

基于线段进行三维重建

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1. <<The 3D Line Motion Matrix and Alignment of Line Reconstructions3D>>中对线的表示

Plucker坐标

3D空间中的线可以用Plucker坐标表示,假设空间中有两个3D点,它们的齐次坐标分别是:

$$\begin{aligned}\mathbf{M}^\top &= (\bar{\mathbf{M}}^\top \quad m) \\ \mathbf{N}^\top &= (\bar{\mathbf{N}}^\top \quad n)\end{aligned}$$

则由它们构成的线的6维Plucker坐标为:

$$\begin{aligned}\mathbf{L}^\top &= (\mathbf{a}^\top \quad \mathbf{b}^\top) \\ \mathbf{a} &= \bar{\mathbf{M}} \times \bar{\mathbf{N}} \\ \mathbf{b} &= m\bar{\mathbf{N}} - n\bar{\mathbf{M}}\end{aligned}$$

其中:

$$\mathbf{a}^\top \mathbf{b} = 0$$

Plucker矩阵

$$\mathbf{L} = \begin{pmatrix} [\mathbf{a}]_\times & \mathbf{b} \\ -\mathbf{b}^\top & 0 \end{pmatrix} = \mathbf{M}\mathbf{N}^\top - \mathbf{N}\mathbf{M}^\top$$

这是一个秩为2的反对称矩阵.

2. 线在3D空间中的变换

单应变换Homomorphy

假设对于3D点的单应变换矩阵为

$$\mathbf{H} = \begin{pmatrix} \tilde{\mathbf{H}} & \mathbf{h}_1 \\ \mathbf{h}_2^\top & h \end{pmatrix}_{4 \times 4}$$

则对于线的单应变换矩阵为:

$$\tilde{\mathbf{H}} = \begin{pmatrix} \det(\tilde{\mathbf{H}})\tilde{\mathbf{H}}^{-\top} & [\mathbf{h}_1]_\times \tilde{\mathbf{H}} \\ -\tilde{\mathbf{H}}[\mathbf{h}_2]_\times & h\tilde{\mathbf{H}} - \mathbf{h}_1\mathbf{h}_2^\top \end{pmatrix}$$

仿射变换

相似变换

刚体变换

设对于3D点的变换矩阵为:

$$\mathbf{T} = \begin{pmatrix} \mathbf{R} & \mathbf{t} \\ \mathbf{0} & 1 \end{pmatrix}$$

则对于线的刚体变换为:

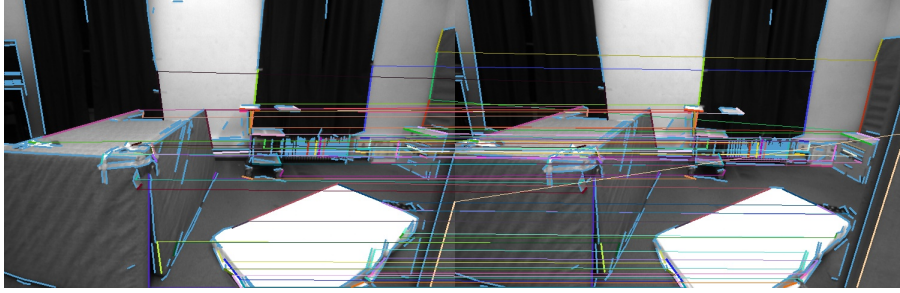
$$\tilde{\mathbf{D}} = \begin{pmatrix} \mathbf{R} & [\mathbf{t}]_{\times} \mathbf{R} \\ \mathbf{0} & \mathbf{R} \end{pmatrix}$$

3. PL-SLAM中对线的操作

线的提取:LSD算法

论文: R. G. von Gioi, J. Jakubowicz, J. M. Morel, and G. Randall. LSD: a line segment detector. IPOL, 2:35-55, 2012

算法集成在OpenCV contrib模块中,可以直接调用.



