

# Appendix

Tiffany Tran

2023-12-08

```
# taking 80% subset of data
library(readr)
countries <- read_csv("C:/Users/Tiffany/Desktop/school/sta 108/countries.csv")

## Rows: 186 Columns: 13
## -- Column specification -----
## Delimiter: ","
## chr (2): Country, Code
## dbl (11): LandArea, Population, Rural, Health, Internet, BirthRate, ElderlyP...
##
## i Use 'spec()' to retrieve the full column specification for this data.
## i Specify the column types or set 'show_col_types = FALSE' to quiet this message.
```

```
n <- nrow(countries)
set.seed(138)
subset_id <- sample(n, 0.8*n)
countries_subset <- countries[subset_id, ]
countries <- countries_subset
n <- nrow(countries)
```

```
# loop to assign variable names
predictors <- names(countries[3:13])
for (x in predictors){
  assign(paste(x), countries[[x]])
}
```

```
# descriptives of all variables
library(psych)
```

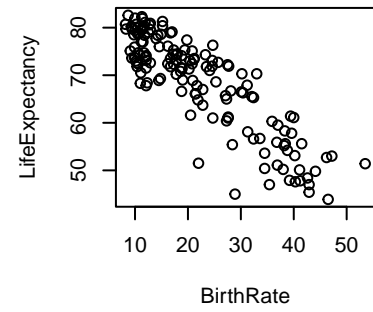
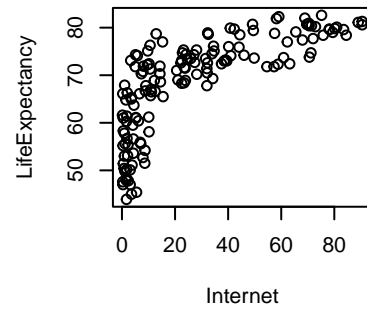
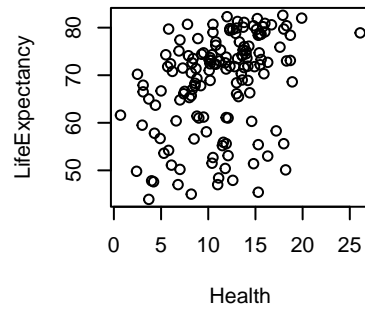
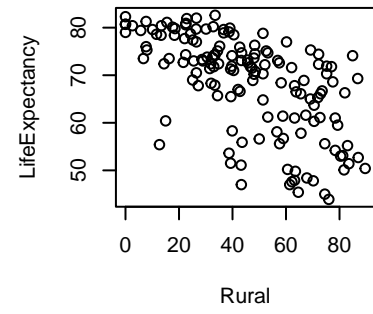
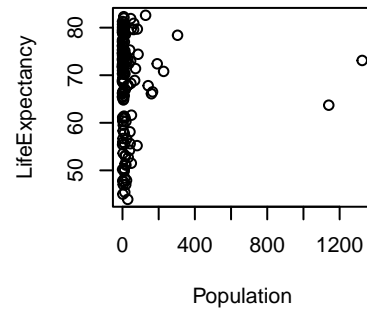
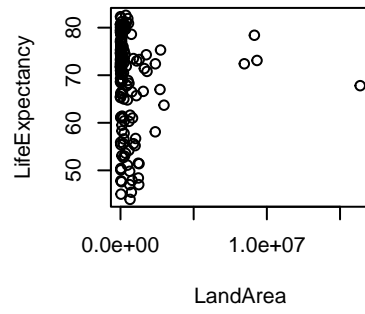
```
## Warning: package 'psych' was built under R version 4.3.2
```

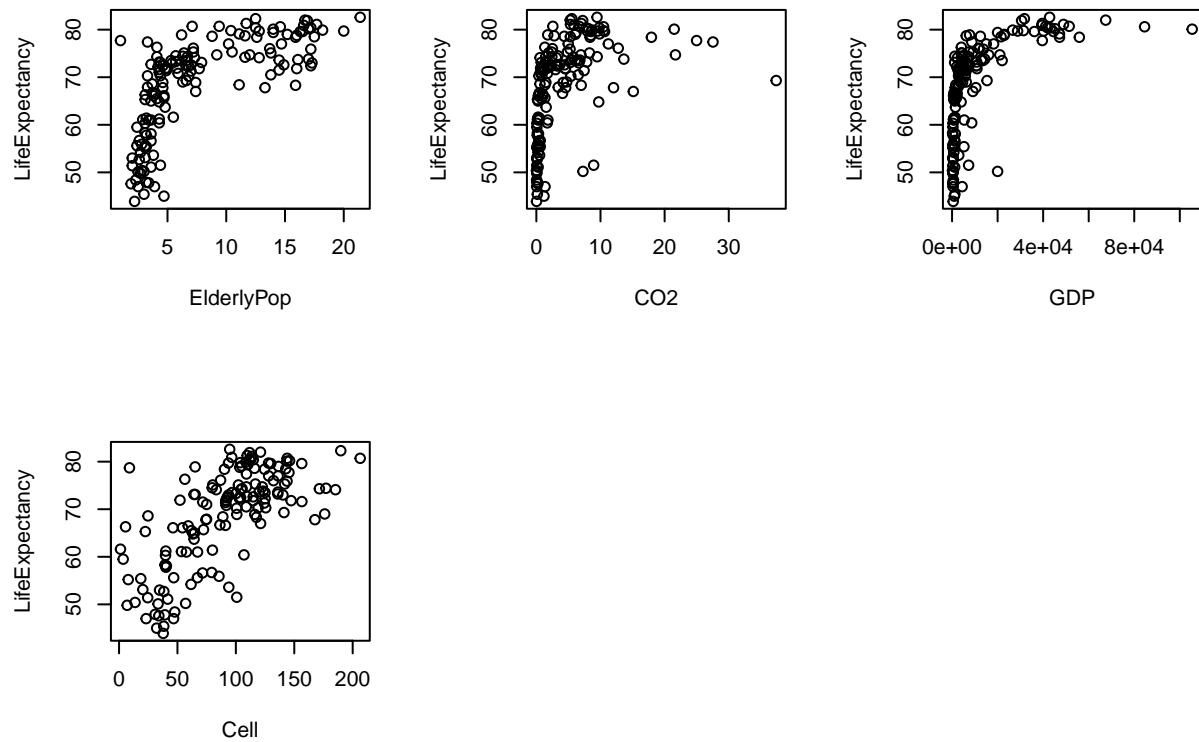
```
variables = cbind(LifeExpectancy, LandArea, Population, Rural, Health, Internet,
                  BirthRate, ElderlyPop, CO2, GDP, Cell)
describe(variables, skew=F)
```

