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Question 1

A. Convert the following numbers to their decimal representation

1. 10011011_2
    a. $1*2^0 + 1*2^1+0*2^2+1*2^3+1*2^4+0*2^5+0*2^6+1*2^7$
    b. 1 + 2 + 0 +8 +16 + 0 + 0 +128
    c. 155
2. 456_7
    a. $6*7^0 + 5*7^1 + 4*7^2$
    b. 6 + 35 + 196
    c. 237
3. $38A_{16}$
    a. $10*16^0+ 8*16^1 +3*16^2$
    b. 10 + 128 + 768
    c. 906
4. 2214_5
    a. $4*5^0+1*5^1+2*5^2+2*5^3$
    b. 4 + 5 + 50 + 250
    c. 309

B. Convert the following numbers to their binary representation

1. 69_{10}	2. 485_{10}	3. $6D1A_{16}$
69/2 R 1 34/2 R 0 17/2 R 1 8/2 R 0 4/2 R 0 2/2 R 0 1/2 R 1 $(1000101)_2$	485/2 R 1 242/2 R 0 121/2 R 1 60/2 R 0 30/2 R 0 15/2 R 1 7/2 R 1 3/2 R 1 1/2 R 1 $(111100101)_2$	6- 0110 D- 1101 1- 0001 A- 1010 $(0110110100011010)_2$

C. Convert the following to their hexadecimal representation

1. 1101011_2
    a. 0110 1011
    b. 6 b
    c. $(6B)_{16}$
2. 895_{10}
    a. 895/16 R 15- F

b. 55/16 R 7
c. 3/16 R 3
d. $(37F)_{16}$

Question 2

1. $7566_8 + 4515_8$ $\begin{array}{r} 7\,5\,6\,6 \\ +\ \ 4\,5\,1\,5 \\ \underline{1\ \,1\,1\ \ \ } \\ 1\,4\,3\,0\,3 \end{array}$ $(14303)_8$	2. $10110011_2 + 1101_2$ $\begin{array}{r} 1\,0\,1\,1\,0\,0\,1\,1 \\ +\qquad 1\,1\,0\,1 \\ \underline{1\,1\,1\,1\,1\,1\ \ } \\ 1\,1\,0\,0\,0\,0\,0\,0 \end{array}$ $(11000000)_2$
3. $7A66_{16} + 45C6_{16}$ $\begin{array}{r} 7\,A\,6\,6 \\ +\qquad 4\,5\,C\,6 \\ \underline{1\,1\ \ \ } \\ C\,0\,2\,C \end{array}$ $(C02C)_{16}$	4. $3022_5 - 2433_5$ $\begin{array}{r} 11\ \ \\ 2\,9\ \ \cancel{1}\,12 \\ \cancel{3}\,\cancel{0}\,\cancel{2}\,\cancel{2} \\ -\quad 2\,4\,3\,3 \\ \hline 0\,0\,3\,4 \end{array}$ $(34)_5$

Question 3

A. Convert the numbers to their 8-bit 2's complement representation

1. 124_{10} 124/2 R 0 62/2 R 0 31/2 R 1 15/2 R 1 7/2 R 1 3/2 R 1 1/2 R 1 01111100	2. -124_{10} 124: 1 1 1 1 1 0 0 0 0 0 0 1 0 0 10000100	3. 109_{10} 109/2 R 1 54/2 R 0 27/2 R 1 13/2 R 1 6/2 R 0 3/2 R 1 1/2 R 1 01101101	4. -79_{10} 79/2 R 1 39/2 R 1 19/2 R 1 9/2 R 1 4/2 R 0 2/2 R 0 1/2 R 1 79: 0 1 0 0 1 1 1 1 1 0 1 1 0 0 0 1 10110001

