Chapter 2

The image, its representations and properties

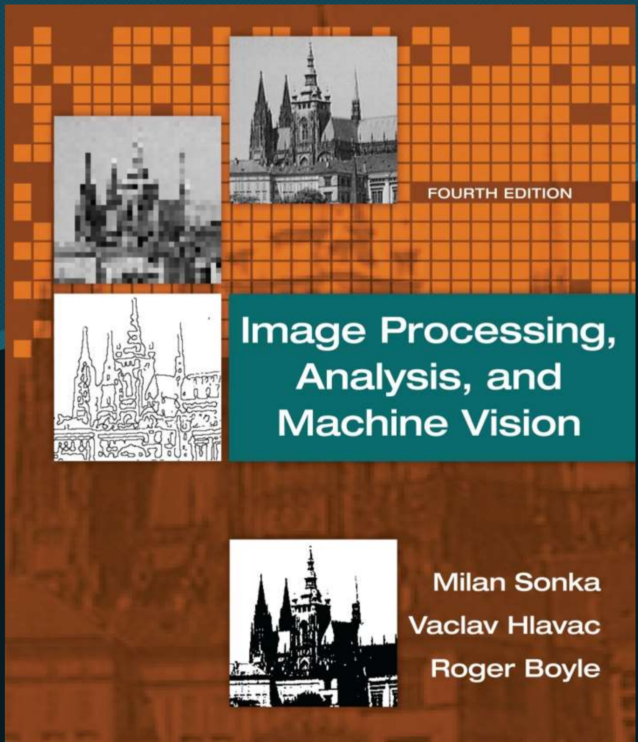

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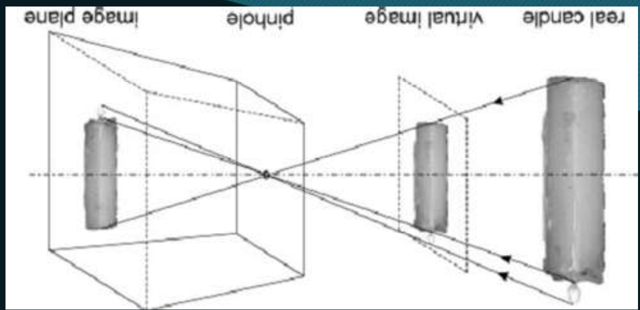
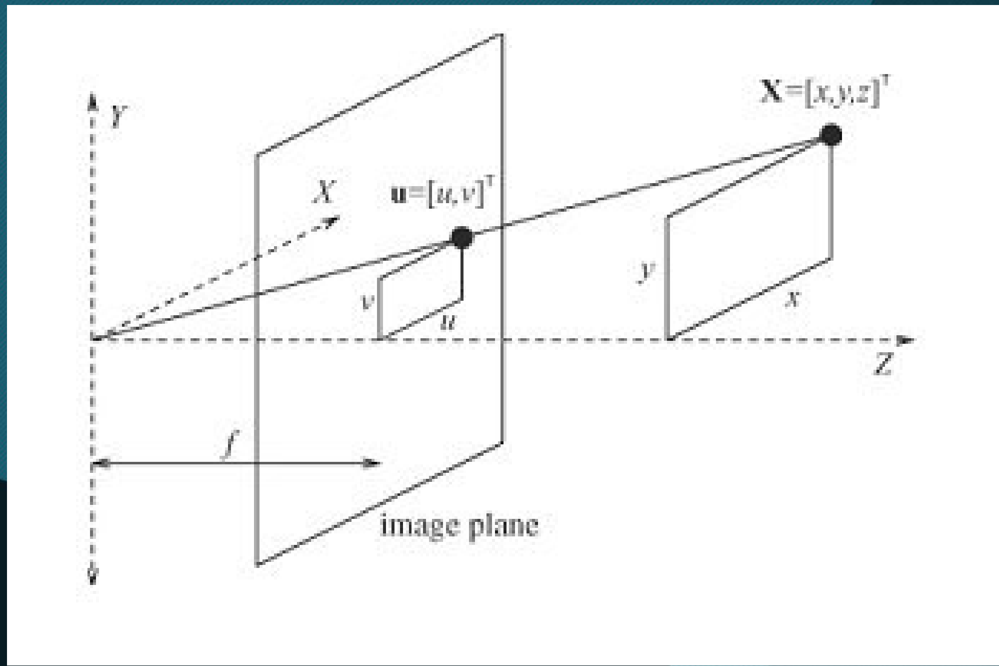
Image representations, a few concepts

- Perspective projection geometry (透視投影幾何公式)

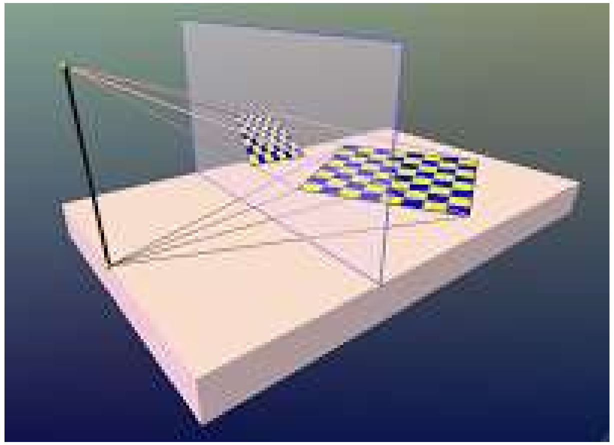
$$u = \frac{xf}{z} \qquad v = \frac{yf}{z}$$

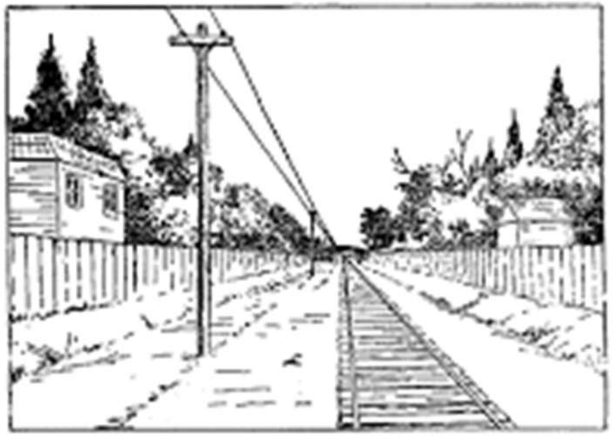
  - f : focal length (焦距)
  - x, y, z: the co-ordinates of the point X in a 3D scene
  - u, v: the co-ordinates in the 2D image plane



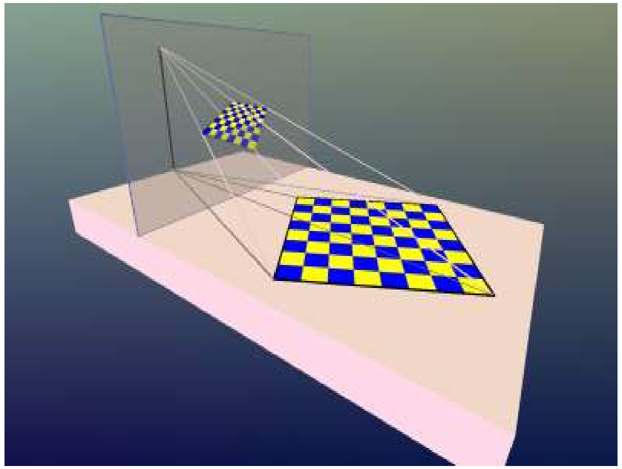


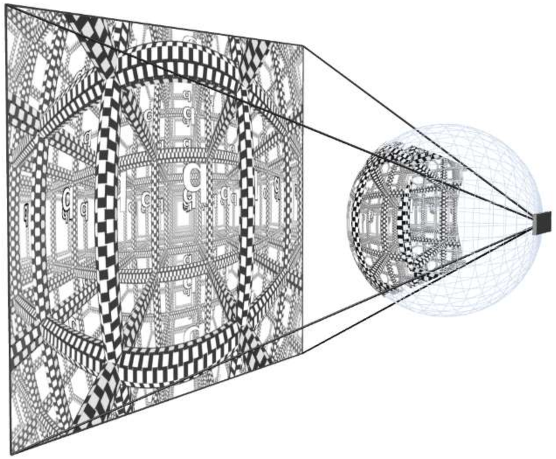
Projection geometry (投影幾何)





http://cgunn3.blogspot.tw/2012/04/where-parallels-meet.html





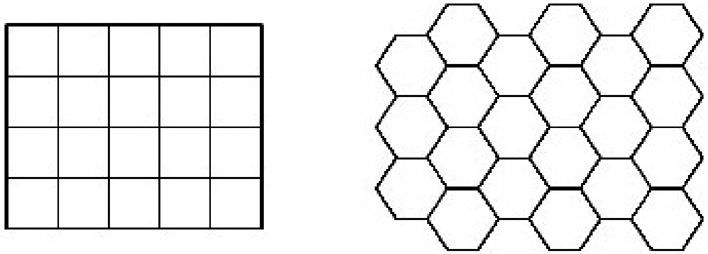
http://page.math.tu-berlin.de/~gunn/pages/PGTalkImages/perspectiveDemo-04.png