##		vars	n	mean	sd	min	max	range
##	LifeExpectancy	1	148	68.47	10.22	43.90	82.60	38.70
##	LandArea	2	148	653690.14	1874764.70	28.00	16376870.00	16376842.00
##	Population	3	148	38.12	147.14	0.06	1324.65	1324.59

```
## Rural      4 148      45.44      23.06      0.00      89.60      89.60
## Health     5 148      11.28       4.37      0.70      26.10      25.40
## Internet   6 148      27.71      26.81      0.20      90.50      90.30
## BirthRate  7 148      22.30      11.12      8.20      53.50      45.30
## ElderlyPop  8 148       7.76       5.23      1.00      21.40      20.40
## CO2        9 148       4.53       5.68      0.02      37.39      37.37
## GDP       10 148    12025.88    17479.17  192.12    105437.67  105245.55
## Cell      11 148      90.29      43.55      1.24      206.43      205.19
##
##                      se
## LifeExpectancy      0.84
## LandArea          154104.71
## Population          12.09
## Rural              1.90
## Health              0.36
## Internet            2.20
## BirthRate           0.91
## ElderlyPop           0.43
## CO2                  0.47
## GDP                1436.78
## Cell                 3.58
```

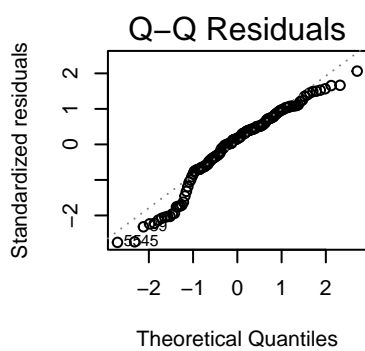
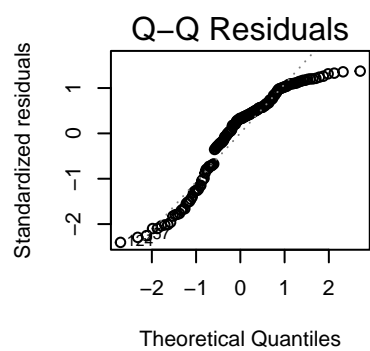
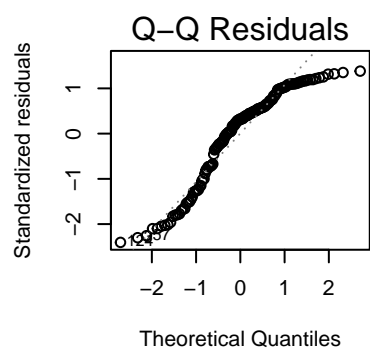
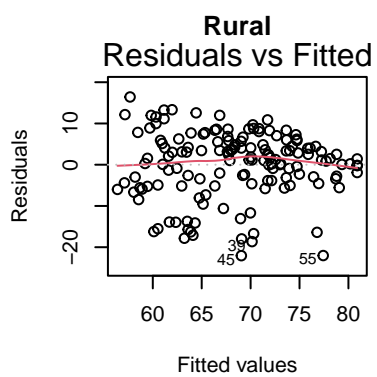
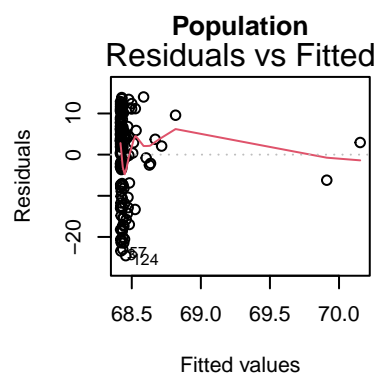
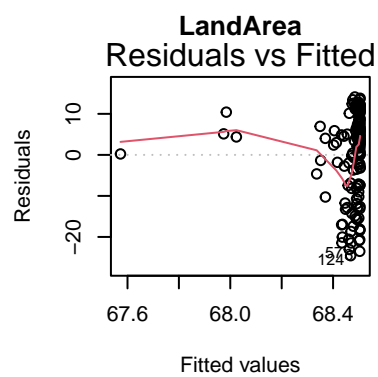
```
# simple linear regression for each variable
# start with scatterplots
par(mfrow=c(2,3))
for (x in predictors){
  if (x == "LifeExpectancy"){
    next
  }
  plot(countries[[x]], LifeExpectancy, xlab = x)
}
```

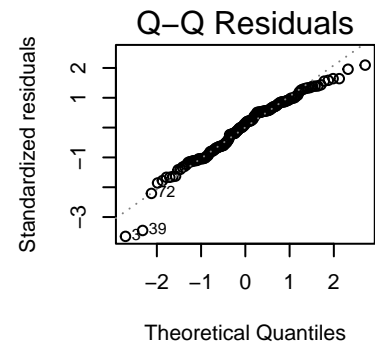
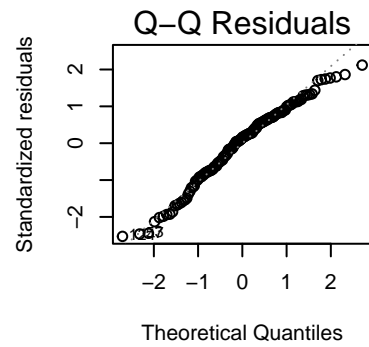
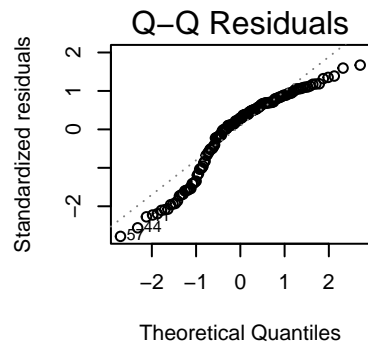
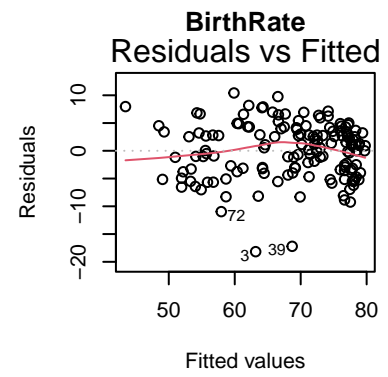
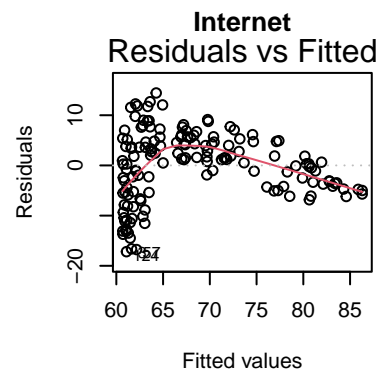
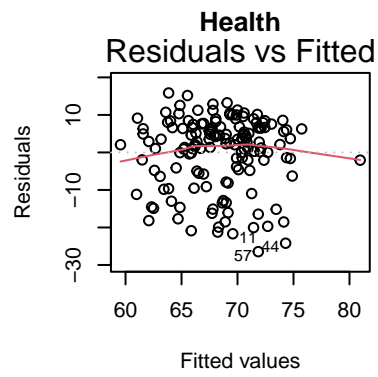


