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<u>Question 7</u>

Solve the following:

A. Exercise 3.1.1, section a, b, c, d, e, f, g

Use the definitions for the sets given to determine whether each statement is true or false.

$$A = \{ x \in \mathbf{Z}: \text{x is an integer multiple of 3} \}$$
$$B = \{ x \in \mathbf{Z}: \text{x is a perfect square} \}$$
$$C = \{ 4, 5, 9, 10 \}$$
$$D = \{ 2, 4, 11, 14 \}$$
$$E = \{ 3, 6, 9 \}$$
$$F = \{ 4, 6, 16 \}$$

a. True
- 27 is a an integer multiple of 3, therefore x is an element of A.

b. False
- 27 is not a perfect square, therefore x is not an element of B

c. True
- 100 is a perfect square, therefore x is an element of B

d. False
- $E \not\subset C$ since $3 \notin C$, $C \not\subset E$ since $4 \notin E$

e. True
- Every element of E is an element of A.

f. False
- $12 \in A$, but $12 \notin E$

g. False
- E is a set of 3 integers, while all elements of A are integers.

B. Exercise 3.1.2, section a, b, c, d, e

Use the definitions for the sets given below to determine whether each statement is true or false.

$$A = \{ x \in \mathbf{Z}: \text{x is an integer multiple of 3} \}$$
$$B = \{ x \in \mathbf{Z}: \text{x is a perfect square} \}$$
$$C = \{ 4, 5, 9, 10 \}$$
$$D = \{ 2, 4, 11, 14 \}$$
$$E = \{ 3, 6, 9 \}$$
$$F = \{ 4, 6, 16 \}$$

a. False

- 15 is not a set.

b. True

- 15 is an integer that is a multiple of 3, there for {15} is a subset of A

c. True

- ∅ is a subset A, where as A is not a subset of ∅

d. True

- Every element of A is in A.

e. False

- ∅ is not an element of B, as ∅ is a set.

C. Exercise 3.1.5, section b, d

Express each set using set builder notation. If the set is finite give its cardinality, otherwise indicate that it is infinite.

b. {3, 6, 9, 12…}

B = { x ∈ $\mathbf{Z}^+$: x is a multiple of 3}. This set is infinite.

d. {0, 10, 20, 30, …, 1000}

D= {x ∈ **N** : x is a multiple of 10 and x <= 1000) }The cardinality is 101.

D. Exercise 3.2.1, section a, b, c, d, e, f, g, h, i, j, k

Let X = {1, {1}, {1, 2}, 2, {3}, 4}. Which statements are true?

a. True
b. True
    i. 2 is an element of x, so {2} is a subset of x
c. False
d. False
e. True
f. True
    i. 1 and 2 are elements of X, therefore {1, 2} is a subset of X
g. True
h. False
i. False
j. False
k. False. The cardinality of X is 6.

<u>Question 8</u>

Solve Exercise 3.2.4 section b

B. Let A = {1, 2, 3}. What is $\{X \in P(A): 2 \in X\}$?

P(A) = {∅ ,1, 2, 3, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3, }}

X is an element of P(A) such as 2 is an element of X.

$\{X \in P(A): 2 \in X\}$ ={{2}, {1, 2}, {2, 3}, {1, 2, 3}}

Question 9

Solve the following questions:

A. Exercise 3.3.1, sections c, d, e

For each of the following set expressions, if the corresponding set is finite, express the set using roster notation. Otherwise indicate that the set is infinite.