Image representations, a few concepts

- The quality of a digital image can be measured by
  - Spatial resolution (space)
    - The proximity (接近) of image samples in the image plane
  - Spectral resolution (color)
    - The bandwidth (頻寬) of the light frequencies captured by the sensor
  - Radiometric (輻射的) resolution (intensity)
    - The number of distinguishable gray-levels
  - Time resolution (temporal resolution)
    - The interval between time samples at which images are captured

Image digitization (影像數位化)

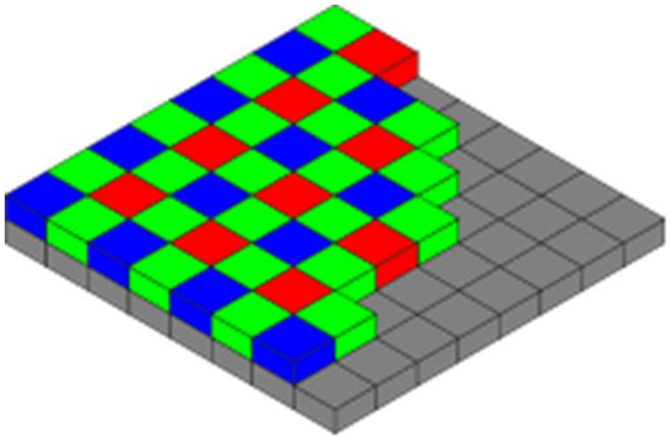
- Sampling (取樣)
  - A continuous image is digitized at sampling points.
    - These sampling points are ordered in the plane, and their geometric relation is called the grid (網格).
    - Grids used in practice are usually square or hexagonal (六邊形).

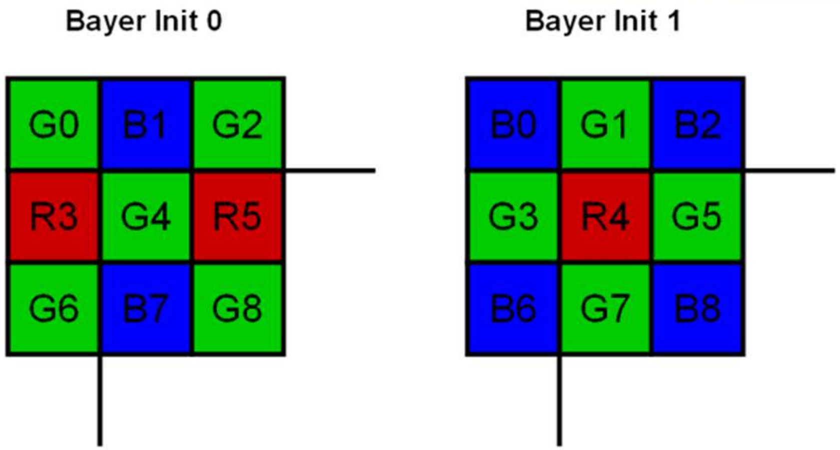


(a) (b)

Figure 2.2: (a) Square grid. (b) Hexagonal grid.

Grids





Bayer Init 0:

$$R = \frac{R3 + R5}{2}$$

$$G = \frac{G0 + G2 + 4G4 + G6 + G8}{8}$$

$$B = \frac{B1 + B7}{2}$$

Bayer Init 1:

$$R = R4$$

$$G = \frac{G1 + G3 + G5 + G7}{4}$$

$$B = \frac{B0 + B2 + B6 + B8}{4}$$

The Bayer color filter mosaic. (Very common RGB filter.) Each two-by-two submosaic contains 2 green, 1 blue and 1 red filter, each covering one pixel sensor.

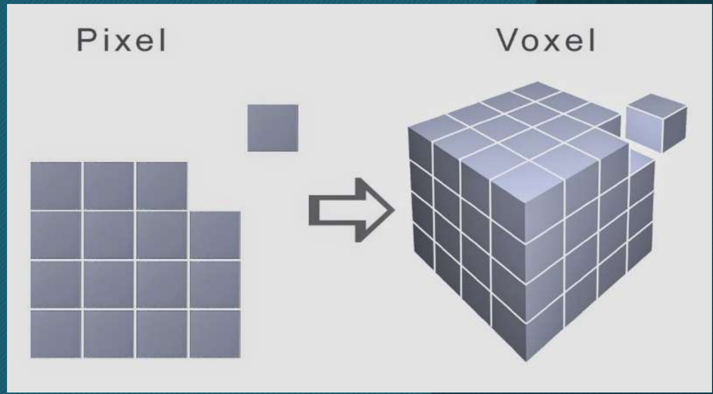
https://en.wikipedia.org/wiki/Color_filter_array

A Bayer filter camera uses a color filter array attached to a monochomatic sensor. The bilinear demosaicing algorithm performs the color reconstruction according to the Figures shown above.

http://www.siliconsoftware.de/download/live_docu/RT5/en/feature_blocks/PSBayer12/PSBayer12_bilinear.html(??)

Image digitization

- Pixel
  - One infinitely small sampling point in the grid corresponds to one picture element also called a pixel or image element in the digital image.
- Voxel (體素)
  - In a 3D image, an image element is called a voxel (volume element)



Quantization

- The transition between continuous values of the image function (brightness) and its digital equivalent is called quantization (量化).

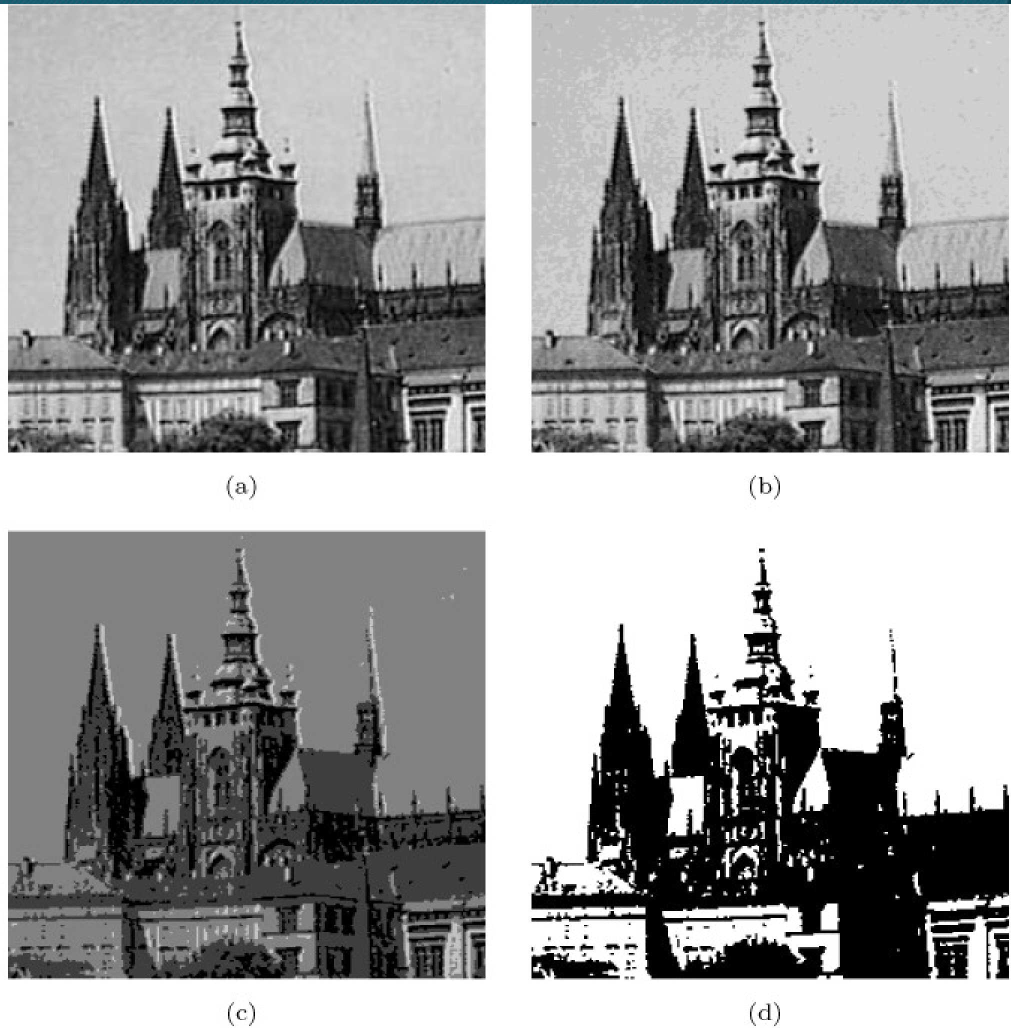


Figure 2.3: Brightness levels. (a) 64. (b) 16. (c) 4. (d) 2. © *Cengage Learning 2015.*

