ADDING AND SUBTRACTING FAST

ESSENTIAL TACTICS TO SPEED UP
MENTAL CALCULATIONS WITH WHOLE NUMBERS

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ADDING AND SUBTRACTING FAST. ESSENTIAL TACTICS TO SPEED UP MENTAL CALCULATIONS WITH WHOLE NUMBERS

This book is dedicated with infinite gratitude to my mother, an exemplary educator and human being, and to the inspiring teachers and professors who instilled in me the love of learning and the passion to teach. I have them to thank for the many ways in which I have improved over the years as a communicator, researcher, and instructor. Their imprint lives in every page of this book.
Of any mistakes or inaccuracies in content, method, or strategy, they are, naturally, completely innocent.

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PREFACE

This book will not teach you how to add or subtract.

I assume that you know how to do that already…. with pen and paper, given ample time.

The goal of this book is to help you improve the speed at which you add and subtract, without pencil and paper.

Learning to add and subtract on your feet will save you time, give your brain a workout, and with a little social finesse, maybe help you impress someone. But that’s only at a bare minimum. You’ll benefit from the methods presented in this book the next time you are doing math homework, but also anytime you are playing games and need to make score calculations to keep things moving, or after you’ve come home from the store, adding ingredient measures for the perfect cooked meal.

The applications of the most essential of arithmetic operations are innumerable. And, although not covered in this book, working with almost any other math skill, including decimals, proportions, percentages, and probabilities all require in some way or another knowledge on how to add or subtract. If you become fluent with the basics, you will be able to deal with anything math related that requires arithmetic, faster than before.

A good analogy for what you would learn here would be learning how to type without looking at the keyboard, or speed reading. We are not aware of it because it is so basic, but the amount of time we save by getting the basics down well is significant.

Mental arithmetic is not just useful in middle and or high school where the applications may be the most obvious. Even after you’ve graduated, whether you are going to college or graduate school, whichever your major, fast mental arithmetic can save you precious next time you are taking that time-sensitive standardized test.

The truth is, we never stop benefiting from knowing arithmetic. You may find it incredibly useful and time saving advanced college math courses like linear algebra where you still need to do arithmetic to do matrix operations for example. Business, science, social science major? You’ll probably use adding and subtracting quite a bit.

But perhaps you didn’t go to school. What use may this be for me then?

Next time you are out with a group of friends, can you quickly estimate how much you should front up when splitting a bill adding the cost of all the items you consumed? Or subtracting that total from the total bill? Or if you are getting a job or changing jobs, can you calculate fast how long it will take you to get to that interview, or the airport if you have a flight to catch, adding the minutes it takes you to walk to the bus/taxi/car, arrive in the airport, and walk in through the terminal to the gate before the plane leaves?

I’m belaboring the point, which is: math is everywhere, and you do – or should – use it every day to optimize your time and do things better, faster, and more effectively. Adding and subtracting fast is the essential first step to improving mental speed and accuracy in arithmetic.

Whatever your reason for coming on this journey with me, thank you, and know that learning this skills has impacted my life positively. Learning this material can do the same thing for you.

Welcome aboard.

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INTRODUCTION

This book is an effort to combine various scattered mental arithmetic tricks that have been taught by teachers many generations behind and in different and distant corners of the world, into a consistent and comprehensive system that will enable you to calculate two, three, and four-digit addition and subtraction operations, as well as mixed operations, with stunning speed and without writing anything down.

Fast mental arithmetic, requires, as does most any other skill, consistent practice and a little flexibility. Math, like traditional or programming languages, and many other arts, cannot be mastered without investing significant time doing it. Adding and subtracting, however basic they may be, is no exception to this rule. Along those lines, the approach I have adopted in this book is that of including both theoretical chapters to present the information, and practice chapters – which are essential, as well as an appendix, to see worked through examples.

Theoretical chapters are used to present the techniques, explaining the logic behind them, along with one or more examples, while practice chapters are intended to encourage you to apply those skills, or at most, helping you start to apply them, since the amount of practice required to master fast arithmetic is beyond the scope of the sixty-so pages of this manuscript.

The worked out answers section in the appendix is not a whimsical addition, it is intended to show you, step by step how the system outlined in the book is applied to the actual examples we ask you to solve in practice chapters. The appendix should be read in tandem with the practice chapters since the exact steps we take in the appendix are outlined there. Such that you can first try to solve the exercise by yourself, and then cross check with the worked out answer in the appendix to see a possible way to solve it, using the methods outlined in the theory chapters.

Still, it should be noted once again, that reading this whole book and doing all the practice exercises several times is no guarantee you will come out adding and subtracting incredibly fast.

Beyond the space limitation, success will depend on many other things such as your previous background and the actual attention you are paying to what you are doing, and the speed and concentration you apply reading.

For those who have not read a mathematics textbook before, one should note that this book, or any book dealing in math or programming languages should not be read at the speed you read a fiction novel or a social science or humanities textbook. Math is read slowly, making pauses and going back to parts you did not fully grasp. This is because, you will be taught things that require understanding and proficiency ini concepts explained maybe just a few lines before, sometimes even within the same page. Skimming and speed reading here will not help you.

In regard to methodolgoy, think of the schema presented in this book, as a system that you can apply, and that you will personalize and tweak according to your own previous math background, as you go about practicing in everyday situations such as grocery shopping, keeping score during a game with friends, or even when working out, counting miles left or total combined weight of free weights. Again, the key to mastery is practice.

Although I show one, and occasionally two ways of going about an operation, as I say perhaps too many times throughout the book, the optimal combination of techniques as well as the best sequencing of techniques for improving your mental calculation speed, will largely depend on the background you bring to this book.

For most people adding 2 to 2 is easier than adding 4 to 3, or for that matter, for some people adding 4 to 3 is easier than adding 3 to 4, but this is not always the case. Use the material presented here as it best fits your personal skill using your judgement when something appears easier done in an alternative way. Do not be a passive learner. Engage with the material.

Except for mental aligning in the theoretical chapters, all number additions, and subtractions (including aligning later on) are presented horizontally, and not in traditional vertical form. This is done not to save space, but to illustrate the sequential thinking you should be doing as you are calculating to gain speed and accuracy. The idea here, is that you think through a math operation in a very linear way, with very little overhead in the form of memorization, so as not to interrupt your mental computing, minimize errors, and optimize time. When you become comfortable you can even stop saying numbers in your head and simply visualize them. Some have argued, quite correctly I think, that calculating multisyllabic numbers takes more time than monosyllabic numbers, but not only because monosyllabic numbers tend to be smaller amounts, but also because it takes more time to “say them” in your head.

The material is presented also, such that very rarely will you be required to retain digits in memory, but when dealing with operations of several digits and/or terms, this may be necessary. The approach for the most part, however, is one of streamlining thought, and resorting to memory, only whenever necessary.

Also, you will notice that as we move to more complicated calculations, we will start to assume that you have assimilated some of the most basic tactics and concepts, and we will not spell them out thoroughly.

For example, by the time you are comfortable calculating three-digit additions, we will not stop to review how useful knowing the table of 2 may be in calculating 8+7, or 81+71, or what the rule of adding to 9 is and how it can be applied to a three-term mixed operation. (Note these are concepts you should not be familiar with at this point).

In all honesty, I do include clues sometimes, to make sure you can work through the exercises, and internalize suggested methods, but I try to balance thoroughness

with keeping the exercises as fluid as possible, assuming you've done the work and are ready to move forward.

Finally, although I have given the techniques names, these are personal, arbitrary, and functional-intuitive – that is, the names are based on what I thought the technique should be called based on what we are doing with the numbers. Many of these techniques have received different names in different parts of the world by different instructors, and some others I have discovered myself through trial and error in thinking of the best method.

The names are important, insomuch they help you identify the method and logic to apply and such that you can know exactly what I am referring to when I speak of applying a particular technique, but after you have them down, the idea is that you don't say in your head,,as you are doing the math…. "now I'm going to 'round and compensate'", or even less so "now I'm going to 'match' these negative numbers to simplify the calculation".

Instead, as you become used to the techniques, move from saying them in your head to simply thinking about them and applying, in other words, I encourage you to make an effort after you've mastered the material, to think about the relationships between numbers, abstractions, ideas, and numbers themselves as visual entities and not something you say in your head.

Are you ready?

Let's begin.

CHAPTER 1: HOW TO ADD FAST - BASIC CONCEPTS

We begin our journey by outlining the basic building blocks, the things you must absolutely master before attempting the exercises outlined in chapters 3 and 5.

As you read the theory, try to spend some time practicing. We will devote Chapter 3 to practicing properly. But the concepts presented in this chapter are simple enough that they don’t require much preparation before you can push yourself to do some calculations in your head. At the end of some sections you will see step by step demonstrations of examples to help you think about adding and subtracting mentally.

It is very important that the foundations are solid. Master the early sections on this chapter to succeed in later ones. Go slowly, steadily, and devote as much time as you need to completely dominate each section.

NUMBER COMPLEMENTS

To add and subtract numbers of many digits, you need to be very comfortable with single digit numbers first. The following ***number complements*** are essential. You must commit them to memory such that you don’t even have to think about the answer.

1 and 9, *2 and 8*, *3 and 7*, *and 4 and 6*, all add to **10**.

1+9 = 9+1 = **10**
2+8 = 8+2 = **10**
3+7 = 7+3 = **10**
4+6 = 6+4 = **10**
5+5 = 5+5 = **10**

You also want to commit to memory the following single digit subtractions,

5-3 = **2**					
	6-3 = **3**				
7-5 = **2**		7-3 = **4**	7-2 = **5**		
	8-5 = **3**		8-3 = **5**	8-2 = **6**	
9-7 = **2**		9-5 = **4**	9-4 = **5**	9-3 = **6**	9-2 = **7**

Knowing these identities by heart is extremely important for the work we will do ahead. We present these subtractions first because they have immediate use even for other fundamental concepts like the ***Rule of Adding to 9*** and the ***Rule of Adding to 8***.

In addition to knowing which numbers add up to 10, It is also useful to know ***number complements*** that add up to 20 and to 100.

You should be comfortable with all positive number additions that result in 20 and 100. Outlined below are the ones you should pay most attention, because they are typically the ones that are hardest to remember.

And within these, bolded additions may be even harder to remember. The goal again, is for these identities to be something you don’t even have to think about.

