ELECTRIC POWER TRANSMISSION SYSTEM ENGINEERING

Analysis and Design

Turan Gönen

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ANALYSIS AND DESIGN

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We are all ignorant, just
about different things.

MARK TWAIN

There is so much good in the worst of us,
And so much bad in the best of us,
That it little behooves any of us,
To talk about the rest of us.

J. M.

For everything you have missed,
You have gained something else;
And for everything you gain,
You lose something else.

R. W. EMERSON

Dedicated to my brother,
Zaim Suat Gönen
for giving me the motivation

PREFACE

The structure of the electric power system is very large and complex. Nevertheless, its main components (or subsystems) can be identified as the generation system, transmission system, and distribution system. These three systems are the basis of the electric power industry. Today, there are various textbooks dealing with a broad range of topics in the power system area of electrical engineering. Some of them are considered to be classics. However, they do not particularly concentrate on the topics specifically dealing with electric power transmission. Therefore, this text is unique in that it is written specifically for in-depth study of modern power transmission engineering.

This book has evolved from the content of courses given by the author at the California State University, Sacramento, the University of Missouri at Columbia, the University of Oklahoma, and the Florida International University. It has been written for senior-level undergraduate and beginning-level graduate students, as well as practicing engineers in the electric power utility industry. It can serve as a text for a two-semester course, or by judicious selection, the material in the text can also be condensed to suit a one-semester course.

This book has been particularly written for a student or practicing engineer who may want to teach himself. Basic material has been explained carefully, clearly, and in detail with numerous examples. Each new term is clearly defined when it is first introduced. Special features of the book include ample numerical examples and problems designed to apply the information presented in each chapter. A special effort has been made to familiarize the reader with the vocabulary and symbols used by the industry.

The addition of the numerous impedance tables for overhead lines, transformers, and underground cables makes the text self-contained.

The text is primarily divided into two parts: electrical design and analysis and mechanical design and analysis. The electrical design and analysis portion of the book includes topics such as transmission system planning; basic concepts; transmission line parameters and the steady-state performance of transmission lines; disturbance of the normal operating conditions and other problems: symmetrical components and sequence impedances; in-depth analysis of balanced and unbalanced faults; extensive review of transmission system protection; detailed study of transient overvoltages and insulation coordination; underground cables; limiting factors for extra-high- and ultrahigh-voltage transmission in terms of corona, radio noise, and audible noise. The mechanical design and analysis portion of the book includes topics such as construction of overhead lines; the factors affecting transmission line route selection; right of way; insulator types; conductor vibration; sag and tension analysis; profile and plan of right of way; templates for locating structures. Also included is a review of the methods for allocating transmission line fixed charges among joint users.

TURAN GÖNEN

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T.G.

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PART I

ELECTRICAL DESIGN AND ANALYSIS

1

TRANSMISSION SYSTEM PLANNING

1.1 INTRODUCTION

An electrical power system can be considered to consist of a generation system, a transmission system, a subtransmission system, and a distribution system. In general, the generation and transmission systems are referred to as *bulk power supply*, and the subtransmission and distribution systems are considered to be the final means to transfer the electric power to the ultimate customer. Bulk power transmission is made of a high-voltage network, generally 138–765 kV alternating current, designed to interconnect power plants and electrical utility systems and to transmit power from the plants to major load centers. Table 1.1 gives the standard transmission voltages as dictated by ANSI Standard C-84 of the American National Standards Institute. The subtransmission refers to a lower voltage network, normally 34.5–115 kV, interconnecting bulk power and distribution substations. The voltages that are in the range of 345–765 kV are classified as extra-high voltages (EHVs). The EHV systems dictate a very thorough system design. Figure 1.1 shows the critical path of steps in an EHV line design. While, on the contrary, the high-voltage transmission systems up to 230 kV can be built in relatively simple and well-standardized designs, the voltages above 765 kV are considered as the ultrahigh voltages (UHVs). Currently, the UHV systems, at 1000-, 1100-, 1500-, and 2250-kV voltage levels, are in the R&D stages. Table 1.2 gives the lengths of the transmission lines installed in the service areas of the regional reliability councils. Figure 1.2 shows the trends in technology and cost of electrical energy (based on 1968 constant dollars). Historically, the decreasing cost of electri-

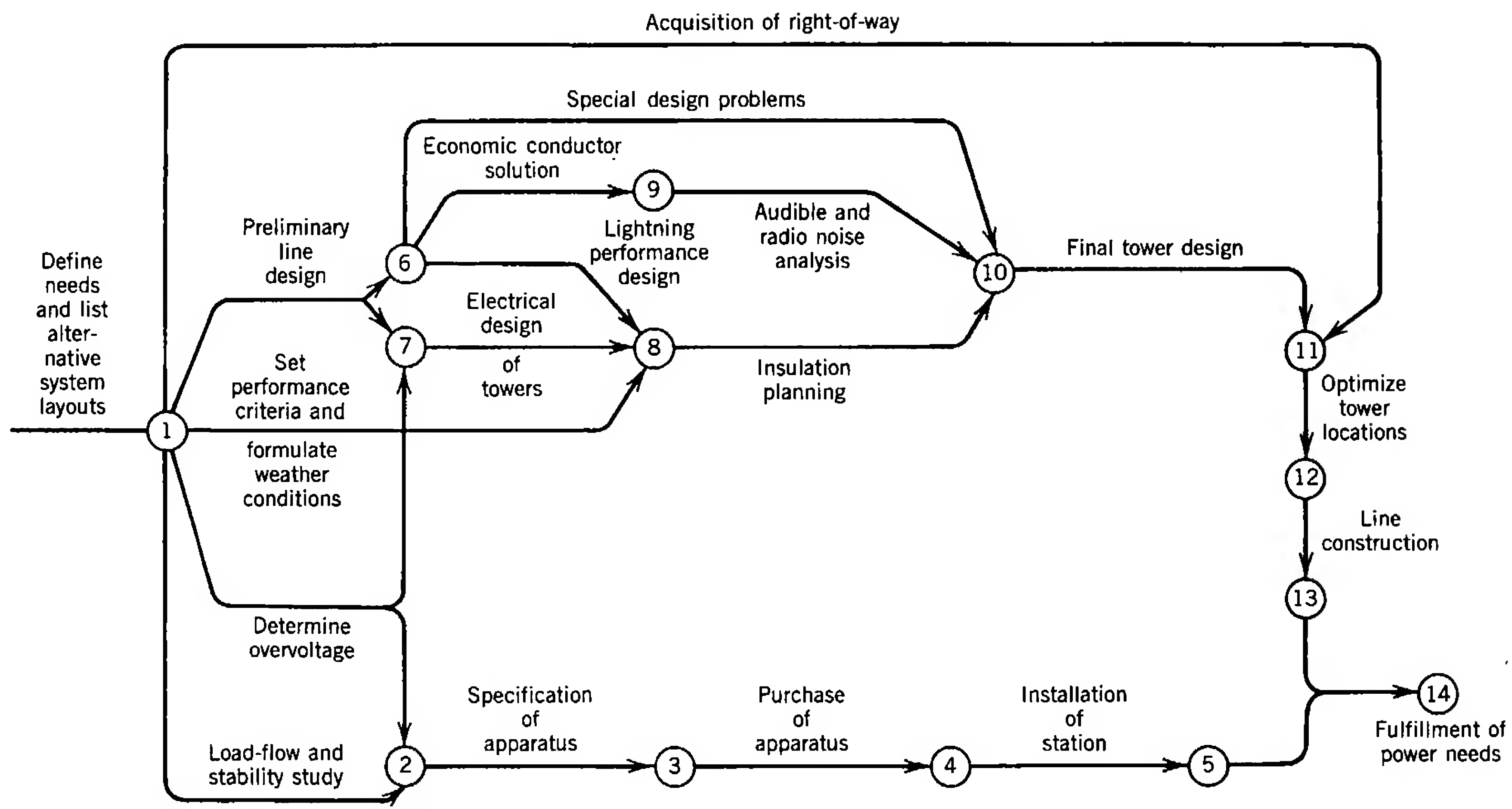

Figure 1.1. Critical path steps in extra-high-voltage line design. (From Electric Power Research Institute, 1979. Used by permission. © 1979 Electric Power Research Institute.)

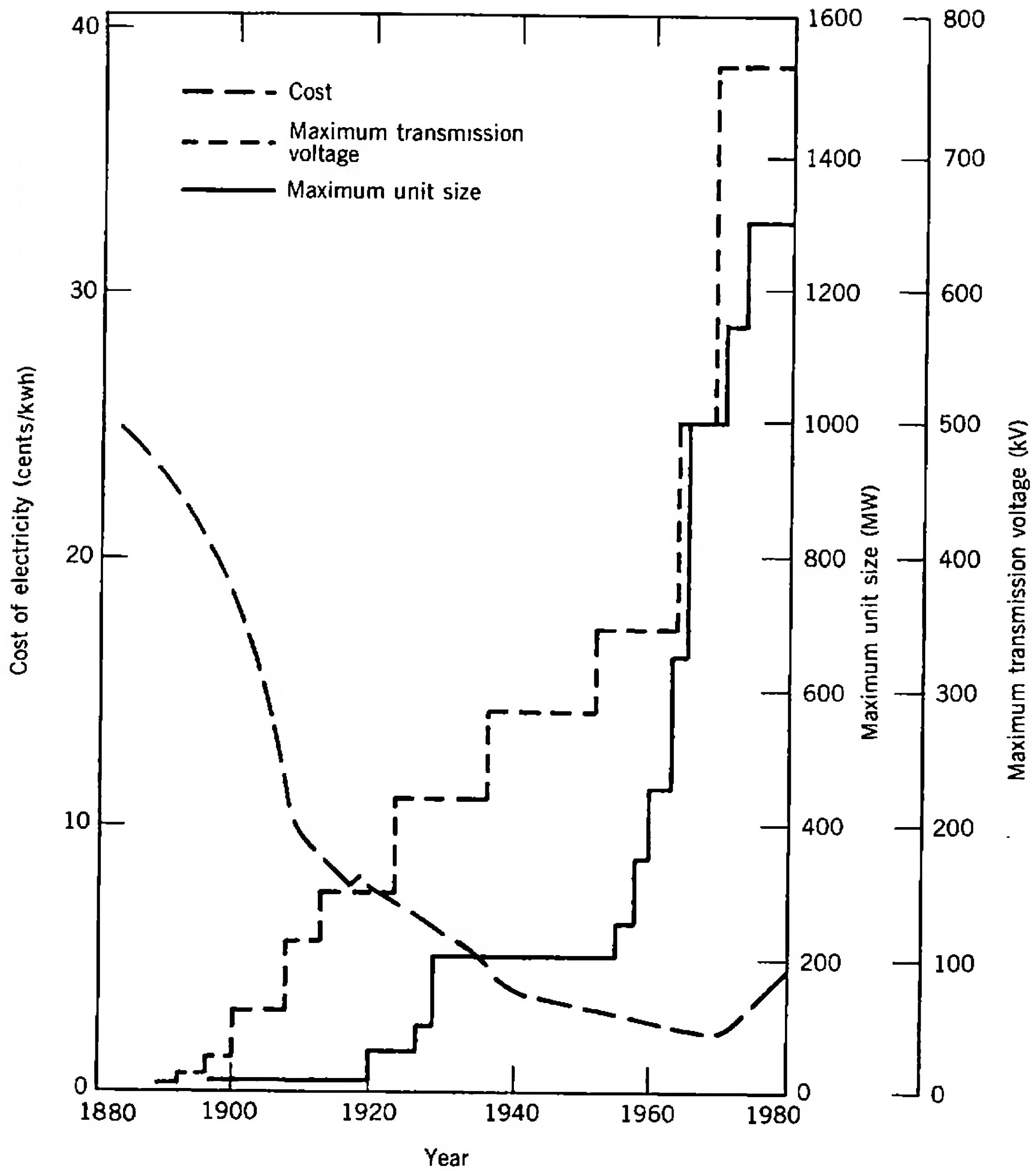

Figure 1.2. Historical trends in technology and cost of electrical energy. (From Electric Power Research Institute, 1979. Used by permission. © 1979 Electric Power Research Institute.)

TABLE 1.1 Standard System Voltages

Rating	
Nominal (kV)	Maximum (kV)
34.5	36.5
46	48.3
69	72.5
115	121
138	145
161	169
230	242
345	362
500	550
700	765

TABLE 1.2 1980 Regional Transmission Lines in Miles

	Voltage (kV)						
	HVAC				HVDC		
Region	230	345	500	765	250	400/450	800
ECAR	934	9.850	796	1.387	0	0	0
ERCOT	0	4.110	0	0	0	0	0
MAAC	4.400	160	1.263	0	0	0	0
MAIN	258	4.852	0	90	0	0	0
MARCA (U.S.)	6.477	3.504	138	0	465	436	0
NPCC (U.S.)	1.557	3.614	5	251	0	0	0
SERC	16.434	2	4.363	0	0	0	0
SPP	3.057	2.843	1.432	0	0	0	0
WSCC (U.S.)	27.892	5.923	7.551	0	0	0	844
NERC (U.S.)	61.009	34.858	15.548	1.728	465	436	844

Source: National Electric Reliability Council 10th Annual Review [32].

cal energy has been due to the technological advances reflected in terms of economies of scale and operating efficiencies.

1.2 PRESENT TRANSMISSION-SYSTEM-PLANNING TECHNIQUES

As aforementioned, the purpose of transmission system planning is to determine the timing and type of new transmission facilities required in order to provide adequate transmission network capability to cope with the future generating capacity additions and load-flow requirements. Figure 1.3

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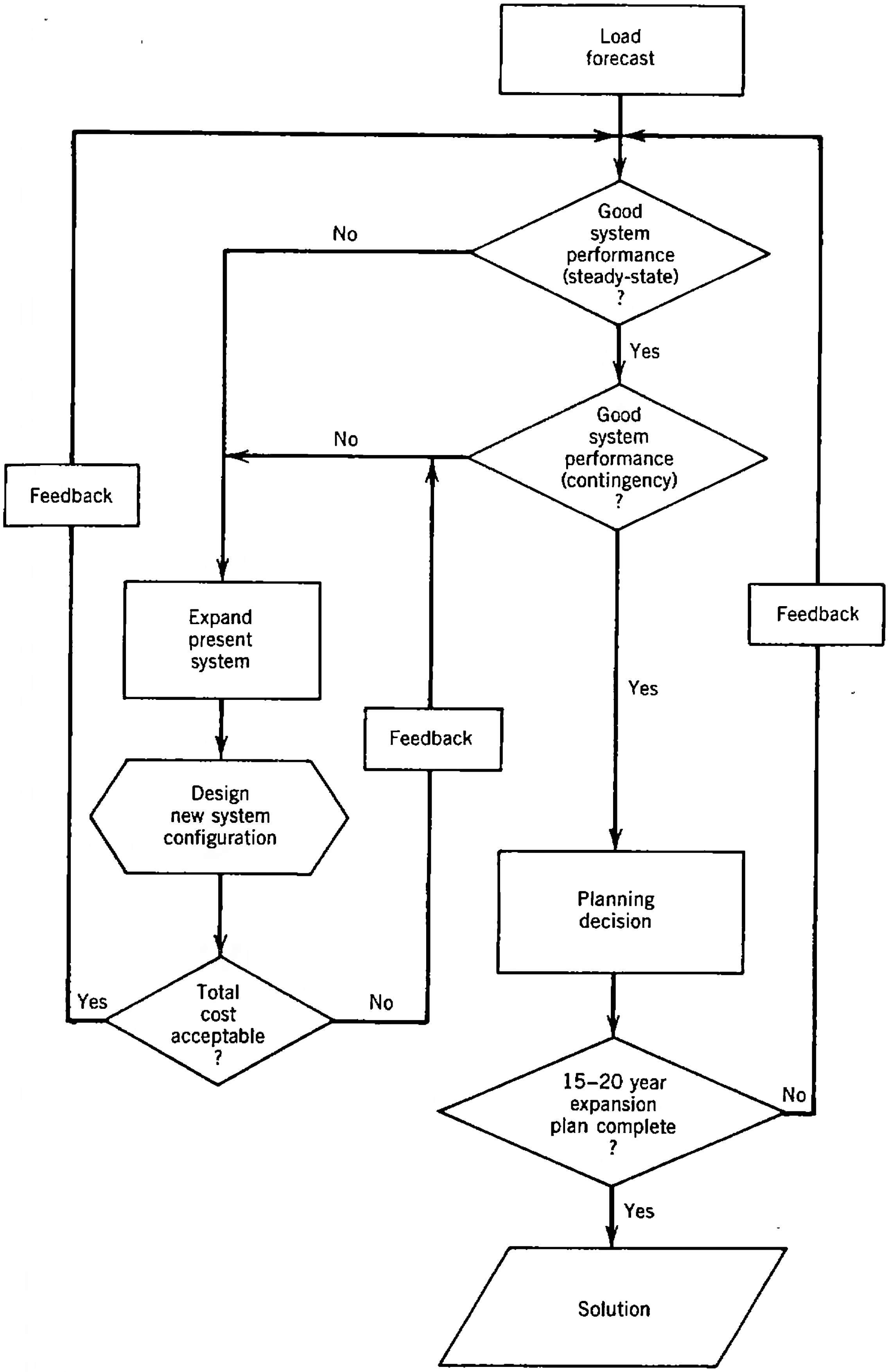

Figure 1.3. Block diagram of typical transmission system planning process.

shows a functional block diagram of a typical transmission-system-planning process. This process may be repeated, with diminishing detail, for each year of a long-range (15–20-year) planning horizon. The key objective is to minimize the long-range capital and operating costs involved in providing an adequate level of system reliability, with due consideration of environmental and other relevant issues. Transmission planning may include not only existing but also new service areas. The starting point of the planning procedure is to develop load forecasts in terms of annual peak demand for the entire system, as well as for each region and each major present and future substation, and then finding specific alternatives that satisfy the new load conditions. The system performance is tested under steady-state and contingency conditions.

The logic diagram for transmission expansion study is shown in Figure 1.4. The main objective is to identify the potential problems, in terms of unacceptable voltage conditions, overloading of facilities, decreasing reliability, or any failure of the transmission system to meet performance criteria. After this analysis stage, the planner develops alternative plans or scenarios that not only will prevent the foreseen problems but also will best meet the long-term objectives of system reliability and economy. The effectiveness of the alternative plans is determined by load-flow studies under both normal and emergency operations. The load-flow programs now in use by the utilities allow the calculation of currents, voltages, and real and reactive power flows, taking into account the voltage-regulating capability of generators, transformers, synchronous condensers, specified generation schedules, as well as net interchange among interconnected systems, automatically. By changing the location, size, and number of transmission lines, the planner can achieve to design an economical system that meets the operating and design criteria.

After determining the best system configuration from load-flow studies, the planner studies the system behavior under fault conditions. The main objectives of short-circuit studies can be expressed as (1) to determine the current-interrupting capacity of the circuit breaker so that the faulted equipment can be disconnected successfully, therefore clearing the fault from the system, and (2) to establish the relay requirements and settings to detect the fault and cause the circuit breaker to operate when the current flowing through it exceeds the maximum allowable current. The short-circuit studies can also be used to (1) calculate voltages during faulted conditions that affect insulation coordination and lightning arrester applications; (2) design the grounding systems, and (3) determine the electromechanical forces affecting the facilities of the system.

Finally, the planner performs stability studies in order to be sure that the system will remain stable following a severe fault or disturbance. Here, the stability analysis is defined as the transient behavior of the power system following a disturbance. It can be classified as transient stability analysis. The transient stability is defined as the ability of the system to maintain

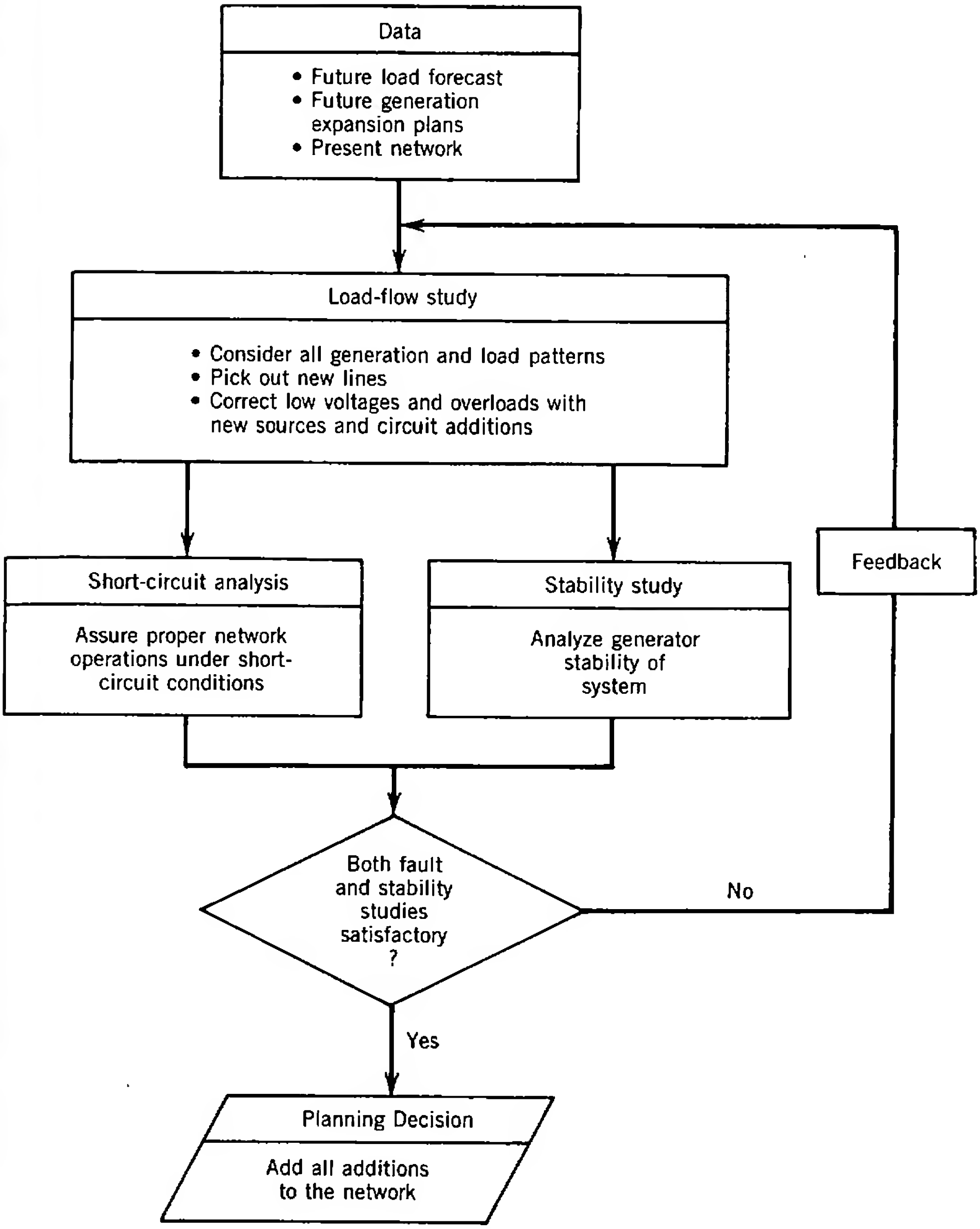

Figure 1.4. Logic diagram for transmission expansion study.

synchronous operation following a disturbance, usually a fault condition. Unless the fault condition is cleared rapidly by circuit breakers, the generators, which are connected to each other through a transmission network, will get out with respect to one another, that is, they will not run in synchronism. This situation, in turn, will cause large currents to flow through the network, transferring power from one generator to another in an oscillating way and causing the power system to become unstable. Consequently, the protective relays will detect these excessive amounts of currents and activate circuit breakers all over the network to open, causing a complete loss of power supply. Usually, the first swing of rotor angles is considered to be an adequate indicator of whether or not the power system remains stable. Therefore, the simulation of the first few seconds following a disturbance is sufficient for transient stability. Whereas steady-state stability analysis is defined as long-term fluctuations in system frequency and power transfers resulting in total blackouts,† in this case, the system is simulated from a few seconds to several minutes.

There are various computer programs available for the planner to study the transient and steady-state stabilities of the system. In general, a transient stability program employs the data, in terms of initial voltages and power flows, provided by a load-flow program as the input and transforms the system to that needed for the transient stability analysis. Usually, the critical switching time, that is, the time during which a faulted system component must be tripped to assure stability, is used as an indicator of stability margin. The critical switching times are calculated for various fault types and locations. The resultant minimum required clearing time is compared to actual relay and circuit breaker operating time. If the relays and circuit breakers cannot operate rapidly enough to maintain stable operation, the planner may consider a change in the network design or a change in the turbine-generator characteristics or perhaps control apparatus.

1.3 MODELS USED IN TRANSMISSION SYSTEM PLANNING

In the past, the transmission system planning and design were rather intuitive and based substantially on the planner's past experience. Today, the planner has numerous analysis and synthesis tools at his disposal. These tools can be used for design and planning activities, such as (1) transmission route identification and selection, (2) transmission network expansion planning, (3) network analysis, and (4) reliability analysis. The first two of these will be discussed in this chapter.

1.4 TRANSMISSION ROUTE IDENTIFICATION AND SELECTION

Figure 1.5 shows a typical transmission route (corridor) selection procedure. The restricting factors affecting the process are safety, engineering and

† The IEEE has redefined *steady-state stability* to include the manifestation formerly included in both *steady-state* and *dynamic* stability. The purpose of this change is to bring American practice into agreement with international practice. Therefore, dynamic stability is no longer found in the IEEE publications unless the reviewers happened to overlook the old usage.

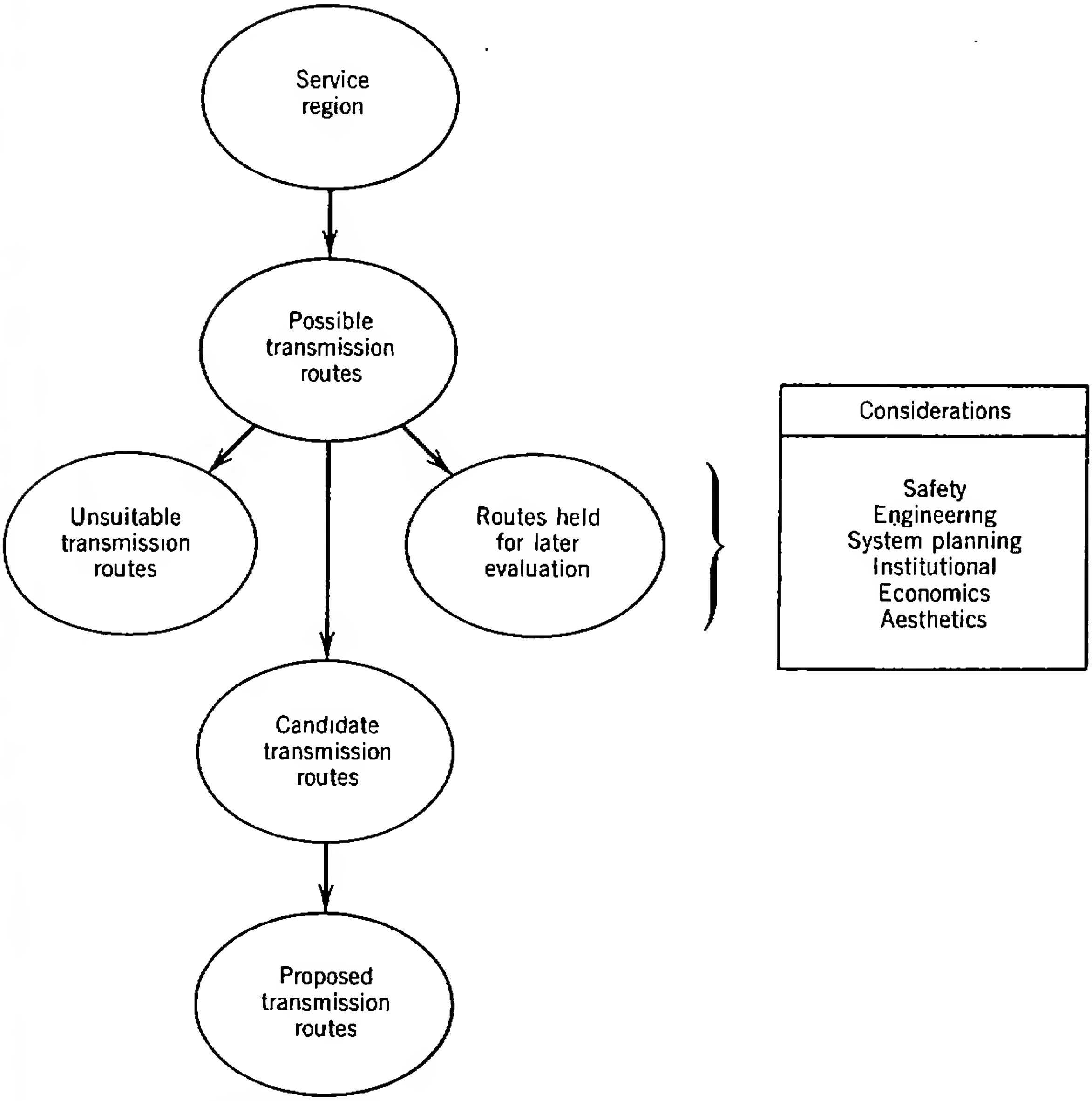

Figure 1.5. Transmission route selection procedure.

technology, system planning, institutional, economics, environmental, and aesthetics. Today, the planner selects the appropriate transmission route based on his knowledge of the system, results of the system analysis, and available rights of way. However, recently, two computer programs, Power and Transthetics, have been developed to aid the planner in transmission route identification and selection [1–3]. The Power computer program can be used to locate not only transmission line corridors but also other types of corridors. Whereas the transthetics computer program is specifically designed for electrical utilities for the purpose of identifying and selecting potential transmission line corridors and purchasing the necessary rights of way.

1.5 TRANSMISSION SYSTEM EXPANSION PLANNING

Today, the system planner, as mentioned in Section 1.2, mostly uses tools such as load-flow, stability, and short-circuit programs in analyzing the performance of specific transmission system alternatives. However, some utilities also employ the use of so-called automatic expansion models to determine the optimum system. Here, the optimality claim is in the mathematical sense; that is, the optimum system is the one that minimizes an objective function (performance function) subject to restrictions. In general, the automatic expansion models can be classified into three basic groups:

1. Heuristic models.
2. Single-stage optimization models.
3. Time-phased optimization models.

1.5.1 Heuristic Models

The primary advantage of the heuristic models is interactive planning; that is, the system planner can observe the expansion process and direct its direction as it is desired. According to Meckiff et al. [4], the characteristics of the heuristic models are (1) simple model and logic, (2) user interaction, and (3) families of *feasible*, *near optimal* plans. Whereas the characteristics of the mathematical programming models are (1) no user interaction, (2) fixed model by program formulation, (3) detailed logic or restriction set definition, and (4) single "global" solution. The heuristic models can be considered to be custom-made, contrary to mathematical models. Some help to simulate the way a system planner employs analytical tools such as load-flow programs [5, 6] and reliability analysis [6] involving simulations of the planning process through automated design logic. The classical paper by Garver [7] describes a method that unites heuristic logic for circuit selection with optimization techniques. The proposed method is to determine the most direct route transmission network from the generation to load without causing any circuit overloads. In heuristic approach, the best circuit addition or exchange is given to the planner by the computer program automatically at each stage of the synthesis process. The planner may select to accept it or modify it as he desires. Further information on heuristic models is given in Baldwin et al. [8–11].

1.5.2 Single-Stage Optimization Models

The single-stage or single-state (or so-called static) optimization models can be used for determining the optimum network expansion from one stage to the next. But they do not give the timing of the expansion. Therefore, even though they provide an optimum solution for year-by-year expansion, they may not give the optimum solution for overall expansion pattern over a time horizon. The mathematical programming techniques used in single-state optimization models include (1) linear programming, (2) integer programming, and (3) gradient search method.

Linear Programming

Linear programming (LP) is a mathematical technique that can be used to minimize or maximize a given linear function, called the objective function in which the variables are subject to linear constraints. The objective function takes the linear form

$$Z = \sum_{i=1}^{n} c_i x_i \tag{1.1}$$

where Z is the value to be optimized. (In expansion studies, Z is the total cost that is to be minimized.) The x_i represents n unknown quantities, and the c_i are the costs associated with one unit of x_i. The c_i may be positive or negative, whereas the x_i must be defined in a manner as to assume only positive values. The constraints, or restrictions, are limitations on the values that the unknowns may assume and must be a linear combination of the unknowns. The constraints assume the form

$$\sum a_{ji} x_i = , \geq , \leq b_j \qquad x_i \geq 0 \tag{1.2}$$

or

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n &= , \geq , \leq b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n &= , \geq , \leq b_2 \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{m1}x_n &= , \geq , \leq b_m \end{aligned}$$

$$x_1 \geq 0,\ x_2 \geq 0, \ldots , x_n \geq 0$$

where

$$j = 1, 2, \ldots , m,\ i = 1, 2, \ldots , n$$

where there are m constraints of which any number may be equalities or inequalities. Also, the number of constraints, m, may be greater than, less than, or equal to the number of unknowns, n. The coefficients of the unknowns, a_{ji}, may be positive, negative, or zero but must be constants. The b_j are also constants, which may be positive, negative, or zero. The constraints define a region of solution feasibility in n-dimensional space. The optimum solution is the point within this space whose x_i values minimize or maximize the objective function Z. In general, the solutions obtained are real and positive.

In 1970, Garver [7] developed a method that uses linear programming and linear flow estimation models in the formulation of an automated transmission-system-planning algorithm. The method helps to determine where capacity shortages exist and where to add new circuits to alleviate overloads. The objective function is the sum of the circuit lengths (guide numbers) times the magnitude of power that they transport. Here, the power flows are calculated employing a *linear loss function network model* that is similar to a transportation model. This model uses Kirchhoff's current law (i.e., at each bus the sum of all flows in and out must sum to zero) but not Kirchhoff's voltage law to specify flows. Instead, the model uses *guide potentials* to assure that conventional circuits are not overloaded. However, the flow model also uses *overload paths*, in which power can flow if required, to determine where circuit additions are to be added. The network is expanded one circuit at a time to eliminate the path with the largest overload until no overload paths exist. After the completion of the network expansion, the system is usually tested, employing an ac load-flow program. As mentioned, the method is also heuristic partly due to the fact that assigning the guide numbers involves a great deal of judgment [12, 13]. A similar method has been suggested by Kaltenbach et al. [14]. However, it treats the problem more rigidly as an optimization problem.

Integer Programming

The term *integer programming* refers to the class of linear programming problems in which some or all of the decision variables are restricted to be integers. For example, in order to formulate the LP program given in equations (1.1) and (1.2) as an integer program, a binary variable can be introduced for each line to denote whether it is selected or not:

$$x_i = 1 \quad \text{if line } i \text{ is selected}$$

$$x_i = 0 \quad \text{if line } i \text{ is not selected}$$

Therefore,

$$\text{Minimize } Z = \sum_{i=1}^{n} c_i x_i \tag{1.3}$$

subject to

$$\sum_{i=1}^{n} a_{ji} x_i \leq b_j \qquad x_i = 0, 1 \tag{1.4}$$

where $j = 1, 2, \ldots, m$ $i = 1, 2, \ldots, n$.

In general, integer programming is more suitable for the transmission expansion problem than linear programming because it takes into account the discrete nature of the problem; that is, a line component is either added or not added to the network. The integer program wherein all variables are restricted to be (0–1) integer valued is called a *pure integer program*. Whereas if the program restricts some of the variables to be integers while others can take continuous (fractional) values, it is called a *mixed-integer program*.

In 1960, Knight [15, 16] applied integer programming to the transmission expansion problem. Adams and Laughton [17] used mixed-integer programming for optimal planning of power networks. Lee et al. [18] and Sjelvgren and Bubenko [19] proposed methods that employ a combination of sensitivity and screening procedures to restrict the search on a limited number of new additions that are most likely to meet all restrictions. The method proposed by Lee et al. [18] starts with a dc load-flow solution to distinguish the overloaded lines as well as to compute the line flow sensitivities to changes in admittances in all transmission corridors. In order to reduce the dimension of the integer programming problem in terms of number of variables and therefore the computer time, it employs a screening process to eliminate ineffective corridors. The resulting problem is then solved by a branch-and-bound technique. It adds capacity only in discrete increments as defined by the optimal capacity cost curves. The process is repeated as many times as necessary until all restrictions are satisfied. Further information on integer programming models is given in Gönen et al. [21–22].

Gradient Search Method

The gradient search method is a nonlinear mathematical programming applicable to so-called automated transmission system planning. Here, the objective function that is to be minimized is a performance index of the given transmission network. The method starts with a dc load-flow solution for the initial transmission network and future load and generation forecasts. The system performance index is calculated and the necessary circuit modifications are made employing the partial derivatives of the performance index with respect to circuit admittances. Again, a dc load-flow solution is obtained, and the procedure is repeated as many times as necessary until a network state is achieved for which no further decrease in the performance index can be obtained. The method proposed by Fischl and Puntel [23] applies Tellegen's theorem. The gradient information necessary to update the susceptances associated with effective line additions as aforementioned was implemented. More detailed information can also be found in Puntel et al. [24, 25].

1.5.3 Time-Phased Optimization Models

The single-stage transmission network expansion models do not take into account the timing of new installations through a given time horizon. Therefore, as Garver [26] points out, there is a need for "a method of finding a sequence of yearly transmission plans which result in the lowest revenue requirements through time but which may be higher in cost than really needed in any one particular year." A time-phased (trough-time, or multistate, or so-called dynamic) optimization model can include inflation, interest rates, as well as yearly operating cost in the comparison of various network expansion plans. Both integer programming and dynamic programming optimization methods have been used to solve the time-phased network expansion models [26a]. The integer programming has been applied by dividing a given time horizon into numerous annual subperiods. Consequently, the objective function in terms of present worth of a cost function is minimized in order to determine the capacity, location, and timing of new facilities subject to defined constraints [17, 22, 27].

The dynamic programming [24] has been applied to network expansion problems by developing a set of network configurations for each year (stage). Only those feasible plans (states) that satisfy the defined restrictions are accepted. However, as Garver [26] points out, "the dynamic programming method has organized the search so that a minimum number of evaluations were necessary to find the lowest cost expansion. However, the dynamic programming method by itself cannot introduce new plans (states), it only links given states together in an optimal manner." Dusonchet and El-Abiad [28] applied dicrete dynamic optimization employing a combination of dynamic programming, a random search, and a heuristic stopping criterion. Henault et al. [27] studied the problem in the presence of uncertainty. Mamandur [29] applied the "k-shortest paths" method to replace dynamic programming for transmission network expansion. The k-shortest paths technique [30] is employed to determine the expansion plans with the minimum costs.

1.6 TRANSMISSION SYSTEM PLANNING IN THE FUTURE†

In the previous sections, some of the past and present techniques used by the system planning engineers of the utility industry performing transmission systems planning have been discussed. Also, the factors affecting the transmission system planning have been reviewed. The purpose of this section is to examine what today's trends are likely to bring for the future of the planning process.

There are several economic factors that will have significant effects on transmission system planning in the 1980s. The first of these is inflation. Fueled by energy shortages, energy source conversion costs, environmental concerns, and large government deficits, inflation will continue to play a major role. The second important economic factor will be the increasing expense of acquiring capital. As long as inflation continues to decrease the

† This section is based on Gönen [31]. Included with permission of McGraw-Hill Book Company.

real value of the dollar, attempts will be made by government to reduce the money supply. This, in turn, will increase the competition for attracting the capital necessary for expansions in power systems. The third factor, which must be considered, is increasing difficulty in increasing customer rates. This rate increase "inertia" also stems in part from inflation as well as from the results of customers being made more sensitive to rate increases by consumer activist groups.

Predictions about the future methods for transmission system planning must necessarily be extrapolations of present methods. Basic algorithms for network analysis have been known for years and are not likely to be improved upon in the near future. However, the superstructure that supports these algorithms and the problem-solving environment used by the system designer is expected to change significantly to take advantage of new methods technology has made possible. Before giving detailed discussion of these expected changes, the changing role of transmission system planning needs to be examined.

For the economic reasons listed above, transmission systems will become more expensive to build, expand, and modify. Thus, it is particularly important that each transmission system design be as cost-effective as possible. This means that the system must be optimal from many points of view over the time period from first day of operation to the planning time horizon. In addition to the accurate load growth estimates, components must be phased in and out of the system so as to minimize capital expenditure, meet performance goals, and minimize losses.

In the utility industry, the most powerful force shaping the future is that of economics. Therefore, any new innovations are not likely to be adopted for their own sake. These innovations will be adopted only if they reduce the cost of some activity or provide something of economic value that previously had been unavailable for comparable costs. In predicting that certain practices or tools will replace current ones, it is necesary that one judge their acceptance on this basis.

The expected innovations that satisfy these criteria are planning tools implemented on a digital computer that deals with transmission systems in network terms. One might be tempted to conclude that these planning tools would be adequate for industry use throughout the 1980s. That this is not likely to be the case may be seen by considering the trends judged to be dominant during this period with those that held sway over the period in which the tools were developed.

1.6.1 New Planning Tools

Tools to be considered fall into two categories: network design tools and network analysis tools. The analysis tools may become more efficient but are not expected to undergo any major changes, although the environment in which they are used will change significantly. This environment will be discussed in the next section.

The design tools, however, are expected to show the greatest development since better planning could have a significant impact on the utility industry. The results of this development will show the following characteristics:

1. Network design will be optimized with respect to many criteria using programming methods of operations research.
2. Network design will be only one facet of transmission system management directed by human engineers using a computer system designed for such management functions.
3. So-called network editors [32] will be available for designing trial networks; these designs in digital form will be passed to extensive simulation programs that will determine if the proposed network satisfies performance and load growth criteria.

1.6.2 Central Role of Computer in Transmission System Planning

As is well known, transmission system planners have used computers for many years to perform the tedious calculations necessary for system analysis. However, it has only been in the past few years that technology has provided the means for planners to truly take a "systems approach" to the total design and analysis. It is the central thesis of this chapter that the development of such an approach will occupy planners in the 1980s and will significantly contribute to their meeting the challenges previously discussed.

1.6.3 Systems Approach

A collection of computer programs to solve the analysis problems of a designer does not necessarily constitute an efficient problem-solving system nor even does such a collection when the output of one can be used as the input of another. The systems approach to the design of a useful tool for the designer begins by examining the types of information required and its sources. The view taken is that this information generates decisions and additional information that pass from one stage of the design process to another. At certain points, it is noted that the human engineer must evaluate the information generated and add his inputs. Finally, the results must be displayed for use and stored for later reference. With this conception of the planning process, the systems approach seeks to automate as much of the process as possible, ensuring in the process that the various transformations of information are made as efficiently as possible. One representation of this information flow is shown in Figures 1.6 and 1.7.

