

W271 Lab 1

Tiffany Jaya, Joanna Huang, Shan He, Robert Deng

```
# add packages
library(dplyr); library(ggplot2); library(knitr); library(stargazer)
# prevent source code from running off the page
opts_chunk$set(tidy.opts=list(width.cutoff=70), tidy=TRUE)
# remove all objects from current workspace
rm(list = ls())
# set seed number to reproduce results
set.seed(1)
# load data
d <- read.csv("./dataset/challenger.csv", header=TRUE, sep=",")
# set min and max for temp, temp at launch, recommended temp to launch
temp.min <- 31; temp.max <- 81; temp.launch <- 31; rec.temp <- 72
# set min and max for pressure, pressure at launch
pressure.min <- 50; pressure.max <- 200; pressure.launch <- 200
```

Introduction

On January 28, 1986, the space shuttle Challenger exploded after a mere 73 seconds into its flight, resulting in the death of all seven crew members. An investigation ensued and found that the accident was caused by the O-rings failing to seal the joints properly in the rocket booster. Two potential variables were investigated: temperature and pressure. Based on findings at the time, researchers surmised that when the temperature is cold, similar to the temperature at the time of the launch at 31°F, O-ring retains its cold-compressed shape, causing the pressurized gas from within to leak out and initiate the disintegration of the vehicle. Furthermore, there was a change in the leak check procedure from 50 psi to 200 psi to check the position of the O-ring.

Based on this background information, we seek to retrospectively analyze and predict how the specific conditions of temperature and pressure may have led to the Challenger explosion. Using logistic regression modelling, we answer the question: Were the temperature at the time of the launch or the change in the leak check procedure contributing factors to the O-rings failure, and consequently the structural breakdown of the space shuttle Challenger?

After a series of analyses, we chose a logistic regression model $\text{logit}(Y) = \beta_0 + \beta_1 * \text{Temp}$ which tells us that for every 1 degree Fahrenheit decrease in temperature, the odds of the O-rings failing for a given launch increases by 26%.

EDA

In order for us to have a better understanding of this relationship, we explored the dataset that contains 23 observations, one for each of the prior flights Challenger took before its final one. Looking at the dataset, there are five numerical attributes with no missing values:

- Flight: the temporal order of flights
- Temp: the temperature at the time of launch (in degree Fahrenheit)
- Pressure: the air pressure used in the leak check procedure (in psi)
- O.ring: the number of O-rings failure on the given flight

- Number: the total number of O-rings, which remains constant at six total, because the rocket booster has three field joints with two O-rings each. This variable has no bearing on O-rings failure.

```
# structure of data
```

```
str(d)
```

```
## 'data.frame':    23 obs. of  5 variables:
## $ Flight   : int  1 2 3 4 5 6 7 8 9 10 ...
## $ Temp     : int  66 70 69 68 67 72 73 70 57 63 ...
## $ Pressure: int  50 50 50 50 50 50 100 100 200 200 ...
## $ O.ring   : int  0 1 0 0 0 0 0 0 1 1 ...
## $ Number   : int  6 6 6 6 6 6 6 6 6 6 ...
```

```
# summary of data
```

```
kable(summary(d))
```

Flight	Temp	Pressure	O.ring	Number
Min. : 1.0	Min. :53.00	Min. : 50.0	Min. :0.0000	Min. :6
1st Qu.: 6.5	1st Qu.:67.00	1st Qu.: 75.0	1st Qu.:0.0000	1st Qu.:6
Median :12.0	Median :70.00	Median :200.0	Median :0.0000	Median :6
Mean :12.0	Mean :69.57	Mean :152.2	Mean :0.3913	Mean :6
3rd Qu.:17.5	3rd Qu.:75.00	3rd Qu.:200.0	3rd Qu.:1.0000	3rd Qu.:6
Max. :23.0	Max. :81.00	Max. :200.0	Max. :2.0000	Max. :6

```
# list row with missing values
```

```
d[!complete.cases(d), ]
```

```
## [1] Flight Temp Pressure O.ring Number
## <0 rows> (or 0-length row.names)
```

Univariate analysis of O.ring

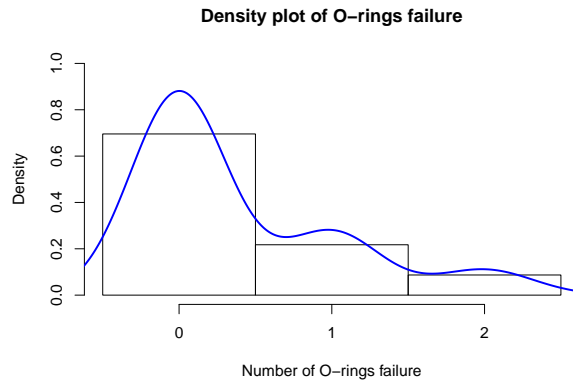
```
# frequency of O-rings failure
```

```
table(d$O.ring)
```

```
##
##  0  1  2
## 16  5  2
```

```
# density of O-rings failure
```

```
hist(d$O.ring, xlab = "Number of O-rings failure", main = "Density plot of O-rings failure",
      breaks = 0:3 - 0.5, xaxt = "n", ylim = c(0, 1), freq = FALSE)
axis(1, at = seq(min(d$O.ring), max(d$O.ring)))
lines(density(d$O.ring), col = "blue", lwd = 2)
```