B. Convert to decimal representation

1. 00011110 $0*2^0+1*2^1+1*2^2+1*2^3+1*2^4$ 0 + 2 + 4 + 8 + 16 30	2. 11100110 1 1 1 0 0 1 1 0 0 0 0 1 1 0 1 0 (11010) will be the negative of this value $0*2^0+ 1*2^1+0*2^2+1*2^3+1*2^4$ 0 + 2 + 0 + 8 + 16 26 -26
3. 00101101 $1*2^0+0*2^1+1*2^2+1*2^3+0*2^4+1*2^5$ 1 + 0 + 4 + 8 + 0 + 32 45	4. 10011110 1 0 0 1 1 1 1 0 0 1 1 0 0 0 1 0 (1100010) will be negative of this value $0*2^0+1*2^1+0*2^2+0*2^3+0*2^4+1*2^5+1*2^6$ 0 + 2 + 0 + 0 + 0 + 32 + 64 98 -98

Question 4

1. Exercise 1.2.4, sections b, c
   B. Write the truth table for
   $\neg(p \lor q)$

p	q	$(p \lor q)$	$\neg(p \lor q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

   C. write the truth table for
   $r \lor (p \land \neg q)$

p	q	r	$\neg q$	$(p \land \neg q)$	$r \lor (p \land \neg q)$
T	T	T	F	F	T
T	T	F	F	F	F
T	F	T	T	T	T
T	F	F	T	T	T
F	T	T	F	F	T
F	T	F	F	F	F
F	F	T	T	F	T
F	F	F	T	F	F

2. Exercise 1.3.4 sections b, d
   B. Give a truth table for
   $(p \to q) \to (q \to p)$

p	q	$(p \to q)$	$(q \to p)$	$(p \to q) \to (q \to p)$
T	T	T	T	T
T	F	F	T	T
F	T	T	F	F
F	F	T	T	T

D. Give a truth table for

$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$

p	q	¬q	$(p \leftrightarrow q)$	$(p \leftrightarrow \neg q)$	$(p \leftrightarrow q) \oplus (p \leftrightarrow \neg q)$
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	F	T	T
F	F	T	T	F	T

Question 5

1. Exercise 1.2.7 sections b, c

B: applicant presents a birth certificate
D: applicant presents a drivers license
M: applicant presents a marriage license

B. Write a logical expression for: the applicant must present at least two of the following forms of identification: birth certificate, driver's license, marriage license

(B or D or M) and (B or D or M)- allows for duplicates,
((B) and (D or M)) exclusive or ((D) and (B or M)) exclusive or ((M) and (B or D))
$(B \land (D \lor M)) \oplus (D \land (B \lor M)) \oplus (M \land (B \lor D))$

C. Write a logical expression for: Applicant must present either a birth certificate or both a driver's license and a marriage license

(B) or (D and M)
$B \lor (D \land M)$

2. Exercise 1.3.7 sections b, c, d, e

S: a person is a senior
Y: a person is at least 17 years old
P: a person is allowed to park in the school parking lot

B. Write a logical expression for: A person can park in the school parking lot if they are a senior or at least seventeen years old

If (S or Y) then (P)
$(S \lor Y) \to P$

C. Write a logical expression for: Being 17 years of age is a necessary condition for being able to park in the school parking lot

If allowed to park, then you're 17
$P \to Y$

D. Write a logical expression for: A person can park in the school parking lot if an only if the person is a senior and at least 17 years old

If and only if allowed to park then you must be a senior and 17 yr old
$P \leftrightarrow (S \land Y)$

E. Write a logical expression for: Being able to park in the school parking lot implies that the person is either a senior or at least 17 years old

If P, then (S or Y)
$P \to (S \lor Y)$

3. Exercise 1.3.9 sections c, d

Y: applicant is at least 18 years old
P: applicant has parental permission
C: the applicant can enroll in the course

C. Write a logical expression for: The applicant can enroll in the course only if the applicant has parental permission.

If (C) then (CP

$C \rightarrow P$

D. Write a logical expression for: Having parental permission is a necessary condition for enrolling in the course

If (C) then (P)

$C \rightarrow P$

Question 6

1. Exercise 1.3.6 sections b, c, d
   Give an english sentence in the the form “If..then..” that is equivalent to each sentence.