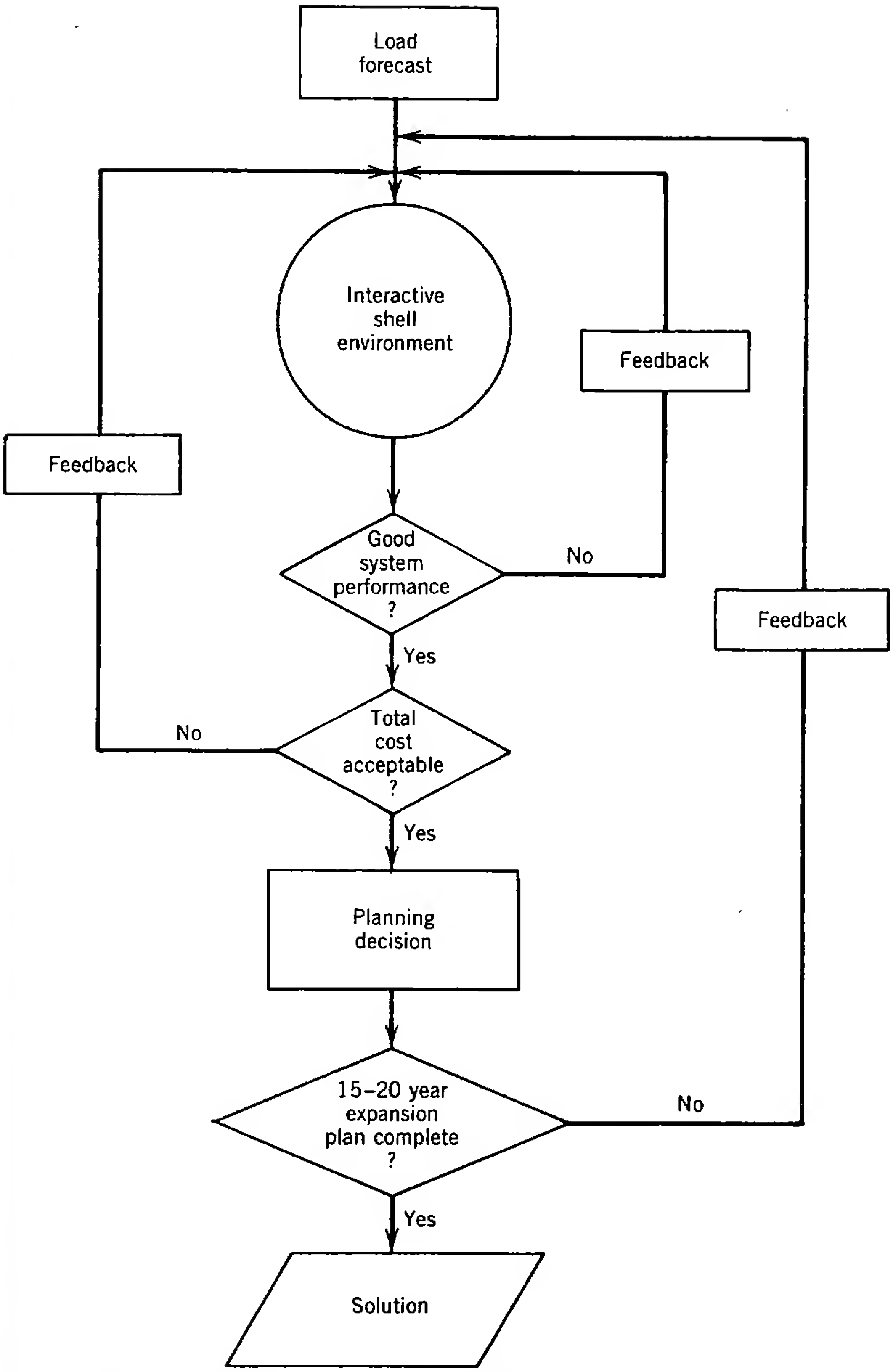

Figure 1.6. Schematic view of transmission planning system.

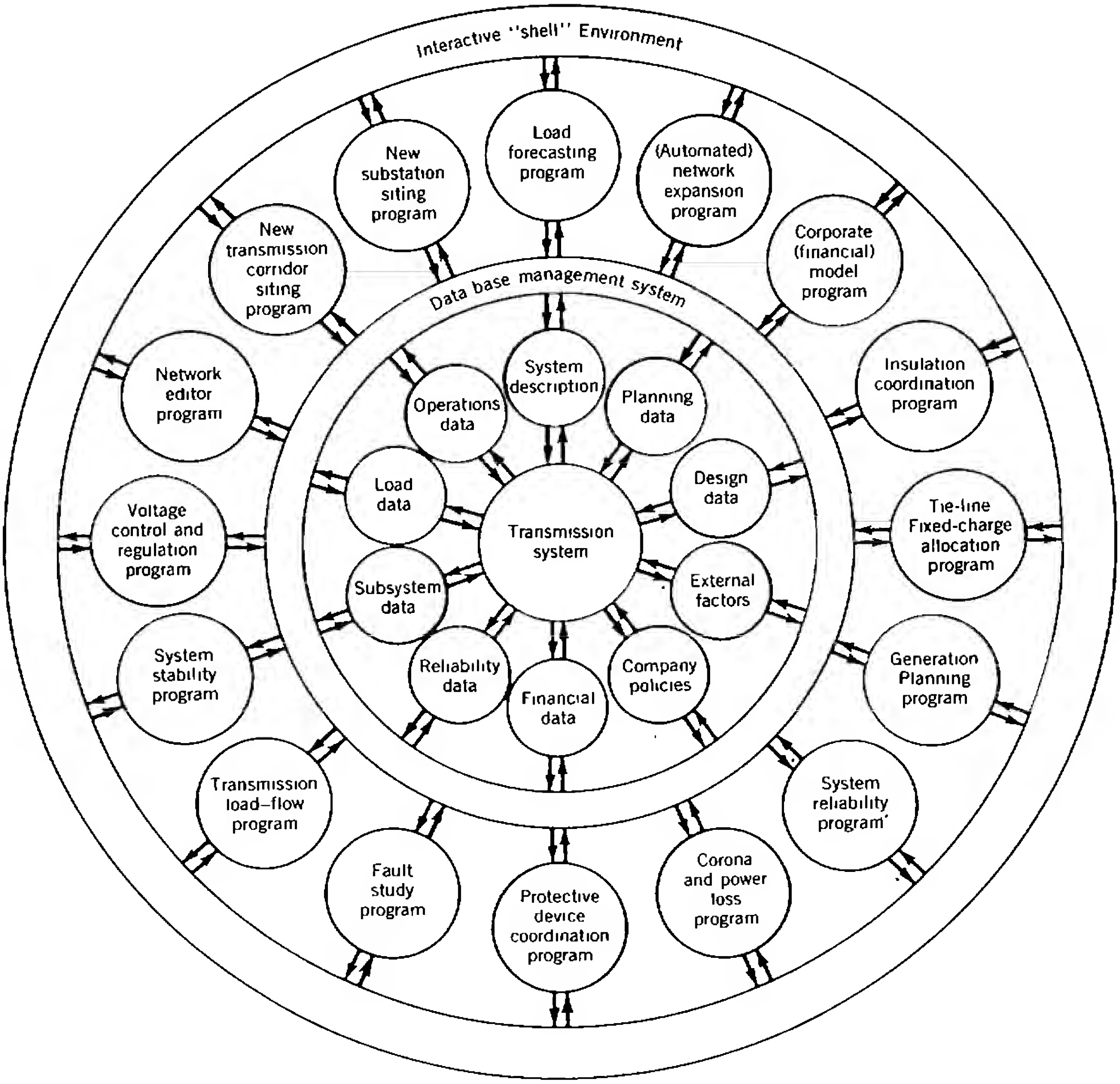

Figure 1.7. Block diagram of transmission system planning process in future.

Here, the outer circle represents the interface between the engineer and the system. Analysis programs forming part of the system are supported by a data base management system that stores, retrieves, and modifies various data on transmission systems.

1.6.4 Data Base Concept

As suggested in Figure 1.6, the data base plays a central role in the operation of such a system. It is in this area that technology has made some significant strides in the past five years so that not only is it possible to store vast quantities of data economically, but it is also possible to retrieve desired data with access times on the order of seconds. The data base management system provides the interface between the process, which requires access to the data, and the data itself. The particular organization likely to emerge as the dominant one in the near future is based on the idea of a relation. Operations on the data base are performed by the data base management system (DBMS).

In addition to the data base management program and the network analysis programs, it is expected that some new tools will emerge to assist the designer in arriving at the optimal design. One such new tool that has appeared in the literature is known as a network editor [42].

1.6.5 Summary

Future transmission systems will be more complex than those of today. This means that the distribution system planner's task will be more complex. If the systems being planned are to be optimal with respect to construction cost, capitalization, performance, and operating efficiency, better planning tools are required.

While it is impossible to foresee all of the effects that technology will have on the way in which transmission system planning will be done, it is possible to identify the major forces beginning to institute a change in the methodology and extrapolate.

The most important single influence is that of the computer, which will permit the automating of more and more of the planning activity. The automation will proceed along two major avenues. First, increased application of operations research techniques will be made to meet performance requirements in the most economical way. Second, improvements in data base technology will permit the planner to utilize far more information in an automatic way than has been possible in the recent past. Interactive computer systems will display network configurations, cost information, device ratings, etc., at the whim of the planner. Moreover, this information will be available to sophisticated planning programs that will modify the data base as new systems are designed and old ones are modified.

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2

STEADY-STATE PERFORMANCE OF TRANSMISSION LINES

2.1 INTRODUCTION

In this chapter, a brief review of fundamental concepts associated with steady-state ac circuits, especially with three-phase circuits, is presented. It is hoped that this brief review is sufficient to provide a common base, in terms of notation and references, that is necessary to be able to follow the subsequent chapters.

2.2 COMPLEX POWER IN BALANCED TRANSMISSION LINES

Figure 2.1(*a*) shows a per-phase representation (or one-line diagram) of a short three-phase balanced transmission line connecting buses i and j. Here, the term *bus* defines a specific nodal point of a transmission network. Assume that the bus voltages $\mathbf{V}_i$ and $\mathbf{V}_j$ are given in phase values (i.e., line-to-neutral values) and that the line impedance is $\mathbf{Z} = R + jX$ per phase. Since the transmission line is a short one, the line current I can be assumed to be approximately the same at any point in the line. However, because of the line losses, the complex powers $\mathbf{S}_{ij}$ and $\mathbf{S}_{ji}$ are not the same.

Therefore, the complex power per phase[†] that is being transmitted from bus i to bus j can be expressed as

$$\mathbf{S}_{ij} = P_{ij} + jQ_{ij} = \mathbf{V}_i \mathbf{I}^* \tag{2.1}$$

[†] For an excellent treatment of the subject, see Elgerd [1].

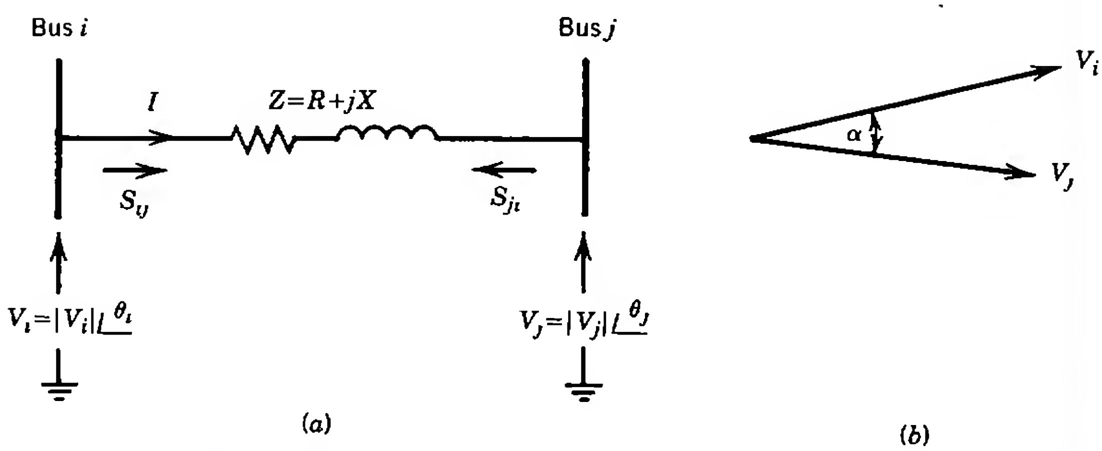


Figure 2.1. Per-phase representation of short transmission line.

Similarly, the complex power per phase that is being transmitted from bus j to bus i can be expressed as

$$\mathbf{S}_{ji} = P_{ji} + Q_{ji} = \mathbf{V}_j(-\mathbf{I})^* \tag{2.2}$$

Since

$$\mathbf{I} = \frac{\mathbf{V}_i - \mathbf{V}_j}{\mathbf{Z}} \tag{2.3}$$

substituting equation (2.3) into equations (2.1) and (2.2),

$$\begin{aligned}\mathbf{S}_{ij} &= \mathbf{V}_i\,\frac{\mathbf{V}_i^* - \mathbf{V}_j^*}{\mathbf{Z}^*}\\ &= \frac{|\mathbf{V}_i|^2 - |\mathbf{V}_i||\mathbf{V}_j|\,\underline{/\theta_i - \theta_j}}{R - jX}\end{aligned} \tag{2.4}$$

and

$$\begin{aligned}\mathbf{S}_{ji} &= \mathbf{V}_j\,\frac{\mathbf{V}_j^* - \mathbf{V}_i^*}{\mathbf{Z}^*}\\ &= \frac{|\mathbf{V}_j|^2 - |\mathbf{V}_i||\mathbf{V}_j|\,\underline{/\theta_j - \theta_i}}{R - jX}\end{aligned} \tag{2.5}$$

However, as shown in Figure 2.1(b), if the power angle (i.e., the phase angle between the two bus voltages) is defined as

$$\gamma = \theta_i - \theta_j \tag{2.6}$$

then real and reactive power per phase values can be expressed, respectively, as

$$P_{ij} = \frac{1}{R^2 + X^2} (R|\mathbf{V}_i|^2 - R|\mathbf{V}_i||\mathbf{V}_j| \cos \gamma + X|\mathbf{V}_i||\mathbf{V}_j| \sin \gamma) \tag{2.7}$$

and

$$Q_{ij} = \frac{1}{R^2 + X^2} (X|\mathbf{V}_i|^2 - X|\mathbf{V}_i||\mathbf{V}_j| \cos \gamma - R|\mathbf{V}_i||\mathbf{V}_j| \sin \gamma) \tag{2.8}$$

Similarly,

$$P_{ji} = \frac{1}{R^2 + X^2} (R|\mathbf{V}_j|^2 - R|\mathbf{V}_i|\mathbf{V}_j| \cos \gamma - X|\mathbf{V}_i||\mathbf{V}_j| \sin \gamma) \tag{2.9}$$

and

$$Q_{ji} = \frac{1}{R^2 + X^2} (X|\mathbf{V}_j|^2 - X|\mathbf{V}_i|\mathbf{V}_j| \cos \gamma + R|\mathbf{V}_i||\mathbf{V}_j| \sin \gamma) \tag{2.10}$$

Of course, the three-phase real and reactive power can directly be found from equations (2.7)–(2.10) if the phase values are replaced by the line values.

In general, the reactance of a transmission line is much greater than its resistance. Therefore, the line impedance value can be approximated as

$$\mathbf{Z} = jX \tag{2.11}$$

by setting $R = 0$. Therefore, equations (2.7)–(2.10) can be expressed as

$$P_{ij} = \frac{|\mathbf{V}_i||\mathbf{V}_j|}{X} \sin \gamma \tag{2.12}$$

$$Q_{ij} = \frac{1}{X} (|\mathbf{V}_i|^2 - |\mathbf{V}_i||\mathbf{V}_j| \cos \gamma) \tag{2.13}$$

and

$$P_{ji} = -\frac{|\mathbf{V}_i||\mathbf{V}_j|}{X} \sin \gamma = -P_{ij} \tag{2.14}$$

$$Q_{ji} = \frac{1}{X} (|\mathbf{V}_j|^2 - |\mathbf{V}_i||\mathbf{V}_j| \cos \gamma) \tag{2.15}$$

Example 2.1

Assume that the impedance of a transmission line connecting buses 1 and 2 is 100 $\angle 60°$ Ω and that the bus voltages are 73,034.8 $\angle 30°$ and 66,395.3 $\angle 20°$ V per phase, respectively. Determine the following:

(a) Complex power per phase that is being transmitted from bus 1 to bus 2.
(b) Active power per phase that is being transmitted.
(c) Reactive power per phase that is being transmitted.

Solution

(a) $$\mathbf{S}_{12} = \mathbf{V}_1 \frac{\mathbf{V}_1^* - \mathbf{V}_2^*}{\mathbf{Z}^*}$$

$$= (73{,}034.8\,\underline{/30^\circ})\,\frac{73{,}034.8\,\underline{/-30^\circ} - 66{,}395.3\,\underline{/-20^\circ}}{100\underline{/-60^\circ}}$$

$$= 10{,}104{,}539.5\,\underline{/3.54^\circ}$$

$$= 10{,}085{,}259.8 + j623{,}908.4 \text{ VA}$$

(b) Therefore,

$$P_{12} = 10{,}085{,}259.8$$

(c) $Q_{12} = 623{,}908.4$ vars

2.3 ONE-LINE DIAGRAM

In general, electric power systems are represented by a one-line diagram, as shown in Figure 2.2(*a*). The one-line diagram is also referred to as the single-line diagram. Figure 2.2(*b*) shows the three-phase equivalent impedance diagram of the system given in Figure 2.2(*a*). However, the need for the three-phase equivalent impedance diagram is almost nil in usual situations. This is due to the fact that a balanced three-phase system can always be represented by an equivalent impedance diagram per phase, as shown in Figure 2.2(*c*). Furthermore, the per-phase equivalent impedance can also be simplified by neglecting the neutral line and representing the system components by standard symbols rather than by their equivalent circuits. The result is, of course, the one-line diagram shown in Figure 2.2(*a*). Table 2.1 gives some of the symbols that are used in one-line diagrams. Additional standard symbols can be found in Neuenswander [2]. At times, as a need arises, the one-line diagram may also show peripheral apparatus such as instrument transformers [i.e., current transformers (CTs) and voltage transformers (VTs)], protective relays, and lighting arrestors. Therefore, the details shown on a one-line diagram depend on its purpose. For example, the one-line diagrams that will be used in load flow studies do not show circuit breakers or relays, contrary to the ones that will be used in stability studies. Furthermore, the ones that will be used in unsymmetrical fault studies may even show the positive-, negative-, and zero-sequence networks separately.

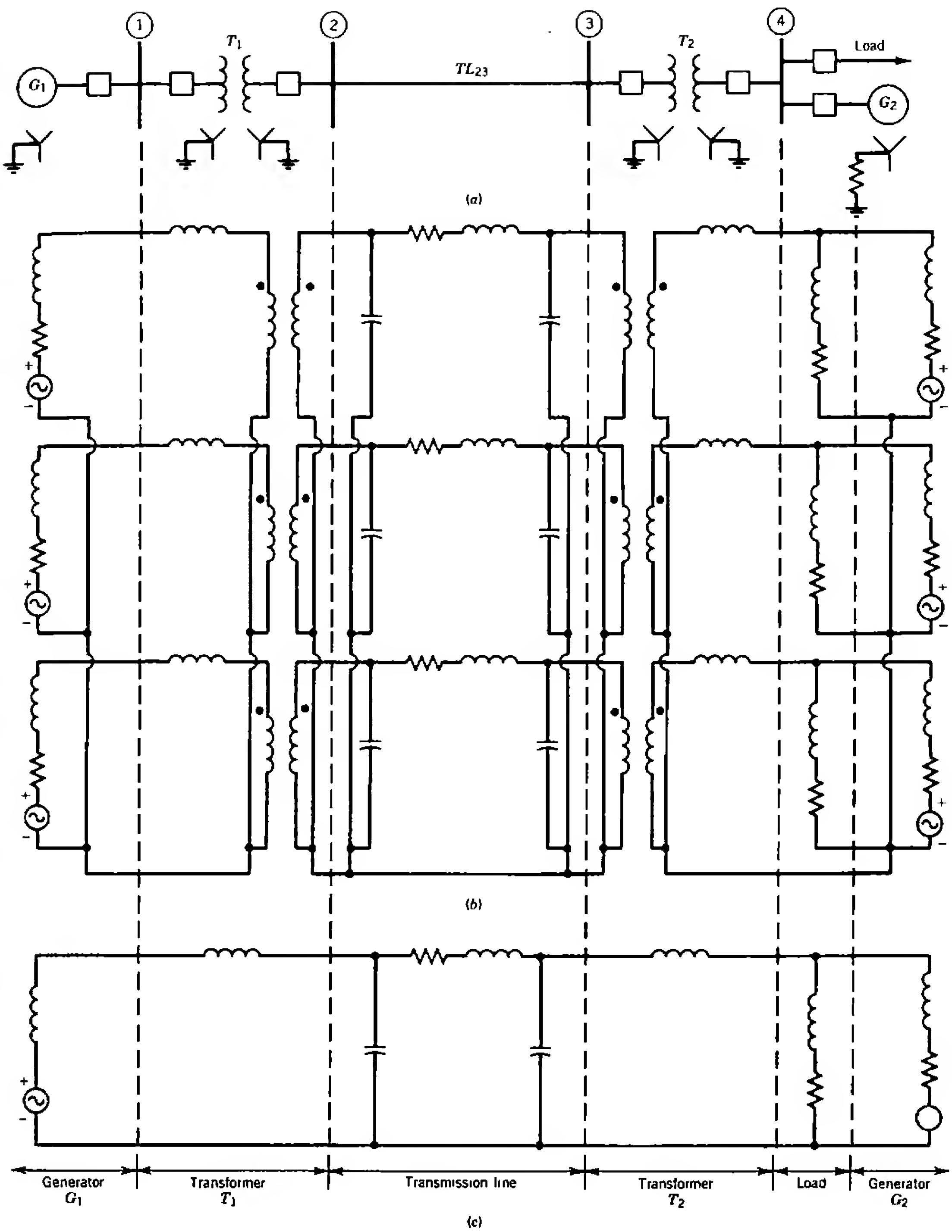
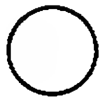
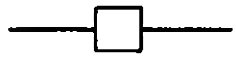
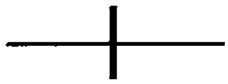
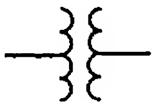
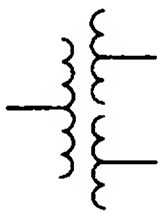
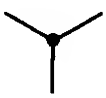
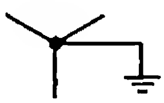
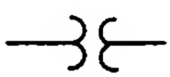
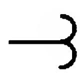
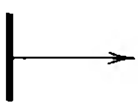
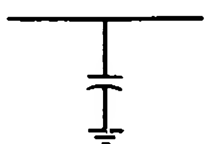


Figure 2.2. Power system representations: (*a*) one-line diagram; (*b*) three-phase equivalent impedance diagram; (*c*) equivalent impedance diagram per phase.

Note that the buses (i.e., the nodal points of the transmission network) that are shown in Figure 2.2(*a*) have been identified by their bus numbers. Also note that the neutral of generator 1 has been "solidly grounded," that is, the neutral point has been directly connected to the earth, whereas the neutral of generator 2 has been "grounded through impedance" using a resistor. Sometimes, it is grounded using an inductance coil. In either case,

TABLE 2.1 Symbols Used in One-Line Diagrams

Symbol	Usage	Symbol	Usage
	Rotating machine		Circuit breaker
	Bus		Circuit breaker (air)
	Two-winding transformer		Disconnect
	Three-winding transformer	 OR 	Fuse
	Delta connection (3Φ, three wire)		Fused disconnect
	Wye connection (3Φ, neutral ungrounded)		Lightning arrester
	Wye connection (3Φ, neutral grounded)		Current transformer (CT)
	Transmission line	 OR 	Voltage transformer (VT)
	Static load		Capacitor

they are used to limit the current flow to ground under fault conditions. Usually, the neutrals of the transformers used in transmission lines are solidly grounded. In general, a proper generator grounding for generators is facilitated by burying a ground electrode system made of grids of buried horizontal wires. Of course, as the number of meshes in the grid is increased, its conductance becomes greater. Sometimes, a metal plate is buried instead of a mesh grid. Transmission lines with overhead ground wires have a ground connection at each supporting structure to which the ground wire is connected. In some circumstances, a "counterpoise," that is, a bare conductor, is buried under a transmission line to decrease the ground resistance. The best known example is the one that has been installed for the transmission line crossing the Mohave Desert. The counterpoise is buried alongside the line and connected directly to the towers and the overhead ground wires.

Note that the equivalent circuit of the transmission line shown in Figure 2.2(*c*) has been represented by a nominal Π. The line impedance, in terms of the resistance and the series reactance of a single conductor for the length of the line, has been lumped. The line-to-neutral capacitance (or shunt capacitive reactance) for the length of the line has been calculated, and half of this value has been put at each end of the line. The transformers have been represented by their equivalent reactances, neglecting their magnetizing currents and consequently their shunt admittances. Also neglected are the resistance values of the transformers and generators due to the fact that their inductive reactance values are much greater than their resistance values. Also not shown in Figure 2.2(*c*) is the ground resistor. This is due to no current flowing in the neutral under balanced conditions. The impedance diagram shown in Figure 2.2(*c*) is also referred to as the positive-sequence network or diagram. The reason is that the phase order of the balanced voltages at any point in the system is the same as the phase order of the generated voltage, and they are positive. The per-phase impedance diagrams may represent a system given either in ohms or in per units.

2.4 PER-UNIT SYSTEM

Because of various advantages involved, it is customary in power system analysis calculations to use impedances, currents, voltages, and powers in per-unit values (which are scaled or normalized values) rather than in physical values of ohms, amperes, kilovolts, and megavoltamperes (or megavars, or megawatts). A per-unit system is a means of expressing quantities for ease in comparing them. The per-unit value of any quantity is defined as the ratio of the quantity to an "arbitrarily" chosen base (i.e., reference) value having the same dimensions. Therefore, the per-unit value of any quantity can be defined as

$$\text{Quantity in per unit} = \frac{\text{physical quantity}}{\text{base value of quantity}} \tag{2.16}$$

where "physical quantity" refers to the given value in ohms, amperes, volts, etc. The base value is also called unit value since in the per-unit system it has a value of 1, or unity. Therefore, a base current is also referred to as a unit current. Since both the physical quantity and base quantity have the same dimensions, the resulting per-unit value expressed as a decimal has no dimension and therefore is simply indicated by a subscript pu. The base quantity is indicated by a subscript B. The symbol for per unit is pu, or 0/1. The percent system is obtained by multiplying the per-unit value by 100. Therefore,

$$\text{Quantity in percent} = \frac{\text{physical quantity}}{\text{base value of quantity}} \times 100 \tag{2.17}$$

However, the percent system is somewhat more difficult to work with and more subject to possible error since it must always be remembered that the quantities have been multiplied by 100. Therefore, the factor 100 has to be continually inserted or removed for reasons that may not be obvious at the time. For example, 40 percent reactance times 100 percent current is equal to 4000 percent voltage, which, of course, must be corrected to 40 percent voltage. Thus, the per-unit system is preferred in power system calculations. The advantages of using the per-unit include the following:

1. Network analysis is greatly simplified since all impedances of a given equivalent circuit can directly be added together regardless of the system voltages.
2. It eliminates the $\sqrt{3}$ multiplications and divisions that are required when balanced three-phase systems are represented by per-phase systems. Therefore, the factors $\sqrt{3}$ and 3 associated with delta and wye quantities in a balanced three-phase system are directly taken into account by the base quantities.
3. Usually, the impedance of an electrical apparatus is given in percent or per unit by its manufacturer based on its nameplate ratings (e.g., its rated voltamperes and rated voltage).
4. Differences in operating characteristics of many electrical apparatus can be estimated by a comparison of their constants expressed in per units.
5. Average machine constants can easily be obtained since the parameters of similar equipment tend to fall in a relatively narrow range and therefore are comparable when expressed as per units based on rated capacity.
6. The use of per-unit quantities is more convenient in calculations involving digital computers.

2.4.1 Single-phase System

In the event that any two of the four base quantities (i.e., base voltage, base current, base voltamperes, and base impedance) are "arbitrarily" specified, the other two can be determined immediately. Here, the term *arbitrarily* is slightly misleading since in practice the base values are selected so as to force the results to fall into specified ranges. For example, the base voltage is selected such that the system voltage is normally close to unity. Similarly, the base voltampere is usually selected as the kilovoltampere or megavoltampere rating of one of the machines or transformers in the system, or a convenient round number such as 1, 10, 100, or 1000 MVA, depending on system size. As aforementioned, on determining the base voltamperes and base voltages, the other base values are fixed. For example, current base can be determined as

$$I_B = \frac{S_B}{V_B} = \frac{VA_B}{V_B} \quad \text{A} \tag{2.18}$$

where I_B = current base in amperes
S_B = selected voltampere base in voltamperes
V_B = selected voltage base in volts

Note that

$$S_B = VA_B = P_B = Q_B = V_B I_B \tag{2.19}$$

Similarly, the impedance base[†] can be determined as

$$Z_B = \frac{V_B}{I_B} \quad \Omega \tag{2.20}$$

where

$$Z_B = X_B = R_B \tag{2.21}$$

Similarly,

$$Y_B = B_B = G_B = \frac{I_B}{V_B} \quad \text{S} \tag{2.22}$$

[†] It is defined as that impedance across which there is a voltage drop that is equal to the base voltage if the current through it is equal to the base current.

Note that by substituting equation (2.18) into equation (2.20), the impedance base can be expressed as

$$Z_B = \frac{V_B}{VA_B / V_B}$$

$$= \frac{V_B^2}{VA_B} \quad \Omega \tag{2.23}$$

or

$$Z_B = \frac{(kV_B)^2}{MVA_B} \quad \Omega \tag{2.24}$$

where kV_B = voltage base in kilovolts
MVA_B = voltampere base in megavoltamperes

Therefore, the per-unit value of any quantity can be found by the normalization process, that is, by dividing the physical quantity by the base quantity of the same dimension. For example, the per-unit impedance can be expressed as

$$Z_{pu} = \frac{Z_{physical}}{Z_B} \quad \text{pu} \tag{2.25}$$

or

$$Z_{pu} = \frac{Z_{physical}}{V_B^2/(kVA_B \times 1000)} \quad \text{pu} \tag{2.26}$$

or

$$Z_{pu} = \frac{(Z_{physical})(kVA_B)(1000)}{V_B^2} \quad \text{pu} \tag{2.27}$$

or

$$Z_{pu} = \frac{(Z_{physical})(kVA_B)}{(kV_B)^2(1000)} \quad \text{pu} \tag{2.28}$$

or

$$Z_{pu} = \frac{Z_{physical}}{(kV_B)^2/MVA_B} \quad \text{pu} \tag{2.29}$$

or

$$Z_{pu} = \frac{(Z_{physical})(MVA_B)}{(kV_B)^2} \quad \text{pu} \tag{2.30}$$

Similarly, the others can be expressed as

$$I_{pu} = \frac{I_{\text{physical}}}{I_B} \quad \text{pu} \tag{2.31}$$

$$V_{pu} = \frac{V_{\text{physical}}}{V_B} \quad \text{pu} \tag{2.32}$$

or

$$kV_{pu} = \frac{kV_{\text{physical}}}{kV_B} \quad \text{pu} \tag{2.33}$$

$$VA_{pu} = \frac{VA_{\text{physical}}}{VA_B} \quad \text{pu} \tag{2.34}$$

or

$$kVA_{pu} = \frac{kVA_{\text{physical}}}{kVA_B} \quad \text{pu} \tag{2.35}$$

or

$$MVA_{pu} = \frac{MVA_{\text{physical}}}{MVA_B} \quad \text{pu} \tag{2.36}$$

Note that the base quantity is always a real number, whereas the physical quantity can be a complex number. For example, if the actual impedance quantity is given as $Z \underline{/\theta}$ Ω, it can be expressed in the per-unit system as *

$$\mathbf{Z}_{pu} \underline{/\theta} = \frac{\mathbf{Z} \underline{/\theta}}{Z_B} \quad \text{pu} \tag{2.37}$$

that is, it is the magnitude expressed in per-unit terms. Alternatively, if the impedance has been given in rectangular form as

$$\mathbf{Z} = R + jX \quad \Omega \tag{2.38}$$

then

$$\mathbf{Z}_{pu} = R_{pu} + jX_{pu} \quad \text{pu} \tag{2.39}$$

where

$$R_{pu} = \frac{R_{\text{physical}}}{Z_B} \quad \text{pu} \tag{2.40}$$

and

$$X_{\text{pu}} = \frac{X_{\text{physical}}}{Z_B} \quad \text{pu} \tag{2.41}$$

Similarly, if the complex power has been given as

$$\mathbf{S} = P + jQ \quad \text{VA} \tag{2.42}$$

then

$$\mathbf{S}_{\text{pu}} = P_{\text{pu}} + jQ_{\text{pu}} \quad \text{pu} \tag{2.43}$$

where

$$P_{\text{pu}} = \frac{P_{\text{physical}}}{S_B} \quad \text{pu} \tag{2.44}$$

and

$$Q_{\text{pu}} = \frac{Q_{\text{physical}}}{S_B} \quad \text{pu} \tag{2.45}$$

Of course, in the event that the actual voltage and current values are given as

$$\mathbf{V} = V \angle \theta_V \quad \text{V} \tag{2.46}$$

and

$$\mathbf{I} = I \angle \theta_I \quad \text{A} \tag{2.47}$$

the complex power can be expressed as

$$\mathbf{S} = \mathbf{V}\mathbf{I}^* \quad \text{VA} \tag{2.48}$$

or

$$S \angle \Phi = (V \angle \theta_V)(I \angle -\theta_I) \quad \text{VA} \tag{2.49}$$

Therefore, dividing through by S_B,

$$\frac{S \angle \Phi}{S_B} = \frac{(V \angle \theta_V)(I \angle -\theta_I)}{S_B} \quad \text{pu} \tag{2.50}$$

However,

$$S_B = V_B I_B \tag{2.51}$$

Thus,

$$\frac{S\,\angle\Phi}{S_B} = \frac{(V\,\angle\theta_V)(I\,\angle-\theta_I)}{V_B I_B} \quad \text{pu} \tag{2.52}$$

or

$$S_{\text{pu}}\,\angle\Phi = (V_{\text{pu}}\,\angle\theta_V)(I_{\text{pu}}\,\angle-\theta_I) \quad \text{pu} \tag{2.53}$$

or

$$\mathbf{S}_{\text{pu}} = \mathbf{V}_{\text{pu}}\mathbf{I}^*_{\text{pu}} \quad \text{pu} \tag{2.54}$$

2.4.2 Converting from Per-Unit Values to Physical Values

The physical values (or system values) and per-unit values are related by the following relationships:

$$\mathbf{I} = \mathbf{I}_{\text{pu}} I_B \quad \text{A} \tag{2.55}$$

$$\mathbf{V} = \mathbf{V}_{\text{pu}} V_B \quad \text{V} \tag{2.56}$$

$$\mathbf{Z} = \mathbf{Z}_{\text{pu}} Z_B \quad \Omega \tag{2.57}$$

$$R = R_{\text{pu}} Z_B \quad \Omega \tag{2.58}$$

$$X = X_{\text{pu}} Z_B \quad \Omega \tag{2.59}$$

$$VA = VA_{\text{pu}} VA_B \quad \text{VA} \tag{2.60}$$

$$P = P_{\text{pu}} VA_B \quad \text{W} \tag{2.61}$$

$$Q = Q_{\text{pu}} VA_B \quad \text{var} \tag{2.62}$$

2.4.3 Change of Base

In general, the per-unit impedance of a power apparatus is given based on its own voltampere and voltage ratings and consequently based on its own impedance base. When such an apparatus is used in a system that has its own bases, it becomes necessary to refer all the given per-unit values to the system base values. Assume that the per-unit impedance of the apparatus is given based on its nameplate ratings as

$$Z_{\text{pu(given)}} = (Z_{\text{physical}}) \frac{MVA_{B(\text{given})}}{[kV_{B(\text{given})}]^2} \quad \Leftarrow \frac{Z_{phy}}{\frac{(kV_{Bg})^2}{(MVA_{Bgiven})}} \tag{2.63}$$

and that it is necessary to refer the very same physical impedance to a new set of voltage and voltampere bases such that

$$Z_{\text{pu(new)}} = (Z_{\text{physical}}) \frac{MVA_{B(\text{new})}}{[kV_{B(\text{new})}]^2} \tag{2.64}$$

By dividing equation (2.63) by equation (2.64) side by side,

$$Z_{\text{pu(new)}} = Z_{\text{pu(given)}} \left[\frac{MVA_{B(\text{new})}}{MVA_{B(\text{given})}} \right] \left[\frac{kV_{B(\text{given})}}{kV_{B(\text{new})}} \right]^2 \quad \text{pu} \tag{2.65}$$

In certain situations it is more convenient to use subscripts 1 and 2 instead of subscripts "given" and "new", respectively. Then equation (2.65) can be expressed as

$$Z_{\text{pu}(2)} = Z_{\text{pu}(1)} \left[\frac{MVA_{B(2)}}{MVA_{B(1)}} \right] \left[\frac{kV_{B(1)}}{kV_{B(2)}} \right]^2 \quad \text{pu} \tag{2.66}$$

In the event that the kilovoltampere bases are the same but the megavoltampere bases are different, from equation (2.65),

$$Z_{\text{pu(new)}} = Z_{\text{pu(given)}} \frac{MVA_{B(\text{new})}}{MVA_{B(\text{given})}} \quad \text{pu} \tag{2.67}$$

Similarly, if the megavoltampere bases are the same but the kilovolt bases are different, from equation (2.65),

$$Z_{\text{pu(new)}} = Z_{\text{pu(given)}} \left[\frac{kV_{B(\text{given})}}{kV_{B(\text{new})}} \right]^2 \quad \text{pu} \tag{2.68}$$

Of course, equations (2.65)–(2.68) must only be used to convert the given per-unit impedance from the base to another but not for referring the physical value of an impedance from one side of the transformer to another [3].

2.4.4 Three-Phase Systems

The three-phase problems involving balanced systems can be solved on a per-phase basis. In that case, the equations that are developed for single-phase systems can be used for three-phase systems as long as per-phase values are used consistently. Therefore,

$$I_B = \frac{S_{B(1\Phi)}}{V_{B(\text{L-N})}} \quad \text{A} \tag{2.69}$$

or

$$I_B = \frac{VA_{B(1\Phi)}}{V_{B(\text{L-N})}} \quad \text{A} \tag{2.70}$$

and

$$Z_B = \frac{V_{B(\text{L-N})}}{I_B} \quad \Omega \tag{2.71}$$

or

$$Z_B = \frac{[kV_{B(\text{L-N})}]^2(1000)}{kVA_{B(1\Phi)}} \quad \Omega \tag{2.72}$$

or

$$Z_B = \frac{[kV_{B(\text{L-N})}]^2}{MVA_{B(1\Phi)}} \quad \Omega \tag{2.73}$$

where the subscripts 1Φ and L-N denote per phase and line to neutral, respectively. Note that, for a balanced system,

$$V_{B(\text{L-N})} = \frac{V_{B(\text{L-L})}}{\sqrt{3}} \quad \text{V} \tag{2.74}$$

and

$$S_{B(1\Phi)} = \frac{S_{B(3\Phi)}}{3} \quad \text{VA} \tag{2.75}$$

However, it has been customary in three-phase system analysis to use line-to-line voltage and three-phase voltamperes as the base values. Therefore,

$$I_B = \frac{S_{B(3\Phi)}}{\sqrt{3}V_{B(\text{L-L})}} \quad \text{A} \tag{2.76}$$

or

$$I_B = \frac{kVA_{B(3\Phi)}}{\sqrt{3}kV_{B(\text{L-L})}} \quad \text{A} \tag{2.77}$$

and

$$Z_B = \frac{V_{B(\text{L-L})}}{I_B} \quad \Omega \tag{2.78}$$

or

$$Z_B = \frac{[kV_{B(\text{L-L})}]^2(1000)}{kVA_{B(3\Phi)}} \quad \Omega \tag{2.79}$$

or

$$Z_B = \frac{[kV_{B(\text{L-L})}]^2}{MVA_{B(3\Phi)}} \quad \Omega \tag{2.80}$$

where the subscripts 3Φ and L-L denote per three phase and line to line, respectively. Furthermore, base admittance can be expressed as

$$Y_B = \frac{1}{Z_B} \quad \text{S} \tag{2.81}$$

or

$$Y_B = \frac{MVA_{B(3\Phi)}}{[kV_{B(\text{L-L})}]^2} \quad \text{S} \tag{2.82}$$

where

$$Y_B = B_B = G_B \quad \text{S} \tag{2.83}$$

The data for transmission lines are usually given in terms of the line resistance R in ohms per mile at a given temperature, the line inductive reactance X_L in ohms per mile at 60 Hz, and the line shunt capacitive reactance X_c in megohms per mile at 60 Hz. Therefore, the line impedance and shunt susceptance in per units for 1 mi of line can be expressed as[†]

$$\mathbf{Z}_{\text{pu}} = (Z, \Omega/\text{mi}) \frac{MVA_{B(3\Phi)}}{[kV_{B(\text{L-L})}]^2} \quad \text{pu} \tag{2.84}$$

where

$$\mathbf{Z} = R + jX_L = Z \underline{/\theta} \quad \Omega/\text{mi}$$

and

$$B_{\text{pu}} = \frac{[kV_{B(\text{L-L})}]^2(10^{-6})}{[MVA_{B(3\Phi)}](X_c, \text{M}\Omega/\text{mi})} \quad \text{pu} \tag{2.85}$$

[†] For further information see Anderson [4].

In the event that the admittance for a transmission line is given in microsiemens per mile, the per-unit admittance can be expressed as

$$Y_{\text{pu}} = \frac{[kV_{B(\text{L-L})}]^2(Y, \mu\text{S})}{[MVA_{B(3\Phi)}](10^6)} \quad \text{pu} \tag{2.86}$$

Similarly, if it is given as reciprocal admittance in megohms per mile, the per-unit admittance can be found as

$$Y_{\text{pu}} = \frac{[kV_{B(\text{L-L})}]^2(10^{-6})}{[MVA_{B(3\Phi)}](Z, \text{M}\Omega)} \quad \text{pu} \tag{2.87}$$

Figure 2.3 shows conventional three-phase transformer connections and associated relationships between the high-voltage and low-voltage side voltages and currents. Of course, the given relationships are correct for a three-phase transformer as well as for a three-phase bank of single-phase transformers. Note that in the figure, n is the turns ratio, that is,

$$n = \frac{N_1}{N_2} = \frac{V_1}{V_2} = \frac{I_2}{I_1} \tag{2.88}$$

where the subscripts 1 and 2 are used for the primary and secondary sides. Therefore, an impedance Z_2 in the secondary circuit can be referred to the primary circuit provided that

$$Z_1 = n^2 Z_2 \tag{2.89}$$

Thus, it can be observed from Figure 2.3 that in an ideal transformer, voltages are transformed in the direct ratio of turns, currents in the inverse ratio, and impedances in the direct ratio squared; and power and voltamperes are, of course, unchanged Note that a balanced delta-connected circuit of Z_Δ Ω/phase is equivalent to a balanced wye-connected circuit of Z_Y Ω/phase as long as

$$Z_\text{Y} = \tfrac{1}{3} Z_\Delta \tag{2.90}$$

Note that the per-unit impedance of a transformer remains the same without taking into account whether it is converted from physical impedance values that are found by referring to the high-voltage side or low-voltage side of the transformer. This can be accomplished by choosing separate appropriate bases for each side of the transformer (whether or not the transformer is connected in wye–wye, delta–delta, delta–wye, or wye–delta since the transformation of voltages is the same as that made by wye–wye transfor-

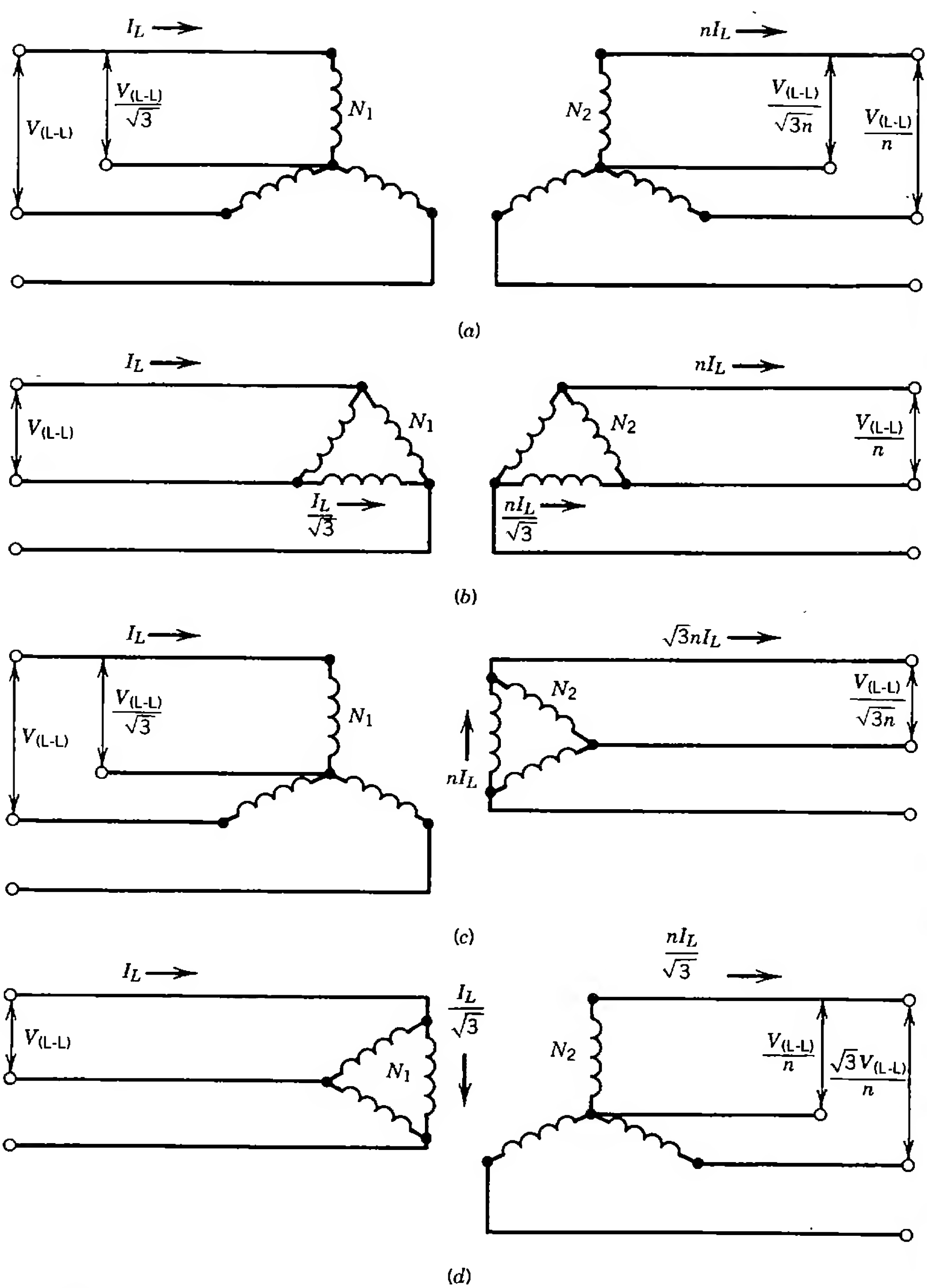


Figure 2.3. Conventional three-phase transformer connections: (*a*) wye–wye connection; (*b*) delta–delta connection; (*c*) wye–delta connection; (*d*) delta–wye connection.

mers as long as the same line-to-line voltage ratings are employed).† In other words, the designated per-unit impedance values of transformers are based on the coil ratings. Since the ratings of coils cannot alter by a simple change in connection (e.g., from wye–wye to delta–wye), the per-unit impedance remains the same regardless of the three-phase connection. Of course, the line-to-line voltage for the transformer will differ. Because of the method of choosing the base in various sections of the three-phase system, the per-unit impedances calculated in various sections can be put together on one impedance diagram without paying any attention to whether the transformers are connected in wye–wye or delta–wye.