Digital image properties

- Matric and topological properties of digital images
  - Distance (Matric)
    - Any function D holding the following three condition is a distance (or a metric)
      - Identity (同一性)

$$D(\boldsymbol{p}, \boldsymbol{q}) = 0 \text{ iff } \boldsymbol{p} = \boldsymbol{q}$$

      - Symmetry (對稱性)

$$D(\boldsymbol{p}, \boldsymbol{q}) = D(\boldsymbol{q}, \boldsymbol{p}),$$

      - Triangular inequatility (三角不等式)

$$D(\boldsymbol{p}, \boldsymbol{z}) \le D(\boldsymbol{p}, \boldsymbol{q}) + D(\boldsymbol{q}, \boldsymbol{z})$$

https://zh.wikipedia.org/wiki/%E5%BA%A6%E9%87%8F%E7%A9%BA%E9%97%B4

$$D(\boldsymbol{p}, \boldsymbol{q}) \ge 0,$$

Matric and topological properties of digital images

- Euclidean distance (歐氏距離)

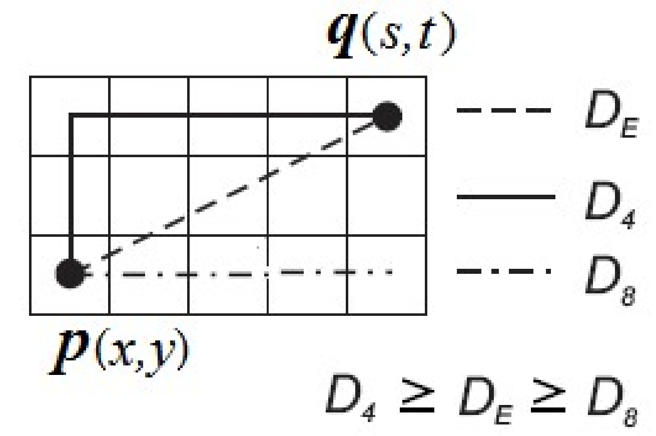
$$D_E(\boldsymbol{p},\boldsymbol{q}) = \sqrt{(x-s)^2 + (y-t)^2}$$

- City block distance (城市街區距離)

$$D_4(\boldsymbol{p},\boldsymbol{q}) = |x-s| + |y-t|$$

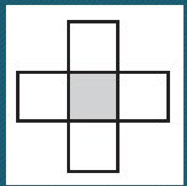
- Chessboard distance (棋盤距離)

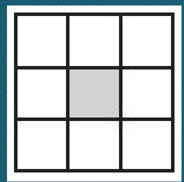
$$D_8(\boldsymbol{p},\boldsymbol{q}) = \max\{|x-s|,|y-t|\}$$



Matric and topological properties of digital images

- Regions (區域)
  - A region is a connected set.
  - A region is a set of pixels in which there is a path between any pair of its pixels, all of those pixels also belong to the set.
  - 4-neighbors vs. 8-neighbors

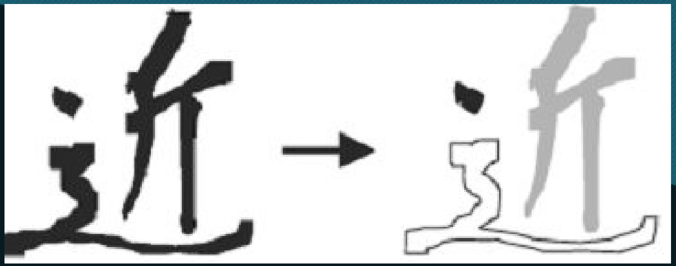




http://ippr-practical.blogspot.com/2012/04/region-segmentation.html

Matric and topological properties of digital images

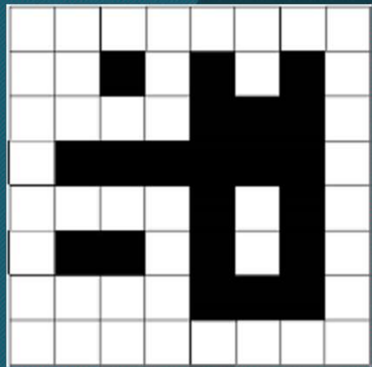
- Contiguous (接觸的;連續的)
  - If there is a path between two pixels in the set of pixels in the image, these pixels are called contiguous.
  - The relation 'to be contiguous' is reflexive (反身), symmetric (對稱), and transitive (遞移).
  - 'To be contiguous' can be used to decompose an image into individual regions.
  - For example, the character is decomposed into 3 regions.



Matric and topological properties of digital images

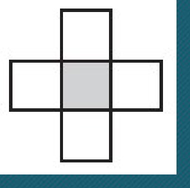
- Multiple Contiguous
  - Let R be the union of all regions R_i (these regions do not touch the image bounds), and

    R^C be the set complement of R with respect to the image.
  - The subset of R^C which is contiguous with the image bound is called the background, and the remainder of the complement R^C is called holes.
  - A region is called simple contiguous if it has no holes.
  - The complement of a simply contiguous region is contiguous.
  - A region with holes is called multiple contiguous.

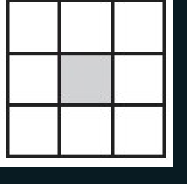


Matric and topological properties of digital images

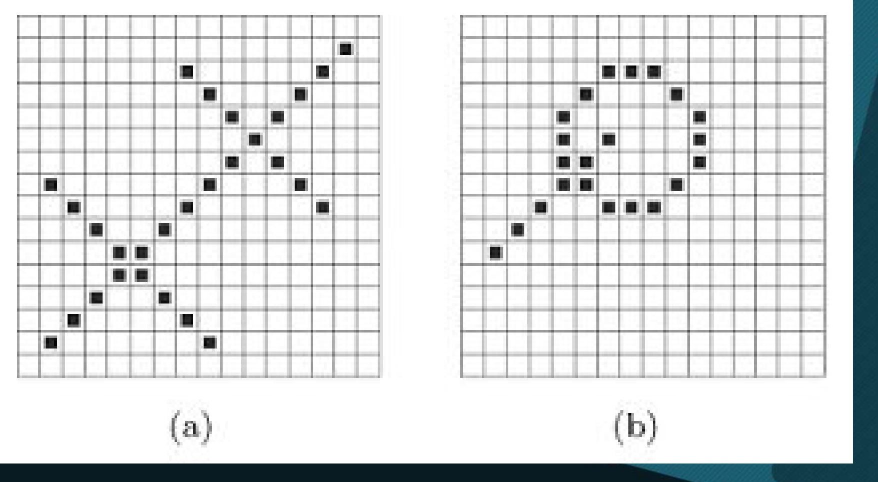
- The figure (a) shows two digital line segments with 45° slope.
  - If 4-connectivity is used, the line are not contiguous at each of their points.
    - Two perpendicular lines do intersect in one case (upper right intersection) and do not intersect in another case (lower left).
  - One possible solution: to treat objects using 4-neighborhoods and background using 8-neighborhoods



4-neighbors



8-neighbors



Matric and topological properties of digital images

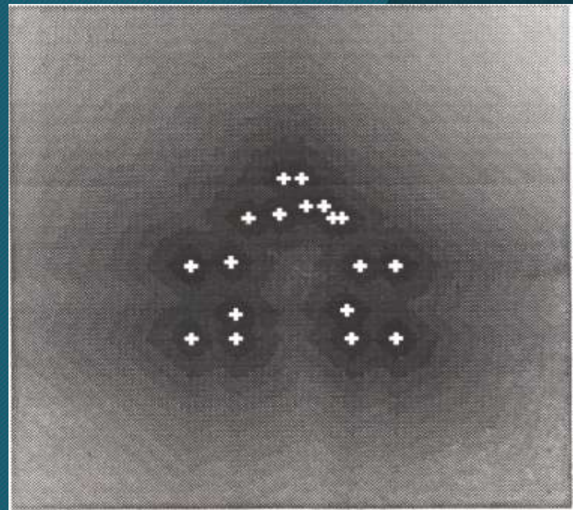
- Chamfer(導角;倒角) distance
  - A simple application of the concept of distance
  - For example, consider a binary image,

0	0	0	0	0	0	1	0
0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	0	0	0	0	1	0	0
0	1	1	0	0	0	1	0
0	1	0	0	0	0	0	1
0	1	0	0	0	0	0	0
0	1	0	0	0	0	0	0

Figure 2.8: Input image: gray pixels correspond to objects and white to background. *© Cengage Learning 2015.*

5	4	4	3	2	1	0	1
4	3	3	2	1	0	1	2
3	2	2	2	1	0	1	2
2	1	1	2	1	0	1	2
1	0	0	1	2	1	0	1
1	0	1	2	3	2	1	0
1	0	1	2	3	3	2	1
1	0	1	2	3	4	3	2

Figure 2.9: Result of the $[D_4]$ distance transform. *© Cengage Learning 2015.*



ALGORITHM 2.1: DISTANCE TRANSFORMATION

Step 1: Choose a distance N_{max} and initialize an image F.

F: the pixels corresponding to the subset to be chamfered to 0, and all others to N_{max}.

Step 2: Pass through image pixels from top to bottom and left to right.

