

Rules of **Probability****Independent**

$$P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$$

$$\begin{aligned} P(X \cap Y) &= P(Y \cap X) = P(X) \cdot P(Y | X) \\ &= P(Y) \cdot P(X | Y) \end{aligned}$$

$$\begin{aligned} P(X \cap Y) &= P(Y \cap X) = P(X) \cdot P(Y) \\ &= P(Y) \cdot P(X) \end{aligned}$$

$$\begin{aligned} P(Y) &= 1 - P(Y') \\ &= P(Y \cap A) + P(Y \cap B) + \dots + P(Y \cap Z) \\ &= P(Y \cap X) + P(Y \cap !X) \\ &= P(X) \cdot P(Y | X) + P(!X) \cdot P(Y | !X) \\ &= P(Y) \cdot P(X | Y) + P(Y) \cdot P(!X | Y) \end{aligned}$$

$$\begin{aligned} P(X | Y) &= \frac{P(X \cap Y)}{P(Y)} & P(X | Y) &= P(X) \\ &= \frac{P(X) \cdot P(Y | X)}{P(Y)} = \frac{P(Y) \cdot P(X | Y)}{P(Y)} \\ &= \frac{P(X) \cdot P(Y | X)}{P(X) \cdot P(Y | X) + P(!X) \cdot P(Y | !X)} = \frac{P(Y) \cdot P(X | Y)}{P(X) \cdot P(Y | X) + P(!X) \cdot P(Y | !X)} \\ &= \frac{P(X) \cdot P(Y | X)}{P(Y) \cdot P(X | Y) + P(Y) \cdot P(!X | Y)} = \frac{P(Y) \cdot P(X | Y)}{P(Y) \cdot P(X | Y) + P(Y) \cdot P(!X | Y)} \end{aligned}$$

$$P(X \cap Y \cap Z) = P(Y) \cdot P(X | Y) \cdot P(Z | X \cap Y)$$

$$P(X \cup Y \cup Z) = P(X) + P(Y) + P(Z) - P(X \cap Y) - P(X \cap Z) - P(Y \cap Z) + P(X \cap Y \cap Z)$$

$$P(A) + P(B) + P(C) = 1 \text{ if mutually exclusive}$$

**discrete**1. **probability mass function (PMF)**

list all possible probabilities of x

$$p_X(x) = P(X = x)$$

- (a) probability of x must be in  $[0, 1]$
- (b) sum of all probabilities must equal to 1
- (c) R: hist(x)  
with x in x-axis and  $p_X(x)$  in y-axis

2. **cumulative distribution function (CDF)**

probability that a random variable is less than or equal to x

$$F_X(x) = P(X \leq x)$$

- (a)  $\lim_{x \rightarrow -\infty} F_X(x) = 0$
- (b)  $\lim_{x \rightarrow \infty} F_X(x) = 1$
- (c) non-decreasing

3.  $E_X(x) = \text{theoretical mean}$ 

For random variable x,

$$E_X(x) = \sum_{all\ x} x \cdot p_X(x)$$

For function g(x),

$$E_X(g(x)) = \sum_{all\ x} g(x) \cdot p_X(x)$$

**continuous**1. **probability density function (PDF)**

list all possible probabilities of x

$$f_X(x) = \frac{\partial}{\partial x} F_X(x)$$

- (a)  $f_X(x) \geq 0$  for all x
- (b) area under the curve is equal to 1
- (c) R: plot(density(x))  
with x in x-axis and  $f_X(x)$  in y-axis

2. **cumulative distribution function (CDF)**

probability that a random variable is less than or equal to x

$$F_X(x) = P(X \leq x) = \int_{-\infty}^x f_X(x) dx$$

$$P(a \leq X \leq b) = F_X(b) - F_X(a) = \int_a^b f_X(x) dx$$

- (a)  $\lim_{x \rightarrow -\infty} F_X(x) = 0$
- (b)  $\lim_{x \rightarrow \infty} F_X(x) = 1$
- (c) non-decreasing

3.  $E_X(x) = \text{theoretical mean}$ 

For random variable x,

$$E_X(x) = \int_{-\infty}^{\infty} x \cdot f_X(x) dx$$

For function g(x),

$$E_X(g(x)) = \int_{-\infty}^{\infty} g(x) \cdot f_X(x) dx$$

1. Rules of **Expectation**

- (a)  $E(aX + b) = a \cdot E(X) + b$
- (b)  $E(Y | X) = \int_{-\infty}^{\infty} y \cdot f_{Y|X}(y | x) dy$
- (c)  $E(XY) = E(E(XY | X)) = E(X \cdot E(Y | X))$
- (d)  $E(XY) = E(X) \cdot E(Y)$  if X, Y independent
- (e)  $E(XY | X) = X \cdot E(Y | X)$
- (f)  $E(Y) = E(E(Y | X))$
- (g)  $E_X(x^2) = \int_{-\infty}^{\infty} x^2 \cdot f_X(x)$

2. Rules of **Variance**

- (a)  $var(X) = \sigma^2 = E(X^2) - E(X)^2$
- (b)  $var(X + c) = var(X)$
- (c)  $var(cX) = c^2 \cdot var(X)$
- (d)  $var(X + Y) = var(X) + var(Y) + 2 \cdot cov(X, Y)$
- (e)  $var(X + Y) = var(X) + var(Y)$  if X, Y independent

3. Rules of **Standard Deviation**

- (a)  $\sigma = \sqrt{var(X)}$

4. Rules of **Normal Distribution**

- (a)  $E(X) = 0$
- (b)  $E(X^2) = 1$
- (c)  $E(X)^2 = 0$
- (d)  $\sigma = 1$
- (e)  $var(X) = 1$

5. Rules of **Joint Distribution**

Remember the rules of probability above (for discrete) applies here.