2+18, **3+17**, **4+16**, 5+15, **6+14**, **7+13**, 8+12, 9+11

All the sums above are equal to **20**. The sums below result in 100. Pay special attention to bolded figures

97+3	96+4	**93+7**	92+8
87+13	86+14	**83+17**	82+18
77+23	76+24	**73+27**	72+28
67+33	66+34	**63+37**	62+38
57+43	56+44	**53+47**	52+48

Example 1	73 + 27 = 100

Example 2	4 + 16 = 20

Example 3	8 - 3 = 5

TABLE OF 2 AND ITS USES

The ***table of 2*** is the easiest to remember and it will come in very handy in at least three ways when adding:

First, when you are adding the same single digit number, you can simply remember the table of 2 and think of the addition as simply the number times two.

2+2 = 2x2 = **4**
3+3 = 3x2 = **6**
4+4 = 4x2 = **8**

and so on and so forth...

Knowing this, when you get an addition of consecutive numbers, like 4 and 5, you can simply remember that you are writing 4x2+1, that is 8+1.

Third, if you are dealing with digits that are at equal distance from each other on the number line, the table of 2 is also useful.

4+2 = 3x2
6+4 = 5x2
7+5 = 6x2
8+6 = 7x2
9+7 = 8x2

For each of the previously listed summand pairs there is only one unit or whole number between them, such that we can think of that number times 2, which may be easier than counting fingers.

And likewise,

6+2 = 4x2
7+3 = 5x2
8+4 = 6x2
9+5 = 7x2

In between each of the previously listed summands there are three units or whole numbers, the middle value of those three digits times two, is the result.

Start thinking of numbers as they relate to other numbers as single entities instead of two added terms, that you will save you valuable time.

Example 1	7 + 5 = 12

Example 2	7 + 7 = 14

Example 3	9 + 5 = 14

RULE OF ADDING TO 9

Remember that adding any whole positive single digit number to 9 will result in a number that has 1 as a digit in the tens, and its own value, minus 1 on the ones (except for 0 of course, which is not a positive number anyway).

Adding any two-digit number to 9, or 9 to any two-digit number - if that's easier to remember, will result in a number that has 1 more unit in the tens, and one less in the ones than the number you added to 9.

9+2 = **11**

Again, 1 in the tens, and 2-1 in the ones.

And ...

9+3 = **12**
9+4 = **13**
9+5 = **14**, and so on and so forth…

And, also,

9+11 = **20**
9+34 = **43**
9+78 = **87**

And for two-digit numbers added to nine that are equal to or greater than **91**, you will get *0 in the tens* and *one in the hundreds*, plus o*ne whole number less in the ones than the two-digit number you were adding to 9*.

9+93 = **102**
9+95 = **104**
9+99 = **108**

And so on and so forth…

This is a different way of looking at addition and thinking about relationships between numbers to immediately calculate the result using the most efficient mental method possible.

Example 1	9 + 13 = 22
Example 2	9 + 45 = 54
Example 3	9 + 97 = 106

RULE OF ADDING TO 8

Eight can be a tricky number to add to as well.

8+1 is easy enough to calculate, as is 8+2.

Along the same lines of the rule to adding to 9 we have,

8+3 = 1 in the tens and 3-2 is 1 in the ones, so **11**
8+4 = 1 in the tens and 4-2 is 2 in the ones, so **12**
8+5 = 1 in the tens and 5-2 is 3 in the ones, so **13**
8+6 = 1 in the tens and 6-2 is 4 in the ones, so **14**

So now instead of subtracting one whole number from the ones, we subtract two.

For two-digit numbers the same rules of the ***Rule of Adding to 9*** with the caveat that instead of one whole less number in the ones position we'll get two less.

8 + 27 = **35**
8 + 53 = **61**
8 + 82 = **90**

Naturally this applies for additions where there are carries.

Not here: 8 + 81 = **89**, since there are no carries,

But here, 8 + 97=**105**.

That's easy to see. Especially if you are proficient in the concepts presented earlier in the chapter, since seeing numbers that add to less than 10 should be second hand to you by now, and you should't spend any signfiicant amount of time trying to distinguish two digit additions with carries from those without.

Finally adding a two-digit summand to eight that is equal or greater than 92 will produce the following results,

8 + 94 = **102**
8 + 96 = **104**
8 + 98 = **106**

It's the same concept presented in the previouse section.

For both the ***Rule of Adding to 9*** and the ***Rule of Adding to 8***, you can see how knowing some of the fundamental single digit subtractions by heart is important to doing mental calculations.

Example 1	8 + 34 = 42

Example 2	8 + 7 = 15

Example 3	8 + 95 = 103

Good job getting here!

We just covered fundamentals. Don’t forget to practice before moving forward.

We will take on Addition Techniques in Chapter 2. Read them carefully, as many of the techniques presented in Chapter 4 (the chapter covering subtraction techniques) are adaptations or variations of the techniques presented in the next pages.

CHAPTER 2: ADDING FAST – TECHNIQUES

In this chapter we'll cover the techniques you must be familiar with to successfully complete the practice section in Chapter 3. The techniques presented here can be applied beyond the regular two term, two or three-digit sum, into additions of more than two numbers with each term having more than three digits.

DECOMPOSING

Decomposing number additions into more manageable parts will be very useful, and not just for the single digits as we will see.

These sums lend themselves to decomposition naturally:

3 + 5 = 3 + (3+2) = **8**
3 + 6 = 3 + (3+3) = **9**
4 + 7 = 4 + (4+3) = **11**
5 + 7 = 5 + (5+2) = **12**

Try not to just memorize the result (although it would be great if you did). Ultimately the faster you can invoke a result, the faster you will be able to compute the addition, and in the case of larger figures and operations with more than two terms, the increased speed will make a world of a difference). At least for the sake of understanding ***decomposing***, try to understand what we are doing with the numbers. The idea here is to break down the addition into easier, more digestible terms.

Example 1	3 + 7 = 3 + (3 + 4) = 10

Example 2	20 + 35 = 20 + (30 + 5) = 55

Example 3	15 + 3 = (10 + 5) + 3= 18

SCANNING

When you are dealing with two or more-digit figure additions you'll be well advised to do a ***quick scan*** before resorting to a more complex or difficult strategy.

Scanning in addition serves to purpose of determining whether we can resolve for instance a two or three term number addition of three digits each, as if we were working with three single digit additions.

For instance

123 + 231 + 245

Scanning the hundreds column we see that 3+1+5 = **9**. scanning the tens 2+3+4 = **9,** 1+2+2 = **5**, and scanning the hundreds So, there's no need to do any strenuous calculations from right to left as you would when writing the sum on paper. The result is just **599**.

You'll be well advised to start ***scanning from right to left***, that is starting with the ones, then the tens and then the hundreds. If all is ok, *perform the addition from left to right*.

Why is this important?

We ***scan from right to left*** because having a carry on the ones is more problematic than having a carry on the tens. But we *calculate from right to left* because the number will be easier to remember if you are doing the calculation mentally.

In other words, the difference from calculating from left to right instead of right to left is that the figure rolls out of your tongue effortlessly, in the end, when there are no carries you are simply making single digit additions and patching them into a three digit number.

Additionally, when we are dealing with operations involving numbers of four or more digits, ***scanning*** can help us break down the operation into sections. Depending on the place value location of the single-digit addition that results in carries, we will decide how to break down the operation.

Example 1	31 + 47 + 20 = 98

Example 2	122 + 250 = 372

Example 3	564 + 111 + 200 = 875

ALIGNING

Something you should also become acquainted with right-off-the-bat is ***aligning***.

That is, mentally ***aligning numbers*** according to their place value position, as in ones, tens, hundreds and so on.

Let's look at an example using the ***scanning technique*** we just learned.

231 + 245 + 456

Immediately you can see there are carries in the ones and the tens place values.

Specifically,

2 + 2 + 4 = **8**
3 + 4 + 5 = **12**
1 + 5 + 6 = **12**

Visually you would map it in your mind something like this,

800 (eight in the hundreds) **12**0 (twelve in the tens) 0**12** (twelve in the ones)

Writing this down is *NOT* compatible with doing mental calculations. And remembering all the carries sequentially is cumbersome.

It is a better idea is to visualize the numbers in your mind according to their place value positions.

By the way, the answer is **932**

We will see through examples how we deal with multiple term additions of more than three digits, but for our purposes right now, it's important that you can mentally map the difference between a 12 in the tens vs a 12 in the ones.

Practice with hard examples, like this one,

123 + 456 + 780

Again, this may be hard, because there is carrying in the ones, and in the tens.

1 + 4 + 7 = **12**

If you are counting with your fingers or spending too much time doing 1 + 4 + 7, I urge you to review the fundamental concepts in Chapter I before moving on

Remember that a way to think about it is:.

*1 + 4 + 7 = 5+7, which **decomposed** is 5 + (5 + 2) = **12***

I'm spelling it out this time, but you should work on getting this down without needing to resort to number decomposition.

1 + 4 + 7 = **12**	2 + 5 + 8 = 10 + 5 = **15**	3 + 6 + 0 = **9**

Mentally align,

1200 (twelve in the hundreds) 0**15**0 (fifteen in the tens) 000**9** (nine in the ones)

Calculate from left to right (say it in your head as you do) … one thousand, three hundred, fifty, nine.

We can see the result as **1,359**.

Example 1	38 + 37 3 + 3 = 6 8 + 7 = 15 60 (six in the tens) 15 (fifteen in the ones) 75

Example 2	381 + 458 3 + 4 = 7 8 + 5 = 13 1 + 8 = 9 700 (seven in the hundreds) 130 (fourteen in the tens) 009 (nine in the ones) 839

Example 3	684 + 941 + 198 6 + 9 + 1 = 16 8 + 4 + 9 = 21 4 + 1 + 8 = 13 1600 (sixteen in the hundreds) 0210 (twenty-one in the tens) 0013 (thirteen in the ones) 1,823

ROUNDING AND COMPENSATION

This is a cornerstone technique that you should resort to and become fluent in if you want to add or subtract large digit numbers fast. You should practice extensively with two-digit additions.

Since we will break down numbers with more than three digits into more manageable pieces later, it is imperative that you do ***rounding and compensation*** with two digits well.

Rounding refers to performing the operation but replacing one of the summands with a number that is a multiple of 10… or 5.

This is where knowing number complements to 10 from the first chapter comes in handy.

Take for instance

39 + 48

Its easy to see that 39 is one away from 40
(or 48 is two away from 50 if you so fancy).

So instead of trying to figure out what 39 + 48 is
(especially since there are carries),
you could do 40 + 48, which is clearly 88. But wait!

We've taken care of the ***rounding*** part. Obviously 39 + 48 is not the same as 40 + 48.

Enter ***compensation***.

Since you borrowed 1 when ***rounding up***, you have now to subtract it to get the actual result:
88 - 1 = 87

And although you could have in theory rounded both summands, say from 39 + 48 to 40 + 50, doing so would have necessitated that you compensated for both, which becomes unnecessarily cumbersome.