EXAMPLE 2.2

Assume that a three-phase transformer has a nameplate ratings of 20 MVA, 345Y–34.5Y kV with a leakage reactance of 12 percent and that the transformer connection is wye–wye. Select a base of 20 MVA and 345 kV on the high-voltage side and determine the following:

(a) Reactance of transformer in per units.
(b) High-voltage side base impedance.
(c) Low-voltage side base impedance.
(d) Transformer reactance referred to high-voltage side in ohms.
(e) Transformer reactance referred to low-voltage side in ohms.

Solution

(a) The reactance of the transformer in per units is 12/100, or 0.12 pu. Note that it is the same whether it is referred to the high-voltage or the low-voltage sides.

(b) The high-voltage side base impedance is

$$Z_{B(\mathrm{HV})} = \frac{[kV_{B(\mathrm{HV})}]^2}{MVA_{B(3\Phi)}}$$

$$= \frac{345^2}{20} = 5951.25\ \Omega$$

(c) The low-voltage side base impedance is

$$Z_{B(\mathrm{LV})} = \frac{[kV_{B(\mathrm{LV})}]^2}{MVA_{B(3\Phi)}}$$

$$= \frac{34.5^2}{20} = 59.5125\ \Omega$$

† This subject has been explained in greater depth in an excellent review by Stevenson [3].

(d) The reactance referred to the high-voltage side is

$$X_{(\mathrm{HV})} = X_{\mathrm{pu}} \times X_{B(\mathrm{HV})}$$
$$= (0.12)(5951.25) = 714.15\ \Omega$$

(e) The reactance referred to the low-voltage side is

$$X_{(\mathrm{LV})} = X_{\mathrm{pu}} \times X_{B(\mathrm{LV})}$$
$$= (0.12)(59.5125) = 7.1415\ \Omega$$

or, from equation (2.233),

$$X_{(\mathrm{LV})} = \frac{X_{(\mathrm{HV})}}{n^2}$$
$$= \frac{714.15\ \Omega}{\left(\dfrac{345/\sqrt{3}}{34.5/\sqrt{3}}\right)^2} = 7.1415\ \Omega$$

where n is defined as the turns ratio of the windings.

EXAMPLE 2.3

Consider Example 2.2 and assume that the voltage ratings are 345Y–34.5Δ kV and that the transformer connection is wye–delta. Determine the following:

(a) Turns ratio of windings.
(b) Transformer reactance referred to low-voltage side in ohms.
(c) Transformer reactance referred to low-voltage side in per units.

Solution

(a) The turns ratio of the windings is

$$n = \frac{345/\sqrt{3}}{34.5} = 5.7735$$

(b) The transformer reactance referred to the delta-connected low-voltage side is

$$X_{(\mathrm{LV})} = \frac{X_{(\mathrm{HV})}}{n^2}$$
$$= \frac{714.15\ \Omega}{(5.7735)^2} = 21.4245\ \Omega$$

(c) From equation (2.90), the reactance of the equivalent wye connection is

$$Z_{\mathrm{Y}} = \frac{Z_{\Delta}}{3}$$
$$= \frac{21.4245\ \Omega}{2} = 7.1415\ \Omega$$

Therefore,

$$X_{\text{pu}} = \frac{7.1415\ \Omega}{Z_{B(\text{LV})}}$$

$$= \frac{7.1415\ \Omega}{59.5125\ \Omega} = 0.12\ \text{pu}$$

Alternatively, if the line-to-line voltages are used,

$$X_{(\text{LV})} = \frac{X_{(\text{HV})}}{n^2}$$

$$= \frac{714.15\ \Omega}{(345/34.5)^2} = 7.1415\ \Omega$$

and therefore,

$$X_{\text{pu}} = \frac{X_{(\text{LV})}}{Z_{B(\text{LV})}}$$

$$= \frac{7.1415\ \Omega}{59.5125\ \text{pu}} = 0.12\ \text{pu}$$

as before.

EXAMPLE 2.4

Figure 2.4 shows a one-line diagram of a three-phase system. Assume that the line length between the two transformers is negligible and the three-phase generator is rated 4160 kVA, 2.4 kV, and 1000 A and that it supplies a purely inductive load of $I_{\text{pu}} = 2.08\,\underline{/-90^\circ}$ pu. The three-phase transformer T_1 is rated 6000 kVA, 2.4Y–24Y kV, with leakage reactance of 0.04 pu. Transformer T_2 is made up of three single-phase transformers and is rated 4000 kVA, 24Y–12Y kV, with leakage reactance of 0.04 pu. Determine the following for all three circuits, 2.4-, 24-, and 12-kV circuits:

(a) Base kilovoltampere values.
(b) Base line-to-line kilovolt values.
(c) Base impedance values.

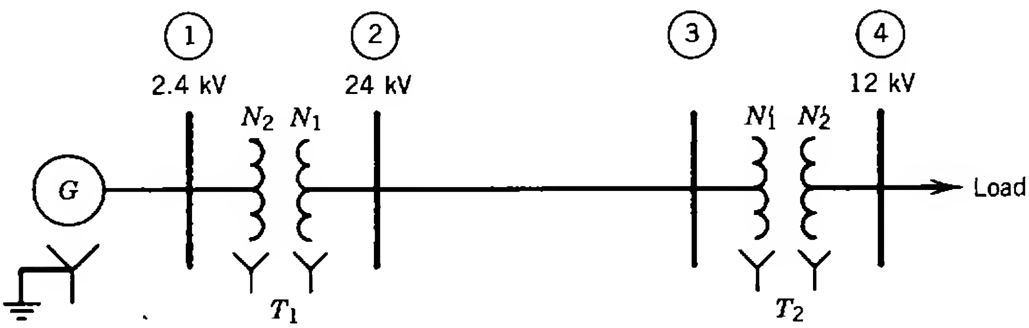


Figure 2.4

(d) Base current values.

(e) Physical current values (neglect magnetizing currents in transformers and charging currents in lines).

(f) Per-unit current values.

(g) New transformer reactances based on their new bases.

(h) Per-unit voltage values at buses 1, 2, and 4.

(i) Per-unit apparent power values at buses 1, 2, and 4.

(j) Summarize results in a table.

Solution

(a) The kilovoltampere base for all three circuits is arbitrarily selected as 2080 kVA

(b) The base voltage for the 2.4-kV circuit is arbitrarily selected as 2.5 kV. Since the turns ratios for transformers T_1 and T_2 are

$$\frac{N_1}{N_2} = 10 \quad \text{or} \quad \frac{N_2}{N_1} = 0.10$$

and

$$\frac{N_1'}{N_2'} = 2$$

the base voltages for the 24- and 12-kV circuits are determined to be 25 and 12.5 kV, respectively.

(c) The base impedance values can be found as

$$Z_B = \frac{[kV_{B(\text{L-L})}]^2(1000)}{kVA_{B(3\Phi)}}$$

$$= \frac{[2.5 \text{ kV}]^2(1000)}{2080 \text{ kVA}} = 3.005 \ \Omega$$

and

$$Z_B = \frac{[25 \text{ kV}]^2(1000)}{2080 \text{ kVA}} = 300.5 \ \Omega$$

and

$$Z_B = \frac{[12.5 \text{ kV}]^2(1000)}{2080 \text{ kVA}} = 75.1 \ \Omega$$

(d) The base current values can be determined as

$$I_B = \frac{kVA_{B(3\Phi)}}{\sqrt{3} kV_{B(\text{L-L})}}$$

$$= \frac{2080 \text{ kVA}}{\sqrt{3}(2.5 \text{ kV})} = 480 \text{ A}$$

and

$$I_B = \frac{2080\text{ kVA}}{\sqrt{3}(25\text{ kV})} = 48\text{ A}$$

and

$$I_B = \frac{2080\text{ kVA}}{\sqrt{3}(12.5\text{ kV})} = 96\text{ A}$$

(e) The physical current values can be found based on the turns ratios as

$$I = 1000\text{ A}$$

$$I = \left(\frac{N_2}{N_1}\right)(1000\text{ A}) = 100\text{ A}$$

$$I = \left(\frac{N_1'}{N_2'}\right)(100\text{ A}) = 200\text{ A}$$

(f) The per-unit current values are the same, 2.08 pu, for all three circuits.

(g) The given transformer reactances can be converted based on their new bases using

$$Z_{\text{pu(new)}} = Z_{\text{pu(given)}}\left[\frac{kVA_{B(\text{new})}}{kVA_{B(\text{given})}}\right]\left[\frac{kV_{B(\text{given})}}{kV_{B(\text{new})}}\right]^2$$

Therefore, the new reactances of transformers T_1 and T_2 can be found as

$$\mathbf{Z}_{\text{pu}(T_1)} = j0.04\left[\frac{2080\text{ kVA}}{6000\text{ kVA}}\right]\left[\frac{2.4\text{ kV}}{2.5\text{ kV}}\right]^2 = j0.0128\text{ pu}$$

and

$$\mathbf{Z}_{\text{pu}(T_2)} = j0.04\left[\frac{2080\text{ kVA}}{4000\text{ kVA}}\right]\left[\frac{12\text{ kV}}{12.5\text{ kV}}\right]^2 = j0.0192\text{ pu}$$

(h) Therefore, the per-unit voltage values at buses 1, 2, and 4 can be calculated as

$$\mathbf{V}_1 = \frac{2.4\text{ kV}\,\underline{/0^\circ}}{2.5\text{ kV}} = 0.96\,\underline{/0^\circ}\text{ pu}$$

$$\mathbf{V}_2 = \mathbf{V}_1 - \mathbf{I}_{\text{pu}}\mathbf{Z}_{\text{pu}(T_1)}$$

$$= 0.96\,\underline{/0^\circ} - (2.08\,\underline{/-90^\circ})(0.0128\,\underline{/90^\circ}) = 0.9334\,\underline{/0^\circ}\text{ pu}$$

$$\mathbf{V}_4 = \mathbf{V}_2 - \mathbf{I}_{\text{pu}}\mathbf{Z}_{\text{pu}(T_2)} = \mathbf{V}_1 - \mathbf{I}_{\text{pu}}[\mathbf{Z}_{\text{pu}(T_1)} + \mathbf{Z}_{\text{pu}(T_2)}]$$

$$= 0.9334\,\underline{/0^\circ} - (2.08\,\underline{/-90^\circ})(0.0192\,\underline{/90^\circ}) = 0.8935\,\underline{/0^\circ}\text{ pu}$$

(i) Thus, the per-unit apparent power values at buses 1, 2, and 4 are

Table 2.2 Results of Example 2.4

Quantity	2.4-kV Circuit	24-kV Circuit	12-kV Circuit
$kVA_{B(3\Phi)}$	2080 kVA	2080 kVA	2080 kVA
$kV_{B(L-L)}$	2.5 kV	25 kV	12.5 kV
Z_B	3005 Ω	300.5 Ω	75.1 Ω
I_B	480 A	48 A	96 A
I_{physical}	1000 A	100 A	200 A
I_{pu}	2.08 pu	2.08 pu	2.08 pu
V_{pu}	0.96 pu	0.9334 pu	0.8935 pu
S_{pu}	2.00 pu	1.9415 pu	1.8585 pu

$$S_1 = 2.00 \text{ pu}$$

$$S_2 = V_2 I_{\text{pu}} = (0.9334)(2.08) = 1.9415 \text{ pu}$$

$$S_4 = V_4 I_{\text{pu}} = (0.8935)(2.08) = 1.8585 \text{ pu}$$

(j) The results are summarized in Table 2.2.

2.5 CONSTANT-IMPEDANCE REPRESENTATION OF LOADS

Usually, the power system loads are represented by their real and reactive powers, as shown in Figure 2.5(*a*). However, it is possible to represent the same load in terms of series or parallel combinations of its equivalent constant-load resistance and reactance values, as shown in Figures 2.5(*b*) and 2.5(*c*), respectively [4].

In the event that the load is represented by the series connection, the equivalent constant impedance can be expressed as

$$\mathbf{Z}_s = R_s + jX_s \quad \Omega \tag{2.91}$$

where

$$R_s = \frac{|V|^2 \cdot P}{P^2 + Q^2} \quad \Omega \tag{2.92}$$

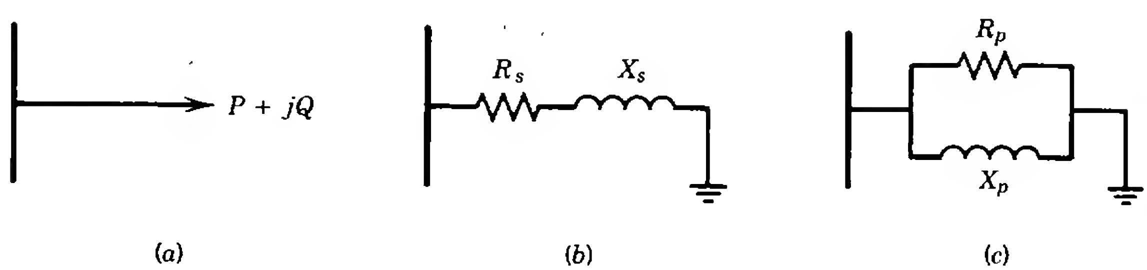


Figure 2.5. Load representations as: (*a*) real and reactive powers; (*b*) constant impedance in terms of series combination; (*c*) constant impedance in terms of parallel combination.

$$X_s = \frac{|V|^2 \cdot Q}{P^2 + Q^2} \quad \Omega \tag{2.93}$$

where R_s = load resistance in series connection in ohms
X_s = load reactance in series connection in ohms
$\mathbf{Z}_s$ = constant-load impedance in ohms
V = load voltage in volts
P = real, or average, load power in watts
Q = reactive load power in vars

The constant impedance in per units can be expressed as

$$\mathbf{Z}_{\text{pu}(s)} = R_{\text{pu}(s)} + jX_{\text{pu}(s)} \quad \text{pu} \tag{2.94}$$

where

$$R_{\text{pu}(s)} = (P_{\text{physical}}) \frac{S_B(V_{\text{pu}})^2}{P^2 + Q^2} \quad \text{pu} \tag{2.95}$$

$$X_{\text{pu}(s)} = (Q_{\text{physical}}) \frac{S_B(V_{\text{pu}})^2}{P^2 + Q^2} \quad \text{pu} \tag{2.96}$$

In the event that the load is represented by the parallel connection, the equivalent constant impedance can be expressed as

$$\mathbf{Z}_p = j \frac{R_p \cdot X_p}{R_p + jX_p} \quad \Omega \tag{2.97}$$

where

$$R_p = \frac{V^2}{P} \quad \Omega$$

$$X_p = \frac{V^2}{Q} \quad \Omega$$

where R_p = load resistance in parallel connection in ohms
X_p = load reactance in parallel connection in ohms
$\mathbf{Z}_p$ = constant-load impedance in ohms

The constant impedance in per units can be expressed as

$$\mathbf{Z}_{\text{pu}(p)} = j \frac{R_{\text{pu}(p)} \cdot X_{\text{pu}(p)}}{R_{\text{pu}(p)} + jX_{\text{pu}(p)}} \quad \text{pu} \tag{2.98}$$

where

$$R_{\text{pu}(p)} = \frac{S_B}{P}\left(\frac{V}{V_B}\right)^2 \quad \text{pu} \tag{2.99}$$

or

$$R_{\text{pu}(p)} = \frac{V_{\text{pu}}^2}{P_{\text{pu}}} \quad \text{pu} \tag{2.100}$$

and

$$X_{\text{pu}(p)} = \frac{S_B}{Q}\left(\frac{V}{V_B}\right)^2 \quad \text{pu} \tag{2.101}$$

or

$$X_{\text{pu}(p)} = \frac{V_{\text{pu}}^2}{Q_{\text{pu}}} \quad \text{pu} \tag{2.102}$$

2.6 THREE-WINDING TRANSFORMERS

Figure 2.6(a) shows a single-phase three-winding transformer. They are usually used in the bulk power (transmission) substations to reduce the transmission voltage to the subtransmission voltage level. If excitation impedance is neglected, the equivalent circuit of a three-winding transformer can be represented by a wye of impedances, as shown in Figure 2.6(b), where the primary, secondary, and tertiary windings are denoted by P, S, and T, respectively. Note that the common point 0 is fictitious and is not related to the neutral of the system. The tertiary windings of a three-phase and three-winding transformer bank is usually connected in delta and may be used for (1) providing a path for zero-sequence currents, (2) in-plant power distribution, and (3) application of power-factor-correcting capacitors or reactors. The impedance of any of the branches shown in Figure 2.6(b) can be determined by considering the short-circuit impedance between pairs of windings with the third open. Therefore,

$$Z_{PS} = Z_P + Z_S \tag{2.103a}$$

$$Z_{TS} = Z_T + Z_S \tag{2.103b}$$

$$Z_{PT} = Z_P + Z_T \tag{2.103c}$$

$$Z_P = \tfrac{1}{2}(Z_{PS} + Z_{PT} - Z_{TS}) \tag{2.104a}$$

$$Z_S = \tfrac{1}{2}(Z_{PS} + Z_{TS} - Z_{PT}) \tag{2.104b}$$

$$Z_T = \tfrac{1}{2}(Z_{PT} + Z_{TS} - Z_{PS}) \tag{2.104c}$$

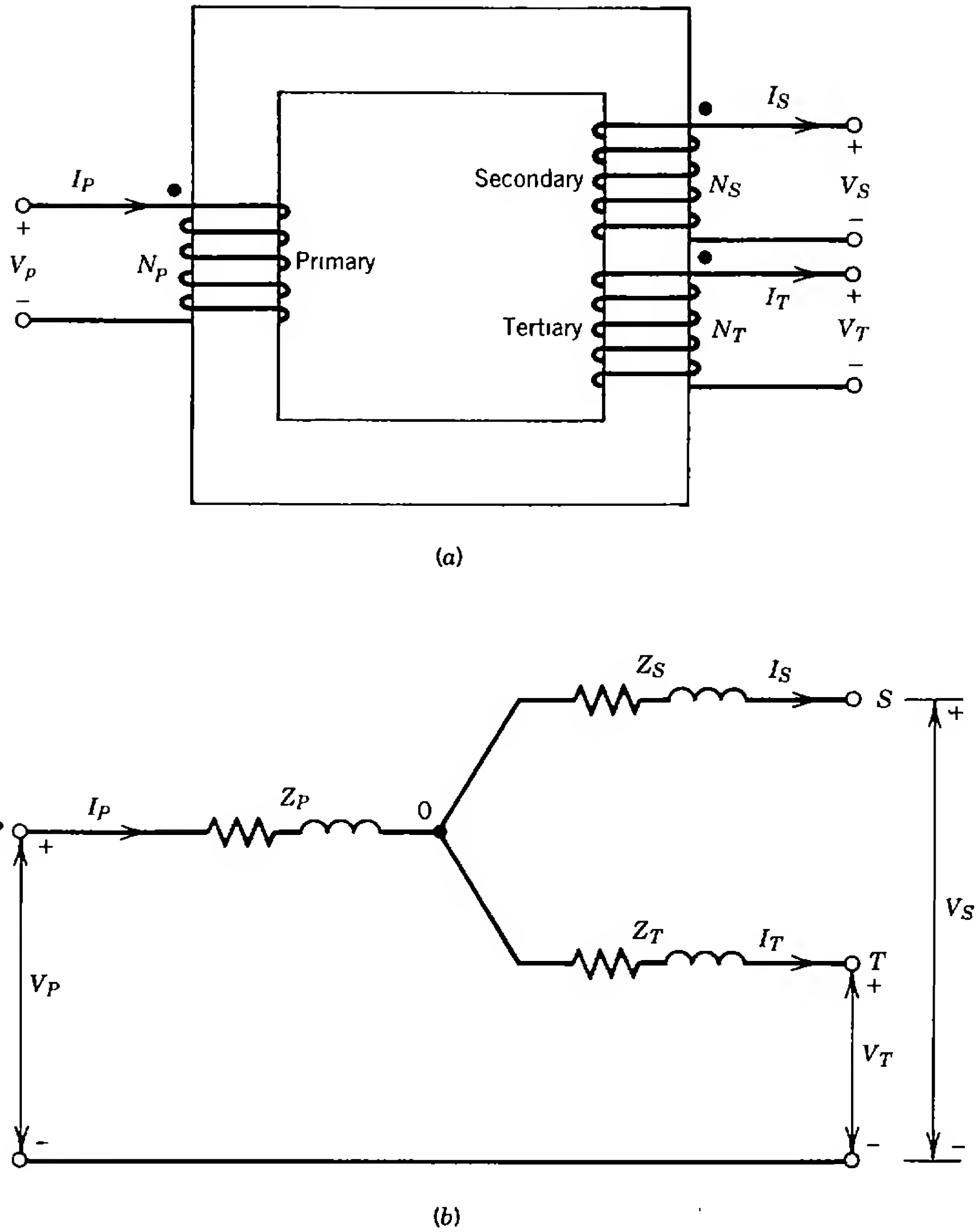


Figure 2.6. A single-phase three-winding transformer: (*a*) winding diagram; (*b*) equivalent circuit.

where Z_{PS} = leakage impedance measured in primary with secondary short-circuited and tertiary open

Z_{PT} = leakage impedance measured in primary with tertiary short-circuited and secondary open

Z_{ST} = leakage impedance measured in secondary with tertiary short-circuited and primary open

Z_P = impedance of primary winding

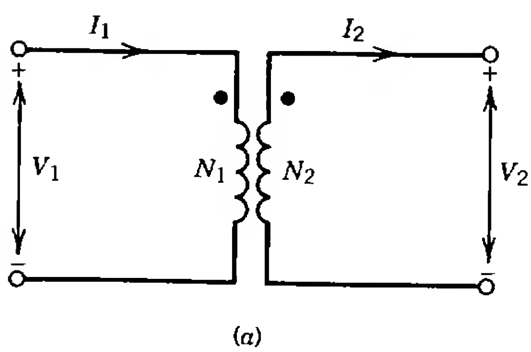
Z_S = impedance of secondary winding

Z_T = impedance of tertiary winding

In most large transformers the value of Z_S is very small and can be negative. Contrary to the situation with a two-winding transformer, the kilovoltampere ratings of the three windings of a three-winding transformer bank are not usually equal. Therefore, all impedances, as defined above, should be expressed on the same kilovoltampere base. For three-winding three-phase transformer banks with delta- or wye-connected windings, the positive- and negative-sequence diagrams are always the same. The corresponding zero-sequence diagrams are shown in Figure 3.10.

2.7 AUTOTRANSFORMERS

Figure 2.7(*a*) shows a two-winding transformer. Viewed from the terminals, the same transformation of voltages, currents, and impedances can be obtained with the connection shown in Figure 2.7(*b*). Therefore, in the autotransformer, only one winding is used per phase, the secondary voltage being tapped off the primary winding, as shown in Figure 2.7(*b*). The *common winding* is the winding between the low-voltage terminals, whereas the remainder of the winding, belonging exclusively to the high-voltage circuit, is called the *series winding* and, combined with the *common winding*, forms the *series–common winding* between the high-voltage terminals. In a sense, an autotransformer is just a normal two-winding transformer connected in a special way. The only structural difference is that the *series winding* must have extra insulation. In a *variable autotransformer* the tap is movable. Autotransformers are increasingly used to interconnect two high-voltage transmission lines operating at different voltages. An autotransformer has two separate sets of ratios, namely, circuit ratios and winding ratios. For



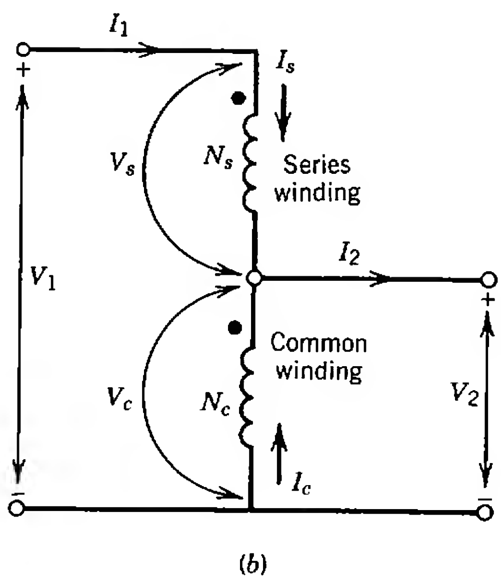


Figure 2.7. Schematic diagram of ideal (stepdown) transformer connected as: (*a*) two-winding transformer; (*b*) autotransformer.

circuit ratios, consider the equivalent circuit of an ideal autotransformer (neglecting losses) shown in Figure 2.7(b). Viewed from the terminals, the voltage and current ratios can be expressed as

$$a = \frac{V_1}{V_2} \tag{2.105a}$$

$$= \frac{N_c + N_s}{N_c} \tag{2.105b}$$

$$= 1 + \frac{N_s}{N_c} \tag{2.105c}$$

and

$$a = \frac{I_2}{I_1} \tag{2.106}$$

From equation (2.105c), it can be observed that the ratio a is always larger than 1.

For winding ratios, consider the voltages and currents of the series and common windings, as shown in Figure 2.7(b). Therefore, the voltage and current ratios can be expressed as

$$\frac{V_s}{V_c} = \frac{N_s}{N_c} \tag{2.107}$$

and

$$\frac{I_c}{I_s} = \frac{I_2 - I_1}{I_1} \tag{2.108a}$$

$$= \frac{I_2}{I_1} - 1 \tag{2.108b}$$

From equation (2.105c),

$$\frac{N_s}{N_c} = a - 1 \tag{2.109}$$

Therefore, substituting equation (2.109) into equation (2.107) yields

$$\frac{V_s}{V_c} = a - 1 \tag{2.110}$$

Similarly, substituting equations (2.106) and (2.109) into equation (2.108b) simultaneously yields

$$\frac{I_c}{I_s} = a - 1 \tag{2.111}$$

For an ideal autotransformer, the voltampere ratings of circuits and windings can be expressed, respectively, as

$$S_{\text{circuits}} = V_1 I_1 = V_2 I_2 \tag{2.112}$$

and

$$S_{\text{windings}} = V_s I_s = V_c I_c \tag{2.113}$$

The advantages of autotransformers are lower leakage reactances, lower losses, smaller exciting currents, and less cost than two-winding transformers when the voltage ratio does not vary too greatly from 1 to 1. For example, if the same core and coils are used as a two-winding transformer and as an autotransformer, the ratio of the capacity as an autotransformer to the capacity as a two-winding transformer can be expressed as

$$\begin{aligned}\frac{\text{Capacity as autotransformer}}{\text{Capacity as two-winding transformer}} &= \frac{V_1 I_1}{V_s I_s} = \frac{V_1 I_1}{(V_1 - V_2) I_1} \\ &= \frac{a}{a-1}\end{aligned} \tag{2.114}$$

Therefore, maximum advantage is obtained with relatively small difference between the voltages on the two sides (e.g., 161 kV/138 kV, 500 kV/700 kV, and 500 kV/345 kV). Therefore, a large saving in size, weight, the cost can be achieved over a two-windings per-phase transformer. The disadvantages of an autotransformer are that there is no electrical isolation between the primary and secondary circuits and there is a greater short-circuit current than the one for the two-winding transformer.

Three-phase autotransformer banks generally have wye-connected main windings, the neutral of which is normally connected solidly to the earth. In addition, it is common practice to include a third winding connected in delta, called the *tertiary winding*.

2.8 DELTA–WYE AND WYE–DELTA TRANSFORMATIONS

The three-terminal circuits encountered so often in networks are the delta and wye[†] configurations, as shown in Figure 2.8. In some problems it is necessary to convert delta to wye or vice versa.

If the impedances $\mathbf{Z}_{ab}$, $\mathbf{Z}_{bc}$, and $\mathbf{Z}_{ca}$ are connected in delta, the equivalent wye impedances $\mathbf{Z}_a$, $\mathbf{Z}_b$, and $\mathbf{Z}_c$ are

[†] In Europe it is called the *star* configuration.

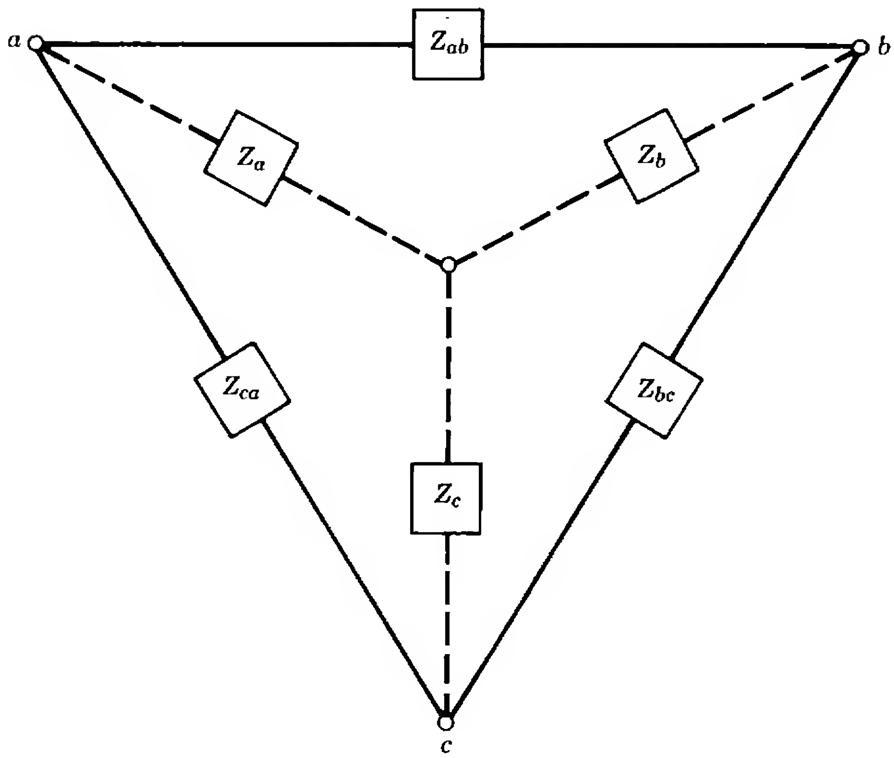


Figure 2.8. Delta-to-wye or wye-to-delta transformations.

$$\mathbf{Z}_a = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{ca}}{\mathbf{Z}_{ab} + \mathbf{Z}_{bc} + \mathbf{Z}_{ca}} \tag{2.115}$$

$$\mathbf{Z}_b = \frac{\mathbf{Z}_{ab}\mathbf{Z}_{bc}}{\mathbf{Z}_{ab} + \mathbf{Z}_{bc} + \mathbf{Z}_{ca}} \tag{2.116}$$

$$\mathbf{Z}_c = \frac{\mathbf{Z}_{bc}\mathbf{Z}_{ca}}{\mathbf{Z}_{ab} + \mathbf{Z}_{bc} + \mathbf{Z}_{ca}} \tag{2.117}$$

Of course, if $\mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca} = \mathbf{Z}$,

$$\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \tfrac{1}{3}\mathbf{Z} \tag{2.118}$$

On the other hand, if the impedances $\mathbf{Z}_a$, $\mathbf{Z}_b$, and $\mathbf{Z}_c$ are connected in wye, the equivalent delta impedances $\mathbf{Z}_{ab}$, $\mathbf{Z}_{bc}$, and $\mathbf{Z}_{ca}$ are

$$\mathbf{Z}_{ab} = \mathbf{Z}_a + \mathbf{Z}_b + \frac{\mathbf{Z}_a\mathbf{Z}_b}{\mathbf{Z}_c} \tag{2.119}$$

$$\mathbf{Z}_{bc} = \mathbf{Z}_b + \mathbf{Z}_c + \frac{\mathbf{Z}_b\mathbf{Z}_c}{\mathbf{Z}_a} \tag{2.120}$$

$$\mathbf{Z}_{ca} = \mathbf{Z}_c + \mathbf{Z}_a + \frac{\mathbf{Z}_c\mathbf{Z}_a}{\mathbf{Z}_b} \tag{2.121}$$

Of course, if $\mathbf{Z}_a = \mathbf{Z}_b = \mathbf{Z}_c = \mathbf{Z}$,

$$\mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca} = 3\mathbf{Z} \tag{2.122}$$

2.9 FACTORS AFFECTING LINE DESIGN

The function of the overhead three-phase electric power transmission line is to transmit bulk power to load centers and large industrial users beyond the primary distribution lines. A given transmission system comprises all land, conversion structures, and equipment at a primary source of supply, including lines, switching, and conversion stations, between a generating or receiving point, and a load center or wholesale point. It includes all lines and equipment whose main function is to increase, integrate, or tie together power supply sources.

The decision to build a transmission system results from system planning studies to determine how best to meet the system requirements. At this stage, the following factors need to be considered and established:

1. Voltage level.
2. Conductor type and size.
3. Line regulation and voltage control.
4. Corona and losses.
5. Proper load-flow and system stability.
6. System protection.
7. Grounding.
8. Insulation coordination.
9. Mechanical design:
   (a) Sag and stress calculations.
   (b) Conductor composition.
   (c) Conductor spacing.
   (d) Insulator and conductor hardware selection.
10. Structural design:
    (a) Structure types.
    (b) Stress calculations.

The basic configuration selection depends on many interrelated factors, including esthetic considerations, economics, performance criteria, company policies and practice, line profile, right-of-way restrictions, preferred materials, and construction techniques. Figure 2.9 shows typical compact configurations. Figures 2.10–2.13 show typical structures used for EHV transmission systems. Figure 2.14 shows a 345-kV line with a single circuit and wood H-frame, whereas Figure 2.15 shows a 345-kV kine with a double circuit and steel tower.

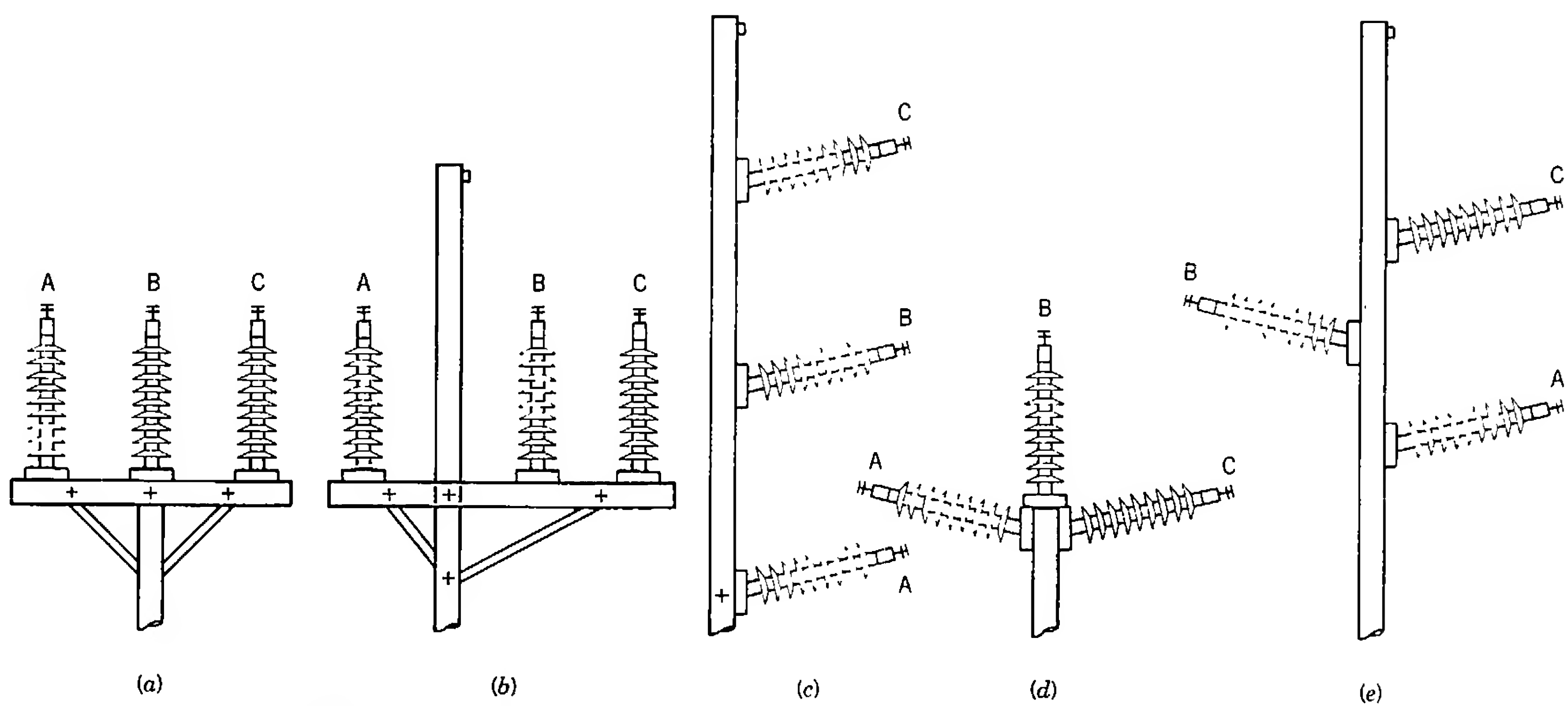


Figure 2.9. Typical compact configurations (not to scale): (*a*) horizontal unshielded; (*b*) horizontal shielded; (*c*) vertical; (*d*) delta; (e) vertical delta. (From Electric Power Research Institute, 1978. Used by permission. © 1978 Electric Power Research Institute.)

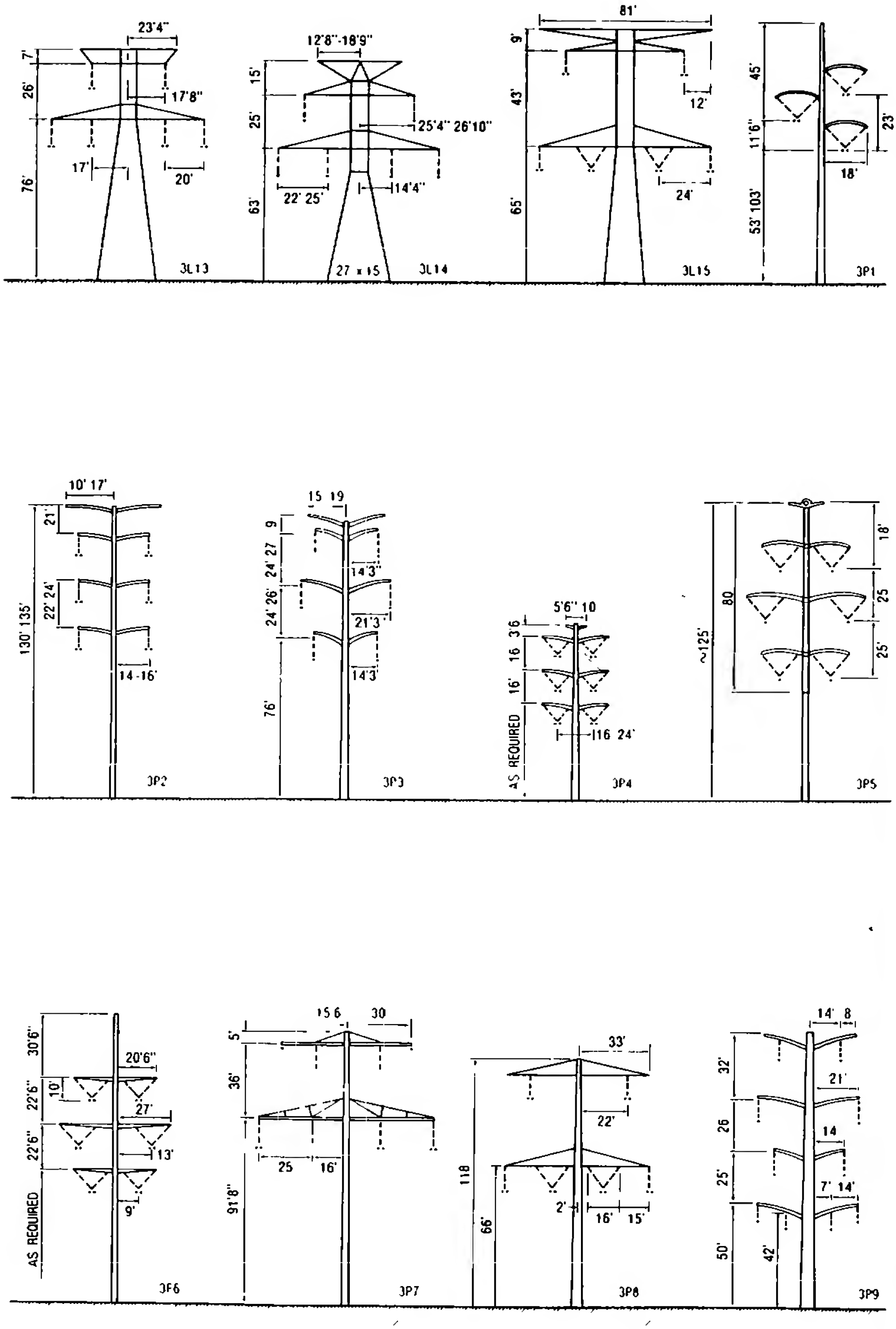


Figure 2.10. Typical pole- and lattice-type structures for 345-kV transmission systems. (Used by permission. © 1979 Electric Power Research Institute.)