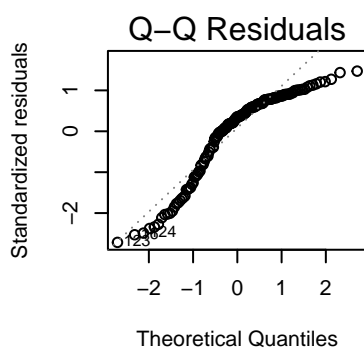
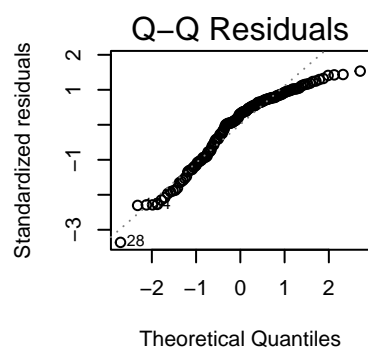
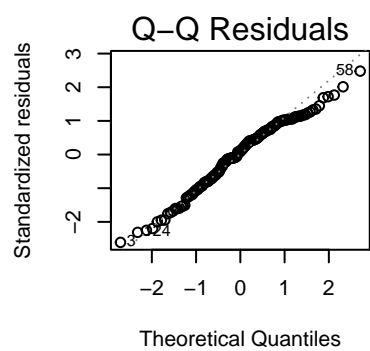
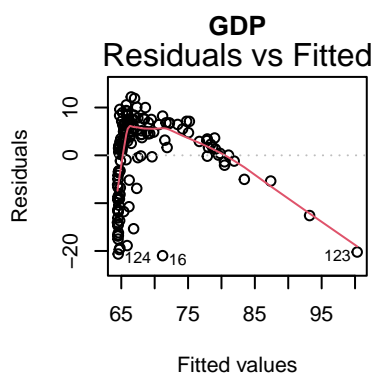
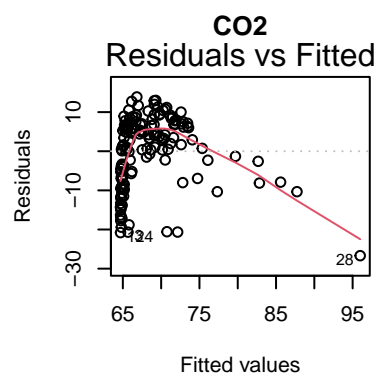
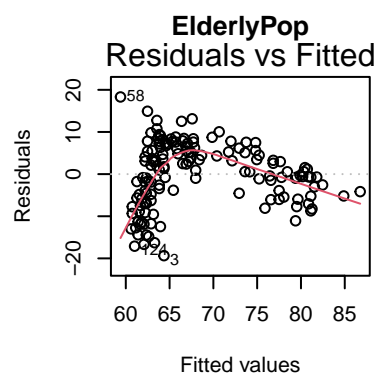


