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Question 5

A. Solve the following questions from zyBook:
1. Exercise 1.12.2 sections b, e

Using the rules of inference and laws of propositional logic, prove that each argument is valid.

B. $p \rightarrow (q \land r)$
$\neg q$
____
$\therefore \neg p$

1.	$\neg q$	Hypothesis
2.	$\neg q \lor \neg r$	Addition, 1
3.	$\neg(q \land r)$	De Morgan's Law
4.	$p \rightarrow (q \land r)$	Hypothesis
5.	$\neg p$	Modus tollens, 3, 4

E. $p \lor q$
$\neg p \lor r$
$\neg q$
____
$\therefore r$

1.	$p \lor q$	Hypothesis
2.	$\neg p \lor r$	Hypothesis
3.	$(q \lor r)$	Resolution, 1, 2
4.	$\neg q$	Hypothesis
5.	r	Disjunctive syllogism, 3, 4

2. Exercise 1.12.3 section c

Some of the rules of inference can be proven using the other rules of inference and the laws of propositional logic

C. Prove without using Disjunctive syllogism.

$p \lor q$
$\neg p$
____
$\therefore q$

1.	$p \lor q$	Hypothesis
2.	$\neg(\neg p) \lor q$	Double Negation 1
3.	$\neg p \rightarrow q$	Conditional, 2
4.	$\neg p$	Hypothesis
5.	q	Modus ponens, 3, 4

3. Exercise 1.12.5 sections c, d
   Give the form of each argument, then prove whether the argument is valid or invalid. If the argument is valid, prove the validity.

C. I will buy a new car and a new house only if I get a job.
I am not going to get a job.
____
$\therefore$ I will not buy a new car.

C: I will buy a new car
H: I will buy a new house
J: i will get a job

$(C \land H) \rightarrow J$
$\neg J$
____
$\therefore \neg C$

The argument is invalid when C= T and H= F and J= F. These values will make the hypothesis true and the conclusion false.

D. I will buy a new car and a new house only if I get a job.
I am not going to get a job.

I will buy a new house.

∴ I will not buy a new car.

C: I will buy a new car
H: I will buy a new house
J: I will get a job

$(C \wedge H) \rightarrow J$
$\neg J$
H

$\therefore \neg C$
This argument is valid. See the proof below

1.	$\neg J$	Hypothesis
2.	$(C \wedge H) \rightarrow J$	Hypothesis
3.	$\neg(C \wedge H)$	Modus tollens, 1, 2
4.	$\neg C \vee \neg H$	De Morgan's Law
5.	H	Hypothesis
6.	$\neg H \vee \neg C$	Commutative
7.	$\neg C$	Disjunctive Syllogism

B. Solve the following questions from zyBook:
1. Exercise 1.13.3 section b
   Show that the given argument is invalid for predicates P and Q, over the domain {a,b}

$\exists x\, (P(x) \vee Q(x))$
$\exists x\, \neg Q(x)$

$\therefore \exists x\, P(x)$

	P	Q
a	F	T
b	F	F

The above truth table shows that hypothesis $\exists x\,(P(x) \lor Q(x))$, and $\exists x\,\neg Q(x)$ are true, but the conclusion $\exists x\,P(x)$ is false, therefore the statement is invalid.

2. Exercise 1.13.5 sections d, e

D. Prove whether each argument is valid or invalid. If the argument is valid, then use the rules of inference to prove that the form is valid

Every student who missed class got a detention.
Penelope is a student in the class.
Penelope did not miss class.

Penelope did not get a detention.

D: Student(s) who got a detention
M: Student(s) who missed a class

$\forall x\,(M(x) \rightarrow D(x))$
Penelope is a student in the class
$\neg M(\text{Penelope})$

$\neg D(\text{Penelope})$

If we assume that that Penelope is the only student in the class, then the following truth values would prove the argument invalid:

M= F
D = T

This would make the hypotheses true and the conclusion false.

E. The domain:

Every student who missed class or got a detention did not get an A.
Penelope is a student in the class.
Penelope got an A.

Penelope did not get a detention.

D: Student(s) who got a detention
S: Student in the class
M: Student(s) who missed a class
A: Student(s) got an A

$\forall x\,((M(x) \lor D(x)) \rightarrow \neg A(x))$
Penelope is a student in the class
$A(\text{Penelope})$

$\neg D(Penelope)$

1.	Penelope is a student in the class	Hypothesis
2.	$\forall x\, ((M(x) \lor D(x)) \rightarrow \neg A(x))$	Hypothesis
3.	$((M(Penelope) \lor D(Penelope)) \rightarrow \neg A(Penelope))$	Universal instantiation
4.	$A(Penelope)$	Hypothesis
5.	$\neg(\neg A(Penelope))$	Double Negation, 4
6.	$\neg(M(Penelope) \lor D(Penelope)$	Modus Tollens, 4, 5
7.	$\neg M(Penelope) \land \neg D(Penelope)$	DeMorgan, 6
8.	$\neg D(Penelope)$	Simplification, 7

The proof above proves the validity of the argument.

