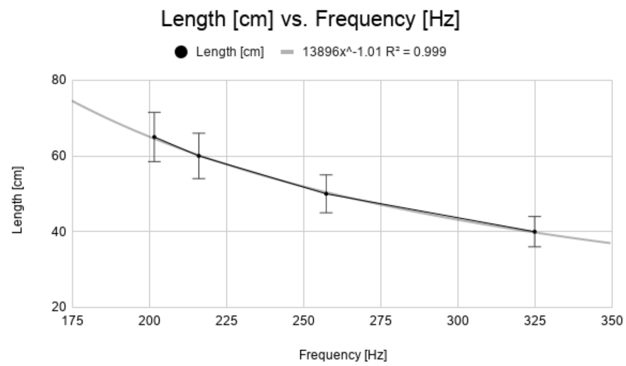

FIG. 7. Sonometer

D. Conclusion

Using a sonometer, Pasco sound sensor, weights, fixed bridge, and movable bridge, we recorded measurements of Fourier transforms as a function of frequency via Capstone software to analyze and define the dependence of frequency on tension as well as length. We found an agreement with the theoretical values. Future replications of this experiment should include repetitions of these processes to determine a more precise functional relationship between these parameters. Collecting frequency data for a greater number of tensions and effective lengths would also yield a better estimate of the relationship. [2]

Errors could have arisen in this experiment due to coupling of the setup with the ground. The Pasco sound sensor uses a quartz to calibrate its measurements, so systematic error could have occurred due to the temperature when the data was taken. [2]

[1] R. Fitzpatrick, *Oscillations and Waves: An Introduction*, 1st ed. (CRC Press, 2017).
[2] P. Bevington and D. K. Robinson, *Data Reduction and Error Analysis for Physical Sciences*, 3rd ed., Vol. 2 (McGraw-Hill Education, 2002).
[3] R. P. Feynman, Phys. Rev. **94**, 262 (1954).
[4] G. C. King, *Vibrations and Waves*, 1st ed. (Wiley, 2009).