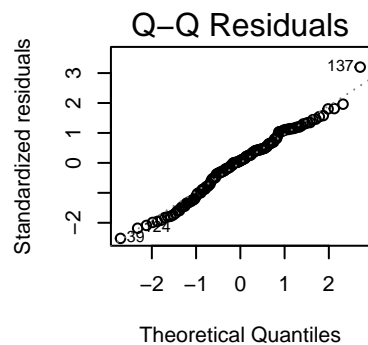
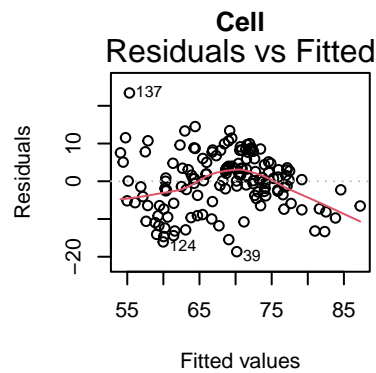
```
# look at residual vs predictor and q-q plots to assess assumptions
par(mfcol=c(2,3))

for (x in predictors){
  if (x == "LifeExpectancy"){
    next
  }
  plot(lm(LifeExpectancy ~ countries[[x]]), which = 1, main = x)
  plot(lm(LifeExpectancy ~ countries[[x]]), which = 2)
}
```









```
# hypothesis test for slope (t-test)
design = cbind(LandArea, Population, Rural, Health, Internet, BirthRate,
              ElderlyPop, CO2, GDP, Cell)
for (x in predictors){
  if (x == "LifeExpectancy"){
    next
  }
  # print(x)
  x <- countries[[x]]
  model <- lm(LifeExpectancy ~ x)
  # print(summary(model))
  # to shorten appendix, not all summaries will be printed
}

# two variables with not significant p-values
summary(lm(LifeExpectancy ~ LandArea))
```

```
##
## Call:
## lm(formula = LifeExpectancy ~ LandArea)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -24.568  -7.329   3.295   7.424  14.115
##
```



```
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6.851e+01  8.929e-01  76.718   <2e-16 ***
## LandArea    -5.695e-08  4.511e-07  -0.126     0.9
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.25 on 146 degrees of freedom
## Multiple R-squared:  0.0001092, Adjusted R-squared:  -0.006739
## F-statistic: 0.01594 on 1 and 146 DF, p-value: 0.8997
```

```
summary(lm(LifeExpectancy ~ Population))
```

```
##
## Call:
## lm(formula = LifeExpectancy ~ Population)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -24.556  -7.253   3.279   7.501  14.014
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept) 68.418315   0.870761  78.573   <2e-16 ***
## Population   0.001310   0.005747   0.228     0.82
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 10.25 on 146 degrees of freedom
## Multiple R-squared:  0.0003556, Adjusted R-squared:  -0.006491
## F-statistic: 0.05193 on 1 and 146 DF, p-value: 0.8201
```

```
# initial full model summary with all variables to determine most significant
model <- lm(LifeExpectancy ~ LandArea+Population+Rural+Health+Internet+
            BirthRate+ElderlyPop+CO2+GDP+Cell)
summary(model)
```

```
##
## Call:
## lm(formula = LifeExpectancy ~ LandArea + Population + Rural +
##      Health + Internet + BirthRate + ElderlyPop + CO2 + GDP +
##      Cell)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.548  -2.521   0.259   2.502  10.058
##
## Coefficients:
##           Estimate Std. Error t value Pr(>|t|)
## (Intercept)  8.098e+01  3.183e+00  25.442 < 2e-16 ***
## LandArea    -3.294e-07  2.372e-07  -1.389  0.16722
## Population   1.808e-03  3.042e-03   0.594  0.55326
## Rural        -2.131e-02  2.334e-02  -0.913  0.36275
```

```
## Health      3.119e-01  9.325e-02  3.345  0.00106 **
## Internet    6.661e-02  3.293e-02  2.023  0.04507 *
## BirthRate   -7.083e-01  6.812e-02 -10.398 < 2e-16 ***
## ElderlyPop  -4.240e-01  1.386e-01  -3.060  0.00266 **
## CO2         -1.157e-01  8.805e-02  -1.314  0.19090
## GDP         4.847e-05  3.658e-05   1.325  0.18744
## Cell        2.514e-02  1.282e-02   1.961  0.05189 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.431 on 137 degrees of freedom
## Multiple R-squared:  0.8248, Adjusted R-squared:  0.812
## F-statistic: 64.47 on 10 and 137 DF,  p-value: < 2.2e-16
```

```
# use leaps to generate order of forward selection
library(leaps)
```

```
## Warning: package 'leaps' was built under R version 4.3.2
```

```
modelforward <- regsubsets(LifeExpectancy ~ LandArea+Population+Rural+Health+
  Internet+BirthRate+ElderlyPop+CO2+GDP+Cell, method = "forward",
  data=countries, nvmax = 9)
summary(modelforward)
```

```
## Subset selection object
## Call: regsubsets.formula(LifeExpectancy ~ LandArea + Population + Rural +
##      Health + Internet + BirthRate + ElderlyPop + CO2 + GDP +
##      Cell, method = "forward", data = countries, nvmax = 9)
## 10 Variables (and intercept)
##      Forced in Forced out
## LandArea      FALSE      FALSE
## Population     FALSE      FALSE
## Rural          FALSE      FALSE
## Health         FALSE      FALSE
## Internet       FALSE      FALSE
## BirthRate      FALSE      FALSE
## ElderlyPop     FALSE      FALSE
## CO2            FALSE      FALSE
## GDP            FALSE      FALSE
## Cell           FALSE      FALSE
## 1 subsets of each size up to 9
## Selection Algorithm: forward
##      LandArea Population Rural Health Internet BirthRate ElderlyPop CO2 GDP
## 1  ( 1 ) " "      " "      " "      " "      "*"      " "      " " " "
## 2  ( 1 ) " "      " "      " "      "*"      " "      "*"      " "      " " " "
## 3  ( 1 ) " "      " "      " "      "*"      "*"      "*"      " "      " " " "
## 4  ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      " " " "
## 5  ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      " " " "
## 6  ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      "*" " "
## 7  ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      "*" "*"
## 8  ( 1 ) "*"      " "      " "      "*"      "*"      "*"      "*"      "*" "*"
## 9  ( 1 ) "*"      " "      "*"      "*"      "*"      "*"      "*"      "*"      "*"
##      Cell
```

```
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
## 5 ( 1 ) "*"
## 6 ( 1 ) "*"
## 7 ( 1 ) "*"
## 8 ( 1 ) "*"
## 9 ( 1 ) "*"
```

```
# assess forward selection
```

```
summary(lm(LifeExpectancy ~ BirthRate + Health + Internet + ElderlyPop + Cell))
```

```
##
## Call:
## lm(formula = LifeExpectancy ~ BirthRate + Health + Internet +
##     ElderlyPop + Cell)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.7525  -2.8776   0.0746   2.8033  10.5680
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  78.39354    2.78182  28.181  < 2e-16 ***
## BirthRate    -0.69085    0.06380 -10.827  < 2e-16 ***
## Health        0.34149    0.09202   3.711  0.000296 ***
## Internet      0.08849    0.02460   3.596  0.000445 ***
## ElderlyPop    -0.39051    0.13518  -2.889  0.004475 **
## Cell          0.02445    0.01225   1.996  0.047872 *
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.46 on 142 degrees of freedom
## Multiple R-squared:  0.816, Adjusted R-squared:  0.8095
## F-statistic: 126 on 5 and 142 DF, p-value: < 2.2e-16
```

```
summary(lm(LifeExpectancy ~ BirthRate + Health + Internet + ElderlyPop +
            Cell + C02))
```

```
##
## Call:
## lm(formula = LifeExpectancy ~ BirthRate + Health + Internet +
##     ElderlyPop + Cell + C02)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -16.1119  -2.8417   0.0953   2.6737  10.3925
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  79.21076    2.82994  27.990  < 2e-16 ***
## BirthRate    -0.70681    0.06454 -10.951  < 2e-16 ***
```

```
## Health      0.32311    0.09258    3.490 0.000645 ***
## Internet    0.10357    0.02669    3.881 0.000159 ***
## ElderlyPop  -0.43028    0.13753   -3.129 0.002134 **
## Cell        0.02645    0.01228    2.153 0.033034 *
## CO2         -0.11997    0.08389   -1.430 0.154915
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 4.443 on 141 degrees of freedom
## Multiple R-squared:  0.8187, Adjusted R-squared:  0.8109
## F-statistic: 106.1 on 6 and 141 DF,  p-value: < 2.2e-16
```

```
# adding more variables does not seem significant! stop at Cell.
```

```
# perform backwards selection with leaps to check model
modelbackward <- regsubsets(LifeExpectancy ~ LandArea+Population+Rural+Health+
  Internet+BirthRate+ElderlyPop+CO2+GDP+Cell, method = "backward",
  data=countries, nvmax = 9)
summary(modelbackward)
```