   B. Maintaining a B average is necessary for Joe to be eligible for the honors program.

If Joe is eligible for the honors program, then he has a B average.

   C. Rajiv can go on the roller coaster only if he is at least 4 feet tall.

If Rajive can go on the roller coaster, then he is at least 4 feet tall.

   D. Rajiv can go on the roller coaster if he is at least four feet tall.

If Rajiv is four feet tall, then he can go on the roller coaster.

2. Exercise 1.3.10 sections c, d, e, f

p: True
q: False
r: ??

C. $(p \lor r) \leftrightarrow (q \land r)$
(T or ?) ↔ [(F and ?)- cannot be true, as both q and r must be true for T]
(T) ↔ (F) can only be true if both is true
(F)
False

D. $(p \land r) \leftrightarrow (q \land r)$
[(T and ?) if r is T, then truth, if r is F, then false] ↔ [(F and ?) must be false, see above]
(?) ↔ (F), if r is F, then the expression is true. If r is T, then the expression is false
(?)
Unknown

E. $p \rightarrow (r \lor q)$
(T) → [(? or F) if r is T, then expression is T. If r is F then expression is F]
(T) → (?), if r is T, then expression is T, if r is F then expression is F
(?)
Unknown

F. $(p \land q) \rightarrow r$
(T and F) → (?)
(F) → [(?), if r is T, then true, if r is F, then also true.]
(T) wil be true regardless if r is T or F
True

Question 7

Exercise 1.4.5 sections b, c, d

j: Sally got the job
l: Sally was late for her interview
r: Sally updated her resume

B. If Sally did not get the job, then she was late for her interview or did not update her resume.
If Sally updated her resume and was not late for her interview, then she got the job.

If (not j), then (l or not r) $(\neg j) \rightarrow (l \vee \neg r)$	If (r and not l), then (j) $(r \wedge \neg l) \rightarrow (j)$

j	l	r	¬j	¬r	$(l \vee \neg r)$	$(\neg j) \rightarrow (l \vee \neg r)$	¬l	$(r \wedge \neg l)$	$(r \wedge \neg l) \rightarrow (j)$
T	T	T	F	F	T	T	F	F	T
T	T	F	F	T	T	T	F	F	T
T	F	T	F	F	F	T	T	T	T
T	F	F	F	T	T	T	T	F	T
F	T	T	T	F	T	T	F	F	T
F	T	F	T	T	T	T	F	F	T
F	F	T	T	F	F	F	T	T	F
F	F	F	T	T	T	T	T	F	T

The two statements are logically equivalent.

C. If Sally got the job then she was not late for her interview.
If Sally did not get the job, then she was late for her interview.

If (j), then (not l) $j \rightarrow (\neg l)$	If (not j), then (l) $(\neg j) \rightarrow (l)$

j	l	$(\neg l)$	$j \rightarrow (\neg l)$	$(\neg j)$	$(\neg j) \rightarrow (l)$
T	T	F	F	F	T
T	F	T	T	F	T
F	T	F	T	T	T

F	F	T	T	T	F

The two statements are not logically equivalent

D. If Sally updated her resume or she was not late for her interview, then she got the job.
If Sally got the job, then she updated her resume and was not late for her interview.

If (r or not l), then (j) $(r \lor \neg l) \rightarrow (j)$	If (j), then (r and not l) $(j) \rightarrow (r \land \neg l)$

r	l	j	¬l	$(r \lor \neg l)$	$(r \lor \neg l) \rightarrow (j)$	¬l	$(r \land \neg l)$	$(j) \rightarrow (r \land \neg l)$
T	T	T	F	T	T	F	F	F
T	T	F	F	T	F	F	F	T
T	F	T	T	T	T	T	T	T
T	F	F	T	T	F	T	T	T
F	T	T	F	F	T	F	F	F
F	T	F	F	F	T	F	F	T
F	F	T	T	T	T	T	F	F
F	F	F	T	T	F	T	F	T

The two statements are not logically equivalent.

