tt2405

Question 7

a. Exercise 6.1.5 sections b, c, d

A 5 card hand is dealt from a perfectly shuffled deck of playing cards. What is the probability of the following events?

b. Hand is a three of a kind? (3 of the same rank, other two have different ranks)
Total combos: C(52, 5)
The three of a kind must be the same: C(13, 1) and 3 suits to choose from C(4, 3)
12 ranks remaining for the last two C(12, 2) and one suit for each C(4, 1) *C(4, 1)

$$\frac{C(13, 1) * C(4, 3) * C(12, 2) * C(4, 1) * C(4, 1)}{C(52, 5)}$$

= 88 / 4165

c. Probability that all 5 cards have the same suit?
4 suits but only choosing 1: C(4, 1)
13 different ranks, choose any 5:
Total combos: C(52, 2)

$$\frac{C(4, 1) * C(13, 5)}{C(52, 5)}$$

= 33/16660

d. Hand has two of a kind?
Total combos: C(52, 5)
Two cards have same rank: C(13, 1) can have 2 suits to choose from C(4, 2)
Remaining 3 cards from the remaining 12 ranks C(12, 3) * $C(4, 1)^3$

= 352 / 833

b. Exercise 6.2.4 sections a, b, c, d

A 5 card hand is dealt from a perfectly shuffled deck of playing cards
a. Hand has at least one club?
Hand has no clubs cannot choose 13 of the cards in the deck
Can pick 5 cards from 39 choices:
C(39, 5)
Total combos: C(52, 5)

$$1 - \frac{C(39, 5)}{C(52, 5)}$$

= 7411 / 9592

b. Hand has at least two cards of the same rank
Hand has only 1 card of a rank
Out of the 13 ranks, must choose 5 cards C(13, 5) and each card has 1 suit $C(4, 1)^5$

$$1 - \frac{C(13, 5) * C(4, 1)^5}{C(52, 5)}$$

= 2053 / 9520

c. Exactly one club or one spade;
One club: C(13, 1) * C(39, 4)- only have 1 suit choice for the first card(13 clubs to pick from), then the other 4 pick from the remaining 39 cards
One spade: C(13, 1) * C(39, 4)

One club and one spade- C(13, 1)*C(13, 1)*C(26, 3) , need to subtract this occurrence out of the probability
Total combo: C(52, 5)

$$\frac{2*(C(13, 1)*C(39, 4)) - C(13, 1)*C(13, 1)*C(26, 3)}{C(52, 5)}$$

= 65351 / 99960

d. Has at least one club or one spade
No clubs and no spades: cannot use 26 cards

$$1 - \frac{C(26, 5)}{C(52, 5)}$$

= 9743 / 9996

Question 8

a. Exercise 6.3.2 sections a, b, c, d, e

The letters {a, b, c, d, e, f, g} are put in a random order. Each permutation is equally likely

A: The letter b falls in the middle (three letters before and three letters after)

B. The letter c appears to the right of b, but does not have to be immediately to the right

C: the letters 'def' occurs together in that order

a. Calculate p(A), p(B), p(C)

A: the position of b is fixed, so there are 6! choices for the rest of the letters

|S| = 7!

p(A) = 1/7

B: combination that c is to the right of b.

7! Total positions

C and b and linked, so there are 5 'free spots' for the other letters to choose

n!/ (n - r)!

n = 7

r = 5

(7! / 2!) / 7! = ½

p(B) = 1/2

C: 'def' are now stuck together in that order , total of 5 units that work together- 5!

|S| = 7!

5! /7!

p(C)= 1/42 = 0.024

b. p(A|C)

B is in the middle and 'def' are together. So there are 5 working units

If b is in the middle, there are 2 choices for 'def' (either before or after b) and 3 choices for the remaining letters

p($A \cap C$) =(2! 3!) / 7!

p(C) = 5! / 7!

p(A|C) = p($A \cap C$) /p(C) = (2! 3!)/ (5!)

= 1/10

c. p(B|C) = p($B \cap C$) /p(C)

p($B \cap C$) = c must be to the right of b, and def are together. C and be are linked so there are 3 free spots

5! / (5 - 3)! = (5! / 2!)

p(C) = 5!

= 5!/2!/5!

= 1/2

d. p(A|B) = p($A \cap B$) /p(B)
p($A \cap B$) = b falls in the middle and c must be to the right of b.
C(5, 3) for the 3 spots in front of b, could be any of the 5 letters but not c,
And C(6, 3) could be any of the 6 letters to the right of b.
p(B) = (7!/2!)

C(5, 3) * C(6, 3) / (7!2!) = 5/63

e. Events A and C and not independent since p(A|C) != p(A). Events B and C are independent since p(B|C) = p(B). Events A and B are not independent since p(A|) != p(A)

b. Exercise 6.3.6 sections b, c

A biased coin is flipped 10 times. In a single flip of the coin the probability of heads is ⅓ the probability of tails is ⅔. THe outcomes are mutually independent.

b. Probability that the first 5 flips come up heads, the last 5 flips come up tails.
$(1/3)^5 * (2/3)^5$

c. Probability that the first flip comes up heads. The rest of the flips come up tails.
$(1/3) * (2/3)^9$

c. Exercise 6.4.2 section a

Assume you have two dice, one of which is fair, and the other is biased toward landing on six, so that 0.25 of the time it lands on six, and 0.15 of the time it lands on each of 1, 2, 3, 4 and 5. You choose a die at random, and roll it six times, getting the values 4, 3, 6, 6, 5, 5. What is the probability that the die you chose is the fair die? The outcomes of the rolls are mutually independent.
p(F) = chance picking fair die = 1/2
p(N) = chance picking not fair die = ½

p(X|F) = ⅙ chance it lands on 6
p(X|N) = ¼ chance it lands on 6

$$p(F|X) \;=\; \frac{(1/6)^6 * (1/2)}{(1/6)^6 * (1/2) + (0.15)^4 (1/4)^2 * ()1/2}$$
= .404