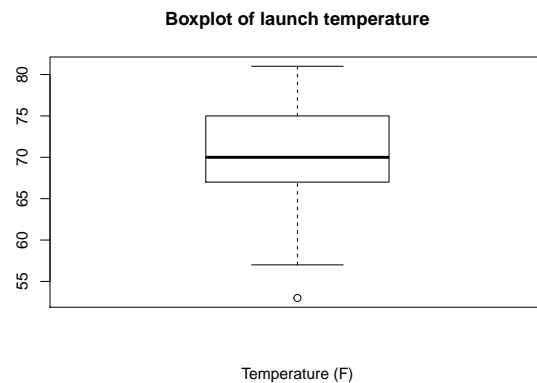
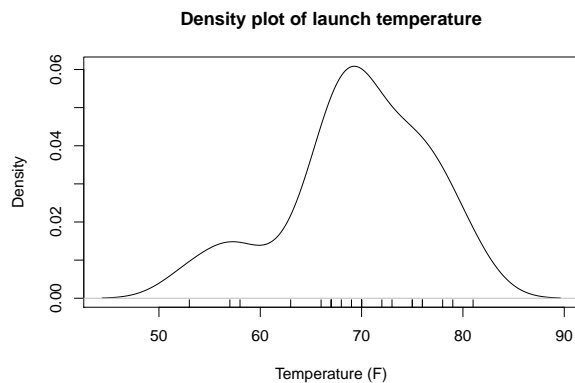


Since the disaster occurred as a result of O-ring failure, the O-ring variable will be the variable that we are trying to predict. From the density plot of the sample, we see an event of no O-ring failure twice as many times in the dataset than a single O-ring failure and seven times more than a second O-ring failure.

Univariate analysis of Temp

```
# density of launch temperature
plot(density(d$Temp), main = "Density plot of launch temperature", xlab = "Temperature (F)")
rug(d$Temp)
# boxplot of launch temperature
boxplot(d$Temp, main = "Boxplot of launch temperature", xlab = "Temperature (F)")
# display temperature outlier
d %>% filter(Temp == min(Temp))
```

```
## Flight Temp Pressure O.ring Number
## 1      14      53      200      2      6
```



As a potential cause of the O-ring failure, Temp is another variable that we analyze. Its density distribution is slightly negatively skewed with the mean being smaller than the median (see summary table), meaning they perform more test launches when the temperature is warmer. It is also interesting to note that two O-rings fail at the outlier temperature of 53 °F and the updated leak check pressure of 200 psi.

Univariate analysis of Pressure

```

# frequency of air pressure used in leak check procedure
table(d$Pressure)

##
## 50 100 200
## 6 2 15

# the leak check procedure used in flights
sapply(unique(d$Pressure), function(p) c(min(which(d$Pressure %in% p)),
max(which(d$Pressure %in% p))))

##      [,1] [,2] [,3]
## [1,]    1    7    9
## [2,]    6    8   23

```

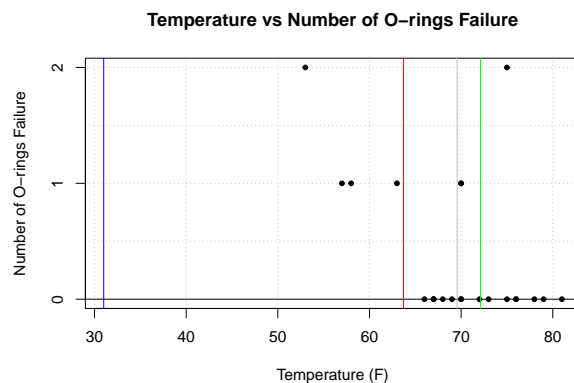
In addition to Temp, we analyze Pressure, the second potential predictor of O-rings failure. The leak check procedure uses air pressure to check the position of the O-ring and changes 3 times: flight 1-6 using an air pressure of 50 psi, flight 7-8 using 100 psi, and flight 9-23 using 200 psi. The increase in air pressure for the leak check procedure is mostly due to a growing desire to ensure that a leak is not masked. Later in the report, we will analyze if the increase in air pressure corresponded to an increase in the number of O-rings failing.

Bivariate analysis

```

# average temperature for flights with at least one O-rings failure
avg.temp.failure <- d %>% filter(., O.ring > 0) %>% .$Temp %>% mean(.)
# average temperature for flights with no O-rings failure
avg.temp.success <- d %>% filter(., O.ring == 0) %>% .$Temp %>% mean(.)
# average temperature for all flights
avg.temp <- mean(d$Temp)
# plot temperature vs number of O-rings failure
plot(main = "Temperature vs Number of O-rings Failure", x = d$Temp, xlab = "Temperature (F)",
      xlim = c(temp.min, temp.max), y = d$O.ring, ylab = "Number of O-rings Failure",
      yaxt = "n", panel.first = grid(col = "gray", lty = "dotted"), pch = 20)
abline(h = 0, v = c(avg.temp.failure, avg.temp.success, avg.temp, temp.launch),
      col = c("black", "red", "green", "gray", "blue"))
axis(side = 2, at = 0:2)

```



When we performed a bivariate analysis of Temp and O.ring variables, we started to notice that if we only looked at the average temperature of flights with at least one O-ring failure (red line), the number of O-rings failing in cold weather is approximately similar to that in the warm weather. It was these specific data points used by two staff members as a reasoning to launch Challenger at 31°F temperature (blue line). But if we took into consideration all previous flights, the average temperature of all 23 launches (gray line) or the average temperature of all successful launches (green line) give us clue that O-rings seem to perform better in warmer temperature.