$$A = \{-3, 0, 1, 4, 17\}$$
$$B = \{-12, -5, 1, 4, 6\}$$
$$C = \{x \in \mathbf{Z}: x \text{ is odd}\}$$
$$D = \{x \in \mathbf{Z}: x \text{ is positive}\}$$

c. $A \cap C$

$A = \{-3, 0, 1, 4, 17\}$

$C = \{x \in \mathbf{Z}: x \text{ is odd}\}$

$A \cap C = \{-3, 1, 17\}$

d. $A \cup (B \cap C)$

$(B \cap C) = \{-5, 1\}$

$B = \{-12, -5, 1, 4, 6\}$

$C = \{x \in \mathbf{Z}: x \text{ is odd}\}$

$A \cup (B \cap C) = \{-5, -3, 0, 1, 4, 17\}$

e. $A \cap B \cap C$

$A \cap B = \{1, 4\}$

$A = \{-3, 0, 1, 4, 17\}$

$B = \{-12, -5, 1, 4, 6\}$

$A \cap B \cap C$

$A \cap B = \{1, 4\}$

$C = \{x \in \mathbf{Z}: x \text{ is odd}\}$

$A \cap B \cap C = \{1\}$

B. Exercise 3.3.3, sections a, b, e, f

Use the following definitions to express each union or intersection. For each definition, $i \in \mathbf{Z}^+$

$$A_i = \{i^0, i^1, i^2\}$$
$$B_i = \{x \in \mathbf{R} : -i \le x \le 1/i\}$$
$$C_i = \{x \in \mathbf{R} : -1/i \le x \le 1/i\}$$

a. $A_i = \{i^0, i^1, i^2\}$

Intersection where range is 5 with integer 2

$= \{1, 2, 4\} \cap \{1, 3, 9\} \cap \{1, 4, 16\}, \{1, 5, 25\}$

$\bigcap_{i=2}^{5} A_i = \{1\}$

b. $A_i = \{i^0, i^1, i^2\}$

Union where range is 5, starting at integer 2

$= \{1, 2, 4\} \cup \{1, 3, 9\} \cup \{1, 4, 16\} \cup \{1, 5, 25\}$

$\cup^5_{1=2} A_i = \{1, 2, 3, 4, 5, 9, 16, 25\}$

e. $C_i = \{x \in \mathbf{R} : -1/i \leq x \leq 1/i\}$

Intersection for the first 100 numbers 100 and start at 1

The largest range for x occurs when i = 1

$\{-1 \leq x \leq 1\}$

The smallest range for x occurs when i = 100

$\{-1/100 \leq x \leq 1/100\}$

$C_b \subseteq C_i$ when $b < i$,

$\cap_{i=1}^{100} C_i = \{x \in \mathbf{R} : -1/100 \leq x \leq 1/100\}$

f. $C_i = \{x \in \mathbf{R} : -1/i \leq x \leq 1/i\}$

Union where range is 100 and integer is 1

The smallest possible set would be {-1}, and the largest possible set would be {1}. And all possibilities in between are valid.

$\cup^{100}_{i=1} C_i = \{x \in \mathbf{R} : -1 \leq x \leq 1\}$

C. Exercise 3.3.4, sections b, d

Use the set definitions to express each set below. Use roster notation in your solutions.

$A = \{a, b\}$

$B = \{b, c\}$

b. $P(A \cup B)$

$A \cup B = \{a, b, c\}$

$P(A \cup B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a, b, c\}\}$

d. $P(A) \cup P(B)$

$P(A) = \{\{\varnothing\}, \{a\}, \{b\}, \{a, b\}\}$

$P(B) = \{\{\varnothing\}, \{b\}, \{c\}, \{c, b\}\}$

$P(A) \cup P(B) = \{\varnothing, \{a\}, \{b\}, \{c\}, \{a, b\}, \{c, b\}\}$

Question 10

Solve the following questions:

a. Exercise 3.5.1, sections b, c

The sets A, B, C are defined as follows:

A= {tall, grande, venti}

B = {foam, no-foam}

C = {non-fat, whole}

Use the definitions for A, B, C to answer the questions. Express the elements using n-tuple notation, not string notation

b. Write an element from the set B x A x C

B x A x C = {foam, tall, non-fat}

c. Write the set B x C using roster notation

B x C = {{foam, non-fat}, {foam, whole}, {no-foam, non-fat}, {no-foam, whole}}

b. Exercise 3.5.3, sections b, c, e

Indicate which of the following statements are true:

b. $\mathbf{Z}^2 \subseteq \mathbf{R}^2$

True, all real integers squared is a set of all real numbers squared.

c. $\mathbf{Z}^2 \cap \mathbf{Z}^3 = \varnothing$

True, the elements of Z^2 are doubles, while Z^3 are all triplets.

e. For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.