线的匹配

1. LBD descriptor(pl-slam)
2. 利用极线约束(line3d++)

线的表示和重投影误差

假设 $\mathbf{P}, \mathbf{Q} \in \mathbf{R}^3$ 是3维空间中一条线段的两个端点, $\mathbf{p}_d, \mathbf{q}_d \in \mathbf{R}^2$ 是它们在图像上被检测到的两个端点, $\mathbf{p}_d^h, \mathbf{q}_d^h \in \mathbf{R}^3$ 是它们的齐次坐标,首先计算归一化直线系数:

$$\mathbf{L} = \frac{\mathbf{p}_d^h \times \mathbf{q}_d^h}{|\mathbf{p}_d^h \times \mathbf{q}_d^h|}$$

于是线段的重投影误差就是重投影的线段的两个端点到该图像上实际提取出来的线的距离,即

$$E_{line}(\mathbf{P}, \mathbf{Q}, \mathbf{L}, \boldsymbol{\theta}, \mathbf{K}) = E_{pl}^2(\mathbf{P}, \mathbf{L}, \boldsymbol{\theta}, \mathbf{K}, \mathbf{K}) + E_{pl}^2(\mathbf{Q}, \mathbf{L}, \boldsymbol{\theta}, \mathbf{K})$$

对应于下图中的 d_1, d_2 .

其中, $\boldsymbol{\theta} = \{\mathbf{R}, \mathbf{t}\}$,

$$E_{pl}(\mathbf{P}, \mathbf{L}, \boldsymbol{\theta}, \mathbf{K}) = \mathbf{L}^\top \boldsymbol{\pi}(\mathbf{P}, \boldsymbol{\theta}, \mathbf{K})$$

然而在实际操作中,由于遮挡等原因,图像上提取的线段的两个端点可能无法与实际空间中线段的两个端点的重投影相匹配,于是把检测到的线段的重投影误差定义为:

$$E_{line}(\mathbf{p}_d, \mathbf{q}_d, \mathbf{L}) = E_{pl,d}^2(\mathbf{p}_d, \mathbf{L}) + E_{pl,d}^2(\mathbf{q}_d, \mathbf{L})$$

其中 \mathbf{L} 是归一化直线系数,检测到的point-to-line error是: $E_{pl,d}(\mathbf{p}_d, \mathbf{L}) = \mathbf{L}^\top \mathbf{T}^\top \mathbf{p}_d$.

对应与下图中的 d'_1, d'_2 .

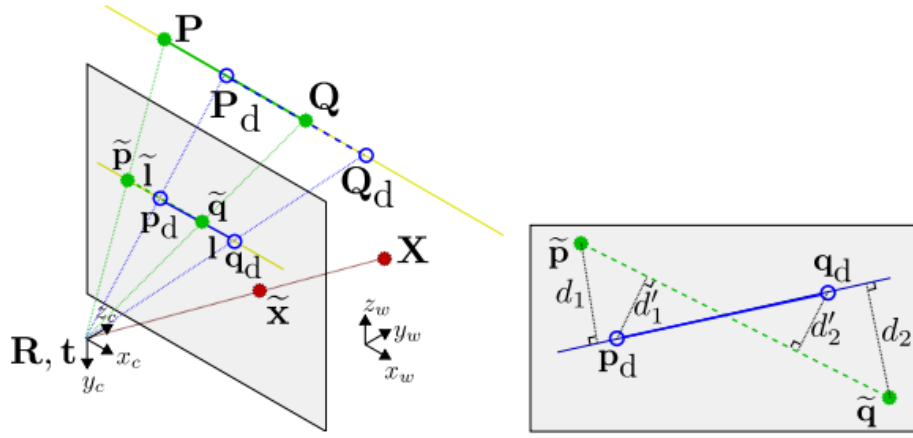


Fig. 3. **Left:** Notation. Let $\mathbf{P}, \mathbf{Q} \in \mathbb{R}^3$ be the 3D endpoints of a 3D line, $\tilde{\mathbf{p}}, \tilde{\mathbf{q}} \in \mathbb{R}^2$ their projected 2D endpoints to the image plane and $\tilde{\mathbf{l}}$ the projected line coefficients. $\mathbf{p}_d, \mathbf{q}_d \in \mathbb{R}^2$ the 2D endpoints of a detected line, $\mathbf{P}_d, \mathbf{Q}_d \in \mathbb{R}^3$ their real 3D endpoints, and \mathbf{l} the detected line coefficients. $\mathbf{X} \in \mathbb{R}^3$ is a 3D point and $\tilde{\mathbf{x}} \in \mathbb{R}^2$ its corresponding 2D projection. **Right:** Line-based reprojection error. d_1 and d_2 represent the *line reprojection error*, and d'_1 and d'_2 the *detected line reprojection error* between a detected 2D line (blue solid) and the corresponding projected 3D line (green dashed).