When we performed a bivariate analysis of Temp and O.ring variables, we started to notice that if we only looked at the average temperature of flights with at least one O-ring failure (red line), the number of O-rings failing in cold weather is approximately similar to that in the warm weather. It was these specific data points used by two staff members as a reasoning to launch Challenger at 31°F temperature (blue line). But if we took into consideration all previous flights, the average temperature of all 23 launches (gray line) or the average temperature of all successful launches (green line) give us clue that O-rings seem to perform better in warmer temperature.

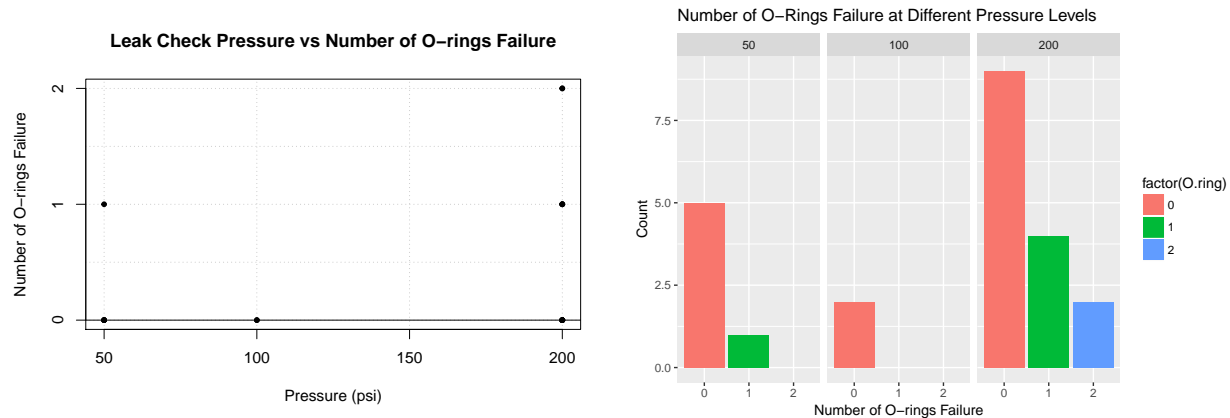
```
summary(aov(Temp ~ O.ring, data = d))
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## O.ring         1  286.2   286.24   7.426 0.0127 *
## Residuals     21  809.4    38.54
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
```

A one-way anova test confirms our intuition that the mean temperature of successful launches is different than the mean temperature of failure launches with a p-value of 0.01. This provides strong evidence to include temperature in the modeling section.

```
# plot pressure vs number of O-rings failure
plot(main = "Leak Check Pressure vs Number of O-rings Failure", x = d$Pressure,
      xlab = "Pressure (psi)", xlim = c(pressure.min, pressure.max), y = d$O.ring,
      ylab = "Number of O-rings Failure", yaxt = "n", panel.first = grid(col = "gray",
                                lty = "dotted"), pch = 20)
abline(h = 0, col = "black")
axis(side = 2, at = 0:2)

# plot number of o-rings failure at different pressure levels
ggplot(d, aes(x = factor(O.ring), fill = factor(O.ring))) + geom_bar() +
  facet_wrap(~Pressure) + labs(title = "Number of O-Rings Failure at Different Pressure Levels",
                                x = "Number of O-rings Failure", y = "Count")
```



Plotting with facets reveals that most O-ring failures occurred at 200 psi. Given that there are more variables in the 200 psi level than in the 50 and 100 psi levels, we may need to be more cautious when interpreting the model at low pressure levels. To investigate pressures between failed and successful O-ring states, a one-way anova test reveals a p-value of 0.188 in which it fails to reject the null hypothesis of no difference between the 3 pressure states. Hence, Pressure may not be a strong candidate as an explanatory variable for O-rings failure.

```
summary(aov(Pressure ~ O.ring, data = d))
```

```
##              Df Sum Sq Mean Sq F value Pr(>F)
## O.ring         1   8297    8297   1.852  0.188
## Residuals     21  94094    4481
```