True, if A is a subset of B, then A x C is a subset of B x C since C is a common denominator in both AC and BC now.

c. Exercise 3.5.6, sections d, e

Express the following sets using the roster method. Express the elements as strings, not n-tuples.

d. {xy: where $x \in \{0\} \cup \{0\}^2$ and $y \in \{1\} \cup \{1\}^2$ }

x= {0, 00}

y = {1, 11}

xy = {01, 011, 001, 0011}

e. {xy: x ∈ {aa, ab} and y ∈ {a} ∪ $\{a\}^2$ }

x = {aa, ab}

y = {a, aa}

xy = {aaa, aaaa, aba, abaa}

d. Exercise 3.5.7, sections c, f, g

Use the following set definitions to specify each set in roster notation. Except where noted, express elements of Cartesian products as string.

A = {a}

B = {b, c}

C = {a, b, d}

c. (A × B) ∪ (A × C)

(A x B) = {ab, ac}

(A x C) = {aa, ab, ad}

(A × B) ∪ (A × C) = {{a, b}, {a, c}, {a, a}, {a, d}}

f. P(A × B)

(A x B) = {ab, ac}

P(A x B) = {∅,{ab}, {ac}, {ab, ac}}

g. P(A) × P(B). Use ordered pair notation for elements of the Cartesian product.

P(A) = {∅, {a}}

P(B) = {∅, {b}, {c}, {bc}}

P(A) × P(B)= {(∅, ∅), (∅, {b}), (∅, {c}), (∅, {bc}), ({a}, ∅), ({a}, {b}), ({a}, {c}), ({a}, {bc})}

<u>Question 11</u>

Solve the following questions

A. Exercise 3.6.2, sections, b, c

b. $(B \cup A) \cap (\overline{B} \cup A) = A$

$(B \cup A) \cap (\overline{B} \cup A)$	
$(B \cap \overline{B}) \cup A$	1. Distributive
$\varnothing \cup A$	2. Complement Law
A	3. Identity Law

See table above.

c. $\overline{A \cap \overline{B}} = \overline{A} \cup B$

$\overline{A \cap \overline{B}}$	
$\overline{A} \cup \overline{\overline{B}}$	1. De Morgans Law
$\overline{A} \cup B$	2. Double Complement Law

See table above.

B. Exercise 3.6.3, sections b, d

Show that each set equation given below is not a set identity.

b. $A - (B \cap A) = A$

Lets say A: {1, 2, 3}

B: {3}

$(B \cap A) = \{1, 2, 3\}$

$A - (B \cap A) = \varnothing$

The above would result in an empty set, not set A.

d. $(B - A) \cup A = A$

Lets say A: {1, 2, 3}

B: {3, 4, 5}

$(B - A) = \{4, 5\}$

$(B - A) \cup A = \{1, 2, 3, 4, 5\}$

The above is not equivalent to set A.

C. Exercise 3.6.4, section b, c

Use the set subtraction law as well as other set identities given to prove each of the following new identities.

b. $A \cap (B - A) = \varnothing$

$A \cap (B - A)$	
$A \cap (B \cap \overline{A})$	1. Set subtraction Law
$A \cap (\overline{A} \cap B)$	2. Commutative Law
$(A \cap \overline{A}) \cap B$	3. Associative Law
$\varnothing \cap B$	4. Complement Law
$\varnothing$	5. Domination Law

See table above.

c. $A \cup (B - A) = A \cup B$

$A \cup (B - A)$	
$A \cup (B \cap \overline{A})$	1. Set subtraction Law
$(A \cup B) \cap (A \cup \overline{A})$	2. Distributive Law

$(A \cup B) \cap U$	3. Complement Law
$(A \cup B)$	4. Identity Law

See table above.