Generally, and again, this depends on your own math background and preferences, it is best to just round one of the two terms, and *in general* the easier for you to round will the one that is closer to a multiple of 10.

Knowing the number complements by heart will aid you in quickly compensating, and since most of the time you will be compensating single digits (in this case on

the ones position), it is also necessary to know by heart the single digit subtractions outlined in Chapter 1.

Remember, typicallly, the number you want to round is the one closest to a multiple of 10.

There are exceptions of course. Take this example,

97 + 59

Here it might be easier for you to round up the 97 instead of 59,
so do 100 + 59 = 159.

In this case, as in the previous because you are rounding up and not down, you must subtract the 3 you borrowed to get the actual result.

So,

159 - 3 = **156**

If you were ***rounding down***, say,

102 + 57

Where you would do 100 + 57 = 157,
the 2 you rounded down would have to be added to get the actual result.

157 + 2 = **159**

Always remember, in addition,

If you are ***rounding up*** a ***positive*** number,
you must ***compensate by subtracting***.

If you are ***rounding down*** a ***positive*** number,
you must ***compensate by adding.***

Example 1	91 + 37 90 + 37 = 127, 127 + 1 = 128

Example 2	58 + 35 60 + 35 = 95, 95 - 2 = 93

Example 3	341 + 520 340 + 520 = 860, 860 + 1 = 861

BALANCING

Balancing is a special case of ***rounding and compensation***. It is faster and easier, and you will be able to do it in a second if you've memorized and mastered the ***uses of the table of 2*** we outlined in Chapter 1.

Suppose you need to mentally add,

97 + 59 + 31

Before you rush in to round the 97 to 100. Look at the ones in 59 and 31, 9 and 1, notice that if you were to round those two numbers, the compensation would cancel itself out. Why?

Because with a 9 you round up 1 to get a multiple of 10, with a 1 you round down 1.

This means effectively that you can do the rounding on two instead of one of the numbers, without having to worry about the compensation.

60 + 30 = 90

And then 100 + 90 = 190, 190 - 3 = **187.**

Example 1	48 + 22 = 50 + 20 = 70

Example 2	182 + 148 + 85 = 180 + 150 + 85 = 330 + 85 = 415

Example 3	13 + 17 + 58 + 12 = 10 + 20 + 60 + 10 = 100

(SEQUENTIAL) GROUPING

The concept of ***grouping*** is fundamental to doing mental calculations whether you are adding single, double, or triple digit figures, and it is particularly important when dealing with multiple terms.

This technique consists of paring up... or sometimes combining figures in groups of 2 or 3 terms so that an addition consisting of multiple terms becomes mentally manageable.

Let's see what we mean with a single digit example.

2 + 5 + 7 + 9 + 3

There are many efficient ways to group these.

1. You could ***group*** 7 and 3, since those are complements and easily add to 10. Then add the 9, conveniently 19, and worry about adding the ***grouped*** (5+2 = 7) which is easy to compute. So, 19+7 (by the ***rule of adding to 9***) is **26**.
2. Or you could just as easily have seen that 3+2 is 5, plus 5 is 10, plus 9 is 19, plus 7, **26**.

The optimal way of ***grouping*** these quantities will depend on what comes easiest to you, and how much you've mastered each of the foundations set out in Chapter 1.

Now what if you are adding five two-digit numbers?

33 + 44 + 45 + 89 + 12

Again, the optimal strategy will vary depending on your background an ease with arithmetic, the number associations you are most comfortable with and/or your proficiency in effectively applying the rules we have covered so far.

By way of example, here is a good strategy:

1. ***Group*** 89 and 12… by the ***rule of adding to 9*** you can see you'll get 101. If that's too complicated think of the 12 ***lending*** a 1 to the 89 to round itself up to 90, so then 90+11 = 101. ***Lending*** is not a technique we cover in of itself under addition and subtraction techniques, but you can use it just like any of the formal ones we present here.
2. Conveniently there's a 44 that needs that 1 to become 45, add 101+44 = 145.
3. We are left with 45 and 33. The easiest to pick is 45, 145+45 = 190. That should be easy enough, if not, review Chapter 1.
4. Finally add 33. Again, resort to rounding and compensation. The 190 borrows 10 from 33 which then makes the sum 200 + 23 = **223**.

Whichever way you choose to ***group*** the numbers. In general, it is advisable that you associate two or three numbers that you can easily mentally add, and then add one by one the numbers that remain. We'll review this later, and we'll refer to it as ***Sequential Grouping***.

We advise against trying to group in three different groups of numbers simultaneously, because you are more likely to misstep and forget or confuse the result of one of the grouped additions you calculated earlier on, getting an incorrect actual result. And again, because we are not writing any of this down, you may be making unreasonable demands on your short term memroy.

Example 1	21 + 13 + 15 + 45 + 50 15 + 45 = 60 60 + 50 = 110 110 + 21 = 131 131 + 13 = 144

Example 2	417 + 487 + 132 + 111 132 + 111 = 243 243 + 417 = 660 660 + 487 = 1147

Example 3	514 + 736 + 198 + 402 198 + 402 = 600 600 + 736 = 1336 1336 + 514 = 1850

Now that you’ve mastered the essential concepts and techniques, let’s get our hands dirty with some practice (with suggestions greyed out within, and with full step by step resolutions in the appendix).

CHAPTER 3: PROGRESSIVE PRACTICE I

This section consists of examples. I will just add references to a preferred way to solve the problem, although you may find it easier to calculate it using a different technique. What matters most is that you have mastered the techniques presented so that you are able to choose the best one, as opposed to simply using the only one you know well.

Remember also two things going forward:

First, if you have trouble recognizing which basic concept or technique is best suited for each step of the calculation you may need a better understanding of concepts and techniques. Concepts like ***the rule of adding to 9*** or ***the rule of adding to 8***, or ***rounding and compensation***, are naturally more complex than single step techniques because they involve more than one step.

Second, the technique that I recommend may not be the one that comes easiest to you, even after attaining mastery. That is OK. As I've said before, which method is best will depend on your familiarity with arithmetic, and even the single or double-digit number associations that come easiest to you.

For now, let's look at a couple of worked out examples.

ADDING AND SUBTRACTING SINGLE DIGIT NUMBERS

3+7 *(tip… adding the smallest number to the largest is easier and faster, so flip to 7+3) (complements)*

6+4 *(complements)*

7-5 *(memorize)*

3+3 *(table of 2)*

7-3 *(memorize)*

7-2 *(memorize)*

8-5 *(memorize)*

8-3 *(memorize)*

4+4 *(table of 2)*

3+5 *(memorize)*

8-2 *(memorize)*

9-5 *(memorize)*

9-4 *(memorize)*

9-2 *(memorize)*

7+5 *(decompose)*

6+8 *(flip)(rule of adding to 8) (or decompose 8 into the complement of 6)*

9+7 *(rule of adding to 9)*

9+3 *(rule of adding to 9)*

9+5 *(rule of adding to 9)*

3+8 *(flip)(rule of adding to 8)*

4+8 *(flip)(decompose/rule of adding to 8)*

8+5 *(rule of adding to 8)*

3+5 *(flip)(decompose)*

3+6 *(flip)*

4+7 *(flip) (decompose)*

1+3+7 *(group 3 and 7) (add 1)*

3+4+5 *(group 4 and 5) (rule of adding to 9)*

2+8+9 *(group 2 and 8 – complements) (add 9)*

5+7+1 *(group 5 and 1) table of 2 (decompose),*

ADDING DOUBLE DIGIT NUMBERS

7 + 23 *(flip)(complements to 10)*

92 + 8 *(complements to 100)*

9 + 11 *(flip)(complements to 100)*

7 + 13 *(flip)(complements to 20)*

53 + 43 *(scan)(table of 2)*

39 + 48 *(round and compensate)*

48 + 52 *(balancing)*

91 + 77 *(round and compensate)*

12 + 39 *(flip)(round and compensate)*

94 + 59 + 36 *(balancing complements 36 and 94)(add the third term)*

71 + 57 + 66 *(group 71 and 66, table of 2) (add third term, decomposition)*

22 + 48 + 29 *(group 22 and 48 – complements to 10)(round and compensate 29)*

97 + 12 + 31 *(group 12 and 31, scan)(round and compensate 97)*

ADDING TRIPLE DIGIT NUMBERS

117 + 23
1. add the tens
2. add the ones (complements to 10)
3. mentally align

233 + 384
1. work with the hundreds as single digit figures: add single digits.
2. work with the ones: (round and compensate)
3. mentally align

124 + 314 *(scan) (add from left to right)*

471 + 229 + 255
1. work with the hundreds as single digit figures: add single digits
2. work with the ones: (group 71 and 29) (balance) (add the remaining term)
3. mentally align

119 + 222 + 777 *(group 111 with 222)(scan) (complements to 1000)(decompose)*

317 + 289 + 122
(group)(lend to round) (scan)

163 + 413 + 720 *(scan) (add from left to right)*

657 + 523 + 566
1. work with the hundreds as single digit figures: add single digits
2. work with the tens: (group 66 and 23 or 57 and23) (add the remaining term)
3. mentally align

310 + 450 + 129 *(scan) (add from left to right)*

151 + 510 + 208 *(scan) (add from left to right)*

973 + 493 + 713
1. work with the hundreds as single digit figures: (group 9 and 7)(rule of adding to 9) (add the remaining term)
2. work with the tens: (group 13 and 73) (round and compensate)
3. mentally align

615+ 213 + 20 *(scan) (add from left to right)*

ADDING NUMBERS OF MORE THAN THREE DIGITS

4194 + 1490
1. work with the hundreds as double-digit figures:
2. work with the tens
3. mentally align

2481 + 7217 *(scan) (add from left to right)*

2757 + 6801
1. work with the hundreds as double-digit figures:
2. work with the tens
3. mentally align

8280 + 5555
1. work with the hundreds as double-digit figures
2. work with the tens as double-digit figures
3 mentally align

8031 + 9102 + 1999
(group 8031and 1999) (round and compensate) (scan) (add from left to right)

1010 + 5050 + 2580
(group 1010 and 5050) (scan) (add from left to right)
2. work with the hundreds as double digits
3. work with the tens as double digits
4. mentally align

1459 + 2451 + 3871
1. work with the hundreds as double-digit figures: (group 14 and 24) (round and compensate)
2. work with the tens as double-digit figures: (balance)
3. mentally align

CHAPTER 4: HOW TO SUBTRACT FAST - TECHNIQUES

If you´ve mastered the chapters we´ve covered so far, the hardest work is behind you.

Although there are a few techniques that are specific to subtracting, most of the techniques you'll use are addition techniques adapted to subtraction. Moreover, except for ***tactics for mixed operations***, which we'll discuss toward the end of this chapter, most subtractions will be of two terms. A positive term and negative term.