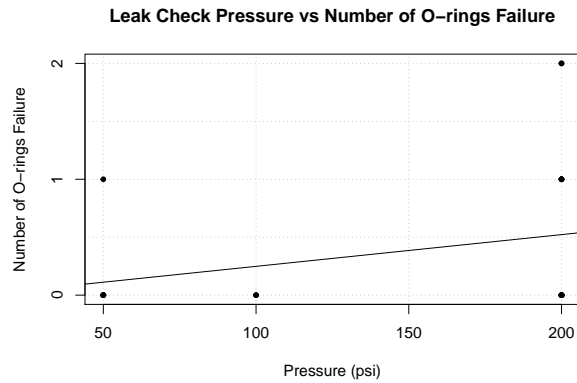
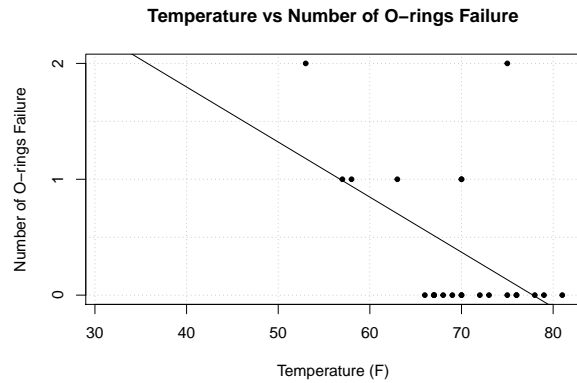
Why not linear regression model

So far, our exploration of the data gives us an intuition that there are more launches tested in warmer climate and in higher leak-check pressure. We would then like to extrapolate from these observations the likelihood that temperature and leak-check pressure play in causing the O-rings to fail on January 28th, the day of the launch.

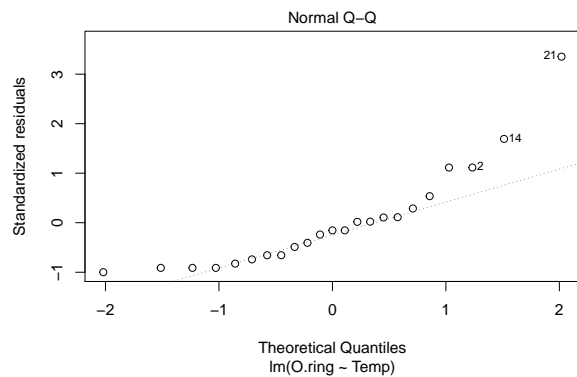
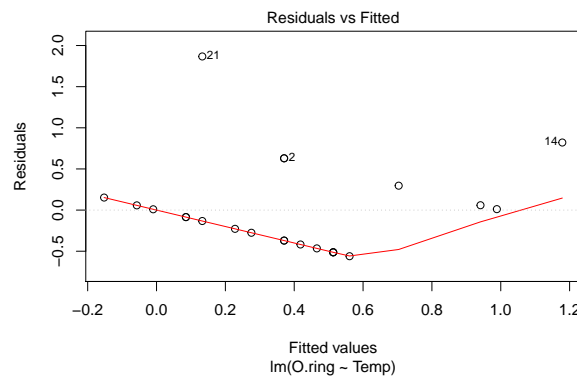
One way we can predict the number of O-ring failure in the physical condition of the launch (31°F and 200 psi) is to employ the linear regression model.

```
plot(main = "Temperature vs Number of O-rings Failure", x = d$Temp, xlab = "Temperature (F)",
     xlim = c(temp.min, temp.max), y = d$O.ring, ylab = "Number of O-rings Failure",
     yaxt = "n", panel.first = grid(col = "gray", lty = "dotted"), pch = 20)
abline(lm(O.ring ~ Temp, data = d))
axis(side = 2, at = 0:2)

plot(main = "Leak Check Pressure vs Number of O-rings Failure", x = d$Pressure,
     xlab = "Pressure (psi)", xlim = c(pressure.min, pressure.max), y = d$O.ring,
     ylab = "Number of O-rings Failure", yaxt = "n", panel.first = grid(col = "gray",
     lty = "dotted"), pch = 20)
abline(lm(O.ring ~ Pressure, data = d))
axis(side = 2, at = 0:2)
```



```
plot(lm(O.ring ~ Temp, data = d), which = c(1, 2))
```



As shown from the fitted model above, linear regression is an inadequate model for prediction. It fails to meet the assumption of homogeneity of variance and linear relation (see residual vs fitted) as well as the normal distribution of residuals (see QQ plot). In reality, it is not possible for the number of O-ring failure to be negative at higher temperature or lower psi or to be a fraction of a failure because an O-ring either fails or does not fail. Knowing that the O.ring variable is more discrete in nature than continuous, a better alternative is to use the logistic regression model.

Applying logistic regression model for a binary response

If we simply wanted to know the probability of at least one O-ring failing, then we can use the logistic regression model with a binary response (success or failure) to predict that probability. A success occurs when there is at least one O-ring failing ($O.ring > 0$), and a failure occurs when there are no O-rings failing ($O.ring == 0$). Unlike linear regression model, logistic regression model binds the predicted probability within zero and one, which eliminates the possibility of getting nonsensical results.

For our analyses, we decided to choose the binary logistic regression model over a binomial logistic regression model for mainly two reasons:

1. We were curious to see the contributing factor for odds that any O-ring failure occurs while we didn't necessarily care for the exact proportion of O-ring that failed
2. A binomial response model assumes the independence between O-rings on different joints while a binary response model is robust against any joint dependence

(Answer 4B and 5A) To ensure that there is a point of reference to compare the effect of the determinants onto the outcome, we employ a base case that takes in no explanatory variable which we will refer to as bm1.