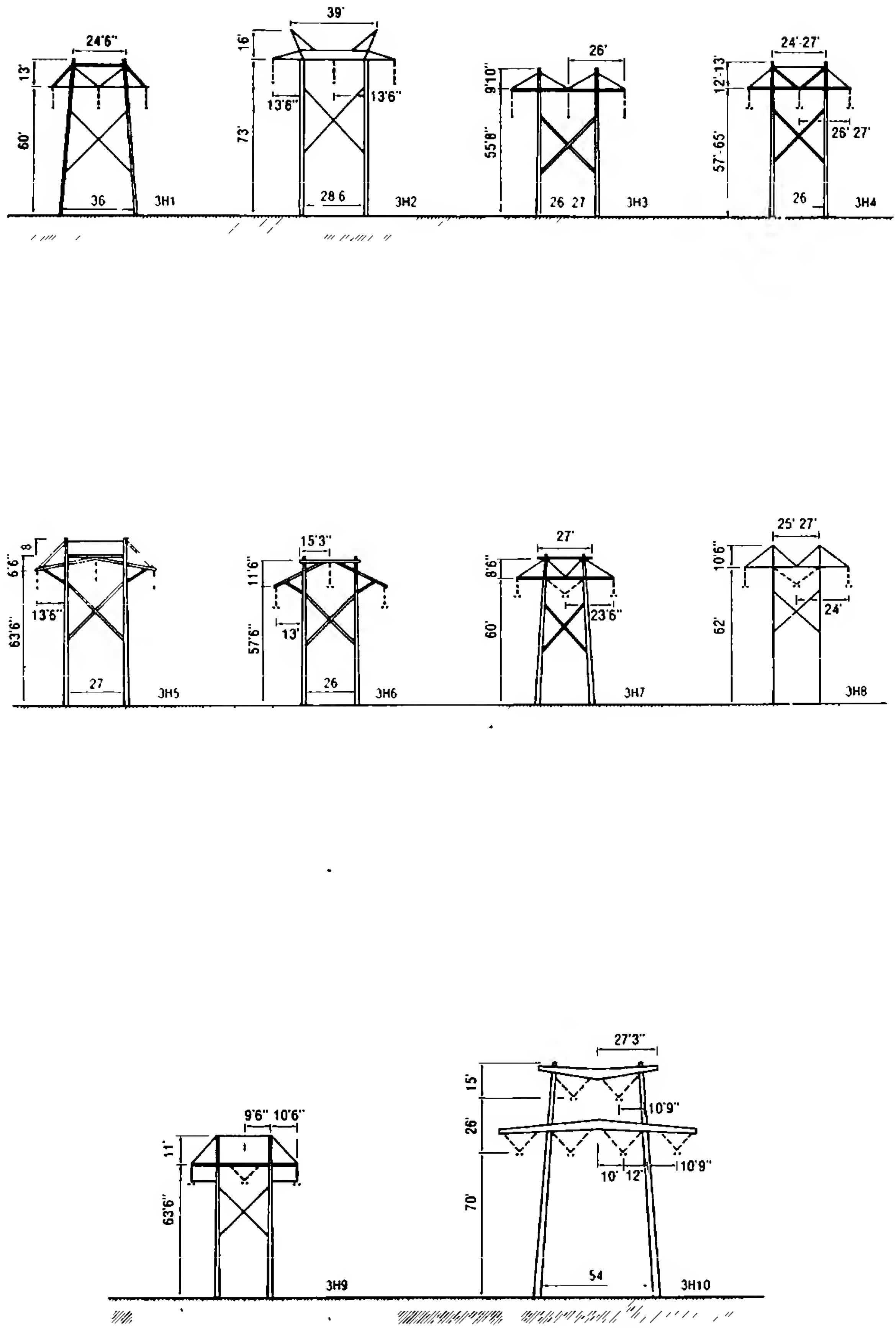


Figure 2.11. Typical wood H-frame-type structures for 345-kV system (From Ref. 14. Used by permission. © 1979 Electric Power Research Institute.)

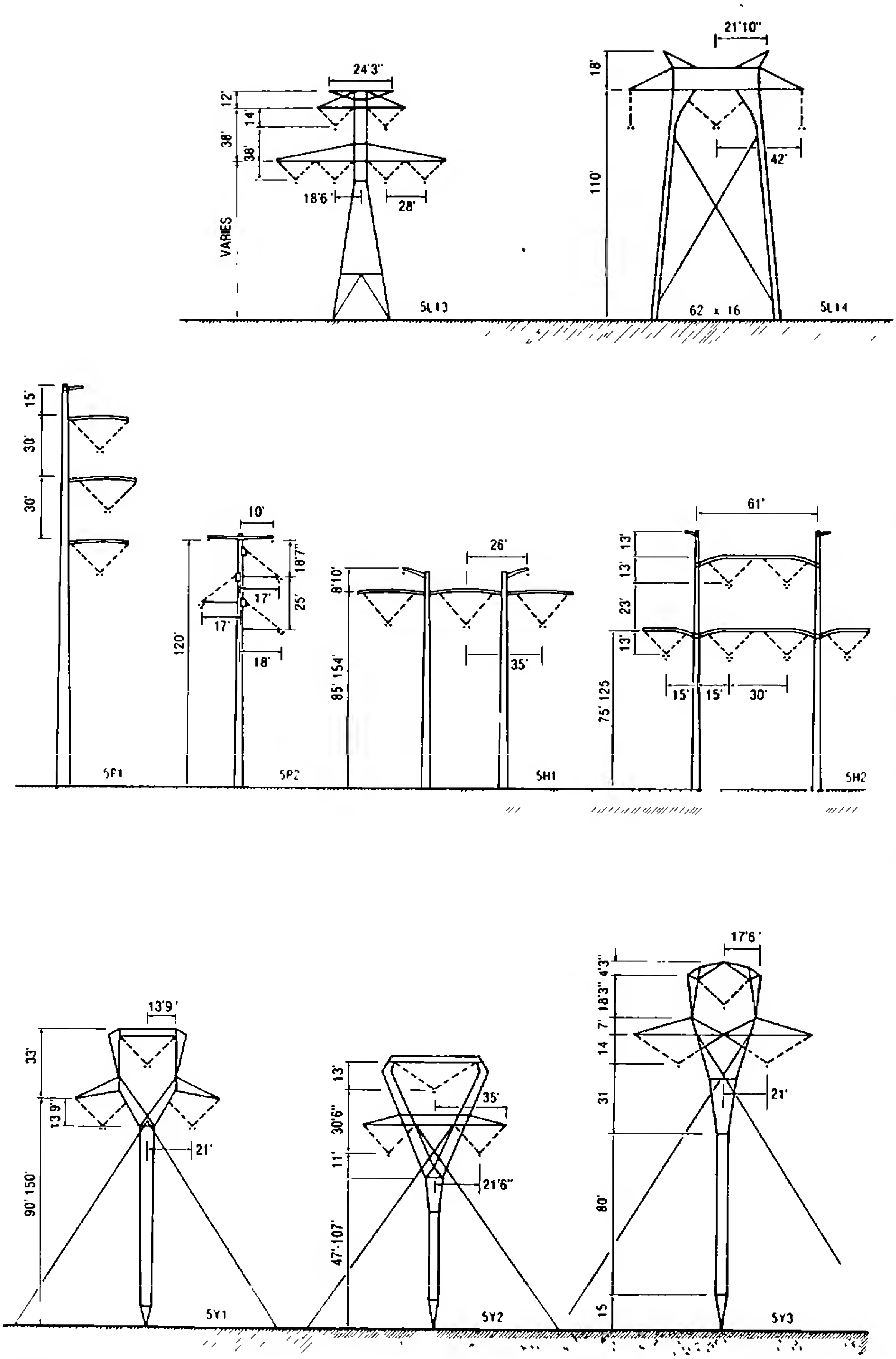


Figure 2.12. Typical 500-kV lattice, pole, guyed V- and Y-type structures (From Ref. 14. Used by permission. © 1979 Electric Power Research Institute.)

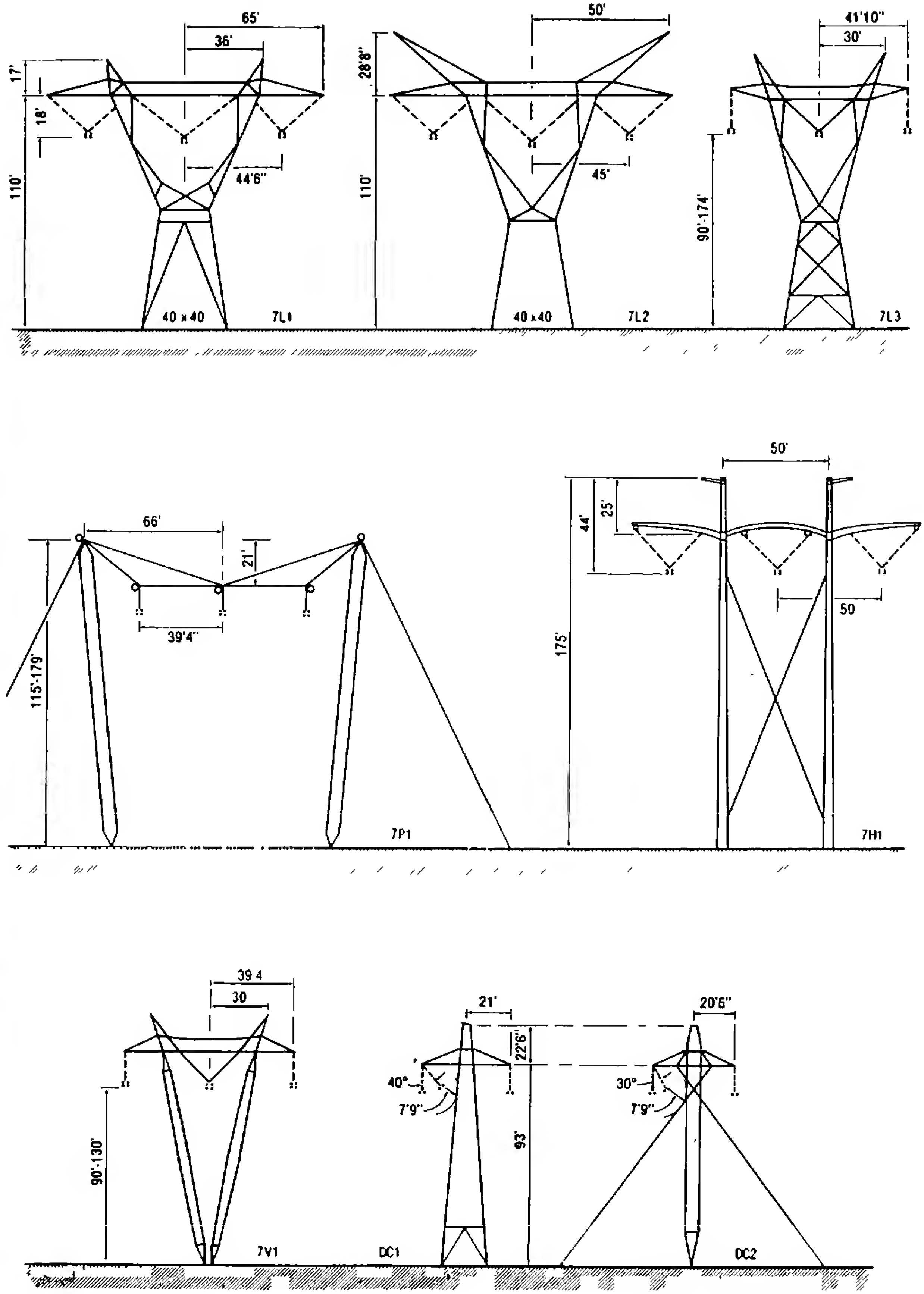


Figure 2.13. Typical 735–800-kV ac designs and dc structures (From Ref. 14. Used by permission: © 1979 Electric Power Research Institute.)

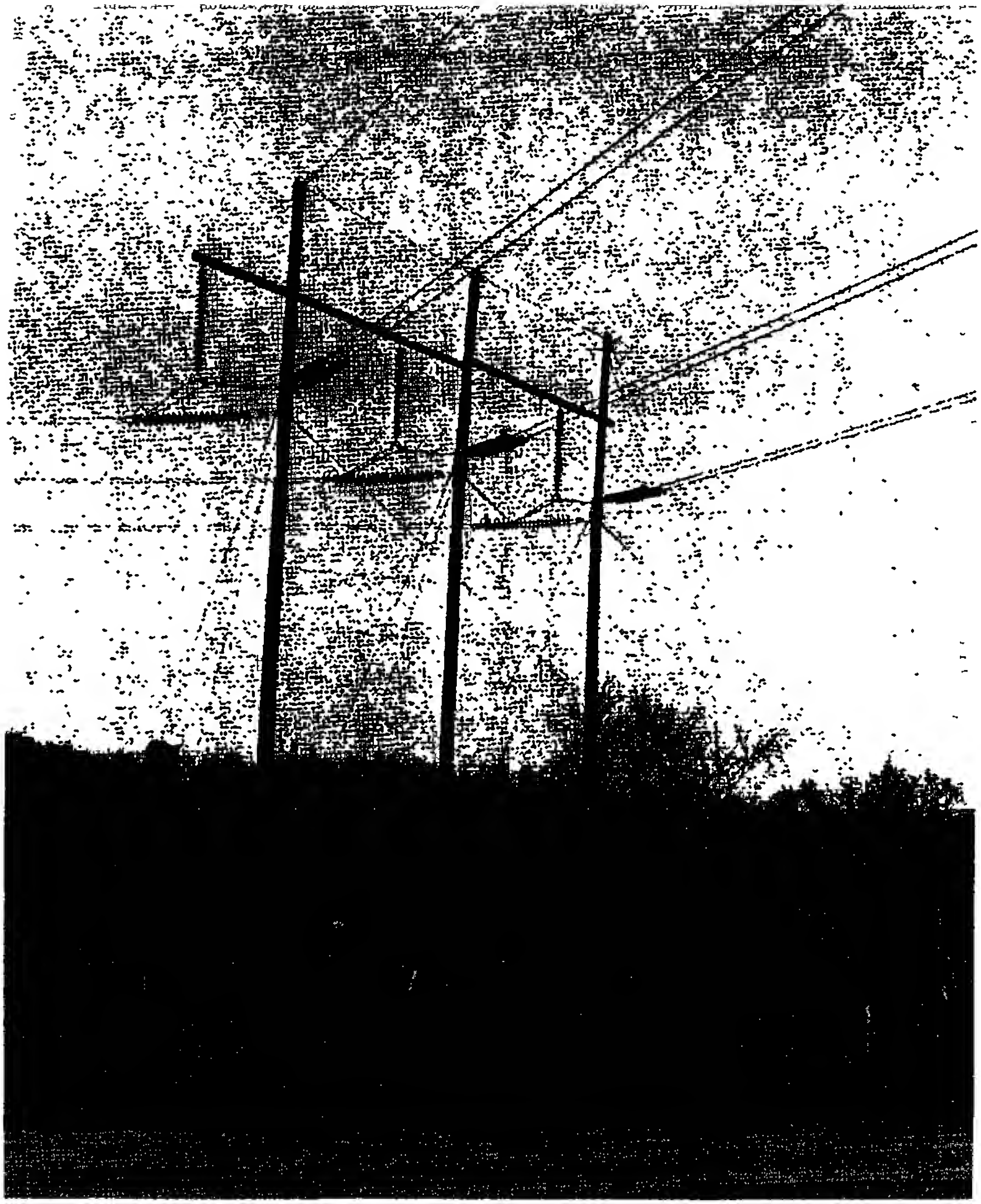


Figure 2.14. Typical 345-kV line with single circuit and wood H-frame (Union Electric Company).

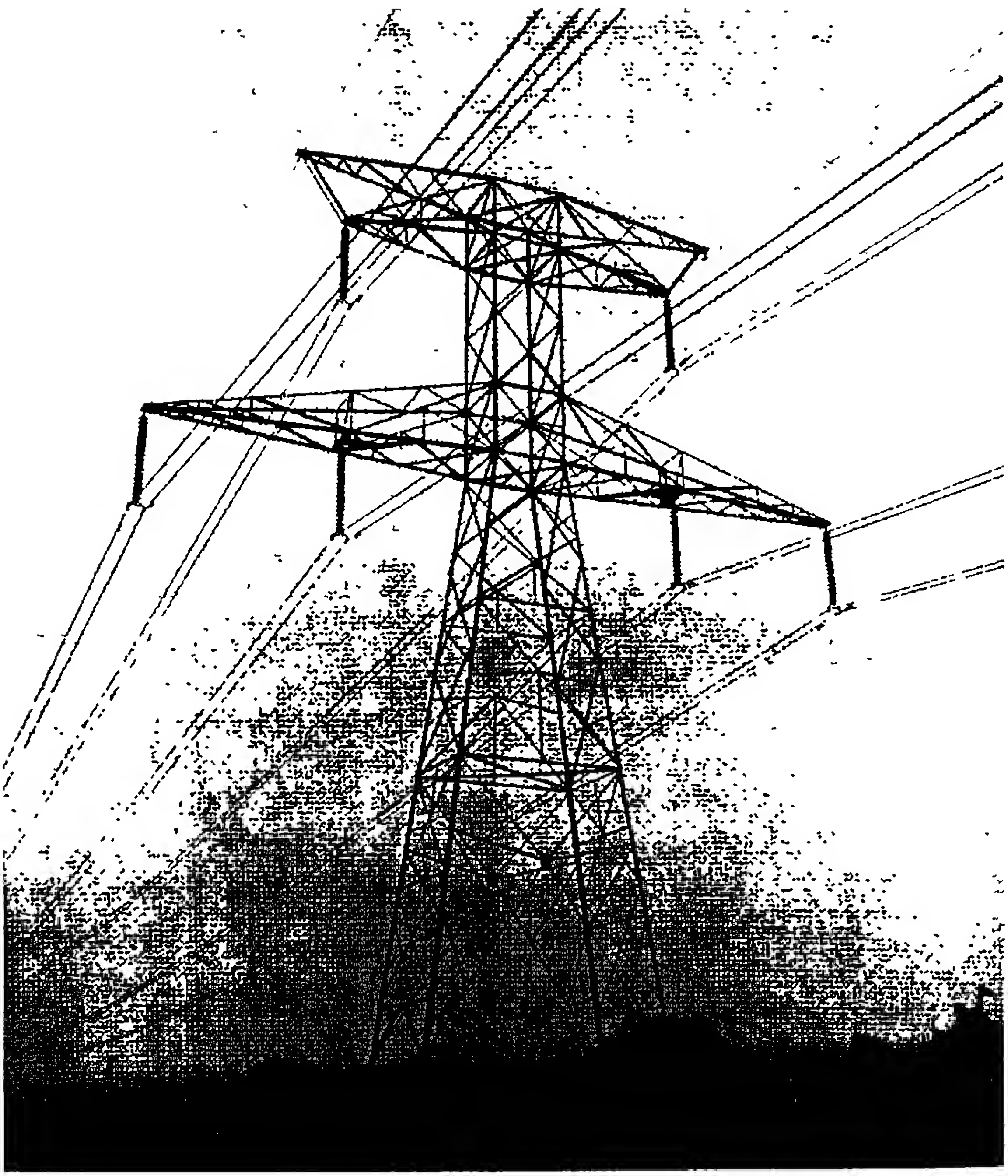


Figure 2.15

2.10 CONDUCTOR SIZE

Conductor sizes are based on the circular mil. A circular mil is the area of a circle that has a diameter of 1 mil. A mil is equal to 1×10^{-3} in. The cross-sectional area of a wire in square inches equals its area in circular mils multiplied by 0.7854×10^{-6}.

For the smaller conductors, up to 211,600 circular mils, the size is usually given by a gage number according to the American Wire Gauge (AWG) standard, formerly known as the Brown and Sharpe Wire Gauge (B&S). Gage sizes decrease as the wire increases in size. The larger the gage size, the smaller the wire. These numbers start at 40, the smallest, which is assigned to a wire with a diameter of 3.145 mils. The largest size is number 0000, written as 4/0, and read as four odds. Above 4/0, the size is determined by cross-sectional area in circular mils. In summary,

$$\begin{aligned}
1 \text{ linear mil} &= 0.001 \text{ inch} \\
&= 0.0254 \text{ millimeter} \\
1 \text{ circular mil} &= \text{area of circle 1 linear mil in diameter} \\
&= \tfrac{1}{4}\pi \text{ square mils} \\
&= \tfrac{1}{4}\pi \times 10^{-6} = 0.7854 \times 10^{-6} \text{ square inch}
\end{aligned}$$

One thousand circular mils is often used as a unit, for exampler, a size given as 250 kcmil or MCM refers to 250,000 circular mils, or 250,000 cmil.

A given conductor may consist of a single strand or several strands. If a single strand, it is solid; if of more than one strand, it is stranded. A solid conductor is often called a wire, whereas a stranded conductor is called a cable. A general formula for the total number of strands in concentrically stranded cables is

$$\text{Number of strands} = 3n^2 - 3n + 1 \tag{2.123}$$

where n is the number of layers, including the single center strand.

In general, distribution conductors larger than 2 AWG are stranded. Insulated conductors for underground distribution or aerial cable lines are classified as cables and usually are stranded. Table 2.3 gives standard conductor sizes.

Conductors may be selected based on Kelvin's law. According to Kelvin's law,[†] "the most economical area of conductor is that for which the annual cost of the energy wasted is equal to the interest on that portion of the capital expense which may be considered as proportional to the weight of the conductor" [5]. Therefore,

$$\text{Annual cost} = \frac{3CI^2R}{1000} + \frac{pwa}{100} \tag{2.124}$$

[†] Expressed by Sir William Thomson (Lord Kelvin) in 1881.

TABLE 2.3 Standard Conductor Sizes

Size (AWG or kcmil)	(Circular mils)	Number of wires	Solid or stranded
18	1,620	1	Solid
16	2,580	1	Solid
14	4,110	1	Solid
12	6,530	1	Solid
10	10,380	1	Solid
8	16,510	1	Solid
7	20,820	1	Solid
6	26,250	1	Solid
6	26,250	3	Stranded
5	33,100	3	Stranded
5	33,100	1	Solid
4	41,740	1	Solid
4	41,740	3	Stranded
3	52,630	3	Stranded
3	52,630	7	Standed
3	52,630	1	Solid
2	66,370	1	Solid
2	66,370	3	Stranded
2	66,370	7	Stranded
1	83,690	3	Stranded
1	83,690	7	Stranded
0 (or 1/0)	105,500	7	Stranded
00 (or 2/0)	133,100	7	Stranded
000 (or 3/0)	167,800	7	Stranded
000 (or 3/0)	167,800	12	Stranded
0000 (or 4/0)	211,600	7	Stranded
0000 (or 4/0)	211,600	12	Stranded
0000 (or 4/0)	211,600	19	Stranded
250	250,000	12	Stranded
250	250,000	19	Stranded
300	300,000	12	Stranded
300	300,000	19	Stranded
350	350,000	12	Stranded
350	350,000	19	Stranded
400	400,000	19	Stranded
450	450,000	19	Stranded
500	500,000	19	Stranded
500	500,000	37	Stranded
600	600,000	37	Stranded
700	700,000	37	Stranded
750	750,000	37	Stranded
800	800,000	37	Stranded
900	900,000	37	Stranded
1,000	1,000,000	37	Stranded

where C = cost of energy wasted in dollars per kilowatt-year
I = current per wire
R = resistance per mile per conductor
p = cost per pound of conductor
w = weight per mile of all conductors
a = percent annual cost of money

Here, the minimum cost is obtained when

$$\frac{3CI^2R}{1000} = \frac{pwa}{100} \tag{2.125}$$

However, in practice, Kelvin's law is seldom used. Instead, the I^2R losses are calculated for the total time horizon.

The conductors used in the modern overhead power transmission lines are bare aluminum conductors, which are classified as follows:

AAC: all-aluminum conductor.
AAAC: all-aluminum-alloy conductor.
ACSR: aluminum conductor steel reinforced.
ACAR: aluminum conductor alloy reinforced.

2.11 TRANSMISSION LINE CONSTANTS

For the purpose of system analysis, a given transmission line can be represented by its resistance, inductance or inductive reactance, capacitance or capacitive reactance, and leakage resistance (which is usually negligible).

2.12 RESISTANCE

The direct-current resistance of a conductor is

$$R_{dc} = \frac{\rho l}{A} \quad \Omega \tag{2.126}$$

where ρ = conductor resistivity
l = conductor length
A = conductor cross-sectional area

In practice, several different sets of units are used in the calculation of the resistance. For example, in the International System of Units (SI units),

l is in meters, A is in square meters, and ρ is in ohm-meters. Whereas in power systems in the United States, ρ is in ohm-circular-mils per foot, l is in feet, and A is in circular mils.

The resistance of a conductor at any temperature may be determined by

$$\frac{R_2}{R_1} = \frac{T_0 + t_2}{T_0 + t_1} \tag{2.127}$$

where R_1 = conductor resistance at temperature t_1
R_2 = conductor resistance at temperature t_2
t_1, t_2 = conductor temperatures in degrees Celsius
T_0 = constant varying with conductor material
= 234.5 for annealed copper
= 241 for hard-drawn copper
= 228 for hard-drawn aluminum

The phenomenon by which alternating current tends to flow in the outer layer of a conductor is called *skin effect.* Skin effect is a function of conductor size, frequency, and the relative resistance of the conductor material.

Tables given in Appendix A provide the dc and ac resistance values for various conductors. The resistances to be used in the positive- and negative-sequence networks are the ac resistances of the conductors.

2.13 INDUCTANCE AND INDUCTIVE REACTANCE

2.13.1 Single-Phase Overhead Lines

Figure 2.16 shows a single-phase overhead line. Assume that a current flows out in conductor a and returns in conductor b. These currents cause magnetic field lines that link between the conductors. A change in current causes a change in flux, which in turn results in an induced voltage in the circuit. In an ac circuit, this induced voltage is called the IX drop. In going around the loop, if R is the resistance of each conductor, the total loss in voltage due to resistance is $2IR$. Therefore, the voltage drop in the single-phase line due to loop impedance at 60 Hz is

$$\mathrm{VD} = 2l\left(R + j0.2794 \log_{10} \frac{D_m}{D_s}\right)I \quad \mathrm{V} \tag{2.128}$$

where VD = voltage drop due to line impedance in volts
l = line length in miles
R = resistance of each conductor in ohms per mile

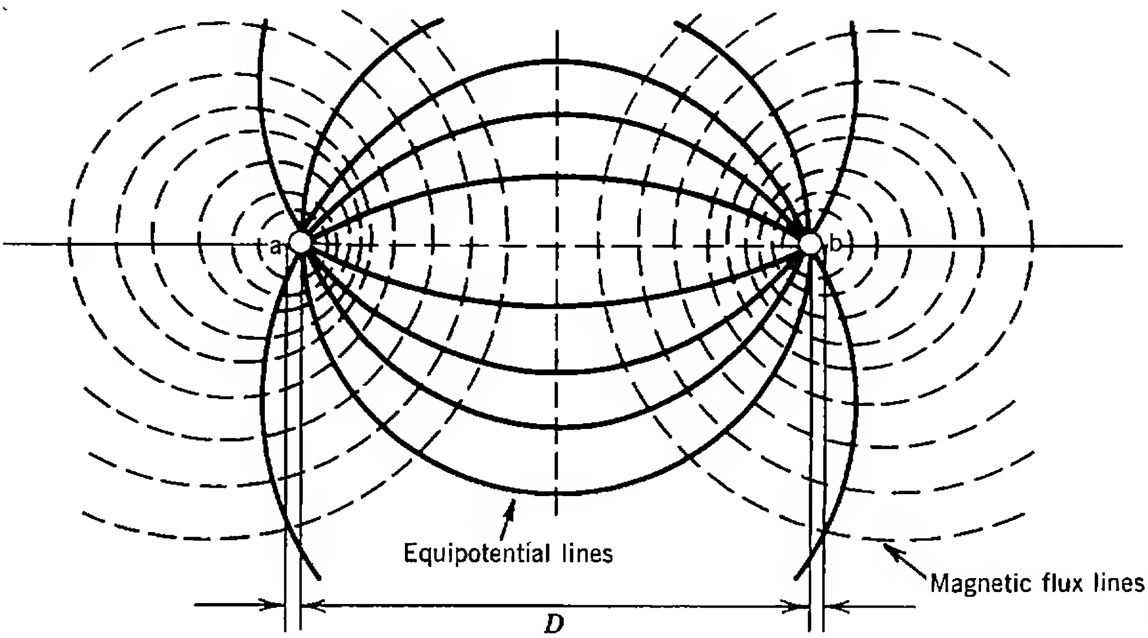


Figure 2.16. Magnetic field of single-phase line.

D_m = equivalent or geometric mean distance (GMD) between conductor centers in inches
D_s = geometric mean radius (GMR) or self-GMD of one conductor in inches,
= $0.7788r$ for cylindrical conductor
r = radius of cylindrical conductor in inches (see Figure 2.16)
I = phase current in amperes

Therefore, the inductance of the conductor is expressed as

$$L = 2 \times 10^{-7} \ln \frac{D_m}{D_s} \quad \text{H/m} \tag{2.129}$$

or

$$L = 0.7411 \log_{10} \frac{D_m}{D_s} \quad \text{mH/mi} \tag{2.130}$$

With the inductance known, the inductive reactance† can be found as

$$X_L = 2\pi f L = 2.02 \times 10^{-3} f \ln \frac{D_m}{D_s} \tag{2.131}$$

or

$$X_L = 4.657 \times 10^{-3} f \log_{10} \frac{D_m}{D_s} \tag{2.132}$$

† It is also the same as the positive- and negative-sequence of a line.

or, at 60 Hz,

$$X_L = 0.2794 \log_{10} \frac{D_m}{D_s} \quad \Omega/\text{mi} \tag{2.133}$$

$$X_L = 0.1213 \ln \frac{D_m}{D_s} \quad \Omega/\text{mi} \tag{2.134}$$

By using the geometric mean radius of a conductor, D_s, the calculation of inductance and inductive reactance can be done easily. Tables give the geometric mean radius of various conductors readily.

2.13.2 Three-Phase Overhead Lines

In general, the spacings D_{ab}, D_{bc}, and D_{ca} between the conductors of three-phase transmission lines are not equal. For any given conductor configuration, the average values of inductance and capacitance can be found by representing the system by one with equivalent equilateral spacing. The "equivalent spacing" is calculated as

$$D_{eq} \triangleq D_m = (D_{ab} \times D_{bc} \times D_{ca})^{1/3} \tag{2.135}$$

In practice, the conductors of a transmission line are transposed, as shown in Figure 2.17. The transposition operation, that is, exchanging the conductor positions, is usually carried out at switching stations.

Therefore, the average inductance per phase is

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s} \quad \text{H/m} \tag{2.136}$$

or

$$L_a = 0.7411 \log_{10} \frac{D_{eq}}{D_s} \quad \text{mH/mi} \tag{2.137}$$

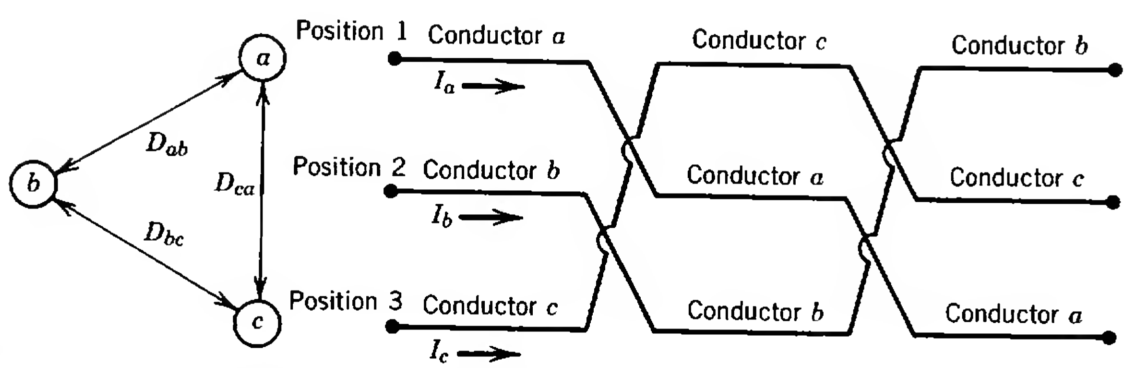


Figure 2.17. Transposition cycle of three-phase line.

and the inductive reactance is

$$X_L = 0.1213 \ln \frac{D_{\text{eq}}}{D_s} \quad \Omega/\text{mi} \tag{2.138}$$

or

$$X_L = 0.2794 \log_{10} \frac{D_{\text{eq}}}{D_s} \quad \Omega/\text{mi} \tag{2.139}$$

2.14 CAPACITANCE AND CAPACITIVE REACTANCE

2.14.1 Single-Phase Overhead Lines

Figure 2.18 shows a single-phase line with two identical parallel conductors a and b of radius r separated by a distance D, center to center, and with a potential difference of V_{ab} volts. Let conductors a and b carry charges of $+q_a$ and $-q_b$ farads per meter, respectively. The capacitance between conductors can be found as

$$C_{ab} = \frac{q_a}{V_{ab}}$$

$$= \frac{2\pi\epsilon}{\ln(D^2/r_a \cdot r_b)} \quad \text{F/m} \tag{2.140}$$

If $r_a = r_b = r$,

$$C_{ab} = \frac{2\pi\epsilon}{2\ln(D/r)} \quad \text{F/m} \tag{2.141}$$

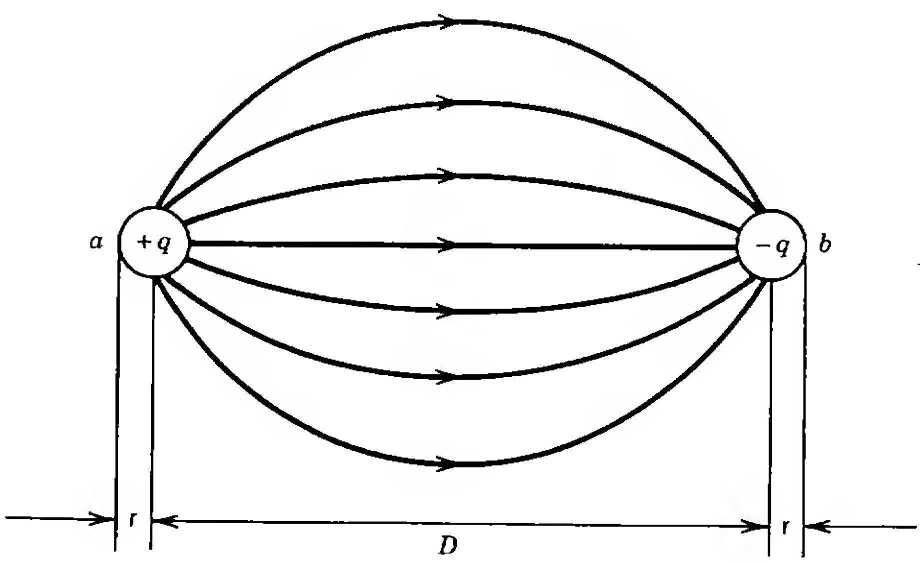


Figure 2.18. Capacitance of single-phase line.

Since

$$\epsilon = \epsilon_0 \cdot \epsilon_r$$

where

$$\epsilon_0 = \frac{1}{36\pi \times 10^9} = 8.85 \times 10^{-12}\ \text{F/m}$$

and

$$\epsilon_r \cong 1 \quad \text{for air}$$

equation (2.141) becomes

$$C_{ab} = \frac{0.0388}{2\log_{10}(D/r)} \quad \mu\text{F/mi} \tag{2.142}$$

or

$$C_{ab} = \frac{0.0345}{2\ln(D/r)} \quad \mu\text{F/mi} \tag{2.143}$$

or

$$C_{ab} = \frac{0.0241}{2\log_{10}(D/r)} \quad \mu\text{F/km} \tag{2.144}$$

Stevenson [3] explains that the capacitance to neutral or capacitance to ground for the two-wire line is twice the line-to-line capacitance or capacitance between conductors, as shown in Figures 2.19 and 2.20. Therefore, the line-to-neutral capacitance is

$$C_N = C_{aN} = C_{bN} = \frac{0.0388}{\log_{10}(D/r)} \quad \mu\text{F/mi to neutral} \tag{2.145}$$

This can easily be verified since C_N must equal $2C_{ab}$ so that the total capacitance between the conductors can be

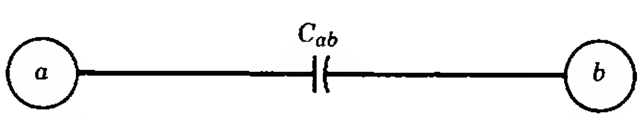


Figure 2.19. Line-to-line capacitance.

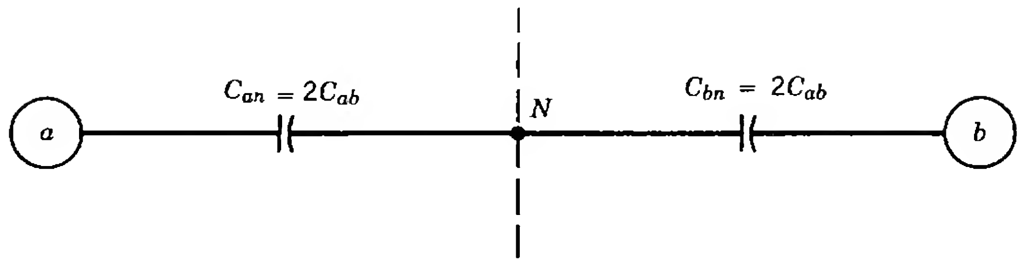


Figure 2.20. Line-to-neutral capacitance.

$$C_{ab} = \frac{C_N \times C_N}{C_N + C_N}$$
$$= \frac{C_N}{2}$$
$$= C_{ab} \quad \text{as before} \tag{2.146}$$

With the capacitance known, the capacitive reactance between one conductor and neutral can be found as

$$X_c = \frac{1}{2\pi f C_N} \tag{2.147}$$

or, for 60 Hz,

$$X_c = 0.06836 \log_{10} \frac{D}{r} \quad \text{M}\Omega/\text{mi to neutral} \tag{2.148}$$

and the line-to-neutral susceptance is

$$b_c = \omega C_N$$

or

$$b_c = \frac{1}{X_c} \tag{2.149}$$

or

$$b_c = \frac{14.6272}{\log_{10}(D/r)} \quad \text{mS/mi to neutral} \tag{2.150}$$

The charging current of the line is

$$\mathbf{I}_c = j\omega C_{ab} V_{ab} \quad \text{A/mi} \tag{2.151}$$

2.14.2 Three-Phase Overhead Lines

Figure 2.21 shows the cross section of a three-phase line with equilateral spacing D. The line-to-neutral capacitance can be found as[†]

$$C_n = \frac{0.0388}{\log_{10}(D/r)} \quad \mu\mathrm{F/mi\ to\ neutral} \tag{2.152}$$

which is identical to equation (2.145).

On the other hand, if the spacings between the conductors of the three-phase line are not equal, the line-to-neutral capacitance is[†]

$$C_n = \frac{0.0388}{\log_{10}(D_{\mathrm{eq}}/r)} \quad \mu\mathrm{F/mi\ to\ neutral} \tag{2.153}$$

where

$$D_{\mathrm{eq}} \triangleq D_m = (D_{ab} \times D_{bc} \times D_{ca})^{1/3}$$

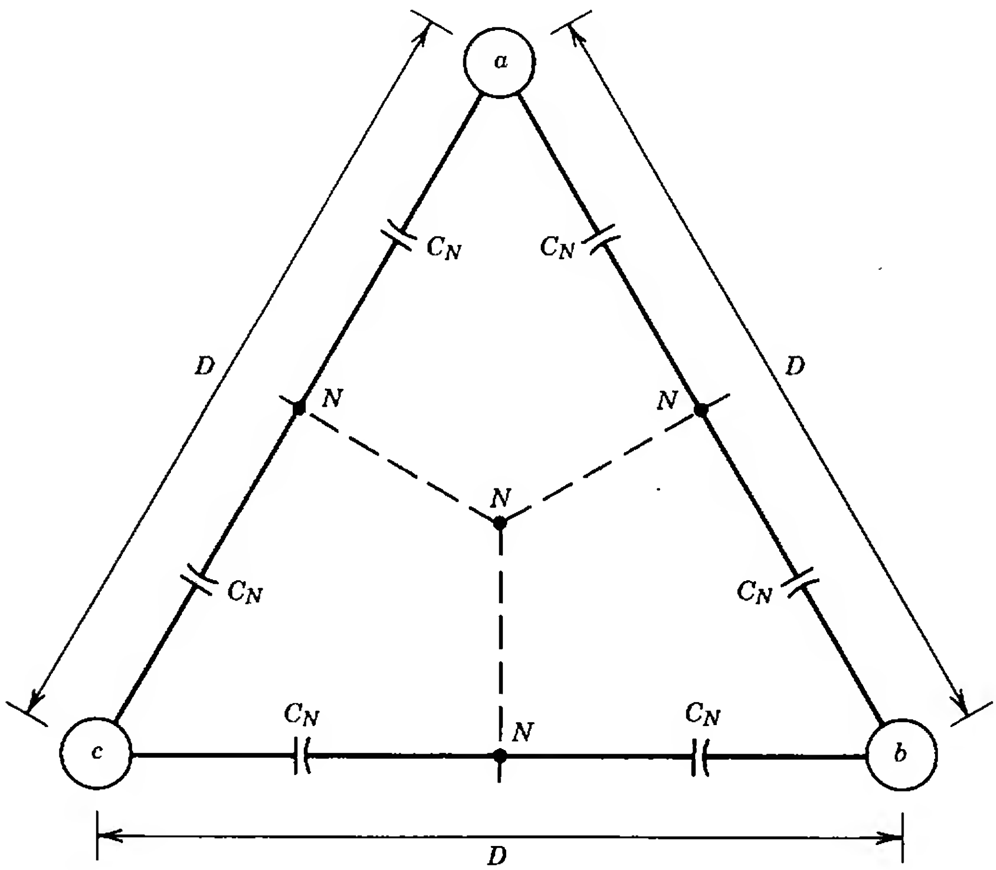


Figure 2.21. Three-phase line with equilateral spacing.

[†] See Stevenson [3] and Anderson [4] for the derivations.

The charging current per phase is

$$I_c = jwC_nV_{an} \quad \text{A/mi} \tag{2.154}$$

2.15 TABLES OF LINE CONSTANTS

Tables provide the line constants directly without using equations for calculation. This concept was suggested by W. A. Lewis [6]. According to this concept, equation (2.134) for inductive reactance at 60 Hz, that is,

$$X_L = 0.1213 \ln \frac{D_m}{D_s} \quad \Omega/\text{mi}$$

can be broken down to

$$X_L = 0.1213 \ln \frac{1}{D_s} + 0.1213 \ln D_m \quad \Omega/\text{mi} \tag{2.155}$$

where D_s = GMR, which can be found from the tables for a given conductor

D_m = GMD between conductor centers

Therefore, equation (2.155) can be rewritten as

$$X_L = x_a + x_d \quad \Omega/\text{mi} \tag{2.156}$$

where

$$x_a = \text{inductive reactance at 1-ft spacing}$$

$$= 0.1213 \ln \frac{1}{D_s} \quad \Omega/\text{mi} \tag{2.157}$$

$$x_d = \text{inductive reactance spacing factor}$$

$$= 0.1213 \ln D_m \quad \Omega/\text{mi} \tag{2.158}$$

For a given frequency, the value of x_a depends only on the GMR, which is a function of the conductor type. Whereas x_d depends only on the spacing D_m. If the spacing is greater than 1 ft, x_d has a positive value that is added to x_a. On the other hand, if the spacing is less than 1 ft, x_d has a negative value that is subtracted from x_a. Tables given in Appendix A give x_a and x_d directly.

Similarly, equation (2.148) for shunt capacitive reactance at 60 Hz, that is,

$$x_c = 0.06836 \log_{10} \frac{D}{r} \quad \text{M}\Omega/\text{mi}$$

can be split into

$$x_c = 0.06836 \log_{10} \frac{1}{r} + 0.06836 \log_{10} D \quad \text{M}\Omega/\text{mi} \tag{2.159}$$

or

$$x_c = x'_a + x'_d \quad \text{M}\Omega/\text{mi} \tag{2.160}$$

where

$$x'_a = \text{capacitive reactance at 1-ft spacing}$$

$$= 0.06836 \log_{10} \frac{1}{r} \quad \text{M}\Omega/\text{mi} \tag{2.161}$$

$$x'_d = \text{capacitive reactance spacing factor}$$

$$= 0.06836 \log_{10} D \quad \text{M}\Omega/\text{mi} \tag{2.162}$$

Tables given in Appendix A provide x'_a and x'_d directly. The term X'_d is added or subtracted from x'_a depending on the magnitude of D.