Question 8

1. Exercise 1.5.2 sections c, f, i
   Prove equivalence
   C. $(p \to q) \land (p \to r) \equiv p \to (q \land r)$

$(p \to q) \land (p \to r)$ Conditional identities: $(p \to q) \equiv \neg p \lor q$ $(p \to r) \equiv \neg p \lor r$ $(\neg p \lor q) \land (\neg p \lor r)$ Distributive law: $(\neg p \lor q) \land (\neg p \lor r) \equiv \neg p \lor (q \land r)$ Conditional identities $\neg p \lor (q \land r) \equiv p \to (q \land r)$	$p \to (q \land r)$

F. $\neg(p \lor (\neg p \land q)) \equiv \neg p \land \neg q$

$\neg(p \lor (\neg p \land q))$ De Morgan's Law: $\neg p \land \neg(\neg p \land q)$ De Morgan's Law $\neg p \land (\neg\neg p \lor \neg q)$ Double Negation Law: $\neg p \land (p \lor \neg q)$ Distributive Law $(\neg p \land p) \lor (\neg p \land \neg q)$ Complement Law $(\neg p \land p) \equiv F$ $F \lor (\neg p \land \neg q)$ Identity Law $F \lor (\neg p \land \neg q) \equiv (\neg p \land \neg q)$	$\neg p \land \neg q$

I. $(p \land q) \to r \equiv (p \land \neg r) \to \neg q$

$(p \land q) \to r$ Conditional : $\neg(p \land q) \lor r$ De Morgan's Law: $(\neg p \lor \neg q) \lor r$ Associative Law:	$(p \land \neg r) \to \neg q$

¬p ∨ (¬q ∨ r) Communicative: ¬p ∨ (¬q ∨ r) (¬q ∨ r)≡ (r ∨¬q) ¬p ∨ (r ∨¬q) Associative Law: (¬p ∨ r) ∨ ¬q Conditional ¬(¬p ∨ r) → ¬q De Morgan's Law: ¬(¬p ∨ r) ≡ (p ∧ ¬r) (p ∧ ¬r) → ¬q	

2. Exercise 1.5.3 sections c, d
   Use the laws of propositional logic to prove that each statement is a tautology (if a compound proposition is always true, regardless of the truth value of the individual propositions)
   C. ¬r ∨ (¬r → p)

r	p	¬r	(¬r → p)	¬r ∨ (¬r → p)
T	T	F	T	T
T	F	F	T	T
F	T	T	T	T
T	F	T	F	T

D. ¬(p → q) → ¬q

p	q	¬q	(p → q)	¬(p → q)	¬(p → q) → ¬q
T	T	F	T	F	T
T	F	T	F	T	T
F	T	F	T	F	T
F	F	T	T	F	T

Question 9

1. Exercise 1.6.3 sections c, d
   Write a logical expression with the same meaning as the statements. The domain is the set of all real numbers.
   C. There is a number that is equal to its square

∃x- there is a value for x, where $x = x^2$

$\exists x(x=x^2)$

   D. Every number is less than or equal to its square plus 1.

∀x- every value of x, $\leq x^2 + 1$

$\forall x(x \leq x^2 + 1)$

2. Exercise 1.7.4 sections b, c, d
   The domain is a set of employees who work at a company. Ingrid is one of the employees at the company. Define the following predicates:

S(x): x was sick yesterday

W(x): x went to work yesterday

V(x): x was on vacation yesterday

   B. Everyone was well and went to work yesterday.

∀x- everyone, was not sick (¬S(x)) and W(x)

$\forall x(\neg S(x) \land W(x))$

   C. Everyone who was sick yesterday did not go to work.

∀x- everyone, if they were sick, then they did not go to work

If S(x), then not W(x)

$\forall x(S(x) \rightarrow \neg W(x))$

   D. Yesterday someone was sick and went to work.

∃x- someone, was S(x) and W(x)

$\exists x(S(x) \land W(x))$

Question 10

1. Exercise 1.7.9 section c, d, e, f, g, h, i
   Using the given truth table, determine whether the following expression is true or false.
   Domain: {a, b, c, d, e}