If both terms are positive, *we add them.*
If both are negative, we also add them ***and pre-pend a minus sign at the resulting number***.

However if the terms have different signs, we resort to a different method:

REVERSE AND NEGATE

8-2 is not too hard to calculate, but what if you get

2-8… that might be a bit more complicated, especially if you are subtracting terms of more digits with inverse signs.

The result of *2-8* is *the negation or additive inverse* of *8-2*

So

Reverse 2-8, we get 8-2 = 6. ***Negating*** it we get **-6**

7-3 is 4, so 3-7 is **-4**

Whenever you have a subtraction where the subtrahend’s absolute value is greater than that of the minuend (the term you are subtracting from), ***reverse the terms*** of the operation and ***negate the result***.

As we combine different techniques for making subtraction easier and faster, you will see that our strategy will be to ***reverse***, but we won't ***negate*** until the very end, after we’ve applied all other techniques to the operation.

For instance, we might ***reverse*** 148 - 252 to 252 - 148 and then do some sort of ***rounding*** say, 252 - 150 = 102, 102 + 2 = 104, before we ***negate*** the 104.

Why didn’t we simply balance the terms?

Why did we add 2 instead of subtracting?

Both answers have to do with the fact that we are subtracting and not adding the second term, but we’ll answer those questions later in more detail.

Example 1	4 - 12 12 – 4 = 8 -8

Example 2	52 - 77 77 – 52 = 25 -25

Example 3	404 - 910 910 – 404 = 506 -506

SUBTRACTING FROM MULTIPLES OF 10

Your level of comfort in dealing with two-digit addition operations and number complements will largely determine how useful you find the technique of ***Subtracting from Multiples of 10***.

60 - 13 may seem a bit intimidating:

At least more intimidating than 10 - 3 which is 7, which we know to calculate really-fast because we are familiar with ***complements***.

Finding the ***complement*** of 13 to make 60 is harder than finding the ***complement*** of 3 to make 10.

Finding the complement of 3 in the ones isn't hard.

But what exactly can you do to remember quickly what happens in the tens?

6 - 1 = 5 in the tens. But the result is not 57, is 47. *It is one unit less.*

With practice it will become second nature to you.

70 - 34 = 6 in the ones (4 is the ***complement*** of 6),
7-3 = 4 in the tens, *minus one*, *3*, so **36.**

90 - 21 = 9 in the ones (1 is the ***complement*** of 9),
9-2 = 7 in the tens, *minus one*, *6*, so **69.**

30 - 9 = 1 in the ones (9 is the ***complement*** of 1),
3-0 = 3 in the tens, *minus one*, *2*, so **21.**

Example 1	100 - 47 three in the ones (7 is the complement of 3) 10 - 4 is 6, minus one is 5 53

Example 2	470 – 34 six in the ones (4 is the complement of 6) 47 - 3 is 44, minus one is 43 436

Example 3	324 - 500 500 – 324 six in the ones (4 is the complement of 6) 50 - 32 is 18, minus one is 17 176 -176

ADDITION TECHNIQUES FOR SUBTRACTION

A lot of what you learned for addition is useful for subtraction, with a few caveats.

Decomposing, ***rounding and compensation***, ***scanning***, and ***grouping*** all apply, albeit a bit differently.

Let’s take a look at ***decomposing***:

63 - 16

Here the six can be split,

63 - 13 - 3 = 50 – 3,
and 50 - 3 is 47, so **47** is the result.

What about this?

16 - 63

reverse
63 - 16

decompose
63 - 13 - 3
50 - 3
47

negate
-47

Then **-47** is the answer.

Ok.

Rounding and compensation works for subtraction as well, only, you will now ***round*** not the number closest to a multiple of 10 as you did in addition, but always (or almost always) ***round*** the term you are subtracting.

In addition, we observed:

If you are ***rounding up*** a ***positive*** number,
you must ***compensate by subtracting***.

If you are ***rounding down*** a ***positive*** number,
you must ***compensate by adding***.

In subtraction, we observe:

If you are ***rounding up*** a ***negative*** number,
you must ***compensate by subtracting***.

If you are ***rounding down*** a ***negative*** number,
you must ***compensate by adding***.

If we ***round up*** a ***negative***, we ***compensate by subtracting***, like so:

117 - 13

117 - 10 = 107 and 107 - 3 = **104**

If we ***round down*** a ***negative***, we ***compensate by adding***, like so:

345 - 28

345 - 30 = 315 and 315 + 2 = **317**

For fast subtracting it is essential you commit these rules to memory, it will make things a lot easier down the road.

Scanning also has a prominent place in subtraction especially when dealing with numbers of three or more digits. Let's look at some examples that lend themselves to easy ***scanning from right to left*** and ***subtracting from left to right***.

379 - 123

Scan, going from right to left to see if any place value single digit subtraction results in less than 0. Since there are no carries we proceed from right to left

3 - 1 = 2
7 - 2 = 5
9 - 3 = 6

so the result is **256**

748 - 999

reverse
999 - 748

scan: No carries.
Now ***subtract from left to right***
9 - 7 = 2
9 - 4 = 5
9 - 8 = 1
251

negate
-251

102 - 273

reverse
273-102
scan: No carries. Now ***subtract from left to right***
2 - 1 = 1
7 - 0 = 7
3 - 2 = 1
171
negate
-171

Always remember, each time you ***reverse***, you ***negate***.

Another one,

387 – 288

Scanning from right to left we see, that although the hundreds and the tens place value figures present no problems. 7-8 is less than 0.

So here, we compute 38 - 28 (on the tens) as a separate operation.
38 - 28 = 10.
So, 10 in the tens.

Because to solve for the ones 7 - 8, we had to reverse 8 - 7 = 1, and negate **-1**.

We can proceed ***aligning*** as we did in addition, only this time we subtract the bottom row term.

100 (ten in the tens) 00**1** (one in the ones)

The result is **99**

Example 1	134 - 264 264 – 134 = 134 + 130 – 134 = 130 -130

Example 2	491 - 377 491 - 380 = 111, 111 + 3 = 114

Example 3	789 – 148 = 641

Before finishing this section, we should say a word about ***grouping*** in the context of subtraction.

Mental subtraction as we are thinking of it, takes place between a positive and a negative number.

Of course, in practice you can subtract a negative from a negative, or three negative terms from each other if you will. But as I mentioned at the beginning of this chapter, to arrive at such figure, you would just treat it as an addition and reverse the sign at the end.

But what if we have four terms, two positive and two negative, or three positives and one negative.

We’ll deal with these cases with a general strategy for dealing with such cases which we’ll refer to as ***Mixed Operations***, since they consist of multiple negative or positive terms. However, first we’ll look at a final technique: ***matching***.

MATCHING

Matching is to subtraction, what ***balancing*** to addition, in the sense that is an effective variation of ***rounding and compensation***.

37 - 22

37 - 22. What happens if we add 3 to both terms?

Two things.

First, we get:

40 - 25 instead of 37 - 22, which is easier to compute.

Second, *we get the same result as the original operation*.

So, when dealing with an operation involving a positive and a negative, we could say that ***matching*** consists of adding the same number to either term to make the operation easier to calculate.

In a sense, we are ***rounding*** the numbers, but unlike ***balancing*** where we ***round up*** one and ***round down*** another, we are ***rounding both up or both down***. And for both ***balancing*** and ***matching***, there's no need to ***compensate*** after ***rounding***, after all, that's what we are doing when adding or subtracting an equal amount from quantities with inverse signs.

Example 1	74 – 39 = 75 – 40 = 35

Example 2	238 – 473 473 – 238 = 475 – 240 = 235 -235

Example 3	799 – 239 800 – 240 = 560

MIXED OPERATIONS

Suppose you need to compute,

83 - 45 - 15 - 38

Here we have a positive term and three negative terms. What strategy should we pursue? If ***grouping*** somehow came to mind, then on you're on the right track.

There is no fixed formula for solving ***mixed operations***, especially if they have more than five terms since the number of combinations in terms of the sign of each number and the number of terms will make the optimum strategy vary.

But this doesn't mean we are left with naught.

Let's look at an easier example first, to illustrate the basic points.

1 + 3 - 2 - 5

In this example, we have two positive terms and two negative terms.

One thing you could do is group the positives and the negatives 1 + 3 = 4 and -2 - 5 = -7. Then do 4 - 7, which we could reverse to 7 - 4 = 3 and then negate to get **-3**.

But that's very cumbersome isn't it?

The reason this is complicated is because you must retain in memory several figures. Granted this example is not overly complicated, but what happened when you are retaining four three digit figures?

The way out?

Always (or in most all cases) start with two, a pair – sometimes grouping three terms may be just as easy for you, and to the preliminary result you get from that group, add, or subtract whichever term is easier to combine that result with, and do that until you reach the end. We introduced this earlier, and call this ***sequential grouping***.

So, in this example it would be
1 + 3 = 4, 4 - 2 = 2, 2 - 5,

reverse (if you need to) 5 - 2 = 3,

negate, **-3**.

Going back to our original mixed operation:

83 - 45 - 15 - 38

Here take 45 and 15 which are the easiest to group to quickly get -60.

-60 - 38 = -98

We are left with 83

83 - 98

reverse
98 - 83

scan, and solve from left to right
9-8 = 1
8-3 = 5
15

negate (always remember to negate if you reversed)
-15

What about

831 - 847 + 391 - 128

For example, 831 and 847 aren't that far apart.

reverse
847 - 831

scan, and ***solve from right to left***
8 - 8 = 0
4 - 3 = 1
7 - 1 = 6
16

negate
-16

Let's add it to -128... not too intimidating
-128 -16 = remember ***the rule of adding to 8*** in single digits, -144

We are left with,

391 - 144

round
391 - 150

scan, ***solve from left to right***
241

compensate
241 + 6
247

That's the result.

That was tough, but if you are familiar with all the techniques presented in this book then this will have been easier than doing the individual operations, grouping by sign, and writing things down vertically.

Example 1	123 - 17 – 58 = 123 – 75 = 100 + 23 – 75 = 25 + 23 = 48

Example 2	478 - 311 - 214 + 31 509 - 311 -214 = 509 – 525 = 525 – 509 = 16 -16

Example 3	417 + 184 - 149 + 200 51 + 417 + 184 = 468 + 184 = 500 140 012 652

Remember to practice. Once you are proficient in the techniques, you won't need to say them in your head… like I'm "***rounding and compensating***" or "I'm ***negating*** because I ***reversed***", or even "I'm using ***the rule of adding to 8***". You'll just do the mental calculations with no meta-information.