We build models that separate out Temp (bm2) and Pressure (bm3, bm4) to better understand their effect individually on the likelihood of at least one O-ring failure. The reason why we separate the model for Pressure further into bm3 and bm4 is because we want to see the effect pressure had collectively on the response variable and at the same time the effect it had when the air pressure changed for the updated leak-check procedure.

In addition, we build bm5 and bm6 to see how Temp and Pressure together affect the probability of at least one O-ring failing.

(Answer 5F) To check if a quadratic term is needed in the model for temperature, we add bm7 into our analysis. As we later discussed in the validation of assumption section, since Temp has a linear relationship with the logit of the outcome, the quadratic term is not necessary. We validated by seeing that adding a quadratic term did not improve the model.

```
bm1 <- glm(formula = 0.ring > 0 ~ 1, family = binomial, data = d)
bm2 <- glm(formula = 0.ring > 0 ~ Temp, family = binomial, data = d)
bm3 <- glm(formula = 0.ring > 0 ~ Pressure, family = binomial, data = d)
bm4 <- glm(formula = 0.ring > 0 ~ factor(Pressure), family = binomial,
  data = d)
bm5 <- glm(formula = 0.ring > 0 ~ Temp + Pressure, family = binomial, data = d)
bm6 <- glm(formula = 0.ring > 0 ~ Temp + factor(Pressure), family = binomial,
  data = d)
bm7 <- glm(formula = 0.ring > 0 ~ Temp + I(Temp^2), family = binomial,
  data = d)
stargazer(bm1, bm2, bm3, bm4, bm5, bm6, bm7, header = FALSE, type = "text")
```

```
##
## =====
##                               Dependent variable:
##                               -----
##                               0.ring > 0
##                               (1)      (2)      (3)      (4)      (5)      (6)      (7)
## -----
## Temp                               -0.232**                               -0.229** -0.221** -2.094
##                               (0.108)                               (0.110)  (0.108)  (2.185)
##
## Pressure                               0.010                               0.010
##                               (0.008)                               (0.009)
##
## factor(Pressure)100                               -15.957                               -15.297
##                               (2,797.442)                               (2,761.759)
##
## factor(Pressure)200                               1.204                               1.377
##                               (1.216)                               (1.315)
##
## I(Temp2)                                         0.014
```



```
## (0.016)
##
## Constant      -0.827* 15.043** -2.455    -1.609    13.292*    13.511*    78.483
##               (0.453) (7.379)  (1.518)    (1.095)    (7.664)    (7.429)    (76.674)
##
## -----
## Observations      23      23      23      23      23      23      23
## Log Likelihood    -14.134 -10.158 -13.268  -12.799   -9.391   -9.107   -9.694
## Akaike Inf. Crit.  30.267  24.315  30.536   31.597   24.782   26.214   25.389
## =====
## Note:                                     *p<0.1; **p<0.05; ***p<0.01
```

(Answer 4C) From the statistical output displayed above, it appears that Temp is a better explanatory variable for the dependent variable than Pressure. We confirmed our intuition by performing a likelihood ratio test of the base case (bm1) against model with only Temp (bm2) and only Pressure (bm3, bm4) and found that bm2 is the better model among the three. We then compared bm2 against bm5 and bm6 to see how the addition of Pressure may improve the model. As it turns out, Pressure is not a statistically significant addition to bm2.

```
# bm1 vs bm2, bm3, bm4
c(anova(bm1, bm2, test = "Chi")[[5]][2], anova(bm1, bm3, test = "Chi")[[5]][2],
  anova(bm1, bm4, test = "Chi")[[5]][2])
```

```
## [1] 0.004803533 0.188303342 0.263149212
```

```
# bm2 vs bm5, bm6
c(anova(bm2, bm5, test = "Chi")[[5]][2], anova(bm2, bm6, test = "Chi")[[5]][2])
```

```
## [1] 0.2156479 0.3497930
```

Before we remove Pressure as one of the explanatory variables, we take a look at the residual deviance of the models. The residual deviance gives us insight into how much the estimated probabilities from our models of interest deviate from the observed proportions of success in the model. The lower the deviance, the better the fit. A difference of 1.53 and 2.10 between the residual deviance of bm2 and bm5 and respectively bm2 and bm6 demonstrate there may be a small effect of pressure present. However, the difference is not significant as the LRT have shown because in general, as more variables are added, the deviance value decreases.

```
c(deviance(bm2), deviance(bm5), deviance(bm2) - deviance(bm5))
```

```
## [1] 20.315193 18.782093 1.533099
```

```
c(deviance(bm2), deviance(bm6), deviance(bm2) - deviance(bm6))
```

```
## [1] 20.315193 18.214366 2.100827
```

For this reason, we apply AIC to assess the quality of our models with consideration of penalizing more complexity. Since our model has a lower AIC, this further confirms our choice to remove Pressure as an explanatory variable.

```
c(bm2$aic, bm5$aic, bm6$aic)
```

```
## [1] 24.31519 24.78209 26.21437
```

The potential issue of removing Pressure is that given a larger dataset with more data points at each psi level, there may be an effect that is not represented within our current model.