C. $\exists x((x = c) \rightarrow P(x))$
$P(C) = F$
False

D. $\exists x(Q(x) \land R(x))$
$Q(e) \land R(e) =$ true
True

E. $Q(a) \land P(d)$
$T \land T$
True

F. $\forall x ((x \neq b) \rightarrow Q(x))$
$Q(a)= T$
$Q(c)= T$
$Q(d)= T$
$Q(e)= T$
True

G. $\forall x (P(x) \lor R(x))$
$P(a) \lor R(a)= T$
$P(b) \lor R(b)= T$
$P(c) \lor R(c)= F$
$P(d) \lor R(d)= T$
$P(e) \lor R(e)= T$
False

H. $\forall x (R(x) \rightarrow P(x))$
$R(a) \rightarrow P(a)= T$
$R(b) \rightarrow P(b)= T$
$R(c) \rightarrow P(c)= T$
$R(d) \rightarrow P(d)= T$
$R(e) \rightarrow P(e)= T$
True

I. $\exists x(Q(x) \lor R(x))$
$Q(a) \lor R(a)= T$
$Q(b) \lor R(b)= F$

Q(c) ∨ R(c)= T
Q(d) ∨ R(d)= T
Q(e) ∨ R(e)= T
True

2. Exercise 1.9.2 section b, c, d, e, f, g, h, i
   Using the given truth table, indicate whether each of the quantified statements is true or false. Domain for x and y is {1, 2, 3}

B. ∃x ∀y Q(x, y)
Q(2, 1) = T
Q(2, 2) = T
Q(2, 3) = T
True

C. ∃y ∀x P(x, y)
P(1, 1) = T
P(2, 1) = T
P(3, 1) = T
True

D. ∃x ∃y S(x, y)
No combination in the truth table for S yields a truth response.
False

E. ∀x ∃y Q(x, y)
Q(1, 1) = F
Q(1,2) = F
Q(1,3) = F
This x has no y where the value is true.
False

F. ∀x ∃y P(x, y)
P(1, 1) = T
P(2, 1) = T
P(3, 1) = T
Every x has at least one y where the value is true.
True

G. ∀x ∀y P(x, y)
P(1, 2) = F
The above is a counter example.
False

H. $\exists x\ \exists y\ Q(x, y)$
$Q(1, 1) = T$
The above shows that there is at least one x where at least one y is true.
True

I. $\forall x\ \forall y\ \neg S(x, y)$

$\neg S$	1	2	3
1	T	T	T
2	T	T	T
3	T	T	T

For every x, every y is true
True

Question 11

1. Exercise 1.10.4 sections c, d, e, f, g
   Translate each of the following into logical expressions.

   C. There are two numbers whose sum is equal to their product.

$\exists x\ \exists y$- there is at least one x and one y

$(x+y) = (x*y)$- the sum of x and y is equal to the product of x and y

$\exists x\ \exists y\ ((x + y) = (x * y))$

   D. The ratio of every two positive numbers is also positive

$\forall x\ \forall y$- every x and every y

$(x > 0)$ x is positive

$(y > 0)$ y is positive

If x and y are positive, then their ratio is positive $((x/y) > 0)$

$\forall x\ \forall y\ (((x > 0) \land (y > 0)) \rightarrow ((x / y) > 0))$

   E. The reciprocal of every positive number less than one is greater than one.

If x is less than 1 and >0, then 1/x is greater than 1

$\forall x$- every x

$\forall x((1 > x) \land (x > 0)) \rightarrow ((1 / x) > 1)$

   F. There is no smallest number.

For every x, there is at least one smaller y

$\forall x\ \exists y\ (x > y)$

   G. Every number other than 0 has a multiplicative inverse.

If x is not equal to 0, then it has a multiplicative inverse

$(x * y) = 1$, multiplicative inverse, there is one y where this is true

$\forall x((x \neq 0) \rightarrow \exists y\ ((x * y) = 1))$

2. Exercise 1.10.7 sections c, d, e, f
   Give the logical expression for each of the following sentences. The domain is a group working on a project at a company. Sam is one of the members.