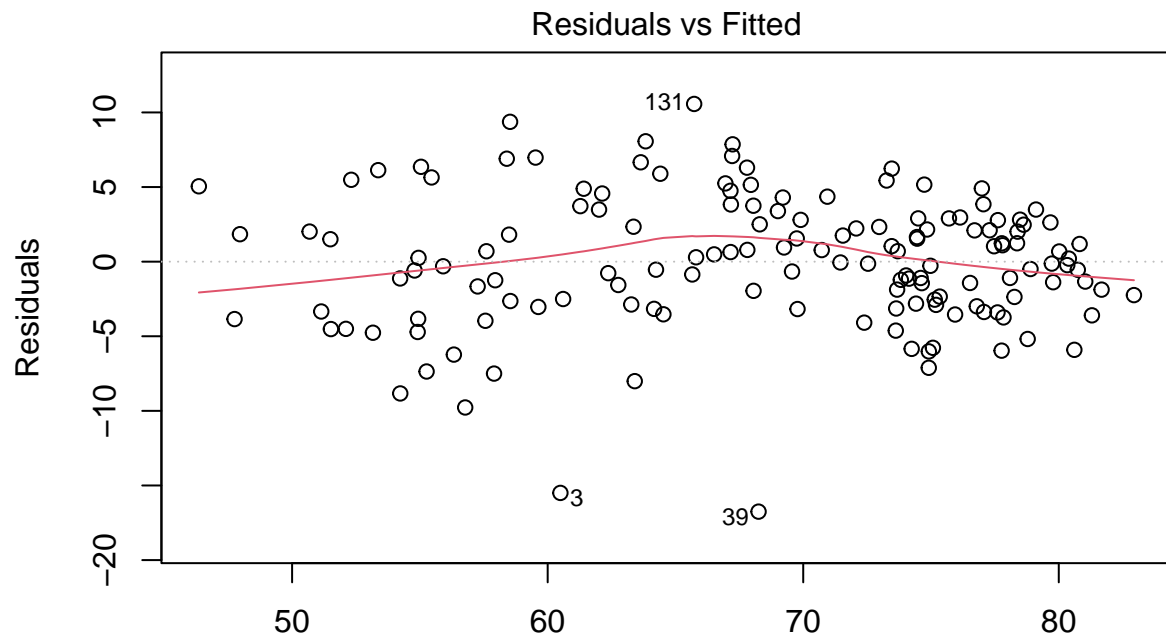
```
## Subset selection object
## Call: regsubsets.formula(LifeExpectancy ~ LandArea + Population + Rural +
##   Health + Internet + BirthRate + ElderlyPop + CO2 + GDP +
##   Cell, method = "backward", data = countries, nvmax = 9)
## 10 Variables (and intercept)
##           Forced in Forced out
## LandArea      FALSE      FALSE
## Population     FALSE      FALSE
## Rural          FALSE      FALSE
## Health         FALSE      FALSE
## Internet       FALSE      FALSE
## BirthRate      FALSE      FALSE
## ElderlyPop     FALSE      FALSE
## CO2            FALSE      FALSE
## GDP            FALSE      FALSE
## Cell           FALSE      FALSE
## 1 subsets of each size up to 9
## Selection Algorithm: backward
##           LandArea Population Rural Health Internet BirthRate ElderlyPop CO2 GDP
## 1 ( 1 ) " "      " "      " "      " "      "*"      " "      " " " "
## 2 ( 1 ) " "      " "      " "      "*"      " "      "*"      " "      " " " "
## 3 ( 1 ) " "      " "      " "      "*"      "*"      "*"      " "      " " " "
## 4 ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      " " " "
## 5 ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      " " " "
## 6 ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      "*" " "
## 7 ( 1 ) " "      " "      " "      "*"      "*"      "*"      "*"      "*" "*"
## 8 ( 1 ) "*"      " "      " "      "*"      "*"      "*"      "*"      "*" "*"
## 9 ( 1 ) "*"      " "      "*"      "*"      "*"      "*"      "*"      "*" "*"
##           Cell
## 1 ( 1 ) " "
## 2 ( 1 ) " "
## 3 ( 1 ) " "
## 4 ( 1 ) " "
```

```
## 5 ( 1 ) "*"
## 6 ( 1 ) "*"
## 7 ( 1 ) "*"
## 8 ( 1 ) "*"
## 9 ( 1 ) "*"
```

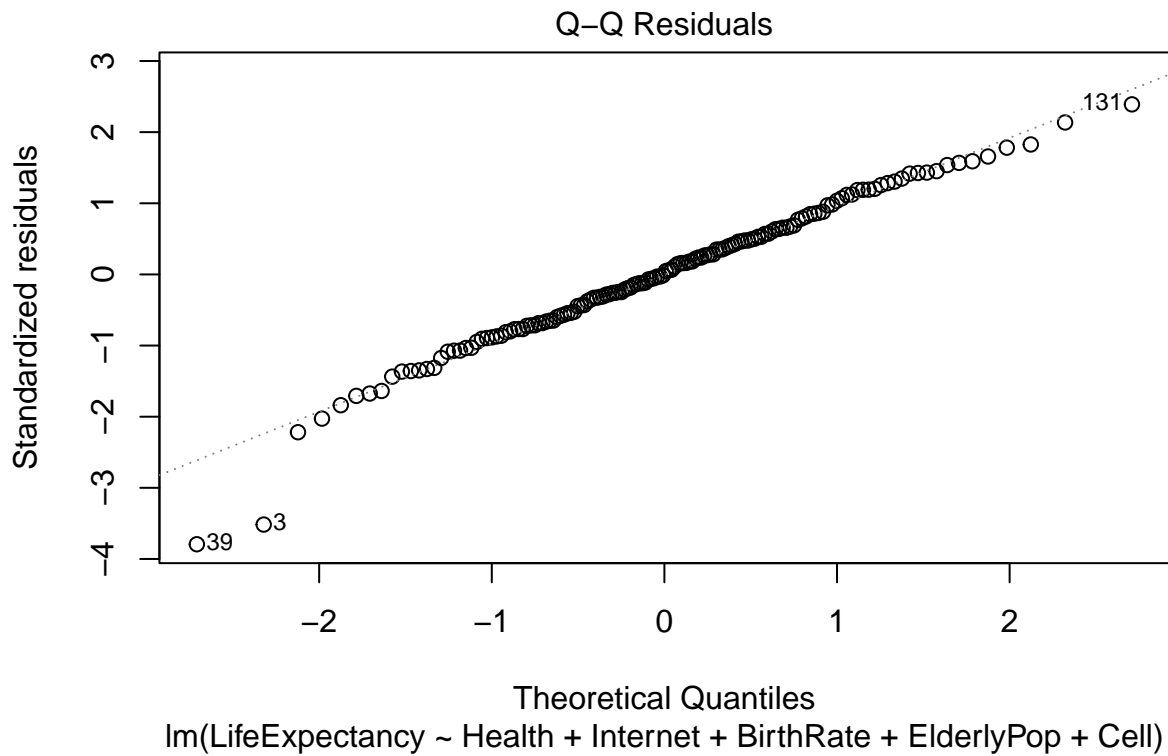
```
# since Population, Rural, and LandArea do not seem significant from our
# observations, remove for criterion testing
redesign = cbind(Health, Internet, BirthRate, ElderlyPop, CO2, GDP, Cell)
leaps(x=redesign, y=LifeExpectancy, names=c("Health", "Internet", "BirthRate",
      "ElderlyPop", "CO2", "GDP", "Cell"), method = "adjr2", nbest = 3)
```

```
## $which
##   Health Internet BirthRate ElderlyPop   CO2   GDP   Cell
## 1  FALSE     FALSE      TRUE     FALSE FALSE FALSE FALSE
## 1  FALSE      TRUE     FALSE     FALSE FALSE FALSE FALSE
## 1  FALSE     FALSE     FALSE     FALSE FALSE FALSE  TRUE
## 2   TRUE     FALSE      TRUE     FALSE FALSE FALSE FALSE
## 2  FALSE      TRUE      TRUE     FALSE FALSE FALSE FALSE
## 2  FALSE     FALSE      TRUE     FALSE FALSE  TRUE FALSE
## 3   TRUE      TRUE      TRUE     FALSE FALSE FALSE FALSE
## 3   TRUE     FALSE      TRUE     FALSE FALSE  TRUE FALSE
## 3   TRUE     FALSE      TRUE     FALSE FALSE FALSE  TRUE
## 4   TRUE      TRUE      TRUE      TRUE FALSE FALSE FALSE
## 4   TRUE     FALSE      TRUE      TRUE FALSE  TRUE FALSE
## 4   TRUE     FALSE      TRUE     FALSE FALSE  TRUE  TRUE
## 5   TRUE      TRUE      TRUE      TRUE FALSE FALSE  TRUE
## 5   TRUE      TRUE      TRUE      TRUE FALSE  TRUE FALSE
## 5   TRUE      TRUE      TRUE      TRUE  TRUE FALSE FALSE
## 6   TRUE      TRUE      TRUE      TRUE  TRUE  TRUE  TRUE
## 6   TRUE      TRUE      TRUE      TRUE  TRUE FALSE  TRUE
## 6   TRUE      TRUE      TRUE      TRUE  TRUE  TRUE  TRUE
## 7   TRUE      TRUE      TRUE      TRUE  TRUE  TRUE  TRUE
##
## $label
## [1] "(Intercept)" "Health"      "Internet"    "BirthRate"  "ElderlyPop"
## [6] "CO2"           "GDP"         "Cell"
##
## $size
## [1] 2 2 2 3 3 3 4 4 4 5 5 5 6 6 6 7 7 8
##
## $adjr2
## [1] 0.7602355 0.5517384 0.4716335 0.7826309 0.7821891 0.7796151 0.7939919
## [8] 0.7931627 0.7926749 0.8055759 0.7999909 0.7998504 0.8095488 0.8063919
## [15] 0.8061010 0.8109402 0.8101789 0.8079946 0.8127930
```