这里我觉得,两种误差都是可行的

4. 线段重建仿真

尝试一

- 仿真数据

用贺一家的vio_simulation_data生成的数据进行仿真,先针对仿真数据中某一条线段,收集n个keyframe中对这条线段的观测.

- 线段的参数化

3D线段用线段的两个端点坐标表示.

- 误差以及雅可比

误差采用上图中 d_1, d_2 表示的部分,检测到的线段(上图右图蓝色线段)可以表示为:

$$\mathbf{l}_d = \mathbf{p}_d^h \times \mathbf{q}_d^h$$

然后进行归一化:

$$\mathbf{l}_d = \frac{\mathbf{l}_d}{\sqrt{\mathbf{l}_d^2[0] + \mathbf{l}_d^2[1]}}$$

假设有一条线段的两个端点为 $\mathbf{P}_0^w, \mathbf{P}_1^w$,图像c上提取的相应的线段为 \mathbf{l}_d ,则重投影误差为:

$$\mathbf{e} = \begin{pmatrix} \mathbf{l}_d \cdot \boldsymbol{\pi}(\boldsymbol{\xi}_{cw}, \mathbf{P}_0^w) \\ \mathbf{l}_d \cdot \boldsymbol{\pi}(\boldsymbol{\xi}_{cw}, \mathbf{P}_1^w) \end{pmatrix}$$

其中 $\boldsymbol{\pi}, \boldsymbol{\xi}_{cw}$ 分别是投影函数和相机pose.

然后就是推导雅可比 $\frac{\partial e_0}{\partial \mathbf{P}_0^w}, \frac{\partial e_1}{\partial \mathbf{P}_1^w}$

$$\begin{aligned}\mathbf{P}_0^c &= \mathbf{T}_{cw} \mathbf{P}_0^w \\ \mathbf{P}_0^k &= \mathbf{K} \mathbf{P}_0^c \\ \mathbf{P}_0^{uw} &= \frac{\mathbf{P}_0^k}{\mathbf{P}_0^k[2]} \\ e_0 &= \mathbf{l}_d^\top \mathbf{P}_0^{uw}\end{aligned}$$

于是

$$\mathbf{J}_{\mathbf{P}_0^w} = \frac{\partial e_0}{\partial \mathbf{P}_0^w} = \frac{\partial e_0}{\partial \mathbf{P}_0^{uw}} \frac{\partial \mathbf{P}_0^{uw}}{\partial \mathbf{P}_0^k} \frac{\partial \mathbf{P}_0^k}{\partial \mathbf{P}_0^c} \frac{\partial \mathbf{P}_0^c}{\partial \mathbf{P}_0^w} = \mathbf{l}_d^\top \begin{pmatrix} \frac{1}{\mathbf{P}_0^k[2]} & 0 & \frac{-\mathbf{P}_0^k[0]}{(\mathbf{P}_0^k[2])^2} \\ 0 & \frac{1}{\mathbf{P}_0^k[2]} & \frac{-\mathbf{P}_0^k[1]}{(\mathbf{P}_0^k[2])^2} \\ 0 & 0 & 0 \end{pmatrix} \mathbf{K} \mathbf{R}_{cw}$$

设线段的状态向量为:

$$\mathbf{X}_{1 \times 6} = (\mathbf{P}_0^{w\top}, \mathbf{P}_1^{w\top})$$

则关于 \mathbf{X} 的雅可比为:

$$\mathbf{J}_{\mathbf{X} 2 \times 6} = \begin{pmatrix} \mathbf{J}_{\mathbf{P}_0^w} & \mathbf{0} \\ \mathbf{0} & \mathbf{J}_{\mathbf{P}_1^w} \end{pmatrix}$$

实验结果:

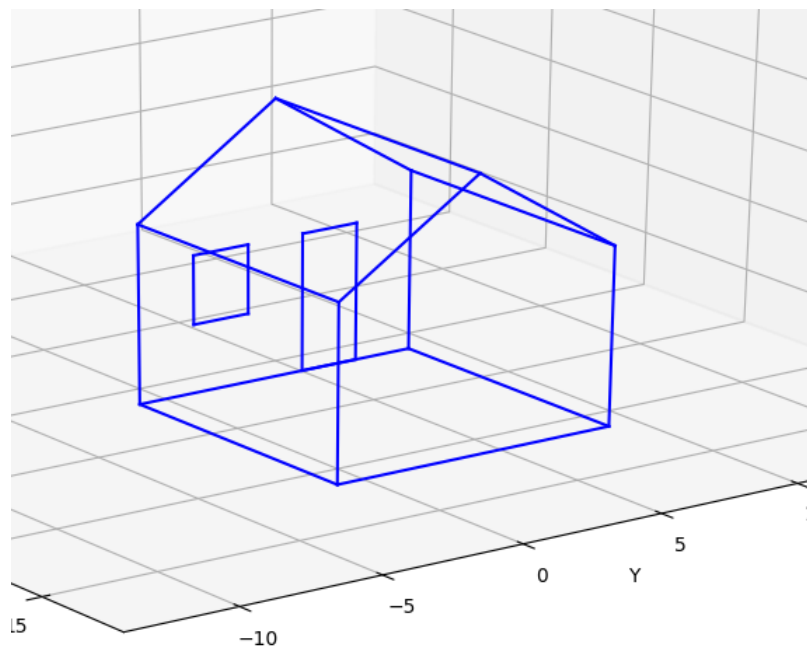
效果较差,实验结果与预期相差较远.

2019-10-24补充:之前是因为导数推错,先已纠正,尚未进行纠正后的代码实验.

尝试二

这次把误差换成了点到点的距离,得到了预期的效果.

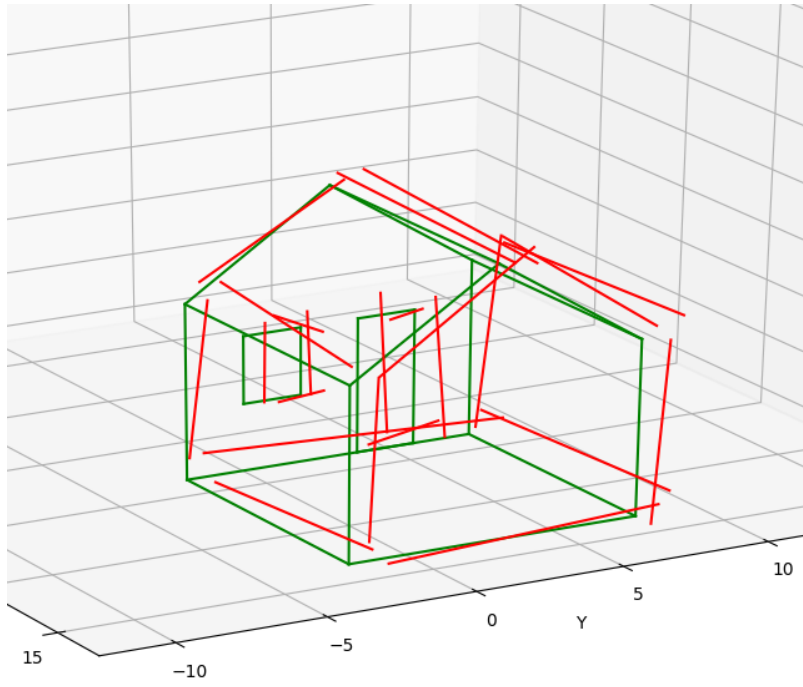
真值:



初值与优化结果:

红色:初始值

绿色:优化结果



尝试三

用线段端点的逆深度对端点进行参数化,误差用端点的重投影误差.

