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<u>Question 3</u>

a. Exercise 4.1.3 sections b, c

Which of the following are functions from **R** to **R?** If f is a function, give its range.

b. f(x) = 1 / (x^2- 4)

As x gets larger, f(x) approaches 0. As x gets larger, f(x) approaches 1/-4.

The function is also not well defined when x = 2 and x= -2.

This is not a well defined function**.**

c. f(x) = $\sqrt{x^{\ 2}}$

f(x) will always be positive. The range is all positive numbers.

The range is $\mathbf{R}^+$ including 0.

b. Exercise 4.1.5 sections b, d, h, i, l

Express the range of each function using roster notation

b.f(x)= x^2, A = {2, 3, 4, 5}

2^2, 3^2, 4^2, 5^2

{4, 9, 16, 25}

d. {1, 2, 3, 4, 5}. A 1 will show up at least once for each of the amount of times for all of the combinations of $\{0,1\}^5$. With the most being 5 times f(11111)= 5, and the least being once f(00001) = 1.

h.{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (3, 1), (3, 2), (3, 3)}

A = {1, 2, 3}, where x and y are both elements of A.

Since f(x)= (y,x) and the target is all integers, the range is all possible combinations of A x A.

i. {(1, 2), (1, 3), (1, 4), (2, 2), (2, 3), (2, 4), (3, 2), (3, 3), (3, 4)}

A = {1, 2, 3}. f(x)= (x, y+1).

y + 1 = {2, 3, 4}. The target is all integers, the range is all possible combinations of A and (y + 1).

l. Domain is P(A)

P(A)= {Ø, {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3}}

If X is a subset of of A, and a P(A) includes all subsets of A, X must be in P(A).

X - {1} are all subsets of a without {1}.

{Ø, {2}, {3}, {2, 3}}

Question 4

I. Solve the following questions from zybooks.

a. Exercise 4.2.2 sections c, g, k

For each of the functions below, indicate whether the function is onto, one to one, neither, or both. If the function is not one-to-one give and example showing why.

c. $h(x) = x^3$ is one to one since every domain will produce a unique element. But it is not onto since not every target will have a domain. No integer x such that h(x)= 11

g. f(x, y) = (x+1, 2y) is one to one since every domain will produce a unique element. But it is not onto since not every target will have a domain (the odd numbers will never have a y domain). No integer pair (x, y) such that f(x, y) = (0, 1)

k. Not onto since not every target will have a domain no positive integer pair (x, y) such that f(x, y) = 1. Not one to one. f(3, 1) = f(2, 5)= 9

b. Exercise 4.2.4 sections b, c, d, g

For each of the functions below, indicate whether the function is onto, one- to- one, neither or both. If the function is not onto or not one to one give an example.

b. Not one to one since not every domain will produce a unique element. f(100) = f(000) = 100. It is not onto since not every element in the target will have a domain. There is no x such that f(x) = 000.

c. This is one to one since every domain will produce a unique element. This is onto since every element in the target will have a domain.

d. This is one to one since every domain will produce a unique element. This is not onto since not every element in the target will have a domain. Eg. There is no x such that f(x) = 000

g. This is not one to one since f({1, 2, 3}) = f({2, 3}) = {2, 3}. This is not onto since not every element in the target will have a domain. Eg. there is no x such that f(x) = {1}

II. Give an example of a function from the set of integers to the set of positive integers that is:

a. One to one, but not onto

$f(x) = x^2 +1$

In this example, every element in the domain will map to a unique element so it is one to one, but not every element in the target has at least one domain, so it is not onto. There will be no x that will produce f(x) = 15.

b. Onto, but not one to one
   f(x) = floor round(x/2)

In this example, every element in the target has at least one domain, so it is onto. But not every element in the domain maps to a unique element, therefore it is not one to one. Eg. f(7) = 3 and f(6) = 3.

c. One to one and onto
   f(x) = x +1

This is an example of a function that is one to one and onto, since every element in the domain will map to a unique element in the target, and for every element in the target, there is at least one element in the domain.

d. Neither one to one nor onto
   f(x)= 3

This is an example of a function that is neither one to one nor onto. Since the elements in the domain do not all map to a unique element, and, not every element in the target will have a domain.