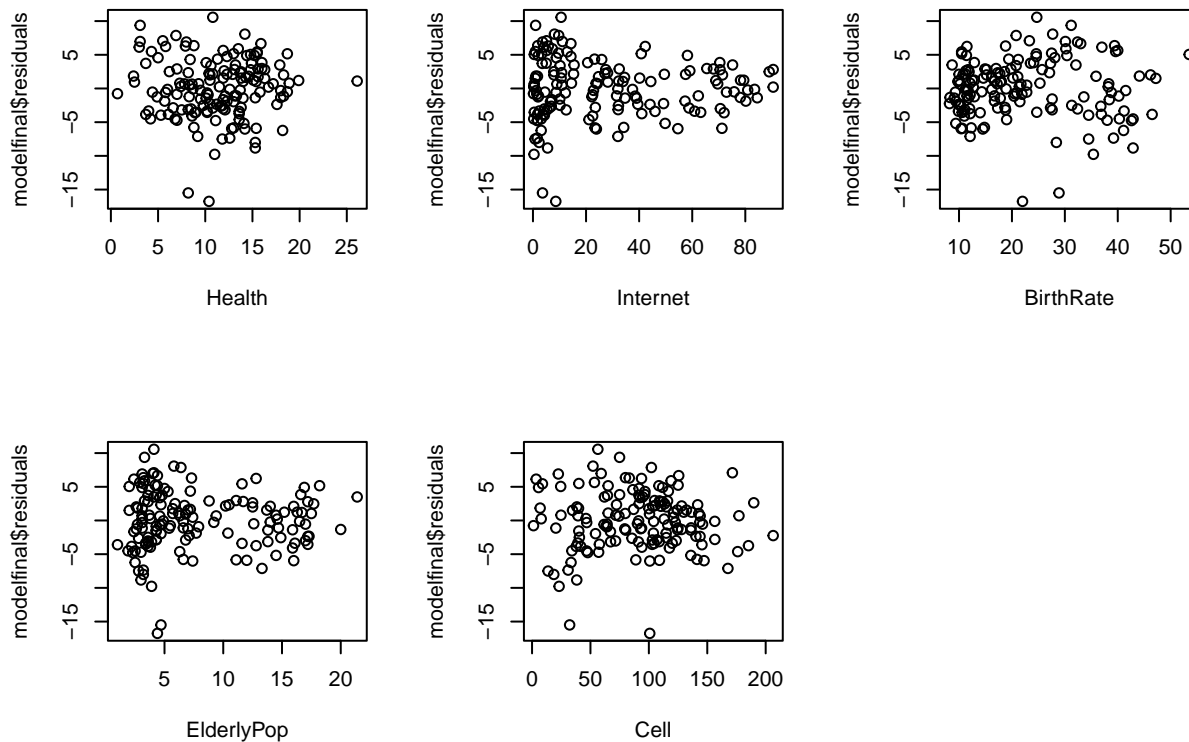
```
# generate residual plots to check assumptions
modelfinal <- lm(LifeExpectancy ~ Health+Internet+BirthRate+ElderlyPop+Cell)
plot(modelfinal, which = c(1, 2))
```



Fitted values  
 $\text{lm}(\text{LifeExpectancy} \sim \text{Health} + \text{Internet} + \text{BirthRate} + \text{ElderlyPop} + \text{Cell})$



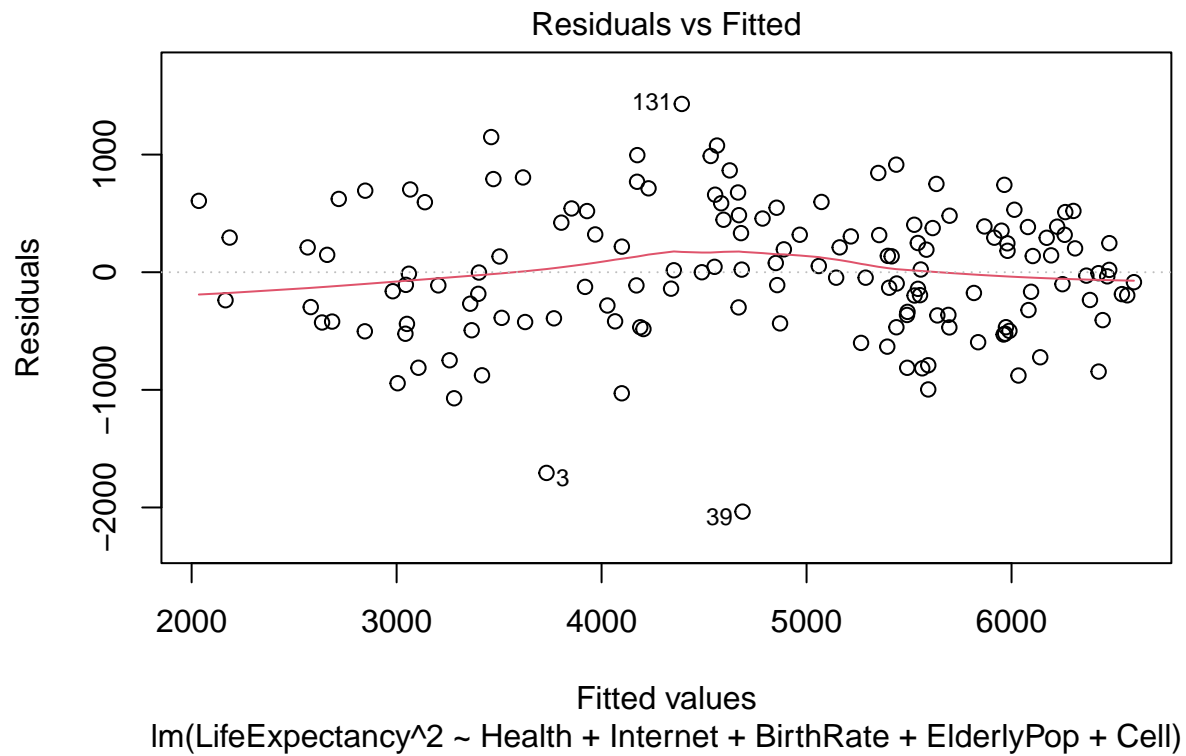
```
# check to see if nonlinearity is caused by predictors  
par(mfrow=c(2,3))  
plot(Health, modelfinal$residuals)  
plot(Internet, modelfinal$residuals)  
plot(BirthRate, modelfinal$residuals)  
plot(ElderlyPop, modelfinal$residuals)  
plot(Cell, modelfinal$residuals)
```