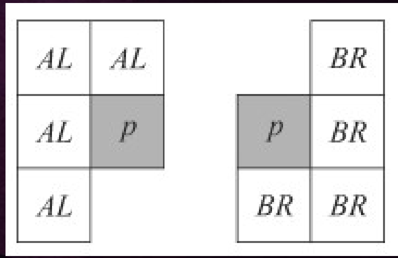
For a given pixel, consider neighbors above and to the left and set

$$F(\mathbf{p}) = \min_{\mathbf{q} \in AL} \left(F(\mathbf{p}), D(\mathbf{p}, \mathbf{q}) + F(\mathbf{q}) \right)$$

Step 3: Pass through image pixels from bottom to top and right to left.

For a given pixel, consider neighbors bottom and to the right and set

$$F(\mathbf{p}) = \min_{\mathbf{q} \in BR} \left(F(\mathbf{p}), D(\mathbf{p}, \mathbf{q}) + F(\mathbf{q}) \right)$$



Step 4: If any pixels remain still set at N_{max}, go to Step 2.

Step 5: The array F now holds a chamfer of the chosen subset(s).

0	0	0	0	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	1	0	0	0	0
0	0	0	1	1	0	0	0
0	0	0	1	1	1	1	0
0	0	0	1	1	1	0	0
0	0	0	0	0	0	0	0

255	255	255	255	255	255	255	255
255	255	255	0	255	255	255	255
255	255	255	0	255	255	255	255
255	255	255	0	255	255	255	255
255	255	255	0	0	255	255	255
255	255	255	0	0	0	0	255
255	255	255	0	0	0	255	255
255	255	255	255	255	255	255	255

255	255	255	255	2	3	4	5
255	255	255	0	255	255	255	255
255	255	255	0	255	255	255	255
255	255	255	0	255	255	255	255
255	255	255	0	0	255	255	255
255	255	255	0	0	0	0	255
255	255	255	0	0	0	255	255
255	255	255	255	255	255	255	255

255	255	255	255	2	3	4	5
255	255	255	0	1	2	3	4
255	255	255	0	1	2	3	4
255	255	255	0	1	2	2	3
255	255	255	0	1	1	1	2
255	255	255	0	0	0	0	1
255	255	255	0	0	0	1	2
4	3	2	1	1	1	2	3

Step 2: Pass through image pixels from top to bottom and left to right.

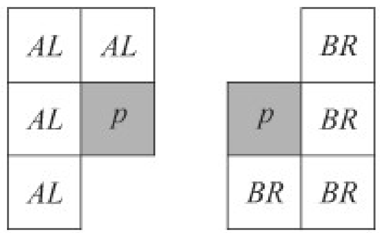
For a given pixel, consider neighbors above and to the left and set

$$F(\mathbf{p}) = \min_{\mathbf{q} \in AL}(F(\mathbf{p}), D(\mathbf{p},\mathbf{q}) + F(\mathbf{q}))$$

Step 3: Pass through image pixels from bottom to top and right to left.

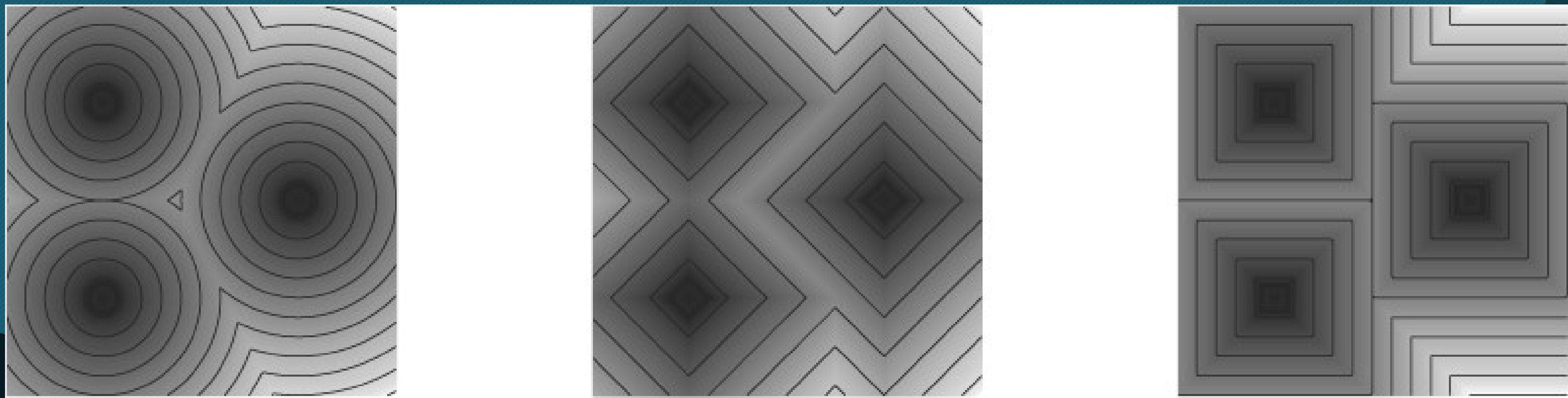
For a given pixel, consider neighbors bottom and to the right and set

$$F(\mathbf{p}) = \min_{\mathbf{q} \in BR}(F(\mathbf{p}), D(\mathbf{p},\mathbf{q}) + F(\mathbf{q}))$$



Matric and topological properties of digital images

- The relationship between different distance functions and algorithm 2.1.
  - Euclidean distance, city block distance, and chessboard distance



(a) (b) (c)

Figure 2.11: Three distances used often in distance transform calculations—the input consists of three isolated 'ones'. Output distance is visualized as intensity; lighter values denote higher distances. Contour plots are superimposed for better visualization. (a) Euclidean distance D_E. (b) City block distance D_4. (c) Chessboard distance D_8. © *Cengage Learning 2015.*

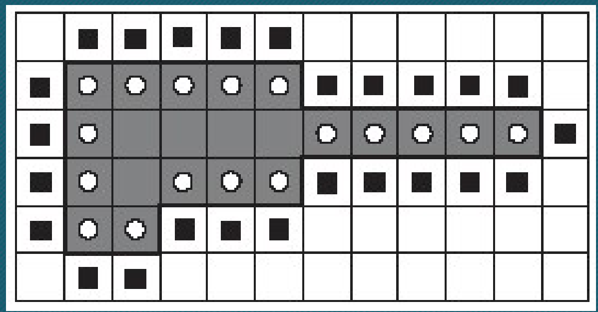
Matric and topological properties of digital images

- Edge
  - A local property of a pixel and its immediate neighborhood
  - It is a vector given by a magnitude and direction which tell us how fast the image intensity varies in a small neighborhood of a pixel.

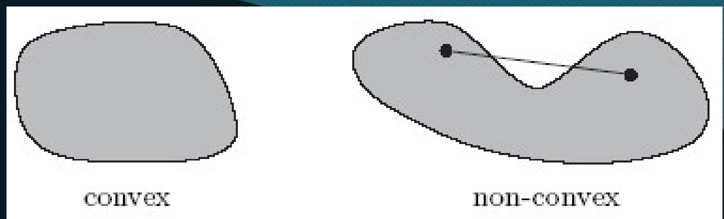
https://docs.opencv.org/3.4/d0/da5/tutorial_ximgproc_prediction.html

Matric and topological properties of digital images

- Inner borders and outer borders of a region
  - Region inner borders shown as white circles and outer borders shown as black squares (using 4-neighborhoods).



- Convex region



Matric and topological properties of digital images

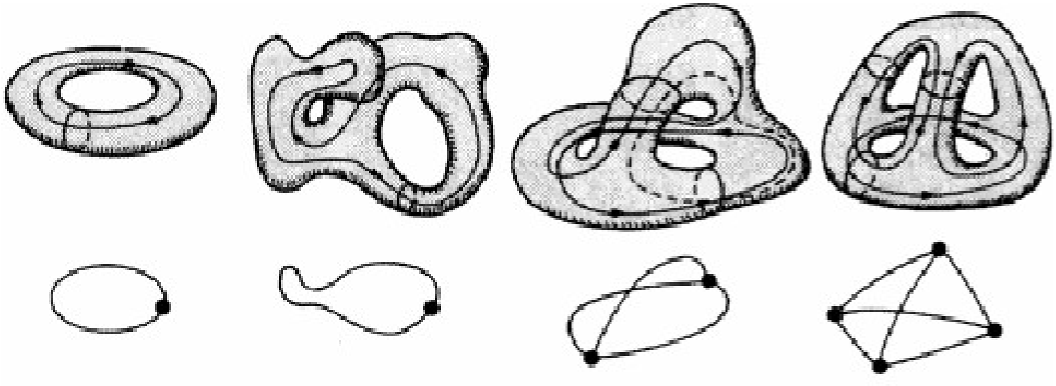
- Convex hull
  - A convex hull of a region is the smallest convex region containing the input region.