For this reason, we conclude that `bm2` is the best estimated model for π_i where π_i represents the probability of at least one O-ring failing.

$$P(Y_i = 1|x_i) = \frac{e^{\beta_0 + \beta_1 x_i}}{1 + e^{\beta_0 + \beta_1 x_i}} \\ \approx \frac{e^{15.043 - 0.232x_i}}{1 + e^{15.043 - 0.232x_i}} = \hat{\pi}_i$$

- i : observation for a single launch
- $Y_i = 1$: failure launch, at least one O-ring failure
- $Y_i = 0$: successful launch, no O-ring failure
- x_i : outside temperature in degrees Fahrenheit

```
summary(bm2)
```

```
##
## Call:
## glm(formula = O.ring > 0 ~ Temp, family = binomial, data = d)
##
## Deviance Residuals:
##      Min       1Q   Median       3Q      Max
## -1.0611  -0.7613  -0.3783   0.4524   2.2175
##
## Coefficients:
##              Estimate Std. Error z value Pr(>|z|)
## (Intercept)  15.0429     7.3786   2.039  0.0415 *
## Temp        -0.2322     0.1082  -2.145  0.0320 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## (Dispersion parameter for binomial family taken to be 1)
##
##      Null deviance: 28.267  on 22  degrees of freedom
## Residual deviance: 20.315  on 21  degrees of freedom
## AIC: 24.315
##
## Number of Fisher Scoring iterations: 5
```

With a p-value of 0.0320, there is sufficient evidence to conclude that outside temperature at the time of the launch does affect the probability of at least one O-ring failing.

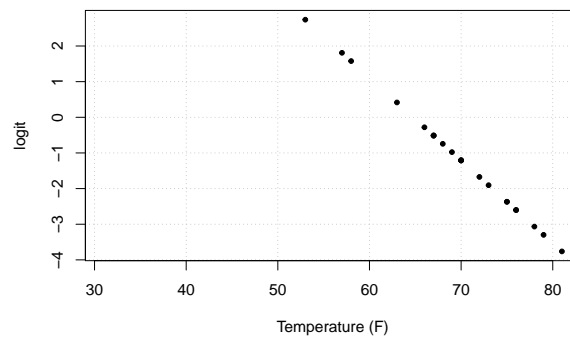
Validating logistic regression assumptions

Before we base our prediction on this model, we should verify whether or not the assumption for using logistic regression model is met. Not doing so will result in a similar situation that Challenger faced: forming decision based on incorrect findings.

Logistic regression method assumes that

1. The dependent variable is binary with a conditional distribution that follows a Bernoulli distribution, which we met since a success occurs when there is at least one O-ring failing ($O.\text{ring} > 0$) and a failure occurs when there are no O-rings failing ($O.\text{ring} == 0$).
2. There is a linear relationship between the predictor variable, in this case Temp, and the logit of the outcome. By plotting out the predicted probability, we can see that this assumption is also met. The current model is sufficient, and we do not need to apply a quadratic term to it.

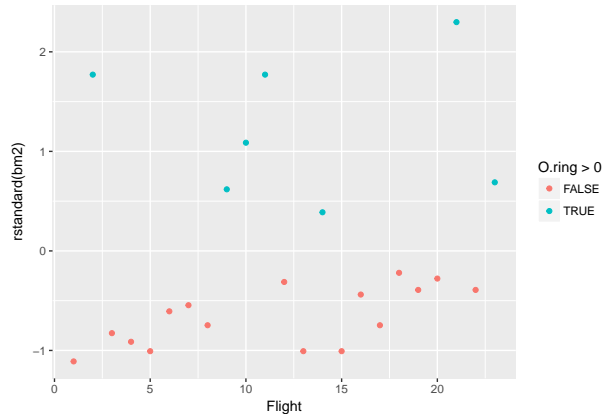
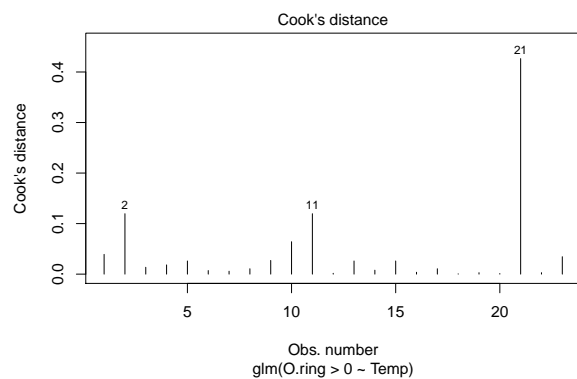
```
pi <- predict(bm2, data.frame(Temp = d$Temp), type = "response")
plot(x = d$Temp, xlab = "Temperature (F)", xlim = c(temp.min, temp.max),
     y = log(pi/(1 - pi)), ylab = "logit", panel.first = grid(col = "gray",
     lty = "dotted"), pch = 20)
```