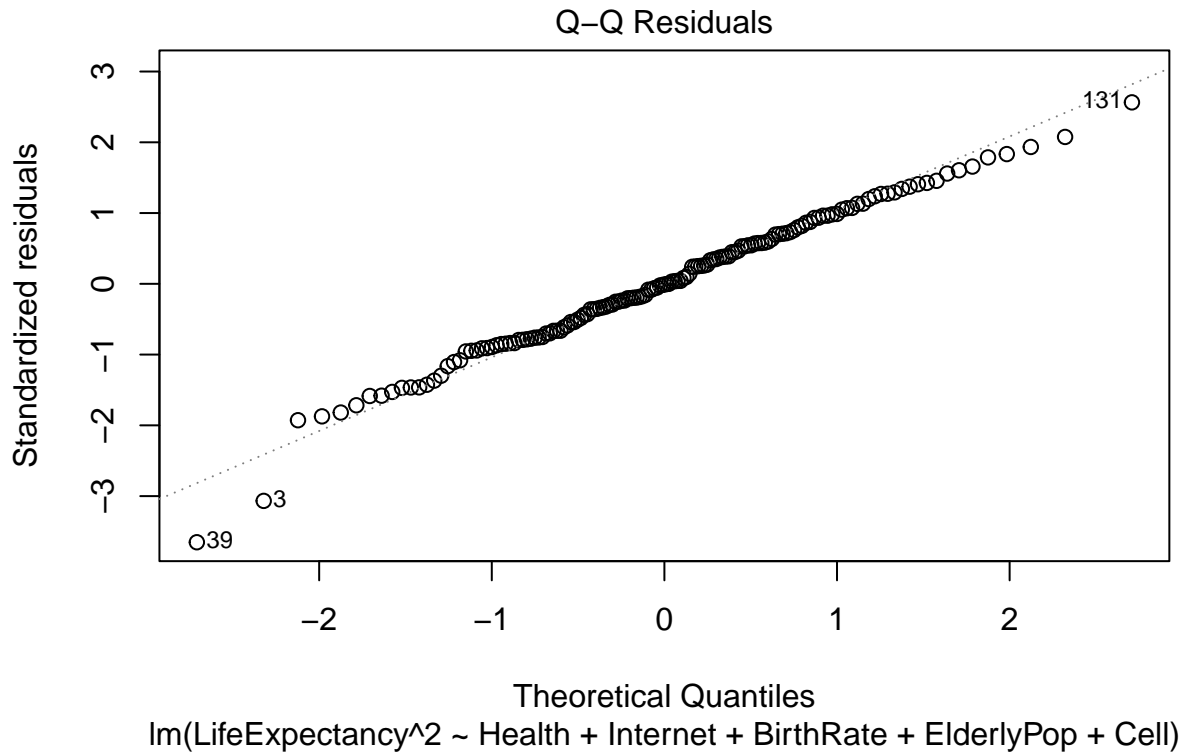


```
# boxcox transformation
#library(MASS)
#boxcox(LifeExpectancy ~ Health+Internet+BirthRate+ElderlyPop+Cell, plotit=T)
# shown in report
```

```
# create new final model
modelnew <- lm(LifeExpectancy^2 ~ Health+Internet+BirthRate+ElderlyPop+Cell)
plot(modelnew, which = c(1,2))
```







```
# reevaluate partial F tests
summary(lm(LifeExpectancy^2 ~ BirthRate + Health + Internet + ElderlyPop + Cell))
```

```
##
## Call:
## lm(formula = LifeExpectancy^2 ~ BirthRate + Health + Internet +
##     ElderlyPop + Cell)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -2035.38  -389.00    -5.52   385.05  1430.43
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  5822.218   351.037  16.586 < 2e-16 ***
## BirthRate    -83.070     8.052 -10.317 < 2e-16 ***
## Health        44.904    11.612   3.867 0.000167 ***
## Internet      13.898     3.105   4.476 1.55e-05 ***
## ElderlyPop   -43.246    17.059  -2.535 0.012324 *
## Cell          2.945     1.546   1.905 0.058800 .
## ---
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 562.8 on 142 degrees of freedom
## Multiple R-squared:  0.826, Adjusted R-squared:  0.8199
## F-statistic: 134.8 on 5 and 142 DF, p-value: < 2.2e-16
```

```

# reevaluate criterion test
reddesign2 = cbind(Health, Internet, BirthRate, ElderlyPop, Cell)
leaps(x=reddesign2, y=LifeExpectancy~2, names=c("Health", "Internet",
"BirthRate", "ElderlyPop", "Cell"), method = "adjr2", nbest = 3)

## $which
##   Health Internet BirthRate ElderlyPop  Cell
## 1  FALSE      FALSE      TRUE      FALSE FALSE
## 1  FALSE      TRUE       FALSE      FALSE FALSE
## 1  FALSE      FALSE      FALSE      TRUE  FALSE
## 2  FALSE      TRUE       TRUE       FALSE FALSE
## 2  TRUE       FALSE      TRUE       FALSE FALSE
## 2  FALSE      FALSE      TRUE       FALSE TRUE
## 3  TRUE       TRUE       TRUE       FALSE FALSE
## 3  FALSE      TRUE       TRUE       TRUE  FALSE
## 3  