- Topological (拓撲學的) properties
  - Topological properties are invariant to homeomorphic (同胚) transforms.
  - For example, rubber sheet transforms
    - Imagine a small rubber balloon with an object painted on it.
    - Topological properties of the object are those which are invariant to arbitrary stretching of the rubber sheet.
    - Stretching does not change contiguity of the object parts and does not change the number of holes in regions.

https://gtpreapgeometry.files.wordpress.com/2016/05/ws-topology-or-rubber-sheet-geometry.pdf

Topological properties

- **Topology** is concerned with the properties of space that are preserved under continuous deformations, such as stretching (拉伸) and bending (彎曲), but not tearing (撕開) or gluing (黏合).



Example of the topological properties: genus and connectivity (a-d) have a connectivity of 1, 1, 2, and 3 respectively(MacDonald et. al. , 1986).

https://www.researchgate.net/figure/267246393_fig10_Fig-211-Example-of-the-topological-properties-genus-and-connectivity-a-d-have-a

https://en.wikipedia.org/wiki/Topology

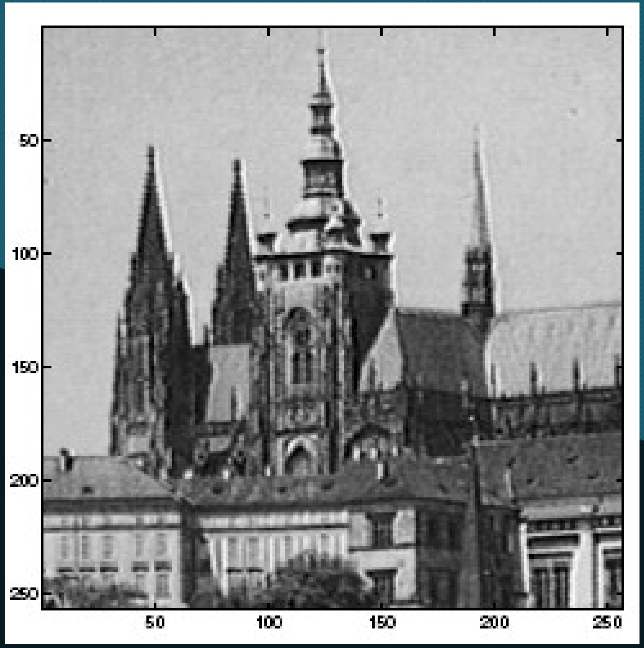
Matric and topological properties of digital images

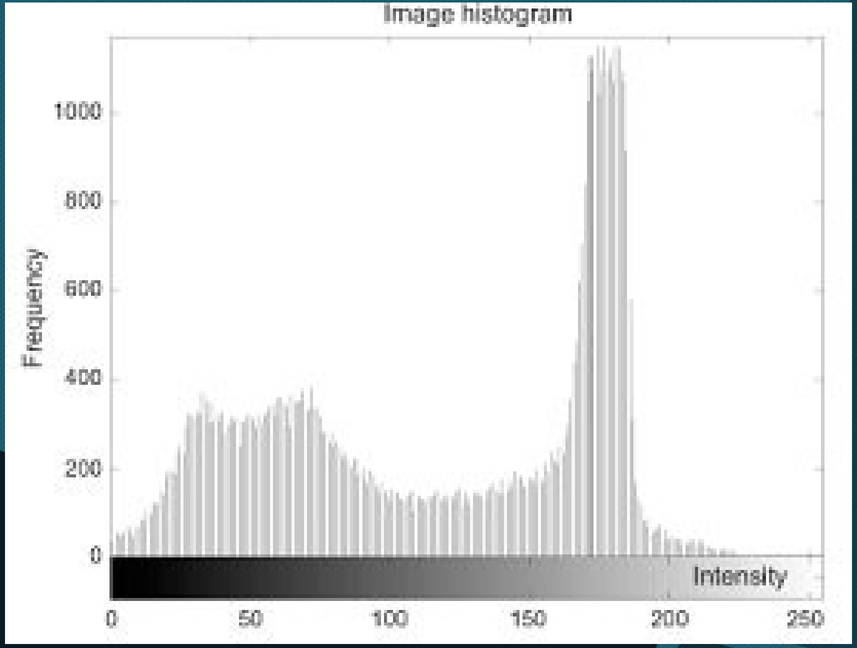
- Topological properties
  - An object with non-regular shape can be represented by a collection of its topological components.
  - The set inside the convex hull which does not belong to an object is called the deficit (不足額) of convexity, including lakes and bays.
    - Lakes are fully surrounded by the object.
    - Bays are contiguous with the border of the convex hull of the object.

Figure 2.15: Description using topological components: An 'R' object, its convex hull, and the associated lakes and bays.

Histograms

- The brightness histogram of an image provides the frequency of the brightness value in the image.
- Local smoothing of the histogram
  - To reduce the number of local minima and maxima





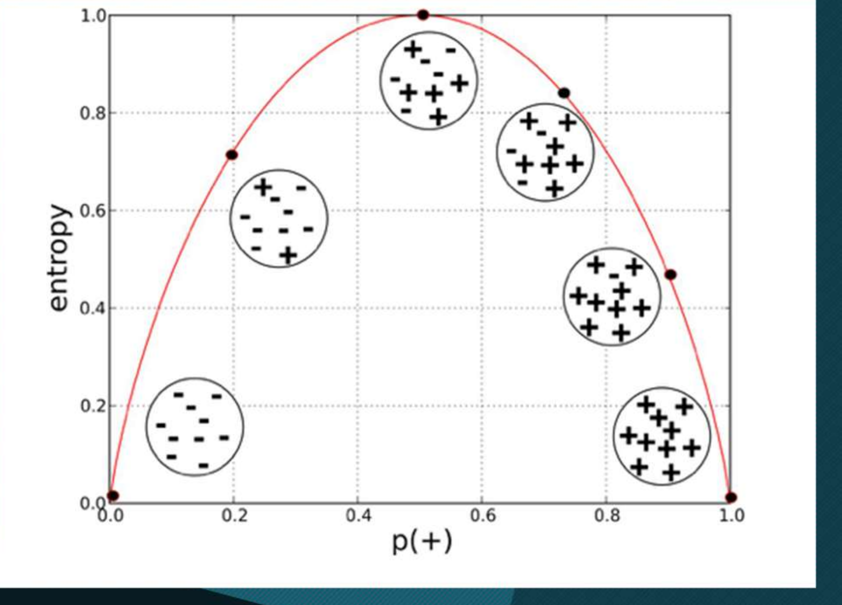
Entropy

- Information entropy
  - The entropy can serve as an measure of ‘disorder’.
  - As the level of disorder rises, entropy increases and events are less predictable.
  - Assuming a discrete random variable X with possible outcomes x_k, $k = 1,\ldots,n$. Then the entropy defined as

$$H(X) = -\sum_{k=1}^{n} p(x_k)\log_2 p(x_k)$$

where $0\log_2 0 = 1\log_2 1 = 0$;

$$\frac{1}{2}\log_2\frac{1}{2} = -\frac{1}{2}$$

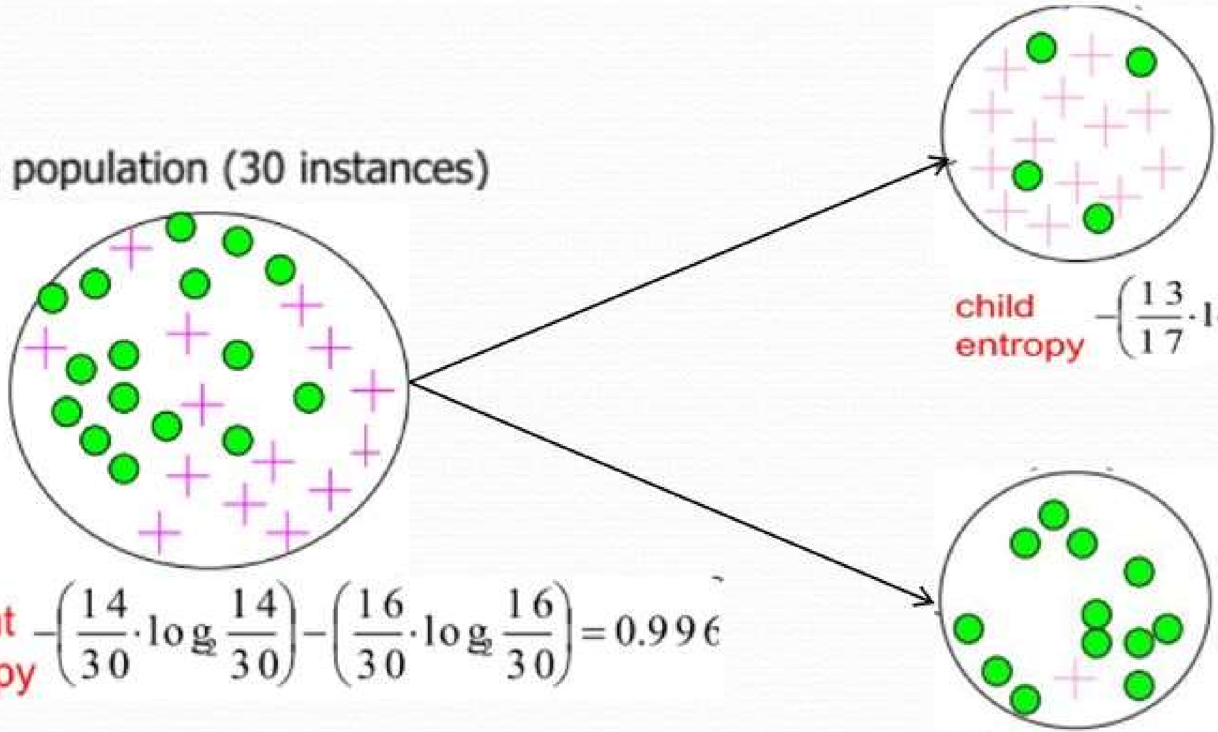


https://towardsdatascience.com/entropy-how-decision-trees-make-decisions-2946b9c18c8