- 误差及雅可比推到

设在两个关键帧 c_0 和 c_1 中对同一个端点的观测的其次坐标为 $\mathbf{P}_{c_0}^h, \mathbf{P}_{c_1}^h$,其中 c_0 是参考帧, ρ 为该端点在参考帧下的逆深度.则:

$$\begin{aligned}\mathbf{P}_{c_0} &= \frac{\mathbf{P}_{c_0}^h}{\rho} \\ \mathbf{P}_{c_1} &= \mathbf{R}_{c_1 c_0} \mathbf{P}_{c_0} + \mathbf{t}_{c_1 c_0} \\ \mathbf{P}_{c_1}^{u_0} &= \frac{\mathbf{P}_{c_1}}{\mathbf{P}_{c_1}[2]} \\ \mathbf{P}_{c_1}^{u_1} &= \mathbf{P}_{c_1}^h \\ \begin{pmatrix} \mathbf{e}_{2 \times 1} \\ 0 \end{pmatrix} &= \mathbf{P}_{c_1}^{u_0} - \mathbf{P}_{c_1}^{u_1}\end{aligned}$$

于是误差关于逆深度的雅可比为:

$$\frac{\partial \mathbf{e}}{\partial \rho} = \frac{\partial \mathbf{e}}{\partial \mathbf{P}_{c_1}} \frac{\partial \mathbf{P}_{c_1}}{\partial \mathbf{P}_{c_0}} \frac{\partial \mathbf{P}_{c_0}}{\partial \rho} = \begin{pmatrix} \frac{1}{\mathbf{P}_{c_1}[2]} & 0 & \frac{-\mathbf{P}_{c_1}[0]}{\mathbf{P}_{c_1}[2]} \\ 0 & \frac{1}{\mathbf{P}_{c_1}[2]} & \frac{-\mathbf{P}_{c_1}[1]}{\mathbf{P}_{c_1}[2]} \end{pmatrix} \mathbf{R}_{c_1 c_0} \left(-\frac{\mathbf{P}_{c_0}^h}{\rho^2} \right)$$

重投影误差关于pose的雅可比推导

设有两个图像帧 c_0, c_1 ,它们的pose分别是 $\mathbf{T}_0, \mathbf{T}_1$,形式为cam-to-world,其中 c_0 是参考帧, p 是在 c_0 上提取的一个点,逆深度为 ρ , p 在 c_0 的归一化平面的坐标的齐次式为 \mathbf{P}_0^h ,令 $\mathbf{P}_0 = \frac{\mathbf{P}_0^h}{\rho}$.

这里 \mathbf{P}_0 是该点在 c_0 坐标系下的坐标,则在 c_1 坐标系下的坐标为:

$$\begin{pmatrix} \mathbf{P}_1 \\ 1 \end{pmatrix} = \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}$$

设 c_1 对该点的观测的归一化坐标为 \mathbf{P}_1^h ,则重投影误差为:

$$\begin{pmatrix} e_0 \\ e_1 \\ 0 \end{pmatrix} = \frac{\mathbf{P}_1}{\mathbf{P}_1[2]} - \mathbf{P}_1^h$$

$$\text{令}\mathbf{e} = \begin{pmatrix} e_0 \\ e_1 \end{pmatrix}$$

$$\text{需要求的是}\frac{\partial \mathbf{e}}{\partial \boldsymbol{\xi}_0}, \frac{\partial \mathbf{e}}{\partial \boldsymbol{\xi}_1}$$

$$\frac{\partial \mathbf{e}}{\partial \boldsymbol{\xi}_0} = \begin{pmatrix} \frac{1}{\mathbf{P}_1[2]} & 0 & -\frac{\mathbf{P}_1[0]}{(\mathbf{P}_1[2])^2} \\ 0 & \frac{1}{\mathbf{P}_1[2]} & -\frac{\mathbf{P}_1[1]}{(\mathbf{P}_1[2])^2} \end{pmatrix} \frac{\partial \mathbf{P}_1}{\partial \boldsymbol{\xi}_0}$$

$$\frac{\partial \mathbf{e}}{\partial \boldsymbol{\xi}_1} = \begin{pmatrix} \frac{1}{\mathbf{P}_1[2]} & 0 & -\frac{\mathbf{P}_1[0]}{(\mathbf{P}_1[2])^2} \\ 0 & \frac{1}{\mathbf{P}_1[2]} & -\frac{\mathbf{P}_1[1]}{(\mathbf{P}_1[2])^2} \end{pmatrix} \frac{\partial \mathbf{P}_1}{\partial \boldsymbol{\xi}_1}$$

$$\text{令}f(\mathbf{T}_0, \mathbf{T}_1) = \begin{pmatrix} \mathbf{P}_1 \\ 1 \end{pmatrix} = \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}, \text{则}$$