(a) **Marginal**

$$\begin{aligned}
 f_X(x) &= \int_{-\infty}^{\infty} f_{X,Y}(x, y) dy \\
 &= \int_{-\infty}^{\infty} f_X(x) \cdot f_{Y|X}(y | x) dy \\
 f_Y(y) &= \int_{-\infty}^{\infty} f_{X,Y}(x, y) dx \\
 &= \int_{-\infty}^{\infty} f_Y(y) \cdot f_{X|Y}(x | y) dx
 \end{aligned}$$

- (b)  $f_{X|Y} = \frac{f_{X,Y}(X,Y)}{f_Y(y)}$
- (c)  $f_{X,Y} = f_X(x) \cdot f_Y(y)$  if X, Y independent

6. Rules of **Covariance**

Measure the strength of linear relationship b/w 2 variables

- (a)  $cov(X, Y) = E((X - \mu_X)(Y - \mu_Y)) = E(XY) - E(X) \cdot E(Y)$
- (b)  $cov(X, X) = var(X)$
- (c)  $cov(aX, bY) = ab \cdot cov(XY)$
- (d)  $cov(X, Y + Z) = cov(X, Y) + cov(X, Z)$

7. Rules of **Correlation**

Normalize Covariance by scaling it b/w -1 and 1

- (a)  $corr(X, Y) = \frac{cov(X, Y)}{\sigma_X \cdot \sigma_Y}$

9. Rules of **Complement**

- (a)  $P(X > 3) = 1 - P(X \leq 3)$
- (b)  $P(2 \leq X \leq 5) = P(X \leq 5) - P(X \leq 1)$
- (c)  $P(2 < X < 5) = P(3 \leq X \leq 4) = P(X \leq 4) - P(X \leq 2)$

10. Rules of **Sum**

- (a) Finite Arithmetic  
i.e. 3, 7, 11, ..., 51  
 $S_n = \frac{n}{2}(a_1 + a_n) = n[a_1 + \frac{n-1}{2}d]$
- (b) Finite Geometric,  $r = \frac{a_n}{a_{n-1}}$   
i.e. 1, 2, 4, ..., 64  
 $S_n = a_1 \frac{1-r^n}{1-r} = \frac{a_1-ra_n}{1-r}$
- (c) Infinite Geometric,  $r = \frac{a_n}{a_{n-1}}$ ,  $-1 < r < 1$   
i.e. 3, 1,  $\frac{1}{3}$ ,  $\frac{1}{9}$ ,  $\frac{1}{27}$ , ...  
 $S = \frac{a_1}{1-r}$
- (d) Infinite Arithmetic and Infinite Geometric w/  
 $r \leq -1$  or  $1 \leq r$   
No formula

11. Rules of **Log**

$$\begin{aligned}
 \log_2(2^k) &= \log_2(9801) \\
 k &= \frac{\log_{10}(9801)}{\log_{10}(2)}
 \end{aligned}$$

**Discrete Joint Probability Table**

	$y_1$	$y_2$	...	$y_n$	
$x_1$	$f_{X,Y}(x_1, y_1)$	$f_{X,Y}(x_1, y_2)$	...	$f_{X,Y}(x_1, y_n)$	$f_X(x_1)$
$x_2$	$f_{X,Y}(x_2, y_1)$	$f_{X,Y}(x_2, y_2)$	...	$f_{X,Y}(x_2, y_n)$	$f_X(x_2)$
...	...	...	...	...	...
$x_m$	$f_{X,Y}(x_m, y_1)$	$f_{X,Y}(x_m, y_2)$	...	$f_{X,Y}(x_m, y_n)$	$f_X(x_m)$
	$f_Y(y_1)$	$f_Y(y_2)$	...	$f_Y(y_n)$	1

Distribution	Type	Description	PMF or PDF	$E_X(x)$	$var(x)$
Bernoulli	Discrete	Binomial with $n = 1$	$p^x(1-p)^{1-x}$	$p$	$1-p$
Binomial	Discrete	$n$ independent Bernoulli trials	$\binom{n}{x}p^x(1-p)^{n-x}$	$np$	$np(1-p)$
Geometric	Discrete	get first success in independent Bernoulli trials	$(1-p)^{x-1}p$	$\frac{1}{p}$	$\frac{1-p}{p^2}$
Negative Binomial	Discrete	get $r^{th}$ success in independent Bernoulli trials	$\binom{x-1}{r-1}p^r(1-p)^{x-r}$	$\frac{r}{p}$	$\frac{r(1-p)}{p^2}$
Hypergeometric	Discrete	sampling objects w/o replacement	$\frac{\binom{A}{a}\binom{B}{b}}{\binom{N}{n}} = \frac{\binom{a}{x}\binom{N-a}{n-x}}{\binom{N}{n}}$	$\frac{n \cdot a}{N}$	$\frac{n \cdot a \cdot (N-a) \cdot (N-n)}{N^2 \cdot (N-1)}$
Poisson	Discrete	# of occurrences of an event	$\frac{\lambda^x \cdot e^{-\lambda}}{x!}$	$\lambda$	$\lambda$
Uniform	Continuous	constant probability	$\frac{1}{b-a}, a \leq x \leq b$	$\frac{a+b}{2}$	$\frac{(b-a)^2}{12}$