3. There is no influential values that can alter the quality of the logistic regression model. We can verify that we met this assumption by first plotting the top 3 largest values of Cook's distance and followed it by filtering any standardized deviance residuals greater than 3; anything greater than 3 are considered influential values. In our case, we do not have any.

```
plot(bm2, which = 4, id.n = 3)
ggplot(d, aes(Flight, rstandard(bm2))) + geom_point(aes(color = O.ring >
0))
abs(rstandard(bm2)) > 3
```

```
##      1      2      3      4      5      6      7      8      9     10     11     12
## FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
##     13     14     15     16     17     18     19     20     21     22     23
## FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE FALSE
```



4. There is no high intercorrelations or multicollinearity among explanatory variables. Since our chosen model only has one explanatory variable, Temp, we have also met this assumption.
5. The outcome variable does not separate the predictor variable completely. We can validate this does not happen because R will give a warning message when it does. In addition, the standard error for the Temp coefficient is not extremely large, and looking at the relationship between the outcome variable and the explanatory variable in the bivariate analysis, we can see that there are failures still occurring at warmer temperature as well as cooler ones.
6. **(Answer 4A)** Each O-ring is independent of each launch. This is an important assumption because it ensures that the logistic regression is interpretable for any O-ring failure and any flight. Specifically, it ensures one failure does not conditionally increase the probability of failure for subsequent O-rings or O-rings on subsequent flights. However, there are details from the case that potentially invalidate this claim.

Pg 947 mentions increases in pressure tests from 50 to 200 which could have caused “blow-holes” leading to thermal distress and O-ring failure. This shows that previous flights might have had more durable O-rings compared to flights after. To alleviate these concerns, additional analysis to investigate the effect of increased pressure on thermal distress (blow holes, erosion) would be needed to show that there wasn’t any significant impact.

(Answer 5E) This model meets the necessary assumptions and we can continue predicting the probability of at least one O-ring failure with greater certainty. Based on our logistic regression model bm2, the probability of at least one O-ring failing when they launch the Challenger at 31°F is approximately $\hat{\pi}_i \approx 0.99$ with 90% confidence interval of (0.999613, 1.000000) found using parametric bootstrap. If they have waited until 72 °F as suggested, it would have reduced the probability of at least one O-ring failure to approximately 0.16 with 90% confidence interval of (0.3968449, 0.9763451) found using the same parametric bootstrap.

```
predict(bm2, data.frame(Temp = c(temp.launch, rec.temp)), type = "response")
```

```
##          1          2
## 0.9996088 0.1580491
```

```
get.pi.hat.star <- function(given.temps, given.model, given.formula, given.weights,
                             sample.size, total.O.rings, which.temp) {
```

```
  # simulate dataset from the given model
```

```

Temp <- sample(given.temps, sample.size, replace = TRUE)
pi.hat <- predict(object = given.model, newdata = data.frame(Temp = Temp),
  type = "response")
O.ring <- rbinom(n = sample.size, size = total.O.rings, pi.hat)
Number <- replicate(sample.size, total.O.rings)
newdata <- data.frame(Temp = Temp, O.ring = O.ring, Number = Number)

# estimate new model from dataset
estimated.model <- glm(formula = given.formula, weights = given.weights,
  family = binomial(link = "logit"), data = newdata)

# predict pi.hat.star at which.temp
return(predict(object = estimated.model, newdata = data.frame(Temp = which.temp),
  type = "response"))
}

get.ci.w.parametric.bootstrap <- function(given.temps, given.model, given.formula,
  given.weights, sample.size, total.O.rings, which.temp, num.datasets,
  percent.ci) {
  mult.pi.hat.star <- replicate(num.datasets, get.pi.hat.star(given.temps,
    given.model, given.formula, given.weights, sample.size, total.O.rings,
    which.temp))
  return(quantile(mult.pi.hat.star, c((1 - percent.ci)/2, percent.ci +
    (1 - percent.ci)/2)))
}

# provide the variables to pass in to bootstrap 1. sample size = 23
sample.size <- nrow(d)

# 2. total number of O-rings = 6
total.O.rings <- unique(d$Number)

# 3. provide the model's formula
given.formula <- paste("O.ring > 0 ~ Temp")
given.weights <- NULL

# 4. provide the model with the formula
given.model <- glm(formula = given.formula, family = binomial(link = "logit"),
  data = d)

# 5. use parametric bootstrap to compute 90% confidence intervals for
# temperature at launch: 31 F
get.ci.w.parametric.bootstrap(given.temps = d$Temp, given.model, given.formula,
  given.weights, sample.size, total.O.rings, which.temp = temp.launch,
  num.datasets = 10000, percent.ci = 0.9)

```

```
##          5%          95%
```

```
## 0.999577 1.000000
# for recommended temperature to launch: 72 F
get.ci.w.parametric.bootstrap(given.temps = d$Temp, given.model, given.formula,
  given.weights, sample.size, total.O.rings, which.temp = rec.temp, num.datasets = 10000,
  percent.ci = 0.9)
```

```
##          5%          95%
## 0.3988924 0.9864637
```

By using bootstrapping to resample the observed data, we were able to calculate a confidence interval of our sample estimates without assuming any specific distribution for the sample estimates. Although such confidence intervals can also be calculated using Wald intervals, such method is prone of violation to the assumption of a normal distribution.