2.16 EQUIVALENT CIRCUITS FOR TRANSMISSION LINES

An overhead line or a cable can be represented as a distributed constant circuit, as shown in Figure 2.22. The resistance, inductance, capacitance, and leakage conductance of a distributed constant circuit are distributed uniformly along the line length. In the figure, L represents the inductance of a line conductor to neutral per unit length, r represents the ac resistance of a line conductor per unit length, C is the capacitance of a line conductor to neutral per unit length, and G is the leakage conductance per unit length.

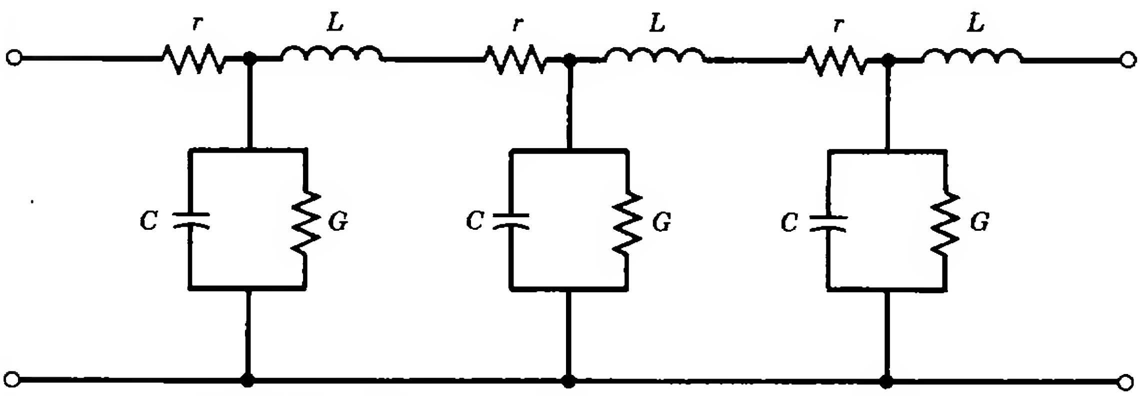


Figure 2.22. Distributed constant equivalent circuit of line.

2.17 SHORT TRANSMISSION LINES (UP TO 50 mi, OR 80 km)

In the case of a short transmission line, the capacitance and leakage resistance to the earth are usually neglected, as shown in Figure 2.23. Therefore, the transmission line can be treated as a simple, lumped, and constant impedance, that is,

$$\begin{aligned} \mathbf{Z} &= R + jX_L \\ &= \mathbf{z}l \\ &= rl + jxl \quad \Omega \end{aligned} \tag{2.163}$$

where $\mathbf{Z}$ = total series impedance per phase in ohms
$\mathbf{z}$ = series impedance of one conductor in ohms per unit length
X_L = total inductive reactance of one conductor in ohms
x = inductive reactance of one conductor in ohms per unit length
l = length of line

The current entering the line at the sending end of the line is equal to the current leaving at the receiving end. Figures 2.24 and 2.25 show vector (or phasor) diagrams for a short transmission line connected to an inductive load and a capacitive load, respectively. It can be observed from the figures that

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{I}_R\mathbf{Z} \tag{2.164}$$

$$\mathbf{I}_S = \mathbf{I}_R = \mathbf{I} \tag{2.165}$$

$$\mathbf{V}_R = \mathbf{V}_S - \mathbf{I}_R\mathbf{Z} \tag{2.166}$$

where $\mathbf{V}_S$ = sending-end phase (line-to-neutral) voltage
$\mathbf{V}_R$ = receiving-end phase (line-to-neutral) voltage
$\mathbf{I}_S$ = sending-end phase current

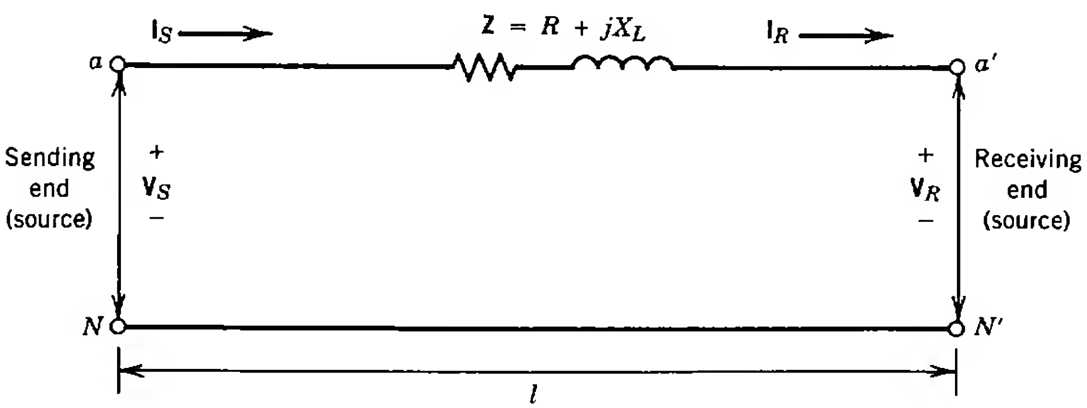


Figure 2.23. Equivalent circuit of short transmission line.

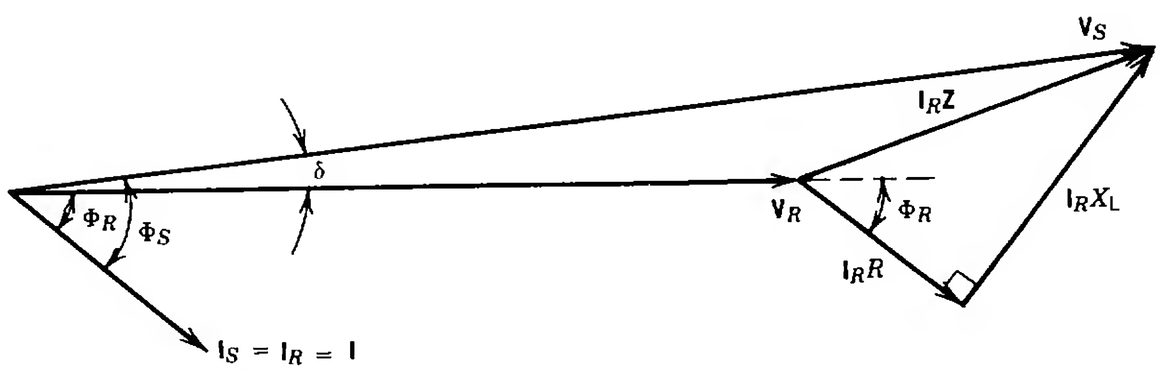


Figure 2.24. Phasor diagram of short transmission line to inductive load.

$\mathbf{I}_R$ = receiving-end phase current
$\mathbf{Z}$ = total series impedance per phase

Therefore, using $\mathbf{V}_R$ as the reference, equation (2.164) can be written as

$$\mathbf{V}_S = \mathbf{V}_R + (I_R \cos \Phi_R \pm jI_R \sin \Phi_R)(R + jX_L) \tag{2.167}$$

where the plus or minus sign is determined by Φ_R, the power factor angle of the receiving end or load. If the power factor is lagging, the minus sign is employed. On the other hand, if it is leading, the plus sign is used.

However, if equation (2.166) is used, it is convenient to use $\mathbf{V}_S$ as the reference. Therefore,

$$\mathbf{V}_R = \mathbf{V}_S - (I_S \cos \Phi_S \pm jI_S \sin \Phi_S)(R + jX) \tag{2.168}$$

where Φ_S is the sending-end power factor angle, that determines, as before, whether the plus or minus sign will be used. Also, from Figure 2.24, using $\mathbf{V}_R$ as the reference vector,

$$\mathbf{V}_S = \sqrt{(\mathbf{V}_R + IR \cos \Phi_R + IX \sin \Phi_R)^2 + (IX \cos \Phi_R \pm IR \sin \Phi_R)^2} \tag{2.169}$$

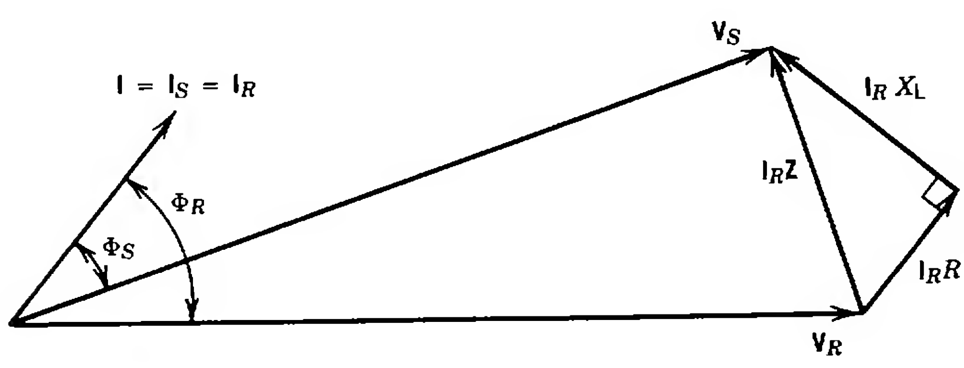


Figure 2.25. Phasor diagram of short transmission line connected to capacitive load.

and load angle

$$\delta = \Phi_S - \Phi_R \tag{2.170}$$

or

$$\delta = \tan^{-1} \frac{IX \cos \Phi_R \pm IR \sin \Phi_R}{\mathbf{V}_R + IR \cos \Phi_R + IX \sin \Phi_R} \tag{2.171}$$

The generalized constants, or **ABCD** parameters, can be determined by inspection of Figure 2.23. Since

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.172}$$

and $\mathbf{AD} - \mathbf{BC} = 1$, where

$$\mathbf{A} = 1 \qquad \mathbf{B} = Z \qquad \mathbf{C} = 0 \qquad \mathbf{D} = 1 \tag{2.173}$$

then

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.174}$$

and

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 1 & -\mathbf{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix}$$

The transmission efficiency of the short line can be expressed as

$$\begin{aligned} \eta &= \frac{\text{output}}{\text{input}} \\ &= \frac{\sqrt{3} V_R I \cos \Phi_R}{\sqrt{3} V_S I \cos \Phi_S} \\ &= \frac{V_R \cos \Phi_R}{V_S \cos \Phi_S} \end{aligned} \tag{2.175}$$

Equation (2.175) is applicable whether the line is single phase or three phase.

The transmission efficiency can also be expressed as

$$\eta = \frac{\text{output}}{\text{output} + \text{losses}}$$

For a single-phase line,

$$\eta = \frac{V_R I \cos \Phi_R}{V_R I \cos \Phi_R + 2I^2 R} \tag{2.176}$$

For a three-phase line,

$$\eta = \frac{\sqrt{3} V_R I \cos \Phi_R}{\sqrt{3} V_R I \cos \Phi_R + 3I^2 R} \tag{2.177}$$

2.17.1 Steady-State Power Limit

Assume that the impedance of a short transmission line is given as $\mathbf{Z} = Z\angle\theta$. Therefore, the real power delivered, at steady state, to the receiving end of the transmission line can be expressed as

$$P_R = \frac{V_S \times V_R}{Z} \cos(\theta - \delta) - \frac{V_R^2}{Z} \cos \theta \tag{2.178}$$

and similarly, the reactive power delivered can be expressed as

$$Q_R = \frac{V_S \times V_R}{Z} \sin(\theta - \delta) - \frac{V_R^2}{Z} \sin \theta \tag{2.179}$$

If V_S and V_R are the line-to-neutral voltages, equations (2.178) and (2.179) give P_R and Q_R values per phase. On the other hand, if the obtained P_R and Q_R values are multiplied by 3 or the line-to-line values of V_S and V_R are employed, the equations give the three-phase real and reactive power delivered to a balanced load at the receiving end of the line.

If, in equation (2.178), all variables are kept constant with the exception of δ, so that the real power delivered, P_R, is a function of δ only, P_R is maximum when $\delta = \theta$, and the maximum power† obtainable at the receiving end for a given regulation can be expressed as

$$P_{R,\max} = \frac{V_R^2}{Z^2} \left(\frac{V_S}{V_R} Z - R \right) \tag{2.180}$$

where V_S and V_R are the phase (line-to-neutral) voltages whether the system is single phase or three phase.

The equation can also be expressed as

$$P_{R,\max} = \frac{V_S \times V_R}{Z} - \frac{V_R^2 \times \cos \theta}{Z} \tag{2.181}$$

† Also called the steady-state power limit.

If $V_S = V_R$,

$$P_{R,\max} = \frac{V_R^2}{Z}(1 - \cos\theta) \tag{2.182}$$

or

$$P_{R,\max} = \left(\frac{V_R}{Z}\right)^2 (Z - R) \tag{2.183}$$

and similarly, the corresponding reactive power delivered to the load is given by

$$Q_R = -\frac{V_R^2}{Z}\sin\theta \tag{2.184}$$

As can be observed, both equations (2.183) and (2.184) are independent of V_S voltage. The negative sign in equation (2.184) points out that the load is a sink of leading vars,[†] that is, going to the load or a source of lagging vars (i.e., from the load to the supply). The total three-phase power transmitted on the three-phase line is three times the power calculated by using the above equations. If the voltages are given in volts, the power is expressed in watts or vars. Otherwise, if they are given in kilovolts, the power is expressed in megawatts or megavars.

In a similar manner, the real and reactive powers for the sending end of a transmission line can be expressed as

$$P_S = \frac{V_S^2}{Z}\cos\theta - \frac{V_S \times V_R}{Z}\cos(\theta + \delta) \tag{2.185}$$

and

$$Q_S = \frac{V_S^2}{Z}\sin\theta - \frac{V_S \times V_R}{Z}\sin(\theta + \delta) \tag{2.186}$$

If, in equation (2.185), as before, all variables are kept constant with the exception of δ, so that the real power at the sending end, P_S, is a function of δ only, P_S is a maximum when

$$\theta + \delta = 180°$$

[†] For many decades, the electrical utility industry has declined to recognize two different kinds of reactive power, *leading* and *lagging vars*. Only *magnetizing vars* are recognized, printed on varmeter scals plates, bought, and sold. Therefore, in the following sections, the leading or lagging vars will be referred to as magnetizing vars.

Therefore, the maximum power at the sending end, the maximum input power, can be expressed as

$$P_{S,\max} = \frac{V_S^2}{Z}\cos\theta + \frac{V_S \times V_R}{Z} \tag{2.187}$$

or

$$P_{S,\max} = \frac{V_S^2 \times R}{Z^2} + \frac{V_S \times V_R}{Z} \tag{2.188}$$

However, if $V_S = V_R$,

$$P_{S,\max} = \left(\frac{V_S}{Z}\right)^2 (Z + R) \tag{2.189}$$

and similarly, the corresponding reactive power at the sending end, the maximum input vars, is given by

$$Q_S = \frac{V_S^2}{Z}\sin\theta \tag{2.190}$$

As can be observed, both equations (2.189) and (2.190) are independent of V_R voltage, and equation (2.190) has a positive sign this time.

2.17.2 Percent Voltage Regulation

The voltage regulation of the line is defined by the rise in voltage when full load is removed, that is,

$$\text{Percentage of voltage regulation} = \frac{|\mathbf{V}_S| - |\mathbf{V}_R|}{|\mathbf{V}_R|} \times 100 \tag{2.191}$$

or

$$\text{Percentage of voltage regulation}^{\dagger} = \frac{|\mathbf{V}_{R,\mathrm{NL}}| - |\mathbf{V}_{R,\mathrm{FL}}|}{|\mathbf{V}_{R,\mathrm{FL}}|} \times 100 \tag{2.192}$$

where $|\mathbf{V}_S|$ = magnitude of sending-end phase (line-to-neutral) voltage at no load

$|\mathbf{V}_R|$ = magnitude of receiving-end phase (line-to-neutral) voltage at full load

† For further information see Stevenson [3, p. 97].

$|\mathbf{V}_{R,\mathrm{NL}}|$ = magnitude of receiving-end voltage at no load
$|\mathbf{V}_{R,\mathrm{FL}}|$ = magnitude of receiving-end voltage at full load with constant $|\mathbf{V}_S|$

Therefore, if the load is connected at the receiving end of the line,

$$|\mathbf{V}_S| = |\mathbf{V}_{R,\mathrm{NL}}|$$

and

$$|\mathbf{V}_R| = |\mathbf{V}_{R,\mathrm{FL}}|$$

An approximate expression for percentage of voltage regulation is

$$\text{Percentage of voltage regulation} \cong I_R \frac{(R \cos \Phi_R \pm X \sin \Phi_R)}{V_R} \times 100 \tag{2.193}$$

EXAMPLE 2.5

A three-phase, 60-Hz overhead short transmission line has a line-to-line voltage of 23 kV at the receiving end, a total impedance of $2.48 \pm j6.57\ \Omega$/phase, and a load of 9 MW with a receiving-end lagging power factor of 0.85.

(a) Calculate line-to-neutral and line-to-line voltages at sending end.
(b) Calculate load angle.

Solution

Method I: Using complex algebra:

(a) The line-to-neutral reference voltage is

$$\mathbf{V}_{R(\mathrm{L\text{-}N})} = \frac{\mathbf{V}_{R(\mathrm{L\text{-}L})}}{\sqrt{3}}$$

$$= \frac{23 \times 10^3 \angle 0^\circ}{\sqrt{3}} = 13{,}294.8 \angle 0^\circ \text{ V}$$

The line current is

$$\mathbf{I} = \frac{9 \times 10^6}{\sqrt{3} \times 23 \times 10^3 \times 0.85} \times (0.85 - j0.527)$$

$$= 266.1(0.85 - j0.527)$$

$$= 226.19 - j140.24 \text{ A}$$

Therefore,

$$\begin{aligned}\mathbf{IZ} &= (226.19 - j140.24)(2.48 + j6.57)\\ &= 266.1 \underline{/-31.8^\circ} \times 7.02 \underline{/69.32^\circ}\\ &= 1868.95 \underline{/37.52^\circ} \text{ V}\end{aligned}$$

Thus, the line-to-neutral voltage at the sending end is

$$\begin{aligned}\mathbf{V}_{S(\text{L-N})} &= \mathbf{V}_{R(\text{L-N})} + \mathbf{IZ}\\ &= 14{,}820 \underline{/4.4^\circ} \text{ V}\end{aligned}$$

The line-to-line voltage at the sending end is

$$\begin{aligned}V_{S(\text{L-L})} &= \sqrt{3} V_{S(\text{L-N})}\\ &\cong 25{,}640 \text{ V}\end{aligned}$$

(b) The load angle is 4.4°.

Method II. Using **I** as the reference:

(a) $V_R \cos \Phi_R + IR = 13{,}294.8 \times 0.85 + 266.1 \times 2.48 \cong 11{,}960$

$V_R \sin \Phi_R + IX = 13{,}294.8 \times 0.527 + 266.1 \times 6.57 \cong 8754$

Then

$$\begin{aligned}V_{S(\text{L-N})} &= (11{,}960.5^2 + 8754.66^2)^{1/2}\\ &\cong 14{,}820 \text{ V/phase}\\ V_{S(\text{L-L})} &\cong 25{,}640 \text{ V}\end{aligned}$$

(b) $\Phi_S = \Phi_R + \delta = \tan^{-1} \dfrac{8754}{11{,}960} = 36.2^\circ$

$\delta = \Phi_S - \Phi_R = 36.2 - 31.8 = 4.4^\circ$

Method III. Using $\mathbf{V}_R$ as the reference:

(a) $$\begin{aligned}V_{S(\text{L-N})} = [(&V_R + IR \cos \Phi_R + IX \sin \Phi_R)^2\\ &+ (IX \cos \Phi_R - IR \sin \Phi_R)^2]^{1/2}\end{aligned}$$

$IR \cos \Phi_R = 266.1 \times 2.48 \times 0.85 = 560.9$

$IR \sin \Phi_R = 266.1 \times 2.48 \times 0.527 = 347.8$

$IX \cos \Phi_R = 266.1 \times 6.57 \times 0.85 = 1486.0$

$IX \sin \Phi_R = 266.1 \times 6.57 \times 0.527 = 921.0$

Therefore,

$$V_{S(\text{L-N})} = [(13{,}294.8 + 560.9 + 921.0)^2 + (1486.0 - 347.8)^2]^{1/2}$$
$$= [14{,}776.7^2 + 1138.2^2]^{1/2}$$
$$\cong 14{,}820 \text{ V}$$
$$V_{S(\text{L-L})} = \sqrt{3} V_{S(\text{L-L})}$$
$$\cong 25{,}640 \text{ V}$$

(b) $\delta = \tan^{-1} \dfrac{1138.2}{14{,}776.7} = 4.4°$

Method IV. Using power relationships:
Power loss in the line is

$$P_{\text{loss}} = 3I^2R$$
$$= 3 \times 266.1^2 \times 2.48 \times 10^{-6} = 0.527 \text{ MW}$$

Total input power to the line is

$$P_T = P + P_{\text{loss}}$$
$$= 9 + 0.527 = 9.527 \text{ MW}$$

Var loss in the line is

$$Q_{\text{loss}} = 3I^2X$$
$$= 3 \times 266.1^2 \times 6.57 \times 10^{-6} = 1.396 \text{ Mvar lagging}$$

Total megavar input to the line is

$$Q_T = \frac{P \sin \Phi_R}{\cos \Phi_R} + Q_{\text{loss}}$$
$$= \frac{9 \times 0.527}{0.85} + 1.396 = 6.976 \text{ Mvar lagging}$$

Total megavoltampere input to the line is

$$S_T = \sqrt{P_T^2 + Q_T^2}$$
$$= \sqrt{9.527^2 + 6.976^2} = 11.81 \text{ MVA}$$

(a) $V_{S(\text{L-L})} = \dfrac{S_T}{\sqrt{3} I}$

$$= \frac{11.81 \times 10^6}{\sqrt{3} \times 266.1} \cong 25{,}640 \text{ V}$$

$$V_{S(\text{L-N})} = \frac{V_{S(\text{L-L})}}{\sqrt{3}} = 14{,}820 \text{ V}$$

(b) $\cos \Phi_S = \dfrac{P_T}{S_T}$

$$= \frac{9.527}{11.81} = 0.807 \text{ lagging}$$

Therefore,

$$\Phi_S = 36.2°$$

$$\delta = 36.2° - 31.8° = 4.4°$$

Method V. Treating the three-phase line as a single-phase line and having V_S and V_R represent line-to-line voltages, not line-to-neutral voltages:

(a) Power delivered is 4.5 MW

$$I_{\text{line}} = \frac{4.5 \times 10^6}{23 \times 10^3 \times 0.85} = 230.18 \text{ A}$$

$$R_{\text{loop}} = 2 \times 2.48 = 4.96\ \Omega$$

$$X_{\text{loop}} = 2 \times 6.57 = 13.14\ \Omega$$

$$V_R \cos \Phi_R = 23 \times 10^3 \times 0.85 = 19{,}550 \text{ V}$$

$$V_R \sin \Phi_R = 23 \times 10^3 \times 0.527 = 12{,}121 \text{ V}$$

$$IR = 230.18 \times 4.96 = 1141.7 \text{ V}$$

$$IX = 230.18 \times 13.14 = 3024.6 \text{ V}$$

Therefore,

$$\begin{aligned} V_{S(\text{L-L})} &= [(V_R \cos \Phi_R + IR)^2 + (V_R \sin \Phi_R + IX)^2]^{1/2} \\ &= [(19{,}550 + 1141.7)^2 + (12{,}121 + 3024.6)^2]^{1/2} \\ &= [20{,}691.7^2 + 15{,}145.6^2]^{1/2} \\ &\cong 25{,}640 \text{ V} \end{aligned}$$

Thus,

$$\begin{aligned} V_{S(\text{L-N})} &= \frac{V_{S(\text{L-L})}}{\sqrt{3}} \\ &= 14{,}820 \text{ V} \end{aligned}$$

(b) $\Phi_S = \tan^{-1} \dfrac{15{,}145.6}{20{,}691.7} = 36.2°$

$\delta = 36{:}2° - 31.8° = 4.4°$

Example 2.6

Calculate percentage of voltage regulation for the values given in Example 2.5.

(a) Using equation (2.191).
(b) Using equation (2.193).

Solution

(a) Using equation (2.191),

$$\text{Percentage of voltage regulation} = \frac{|\mathbf{V}_S| - |\mathbf{V}_R|}{|\mathbf{V}_R|} \times 100$$

$$= \frac{14{,}820 - 13{,}294.8}{13{,}294.8} \times 100$$

$$= 11.5$$

(b) Using equation (2.193),

Percentage of voltage regulation

$$= I_R \frac{R \cos \Phi_R + X \sin \Phi_R}{V_R} \times 100$$

$$= 266.1 \frac{2.48 \times 0.85 + 6.57 \times 0.527}{13{,}294.8} \times 100$$

$$= 11.2$$

2.17.3 Representation of Mutual Impedance of Short Lines

Figure 2.26(a) shows a circuit of two lines, x and y, that have self-impedances of $\mathbf{Z}_{xx}$ and $\mathbf{Z}_{yy}$ and mutual impedance of $\mathbf{Z}_{xy}$. Its equivalent circuit is shown in Figure 2.26(b). Sometimes, it may be required to preserve the electrical identity of the two lines, as shown in Figure 2.27. The mutual impedance $\mathbf{Z}_{xy}$ can be in either line and transferred to the other by means of a transformer that has a 1:1 turns ratio. This technique is also applicable for three-phase lines.

Example 2.7

Assume that the mutual impedance between two parallel feeders is $0.09 + j0.3\ \Omega/\text{mi}$ per phase. The self-impedances of the feeders are $0.604 \underline{/50.4^\circ}$ and $0.567 \underline{/52.9^\circ}\ \Omega/\text{mi}$ per phase, respectively. Represent the mutual impedance between the feeders as shown in Figure 2.26(b).

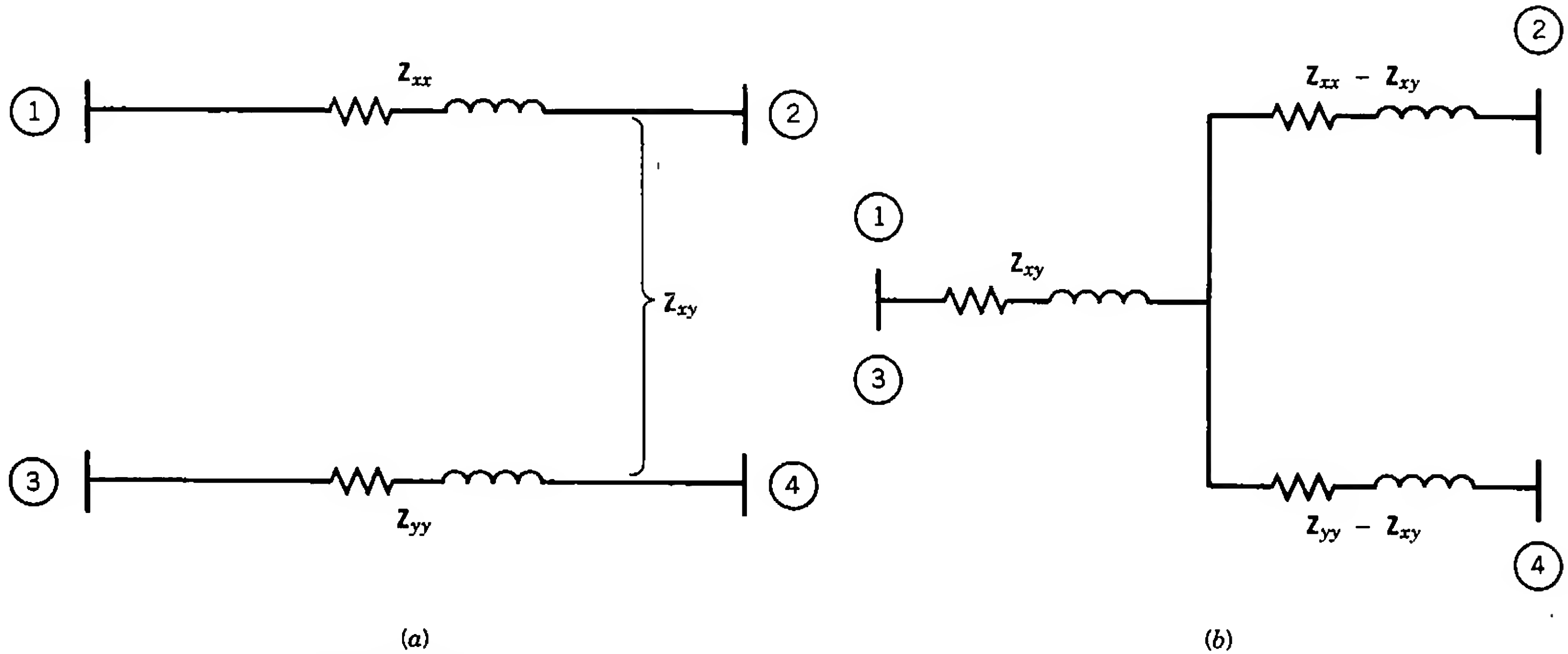


Figure 2.26. Representation of mutual impedance between two circuits.

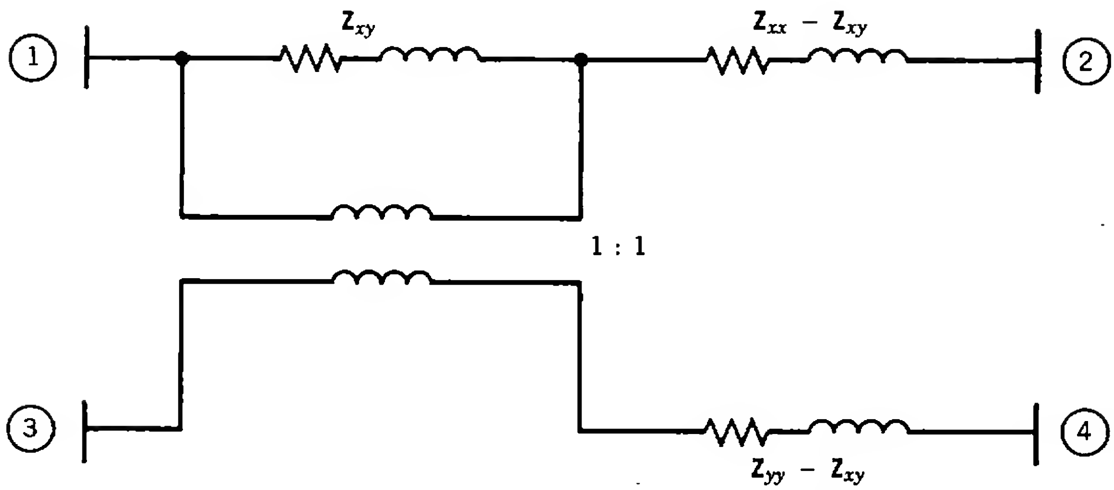


Figure 2.27. Representation of mutual impedance between two circuits by means of 1:1 transformer.

Solution

$$\mathbf{Z}_{xy} = 0.09 + j0.3\ \Omega$$

$$\mathbf{Z}_{xx} = 0.604\ \underline{/50.4^\circ} = 3.85 + j0.465\ \Omega$$

$$\mathbf{Z}_{yy} = 0.567\ \underline{/52.9^\circ} = 0.342 + j0.452\ \Omega$$

Therefore,

$$\mathbf{Z}_{xx} - \mathbf{Z}_{xy} = 0.295 + j0.165\ \Omega$$

$$\mathbf{Z}_{yy} - \mathbf{Z}_{xy} = 0.252 + j0.152\ \Omega$$

Hence, the resulting equivalent circuit is shown in Figure 2.28.

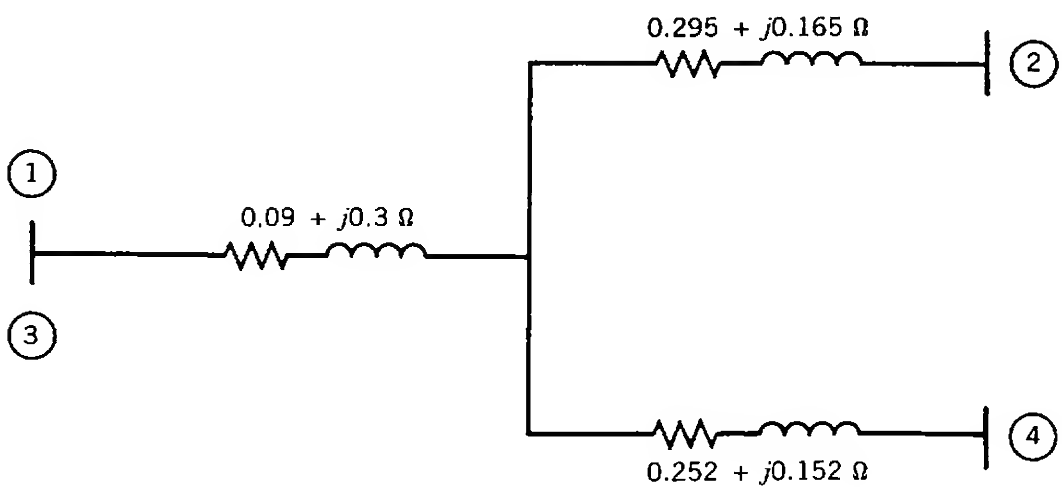


Figure 2.28

2.18 MEDIUM-LENGTH TRANSMISSION LINES (UP TO 150 mi, OR 240 km)

As the line length and voltage increase, the use of the formulas developed for the short transmission lines give inaccurate results. Therefore, the effect of the current leaking through the capacitance must be taken into account for a better approximation. Thus, the shunt admittance is "lumped" at a few points along the line and represented by forming either a T or a Π network, as shown in Figures 2.29 and 2.30.

In the figures,

$$\mathbf{Z} = \mathbf{z}l$$

For the T circuit shown in Figure 2.29,

$$\begin{aligned}\mathbf{V}_S &= \mathbf{I}_S \times \tfrac{1}{2}\mathbf{Z} + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z} + \mathbf{V}_R \\ &= [\mathbf{I}_R + (\mathbf{V}_R + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z})\mathbf{Y}]\tfrac{1}{2}\mathbf{Z} + \mathbf{V}_R + \mathbf{I}_R\tfrac{1}{2}\mathbf{Z}\end{aligned}$$

or

$$\mathbf{V}_S = \underbrace{(1 + \tfrac{1}{2}\mathbf{ZY})}_{\mathbf{A}}V_R + \underbrace{(\mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2)}_{\mathbf{B}}I_R \tag{2.194}$$

and

$$\mathbf{I}_S = \mathbf{I}_R + (\mathbf{V}_R + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z})\mathbf{Y}$$

or

$$\mathbf{I}_S = \underbrace{\mathbf{Y}}_{\mathbf{C}} \times \mathbf{V}_R + \underbrace{(1 + \tfrac{1}{2}\mathbf{ZY})}_{\mathbf{D}}\mathbf{I}_R \tag{2.195}$$

Alternatively, neglecting conductance so that

$$\mathbf{I}_C = \mathbf{I}_Y$$

and

$$\mathbf{V}_C = \mathbf{V}_Y$$

yields

$$\begin{aligned}\mathbf{I}_C &= \mathbf{V}_C \times \mathbf{Y} \\ \mathbf{V}_C &= \mathbf{V}_R + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z}\end{aligned}$$

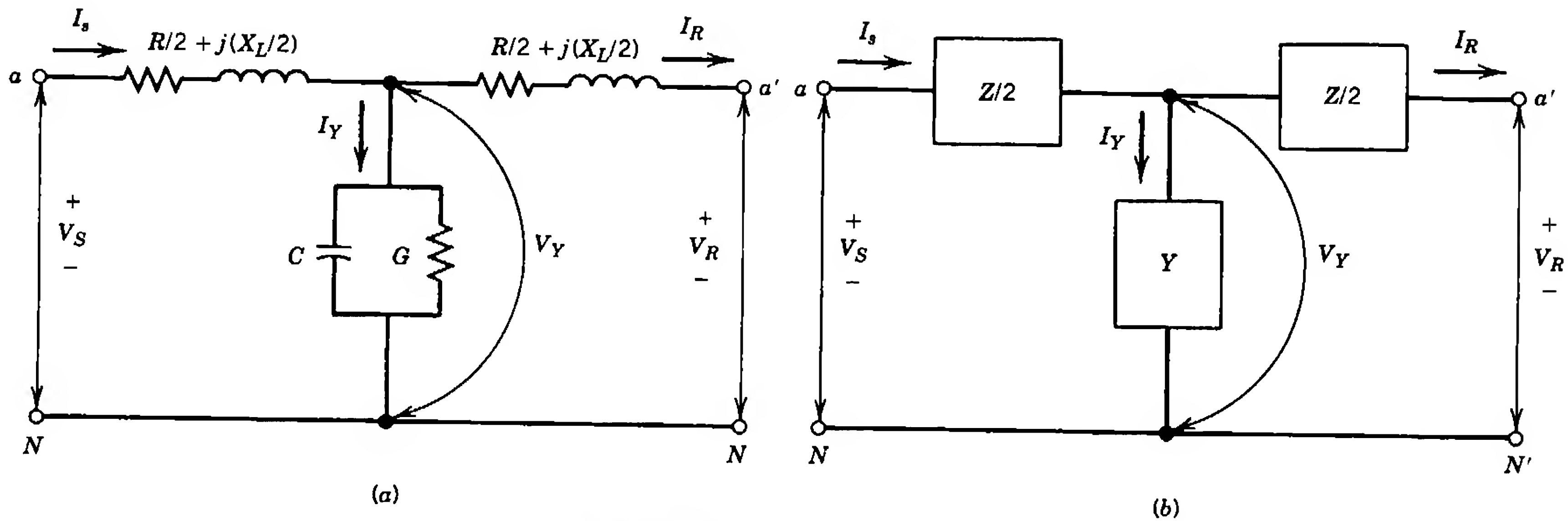


Figure 2.29. Nominal-T circuit.

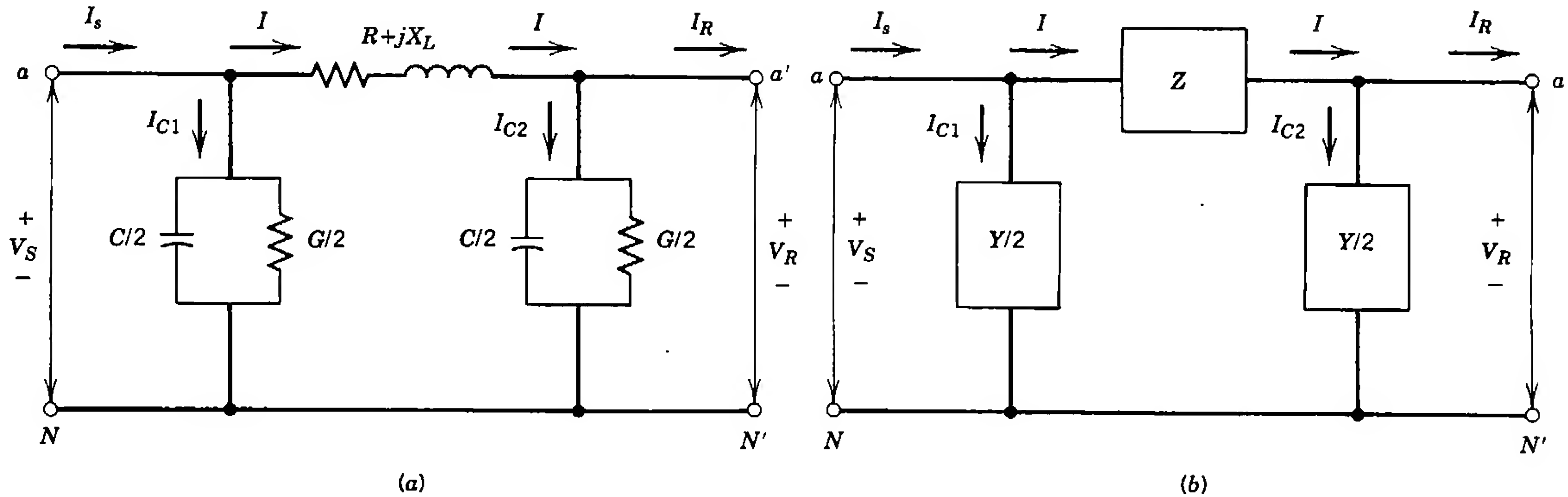


Figure 2.30. Nominal-Π circuit.