   P(x,y): x knows y's phone number. (A person may or may not know their own phone number)

   D(x): x missed the deadline

   N(x): x is a new employee.

   C. There is at least one new employee who missed the deadline

$\exists x$- there is at least one employee

The employee is new and missed the deadline

$\exists x(N(x) \land D(x))$

D. Sam knows the phone number of everyone who missed the deadline.

X input is (Sam), who knows the phone number of of everyone ($\forall$y) who missed the deadline.

If you missed the deadline, then Sam knows your number.

$\forall y\ (D(y) \rightarrow (P(Sam,y)))$

E. There is a new employee who knows everyone's phone number

There is one x, who is a new employee

You're that new employee and you know everyone's phone number (every y)

$\exists x \forall y\ (N(x) \land (P(x,y)))$

F. Exactly one new employee missed the deadline.

$\exists$x- there is one person that is a new employee and missed the deadline

$\exists x\ (N(x) \land D(x))$- there is one person that is new and missed the deadline

And, for every y that is not x, y is not new and missed the deadline.

$\forall y\ ((y \neq x) \land \neg(N(y) \land D(y))$

$\exists x\ ((N(x) \land D(x)) \land \forall y\ ((y \neq x) \land \neg(N(y) \land D(y)))$

3. Exercise 1.10.10 sections c, d, e, f

   Give a logical expression for each sentence.

   The domain x is a set of students at a university. The domain y is the set of math classes offered at the university.

T(x,y): Suggests student x has taken class y.

Sam is a student

C. Every student has taken at least one class other than Math 101

$\forall$x- every student

Has taken at least one Y, where y is not math 101. $\exists y\ ((y \neq \text{Math } 101)) \land (T(x,y)))$

$\forall x \exists y\ ((y \neq \text{Math } 101)) \land (T(x,y)))$

D. There is a student who has taken every math class other than Math 101

$\exists$x- there is one student

Taken every math class, y, where y is not Math 101

If y is not Math 101, then there is one student who has taken every math class.

$\exists x \forall y\ ((y \neq \text{Math } 101)) \rightarrow (T(x,y)))$

E. Everyone other than Sam has taken at least two different math classes.

Every x , where x is not Sam, has taken at least two math classes.

T(x, y) and T(x, z)- two different math classes

If (x is not Sam) then x has taken class Y and class Z, but y is not equal to z.

$\forall x \exists y((x \neq Sam) \rightarrow (T(x,y) \land T(x,z) \land (y \neq z))$

F. Sam has taken exactly two math classes.

There is one x(Sam), who has taken only class Y and class z

Taken only two classes- there is a y and a z, where they are not equal, and T(Sam, y) and T(Sam, Z)

¬T(Sam, a)- Sam has not taken the third class, and this A, is not equal to y and not equal to z

If this class a is not class y or z, then Sam has not taken the class.

$\exists y \exists z \exists a\ ((y \neq z) \wedge T(Sam,y) \wedge T(Sam,z)) \wedge (((a \neq y) \wedge (a \neq z)) \rightarrow \neg T(Sam, a))$

Question 12

1. Exercise 1.8.2 sections b, c, d, e

   The domain is a set of male patients in a clinical study.

   P(x): x was given the placebo

   D(x): x was given the medication

   M(x): x had migraines

Translate each statement into a logical expression, then negate the expression by adding a negation operation to the beginning of the expression. Apply De Morgan's Law until each negation operation applies directly to a predicate and then translate the logical expression back to English.