Question 9

a. Exercise 6.5.2 sections, a, b

A hand of 5 cards is dealt from a perfectly shuffled deck of playing cards. Let the random variable A denote the number of aces in the hand.

a. A five card hand could have 0 aces, or 4 aces.
Range: {0, 1, 2, 3, 4}

b. Distribution of A
Total possible combinations: C(52, 5)
0 aces: must be 48 of the other cards in the deck
1 ace: C(4, 1) suit *C(48, 4) for the other cards
2 aces: C(4, 2) suit for the ace * C(48, 3) for the other cards
3 aces: C(4, 3) suit for the ace * C(48, 2) for the other cards
4 aces: C(4, 4) (is just 1) and 48 other options for the other cards

{(0, C(48, 5)/ C(52, 5)) , (1, C(4, 1)*C(48, 4)/ C(52, 5)) , (2, C(4, 2)*C(48, 4)/ C(52, 5)
(3, C(4, 3)*C(48, 2)/ C(52, 5)), (4, 48/ C(52, 5))}

b. Exercise 6.6.1 section a

a. Two student council representatives are chosen at random from a group of 7 girls and 3 boys. Let G be the random variable denoting the number of girls chosen. What is E[G]?
Total choices: C(10, 2)
Choice of 2 girls: C(7, 2)
Choice 1 girl: C(7, 1) * C(3, 1)
Choice no girls: C(3, 2)
2 * (C(7, 2)/ C(10, 2)) + 1 * (C(7, 1)* C(3, 1)/ C(10, 2)) + 0 * (C(3, 2)/ C(10, 2))
= 7/5 girls

c. Exercise 6.6.4 sections a, b

a. A fair die is rolled once. Let X be the random variable that denotes the square of the number that shows up on the die. For example, if the die comes up 5, then X = 25. What is E[X]?
Chance 1: 1 * (1/6)
Chance 4: 4 * (1/6)
Chance 9: 9 * (1/6)
Chance 16: 16 * 1/6
Chance 25: 25 * (1/6)
Chance 36: 36 * ⅙
= (1/6) + (4/6) + (9/ 6) + (16 / 6) + (25/ 6) + (36/ 6)
= 91 /6

b. A fair coin is tossed three times. Let Y be the random variable that denotes the square of the number of heads. For example, in the outcome HTH, there are two heads and Y = 4. What is E[Y]?

Total combos: 8

3 heads: 9 * (1/8)

2 heads: 4 * (C(3, 2) / 8) = 4 * (3/8)

1 head: 1 * (C(3, 1) / 8) = 1 * (3/8)

No heads: 0 * (1/ 8)

= 9/8 + 12/ 8 + ⅜ + 0

= 3

d. Exercise 6.7.4 section a

a. A class of 10 students hang up their coats when they arrive at school. Just before recess, the teacher hands one coat selected at random to each child. What is the expected number of children who get his or her own coat?

Each child gets 1/10 chance of getting their coats.

For 10 children: 10 * 1/10 = 1

Expect 1 child to get their coat.

Question 10

a. Exercise 6.8.1 sections a, b, c, d

The probability that a circuit board produced by a particular manufacturer has a defect is 1%. You can assume that errors are independent, so the event that one circuit board has a defect is independent of whether a different circuit board has a defect.

a. Probability that 100 circuit boards have 2 defects?

$C(100, 2)* (.99)^{98}(.01)^{2}$

$b(2;100, .01) = C(100, 2)* (.99)^{98}(.01)$

b. Probability that 100 circuit boards have at least 2 defects?

1 - probability that 100 circuit boards have 1 defect and no defect.

1 defect = $C(100, 1) * (.99)^{99} (.01) = (.99)^{99}$

0 defects = $C(100, 0) * (.99)^{100}(.01)^{0} = (.99)^{100}$

$1- ((.99)^{99} + (.99)^{100})$

c. Expected number of circuit boards with defects out of the 100 made?

100 * .01 = 1

1

d. Now suppose that the circuit boards are made in batches of two. Either both circuit boards in a batch have a defect or they are both free of defects. The probability that a batch has a defect is 1%. What is the probability that out of 100 circuit boards (50 batches) at least 2 have defects? What is the expected number of circuit boards with defects out of the 100 made? How do your answers compared to the situation in which each circuit board is made separately?

What is the probability that out of 50 batches at least 1 has a defect?

1- no defect

No batch has defect: $C(50, 0)* (.99)^{50}(.01)^{0}$

$1- (.99)^{50}$

The 50 batches has a higher probability of expected defects.

b. Exercise 6.8.3 section b

A gambler has a coin which is either fair (equal probability heads or tails) or is biased with a probability of heads equal to 0.3. Without knowing which coin he is using, you ask him to flip the coin 10 times. If the number of heads is at least 4, you conclude that the coin is fair. If the number of heads is less than 4, you conclude that the coin is biased.

b. What is the probability that you reach an incorrect conclusion if the coin is biased?

Heads: 0.3

Tails: 0.7

If more than 3 heads, then incorrect conclusion

1 - (chance of 3 or less heads)

3 heads: $C(10, 3) * (.3)^3(.7)^7$
2 heads: $C(10, 2) * (.3)^2(.7)^8$
1 head: $C(10, 1) * (.3)^1(.7)^9$
0 head: $C(10, 0) * (.3)^0(.7)^{10}$

$1 -((0.7)^{10} + C(10, 1) * (0.3)^1(0.7)^9 + C(10, 2) * (0.3)^2(0.7)^8 + C(10, 3) * (.3)^3(.7)^7)$