Algorithm Analysis Homework 2

Due by 3/28(Fri.) through LMS

1. The Merge Sort algorithm is modified so that the array is divided into three equal-sized subarrays instead of two. When merging three sorted subarrays, the smallest element among the three candidates (one from each subarray) is placed in the sorted array first.

   (a) Formulate the recurrence relation for the modified merge sort algorithm, denoted as T(n).

   (b) Solve the recurrence relation using the Master Theorem and express the time complexity in Θ (Theta) notation.

   (c) To verify the solution obtained in part (b) using the recursion tree method, perform the following steps:
   (i) Determine the height of the recursion tree.
   (ii) Find the number of nodes at level 2, assuming the root is at level 0.
   (iii) Compute the sum of work done at level 1 and level 2 separately.
   (iv) Derive the asymptotic tight bound from the recursion tree and compare it with the result from part (b). Are the results the same?

2. Use the Master Theorem to determine the asymptotic tight bounds for the following recurrence relations. Express the results in Θ (Theta) notation.

   (a) $T(n) = 4T(n/2) + \Theta(n)$

   (b) $T(n) = 4T(n/2) + \Theta(n^2)$

   (c) $T(n) = 4T(n/2) + \Theta(n^2 \log n)$