CHAPTER 5: PROGRESSIVE PRACTICE II

We continue practicing mental arithmetic but now our focus is on subtraction. We’ll move progressively as we did in Chapter 3 and outline suggested ways to tackle the problem in greyed out italics to the right of the operation.

This time try to see what you come up with. Don’t focus on the greyed-out portion. Read it only after you’ve tried to solve the operation and determine if your technique resulted in a fast calculation.

If it did, then, Congratulations! It means that both,
a) you´ve mastered a technique or set of techniques appropriate for the problem.
b) you found a technique or combination of techniques that best suits you and your skill profile for similar problems

We have not included a section subtracting single-digit numbers.

That was covered in Chapter 3.

Also, for two-digit subtractions. If you are not there yet, keep practicing double digit number subtraction until you’ve become comfortable with it.

SUBTRACTING DOUBLE DIGIT NUMBERS

4 - 40	*(reverse) (subtract from multiples of 10) (negate)*
3 - 90	*(reverse) (subtract from multiples of 10) (negate)*
9 - 66	*(reverse) (decompose) (negate)*
9 - 57	*(reverse) (decompose) (negate)*
23 - 77	*(reverse) (round and compensate the negative term) (negate)*
81 - 13	*(decompose))*
21 - 79	*(reverse) (round and compensate(negate)*
28 - 43	*(reverse) (match) (negate)*
-50 - 4 - 5	*(remember here you are adding and then negating the result)*
-43 - 2 - 8	*(group) (negate)*
-88 - 97 - 10	*(round and compensate)*
-22 - 47 - 17	*(match)*

SUBTRACTING TRIPLE DIGIT NUMBERS

137 - 457	*(reverse) (scan) (subtract single digits from left to right)*
753 - 142	*(scan) (subtract single digits from left to right)*
329 - 179	*(scan) (work with the tens breaking up the terms) (match) (align)*
684 - 592	*(scan) (break up the terms) (align)*
348 - 764	*(reverse) (decompose) (work with the hundreds) (work with the ones) (align)(negate)*
921 - 524	*(round and compensate) (decompose)*
282 - 472	*(work with the hundreds, reverse and negate) (work with the ones) (reverse and negate) (align)*
724 - 822	*(reverse) (take on the hundreds and tens a number and subtract) (reverse and negate the ones) (align)*

SUBTRACTING NUMBERS OF MORE THAN THREE DIGITS

1,739 – 9,371
1. work with thousands and hundreds as a double-digit figure: (reverse) (round and compensate) (negate)
2. work with the tens and ones a double-digit figure (reverse) (round and compensate) (negate)
3. mentally align

8,260 - 2,740
1. work with the hundreds as a double-digit figure: match)
2. work with the ones a double-digit figure
3. mentally align

9,888 - 1,119
1. work with the tens as a triple-digit figure: (subtract from right to left)
2. work with ones a single-digit figure (reverse and negate)
3. mentally align

5,173 - 5,977 *(reverse) (scan) (subtract from left to right) (negate)*

4,062 - 6,048
1. (reverse)
2. work with the hundreds as a double-digit figure
3. work with the ones a double-digit figure (reverse) (round and compensate) (negate)
4. mentally align
5. negate

6,284 - 4,826
1. work with the hundreds as a double-digit figure: (round and compensate)
2. work with the ones a double-digit figure (decompose)
3. mentally align

MIXED OPERATIONS

88 - 25 + 28 *(group sequentially)*

381 - 245 + 481 *(group positive terms)*
1. work with the hundreds as a single digit figure.
2. work with tens and ones as double-digit figures (round and compensate)
3. mentally align
(group remaining term)

93 - 82 - 63 - 23 *(group the second and fourth term) (group the third term) (group the remaining term) (reverse the remaining two terms) (round and compensate) (negate)*

381 + 381 + 381 - 38 *(by the rule of adding to eight, you can see how grouping the first two terms is easy) (group the third term using the rule of 8)*
(group the remaining term, round and compensate)

750 - 210 + 381 - 575 *(group multiples of 10)*
1. (work with the tens as two-digit terms) (round and compensate)
2. (align)
(group result with fourth term) (decompose)
(group the remaining term) (round and compensate)

2,394 - 1,827 + 3,817 - 3,919 *(group the third and fourth terms) (reverse) (negate) (group result to first term) (group the remaining term)*
1. work with the hundreds as a two-digit figure
2. work with the ones as a two-digit figure (round and compensate)
3. mentally align

PARTING WORDS

Thank you for reading this book. I really do hope you found it useful and enjoyable. Math is challenging, that's what makes it fun.

If you review and re-do the exercises in the practice chapters, make sure you also review the appendix.

Did you find a better way to arrive at the result? Congratulations!

Success is all about learning to do stuff well, and then taking matters into your hands!

Did you like the book?
Let us know, we love to hear what we did well.
It encourages us to pursue greater challenges.

Did you find things you didn't like?
Do let us know also.
We are human, so we make mistakes, however painstakingly we may have reviewed this book before it was published.

In any case, I wish you the best in your journey to achieving mastery in this or any other areas of your life.

And I hope that you will join us as we explore the world of mathematics in future publications.

With warm regards, all the best!

The Sigmacasts Team

APPENDIX A1: WORKED OUT ANSWERS TO PROBLEMS (CHAPTER 3)

ADDING AND SUBTRACTING SINGLE DIGIT NUMBERS

3+7 = 7+3 = **10**	**8-5 =** **3**	**9-4 =** **5**	**9+5 =** **14**	**3+6 =** 6+3 = **9**
6+4 = **10**	**8-3 =** **5**	**9-2 =** **7**	**3+8 =** 8+3 = **11**	**4+7 =** 7+4 = **11**
7-5 = **2**	**4+4 =** 4x2 = **8**	**7+5 =** (7+3)+2 = 10+2 = **12**	**4+8 =** 8+4 = 8+2+2 = 10+2 = **12**	**1+3+7 =** 1+(7+3) = 1+10 = **11**
3+3 = 3x2 = **6**	**3+5 =** 3+(3+2) = **8**	**6+8 =** 8+6 = **14**	**8+5 =** **13**	**3+4+5 =** 3+9 = **12**
7-3 = **4**	**8-2** = **6**	**9+7 =** **16**	**3+5 =** 5+3 = **8**	**2+8+9** = 10+9 = **19**

7-2 =	**9-5 =**	**9+3 =**
5	**4**	**12**

ADDING DOUBLE DIGIT NUMBERS

7 + 23 =

23+7 =

30

39 + 48

40 + 48 = 88, 88 – 1

87

94 + 59 + 36

100 + 30 + 59 =
130 + 59 =

189

92 + 8 =

100

48 + 52 =

50 + 50 =

100

71 + 57 + 66

71 + 66 + 57
(6x2 + 1= 13) thirteenin the tens
(6+1 = 7) seven in the ones)
137 + 57 =
137 + (50 + 7) =
187 + 7 =

194

9 + 11 =

11+9 =

20

91 + 77

90 + 77 = 167,
167 + 1

168

22 + 48 + 29

70 + 29
70 + 30 = 100, 100 - 1

99

7 + 13 =

13+7 =

20

12 + 39

39 + 12
40 + 12 = 52, 52 – 1

51

97 + 12 + 31

97 + 43
100 + 43 = 143 - 3

140

53 + 43

(5+4 = 9), nine in the tens
(3x2 = 6) six in the ones
90 + 6 =

96

ADDING TRIPLE DIGIT NUMBERS

117 + 23

11+2 = 13
7+3 = 10

130 + 10

140

119 + 222 + 777 =

(222 + 777) + 119 =
999 + 119 =
999 + 1 + 118 =

1,118

310 + 450 + 129

3+4+1 = 8, eight in the hundreds
1+5+2 = 8, eight in the tens
9, nine in the ones

889

233 + 384

2+3= 5 in the hundreds

33 + 80 = 113
113 + 4 = 117
500 + 117

617

317 + 289 + 122

289 + 11 + 111= 411
317 + 411 =
728

151 + 510 + 208

1+5+2 = 8
5+1 = 6
1+8 = 9

869

124 + 314

1+3 = 4
2+1 = 3
4+4 = 8

438

163 + 413 + 720

1+4+7 = 12
6+1+2 = 9
3+3 = 6

1,296

973 + 493 + 713

9+4+7 = 20, twenty in the hundreds

73 + 93 + 13 =
86 + 93

100 + 86 = 186,
186 - 7 = 179
2,000 + 179

2,179

471 + 229 + 255

4+2+2 = 8, eight in the hundreds

70 + 30 + 55
100 + 55 = 155
800 + 155 =

955

57 + 523 + 566

6+5+5 = 16, sixteen in the hundreds

57 + 23 + 66
80 + 66 = 146
1600 + 146 =

1746

651 + 213 + 20

6+2 = 8
5+1+2 = 8
1+3 = 4

884

ADDING NUMBERS OF MORE THAN THREE DIGITS

4,194 + 1,490

41 + 14 = 55, fifty-five in the hundreds,
94 + 90 = 184 in the ones

5,500 + 184 =

5,684

8,280 + 5,555

82 + 55 = 137 in the hundreds

80 + 55 = 135
13,700 + 135 =

13,835

1,010 + 5,050 + 2,5800 =

(1,010 + 5,050) + 2,580
6,060 + 2,580

60 + 25 = 85, eighty five in the hundreds
60 + 80 = 140, one hundred and forty in the ones

8,500 + 140 =

8,640

2,481 + 7,217

2+7 = 9
4+2= 6
8+1 = 9
1+7 = 8

9,698

8,031 + 9,102 + 1,999

8,031 + 2,000 = 10,031,
10,031-1 = 10,030
10,030 + 9,102 =

19,132

1,459 + 2,451 + 3,871

38 + 38 = 76, seventy six in the hundreds

60 + 50 + 71 =
110 + 71 = 181 in the ones

7,600 + 181 =
7,781

2,757 + 6,801

27 + 68 = 95, ninety five in the hundreds

57 + 1 = 58

9,500 + 58 =

9,558

APPENDIX A2: WORKED OUT ANSWERS TO PROBLEMS (CHAPTER 5)

SUBTRACTING DOUBLE DIGIT NUMBERS

137 – 457

457 – 137 =
320

-320

329 - 179

32 - 17 =
35 - 20 = 15, fifteen in the tens
9 - 9 = 0

150

348 - 764

7 - 3 = 4, four in the hundreds
64 - 48 =
(64 – 44) – 4 =
20 - 4= 16, sixteen in the ones

