tt2405

Question 3

a. Solve exercise 8.2.2 section b

$f(n) = n^3 + 3n^2 + 4$. Prove that $f = \Theta(n^3)$

$T(N) = \Theta(h(N))$ if and only if $T(N) = O(h(N))$ and $T(N) = \Omega(h(N))$

$T(N) = O(h(N))$ if there are positive constants c and n_0 such that $T(N) <= c*h(N)$when $n >= n_0$

Let c = 5 and n_0= 1

Then for any $n>= n_0$ we have:

$n^3 + 3n^2 + 4 < 5n^3$

$f(n) = O(n^3)$

$T(N) = \Omega(h(N))$ if there are positive constants c and n_0 such that $T(N) >= c*h(N)$ when $n >= n_0$

Let c = 1 and n_0= 1

Then for any $n >= n_0$ we have:

$n^3 + 3n^2 + 4 > n^3$

$f(n) = \Omega(n^3)$

Since $O(n^3) = \Omega(n^3)$, $\Theta(n^3)$

b. Solve exercise 8.3.5 section a, b, c, d, e

a. This algorithm works by taking a series of numbers from the user and having a 'target value' (the variable p). There are two counters, i and j, i counts forward, and j counts backwards.). There are three loops, two inner loops and outer loop. The first inner loop looks for the values less than p and the outer loop finds the element that is greater than or equal to p. If the value exists, it will swap those two values and execute the greater outer loop. The output from all of this is that while 1 < j, and a_j is less than or equal to p a swap will occur, .

b. The total amount of times i = i + 1 and j = j -1 is executed is n -1 times.

c. The minimum amount of times the swap occurs is floor round (n /2) times when p is in the middle of the series. Since the swap will only occur when i is less than j and with each run of the loop, both i and j move closer to each other by one. The maximum number of swaps occur when all values are less than p (n-1) times. J will never decrease and i will be the only one increasing, and the outer loop runs when i < j.

d. For any n, the two inner loops are executed n - 1 times, and the swap is executed at minimum floor round (n/2) times. $\Omega(n)$ since the greatest order in both terms is n.

e. The upper bound is $O(n)$ since the inner loops will run (n - 1) + (n - 1) times. The highest order term from that operation is n.

Question 4
Solve the following questions from zybooks:

a. Exercise 5.1.2, sections b, c

b. Strings of length 7, 8, or 9. Characters can be special characters, digits or letters.
Characters: 26
Digits: 10
Special characters: 4
Total: 40

Strings of 7: 40^7
Strings of 8: 40^8
Strings of 9: 40^9

Total: $40^7 + 40^8 + 40^9$

c. Strings of 7, 8, 9 but the first character cannot be a letter.
Digits and special characters: 14
Since the first character cannot be a letter, there are 14 choices for the first slot, and 40 for the rest.

Strings of 7: $14 * 40^6$
Strings of 8: $14 * 40^7$
Strings of 9: $14 * 40^8$

Total = $14(40^6 + 40^7 + 40^8)$

b. Exercise 5.3.2, section a

a. String with 10 length 10 that has no repetitions.
In the first slot you would have 3 choices (a, b, c), then following you would only have 2(if the first spot was a, then the next would have to be b or c). Following that you would only have two choices as well

$3 * 2^9$ strings

c. Exercise 5.3.3, sections b, c

b. How many license plate numbers are possible if no digit appears more than once?
First character is a digit: 10
Next four characters are capital letters: 26
Last two characters are digits: 10

First digit you have 10 choices, second digit you have 9 choices, last digit you have 8 choices
All of the letters can repeat so it is 26^4 for those 4 slots.

Total combinations: 10 * 9 * 8 * 26^4

c. How many license plate numbers are possible if no digit or letter appears more than once?
10 for the first digit, 9 for the second, 8 for the third.
26 for the first letter, 25 for the second, 24 for the third, 23 for the fourth.

Total combinations: 10 * 9 * 8 * 26 * 25 * 24 * 23

d. Exercise 5.2.3, sections a, b

a. Show a bijection between B^9 and E^{10}. Explain why your function is a bijection.
B^9 is the set of binary strings with 9 bits.
E_{10} is the set of binary strings with 10 bits that have an even number of 1's.

A function that would make an element in B into an element of E would be to evaluate how many ones there are in B, then if the amount is even, add a 0 to the end (so it will be 10 bits with even amount of 1's). If the amount is odd, add a 1 (so it will be 10 bits with even amount of 1's).
f is one to one, since for every x != y, f(x) != f(y),
f is onto because the range of E (all 10 bit strings with an even amount of 1's) is equal to the target (all 10 bit strings with an even amount of 1's)

- All 10 bit strings in E have their 9 bit counterpart in B, by removing the last digit.

b. $|E_{10}| = |B^9|$
$B^9 = 2^9$, since there are 2 choices for all 9 positions
The bijection rules states that E_{10} would also have a cardinality of 2^9

Question 5
Solve the following questions from zyBooks:

a. Exercise 5.4.2, sections a, b

a. How many different phone numbers are possible if they start with either 824 or 825 and are 7 digits long
2 choices to begin (824 or 825), then following there 10 digits, for 4 positions left: 10^4

