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Question 5

a. $5n^3 + 2n^2 + 3n = \Theta(n^3)$

T(N) = Θ(h(N)) if and only if T(N) = O(h(N)) and T(N) = Ω(h(N))
T(N) = O(h(N)) if there are positive constants c and n_0 such that T(N) <= c*h(N)when n >= n_0

Let c = 10 and n_0 = 1
$5n^3 + 2n^2 + 3n <= 5n^3 + 5n^3$
$= 10n^3$
$= c*(n^3)$
$5n^3 + 2n^2 + 3n = O(n^3)$

T(N) = Ω(h(N)) if there are positive constants c and n_0 such that T(N) >= c*h(N) when n >= n_0

Let c = 1 and n_0 = 1
$5n^3 + 2n^2 + 3n >= 1n^3$
$= c*(n^3)$
$5n^3 + 2n^2 + 3n = \Omega(n^3)$

Since $5n^3 + 2n^2 + 3n = O(n^3)$ and $5n^3 + 2n^2 + 3n = \Omega(n^3)$, then $5n^3 + 2n^2 + 3n = \Theta(n^3)$

b. $\sqrt{7n^2 + 2n - 8} = \Theta(n)$

T(N) = Θ(h(N)) if and only if T(N) = O(h(N)) and T(N) = Ω(h(N))
T(N) = O(h(N)) if there are positive constants c and n_0 such that T(N) <= c*h(N)when N >= n_0

Let c = 3 and n_0 = 1
$\sqrt{7n^2 + 2n - 8} <= \sqrt{9n^2}$
= 3n
= c *(n)
$\sqrt{7n^2 + 2n - 8}$ = O(n)

T(N) = Ω(h(N)) if there are positive constants c and n_0 such that T(N) >= c*h(N) when N >= n_0

Let c = 1 and n_0 = 1
$\sqrt{7n^2 + 2n - 8} >= \sqrt{1n^2}$
= 1n
= c*(n)
$\sqrt{7n^2 + 2n - 8}$ = Ω(n)

Since $\sqrt{7n^2 + 2n - 8}$ = O(n) and $\sqrt{7n^2 + 2n - 8}$ = Ω(n),
then $\sqrt{7n^2 + 2n - 8}$ = Θ(n)