416

-416

282 - 472

4 - 2 = 2, two in the hundreds

82 - 72 = 10
200 – 10 =

190

-190

753 – 142 =

611

684 – 592

68 - 60 = 8
8 + 1 = 9
4 - 2 = 2

92

921 - 524

921 - 500 = 421, 421 – 24 =
421 - 21 – 3 =
400 – 3 =

397

724 - 822

82 - 72 = 10, ten in the tens
4 - 2 = 2, two in the ones

100 - 2
98

-98

SUBTRACTING TRIPLE DIGIT NUMBERS

137 – 457

457 – 137 =
320

-320

684 - 592

68 - 59 = 9. nine in the tens
4 - 2 = 2, two in the ones

92

282 - 472

47 – 28 = 19, nineteen in the hundreds
2 - 2 = 0, zero in the ones

190

- 190

753 – 142 =

611

348 - 764

76 – 34 = 42, forty two in the tens
8 - 4 = 4, four in the ones

420 – 4 =
416

-416

724 - 822

82 – 72 = 10, ten in the tens
4 - 2 = 2, two in the ones

100 – 2 =

-98

329 – 179 =

32 - 17 = 15
9 - 9 = 0

150

921 - 524

900 - 524 = 376
376 + 21 =

397

SUBTRACTING NUMBERS OF MORE THAN THREE DIGITS

1,739 - 9,371

93 - 17 =
93 - 20 = 73, 73 + 3 =
76, seventy six in the hundreds

71 - 39 =
71 - 40 = 31, 31 + 1 = 32, thirty two in the ones

7600 + 32 =
7632

-7,632

5,173 - 5,977

5,977 – 5,173 =
804

-804

8,260 – 2,740

82 - 27 = 85 - 30 = 55, fifty five in the hundreds
60 - 40 = 20, twenty in the ones

5500 + 20

5,520

4,062 – 6,048

6,048 – 4,062

60 - 40 = 20, twenty in the hundreds

62 – 48
62 - 50 = 12, 12 + 2 = 14, fourteen in the ones

1,986

-1,986

9,888 - 1,119

988 - 111 = 877, eight hundred seventy seven in the tens
9 - 8 = 1, one in the ones

8,770 – 1

8,769

6,284 – 4,826

62 – 48
62 - 50 = 12, 12 + 2 = 14, fourteen in the tens

84 – 26 =
84 - 24 - 2 = 60 - 2 = 58, fifty eight in the ones

1,458

MIXED OPERATIONS

88 - 25 + 28

28 - 25 = 3
88 + 3 =

91

381 - 245 + 481

481 + 381
4 + 3 = 7, seven in the hundreds
81 x 2 = 162, one hundred and sixty two in the ones

700 + 162 = 862
862 - 245
8 – 2 = 6 in the hundreds
62 – 45 = 17, seventeen in the ones

617

381 + 381 + 381 – 38

381 + 381
3+3 = 6, six in the hundreds
81 x 2 = 162, one hundred and sixty two in the ones
600 + 162 = 762
762 + 381
7 + 3 = 10, ten in the hundreds
81 + 62 = 143, one hundred and forty three in the ones

1,000 + 143 = 1,143
1,143 - 38, 1,143 - 40 = 1,103
1,103 +2 =

1,105

750 - 210 + 381 - 575

750 - 210
75 - 20 = 55, 55 - 1 = 54, fifty four in the tens

540
540 - 575 =
540 - (540 - 35) = -35

381 - 35
381 - 40 = 341, 341 + = 346

346

93 - 82 - 63 – 23

82 + 23 = 105 (-105)
105 + 63 = 168 (-168)

168 - 93
168 - 90 = 78 , 78 – 3 = 75

-75

2,394 - 1,827 + 3,817 - 3,919

3,919 + 3,817 = 102 (-102)
2,394 - 102 = 2292

2,292 - 1,827
22 - 18 = 4, four in the hundreds
92 - 27, 92 - 30 = 62, 62 + 3 = 65, sixty five in the ones

465

APPENDIX B: ALGEBRAIC PROPERTIES OF ADDITION AND SUBTRACTION

Algebraic Properties of Addition and Subtraction	
COMMUTATIVE PROPERTY	
a + b = b + a	2 + 3 = 3 + 2
ASSOCIATIVE PROPERTY	
(a + b) + c = a + (b + c)	(2 + 3) + 5 = 2 + (3 + 5) 5 + 5 = 2 + 8
IDENTITY PROPERTY	
a + 0 = a	2 + 0 = 2
INVERSE OPERATIONS	
a - a = 0	2 - 2 = 0

APPENDIX C1: ADDITION TABLES

Addition Tables

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	+
2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	1
3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	2
4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	3
5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	4
6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	5
7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	6
8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	7
9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	8
10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	9
11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	10
12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	11
13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	12
14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	13
15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	14
16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	15
17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	16
18	19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	17
19	20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	18
20	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	19
21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40	2

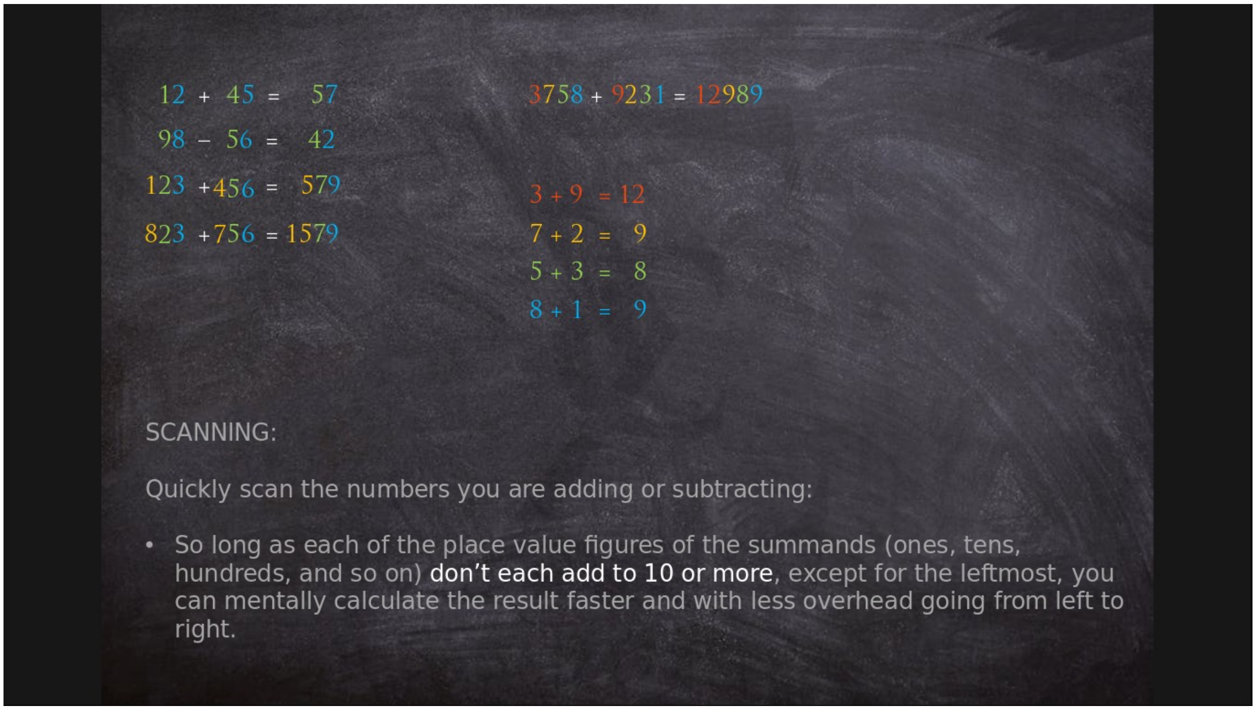
APPENDIX C2: SUBTRACTION TABLES

Subtraction Tables

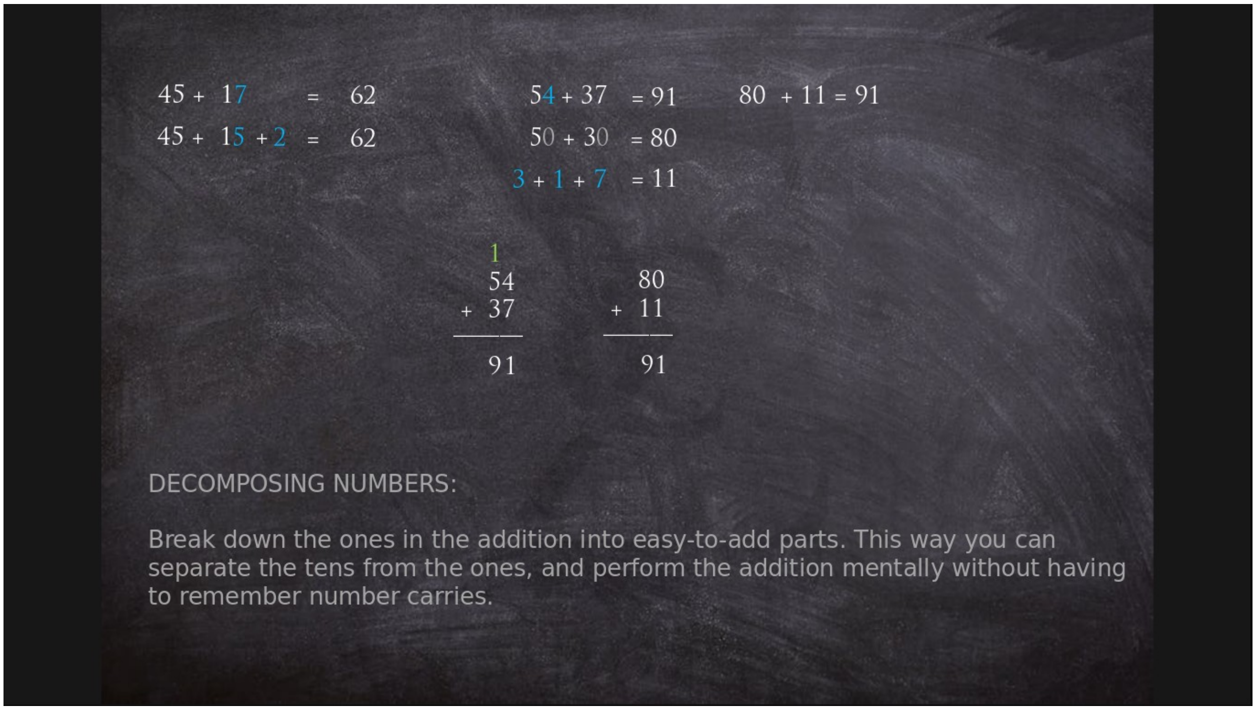
20	19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	-
19	18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	1
18	17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	2
17	16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	3
16	15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	4
15	14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	5
14	13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	6
13	12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	7
12	11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	8
11	10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	9
10	9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	10
9	8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	11
8	7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	12
7	6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	13
6	5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	14
5	4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	15
4	3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	16
3	2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	17
2	1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	18
1	0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	19
0	-1	-2	-3	-4	-5	-6	-7	-8	-9	-10	-11	-12	-13	-14	-15	-16	-17	-18	-19	20