Hence,

$$\begin{aligned}\mathbf{V}_S &= \mathbf{V}_C + \mathbf{I}_S \times \tfrac{1}{2}\mathbf{Z} \\ &= \mathbf{V}_R + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z} + [\mathbf{V}_R\mathbf{Y} + \mathbf{I}_R(1 + \tfrac{1}{2}\mathbf{YZ})](\tfrac{1}{2}\mathbf{Z})\end{aligned}$$

or

$$\mathbf{V}_S = \underbrace{(1 + \tfrac{1}{2}\mathbf{YZ})}_{\mathbf{A}}\mathbf{V}_R + \underbrace{(\mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2)}_{\mathbf{B}}\mathbf{I}_R \tag{2.196}$$

Also,

$$\begin{aligned}\mathbf{I}_S &= \mathbf{I}_R + \mathbf{I}_C \\ &= \mathbf{I}_R + \mathbf{V}_C \times \mathbf{Y} \\ &= \mathbf{I}_R + (\mathbf{V}_R + \mathbf{I}_R \times \tfrac{1}{2}\mathbf{Z})\mathbf{Y}\end{aligned}$$

Again,

$$\mathbf{I}_S = \underbrace{\mathbf{Y}}_{\mathbf{C}} \times \mathbf{V}_R + \underbrace{(1 + \tfrac{1}{2}\mathbf{ZY})}_{\mathbf{D}}\mathbf{I}_R \tag{2.197}$$

Since

$$\mathbf{A} = 1 + \tfrac{1}{2}\mathbf{YZ} \tag{2.198}$$

$$\mathbf{B} = \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \tag{2.199}$$

$$\mathbf{C} = \mathbf{Y} \tag{2.200}$$

$$\mathbf{D} = 1 + \tfrac{1}{2}\mathbf{YZ} \tag{2.201}$$

for a nominal-T circuit, the general circuit parameter matrix, or transfer matrix, becomes

$$\begin{bmatrix}\mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D}\end{bmatrix} = \begin{bmatrix}1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \\ \mathbf{Y} & 1 + \tfrac{1}{2}\mathbf{YZ}\end{bmatrix}$$

Therefore,

$$\begin{bmatrix}\mathbf{V}_S \\ \mathbf{I}_S\end{bmatrix} = \begin{bmatrix}1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \\ \mathbf{Y} & 1 + \tfrac{1}{2}\mathbf{YZ}\end{bmatrix}\begin{bmatrix}\mathbf{V}_R \\ \mathbf{I}_R\end{bmatrix} \tag{2.202}$$

and

$$\begin{bmatrix}\mathbf{V}_R \\ \mathbf{I}_R\end{bmatrix} = \begin{bmatrix}1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \\ \mathbf{Y} & 1 + \tfrac{1}{2}\mathbf{YZ}\end{bmatrix}^{-1}\begin{bmatrix}\mathbf{V}_S \\ \mathbf{I}_S\end{bmatrix} \tag{2.203}$$

For the Π circuit shown in Figure 2.30,

$$\mathbf{V}_S = (\mathbf{V}_R \times \tfrac{1}{2}\mathbf{Y} + \mathbf{I}_R)\mathbf{Z} + \mathbf{V}_R$$

or

$$\mathbf{V}_S = \underbrace{(1 + \tfrac{1}{2}\mathbf{YZ})}_{\mathbf{A}}\mathbf{V}_R + \underbrace{\mathbf{Z}}_{\mathbf{B}} \times \mathbf{I}_R \tag{2.204}$$

and

$$\mathbf{I}_S = \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_S + \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_R + \mathbf{I}_R \tag{2.205}$$

By substituting equation (2.204) into equation (2.205),

$$\mathbf{I}_S = [(1 + \tfrac{1}{2}\mathbf{YZ})\mathbf{V}_R + \mathbf{Z}\mathbf{I}_R]\tfrac{1}{2}\mathbf{Y} + \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_R + \mathbf{I}_R$$

or

$$\mathbf{I}_S = \underbrace{(\mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z})}_{\mathbf{C}}\mathbf{V}_R + \underbrace{(1 + \tfrac{1}{2}\mathbf{YZ})}_{\mathbf{D}}\mathbf{I}_R \tag{2.206}$$

Alternatively, neglecting conductance,

$$\mathbf{I} = \mathbf{I}_{C2} + \mathbf{I}_R$$

where

$$\mathbf{I}_{C2} = \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_R$$

yields

$$\mathbf{I} = \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_R + \mathbf{I}_R \tag{2.207}$$

Also,

$$\mathbf{V}_S = \mathbf{V}_R + \mathbf{IZ} \tag{2.208}$$

By substituting equation (2.207) into equation (2.208),

$$\mathbf{V}_S = \mathbf{V}_R + (\tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_R + \mathbf{I}_R)\mathbf{Z}$$

or

$$\mathbf{V}_S = \underbrace{(1 + \tfrac{1}{2}\mathbf{YZ})}_{\mathbf{A}}\mathbf{V}_R + \underbrace{\mathbf{Z}}_{\mathbf{B}} \times \mathbf{I}_R \tag{2.209}$$

and

$$\mathbf{I}_{C1} = \tfrac{1}{2}\mathbf{Y} \times \mathbf{V}_S \tag{2.210}$$

By substituting equation (2.209) into equation (2.210),

$$\mathbf{I}_{C1} = \tfrac{1}{2}\mathbf{Y}(1 + \tfrac{1}{2}\mathbf{YZ})\mathbf{V}_R + \tfrac{1}{2}\mathbf{Y} \times \mathbf{Z}\mathbf{I}_R \tag{2.211}$$

and since

$$\mathbf{I}_S = \mathbf{I} + \mathbf{I}_{C1} \tag{2.212}$$

by substituting equation (2.207) into equation (2.212),

$$\mathbf{I}_S = \tfrac{1}{2}\mathbf{Y}\mathbf{V}_R + \mathbf{I}_R + \tfrac{1}{2}\mathbf{Y}(1 + \tfrac{1}{2}\mathbf{YZ})\mathbf{V}_R + \tfrac{1}{2}\mathbf{YZ}\mathbf{I}_R$$

or

$$\mathbf{I}_S = \underbrace{(\mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z})}_{\mathbf{C}}\mathbf{V}_R + \underbrace{(1 + \tfrac{1}{2}\mathbf{YZ})}_{\mathbf{D}}\mathbf{I}_R \tag{2.213}$$

Since

$$\mathbf{A} = 1 + \tfrac{1}{2}\mathbf{YZ} \tag{2.214}$$

$$\mathbf{B} = \mathbf{Z} \tag{2.215}$$

$$\mathbf{C} = \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z} \tag{2.216}$$

$$\mathbf{D} = 1 + \tfrac{1}{2}\mathbf{YZ} \tag{2.217}$$

for a nominal-Π circuit, the general circuit parameter matrix becomes

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} \\ \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z} & 1 + \tfrac{1}{2}\mathbf{YZ} \end{bmatrix} \tag{2.218}$$

Therefore,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} \\ \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z} & 1 + \tfrac{1}{2}\mathbf{YZ} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.219}$$

and

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} 1 + \tfrac{1}{2}\mathbf{YZ} & \mathbf{Z} \\ \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z} & 1 + \tfrac{1}{2}\mathbf{YZ} \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} \tag{2.220}$$

As can be proved easily by using a delta–wye transformation, the nominal-T and nominal-Π circuits are not equivalent to each other. This result is to be expected since two different approximations are made to the actual circuit, neither of which is absolutely correct. More accurate results can be obtained by splitting the line into several segments, each given by its nominal-T or nominal-Π circuits and cascading the resulting segments.

Here, the power loss in the line is given as

$$P_{\text{loss}} = I^2R \tag{2.221}$$

which varies approximately as the square of the through-line current. The reactive powers absorbed and supplied by the line are given as

$$Q_L = I^2X_L \tag{2.222}$$

and

$$Q_C = V^2b \tag{2.223}$$

respectively. The Q_L varies approximately as the square of the through line current, whereas the Q_C varies approximately as the square of the mean line voltage. The result is that increasing transmission voltages decrease the reactive power absorbed by the line for heavy loads and increase the reactive power supplied by the line for light loads.

The percentage of voltage regulation for the medium-length transmission lines is given by Stevenson [3] as

$$\text{Percentage of voltage regulation} = \frac{|\mathbf{V}_S|/|\mathbf{A}| - |\mathbf{V}_{R,\text{FL}}|}{|\mathbf{V}_{R,\text{FL}}|} \times 100 \tag{2.224}$$

where $|\mathbf{V}_S|$ = magnitude of sending-end phase (line-to-neutral) voltage

$|\mathbf{V}_{R,\text{FL}}|$ = magnitude of receiving-end phase (line-to-neutral) voltage at full load with constant $|\mathbf{V}_S|$

$|\mathbf{A}|$ = magnitude of line constant A

Example 2.8

A three-phase 138-kV transmission line is connected to a 49-MW load at a 0.85 lagging power factor. The line constants of the 52-mi-long line are $\mathbf{Z} = 95\,\underline{/78^\circ}\ \Omega$ and $\mathbf{Y} = 0.001\,\underline{/90^\circ}$ S. Using nominal-T circuit representation, calculate:

(a) The **A**, **B**, **C**, and **D** constants of the line.

(b) Sending-end voltage.

(c) Sending-end current.
(d) Sending-end power factor.
(e) Efficiency of transmission.

Solution

$$V_{R(\text{L-N})} = \frac{138 \text{ kV}}{\sqrt{3}} = 79{,}768.8 \text{ V}$$

Using the receiving-end voltage as the reference,

$$\mathbf{V}_{R(\text{L-N})} = 79{,}768.8 \underline{/0°} \text{ V}$$

The receiving-end current is

$$I_R = \frac{49 \times 10^6}{\sqrt{3} \times 138 \times 10^3 \times 0.85} = 241.46 \text{ A} \quad \text{or} \quad 241.46 \underline{/-31.8°} \text{ A}$$

(a) The **A**, **B**, **C**, and **D** constants for the nominal-T circuit representation are

$$\begin{aligned}
\mathbf{A} &= 1 + \tfrac{1}{2}\mathbf{YZ} \\
&= 1 + \tfrac{1}{2}(0.001 \underline{/90°})(95 \underline{/78°}) \\
&= 0.9535 + j0.0099 \\
&= 0.9536 \underline{/0.6°} \\
\mathbf{B} &= \mathbf{Z} + \tfrac{1}{4}\mathbf{YZ}^2 \\
&= 95 \underline{/78°} + \tfrac{1}{4}(0.001 \underline{/90°})(95 \underline{/78°})^2 \\
&= 18.83 + j90.86 \\
&= 92.79 \underline{/78.3°} \ \Omega \\
\mathbf{C} &= \mathbf{Y} = 0.001 \underline{/90°} \text{ S} \\
\mathbf{D} &= 1 + \tfrac{1}{2}\mathbf{YZ} = \mathbf{A} \\
&= 0.9536 \underline{/0.6°}
\end{aligned}$$

(b) $$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 0.9536 \underline{/0.6°} & 92.79 \underline{/78.3°} \\ 0.001 \underline{/90°} & 0.9536 \underline{/0.6°} \end{bmatrix} \begin{bmatrix} 79{,}768.8 \underline{/0°} \\ 241.46 \underline{/-31.8°} \end{bmatrix}$$

The sending-end voltage is

$$\begin{aligned}
\mathbf{V}_{S(\text{L-N})} &= 0.9536 \underline{/0.6°} \times 79{,}768.8 \underline{/0°} + 92.79 \underline{/78.3°} \times 241.46 \underline{/-31.8°} \\
&= 91{,}486 + j17{,}0486 = 93{,}060.9 \underline{/10.4°} \text{ V}
\end{aligned}$$

or

$$\mathbf{V}_{S(\text{L-L})} = 160{,}995.4 \underline{/40.4°} \text{ V}$$

(c) The sending-end current is

$$\mathbf{I}_S = 0.001\,\underline{/90^\circ} \times 79{,}768.8\,\underline{/0^\circ} + 0.9536\,\underline{/0.6^\circ} \times 241.46\,\underline{/-31.8^\circ}$$

$$= 196.95 - j39.5 = 200.88\,\underline{/-11.3^\circ}\text{ A}$$

(d) The sending-end power factor is

$$\Phi_S = 10.4^\circ + 11.3^\circ = 21.7^\circ$$

$$\cos \Phi_S = 0.929$$

(e) The efficiency of transmission is

$$\eta = \frac{\text{output}}{\text{input}}$$

$$= \frac{\sqrt{3} V_R I_R \cos \Phi_R}{\sqrt{3} V_S I_S \cos \Phi_S} \times 100$$

$$= \frac{138 \times 10^3 \times 241.46 \times 0.85}{160{,}995.4 \times 200.88 \times 0.929} \times 100$$

$$= 94.27\%$$

Example 2.9

Repeat Example 2.8 using nominal-Π circuit representation.

Solution

(a) The **A**, **B**, **C**, and **D** constants for the nominal-Π circuit representation are

$$\mathbf{A} = 1 + \tfrac{1}{2}\mathbf{YZ}$$

$$= 0.9536\,\underline{/0.6^\circ}$$

$$\mathbf{B} = \mathbf{Z} = 95\underline{/78^\circ}\ \Omega$$

$$\mathbf{C} = \mathbf{Y} + \tfrac{1}{4}\mathbf{Y}^2\mathbf{Z}$$

$$= 0.001\,\underline{/90^\circ} + \tfrac{1}{4}(0.001/90^\circ)^2(95\,\underline{/78^\circ})$$

$$= -4.9379 \times 10^{-6} + j102.375 \times 10^{-5} \cong 0.001\underline{/90.3^\circ}\text{ S}$$

$$\mathbf{D} = 1 + \tfrac{1}{2}\mathbf{YZ} = \mathbf{A}$$

$$= 0.9536\,\underline{/0.6^\circ}$$

(b)
$$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} 0.9536\,\underline{/0.6^\circ} & 95\,\underline{/78^\circ} \\ 0.001\,\underline{/90.3^\circ} & 0.9536\,\underline{/0.6^\circ} \end{bmatrix} \begin{bmatrix} 79{,}768.8\,\underline{/0^\circ} \\ 241.46\,\underline{/-31.8^\circ} \end{bmatrix}$$

Therefore,

$$\mathbf{V}_{S(\text{L-N})} = 0.9536\,\underline{/0.6^\circ} \times 79{,}768.8\,\underline{/0^\circ} + 95\,\underline{/78^\circ} \times 241.46\,\underline{/-31.8^\circ}$$

$$= 91{,}940.2 + j17{,}352.8 = 93{,}563.5\,\underline{/10.7^\circ}\text{ V}$$

or

$$\mathbf{V}_{S(\text{L-L})} = 161{,}864.9 \underline{/40.7^\circ} \text{ V}$$

(c) The sending-end current is

$$\mathbf{I}_S = 0.001 \underline{/90.3^\circ} \times 79{,}768.8 \underline{/0^\circ} + 0.9536\underline{/0.6^\circ} \times 241.46 \underline{/-31.8^\circ}$$
$$= 196.53 - j39.51 = 200.46 \underline{/-11.37^\circ} \text{ A}$$

(d) The sending-end power factor is

$$\Phi_S = 10.7^\circ + 11.37^\circ = 22.07^\circ$$
$$\cos \Phi_S = 0.927$$

(e) The efficiency of transmission is

$$\eta = \frac{\sqrt{3} V_R I_R \cos \Phi_R}{\sqrt{3} V_S I_S \cos \Phi_S} \times 100$$
$$= \frac{138 \times 10^3 \times 241.46 \times 0.85}{161{,}864.9 \times 200.46 \times 0.927} \times 100$$
$$= 94.16\%$$

The discrepancy between these results and the results of Example 2.8 is due to the fact that the nominal-T and nominal-Π circuits of a medium-length line are not equivalent to each other. In fact, neither the nominal-T nor the nominal-Π equivalent circuit exactly represent the actual line.

2.19 LONG TRANSMISSION LINES (ABOVE 150 mi, OR 240 km)

A more accurate analysis of the transmission lines require that the parameters of the lines are not lumped, as before, but are distributed uniformly throughout the length of the line.

Figure 2.31 shows a uniform long line with an incremental section dx at a distance x from the receiving end, its series impedance is $\mathbf{z}\, dx$, and its shunt admittance is $\mathbf{y}\, dx$, where $\mathbf{z}$ and $\mathbf{y}$ are the impedance and admittance per unit length, respectively.

The voltage drop in the section is

$$d\mathbf{V}_x = (\mathbf{V}_x + d\mathbf{V}_x) - \mathbf{V}_x = d\mathbf{V}_x$$
$$= (\mathbf{I}_x + d\mathbf{I}_x)\mathbf{z}\, dx$$

or

$$d\mathbf{V}_x \cong \mathbf{I}_x \mathbf{z}\, dx \tag{2.225}$$

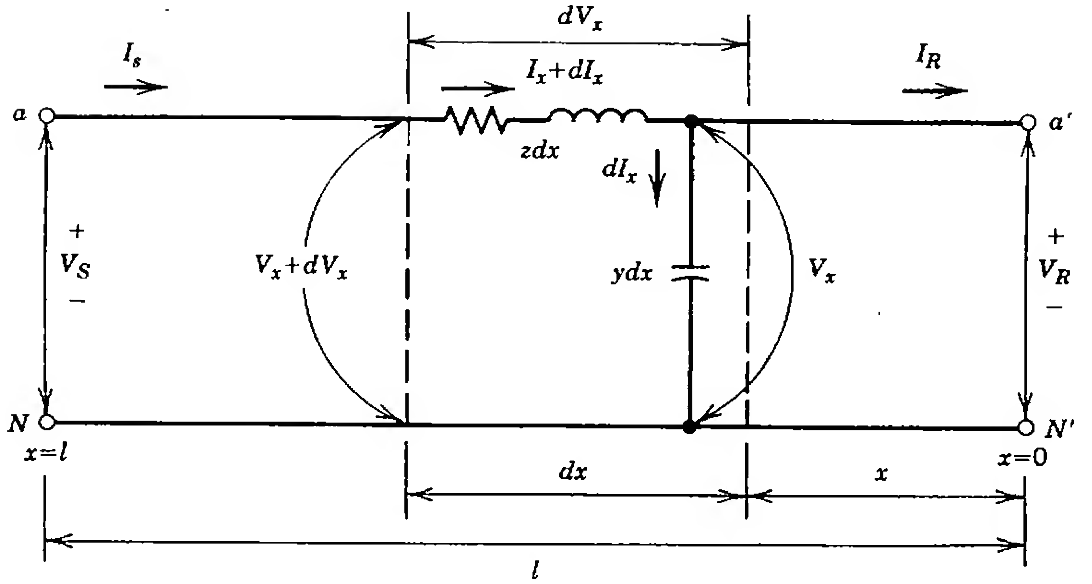


Figure 2.31. One phase and neutral connection of three-phase transmission line.

Similarly, the incremental charging current is

$$d\mathbf{I}_x \cong \mathbf{V}_x \mathbf{y}\, dx \tag{2.226}$$

Therefore,

$$\frac{d\mathbf{V}_x}{dx} = \mathbf{z}\mathbf{I}_x \tag{2.227}$$

and

$$\frac{d\mathbf{I}_x}{dx} = \mathbf{y}\mathbf{V}_x \tag{2.228}$$

Differentiating equations (2.227) and (2.228) with respect to x,

$$\frac{d^2\mathbf{V}_x}{dx^2} = \mathbf{z}\,\frac{d\mathbf{I}_x}{dx} \tag{2.229}$$

and

$$\frac{d^2\mathbf{I}_x}{dx^2} = \mathbf{y}\,\frac{d\mathbf{V}_x}{dx} \tag{2.230}$$

Substituting the values of $d\mathbf{I}_x/dx$ and $d\mathbf{V}_x/dx$ from equations (2.228) and (2.229) in equations (2.227) and (2.230), respectively,

$$\frac{d^2\mathbf{V}_x}{dx^2} = \mathbf{y}\mathbf{z}\mathbf{V}_x \tag{2.231}$$

and

$$\frac{d^2\mathbf{I}}{dx^2} = \mathbf{yzI}_x \tag{2.232}$$

At $x = 0$, $\mathbf{V}_x = \mathbf{V}_R$ and $\mathbf{I}_x = \mathbf{I}_R$. Therefore, the solution of the ordinary second-order differential equations (2.231) and (2.232) gives†

$$\mathbf{V}(x) = \underbrace{(\cosh \sqrt{\mathbf{yz}}x)}_{\mathbf{A}}\mathbf{V}_R + \underbrace{\left(\sqrt{\frac{\mathbf{z}}{\mathbf{y}}} \sinh \sqrt{\mathbf{yz}}x\right)}_{\mathbf{B}}\mathbf{I}_R \tag{2.233}$$

Similarly,

$$\mathbf{I}(x) = \underbrace{\left(\frac{\sqrt{\mathbf{y}}}{\mathbf{z}} \sinh \sqrt{\mathbf{yz}}x\right)}_{\mathbf{C}}\mathbf{V}_R + \underbrace{(\cosh \sqrt{\mathbf{yz}}x)}_{\mathbf{D}}\mathbf{I}_R \tag{2.234}$$

Equations (2.233) and (2.234) can be rewritten as

$$\mathbf{V}(x) = (\cosh \gamma x)\mathbf{V}_R + (\mathbf{Z}_c \sinh \gamma x)\mathbf{I}_R \tag{2.235}$$

and

$$\mathbf{I}(x) = (\mathbf{Y}_c \sinh \gamma x)\mathbf{V}_R + (\cosh \gamma x)\mathbf{I}_R \tag{2.236}$$

where γ = propagation constant per unit length, $= \sqrt{\mathbf{yz}}$

$\mathbf{Z}_c$ = characteristic (or surge or natural) impedance of line per unit length, $= \sqrt{\mathbf{z}/\mathbf{y}}$

$\mathbf{Y}_c$ = characteristic (or surge or natural) admittance of line per unit length, $= \sqrt{\mathbf{y}/\mathbf{z}}$

Further,

$$\gamma = \alpha + j\beta \tag{2.237}$$

where α = attenuation constant (measuring decrement in voltage and current per unit length in direction of travel) in nepers per unit length

β = phase (or phase change) constant in radians per unit length (i.e., change in phase angle between two voltages, or currents, at two points one per unit length apart on infinite line)

† See Stevenson [3, p. 103] and Neuenswander [2, p. 35].

When $x = l$, equations (2.235) and (2.236) become

$$\mathbf{V}_S = (\cosh \gamma l)\mathbf{V}_R + (\mathbf{Z}_c \sinh \gamma l)\mathbf{I}_R \tag{2.238}$$

and

$$\mathbf{I}_S = (\mathbf{Y}_c \sinh \gamma l)\mathbf{V}_R + (\cosh \gamma l)\mathbf{I}_R \tag{2.239}$$

Equations (2.238) and (2.239) can be written in matrix form as

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & \mathbf{Z}_c \sinh \gamma l \\ \mathbf{Y}_c \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.240}$$

and

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & \mathbf{Z}_c \sinh \gamma l \\ \mathbf{Y}_c \sinh \gamma l & \cosh \gamma l \end{bmatrix}^{-1} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} \tag{2.241}$$

or

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \cosh \gamma l & -\mathbf{Z}_c \sinh \gamma l \\ -\mathbf{Y}_c \sinh \gamma l & \cosh \gamma l \end{bmatrix} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} \tag{2.242}$$

Therefore,

$$\mathbf{V}_R = (\cosh \gamma l)\mathbf{V}_S - (\mathbf{Z}_c \sinh \gamma l)\mathbf{I}_S \tag{2.243}$$

and

$$\mathbf{I}_R = -(\mathbf{Y}_c \sinh \gamma l)\mathbf{V}_S + (\cosh \gamma l)\mathbf{I}_S \tag{2.244}$$

In terms of **ABCD** constants,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.245}$$

and

$$\begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ -\mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & -\mathbf{B} \\ -\mathbf{C} & \mathbf{A} \end{bmatrix} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} \tag{2.246}$$

where

$$\mathbf{A} = \cosh \gamma l = \cosh \sqrt{\mathbf{YZ}} = \cosh \boldsymbol{\theta} \tag{2.247}$$

$$\mathbf{B} = \mathbf{Z}_c \sinh \gamma l = \sqrt{\mathbf{Z}/\mathbf{Y}} \sinh \sqrt{\mathbf{YZ}} = \mathbf{Z}_c \sinh \boldsymbol{\theta} \tag{2.248}$$

$$\mathbf{C} = \mathbf{Y}_c \sinh \gamma l = \sqrt{\mathbf{Y}/\mathbf{Z}} \sinh \sqrt{\mathbf{YZ}} = \mathbf{Y}_c \sinh \boldsymbol{\theta} \tag{2.249}$$

$$\mathbf{D} = \mathbf{A} = \cosh \gamma l = \cosh \sqrt{\mathbf{YZ}} = \cosh \boldsymbol{\theta} \tag{2.250}$$

$$\boldsymbol{\theta} = \sqrt{\mathbf{YZ}} \tag{2.251}$$

$$\sinh \gamma l = \tfrac{1}{2}(e^{\gamma l} - e^{-\gamma l}) \tag{2.252}$$

$$\cosh \gamma l = \tfrac{1}{2}(e^{\gamma l} + e^{-\gamma l}) \tag{2.253}$$

Equations (2.238)–(2.251) can be used if tables of complex hyperbolic functions or pocket calculators with complex hyperbolic functions are available. Alternatively, the following expansions can be used:

$$\sinh \gamma l = \sinh(\alpha l + j\beta l) = \sinh \alpha l \cos \beta l + j \cosh \alpha l \sin \beta l \tag{2.254}$$

$$\cosh \gamma l = \cosh(\alpha l + j\beta l) = \cosh \alpha l \cos \beta l + j \sinh \alpha l \sin \beta l \tag{2.255}$$

Furthermore, by substituting for γl and $\mathbf{Z}_c$ in terms of $\mathbf{Y}$ and $\mathbf{Z}$, that is, the total line shunt admittance per phase and the total line series impedance per phase, in equation (2.245) gives

$$\mathbf{V}_S = (\cosh \sqrt{\mathbf{YZ}})\mathbf{V}_R + \left(\sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} \sinh \sqrt{\mathbf{YZ}}\right)\mathbf{I}_R \tag{2.256}$$

and

$$\mathbf{I}_S = \left(\sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}} \sinh \sqrt{\mathbf{YZ}}\right)\mathbf{V}_R + (\cosh \sqrt{\mathbf{YZ}})\mathbf{I}_R \tag{2.257}$$

or, alternatively,

$$\mathbf{V}_S = (\cos \sqrt{\mathbf{YZ}})\mathbf{V}_R + \left(\frac{\sinh \sqrt{\mathbf{YZ}}}{\sqrt{\mathbf{YZ}}}\right)\mathbf{ZI}_R \tag{2.258}$$

and

$$\mathbf{I}_S = \left(\frac{\sinh \sqrt{\mathbf{YZ}}}{\sqrt{\mathbf{YZ}}}\right)\mathbf{YV}_R + (\cosh \sqrt{\mathbf{YZ}})\mathbf{I}_R \tag{2.259}$$

The factors in parentheses in equations (2.256)–(2.259) can readily be found by using Woodruff's charts, which are not included here but can be found in L. F. Woodruff, *Electric Power Transmission* (Wiley, NY, 1952).

The **ABCD** parameters in terms of infinite series can be expressed as

$$\mathbf{A} = 1 + \frac{\mathbf{YZ}}{2} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{24} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{720} + \frac{\mathbf{Y}^4\mathbf{Z}^4}{40{,}320} + \cdots \tag{2.260}$$

$$\mathbf{B} = \mathbf{Z}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{120} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{5040} + \frac{\mathbf{Y}^4\mathbf{Z}^4}{362{,}880} + \cdots\right) \tag{2.261}$$

$$\mathbf{C} = \mathbf{Y}\left(1 + \frac{\mathbf{YZ}}{6} + \frac{\mathbf{Y}^2\mathbf{Z}^2}{120} + \frac{\mathbf{Y}^3\mathbf{Z}^3}{5040} + \frac{\mathbf{Y}^4\mathbf{Z}^4}{362{,}880} + \cdots\right) \tag{2.262}$$

where

$$\begin{aligned}\mathbf{Z} &= \text{total line series impedance per phase}\\ &= \mathbf{z}l\\ &= (r + jx_L)l \quad \Omega\\ \mathbf{Y} &= \text{total line shunt admittance per phase}\\ &= \mathbf{y}l\\ &= (g + jb)l \quad \text{S}\end{aligned}$$

In practice, usually not more than three terms are necessary in equations (2.260)–(2.262). Weedy [7] suggests the following approximate values for the **ABCD** constants if the overhead transmission line is less than 500 km in length:

$$\mathbf{A} = \mathbf{D} = 1 + \tfrac{1}{2}\mathbf{YZ} \tag{2.263}$$

$$\mathbf{B} = \mathbf{Z}(1 + \tfrac{1}{6}\mathbf{YZ}) \tag{2.264}$$

and

$$\mathbf{C} = \mathbf{Y}(1 + \tfrac{1}{6}\mathbf{YZ}) \tag{2.265}$$

However, the error involved may be too large to be ignored for certain applications.

EXAMPLE 2.10

A single-circuit, 60-Hz, three-phase transmission line is 150 mi long. The line is connected to a load of 50 MVA at a lagging power factor of 0.85 at 138 kV. The line constants are given as $R = 0.1858\ \Omega/\text{mi}$, $L = 2.60\ \text{mH/mi}$, and $C = 0.012\ \mu\text{F/mi}$. Calculate the following:

(a) **A**, **B**, **C**, and **D** constants of line.
(b) Sending-end voltage.
(c) Sending-end current.

(d) Sending-end power factor.
(e) Sending-end power.
(f) Power loss in line.
(g) Transmission line efficiency.
(h) Percentage of voltage regulation.
(i) Sending-end charging current at no load.
(j) Value of receiving-end voltage rise at no load if sending-end voltage is held constant.

Solution

$$\begin{aligned}\mathbf{z} &= 0.1858 + j2\pi \times 60 \times 2.6 \times 10^{-3}\\ &= 0.1858 + j0.9802 = 0.9977 \underline{/79.27^\circ}\ \Omega/\text{mi}\end{aligned}$$

$$\mathbf{y} = j2\pi \times 60 \times 0.012 \times 10^{-6} = 4.5239 \times 10^{-6} \underline{/90^\circ}\ \text{S/mi}$$

The propagation constant of the line is

$$\begin{aligned}\gamma &= \sqrt{\mathbf{yz}}\\ &= [(4.5239 \times 10^{-6} \underline{/90})(0.9977 \underline{/79.27^\circ})]^{1/2}\\ &= [4.5135 \times 10^{-6}]^{1/2} \underline{/(\tfrac{1}{2}90^\circ + 79.27^\circ)} = 0.002144 \underline{/84.63^\circ}\end{aligned}$$

The characteristic impedance of the line is

$$\begin{aligned}\mathbf{Z}_c &= \sqrt{\frac{\mathbf{z}}{\mathbf{y}}} = \left(\frac{0.9977 \underline{/79.27^\circ}}{4.5239 \times 10^{-6} \underline{/90^\circ}}\right)^{1/2}\\ &= \left(\frac{(0.9977 \times 10^{6})}{4.5239}\right)^{1/2} \underline{/\tfrac{1}{2}(79.27^\circ - 90^\circ)} = 469.62 \underline{/-5.37^\circ}\ \Omega\end{aligned}$$

The receiving-end line-to-neutral voltage is

$$\mathbf{V}_{R(\text{L-N})} = \frac{138\ \text{kV}}{\sqrt{3}} = 79{,}674.34\ \text{V}$$

Using the receiving-end voltage as the reference,

$$\mathbf{V}_{R(\text{L-N})} = 79{,}674.34 \underline{/0^\circ}\ \text{V}$$

The receiving-end current is

$$\mathbf{I}_R = \frac{50 \times 10^6}{\sqrt{3} \times 138 \times 10^3} = 209.18\ \text{A} \quad \text{or } 209.18 \underline{/-31.8^\circ}\ \text{A}$$

(a) The **A**, **B**, **C**, and **D** constants of the line:

$$\begin{aligned}\mathbf{A} &= \cosh \gamma l\\ &= \cosh(\alpha + j\beta)l\\ &= \tfrac{1}{2}(e^{\alpha l} e^{j\beta l} + e^{-\alpha l} e^{-j\beta l}) = \tfrac{1}{2}(e^{\alpha l} \underline{/\beta l} + e^{-\alpha l} \underline{/-\beta l})\end{aligned}$$

Therefore,

$$\mathbf{A} = \tfrac{1}{2}(e^{0.0301}\, e^{j0.3202} + e^{-0.0301}\, e^{-j0.3202})$$
$$= \tfrac{1}{2}(e^{0.0301}\,\angle 18.35^\circ + e^{-0.0301}\,\angle -18.35^\circ)$$
$$= \tfrac{1}{2}(1.0306\,\angle 18.35^\circ + 0.9703\,\angle -18.35^\circ)$$
$$= 0.9496 + j0.0095 = 0.9497\,\angle 0.57^\circ$$

$$\mathbf{B} = \mathbf{Z}_c \sinh \gamma l = \mathbf{Z}_c \sinh(\alpha + j\beta)l$$
$$= \mathbf{Z}_c[\tfrac{1}{2}(e^{\alpha l}\, e^{j\beta l} - e^{-\alpha l}\, e^{-j\beta l})] = \tfrac{1}{2}\mathbf{Z}_c[e^{\alpha l}\,\angle \beta l - e^{-\alpha l}\,\angle -\beta l]$$
$$= \tfrac{1}{2}(469.62\,\angle 5.37^\circ)[e^{0.0301}\, e^{j0.3202} - e^{-0.0301}\, e^{-j0.3202}]$$
$$= 234.81\,\angle -5.37^\circ[1.0306\,\angle 18.35^\circ - 0.9703\,\angle -18.35^\circ]$$
$$= (234.81\,\angle -5.37^\circ)(0.0572 + j0.6300)$$
$$= (234.81\,\angle -5.37^\circ)(0.6326\,\angle 84.81^\circ)$$
$$= 148.54\,\angle 79.44^\circ\ \Omega$$

$$\mathbf{C} = \mathbf{Y}_c \sinh \gamma l = \frac{1}{\mathbf{Z}_c} \sinh \gamma l$$
$$= \frac{1}{469.62\,\angle -5.37^\circ} \times 0.3163\,\angle 84.81^\circ = 0.00067\,\angle 90.18^\circ\ \text{S}$$

$$\mathbf{D} = \mathbf{A} = \cosh \gamma l = 0.9497\,\angle 0.57^\circ$$

(b) $$\begin{bmatrix} \mathbf{V}_{S(\text{L-N})} \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_{R(\text{L-N})} \\ \mathbf{I}_R \end{bmatrix}$$

$$= \begin{bmatrix} 0.9497\,\angle 0.57^\circ & 148.54\,\angle 79.44^\circ \\ 0.00067\,\angle 90.18^\circ & 0.9497\,\angle 0.57^\circ \end{bmatrix} \begin{bmatrix} 79{,}674.34\,\angle 0^\circ \\ 209.18\,\angle -31.8^\circ \end{bmatrix}$$

Thus, the sending-end voltage is

$$\mathbf{V}_{S(\text{L-N})} = (0.9497\,\angle 0.57^\circ)(79.674.34\,\angle 0^\circ) + (148.54\,\angle 79.44^\circ)(209.18\,\angle -31.8^\circ)$$
$$= 99{,}470.05\,\angle 13.79^\circ\ \text{V}$$
$$\mathbf{V}_{S(\text{L-L})} = 172{,}287.18\ \text{V}$$

(c) The sending-end current is

$$\mathbf{I}_S = (0.0067\,\angle 90.18^\circ)(79{,}674.34\angle 0^\circ) + (0.9497\,\angle 0.57^\circ)$$
$$\times (209.18\,\angle -31.8^\circ)$$
$$= 176.8084\,\angle -16.3^\circ$$

(d) The sending-end power factor is

$$\Phi_S = 13.79^\circ + 16.3^\circ = 30.09^\circ$$
$$\cos \Phi_S = 0.8653$$

(e) The sending-end power is

$$P_S = \sqrt{3} V_{S(\text{L-L})} I_S \cos \Phi_S$$

$$= \sqrt{3} \times 172{,}287.18 \times 176.8084 \times 0.8653 = 45{,}654.46 \text{ kW}$$

(f) The receiving-end power is

$$P_R = \sqrt{3} V_{R(\text{L-L})} I_R \cos \Phi_R$$

$$= \sqrt{3} \times 138 \times 10^3 \times 209.18 \times 0.85 = 42{,}499 \text{ kW}$$

Therefore, the power loss in the line is

$$P_L = P_S - P_R = 3155.46 \text{ kW}$$

(g) The transmission line efficiency is

$$\eta = \frac{P_R}{P_S} \times 100 = \frac{42{,}499}{45{,}654.46} \times 100 = 93.1\%$$

(h) The percentage of voltage regulation is

$$\text{Percentage of voltage regulation} = \frac{99{,}470.04 - 79{,}674.34}{79{,}674.34} \times 100 = 24.9\%$$

(i) The sending-end charging current at no load is

$$I_c = \tfrac{1}{2} Y V_{S(\text{L-N})} = (339.2925 \times 10^{-6})(99{,}470.05) = 33.75 \text{ A}$$

(j) The receiving-end voltage rise at no load is

$$\mathbf{V}_{R(\text{L-N})} = \mathbf{V}_{S(\text{L-N})} - \mathbf{Z}\mathbf{I}_c$$

$$= 99{,}470.05 \underline{/13.79^\circ} - (149.66 \underline{/79.27^\circ})(33.75 \underline{/103.79^\circ})$$

$$= 104{,}436.74 \underline{/13.27^\circ} \text{ V}$$

Therefore, the line-to-line voltage at the receiving end is

$$V_{R(\text{L-L})} = \sqrt{3} V_{R(\text{L-N})} = 180{,}889.74 \text{ V}$$

2.19.1 Equivalent Circuit of Long Transmission Line

Using the values of the **ABCD** parameters obtained for a transmission line, it is possible to develop an exact Π or an exact T, as shown in Figure 2.32.

For the equivalent-Π circuit,

$$\mathbf{Z}_{\Pi} = \mathbf{B} = \mathbf{Z}_c \sinh \theta \tag{2.266}$$

$$= \mathbf{Z}_c \sinh \gamma l \tag{2.267}$$

$$= \mathbf{Z} \frac{\sinh \sqrt{\mathbf{YZ}}}{\sqrt{\mathbf{YZ}}} \tag{2.268}$$

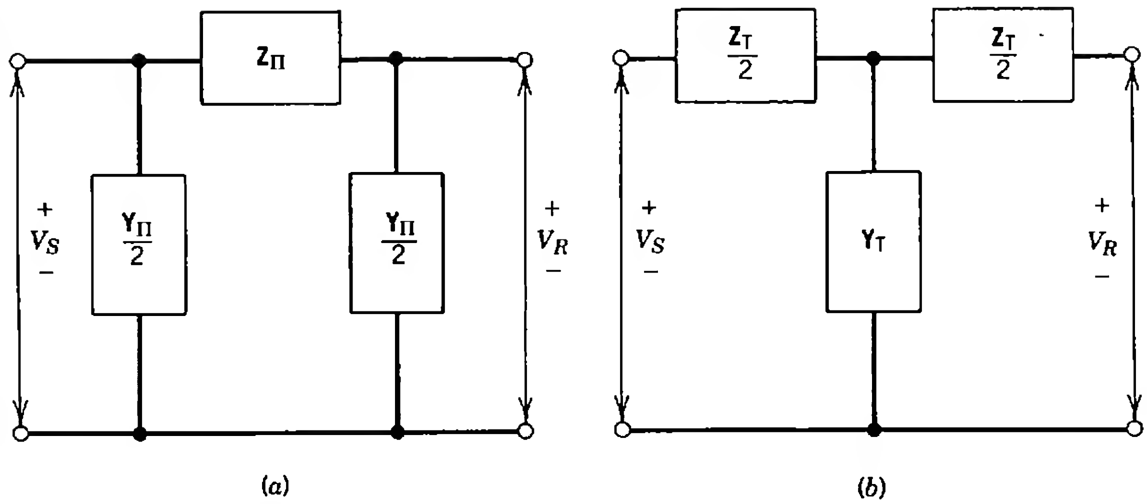


Figure 2.32. Equivalent and T circuits for long transmission line.

and

$$\frac{\mathbf{Y}_\Pi}{2} = \frac{\mathbf{A}-1}{\mathbf{B}} = \frac{\cosh \boldsymbol{\theta} - 1}{\mathbf{Z}_c \sinh \boldsymbol{\theta}} \tag{2.269}$$

or

$$\mathbf{Y}_\Pi = \frac{2\tanh[(1/2)\gamma l]}{\mathbf{Z}_c} \tag{2.270}$$

or

$$\frac{\mathbf{Y}_\Pi}{2} = \frac{\mathbf{Y}}{2}\,\frac{\tanh(1/2)\sqrt{\mathbf{YZ}}}{(1/2)\sqrt{\mathbf{YZ}}} \tag{2.271}$$

For the equivalent-T circuit,

$$\frac{\mathbf{Z}_T}{2} = \frac{\mathbf{A}-1}{\mathbf{C}} = \frac{\cosh \theta - 1}{\mathbf{Y}_c \sinh \theta} \tag{2.272}$$

or

$$\mathbf{Z}_T = 2\mathbf{Z}_c \tanh \tfrac{1}{2}\gamma l \tag{2.273}$$

or

$$\frac{\mathbf{Z}_T}{2} = \frac{\mathbf{Z}}{2}\,\frac{\tanh(1/2)\sqrt{\mathbf{YZ}}}{(1/2)\sqrt{\mathbf{YZ}}} \tag{2.274}$$

and

$$\mathbf{Y}_T = \mathbf{C} = \mathbf{Y}_c \sinh \theta \tag{2.275}$$

or

$$\mathbf{Y}_\mathrm{T} = \frac{\sinh \gamma l}{\mathbf{Z}_c} \tag{2.276}$$

or

$$\mathbf{Y}_\mathrm{T} = \mathbf{Y}\,\frac{\sinh \sqrt{\mathbf{YZ}}}{\sqrt{\mathbf{YZ}}} \tag{2.277}$$

EXAMPLE 2.11

Find the equivalent-Π and the equivalent-T circuits for the line described in Example 2.10 and compare them with the nominal-Π and the nominal-T circuits.

Solution

Figures 2.33 and 2.34 show the equivalent-Π and the nominal-Π circuits, respectively. For the equivalent-Π circuit,

$$\mathbf{Z}_\Pi = \mathbf{B} = 148.54\,\underline{/79.44^\circ}\,\Omega$$

$$\frac{\mathbf{Y}_\Pi}{2} = \frac{\mathbf{A}-1}{\mathbf{B}} = \frac{0.9497\,\underline{/0.57^\circ}-1}{148.54\,\underline{/79.44^\circ}} = 0.000345\,\underline{/89.89^\circ}\,\mathrm{S}$$

For the nominal-Π circuit,

$$\mathbf{Z} = 150 \times 0.9977\,\underline{/79.27^\circ} = 149.655\,\underline{/79.27^\circ}\,\Omega$$

$$\tfrac{1}{2}\mathbf{Y} = \tfrac{1}{2}[150(4.5239 \times 10^{-6}\,\underline{/90^\circ})] = 0.000339\,\underline{/90^\circ}\,\mathrm{S}$$

Figures 2.35(*a*) and 2.35(*b*) show the equivalent-T and nominal-T circuits, respectively. For the equivalent-T circuit,

$$\frac{\mathbf{Z}_\mathrm{T}}{2} = \frac{\mathbf{A}-1}{\mathbf{C}} = \frac{0.9497\,\underline{/0.57^\circ}-1}{0.00067\,\underline{/90.18^\circ}} = 76.57\,\underline{/79.15^\circ}\,\Omega$$

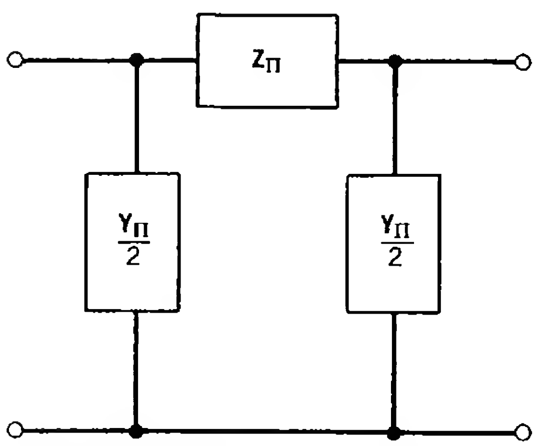


Figure 2.33. Equivalent-Π circuit.

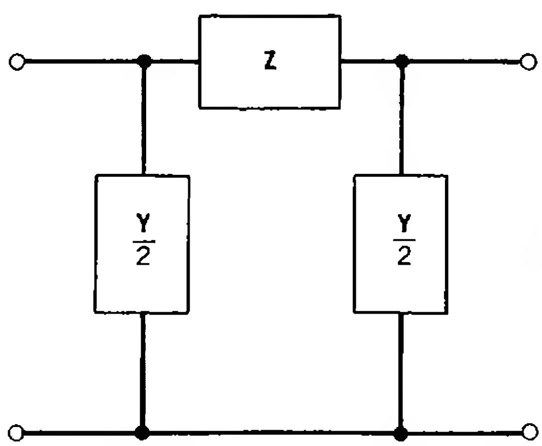


Figure 2.34. Nominal-Π circuit.

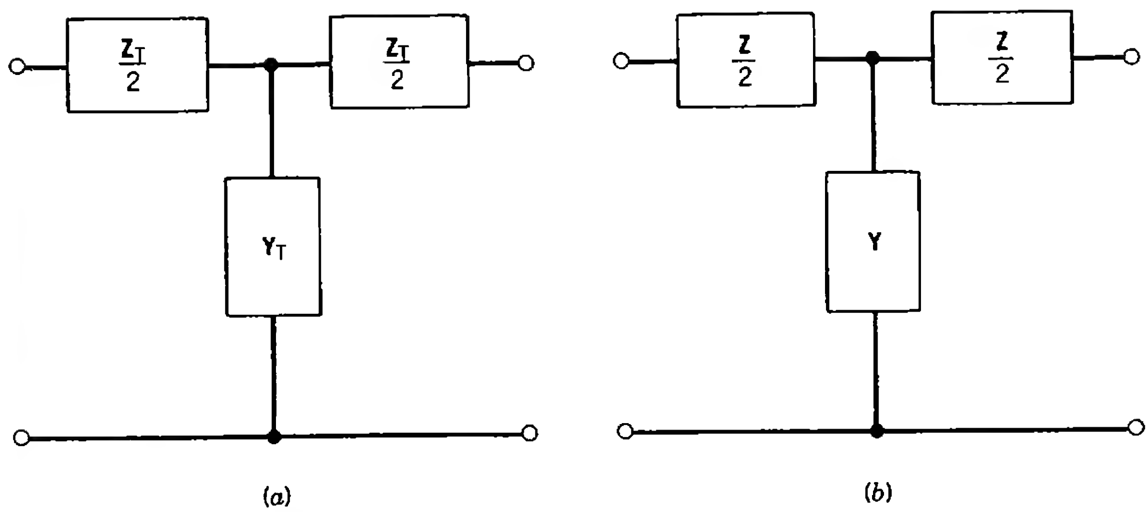


Figure 2.35. The T circuits: (*a*) equivalent T; (*b*) nominal T.

$$\mathbf{Y}_T = \mathbf{C} = 0.00067\ \underline{/90.18^\circ}\ \mathrm{S}$$

For the nominal-T circuit,

$$\tfrac{1}{2}\mathbf{Z} = \tfrac{1}{2}(149.655\ \underline{/79.27^\circ}) = 74.83\ \underline{/79.27^\circ}\ \Omega$$

$$\mathbf{Y} = 0.000678\ \underline{/90^\circ}\ \mathrm{S}$$

As can be observed from the results, the difference between the values for the equivalent and nominal circuits is very small for a 150-mi-long transmission line.