To interpret our model, as we can see below, for every 1 degree Fahrenheit decrease in temperature, the odds of the O-rings failing for a given launch increases by 26% with the true population effect between 1.02 and 1.56.

```
# odds
bm2.b0 <- bm2$coefficients[1]
bm2.b1 <- bm2$coefficients[2]
c(100 * (exp(-bm2.b1) - 1), exp(-bm2.b1))
```

```
##      Temp      Temp
## 26.132499  1.261325
```

```
# ci of odds
get.glm.se <- function(model) sqrt(diag(vcov(model)))
c(exp(-bm2.b1 - 1.96 * get.glm.se(bm2)["Temp"]), exp(-bm2.b1 + 1.96 * get.glm.se(bm2)["Temp"]))
```

```
##      Temp      Temp
## 1.020221  1.559408
```

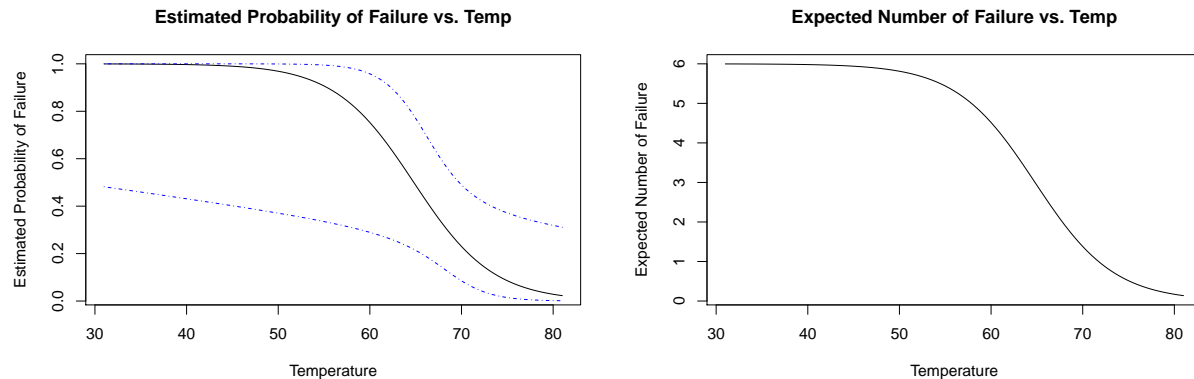
Answer 5B and 5C With a selected model `bm2`, we can then investigate how the estimated probability changes over different temperatures.

```
# Plot estimated pi
curve(expr = predict(object = bm2, newdata = data.frame(Temp = x), type = "response"),
  xlim = c(31, 81), xlab = "Temperature", ylab = "Estimated Probability of Failure",
  main = "Estimated Probability of Failure vs. Temp")
# Create C.I. function
conf.int <- function(newdata, mod, alpha) {
  linear.pred <- predict(object = mod, newdata = newdata, type = "link",
    se = TRUE)
  CI.lin.pred.lower <- linear.pred$fit - qnorm(p = 1 - alpha/2) * linear.pred$sse
  CI.lin.pred.upper <- linear.pred$fit + qnorm(p = 1 - alpha/2) * linear.pred$sse
  CI.pi.lower <- exp(CI.lin.pred.lower)/(1 + exp(CI.lin.pred.lower))
  CI.pi.upper <- exp(CI.lin.pred.upper)/(1 + exp(CI.lin.pred.upper))
  list(lower = CI.pi.lower, upper = CI.pi.upper)
}
# Plot confidence interval
```

```

curve(expr = conf.int(newdata = data.frame(Temp = x), mod = bm2, alpha = 0.05)$lower,
      col = "blue", lty = "dotdash", add = TRUE, xlim = c(31, 81))
curve(expr = conf.int(newdata = data.frame(Temp = x), mod = bm2, alpha = 0.05)$upper,
      col = "blue", lty = "dotdash", add = TRUE, xlim = c(31, 81))
# Plot expected number of failures
curve(expr = 6 * predict(object = bm2, newdata = data.frame(Temp = x),
      type = "response"), xlim = c(31, 81), xlab = "Temperature", ylab = "Expected Number of Failure",
      main = "Expected Number of Failure vs. Temp")

```



Since we assume this to be a binomial distribution, we can also compute and plot the expected number of O-ring failures using equation $n * p_i$

As we can see, the confidence interval is much wider at low temperatures than high ones. This is because the variance at the lower temperature is much higher, which is due to a lack of data.

It seems like the expected number of failures starts dropping in the mid 50's (temperature) and approaches zero at around 80 degrees.

Conclusion

From the observed data sample, our analyses showed that the temperature at launch was a significant factor in predicting the O ring failure. We chose to use a logistic regression model after explaining the inadequacy of using a linear regression in this case. And with our chosen model, $\text{logit}(Y) = \beta_0 + \beta_1 * \text{Temp}$, we saw that for every 1 degree Fahrenheit decrease in temperature, the odds of the O-rings failing for a given launch increases by 26% with the true population effect between 1.02 and 1.56. We also used bootstrapping which estimated the probability of O ring failure at launch when it's 31 F to be between 0.9996 and 1.0000. According to our model, such probability when it's 72 F would be in the range between 0.3968 and 0.9763.