APPENDIX D: E-BLACKBOARD ILLUSTRATIONS

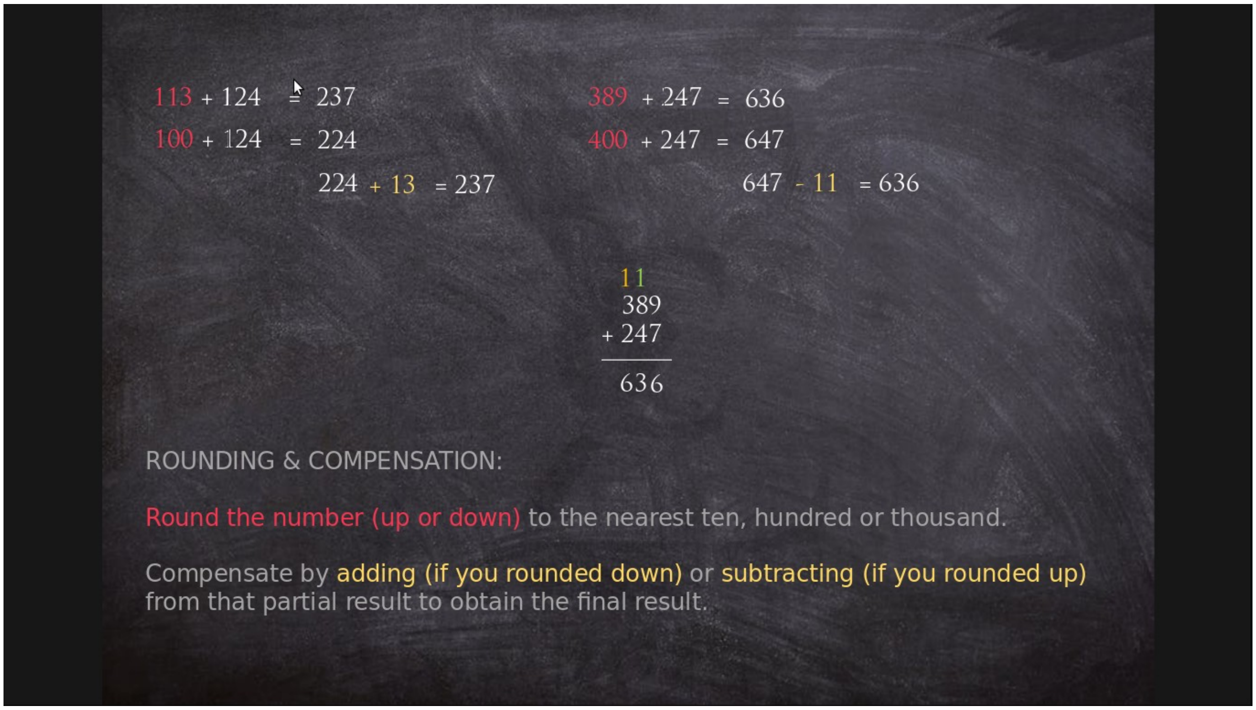
FAST ADDITION TECHNIQUES: SCANNING



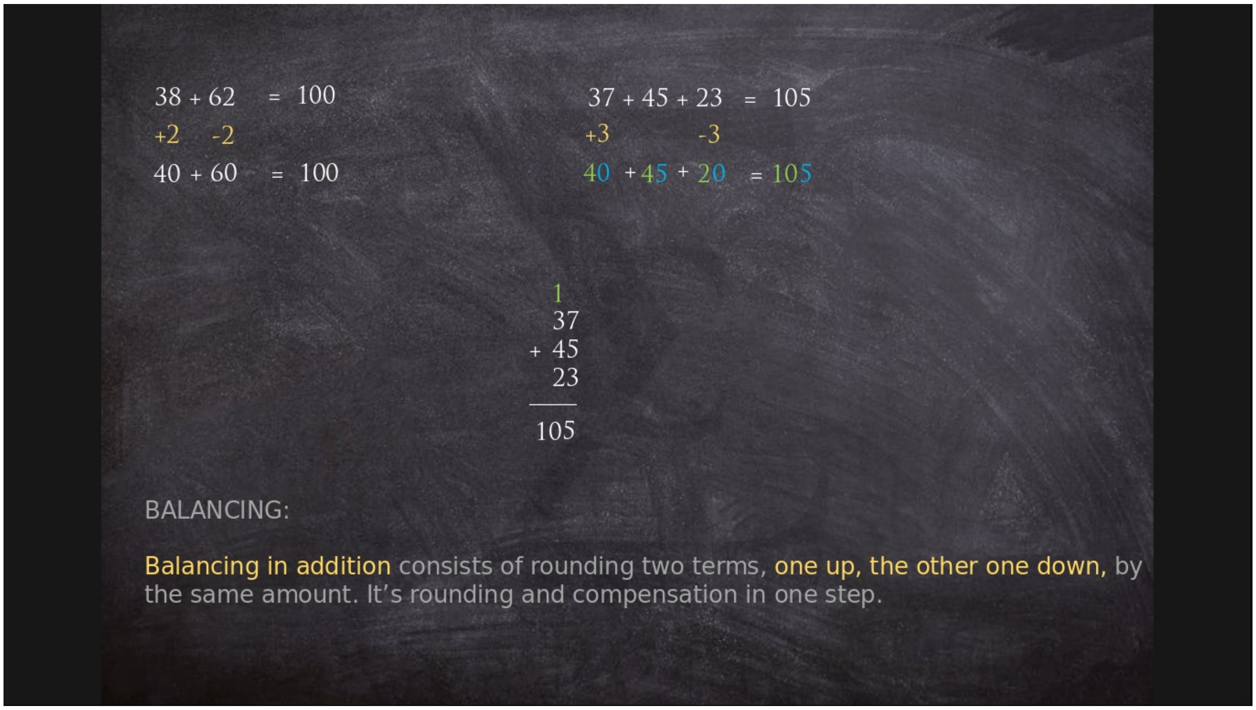
FAST ADDITION TECHNIQUES: DECOMPONG



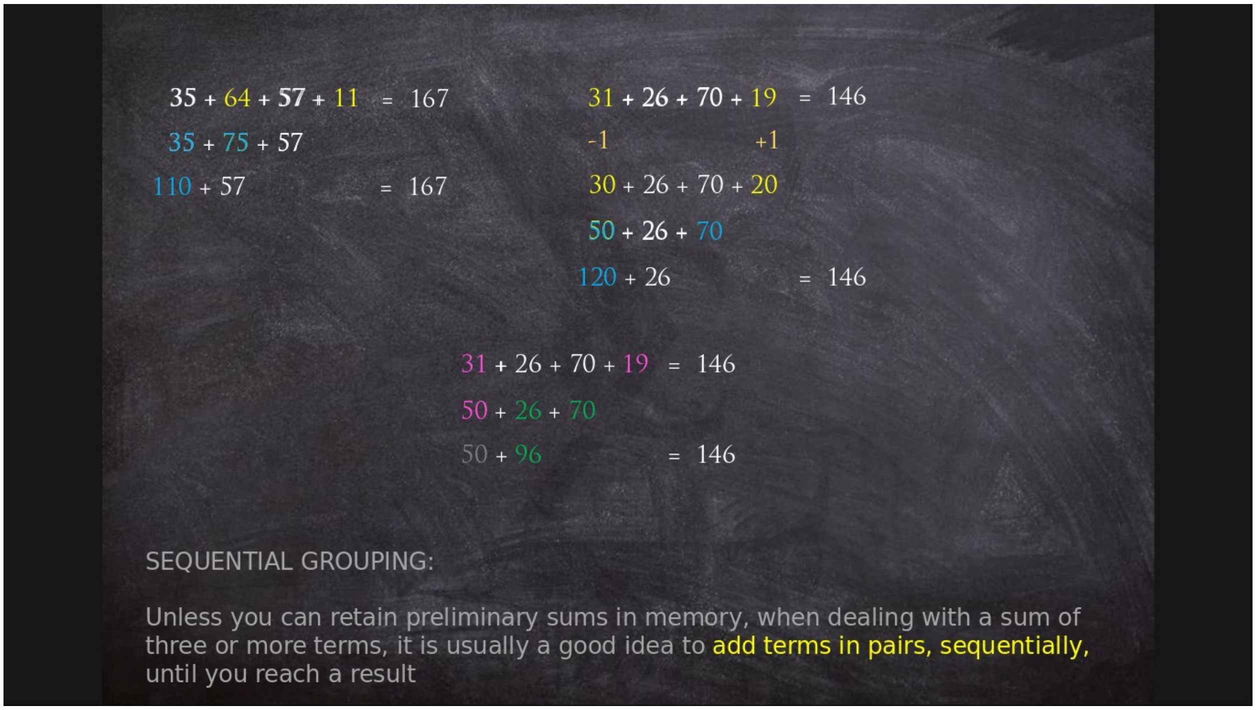
FAST ADDITION TECHNIQUES: ROUNDING AND COMPENSATION



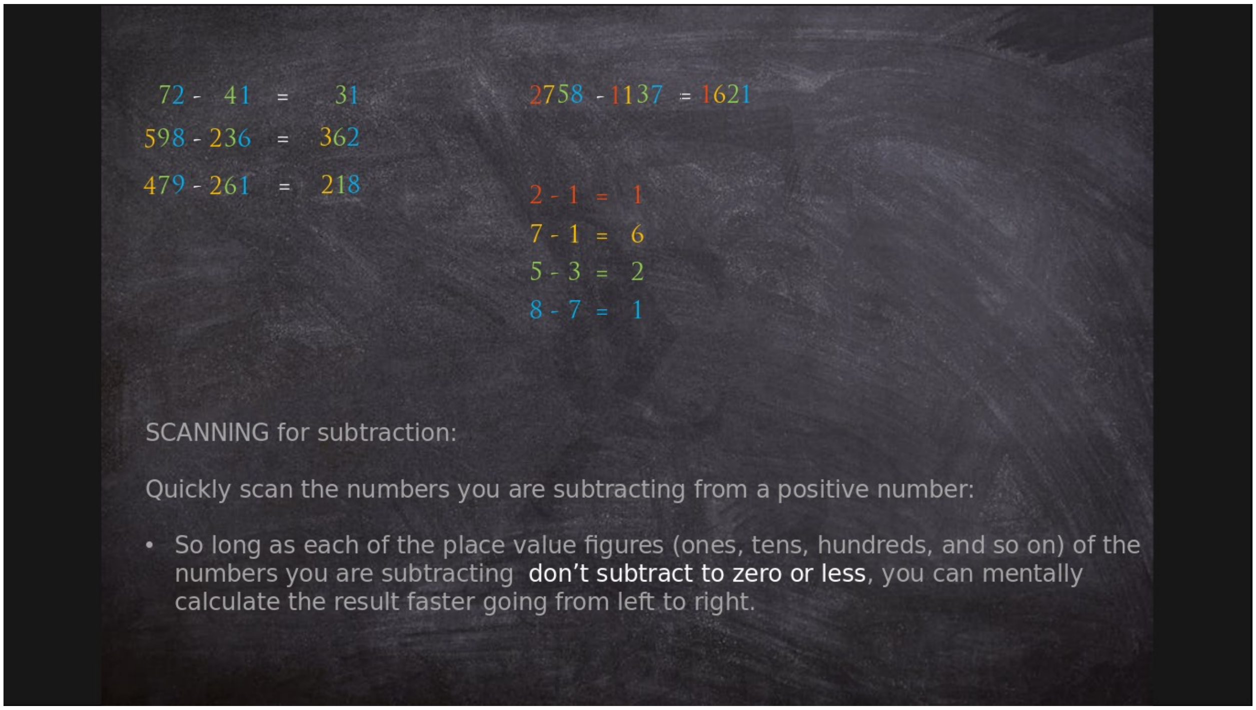
FAST ADDITION TECHNIQUES: BALANCING