FALSE      TRUE       TRUE       FALSE TRUE
## 4  TRUE       TRUE       TRUE       TRUE  FALSE
## 4  TRUE       TRUE       TRUE       FALSE TRUE
## 4  FALSE      TRUE       TRUE       TRUE  TRUE
## 5  TRUE       TRUE       TRUE       TRUE  TRUE
##
## $label
## [1] "(Intercept)" "Health"      "Internet"    "BirthRate"  "ElderlyPop"
## [6] "Cell"
##
## $size
## [1] 2 2 2 3 3 3 4 4 4 5 5 5 6
##
## $adjr2
## [1] 0.7587234 0.5939574 0.4975963 0.7953208 0.7865722 0.7689083 0.8082976
## [8] 0.7999759 0.7987442 0.8165622 0.8130380 0.8022978 0.8198738

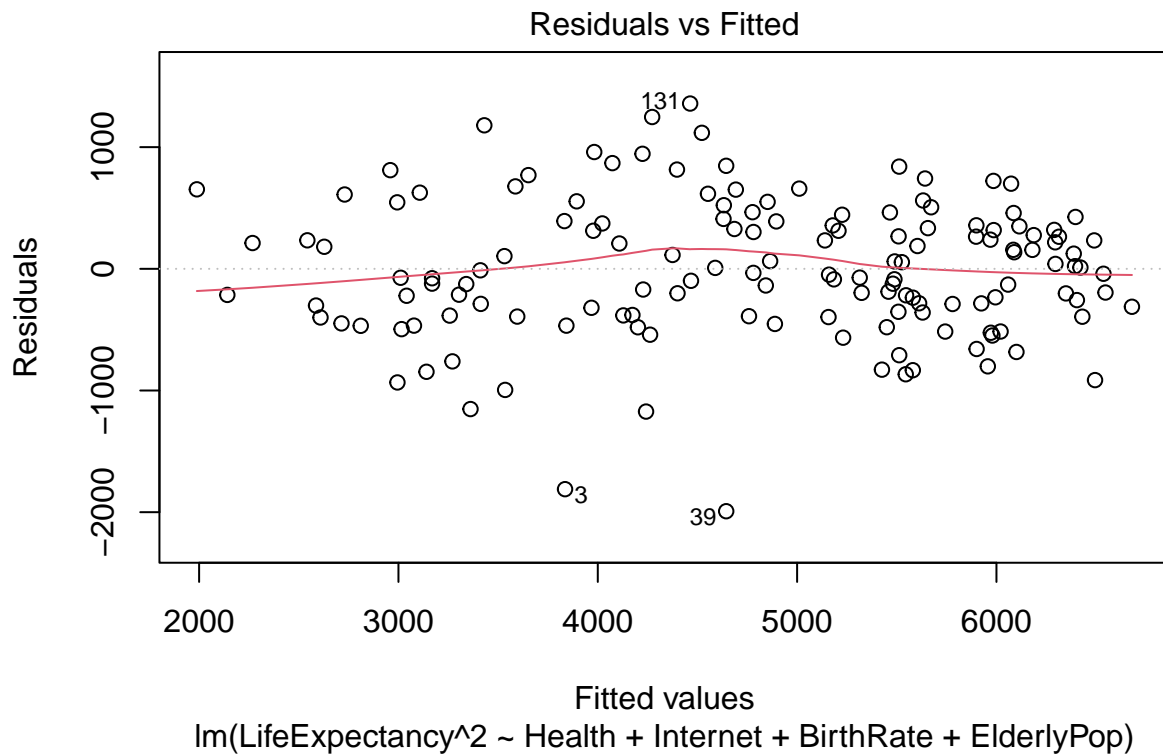
# remove cell for new final model
modelnew2 <- lm(LifeExpectancy~2 ~ Health+Internet+BirthRate+ElderlyPop)
summary(modelnew2)

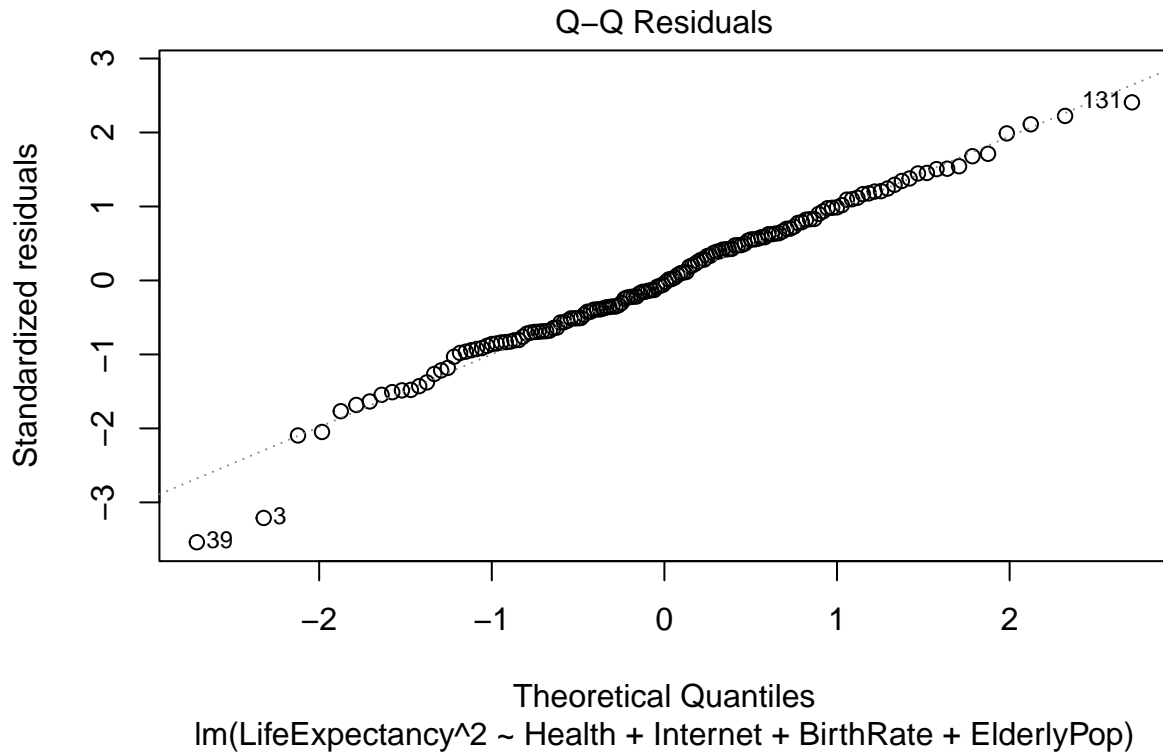
##
## Call:
## lm(formula = LifeExpectancy~2 ~ Health + Internet + BirthRate +
##     ElderlyPop)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1992.02  -380.16   -22.69   361.41  1358.30
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  6243.653    275.050   22.700 < 2e-16 ***
## Health        43.826     11.705    3.744 0.000261 ***
## Internet      15.343      3.038    5.050 1.33e-06 ***
## BirthRate    -90.051      7.235  -12.447 < 2e-16 ***
## ElderlyPop   -46.820     17.110   -2.736 0.007000 **
## ---

```

```
## Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 567.9 on 143 degrees of freedom
## Multiple R-squared:  0.8216, Adjusted R-squared:  0.8166
## F-statistic: 164.6 on 4 and 143 DF,  p-value: < 2.2e-16
```

```
plot(modelnew2, which = c(1,2))
```





```
# multicollinearity
library(faraway)
```

```
## Warning: package 'faraway' was built under R version 4.3.2
```

```
##
## Attaching package: 'faraway'
```

```
## The following object is masked from 'package:psych':
```

```
##
##      logit
```

```
vif(modelnew2)
```

```
##      Health      Internet      BirthRate      ElderlyPop
##      1.190737      3.024301      2.949096      3.652759
```