Information gain calculation example

Information Gain = entropy(parent) – [average entropy(children)]

Entire population (30 instances)



17 instances

child entropy $-\left(\frac{13}{17}\cdot\log_2\frac{13}{17}\right)-\left(\frac{4}{17}\cdot\log_2\frac{4}{17}\right)=0.787$

13 instances

child entropy $-\left(\frac{1}{13}\cdot\log_2\frac{1}{13}\right)-\left(\frac{12}{13}\cdot\log_2\frac{12}{13}\right)=0.391$

parent entropy $-\left(\frac{14}{30}\cdot\log_2\frac{14}{30}\right)-\left(\frac{16}{30}\cdot\log_2\frac{16}{30}\right)=0.996$

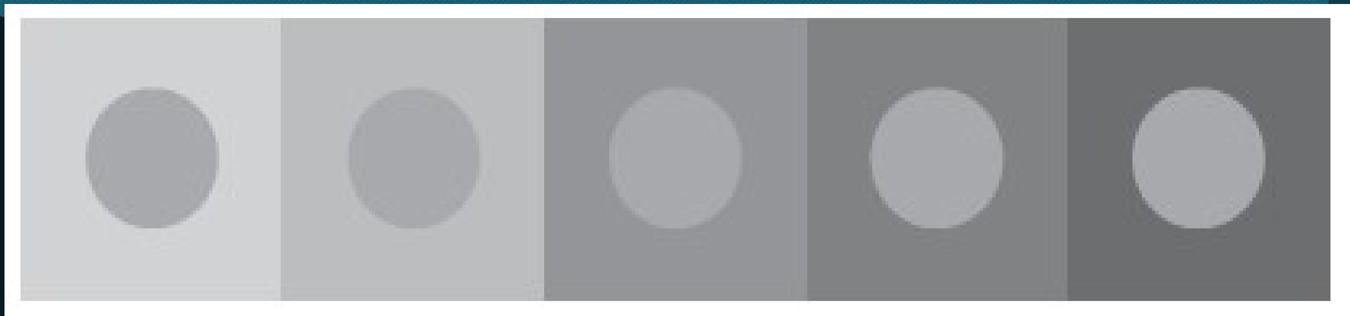
(Weighted) Average Entropy of Children = $\left(\frac{17}{30}\cdot 0.787\right)+\left(\frac{13}{30}\cdot 0.391\right)=0.615$

Information Gain= 0.996 - 0.615 = 0.38

https://devopedia.org/images/article/168/4250.1555312917.jpg

Visual perception of the image

- Contrast
  - Contrast is the local change in brightness and is defined as the ratio between average brightness of an object and the background.
  - Conditional contrast effect
    - Circles inside squares have the same brightness.
    - Human perceive the brightness of the small circles as different.

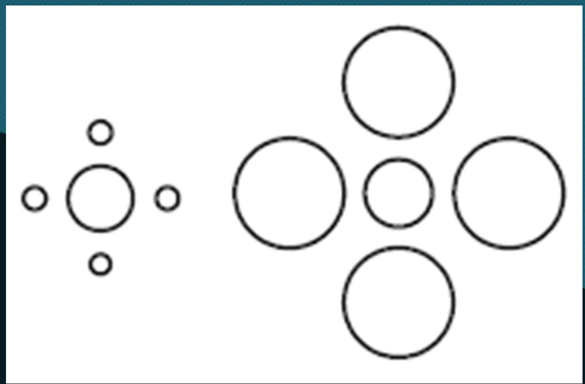
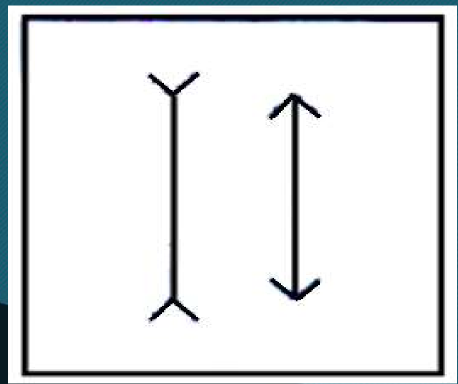


Visual perception of the image

- Acuity (敏銳度)
  - Acuity is the ability to detect details in an image.
    - The human eye is
      less sensitive to slow and fast changes in brightness in the image plane but
      more sensitive to intermediate (中間的) changes.
    - Acuity also decreases with increasing distance from the optical axis (光軸).
  - Resolution in an image is bounded by the resolution ability of the human eye.

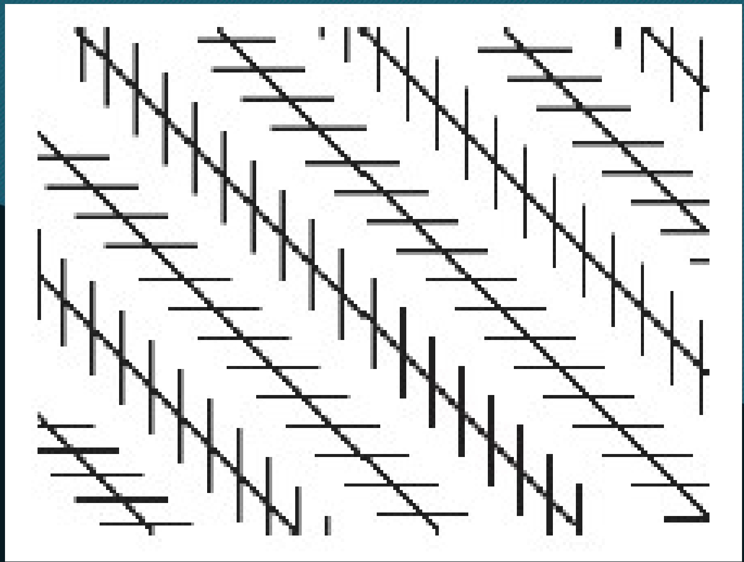
Visual perception of the image

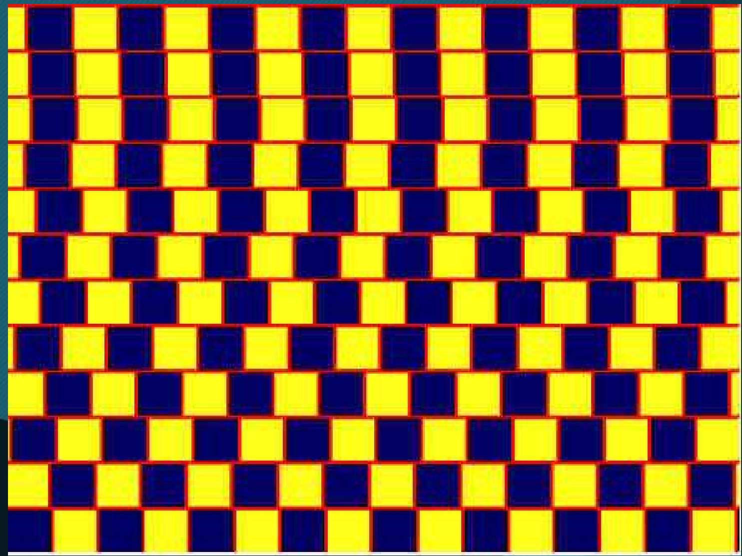
- Some visual illusions (錯覺)
  - Human perception of images is prone (易於) to many illusions.
    - Object borders
      - Boundaries of objects and simple patterns such as blobs or lines enable adaptation effects similar to conditional contrast.
      - For example, the Ebbinghaus illusion (艾賓浩斯錯覺)