Question 5

Solve the following questions from the zybooks.

a. Exercise 4.3.2 sections c, d, g, i

c. $f(x) = 2x + 3$

$f^{-1}(y) = (y - 3) / 2$

This is a well defined inverse. f(x) is one to one (every domain maps to a unique element) and onto (every element in the target has a domain).

d. $X \subseteq A$, $f(X) = |X|$, which will map the cardinality of A.

A = {1, 2, 3, 4, 5, 6, 7, 8}

For |P(A)|, at the least it will be 0 {empty set}, or it will be 8 {1, 2, 3, 4, 5, 6, 7, 8}.

Therefore for f: P(A) → {0, 1, 2, 3, 4, 5, 6, 7, 8}, every element in the target will have a domain (onto). But will not be one to one, as not every domain will map to a unique element.

Eg. f({2, 3}) = f({4, 5}) = 2.

This is not a well defined function.

g. This is a well defined function.

f^{-1} is obtained by and taking the input and reversing the strings. ($f^{-1} = f$).

i. This is a well defined function. As it is one to one (every element in the domain will map to a unique element). And it is onto (every element in the target maps to a unique domain).

$f^{-1}(x, y) = (x - 5, y + 2)$

b. Exercise 4.4.8 sections c, d

The domain and target set of function f, g, and h are **Z**. The functions are defined:

$f(x) = 2x + 3$

$g(x) = 5x + 3$

$h(x) = x^2 + 1$

c. f o h:

$f(x) = 2x + 3$

$h(x) = x^2 + 1$

$f(h(x)) = 2(x^2 + 1) + 3$

$f \circ h = 2x^2 + 5$

d. h o f:

$h(x) = x^2 + 1$

$f(x) = 2x + 3$

$h(f(x) = (2x + 3)^2 + 1$
$h \circ f = 4x^2 + 12x + 10$

c. Exercise 4.4.2, sections b, c, d

b. $(f \circ h)(52)$
$f(x) = x^2$
$h(x)$ = ceiling round $(x / 5)$
$h(52)$ = ceiling round$(52 / 5) = 11$
$f(11) = x^2 = 121$
$f \circ h\ (52) = 121$

c. $(g \circ h \circ f)(4)$
$f(4) = 16$
$h(16)$ = ceiling round$(16 / 5) = 4$
$g(4) = 2^4 = 16$
$(g \circ h \circ f)(4) = 16$

d. $(h \circ f)(x)$ = ceiling round$(x^2 / 5)$

d. Exercise 4.4.6, sections c, d, e

c. h(x) = The output of h is obtained by taking the input string x and replacing the last bit with a copy of the first bit.

f(x)= The output of f is obtained by taking the input string and replacing the first bit by 1, regardless of whether the first bit is 0 or 1

f(010) = 110
h(110) = 111
(h o f)(010) = 111

d. The range of f(x) = (100) to (111)
The range of h o f would be 101 and 111.
Range of h o f: {101, 111}

e. Range of f(x) = (100) (101) (110) (111)
Range of g o f = {001, 101, 011, 111}

e. Exercise 4.4.4 sections c, d

c. f(x) is not one to one. Eg. f(x) = |x| (not one to one since f(2) = f(-2) = 2)
Can g o f be one to one?
No that would not be possible as it would require g(x) to have multiple elements mapped from the same domain.

If g o f(x) is one to one, then f(x) must be one to one.
If $x_1 \in X$ and $x_2 \in X$ such that $x_1 \neq x_2$, and $(g(f(x_1))) \neq (g(f(x_2)))$
The $f(x_1) \neq f(x_2)$, and f(x) must be one to one.

d. Yes it can be possible that g(x) is not one to one and g o f is one-to-one.
f: X → Y
Eg. the target Y is {1, 2, 3, 4, 5}, but the range for f is {1, 2, 3, 4}
g: Y→ Z
Eg. If g(4) = g(5) then g would not be one to one,
But in this scenario, g o f, g(5) would never occur since that is outside the range of f.