2.19.2 Incident and Reflected Voltages of Long Transmission Line

Previously, the propagation constant has been given as

$$\gamma = \alpha + j\beta \quad \text{per-unit length} \tag{2.278}$$

and also

$$\cosh \gamma l = \tfrac{1}{2}(e^{\gamma l} + e^{-\gamma l}) \tag{2.279}$$

$$\sinh \gamma l = \tfrac{1}{2}(e^{\gamma l} - e^{-\gamma l}) \tag{2.280}$$

The sending-end voltage and current have been expressed as

$$\mathbf{V}_S = (\cosh \gamma l)\mathbf{V}_R + (\mathbf{Z}_c \sinh \gamma l)\mathbf{I}_R \tag{2.281}$$

and

$$\mathbf{I}_S = (\mathbf{Y}_c \sinh \gamma l)\mathbf{V}_R + (\cosh \gamma l)\mathbf{I}_R \tag{2.282}$$

By substituting equations (2.278)–(2.280) in equations (2.281) and (2.282),

$$\mathbf{V}_S = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_c)e^{\alpha l}\, e^{j\beta l} + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_c)e^{-\alpha l}\, e^{-j\beta l} \tag{2.283}$$

and

$$\mathbf{I}_S = \tfrac{1}{2}(\mathbf{V}_R\mathbf{Y}_c + \mathbf{I}_R)e^{\alpha l}\, e^{j\beta l} - \tfrac{1}{2}(\mathbf{V}_R\mathbf{Y}_c - \mathbf{I}_R)e^{-\alpha l}\, e^{-j\beta l} \tag{2.284}$$

In equation (2.283), the first and the second terms are called the *incident voltage* and the *reflected voltage*, respectively. They act like traveling waves as a function of l. The incident voltage increases in magnitude and phase as the l distance from the receiving end increases and decreases in magnitude and phase as the distance from the sending end toward the receiving end decreases. Whereas the reflected voltage decreases in magnitude and phase as the l distance from the receiving end toward the sending end increases. Therefore, for any given line length l, the voltage is the sum of the corresponding incident and reflected voltages. Here, the term $e^{\alpha l}$ changes as a function of l, whereas $e^{j\beta l}$ always has a magnitude of 1 and causes a phase shift of β radians per unit length of line.

In equation (2.283), when the two terms are 180° out of phase, a cancellation will occur. This happens when there is no load on the line, that is, when

$$\mathbf{I}_R = 0 \quad \text{and} \quad \alpha = 0$$

and when $\beta x = \frac{1}{2}\pi$ radians, or one-quarter wavelengths.

The *wavelength* λ is defined as the distance l along a line between two points to develop a phase shift of 2π radians, or 360°, for the incident and reflected waves. If β is the phase shift in radians per mile, the wavelength in miles is

$$\lambda = \frac{2\pi}{\beta} \tag{2.285}$$

Since the propagation velocity is

$$\nu = \lambda f \quad \text{mi/s} \tag{2.286}$$

and is approximately equal to the speed of light, that is, 186,000 mi/s, at a frequency of 60 Hz, the wavelength is

$$\lambda = \frac{186{,}000 \text{ mi/s}}{60 \text{ Hz}}$$
$$= 3100 \text{ mi}$$

Whereas, at a frequency of 50 Hz, the wavelength is approximately 6000 km.

If a finite line is terminated by its characteristic impedance $\mathbf{Z}_c$, that impedance could be imagined replaced by an infinite line. In this case, there is no refected wave of either voltage or current since

$$\mathbf{V}_R = \mathbf{I}_R \mathbf{Z}_c$$

in equations (2.283) and (2.284), and the line is called an infinite (or flat) line.

Stevenson [3] gives the typical values of $\mathbf{Z}_c$ as 400 Ω for a single-circuit line and 200 Ω for two circuits in parallel. The phase angle of $\mathbf{Z}_c$ is usually between 0 and −15° [3].

Example 2.12

Using the data given in Example 2.10, determine the following:

(a) Attenuation constant and phase change constant per mile of line.
(b) Wavelength and velocity of propagation.
(c) Incident and reflected voltages at receiving end of line.
(d) Line voltage at receiving end of line.
(e) Incident and reflected voltages at sending end of line.
(f) Line voltage at sending end.

Solution

(a) Since the propagation constant of the line is

$$\gamma = \sqrt{\mathbf{yz}} = 0.0002 + j0.0021$$

the attenuation constant is 0.0002 Np/mi, and the phase change constant is 0.0021 rad/mi.

(b) The wavelength of propagation is

$$\lambda = \frac{2\pi}{\beta} = \frac{2\pi}{0.0021} = 2991.99 \text{ mi}$$

and the velocity of propagation is

$$\nu = \lambda f = 2991.99 \times 60 = 179{,}519.58 \text{ mi/s}$$

(c) From equation (2.283),

$$\mathbf{V}_S = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R \mathbf{Z}_C)e^{\alpha l}\, e^{j\beta l} + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R \mathbf{Z}_C)e^{-\alpha l}\, e^{-j\beta l}$$

Since, at the receiving end, $l = 0$,

$$\mathbf{V}_S = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R \mathbf{Z}_C) + \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R \mathbf{Z}_C)$$

Therefore, the incident and reflected voltage at the receiving end are

$$\begin{aligned} \mathbf{V}_{R(\text{incident})} &= \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R \mathbf{Z}_C) \\ &= \tfrac{1}{2}[79{,}674.34\,\underline{/0^\circ} + (209.18\,\underline{/-31.8^\circ})(469.62\,\underline{/-5.37^\circ})] \\ &= 84{,}367.77\,\underline{/-20.59^\circ} \text{ V} \end{aligned}$$

$$\mathbf{V}_{R(\text{reflected})} = \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)$$

$$= \tfrac{1}{2}[79{,}674.34\ \underline{/0^\circ} - (209.18\ \underline{/-31.8^\circ})(469.62\ \underline{/-5.37^\circ})]$$

$$= 29{,}684.15\ \underline{/88.65^\circ}\ \text{V}$$

(d) The line-to-neutral voltage at the receiving end is

$$\mathbf{V}_{R(\text{L-N})} = \mathbf{V}_{R(\text{incident})} + \mathbf{V}_{R(\text{reflected})} = 79{,}674\ \underline{/0^\circ}\ \text{V}$$

Therefore, the line voltage at the receiving end is

$$V_{R(\text{L-L})} = \sqrt{3}V_{R(\text{L-N})} = 138{,}000\ \text{V}$$

(e) At the sending end,

$$\mathbf{V}_{S(\text{incident})} = \tfrac{1}{2}(\mathbf{V}_R + \mathbf{I}_R\mathbf{Z}_C)e^{\alpha l}e^{j\beta l}$$

$$= (84{,}367.77\ \underline{/-20.59^\circ})\ e^{0.301}\ \underline{/18.35^\circ} = 86{,}946\ \underline{/-2.24^\circ}\ \text{V}$$

$$\mathbf{V}_{S(\text{reflected})} = \tfrac{1}{2}(\mathbf{V}_R - \mathbf{I}_R\mathbf{Z}_C)e^{-\alpha l}e^{-j\beta l}$$

$$= (29.684.15\ \underline{/88.65^\circ})e^{-0.0301}\ \underline{/-18.35^\circ} = 28{,}802.5\ \underline{/70.3^\circ}\ \text{V}$$

(f) The line-to-neutral voltage at the sending end is

$$\mathbf{V}_{S(\text{L-N})} = \mathbf{V}_{S(\text{incident})} + \mathbf{V}_{S(\text{reflected})}$$

$$= 86{,}946\ \underline{/-2.24^\circ} + 28{,}802.5\ \underline{/70.3^\circ} = 99.458.1\ \underline{/13.8^\circ}\ \text{V}$$

Therefore, the line voltage at the sending end is

$$V_{S(\text{L-L})} = \sqrt{3}V_{S(\text{L-N})} = 172{,}266.5\ \text{V}$$

2.19.3 Surge Impedance Loading (SIL) of Transmission Line

In power systems, if the line is lossless,† the characteristic impedance Z_c of a line is sometimes called *surge impedance*. Therefore, for a loss-free line,

$$R = 0 \quad \text{and} \quad G = 0$$

Thus,

$$Z_c^* = \sqrt{\frac{Z}{Y}} \cong \sqrt{\frac{L}{C}}\quad \Omega \tag{2.287}$$

and is a pure resistor. It is a function of the line inductance and capacitance as shown and is indepenent of the line length.

The surge impedance loading (SIL) (or the natural loading) of a transmis-

† When dealing with high frequencies or with surges due to lightning, losses are often ignored [3].

sion line is defined as the power delivered by the line to a purely resistive load equal to its surge impedance. Therefore,

$$\mathrm{SIL} = \frac{|k\mathbf{V}_{R(\mathrm{L\text{-}L})}|^2}{Z_c^*} \quad \mathrm{MW} \tag{2.288}$$

or

$$\mathrm{SIL} \cong \frac{|k\mathbf{V}_{R(\mathrm{L\text{-}L})}|^2}{\sqrt{L/C}} \quad \mathrm{MW} \tag{2.289}$$

or

$$\mathrm{SIL} = \sqrt{3}|\mathbf{V}_{R(\mathrm{L\text{-}L})}||\mathbf{I}_L| \quad \mathrm{W} \tag{2.290}$$

where

$$|\mathbf{I}_L| = \frac{|\mathbf{V}_{R(\mathrm{L\text{-}L})}|}{\sqrt{3} \times \sqrt{L/C}} \quad \mathrm{A} \tag{2.291}$$

and

SIL = surge impedance loading in megawatts or watts
$|k\mathbf{V}_{R(\mathrm{L\text{-}L})}|$ = magnitude of line-to-line receiving-end voltage in kilovolts
$|\mathbf{V}_{R(\mathrm{L\text{-}L})}|$ = magnitude of line-to-line receiving-end voltage in volts
Z_c = surge impedance in ohms,
$\cong \sqrt{L/C}$
$\mathbf{I}_L$ = line current at surge impedance loading in amperes

In practice, the allowable loading of a transmission line may be given as a fraction of its SIL. Thus, SIL is used as a means of comparing the load-carrying capabilities of lines.

However, the SIL in itself is not a measure of the maximum power that can be delivered over a line. For the maximum delivered power, the line length, the impedance of sending- and receiving-end apparatus, and all of the other factors affecting stability must be considered.

Since the characteristic impedance of underground cables is very low, the SIL (or natural load) is far larger than the rated load of the cable. Therefore, a given cable acts as a source of lagging vars.

The best way of increasing the SIL of a line is to increase its voltage level, since, as it can be seen from equation (2.288), the SIL increases with its square. However, increasing voltage level is expensive. Therefore, instead, the surge impedance of the line is reduced. This can be accomplished by adding capacitors or induction coils. There are four possible ways of changing the line capacitance or inductance, as shown in Figures 2.36 and 2.37.

For a lossless line, the characteristic impedance and the propagation constant can be expressed as

$$Z_c = \sqrt{\frac{L}{C}} \tag{2.292}$$

and

$$\gamma = \sqrt{LC} \tag{2.293}$$

Therefore, the addition of lumped inductances in series will increase the line inductance, and thus, the characteristic impedance and the propagation constant will be increased, which is not desirable.

The addition of lumped inductances in parallel will decrease the line capacitance. Therefore, the propagation constant will be decreased, but the characteristic impedance will be increased, which again is not desirable.

The addition of capacitances in parallel will increase the line capacitance. Hence, the characteristic impedance will be decreased, but the propagation constant will be increased, which affects negatively the system stability. However, for the short lines, this method can be used effectively.

Finally, the addition of capacitances in series will decrease the line inductance. Therefore, the characteristic impedance and the propagation constant will be reduced, which is desirable. Thus, the series capacitor compensation of transmission lines is used to improve stability limits and voltage regulation, to provide a desired load division, and to maximize the load-carrying capability of the system. However, having the full line current going through the capacitors connected in series causes harmful overvoltages

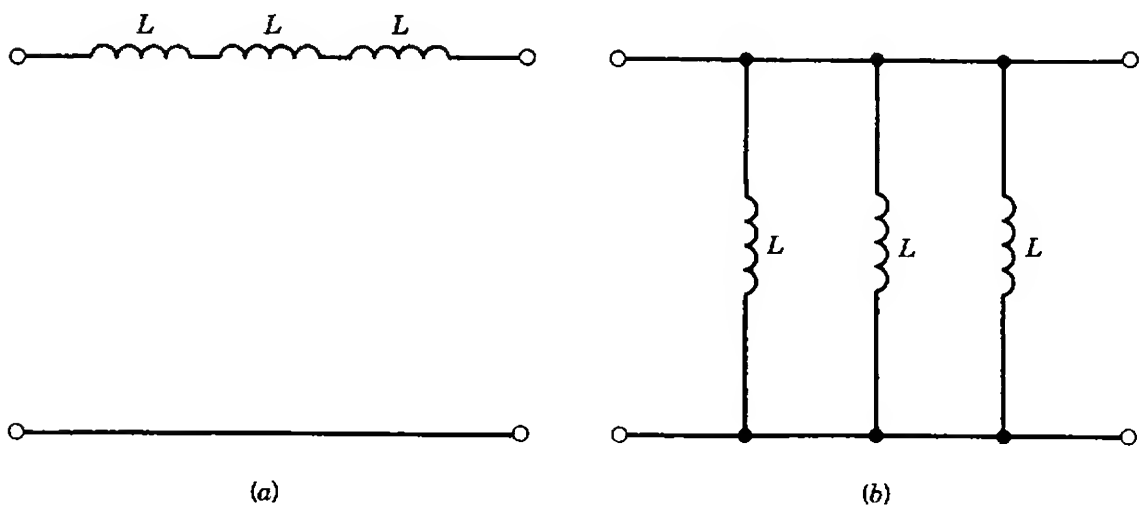


Figure 2.36. Transmission line compensation by adding lump inductances in: (*a*) series; (*b*) parallel (i.e., shunt).

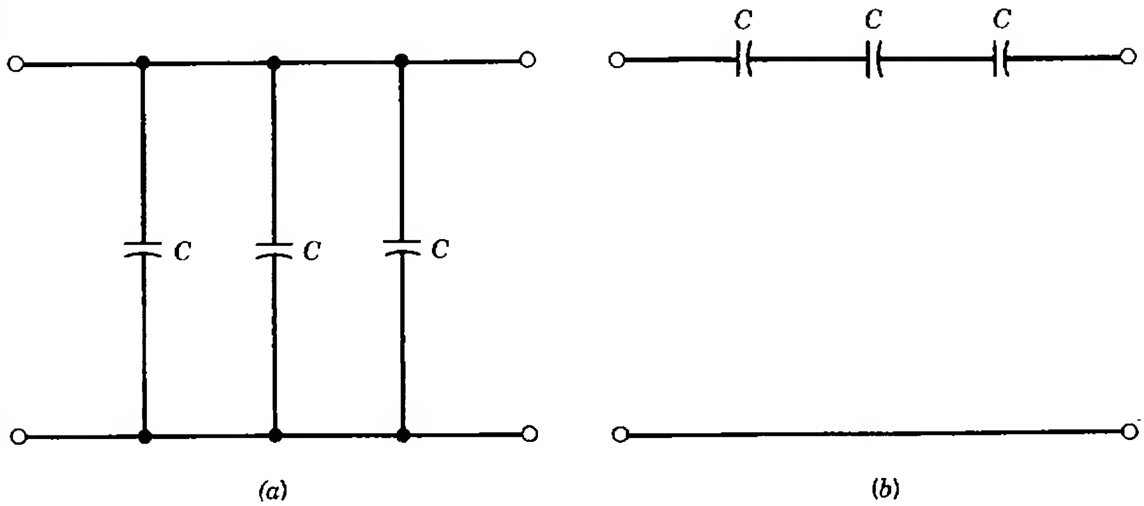


Figure 2.37. Transmission line compensation by adding capacitances in: (*a*) parallel (i.e., shunt); (*b*) series.

on the capacitors during short circuits. Therefore, they introduce special problems for line protective relaying.† Under fault conditions, they introduce an impedance discontinuity (negative inductance) and subharmonic currents, and when the capacitor protective gap operates, they impress high-frequency currents and voltages on the system. All of these factors result in incorrect operation of the conventional relaying schemes. The series capacitance compensation of distribution lines has been attempted from time to time for many years. However, it is not widely used.

Example 2.13

Determine the SIL of the transmission line given in Example 2.10.

Solution

The approximate value of the surge impedance of the line is

$$Z_c \cong \sqrt{\frac{L}{C}} = \left(\frac{2.6 \times 10^{-3}}{0.012 \times 10^{-6}}\right)^{1/2} = 465.5\ \Omega$$

† The application of series compensation on the new EHV lines has occasionally caused a problem known as "subsynchronous resonance." It can be briefly defined as an oscillation due to the interaction between a series capacitor compensated transmission system in electrical resonance and a turbine generator mechanical system in torsional mechanical resonance. As a result of the interaction, a negative resistance is introduced into the electric circuit by the turbine generator. If the effective resistance magnitude is sufficiently large to make the net resistance of the circuit negative, oscillations can increase until mechanical failures take place in terms of flexing or even breaking of the shaft. The event occurs when the electrical subsynchronous resonance frequency is equal or close to 60 Hz minus the frequency of one of the natural torsional modes of the turbine generator. The most well-known subsynchronous resonance problem took place at Mojave Generating Station [8–11].

Therefore,

$$\text{SIL} \cong \frac{|k\mathbf{V}_{R(\text{L-L})}|^2}{\sqrt{L/C}} = \frac{|138|^2}{469.62} = 40.913 \text{ MW}$$

which is an approximate value of the SIL of the line. The exact value of the SIL of the line can be determined as

$$\text{SIL} = \frac{|k\mathbf{V}_{R(\text{L-L})}|^2}{Z_c} = \frac{|138|^2}{469.62} = 40.552 \text{ MW}$$

2.20 GENERAL CIRCUIT CONSTANTS

Figure 2.38 shows a general two-port, four-terminal network consisting of passive impedances connected in some fashion. From general nework theory,

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R \tag{2.294}$$

and

$$\mathbf{I}_S = \mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R \tag{2.295}$$

Also,

$$\mathbf{V}_R = \mathbf{D}\mathbf{V}_S - \mathbf{B}\mathbf{I}_S \tag{2.296}$$

and

$$\mathbf{I}_R = -\mathbf{C}\mathbf{V}_S + \mathbf{A}\mathbf{I}_S \tag{2.297}$$

It is always true that the determinant of equations (2.294) and (2.295) or (2.296) and (2.297) is always unity, that is,

$$\mathbf{AD} - \mathbf{BC} = 1 \tag{2.298}$$

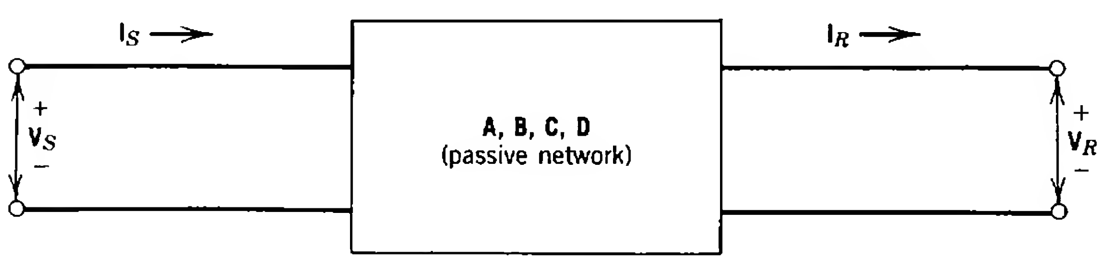


Figure 2.38. General two-part, four-terminal network.

In the above equations, the **A**, **B**, **C**, and **D** are constants for a given network and are called general circuit constants. Their values depend on the parameters of the circuit concerned and the particular representation chosen. In general, they are complex numbers. For a network that has the symmetry of the uniform transmission line,

$$\mathbf{A} = \mathbf{D} \tag{2.299}$$

2.20.1 Determination of A, B, C, and D Constants

The **A**, **B**, **C**, and **D** constants can be calculated directly by network reduction. For example, when $\mathbf{I}_R = 0$, from equation (2.294),

$$\mathbf{A} = \frac{\mathbf{V}_S}{\mathbf{V}_R} \tag{2.300}$$

and from equation (2.295),

$$\mathbf{C} = \frac{\mathbf{I}_S}{\mathbf{V}_R} \tag{2.301}$$

Therefore, the **A** constant is the ratio of the sending- and receiving-end voltages, whereas the **C** constant is the ratio of sending-end current to receiving-end voltage when the receiving end is open-circuited. When $\mathbf{V}_R = 0$, from equation (2.294),

$$\mathbf{B} = \frac{\mathbf{V}_S}{\mathbf{I}_R} \tag{2.302}$$

When $\mathbf{V}_R = 0$, from equation (2.295),

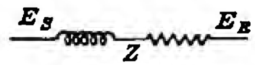
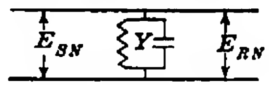
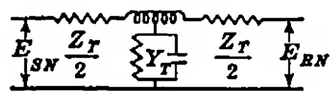
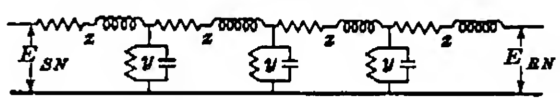
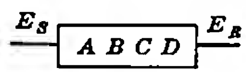
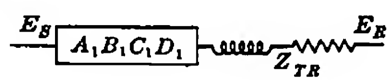
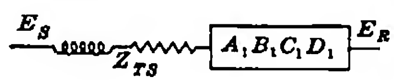
$$\mathbf{D} = \frac{\mathbf{I}_S}{\mathbf{I}_R} \tag{2.303}$$

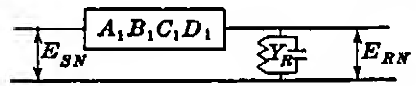
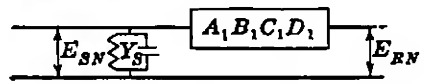
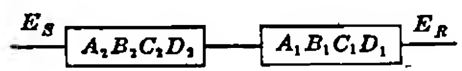
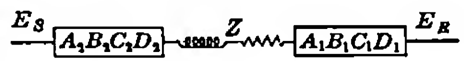
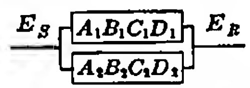
Therefore, the **B** constant is the ratio of the sending-end voltage to the receiving-end current when the receiving end is short-circuited. Whereas the **D** constant is the ratio of the sending-end and receiving-end currents when the receiving end is short-circuited.

Alternatively, the **A**, **B**, **C**, and **D** generalized circuit constants can be calculated indirectly from a knowledge of the system impedance parameters as shown in the previous sections. Table 2.4 gives general circuit constants for different network types. Table 2.5 gives network conversion formulas to convert a given parameter set into another one.

As can be observed in equations (2.294) and (2.295), the dimensions of the **A** and **D** constants are numeric. The dimension of the **B** constant is impedance in ohms, whereas the dimension of the **C** constant is admittance in siemens.

TABLE 2.4 General Circuit Constants for Different Network Types

Network number	Type of network		Equations for general circuit constants in terms of constants of component networks			
			$A =$	$B =$	$C =$	$D =$
1	Series impedance		1	Z	0	1
2	Shunt admittance		1	0	Y	1
3	Transformer		$1+\frac{Z_T Y_T}{2}$	$Z_T\left(1+\frac{Z_T Y_T}{4}\right)$	Y_T	$1+\frac{Z_T Y_T}{2}$
4	Transmission line		$\cosh\sqrt{ZY} = \left(1+\frac{ZY}{2}+\frac{Z^2Y^2}{24}+\cdots\right)$	$\sqrt{Z/Y}\,\sinh\sqrt{ZY} = Z\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\cdots\right)$	$\sqrt{Y/Z}\,\sinh\sqrt{ZY} = Y\left(1+\frac{ZY}{6}+\frac{Z^2Y^2}{120}+\cdots\right)$	Same as A
5	General network		A	B	C	D
6	General network and transformer impedance at receiving end		A_1	$B_1 + A_1 Z_{TR}$	C_1	$D_1 + C_1 Z_{TR}$
7	General network and transformer impedance at sending end		$A_1 + C_1 Z_{TS}$	$B_1 + D_1 Z_{TS}$	C_1	D_1
8	General network and transformer impedance at both ends—referred to high voltage		$A_1 + C_1 Z_{TS}$	$B_1 + A_1 Z_{TR} + D_1 Z_{TS} + C_1 Z_{TR} Z_{TS}$	C_1	$D_1 + C_1 Z_{TR}$

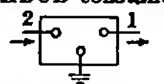
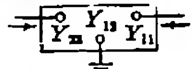
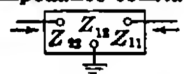
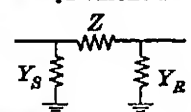
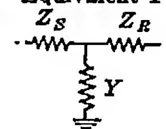
9	General network and transformer impedance at both ends—transformers having different ratios T_R and T_S referred to low voltage		$\frac{T_R}{T_S}(A_1 + C_1Z_{TS})$	$\frac{1}{T_RT_S}(B_1 + A_1Z_{TR} + D_1Z_{TS} + C_1Z_{TR}Z_{TS})$	$C_1T_RT_S$	$\frac{T_S}{T_R}(D_1 + C_1Z_{TR})$
10	General network and shunt admittance at receiving end		$A_1 + B_1Y_R$	B_1	$C_1 + D_1Y_R$	D_1
11	General network and shunt admittance at sending end		A_1	B_1	$C_1 + A_1Y_S$	$D_1 + B_1Y_S$
12	General network and shunt admittance at both ends		$A_1 + B_1Y_R$	B_1	$C_1 + A_1Y_S + D_1Y_R + B_1Y_RY_S$	$D_1 + B_1Y_S$
13	Two general networks in series		$A_1A_2 + C_1B_2$	$B_1A_2 + D_1B_2$	$A_1C_2 + C_1D_2$	$B_1C_2 + D_1D_2$
14	Two general networks in series with intermediate impedance		$A_1A_2 + C_1B_2 + C_1A_2Z$	$B_1A_2 + D_1B_2 + D_1A_2Z$	$A_1C_2 + C_1D_2 + C_1C_2Z$	$B_1C_2 + D_1D_2 + D_1C_2Z$
15	Two general networks in series with intermediate shunt admittance		$A_1A_2 + C_1B_2 + A_1B_2Y$	$B_1A_2 + D_1B_2 + B_1B_2Y$	$A_1C_2 + C_1D_2 + A_1D_2Y$	$B_1C_2 + D_1D_2 + B_1D_2Y$
16	Three general networks in series		$A_3(A_1A_2 + C_1B_2) + B_3(A_1C_2 + C_1D_2)$	$A_3(B_1A_2 + D_1B_2) + B_3(B_1C_2 + D_1D_2)$	$C_3(A_1A_2 + C_1B_2) + D_3(A_1C_2 + C_1D_2)$	$C_3(B_1A_2 + D_1B_2) + D_3(B_1C_2 + D_1D_2)$
17	Two general networks in parallel		$\frac{A_1B_2 + B_1A_2}{B_1 + B_2}$	$\frac{B_1B_2}{B_1 + B_2}$	$\frac{C_1 + C_2 + (A_1 - A_2)(D_2 - D_1)}{B_1 + B_2}$	$\frac{B_1D_2 + D_1B_2}{B_1 + B_2}$

NOTE. The exciting current of the receiving end transformers should be added vectorially to the load current, and the exciting current of the sending end transformers should be added vectorially to the sending end current.

General equations: $E_S = E_RA + I_RB$; $E_R = E_SD - I_SB$; $I_S = I_RD + E_RC$; $I_R = I_SA - E_SC$. As a check in the numerical calculation of the A, B, C, and D constants note that in all cases $AD - BC = 1$.

Source: Wagner and Evans [6]. Copyright McGraw-Hill Co., 1933. Used with permission of McGraw-Hill Co.

TABLE 2.5 Network Conversion Formulas

To	To convert from: ABCD	Admittance	Impedance	Equivalent π	Equivalent T
	ABCD constants  $E_2 = AE_1 + BI_1$ $I_2 = CE_1 + DI_1$ $E_1 = DE_2 - BI_2$ $I_1 = -CE_2 + AI_2$				
$A =$		$\frac{Y_{11}}{Y_{12}}$	$-\frac{Z_{22}}{Z_{12}}$	$1 + ZY_R$	$1 + Z_SY$
$B =$		$\frac{1}{Y_{12}}$	$-\frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}}$	Z	$Z_R + Z_S + YZ_RZ_S$
$C =$		$\frac{Y_{11}Y_{22} - Y_{12}^2}{Y_{12}}$	$-\frac{1}{Z_{12}}$	$Y_R + Y_S + ZY_RY_S$	Y
$D =$		$\frac{Y_{22}}{Y_{12}}$	$-\frac{Z_{11}}{Z_{12}}$	$1 + ZY_S$	$1 + Z_RY$
		Admittance constants  $I_1 = Y_{11}E_1 - Y_{12}E_2$ $I_2 = Y_{22}E_2 - Y_{12}E_1$			
$Y_{11} =$	$\frac{A}{B}$		$\frac{Z_{22}}{Z_{11}Z_{22} - Z_{12}^2}$	$Y_R + \frac{1}{Z}$	$\frac{1 + Z_SY}{Z_R + Z_S + YZ_RZ_S}$
$Y_{12} =$	$\frac{1}{B}$		$-\frac{Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}$	$\frac{1}{Z}$	$\frac{1}{Z_R + Z_S + YZ_RZ_S}$
$Y_{22} =$	$\frac{D}{B}$		$\frac{Z_{11}}{Z_{11}Z_{22} - Z_{12}^2}$	$Y_S + \frac{1}{Z}$	$\frac{1 + YZ_R}{Z_R + Z_S + YZ_RZ_S}$
			Impedance constants  $E_1 = Z_{11}I_1 - Z_{12}I_2$ $E_2 = Z_{22}I_2 - Z_{12}I_1$		
$Z_{11} =$	$\frac{D}{C}$	$\frac{Y_{22}}{Y_{11}Y_{22} - Y_{12}^2}$		$\frac{1 + ZY_S}{Y_R + Y_S + ZY_RY_S}$	$Z_R + \frac{1}{Y}$
$Z_{12} =$	$-\frac{1}{C}$	$-\frac{Y_{12}}{Y_{11}Y_{22} - Y_{12}^2}$		$\frac{1}{Y_R + Y_S + ZY_RY_S}$	$-\frac{1}{Y}$
$Z_{22} =$	$\frac{A}{C}$	$\frac{Y_{11}}{Y_{11}Y_{22} - Y_{12}^2}$		$\frac{1 + ZY_R}{Y_R + Y_S + ZY_RY_S}$	$Z_S + \frac{1}{Y}$
				Equivalent π 	
$Y_R =$	$\frac{A - 1}{B}$	$Y_{11} - Y_{12}$	$\frac{Z_{22} + Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}$		$\frac{YZ_S}{Z_R + Z_S + YZ_RZ_S}$
$Z =$	B	$\frac{1}{Y_{12}}$	$-\frac{Z_{11}Z_{22} - Z_{12}^2}{Z_{12}}$		$Z_R + Z_S + YZ_RZ_S$
$Y_S =$	$\frac{D - 1}{B}$	$Y_{22} - Y_{12}$	$\frac{Z_{11} + Z_{12}}{Z_{11}Z_{22} - Z_{12}^2}$		$\frac{YZ_R}{Z_R + Z_S + YZ_RZ_S}$
					Equivalent T 
$Z_R =$	$\frac{D - 1}{C}$	$\frac{Y_{22} - Y_{12}}{Y_{11}Y_{22} - Y_{12}^2}$	$Z_{11} + Z_{12}$	$\frac{ZY_S}{Y_R + Y_S + ZY_RY_S}$	
$Y =$	C	$\frac{Y_{11}Y_{22} - Y_{12}^2}{Y_{12}}$	$-\frac{1}{Z_{12}}$	$Y_R + Y_S + ZY_RY_S$	
$Z_S =$	$\frac{A - 1}{C}$	$\frac{Y_{11} - Y_{12}}{Y_{11}Y_{22} - Y_{12}^2}$	$Z_{22} + Z_{12}$	$\frac{ZY_R}{Y_R + Y_S + ZY_RY_S}$	

NOTE 1. P_1 and P_2 are positive in all cases for power flowing *into* the network from the point considered.
NOTE 2. P and Q of same sign indicates lagging power factor; that is $P + jQ = E\hat{I}$.

ABCD	Admittance	Impedance
$P_1 + jQ_1 = \frac{\hat{A}}{\hat{B}}\bar{E}_1^2 - \frac{1}{\hat{B}}E_1\hat{E}_2$	$= \hat{Y}_{11}\bar{E}_1^2 - \hat{Y}_{12}E_1\hat{E}_2$	$= Z_{11}\bar{I}_1^2 - Z_{12}\hat{I}_1I_2$
$P_2 + jQ_2 = \frac{\hat{C}}{\hat{B}}\bar{E}_2^2 - \frac{1}{\hat{B}}\hat{E}_1E_2$	$= \hat{Y}_{22}\bar{E}_2^2 - \hat{Y}_{12}\hat{E}_1E_2$	$= Z_{22}\bar{I}_2^2 - Z_{12}I_1\hat{I}_2$

Source: Wagner and Evans [6]. Copyright McGraw-Hill Co., 1933. Used with permission of McGraw-Hill Co.

2.20.2 A, B, C, and D Constants of Transformer

Figure 2.39 shows the equivalent circuit of a transformer at no load. Neglecting its series impedance,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.304}$$

where the transfer matrix is

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \mathbf{Y}_T & 1 \end{bmatrix} \tag{2.305}$$

since

$$\mathbf{V}_S = \mathbf{V}_R$$

and

$$\mathbf{I}_S = \mathbf{Y}_T \mathbf{V}_R + \mathbf{I}_R \tag{2.306}$$

and where $\mathbf{Y}_T$ is the magnetizing admittance of the transformer.

Figure 2.40 shows the equivalent circuit of a transformer at full load that has a transfer matrix of

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}\mathbf{Z}_T\mathbf{Y}_T & \mathbf{Z}_T(1 + \frac{1}{4}\mathbf{Z}_T\mathbf{Y}_T) \\ \mathbf{Y}_T & 1 + \frac{1}{2}\mathbf{Z}_T\mathbf{Y}_T \end{bmatrix} \tag{2.307}$$

since

$$\mathbf{V}_S = [1 + \tfrac{1}{2}\mathbf{Z}_T\mathbf{Y}_T]\mathbf{V}_R + [\mathbf{Z}_T(1 + \tfrac{1}{4}\mathbf{Z}_T\mathbf{Y}_T)\mathbf{I}_R \tag{2.308}$$

and

$$\mathbf{I}_S = [\mathbf{Y}_T]\mathbf{V}_R + [1 + \tfrac{1}{2}\mathbf{Z}_T\mathbf{Y}_T]\mathbf{I}_R \tag{2.309}$$

where $\mathbf{Z}_T$ is the total equivalent series impedance of the transformer.

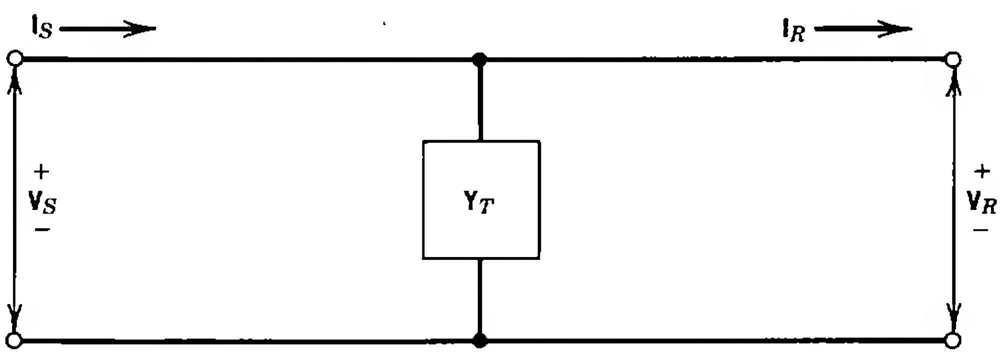


Figure 2.39. Transformer equivalent circuit at no load.

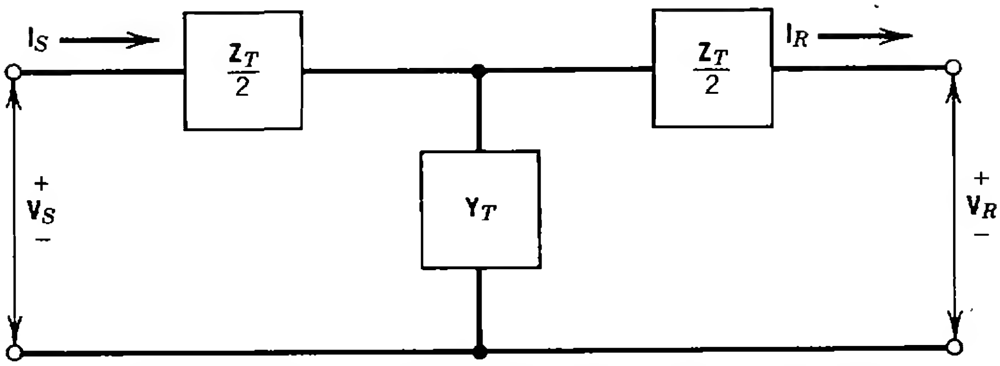


Figure 2.40. Transformer equivalent circuit at full load.

2.20.3 Asymmetrical Π and T Networks

Figure 2.41 shows an asymmetrical Π network that can be thought of as a series (or cascade, or tandem) connection of a shunt admittance, a series impedance, and a shunt admittance.

The equivalent transfer matrix can be found by multiplying together the transfer matrices of individual components. Therefore,

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \mathbf{Y}_1 & 1 \end{bmatrix} \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \mathbf{Y}_2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \mathbf{ZY}_2 & \mathbf{Z} \\ \mathbf{Y}_1 + \mathbf{Y}_2 + \mathbf{ZY}_1\mathbf{Y}_2 & 1 + \mathbf{ZY}_1 \end{bmatrix} \tag{2.310}$$

When the Π network is symmetrical,

$$\mathbf{Y}_1 = \mathbf{Y}_2 = \tfrac{1}{2}\mathbf{Y}$$

and the transfer matrix becomes

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}\mathbf{ZY} & \mathbf{Z} \\ \mathbf{Y} + \frac{1}{4}\mathbf{ZY}^2 & 1 + \frac{1}{2}\mathbf{YZ} \end{bmatrix} \tag{2.311}$$

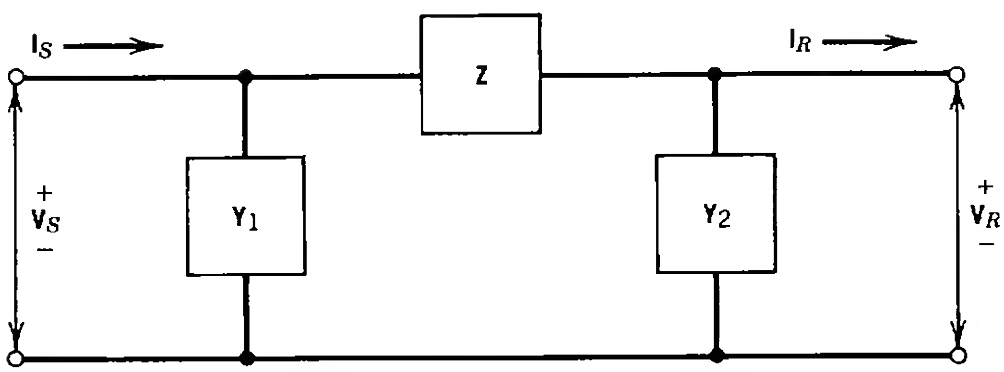


Figure 2.41. Asymmetrical-Π network.

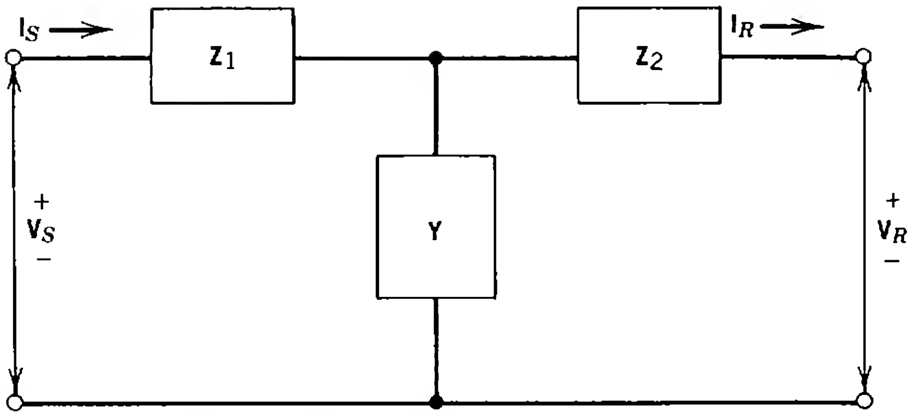


Figure 2.42. Asymmetrical-T network.

which is the same as equation (2.218) for a nominal-Π circuit of a medium-length transmission line.

Figure 2.42 shows an asymmetrical T network that can be thought of as a cascade connection of a series impedance, a shunt admittance, and a series impedance.

Again, the equivalent transfer matrix can be found by multiplying together the transfer matrices of individual components. Therefore,

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 & \mathbf{Z}_1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \mathbf{Y} & 1 \end{bmatrix} \begin{bmatrix} 1 & \mathbf{Z}_2 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + \mathbf{Z}_1\mathbf{Y} & \mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_1\mathbf{Z}_2\mathbf{Y} \\ \mathbf{Y} & 1 + \mathbf{Z}_2\mathbf{Y} \end{bmatrix} \tag{2.312}$$

When the T network is symmetrical,

$$\mathbf{Z}_1 = \mathbf{Z}_2 = \tfrac{1}{2}\mathbf{Z}$$

and the transfer matrix becomes

$$\begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} = \begin{bmatrix} 1 + \frac{1}{2}\mathbf{Z}\mathbf{Y} & \mathbf{Z} + \frac{1}{4}\mathbf{Z}^2\mathbf{Y} \\ \mathbf{Y} & 1 + \frac{1}{2}\mathbf{Z}\mathbf{Y} \end{bmatrix} \tag{2.313}$$

which is the same as the equation for a nominal-T circuit of a medium-length transmission line.

2.20.4 Networks Connected in Series

Two four-terminal transmission networks may be connected in series, as shown in Figure 2.43, to form a new four-terminal transmission network.

For the first four-terminal network,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix} \begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} \tag{2.314}$$

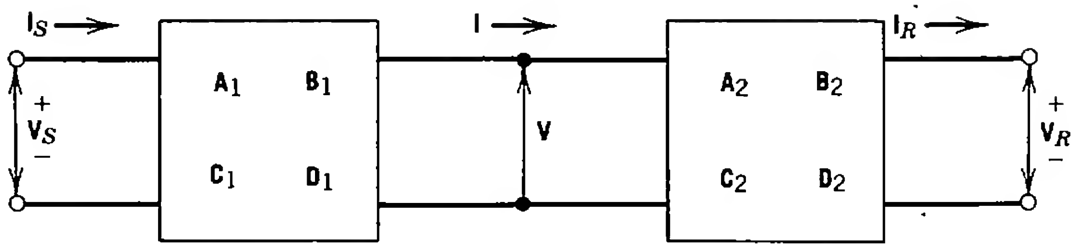


Figure 2.43. Transmission networks in series.

and for the second four-terminal network,

$$\begin{bmatrix} \mathbf{V} \\ \mathbf{I} \end{bmatrix} = \begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.315}$$

By substituting equation (2.315) into equation (2.314),

$$\begin{aligned} \begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} &= \begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix} \begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{A}_1\mathbf{A}_2 + \mathbf{B}_1\mathbf{C}_2 & \mathbf{A}_1\mathbf{B}_2 + \mathbf{B}_1\mathbf{D}_2 \\ \mathbf{C}_1\mathbf{A}_2 + \mathbf{D}_1\mathbf{C}_2 & \mathbf{C}_1\mathbf{B}_2 + \mathbf{D}_1\mathbf{D}_2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \end{aligned} \tag{2.316}$$

Therefore, the equivalent **A**, **B**, **C**, and **D** constants for two networks connected in series are

$$\mathbf{A} = \mathbf{A}_1\mathbf{A}_2 + \mathbf{B}_1\mathbf{C}_2 \tag{2.317}$$

$$\mathbf{B} = \mathbf{A}_1\mathbf{B}_2 + \mathbf{B}_1\mathbf{D}_2 \tag{2.318}$$

$$\mathbf{C} = \mathbf{C}_1\mathbf{A}_2 + \mathbf{D}_1\mathbf{C}_2 \tag{2.319}$$

$$\mathbf{D} = \mathbf{C}_1\mathbf{B}_2 + \mathbf{D}_1\mathbf{D}_2 \tag{2.320}$$

EXAMPLE 2.14

Figure 2.44 shows two networks connected in cascade. Determine the equivalent **A**, **B**, **C**, and **D** constants.