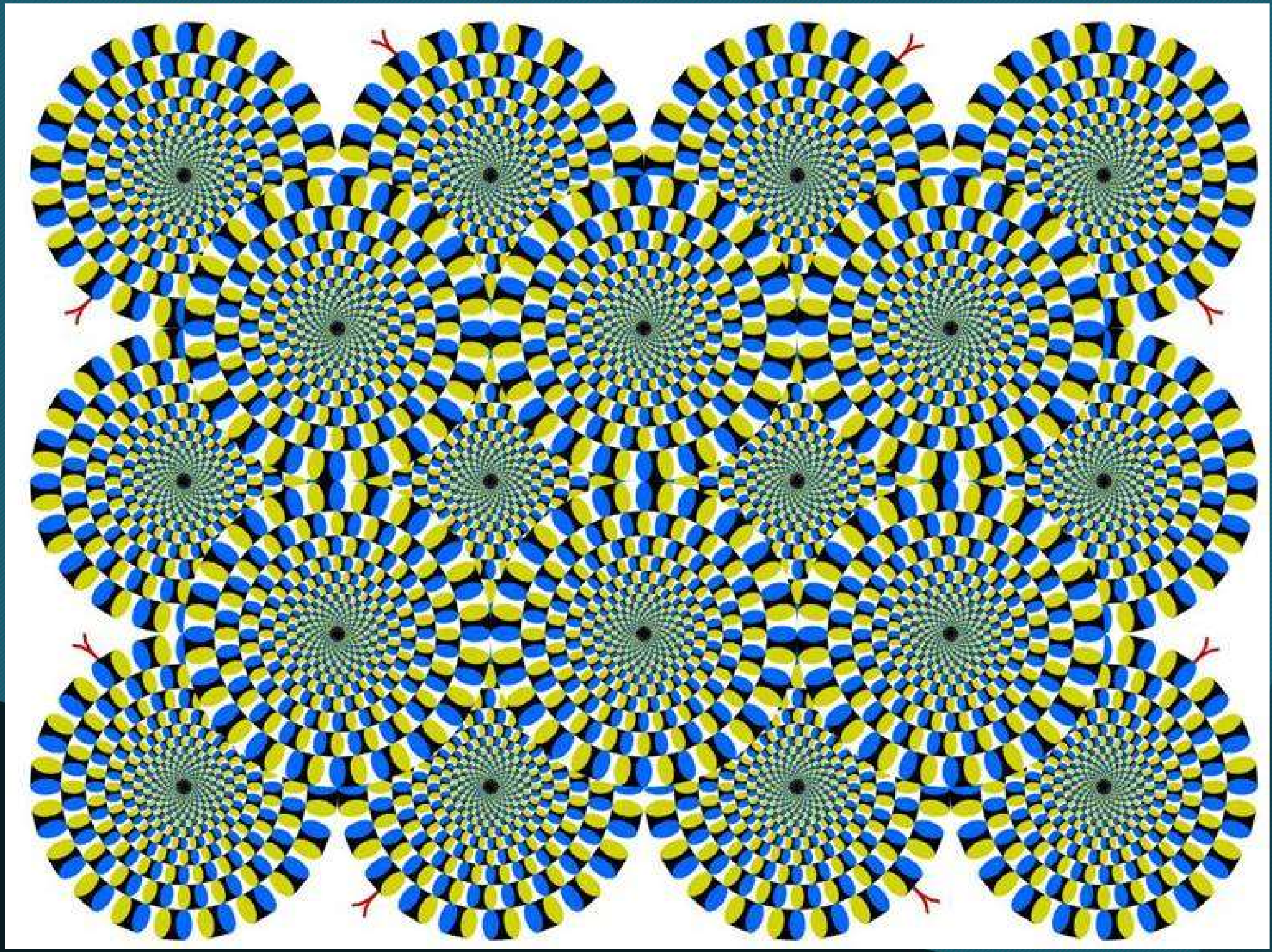
Visual perception of the image

- Some visual illusions (錯覺)
  - Parallel diagonal line segments which are not perceived as parallel. (left image)
  - Horizontal lines are parallel, although not perceived as such. (right image)





If something is rotating – go home, you need a break!

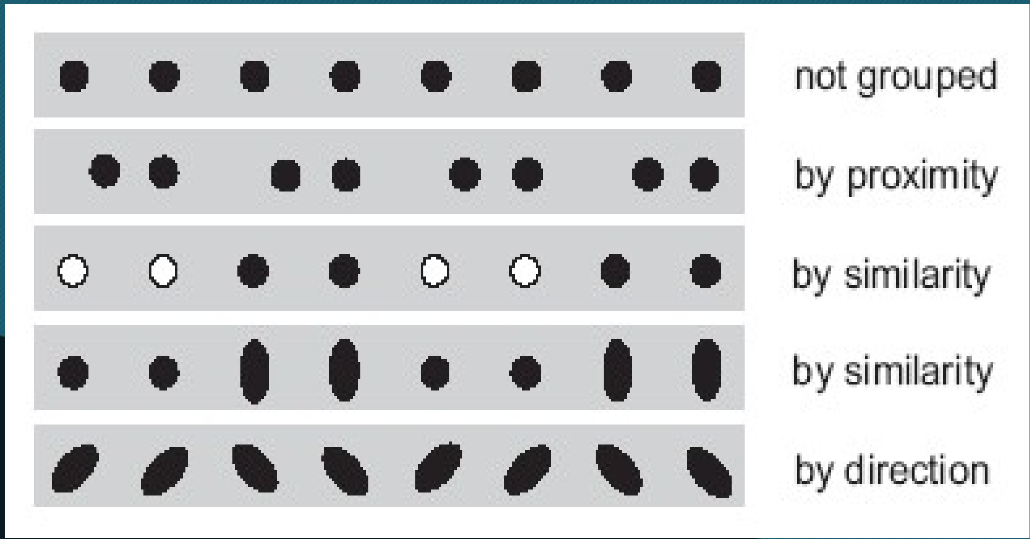


Visual illusions

https://www.chinatimes.com/realtimenews/20160704005335-262903?chdtv

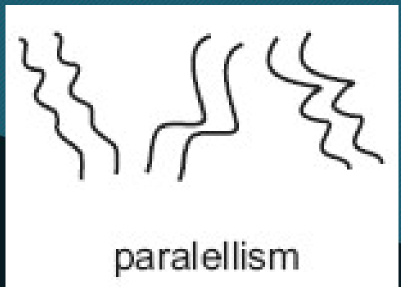
Visual perception of the image

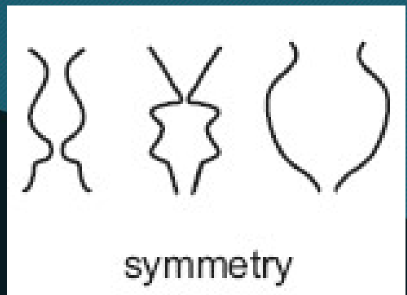
- Perceptual (知覺) grouping
  - The human ability to group items according to various properties is illustrated in the figure.

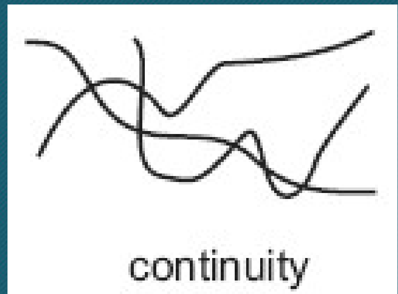


Visual perception (視覺感知) of the image

- Perceptual grouping (知覺分組)
  - Perceived properties (感知屬性) help people to connect elements together (in cluttered scenes) based on strongly perceived properties as parallelism, symmetry, continuity and closure (封閉性).







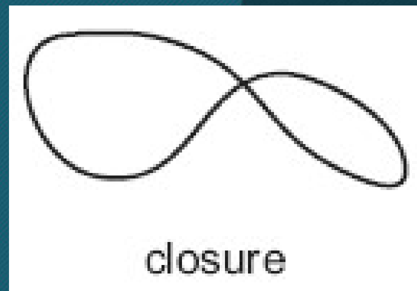
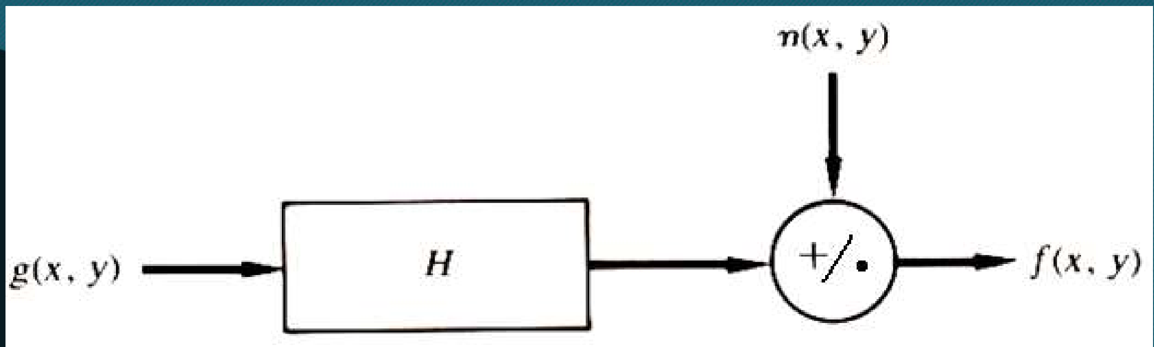


Image quality

- Image degradation (退化;下降)
  - An image might be degraded (退化) during capture, transmission, or processing.
  - Measures of image quality can be used to assess the degree of degradation.
  - Image degradation (影像劣化;影像衰退) model H