Total combinations $2 * 10^4$

b. How many different phone numbers where the last four digits are all different?
2 choices, to begin, then 10 choices, 9 choices, 8 choices, 7 choices

Total combinations: 2 * 10 * 9 * 8 * 7

b. Exercise 5.5.3, sections a, b, c, d, e, f, g

How many 10 bit string sare there subject to each of the following restrictions?

a. No restrictions;
There are 2 possible combinations for 10 places
2^{10}

b. The string starts with 001
This leaves 2 possible combinations for the last 7 places.
$1 * 2^7$
2^7

c. The string starts with 001 or 10
For start with 001: there are 2 choices for the last 7 places (2^7)
For start with 10: there are 2 choices for the last 8 places (2^8)
$2^7 + 2^8$

d. The first two bits are the same as the last two bits:
Since the first two positions are tied to the last two positions, we can think of them as 'stuck together'
So for the first two positions there are 2^2 choices (and this dictates the choices for the last two positions) then the remaining 6 positions have 2^6 choices
$2^2 * 2^6 = 2^8$
2^8

e. The string has exactly 6 zeros.
There are only 4 spots to choose from out of 10 C(10, 4)
$C(10,\ 4)\ =\ \frac{10!}{4!(10-4)!}\ =\ \frac{10!}{4!6!}$
210 choices

f. The string has exactly 6 zeros and the first bit is 1
There are 9 spaces to choose from, but 6 are 0's.
C(9, 3)
$= \frac{9!}{3!6!}$
84

g. There is exactly one 1 in the first half and three 1's in the second half
For the first half: out of 5 spaces there are 4 choices
C(5, 4)
$= \frac{5!}{4!1!}$
5

For the second lf: out of 5 spaces there are 2 choices
C(5, 2)
$\frac{5!}{3!2!}$
10

The two get multiplied (product rule)
50 combinations

c. Exercise 5.5.5, section a

a. There are 30 boys and 35 girls that try out for a chorus. Only 10 girls and 10 boys will be selected.
For the boys: C(30, 10)
$= \frac{30!}{10!20!}$

For the girls: C(35, 10)
$= \frac{35!}{10!15!}$

Apply the product rule: (C(30, 10) * C(35, 10))

d. Exercise 5.5.8, sections c, d, e, f

Standard playing cards have 52 cards.

c. How many five-card hands are made entirely of hearts and diamonds?
Hearts: 13
Diamonds: 13
So choose 5 five cards from 26.
C(26, 5)
65780 combinations

d. How many five-card hand have 4 cards of the same rank?

There are 4 cards in a rank
So one slot you can choose any of the remaining 48 cards. C(48, 1)
13 cards have the same rank, 13 cases for each of the 48 remaining cards.

(C(13, 1) * C(48, 1))
624 combinations

e. A full house is a five card hand that has 2 cards of the same rank and three cards of the same rank.
How many five card hands contain a full house?
13 ranks for the first option C(13, 1),
There are 4 cards of different suits (for each rank) and need to select 3 C(4, 3)
Then after, there are 12 ranks for the second option. C(12, 1)
There are 4 suits for this rank and need to select 2 C(4, 2)
Apply the product rule to all 4:
C(13, 1) * C(4, 3) * C(12, 1) * C(4, 2)
3744

f. How many five card hands do not have any two cards of the same rank?
C(13, 5). Choose 5 cards from the 13 ranks. Each rank must choose one of the 4 suits (for each 5 cards).

C(13, 5) * 4^5

e. Exercise 5.6.6, sections, a, b

Suppose that a national senate contains 100 members, 44 of which are D and 56 of which are R
a. Choose 10 members from the 44 D, and choose 10 members from the 56 R.
C(44, 5) * C(56, 5)

b. Each party must select a speaker and a vice speaker.
From the 44 D, 2 choices are made, but the ‘order matters’
P(44, 2)

From the 56 R, 2 choices are made and ‘order matters’
P(56, 2)

P(44, 2) * P(56, 2)

Question 6
Solve the following questions from zybooks:

a. Exercise 5.7.2, sections a, b

a. How many 5 card hands have at least one club?
Total combinations: C(52, 5)
There are 13 club cards.
Chance to draw no club cards C(52 - 13, 5) = C(39, 5)

C(52, 5) - C(39, 5)

b. At least two cards with the same rank?
Cards without the same rank: C(13, 5)*4^5

C(52, 5) - C(13, 5)*4^5

b. Exercise 5.8.4, sections a, b

20 comic books are distributed to 5 kids.
a. How many ways are there to distribute the comic books if there are no restrictions on how many go to each kid?
5^{20}

b. How many ways are there to distribute the comic books if they are divided evenly so that 4 go to each kid?
$\frac{20!}{4!4!4!4!}$

Question 7 - How many one to one functions are there from a set with five elements, to sets with the following number of elements:

a. 4

None, since the cardinality of a set with 4 elements (target) is less than the cardinality of a set with 5 elements (domain). This can never be one to one.

b. 5,

5 for the first element, 4 for the second, all the way down

5!

120

c. 6

Order of which element you choose matters. So P(6, 5)

= 720

d. 7

Order matters, need to use a permutation P(7, 5)

= 2520