B. Every patient was given the medication or the placebo or both.

Statement:	∀x- every patient, was given (D(x) or (P(x)) ∀x((D(x) ∨ P(x))
Negation:	¬∀x((D(x) ∨ ¬P(x))
De Morgan's Law:	∃x(¬(D(x) ∧ ¬P(x))
English:	There is a patient was not given the medication and not given the placebo

C. There is a patient who took the medication and had migraines.

Statement:	∃x- there is a patient (D(x) and M(x))- that took the medication and had migraines ∃x(D(x) ∧ M(x))
Negation:	¬∃x(D(x) ∧ M(x))
De Morgan's Law:	∀x(¬D(x) ∨ ¬M(x))
English:	Every person did not have the medication or did not have migraines

D. Every patient who took the placebo had migraines.

Statement:	Every person, if they took the placebo, then they had migraines ∀x(P(x) → M(x))
Negation:	¬∀x(P(x) → M(x))

De Morgan's Law	$\neg\forall x(P(x) \rightarrow M(x))$ Conditional Identity: $(P(x) \rightarrow M(x)) = \neg P(x) \lor M(x)$ $\neg\forall x(\neg P(x) \lor M(x))$ $\exists x(\neg\neg P(x) \land \neg M(x))$ $\exists x(P(x) \land \neg M(x))$
English:	There is a patient who took the placebo and did not have migraines.

E. There is a patient who had migraines and was given the placebo.

Statement:	$\exists x$- there is a patient (M(x) and P(x)) $\exists x(M(x) \land P(x))$
Negation:	$\neg\exists x(M(x) \land P(x))$
De Morgan's:	$\forall x(\neg M(x) \lor \neg P(x))$
English:	Every patient did not have a migraine or they did not take the placebo.

2. Exercise 1.9.4 sections c, d, e
   Write the negation of each of the following logical expressions so that all negations immediately precede predicates.
   C. $\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$

$\neg\exists x\ \forall y\ (P(x, y) \rightarrow Q(x, y))$
$\forall x \neg\forall y\ (P(x, y) \rightarrow Q(x, y))$
$\forall x \exists y\ \neg(P(x, y) \rightarrow Q(x, y))$
Conditional Identity: $(P(x, y) \rightarrow Q(x, y)) = \neg P(x, y) \lor Q(x, y)$
$\forall x \exists y\ \neg(\neg P(x, y) \lor Q(x, y))$
$\forall x \exists y\ (\neg\neg P(x, y) \land \neg Q(x, y))$
$\forall x \exists y(P(x, y) \land \neg Q(x, y))$

D. $\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$

$\neg\exists x\ \forall y\ (P(x, y) \leftrightarrow P(y, x))$
$\forall x \neg\forall y\ (P(x, y) \leftrightarrow P(y, x))$
$\forall x \exists y\ \neg\ (P(x, y) \leftrightarrow P(y, x))$
Conditional Identity: $(P(x, y) \leftrightarrow P(y, x)) = (P(x, y) \rightarrow P(x, y)) \land (P(x, y) \rightarrow P(x, y))$
$(P(x, y) \rightarrow P(x, y)) = (\neg P(x, y) \lor P(x, y))$
$(P(x, y) \rightarrow P(x, y)) = (\neg P(x, y) \lor P(x, y))$
$(\neg P(x, y) \lor P(x, y)) \land (\neg P(x, y) \lor P(x, y))$
$\forall x \exists y\ \neg(\neg P(x, y) \lor P(x, y)) \land (\neg P(x, y) \lor P(x, y)))$

$\forall x \exists y(\neg(\neg P(x, y) \lor P(x, y)) \lor \neg(\neg P(x, y) \lor P(x, y)))$

$\forall x \exists y(\neg\neg P(x, y) \land \neg P(x, y)) \lor (\neg\neg P(x, y) \land \neg P(x, y))$

$\forall x \exists y(P(x, y) \land \neg P(x, y)) \lor (P(x, y) \land \neg P(x, y))$

E. $\exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

$\neg \exists x \exists y P(x, y) \land \forall x \forall y Q(x, y)$

$\forall x \neg \exists y P(x, y) \land \forall x \forall y Q(x, y)$

$\forall x \forall y \neg(P(x, y) \land \forall x \forall y Q(x, y))$

$\forall x \forall y(\neg P(x, y)) \lor \neg(\forall x \forall y Q(x, y))$

$\forall x \forall y(\neg P(x, y)) \lor \exists x \exists y \neg Q(x, y)$