(A)

In geometric distribution, its random variable X represents the number of Bernoulli tosses needed for a head to come up for the first time.
The geometric probability function is

$$P(x) = (1-p)^{x-1}p, x = 1, \dots,$$

in which p is the only parameter.
Given a sample $X = \{X_t\}_{t=1}^{n}$,
derive the maximum likelihood estimate of p.

log likelihood :

$$\begin{aligned} L(\theta|x) &= \sum_{t=1}^{n} logP(x_t|\theta) \\ &= \sum_{t=1}^{n} log\{(1-p)^{x_t-1}p\} \\ &= \sum_{t=1}^{n} log(1-p)^{x_t-1} + \sum_{t=1}^{n} log(p) \\ &= log(1-p)^{\sum_{t=1}^{n} x_t - n} + nlog(p) \\ &= (\sum_{t=1}^{n} x_t - n) log(1-p) + nlog(p) \end{aligned}$$

MLE :
let $\frac{dL}{dp} = 0$

$$\begin{aligned} \frac{\partial L}{\partial p} &= \frac{\partial}{\partial p}[(\sum_{t=1}^{n} x_t - n)log(1-p) + nlog(p)] \\ &= \frac{-(\sum_{t=1}^{n} x_t - n)}{1-p} + \frac{n}{p} \\ &= \frac{-p(\sum_{t=1}^{n} x_t - n) + n(1-p)}{p(1-p)} = 0 \end{aligned}$$

$$-p(\sum_{t=1}^{n} x_t - n) + n(1-p) = 0$$

$$-p\sum_{t=1}^{n} x_t + np + n - np = n - p\sum_{t=1}^{n} x_t = 0$$

$$p = n/\sum_{t=1}^{n} x_t$$

(B)

Let $R = \begin{bmatrix} 3 & 1 \\ 1 & 4 \end{bmatrix}$, $P = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$, $\langle d_k^2 \rangle = 10$

for $\xi(w) = \langle d_k^2 \rangle + w^T R w - 2p^T w$, which reveals a paraboloid in the space (ξ, w).
Find
i)

$$\frac{d\xi(w)}{dw}$$

ii) the optimum weight weight vector $w^\star$
iii) the minimum mean square error ξ_{min}.