Question 6

1. Solve exercise 2.4.1 section d

   D. Prove using a direct proof: The product of two odd integers is an odd integer

Proof: Assume x and y are odd, we will prove that the product of x and y is also odd.

Since x is odd, $x = 2k + 1$, for some integer k

Since y is odd, $y = 2z + 1$, for some integer z

Therefore, $xy = (2k + 1)(2z + 1)$

$= 4kz + 2k + 2z + 1$

$= 2(2kz + k + z) + 1$

Since k and z are integers, $2kz + k + z$ is also an integer.

$xy = 2(k) + 1$, therefore xy is odd ■

2. Solve exercise 2.4.3 section b

   B. Prove using a direct proof. If x is a real number and $x \le 3$, then $12 - 7x + x^2 \ge 0$

Proof: Assume that x is a real number and $x \le 3$, we will prove that $12 - 7x + x^2 \ge 0$

If $x \le 3$, then $x - 3 \le 0$

If $x - 3 \le 0$, then $x - 4 \le 0$

$12 - 7x + x^2 \ge 0$ is $(x - 3)(x - 4) \ge 0$ ■

Question 7

1. Solve exercise 2.5.1 section d

D. Prove the statement by contrapositive:

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

Proof: Let n be even, we will prove that $n^2 - 2n + 7$ is odd

If n is even, then n = 2k, for some integer k

$(2k)^2 - 2(2k) + 7 = 2(k^2 - 2k + 3) + 1$

Since k is an integer, then $(k^2 - 2k + 3)$ is also an integer.

$n^2 - 2n + 7 = 2k + 1$

Therefore, the statement is odd. ■

2. Solve exercise 2.5.4 sections a, b

A. Prove the statement by contrapositive:

For every pair of real numbers x and y if $x^3 + xy^2 \leq x^2y + y^3$, then $x \leq y$

Proof: Let $x > y$, we will prove that $x^3 + xy^2 > x^2y + y^3$

If $x > y$, then $x(x^2 + y^2) > y(x^2 + y^2)$

Therefore $x^3 + xy^2 > x^2y + y^3$ ■

B. Prove the statement by contrapositive:

For every pair of real numbers x and y, if $x + y > 20$, then $x > 10$, or $y > 10$

Proof: Let $x \leq 10$ and $y \leq 10$, we will prove that $x + y \leq 20$

$x + y \leq 10 + 10$, and therefore,

$x + y \leq 20$ ■

3. Solve exercise 2.5.5 section c

C. Prove using a direct proof or by contrapositive:

For every non zero real number x, if x is irrational, then 1/x is also irrational.

Proof: Let 1/x be rational, we will prove that x is also rational

If 1/x is rational, then 1/x = a/b for some integers a and b

x is the inverse of 1/x, and b/a is the inverse of a/b.

Since a and b are integers, then b/a must be rational and x is a rational number. ■

Question 8

Solve exercise 2.6.6 sections c, d

C. Give a proof for the statement:

The average of three real numbers is greater than or equal to at least one of the numbers

Proof: Suppose that the average of three real numbers, x, y, and z, is less than each of these numbers.

$$\frac{x + y + z}{3} < x + y + z$$

The above is a contradiction. ■

D. There is no smallest integer

Proof: suppose that there is a smallest integer x,

x - 1 is also an integer, this would suppose that x < (x - 1),

The above is a contradiction. ■

<u>Question 9</u>

Solve exercise 2.7.2 section b

B. If integers x and y have the same parity, then x + y is even

Proof:

Case 1: x and y are both even. Since x is even, $x = 2k$, for some integer k.

Since y is even, $y = 2j$ for some integer j.

$$x + y = (2k + 2j) = 2(k + j).$$

Since k and j are integers, $k + j$ must be an integer, and therefore the sum of 2 even numbers is $2(k+j)$ is even.

Case 2: x and y are both odd. Since x is odd, $x = 2k + 1$ for some integer k,

Since y is odd, $y = 2j + 1$ for integer j.

$$x + y = 2k + 1 + 2j + 1 = 2(k + j + 1)$$

Since k and j are integers, $(k + j + 1)$ is also an integer. Therefore $2(k + j + 1)$ is even, and the sum of two odd integers is even. ■