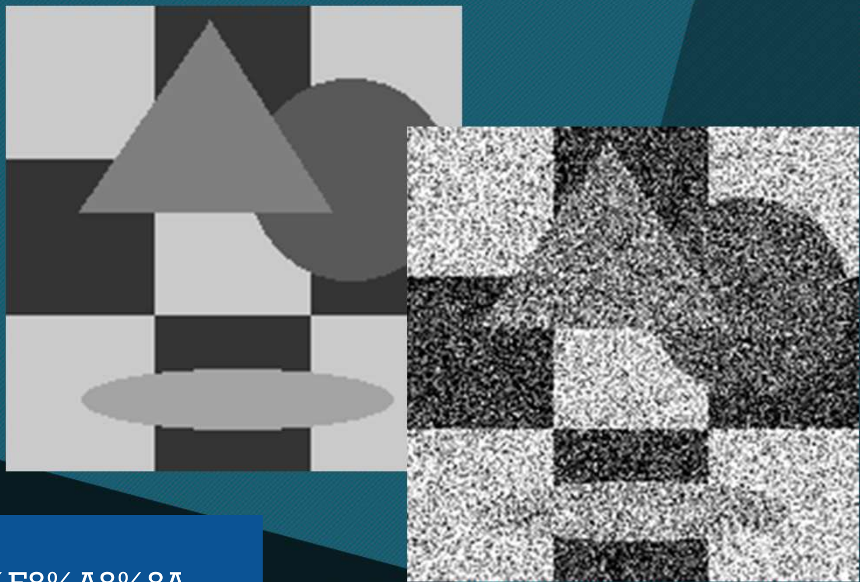
$g(x, y)$: input image
$f(x, y)$: degraded image
H: degradation process
$n(x, y)$: noise

Noise in images

- Noise is one kind of image degradations.
  - Noise is usually described by its probabilistic characteristics.
  - White noise has a constant power spectrum.
  - Gaussian noise: a special case of white noise

$$p(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}},$$

where μ is the mean and
σ is the standard deviation
of the random variable



white noise:
https://zh.wikipedia.org/wiki/%E7%99%BD%E9%9B%9C%E8%A8%8A

Noise in images

- When an image is transmitted through some channel, noise which is usually independent of the image signal occurs.
  - Additive noise model (加性雜訊模型)

$$f(x, y) = g(x, y) + \nu(x, y)$$

  where ν is the noise and g is the input image.

  - Algorithm 2.3: Generation of additive, zero mean Gaussian noise

Algorithm 2.3: Generation of additive, zero mean Gaussian noise

Step 1: Suppose an image has gray-level range $[0, G-1]$.

Select $\sigma > 0$; low values generate less noise.

Step 2: For each pair of horizontally neighboring pixels $(x, y), (x, y+1)$ generate a pair of independent random number r, φ in the range $[0, 1]$.

Step 3: Calculate $z_1 = \sigma \cos(2\pi\varphi)\sqrt{-2\ln r}$, $z_2 = \sigma \sin(2\pi\varphi)\sqrt{-2\ln r}$,

(This is the Box-Muller transform which assumes that z_1, z_2 are independently normally distributed with zero mean and variance σ^2)

Step 4: Set $f'(x,y) = g(x,y) + z_1$ and $f'(x,y+1) = g(x,y+1) + z_2$, where g is the input image.

Step 5: Set

$$f(x,y) = \begin{cases} 0 & \text{if } f'(x,y) < 0 \\ G-1 & \text{if } f'(x,y) > G-1 \\ f'(x,y) & \text{otherwise} \end{cases}; \quad f(x,y+1) = \begin{cases} 0 & \text{if } f'(x,y+1) < 0 \\ G-1 & \text{if } f'(x,y+1) > G-1 \\ f'(x,y+1) & \text{otherwise} \end{cases}$$

Step 6: Go to Step 2 until all pixels have been scanned.

Box-Muller transform

Suppose U_1 and U_2 are independent random variables that are uniformly distributed in the interval (0, 1). Let

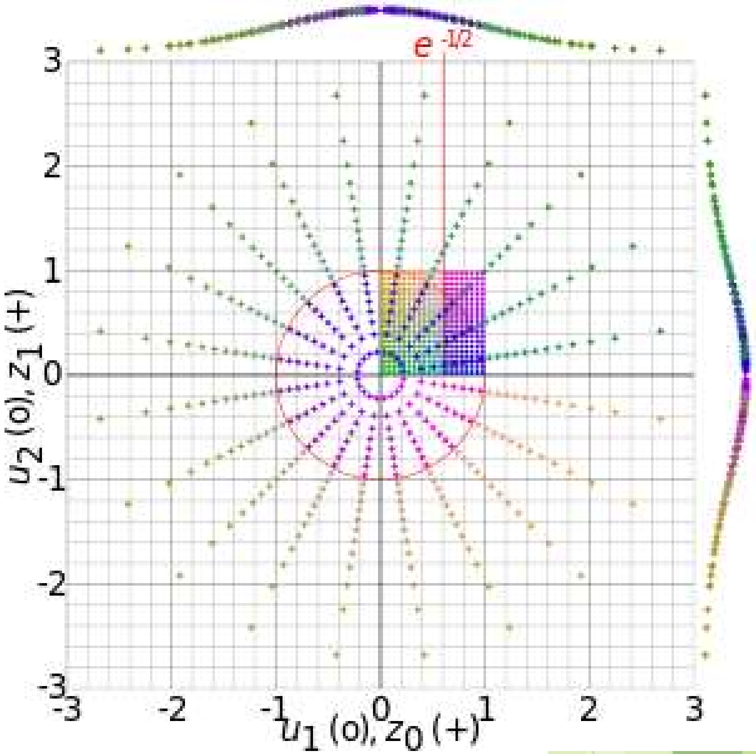
$$Z_0 = R\cos(\theta) = \sqrt{-2lnU_1}\cos(2\pi U_2)$$

and

$$Z_1 = R\sin(\theta) = \sqrt{-2lnU_1}\sin(2\pi U_2)$$

Then Z_0 and Z_1 are independent random variables with a standard normal distribution.

https://en.wikipedia.org/wiki/Box%E2%80%93Muller_transform#cite_note-4



Visualisation of the Box–Muller transform — the colored points in the unit square (u_1, u_2), drawn as circles, are mapped to a 2D Gaussian (z_0, z_1), drawn as crosses. The plots at the margins are the probability distribution functions of z_0 and z_1. Note that z_0 and z_1 are unbounded; they appear to be in [-3,3] due to the choice of the illustrated points.

Noise in images

- Signal-to-noise ratio (SNR;信噪比)

$$E = \sum_{(x,y)} \nu^2(x,y); \qquad F = \sum_{(x,y)} f^2(x,y)$$

$$\mathrm{SNR} = \frac{F}{E}$$

where ν is the noise and f is the image.

  - SNR represents a measure of image quality, with high values being 'good'.
  - It is often expressed in the logarithmic scale.

$$\mathrm{SNR}_{dB} = 10 \log_{10} \mathrm{SNR}$$

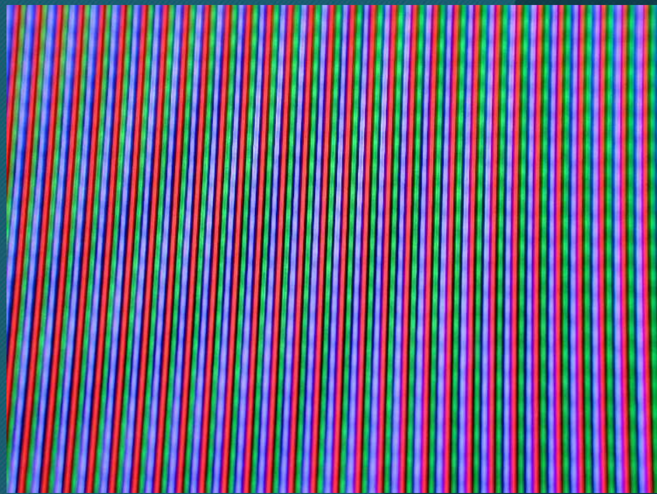
Noise in images

- Multiplicative noise model (乘性雜訊模型)

$$f = gv$$

where v is the noise and g is the input image.

- The noise magnitude depends in many cases on the signal magnitude itself.
- Television raster (光柵) degradation
  - An example of multiplicative noise
  - Degradation depends on TV lines
    - In the area of a line this noise is maximal, and between two lines it is minimal.



https://taratrangmarvideoproduction.wordpress.com/2012/10/16/the-tv-lines/

Noise in images

- Quantization noise (量化雜訊)
  - Quantization noise occurs when insufficient quantization levels are used.
  - For example, 50 levels for a monochromatic (單色) image.
- Impulse noise (脈衝雜訊)
  - Impulse noise means that an image is corrupted with individual noisy pixels whose brightness differs significantly from that of the neighborhood.
  - For example, salt-and-pepper noise (椒鹽雜訊)
    - An image is corrupted with white and/or black pixels