$$\begin{aligned} \frac{\partial f}{\partial \boldsymbol{\xi}_0} &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{f(\delta \boldsymbol{\xi}_0 \otimes \mathbf{T}_0, \mathbf{T}_1) - f(\mathbf{T}_0, \mathbf{T}_1)}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \exp(\delta \boldsymbol{\xi}_0^\wedge) \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix} - \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} (\mathbf{I} + \delta \boldsymbol{\xi}_0^\wedge) \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix} - \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \delta \boldsymbol{\xi}_0^\wedge \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \begin{pmatrix} \delta \phi_0^\wedge & \delta \rho_0 \\ \mathbf{0}^\top & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0 \\ 1 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \begin{pmatrix} \delta \phi_0^\wedge (\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0) + \delta \rho_0 \\ 0 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \begin{pmatrix} \delta \rho_0 - (\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \delta \phi_0 \\ 0 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \lim_{\delta \boldsymbol{\xi}_0 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & 0 \end{pmatrix} \begin{pmatrix} \delta \rho_0 \\ \delta \phi_0 \end{pmatrix}}{\delta \boldsymbol{\xi}_0} \\ &= \mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & \mathbf{0}^\top \end{pmatrix} \end{aligned}$$

$$\begin{aligned}
\frac{\partial f}{\partial \xi_1} &= \lim_{\delta \xi_1 \rightarrow 0} \frac{(\exp(\delta \xi_1^\wedge) \mathbf{T}_1)^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix} - \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{\mathbf{T}_1^{-1} \exp(-\delta \xi_1^\wedge) \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix} - \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{\mathbf{T}_1^{-1} (\mathbf{I} - \delta \xi_1^\wedge) \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix} - \mathbf{T}_1^{-1} \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{-\mathbf{T}_1^{-1} \delta \xi_1^\wedge \mathbf{T}_0 \begin{pmatrix} \mathbf{P}_0 \\ 1 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{-\mathbf{T}_1^{-1} \begin{pmatrix} \delta \phi_1^\wedge & \delta \rho_1 \\ \mathbf{0}^\top & 0 \end{pmatrix} \begin{pmatrix} \mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0 \\ 1 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{-\mathbf{T}_1^{-1} \begin{pmatrix} \delta \phi_0^\wedge (\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0) + \delta \rho_0 \\ 0 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{-\mathbf{T}_1^{-1} \begin{pmatrix} \delta \rho_0 - (\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \delta \phi_0 \\ 0 \end{pmatrix}}{\delta \xi_1} \\
&= \lim_{\delta \xi_1 \rightarrow 0} \frac{-\mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & 0 \end{pmatrix} \begin{pmatrix} \delta \rho_0 \\ \delta \phi_0 \end{pmatrix}}{\delta \xi_1} \\
&= -\mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & \mathbf{0}^\top \end{pmatrix}
\end{aligned}$$

于是:

$$\frac{\partial \mathbf{e}}{\xi_0} = \begin{pmatrix} \frac{1}{\mathbf{P}_1[2]} & 0 & -\frac{\mathbf{P}_1[0]}{(\mathbf{P}_1[2])^2} \\ 0 & \frac{1}{\mathbf{P}_1[2]} & -\frac{\mathbf{P}_1[1]}{(\mathbf{P}_1[2])^2} \end{pmatrix} (\mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & \mathbf{0}^\top \end{pmatrix})_{(\text{取前 3 行})}$$

$$\frac{\partial \mathbf{e}}{\xi_1} = - \begin{pmatrix} \frac{1}{\mathbf{P}_1[2]} & 0 & -\frac{\mathbf{P}_1[0]}{(\mathbf{P}_1[2])^2} \\ 0 & \frac{1}{\mathbf{P}_1[2]} & -\frac{\mathbf{P}_1[1]}{(\mathbf{P}_1[2])^2} \end{pmatrix} (\mathbf{T}_1^{-1} \begin{pmatrix} \mathbf{I} & -(\mathbf{R}_0 \mathbf{P}_0 + \mathbf{t}_0)^\wedge \\ \mathbf{0}^\top & \mathbf{0}^\top \end{pmatrix})_{(\text{取前 3 行})}$$