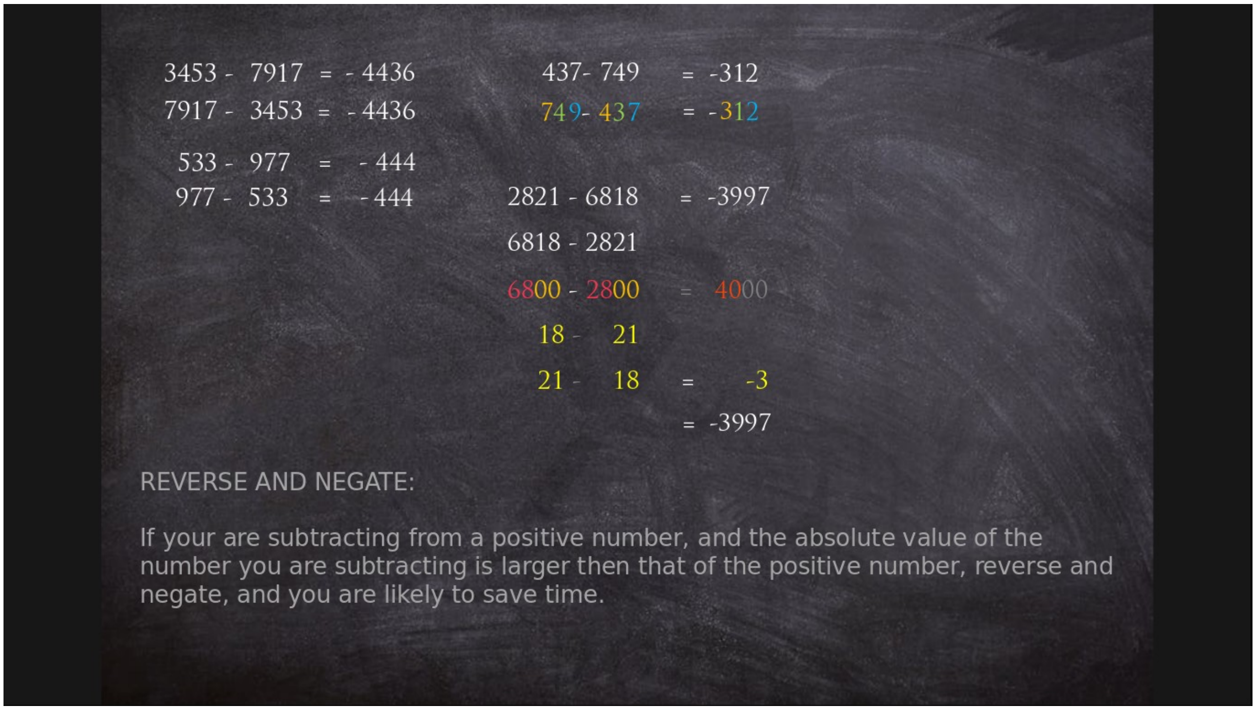
FAST ADDITION TECHNIQUES: SEQUENTIAL GROUPING



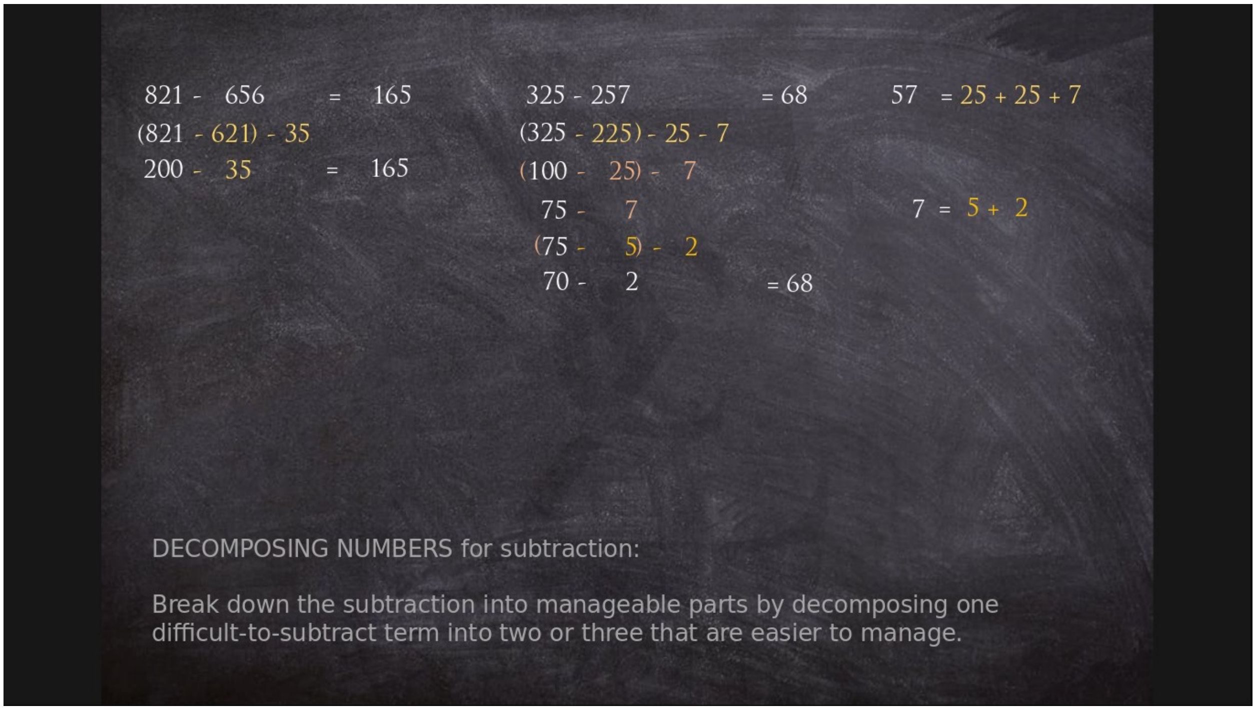
FAST SUBTRACTION TECHNIQUES: SCANNING



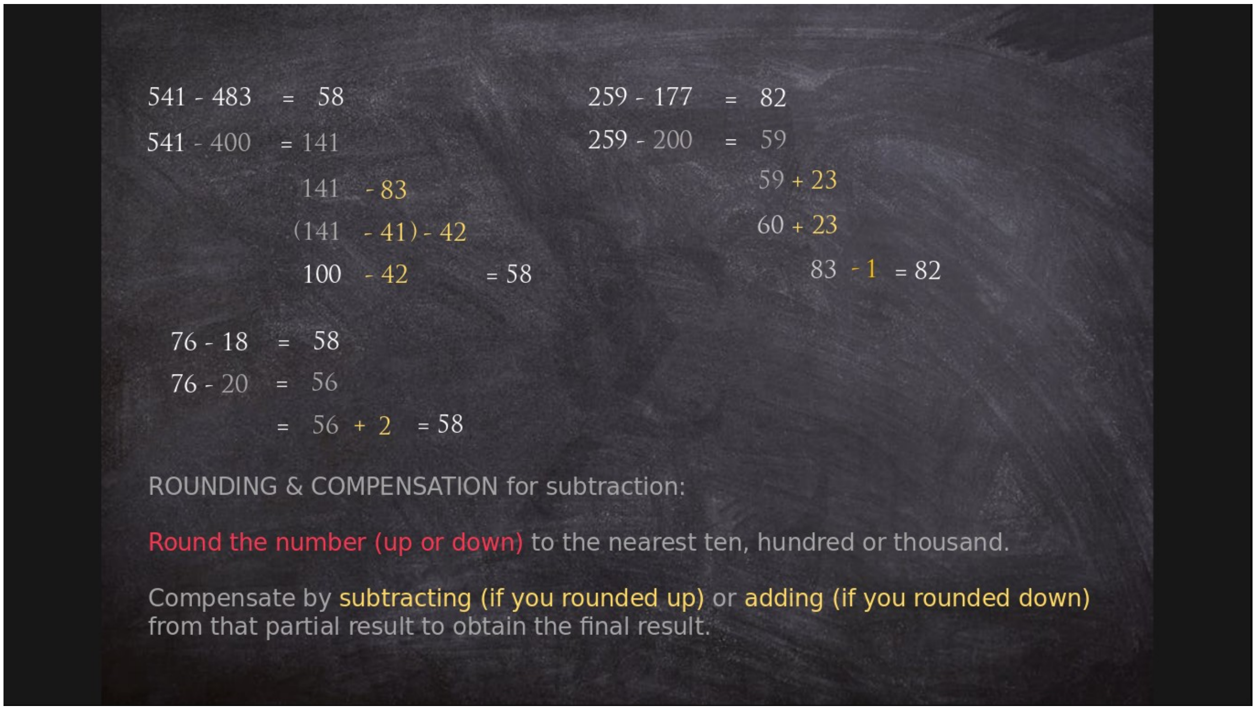
FAST SUBTRACTION TECHNIQUES: REVERSE AND NEGATE



FAST SUBTRACTION TECHNIQUES: DECOMPOSING



FAST SUBTRACTION TECHNIQUES: ROUNDING AND COMPENSATION



FAST SUBTRACTION TECHNIQUES: MATCHING