Solution

For network 1,

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix} = \begin{bmatrix} 1 & 10\,\underline{/30^\circ} \\ 0 & 1 \end{bmatrix}$$

For network 2,

$$\mathbf{Y}_2 = \frac{1}{\mathbf{Z}_2} = \frac{1}{40\,\underline{/-45^\circ}} = 0.025\,\underline{/45^\circ}\text{ S}$$

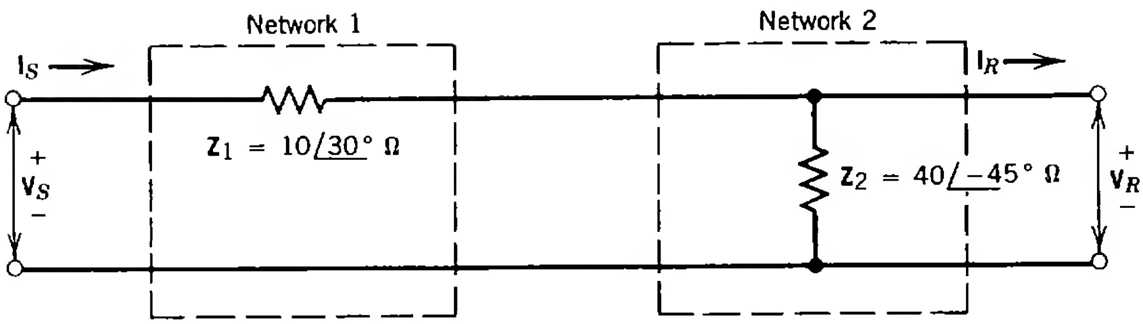


Figure 2.44

Then

$$\begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\underline{/45°} & 1 \end{bmatrix}$$

Therefore,

$$\begin{bmatrix} \mathbf{A}_{eq} & \mathbf{B}_{eq} \\ \mathbf{C}_{eq} & \mathbf{D}_{eq} \end{bmatrix} = \begin{bmatrix} 1 & 10\underline{/30°} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0.025\underline{/45°} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1.09\underline{/12.8°} & 10\underline{/30°} \\ 0.025\underline{/45°} & 1 \end{bmatrix}$$

2.20.5 Networks Connected in Parallel

Two four-terminal transmission networks may be connected in parallel, as shown in Figure 2.45, to form a new four-terminal transmission network.

Since

$$\begin{aligned} \mathbf{V}_S &= \mathbf{V}_{S1} = \mathbf{V}_{S2} \\ \mathbf{V}_R &= \mathbf{V}_{R1} = \mathbf{V}_{R2} \end{aligned} \tag{2.321}$$

and

$$\begin{aligned} \mathbf{I}_S &= \mathbf{I}_{S1} + \mathbf{I}_{S2} \\ \mathbf{I}_R &= \mathbf{I}_{R1} + \mathbf{I}_{R2} \end{aligned} \tag{2.322}$$

for the equivalent four-terminal network,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2} & \dfrac{\mathbf{B}_1\mathbf{B}_2}{\mathbf{B}_1 + \mathbf{B}_2} \\ \mathbf{C}_1 + \mathbf{C}_2 + \dfrac{(\mathbf{A}_1 - \mathbf{A}_2)(\mathbf{D}_2 - \mathbf{D}_1)}{\mathbf{B}_1 + \mathbf{B}_2} & \dfrac{\mathbf{D}_1\mathbf{B}_2 + \mathbf{D}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.323}$$

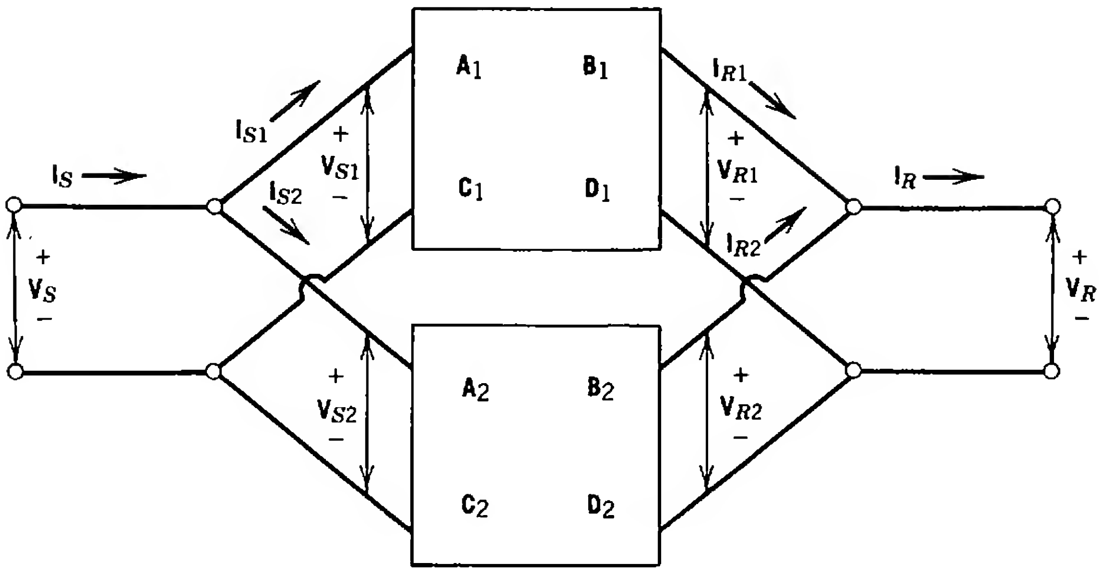


Figure 2.45. Transmission networks in parallel.

where the equivalent **A**, **B**, **C**, and **D** constants are

$$\mathbf{A} = \frac{\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2} \tag{2.324}$$

$$\mathbf{B} = \frac{\mathbf{B}_1\mathbf{B}_2}{\mathbf{B}_1 + \mathbf{B}_2} \tag{2.325}$$

$$\mathbf{C} = \mathbf{C}_1 + \mathbf{C}_2 + \frac{(\mathbf{A}_1 - \mathbf{A}_2)(\mathbf{D}_2 - \mathbf{D}_1)}{\mathbf{B}_1 + \mathbf{B}_2} \tag{2.326}$$

$$\mathbf{D} = \frac{\mathbf{D}_1\mathbf{B}_2 + \mathbf{D}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2} \tag{2.327}$$

Example 2.15

Assume that the two networks given in Example 2.14 are connected in parallel, as shown in Figure 2.46. Determine the equivalent **A**, **B**, **C**, and **D** constants.

Solution

Using the **A**, **B**, **C**, and **D** parameters found previously for networks 1 and 2, that is,

$$\begin{bmatrix} \mathbf{A}_1 & \mathbf{B}_1 \\ \mathbf{C}_1 & \mathbf{D}_1 \end{bmatrix} = \begin{bmatrix} 1 & 10\,\underline{/30^\circ} \\ 0 & 1 \end{bmatrix}$$

and

$$\begin{bmatrix} \mathbf{A}_2 & \mathbf{B}_2 \\ \mathbf{C}_2 & \mathbf{D}_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\,\underline{/45^\circ} & 1 \end{bmatrix}$$

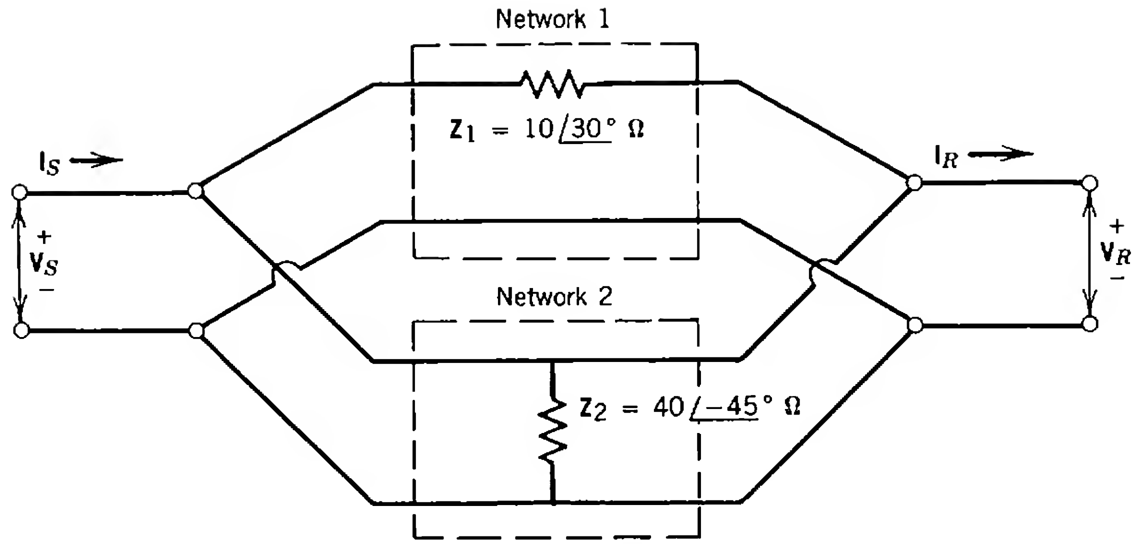


Figure 2.46

the equivalent **A**, **B**, **C**, and **D** constants can be calculated as

$$\mathbf{A}_{eq} = \frac{\mathbf{A}_1\mathbf{B}_2 + \mathbf{A}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0 + 1 \times 10\,\underline{/30^\circ}}{\cdot\,10\,\underline{/30^\circ} + 0} = 1$$

$$\mathbf{B}_{eq} = \frac{\mathbf{B}_1\mathbf{B}_2}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0}{1 + 0} = 0$$

$$\mathbf{C}_{eq} = \mathbf{C}_1 + \mathbf{C}_2 + \frac{(\mathbf{A}_1 - \mathbf{A}_2)(\mathbf{D}_2 - \mathbf{D}_1)}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= 0 + 0.025\,\underline{/45^\circ} + \frac{(1-1)(1-1)}{10\,\underline{/30^\circ} - 0} = 0.025\,\underline{/45^\circ}$$

$$\mathbf{D}_{eq} = \frac{\mathbf{D}_1\mathbf{B}_2 + \mathbf{D}_2\mathbf{B}_1}{\mathbf{B}_1 + \mathbf{B}_2}$$

$$= \frac{1 \times 0 + 1 \times 10\,\underline{/30^\circ}}{10\,\underline{/30^\circ} + 0} = 1$$

Therefore,

$$\begin{bmatrix} \mathbf{A}_{eq} & \mathbf{B}_{eq} \\ \mathbf{C}_{eq} & \mathbf{D}_{eq} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0.025\,\underline{/45^\circ} & 1 \end{bmatrix}$$

2.20.6 Terminated Transmission Line

Figure 2.47 shows a four-terminal transmission network connected to (i.e., terminated by) a load $\mathbf{Z}_L$.

For the given network,

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix} \tag{2.328}$$

or

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R \tag{2.329}$$

and

$$\mathbf{I}_S = \mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R \tag{2.330}$$

and also

$$\mathbf{V}_R = \mathbf{Z}_L \mathbf{I}_R \tag{2.331}$$

Therefore, the input impedance is

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_S}{\mathbf{I}_S} = \frac{\mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R}{\mathbf{C}\mathbf{V}_R + \mathbf{D}\mathbf{I}_R} \tag{2.332}$$

or by substituting equation (2.331) into equation (2.332),

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{A}\mathbf{Z}_L + \mathbf{B}}{\mathbf{C}\mathbf{Z}_L + \mathbf{D}} \tag{2.333}$$

Since for the symmetrical and long transmission line,

$$\mathbf{A} = \cosh \sqrt{\mathbf{YZ}} = \cosh \boldsymbol{\theta}$$

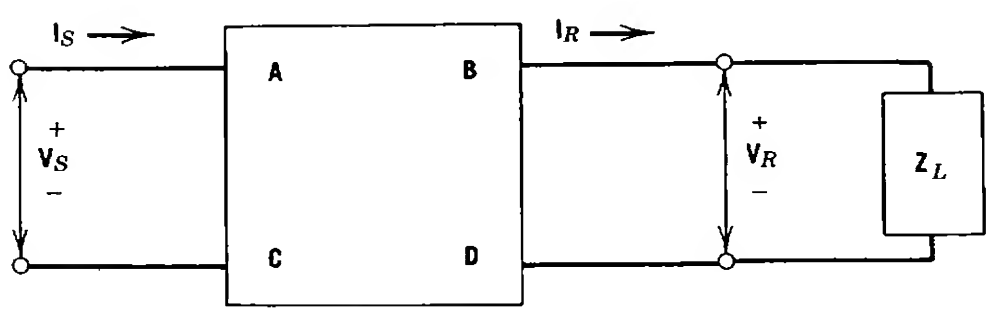


Figure 2.47. Terminated transmission line.

$$\mathbf{B} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}} \sinh \sqrt{\mathbf{YZ}} = \mathbf{Z}_c \sinh \boldsymbol{\theta}$$

$$\mathbf{C} = \sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}} \sinh \sqrt{\mathbf{YZ}} = \mathbf{Y}_c \sinh \boldsymbol{\theta}$$

$$\mathbf{D} = \mathbf{A} = \cosh \sqrt{\mathbf{YZ}} = \cosh \boldsymbol{\theta}$$

the input impedance, from equation (2.333), becomes

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L \cosh \boldsymbol{\theta} + \mathbf{Z}_c \sinh \boldsymbol{\theta}}{\mathbf{Z}_L \mathbf{Y}_c \sinh \boldsymbol{\theta} + \cosh \boldsymbol{\theta}} \tag{2.334}$$

or

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{Z}_L[(\mathbf{Z}_c/\mathbf{Z}_L) \sinh \boldsymbol{\theta} + \cosh \boldsymbol{\theta}]}{(\mathbf{Z}_L/\mathbf{Z}_c) \sinh \boldsymbol{\theta} + \cosh \boldsymbol{\theta}} \tag{2.335}$$

If the load impedance is chosen to be equal to the characteristic impedance, that is,

$$\mathbf{Z}_L = \mathbf{Z}_c \tag{2.336}$$

the input impedance, from equation (2.335), becomes

$$\mathbf{Z}_{\text{in}} = \mathbf{Z}_c \tag{2.337}$$

which is independent of $\boldsymbol{\theta}$ and the line length. The value of the voltage is constant all along the line.

EXAMPLE 2.16

Figure 2.48 shows a short transmission line that is terminated by a load of 200 kVA at a lagging power factor of 0.866 at 2.4 kV. If the line impedance is $2.07 + j0.661\ \Omega$, calculate:

(a) Sending-end current.
(b) Sending-end voltage.
(c) Input impedance.
(d) Real and reactive power loss in line.

Solution

(a) From equation (2.328),

$$\begin{bmatrix} \mathbf{V}_S \\ \mathbf{I}_S \end{bmatrix} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \mathbf{Z} \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \mathbf{V}_R \\ \mathbf{I}_R \end{bmatrix}$$

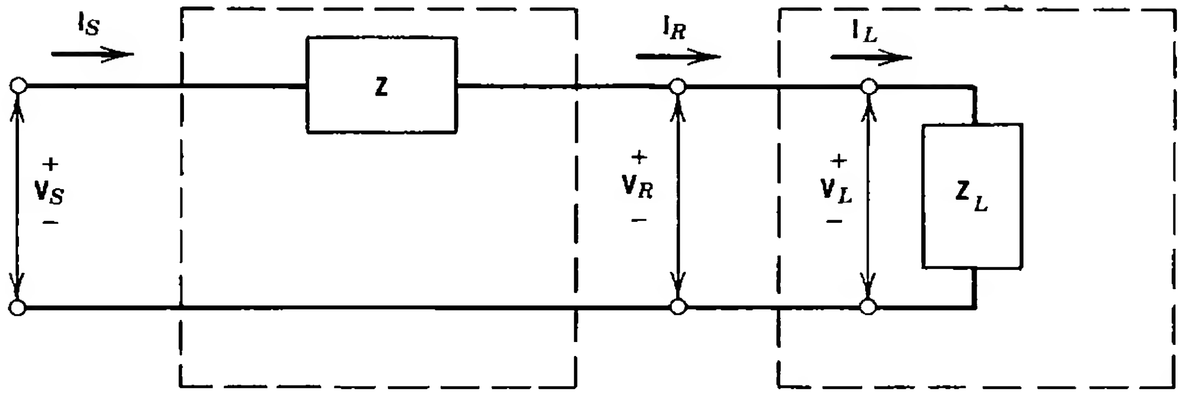


Figure 2.48

where

$$\mathbf{Z} = 2.07 + j0.661 = 2.173 \underline{/17.7°}\ \Omega$$

$$\mathbf{I}_R = \mathbf{I}_S = \mathbf{I}_L$$

$$\mathbf{V}_R = \mathbf{Z}_L \mathbf{I}_R$$

Since

$$\mathbf{S}_R = 200 \underline{/30°} = 173.2 + j100 \text{ kVA}$$

and

$$\mathbf{V}_L = 2.4 \underline{/0°} \text{ kV}$$

then

$$\mathbf{I}_L^* = \frac{\mathbf{S}_R}{\mathbf{V}_L} = \frac{200 \underline{/30°}}{2.4 \underline{/0°}} = 83.33 \underline{/30°} \text{ A}$$

or

$$\mathbf{I}_L = 83.33 \underline{/-30°} \text{ A}$$

Therefore,

$$\mathbf{I}_S = \mathbf{I}_R = \mathbf{I}_L = 83.33 \underline{/-30°} \text{ A}$$

(b)

$$\mathbf{Z}_L = \frac{\mathbf{V}_L}{\mathbf{I}_L} = \frac{2.4 \times 10^3 \underline{/0°}}{83.33 \underline{/-30°}} = 28.8 \underline{/30°}\ \Omega$$

and

$$\mathbf{V}_R = \mathbf{Z}_L \mathbf{I}_R = 28.8 \underline{/30°} \times 83.33 \underline{/-30°} = 2404 \underline{/0°} \text{ kV}$$

Therefore,

$$\mathbf{V}_S = \mathbf{A}\mathbf{V}_R + \mathbf{B}\mathbf{I}_R$$

$$= 2400\,\underline{/0^\circ} + 2.173\,\underline{/17.7^\circ} \times 83.33\,\underline{/-30^\circ}$$
$$= 2576.9 - j38.58$$
$$= 2.577.2\,\underline{/-0.9^\circ}\text{ V}$$

(c) The input impedance is:

$$\mathbf{Z}_{\text{in}} = \frac{\mathbf{V}_S}{\mathbf{I}_S} = \frac{\mathbf{AV}_R + \mathbf{BI}_R}{\mathbf{CV}_R + \mathbf{DI}_R}$$
$$= \frac{2577.2\,\underline{/-0.9^\circ}}{83.33\,\underline{/-30^\circ}} = 30.93\,\underline{/29.1^\circ}\,\Omega$$

(d) The real and reactive power loss in the line:

$$\mathbf{S}_L = \mathbf{S}_S - \mathbf{S}_R$$

where

$$\mathbf{S}_S = \mathbf{V}_S\mathbf{I}_S^* = 2.577.2\,\underline{/-0.9^\circ} \times 83.33\,\underline{/+30^\circ} = 214{,}758\,\underline{/29.1^\circ}$$

or

$$\mathbf{S}_S = \mathbf{I}_S \times \mathbf{Z}_{\text{in}} \times \mathbf{I}_S^* = 214{,}758\,\underline{/29.1^\circ}\text{ VA}$$

Therefore,

$$\mathbf{S}_L = 214{,}758\,\underline{/29.1^\circ} - 200{,}000\,\underline{/30^\circ}$$
$$= 14{,}444.5 + j4444.4$$

that is, the active power loss is 14,444.5 W, and the reactive power loss is 4444.4 vars.

2.20.7 Power Relations Using A, B, C, and D Line Constants

For a given long transmission line, the complex power at the sending and receiving ends are

$$\mathbf{S}_S = P_S + jQ_S = \mathbf{V}_S\mathbf{I}_S^* \tag{2.338}$$

and

$$\mathbf{S}_R = P_R + jQ_R = \mathbf{V}_R\mathbf{I}_R^* \tag{2.339}$$

Also, the sending- and receiving-end voltages and currents can be expressed as

$$\mathbf{V}_S = \mathbf{AV}_R + \mathbf{BI}_R \tag{2.340}$$
$$\mathbf{I}_S = \mathbf{CV}_R + \mathbf{DI}_R \tag{2.341}$$

and

$$\mathbf{V}_R = \mathbf{A}\mathbf{V}_S - \mathbf{B}\mathbf{I}_S \tag{2.342}$$

$$\mathbf{I}_R = -\mathbf{C}\mathbf{V}_S + \mathbf{D}\mathbf{I}_S \tag{2.343}$$

where

$$\mathbf{A} = A\,\underline{/\alpha} = \cosh\sqrt{\mathbf{YZ}} \tag{2.344}$$

$$\mathbf{B} = B\,\underline{/\beta} = \sqrt{\frac{\mathbf{Z}}{\mathbf{Y}}}\,\sinh\sqrt{\mathbf{YZ}} \tag{2.345}$$

$$\mathbf{C} = C\,\underline{/\delta} = \sqrt{\frac{\mathbf{Y}}{\mathbf{Z}}}\,\sinh\sqrt{\mathbf{YZ}} \tag{2.346}$$

$$\mathbf{D} = \mathbf{A} = \cosh\sqrt{\mathbf{YZ}} \tag{2.347}$$

$$\mathbf{V}_R = V_R\,\underline{/0^\circ} \tag{2.348}$$

$$\mathbf{V}_S = V_S\,\underline{/\delta} \tag{2.349}$$

From equation (2.342),

$$\mathbf{I}_S = \frac{\mathbf{A}}{\mathbf{B}}\,\mathbf{V}_S - \frac{\mathbf{V}_R}{\mathbf{B}} \tag{2.350}$$

or

$$\mathbf{I}_S = \frac{AV_S}{B}\,\underline{/\alpha + \delta - \beta} - \frac{V_R\,\underline{/-\beta}}{B} \tag{2.351}$$

and

$$\mathbf{I}_S^* = \frac{AV_S}{B}\,\underline{/-\alpha - \delta + \beta} - \frac{V_R\,\underline{/\beta}}{B} \tag{2.352}$$

and from equation (2.340)

$$\mathbf{I}_R = \frac{\mathbf{V}_S}{\mathbf{B}} - \frac{\mathbf{A}}{\mathbf{B}}\,\mathbf{V}_R \tag{2.353}$$

or

$$\mathbf{I}_R = \frac{V_S}{B}\,\underline{/\delta - \beta} - \frac{AV_R}{B}\,\underline{/\alpha - \beta} \tag{2.354}$$

and

$$\mathbf{I}_R^* = \frac{V_S}{B}\,\underline{/-\delta + \beta} - \frac{AV_R}{B}\,\underline{/-\alpha + \beta} \tag{2.355}$$

By substituting equation (2.352) and (2.355) into equations (2.338) and (2.339), respectively,

$$\mathbf{S}_S = P_S + jQ_S = \frac{AV_S^2}{B} \underline{/\beta - \alpha} - \frac{V_S V_R}{B} \underline{/\beta + \delta} \tag{2.356}$$

and

$$\mathbf{S}_R = P_R + jQ_R = \frac{V_S V_R}{B} \underline{/\beta - \delta} - \frac{AV_R^2}{B} \underline{/\beta - \alpha} \tag{2.357}$$

Therefore, the real and reactive powers at the sending end are

$$P_S = \frac{AV_S^2}{B} \cos(\beta - \alpha) - \frac{V_S V_R}{B} \cos(\beta + \delta) \tag{2.358}$$

and

$$Q_S = \frac{AV_S^2}{B} \sin(\beta - \alpha) - \frac{V_S V_R}{B} \sin(\beta + \delta) \tag{2.359}$$

and the real and reactive powers at the receiving end are

$$P_R = \frac{V_S V_R}{B} \cos(\beta - \delta) - \frac{AV_R^2}{B} \cos(\beta - \alpha) \tag{2.360}$$

and

$$Q_R = \frac{V_S V_R}{B} \sin(\beta - \delta) - \frac{AV_R^2}{B} \sin(\beta - \alpha) \tag{2.361}$$

For constant V_S and V_R, for a given line, the only variable in equations (2.358)–(2.361) is δ, the power angle. Therefore, treating P_S as a function of δ only in equation (2.358), P_S is maximum when $\beta + \delta = 180°$. Therefore, the maximum power at the sending end, the maximum input power, can be expressed as

$$P_{S,\max} = \frac{AV_S^2}{B} \cos(\beta - \alpha) + \frac{V_S V_R}{B} \tag{2.362}$$

and similarly the corresponding reactive power at the sending end, the maximum input vars, is

$$Q_{S,\max} = \frac{AV_S^2}{B} \sin(\beta - \alpha) \tag{2.363}$$

On the other hand, P_R is maximum when $\delta = \beta$. Therefore, the maximum power† obtainable at the receiving end can be expressed as

$$P_{R,\max} = \frac{V_S V_R}{B} - \frac{A V_R^2}{B} \cos(\beta - \alpha) \tag{2.364}$$

and similarly, the corresponding reactive power delivered at the receiving end is

$$Q_{R,\max} = -\frac{A V_R^2}{B} \sin(\beta - \alpha) \tag{2.365}$$

In the above equations, V_S and V_R are the phase (line-to-neutral) voltages whether the system is single phase or three phase. Therefore, the total three-phase power transmitted on the three-phase line is three times the power calculated by using the above equations. If the voltages are given in volts, the power is expressed in watts or vars. Otherwise, if they are given in kilovolts, the power is expressed in megawatts or megavars.

For a given value of γ, the power loss P_L in a long transmission line can be calculated as the difference between the sending- and the receiving-end real powers,

$$P_L = P_S - P_R \tag{2.366}$$

and the lagging vars loss is

$$Q_L = Q_S - Q_R \tag{2.367}$$

EXAMPLE 2.17

Figure 2.49 shows a three-phase, 345-kV ac transmission line with bundled conductors connecting two buses that are voltage regulated. Assume that series capacitor and shunt reactor compensation are to be considered. The bundled conductor line has two 795-kcmil ACSR conductors per phase. The subconductors are separated 18 in., and the phase spacing of the flat configuration is 24, 24, and 48 ft. The resistance inductive reactance and susceptance of the line are given as 0.059 Ω/mi per phase, 0.588 Ω/mi per phase, and 7.20×10^{-6} S phase to neutral per phase per mile, respectively. The total line length is 200 mi, and the line resistance may be neglected because simple calculations and approximate answers will suffice. First assume that there is no compensation in use; that is, both reactors are disconnected and the series capacitor is bypassed. Determine the following:

(a) Total three-phase SIL of line in megavoltamperes.
(b) Maximum three-phase theoretical steady-state power flow limit in megawatts.

† Also called the steady-state power limit.

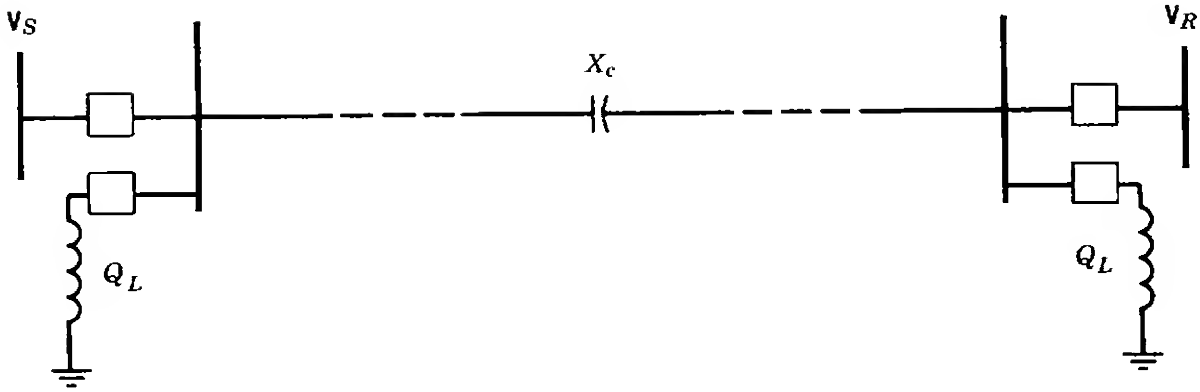


Figure 2.49

(c) Total three-phase magnetizing var generation by line capacitance.

(d) Open-circuit receiving-end voltage if line is open at receiving end.

Solution

(a) The surge impedance of the line is

$$Z_c = (\psi_L \times \psi_c)^{1/2} \tag{2.368}$$

where

$$\psi_c = \frac{1}{b_c} \quad \Omega/\text{mi phase} \tag{2.369}$$

Therefore,

$$Z_c = \left(\frac{\psi_L}{b_c}\right)^{1/2}$$

$$= \left(\frac{0.588}{7.20 \times 10^{-6}}\right)^{1/2} = 285.77\ \Omega/\text{mi phase} \tag{2.370}$$

Thus, the total three-phase SIL of the line is

$$\text{SIL} = \frac{|k\mathbf{V}_{(\text{L-L})}|^2}{Z_c}$$

$$= \frac{345^2}{285.77} = 416.5 \quad \text{MVA/mi}$$

(b) Neglecting the line resistance,

$$P = P_S = P_R$$

or

$$P = \frac{V_S V_R}{X_L} \sin \delta \tag{2.371}$$

When $\delta = 90°$, the maximum three-phase theoretical steady-state power flow limit is

$$P_{\max} = \frac{V_S V_R}{X_L}$$

$$= \frac{(345\ \text{kV})^2}{117.6} = 1012.1\ \text{MW} \tag{2.372}$$

(c) Using a nominal-Π circuit representation, the total three-phase magnetizing var generated by the line capacitance can be expressed as

$$Q_c = V_S^2 \frac{b_c l}{2} + V_R^2 \frac{b_c l}{2}$$

$$= V_S^2 \frac{B_c}{2} + V_R^2 \frac{B_c}{2}$$

$$= (345 \times 10^3)^2[\tfrac{1}{2}(7.20 \times 10^{-6})(200)]$$

$$+ (345 \times 10^3)^2[\tfrac{1}{2}(7.20 \times 10^{-6})(200)]$$

$$= 171.4\ \text{Mvar} \tag{2.373}$$

(d) If the line is open at the receiving end, the open-circuit receiving-end voltage can be expressed as

$$\mathbf{V}_S = V_{R(\text{oc})} \cosh \gamma l \tag{2.374}$$

or

$$\mathbf{V}_{R(\text{oc})} = \frac{\mathbf{V}_S}{\cosh \gamma l} \tag{2.375}$$

where

$$\gamma = jw\sqrt{LC}$$

$$= jw\left[\left(\frac{\psi_L}{w} \frac{1}{w\psi_c}\right)\right]^{1/2}$$

$$= j\left(\frac{\psi_L}{\psi_c}\right)^{1/2}$$

$$= j[(0.588)(7.20 \times 10^{-6})]^{1/2} = 0.0021\ \text{rad/mile} \tag{2.376}$$

and

$$\gamma l = j(0.0021)(200) = j0.4115\ \text{rad}$$

Thus,

$$\cos \gamma l = \cosh(0 + j0.4115)$$

$$= \cosh(0)\cos(0.4115) + j\sinh(0)\sin(0.4115)$$

$$= 0.9164$$

Therefore,

$$V_{R(\mathrm{oc})} = \frac{345\ \mathrm{kV}}{0.9165} = 376.43\ \mathrm{kV}$$

Alternatively,

$$\begin{aligned} V_{R(\mathrm{oc})} &= V_S\, \frac{X_c}{X_c + X_L} \\ &= (345\ \mathrm{kV})\, \frac{-j1388.9}{-j1388.9 + j117.6} \\ &= 376.74\ \mathrm{kV} \end{aligned} \tag{2.377}$$

EXAMPLE 2.18

Use the data given in Example 2.17 and assume that the shunt compensation is now used. Assume also that the two shunt reactors are connected to absorb 60 percent of the total three-phase magnetizing var generation by line capacitance and that half of the total reactor capacity is placed at each end of the line. Determine the following:

(a) Total three-phase SIL of line in megavoltamperes.
(b) Maximum three-phase theoretical steady-state power flow limit in megawatts.
(c) Three-phase megavoltampere rating of each shunt reactor.
(d) Cost of each reactor at $10/kVA
(e) Open-circuit receiving-end voltage if line is open at receiving end.

Solutions

(a) SIL = 416.5, as before, in Example 2.17.
(b) $P_{\max} = 1012.1$ MW, as before.
(c) The three-phase megavoltampere rating of each shunt reactor is

$$\begin{aligned} \tfrac{1}{2}Q_L &= \tfrac{1}{2}0.60Q_c \\ &= \tfrac{1}{2}0.60(171.4) = 51.42\ \mathrm{MVA} \end{aligned}$$

(d) The cost of each reactor at $10/kVA is

$$(51.42\ \mathrm{MVA/reactor})(\$10/\mathrm{kVA}) = \$514{,}200$$

(e) Since

$$\gamma l = j0.260\ \mathrm{rad}$$

and

$$\cosh \gamma l = 0.9663$$

then

$$V_{R(\mathrm{oc})} = \frac{345\ \mathrm{kV}}{0.9663} = 357.03\ \mathrm{kV}$$

Alternatively,

$$V_{R(\text{oc})} = V_S \frac{-j3472}{-j3472 + j117.6}$$
$$= (345\ \text{kV})(1.035) = 357.1\ \text{kV}$$

Therefore, the inclusion of the shunt reactor causes the receiving-end open-circuit voltage to decrease.

2.21 UNDERGROUND CABLE TRANSMISSION

As discussed in the previous sections, the inductive reactance of an overhead high-voltage ac line is much greater than its capacitive reactance. Whereas the capacitive reactance of an underground high-voltage ac cable is much greater than its inductive reactance due to the fact that the three-phase conductors are located very close to each other in the same cable. The approximate values of the resultant vars (reactive power) that can be generated by ac cables operating at the phase-to-phase voltages of 132, 220, and 400 kV are 2000, 5000, and 15,000 kVA/mi, respectively. This var generation, due to the capacitive charging currents, sets a practical limit to the possible noninterrupted length of an underground ac cable. This situation can be compensated for by installing appropriate inductive shunt reactors along the line. This "critical length" of the cable can be defined as the length of cable line that has a three-phase charging reactive power equal in magnitude to the thermal rating of the cable line. For example, the typical critical lengths of ac cables operating at the phase-to-phase voltages of 132, 200, and 400 kV can be given approximately as 40, 25 and 15 mi, respecitvely.

The study done by Schifreen and Marble [12] illustrated the limitations in the operation of high-voltage ac cable lines due to the charging current. For example, Figure 2.50 shows that the magnitude of the maximum permissible power output decreases as a result of an increase in cable length. Figure 2.51 shows that increasing lengths of cable line can transmit full-rated current (1.0 pu) only if the load power factor is decreased to resolve lagging values. Note that the critical length is used as the base length in the figures. Table 2.6 [3] gives characteristics of a 345-kV pipe-type cable. Figure 2.52 shows the permissible variation in per-unit vars delivered to the electric system at each terminal of cable line for a given power transmission.

EXAMPLE 2.19

Consider a high-voltage open-circuit three-phase insulated power cable of length l shown in Figure 2.53. Assume that a fixed sending-end voltage is to be supplied; the receiving-end voltage floats, and it is an overvoltage. Furthermore, assume that at some critical length ($l = l_0$), the sending-end

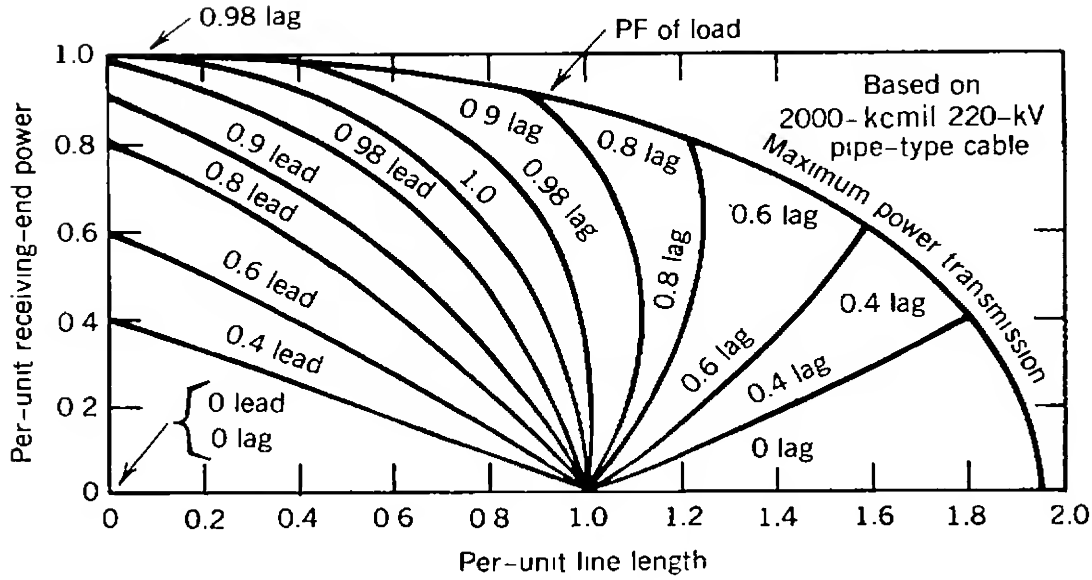


Figure 2.50. Power transmission limits of high-voltage ac cable lines. Curved lines: Sending-end current equal to rated or base current of cable. Horizontal lines: Receiving-end current equal to rated or base current of cable. (From Ref. 12. Used by permission. © 1956 IEEE.)

current $\mathbf{I}_S$ is equal to the ampacity of the cable circuit, $\mathbf{I}_{l0}$. Therefore, if the cable length is l_0, no load whatever of 1.0 or leading power factor can be supplied without overloading the sending end of the cable. Use the general long-transmission-line equations, which are valid for steady-state sinusoidal operation, and verify that the approximate critical length can be expressed as

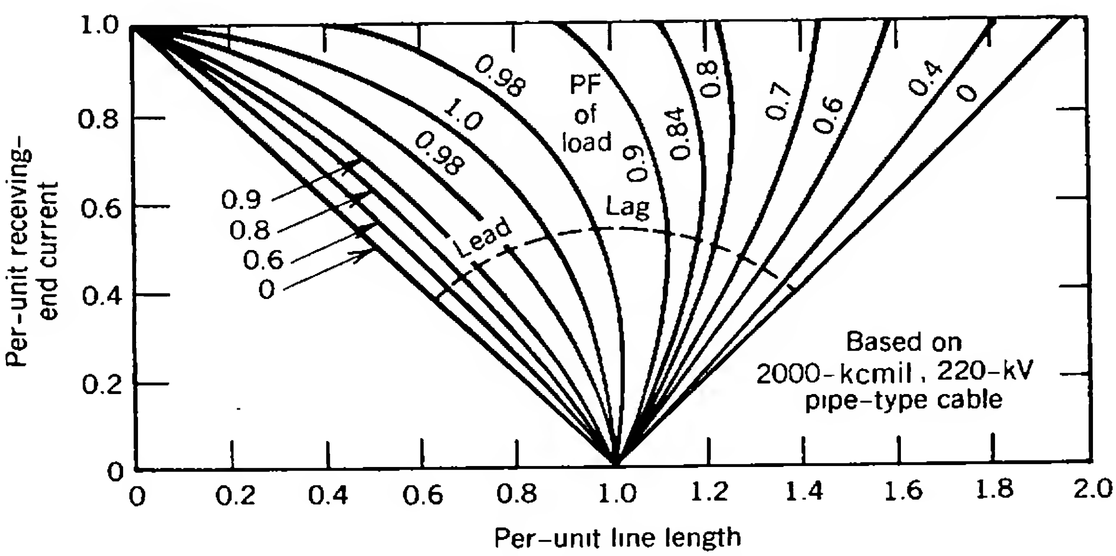


Figure 2.51. Receiving-end current limits of high-voltage ac cable lines. Curved lines: Sending-end current equal to rated or base current of cable. (From Ref. 12. Used by permission. © 1956 IEEE.)

TABLE 2.6 Characteristics of 345-kV Pipe-Type Cable

Characteristics	Power Factor (%)	Maximum Electric Stress, 300 V/mil				Power Factor (%)	Maximum Electric Stress, 350 V/mil			
Conductor size, thousand cmils		1,000	1,250	1,500	2,000		1,000	1,230	1,300	2,000
Insulation thickness, mils		1,250	1,173	1,110	1,035		980	915	885	835
E_R kV, 200 kV										
I_T A	0.3	383	638	680	730	0.3	576	623	636	688
	0.3	653	721	780	860	0.3	637	724	776	847
Rated 3-phase MVA	0.3	350	381	406	436	0.3	344	372	392	413
	0.3	390	431	466	516	0.3	393	432	463	508
Z Ω/mi		$0.403\angle 78.0$	$0.381\angle 78.8$	$0.363\angle 80.0$	$0.338\angle 80.0$		$0.377\angle 77.2$	$0.355\angle 78.5$	$0.367\angle 79.4$	$0.319\angle 80.0$
Y S, mi $\times 10^{-4}$		$1.08\angle 89.7$	$1.20\angle 89.7$	$1.32\angle 89.7$	$1.53\angle 89.7$		$1.26\angle 89.7$	$1.41\angle 89.7$	$1.54\angle 89.7$	$1.78\angle 89.7$
A numeric/mi $\times 10^{-8}$		$6.6\angle 83.9$	$6.8\angle 84.3$	$6.9\angle 84.9$	$7.19\angle 84.8$		$6.9\angle 83.5$	$7.1\angle 84.1$	$7.5\angle 84.6$	$7.54\angle 84.8$
Z_0 Ω		$61.2\angle 5.9$	$56.4\angle 5.5$	$52.5\angle 4.9$	$67.1\angle 6.9$		$54.7\angle 6.3$	$50.1\angle 3.6$	$48.9\angle 3.2$	$42.3\angle 4.9$
0 A/mi		21.6	24.1	26.5	30.5		25.1	28.1	30.8	3.37
3Φ charging kVA/mi		12,900	14,400	13,800	18,300		15,000	16,800	18,000	21,300
S_4, mi	0.3	27.1	26.5	25.7	24.0	0.5	22.9	22.2	21.3	19.4
	0.3	30.3	30.0	29.3	28.2	0.3	26.2	25.7	25.2	23.9
Nominal pipe size (in) 10^8/in.										
Earth resistivity (thermal ohm cm)		80	80	80	80		80	80	80	80
Conductor temperature. °C		70	70	70	70		70	70	70	70
3Φ dielectric loss W/ft	0.5	12.2	13.7	14.9	17.3	0.5	14.2	15.8	17.3	20.1
	0.3	7.3	81	9.0	10.3	0.3	8.5	9.5	10.4	12.0
3Φ total loss. W/ft	0.3	29.0	30.8	32.2	34.3	0.5	30.5	32.1	33.5	35.2
	0.3	28.3	29.9	31.8	33.8	0.3	29.7	31.5	33.0	34.8
Ratio watts dielectric loss Φ total loss	0.5	42.0	44.5	46.0	50.5	0.5	46.3	49.3	52.6	37.0
	0.3	38.8	36.9	35.3	30.5	0.3	35.0	33.1	31.7	34.5

Source: From Wiseman [13]. Used with permission. © 1956 IEEE.

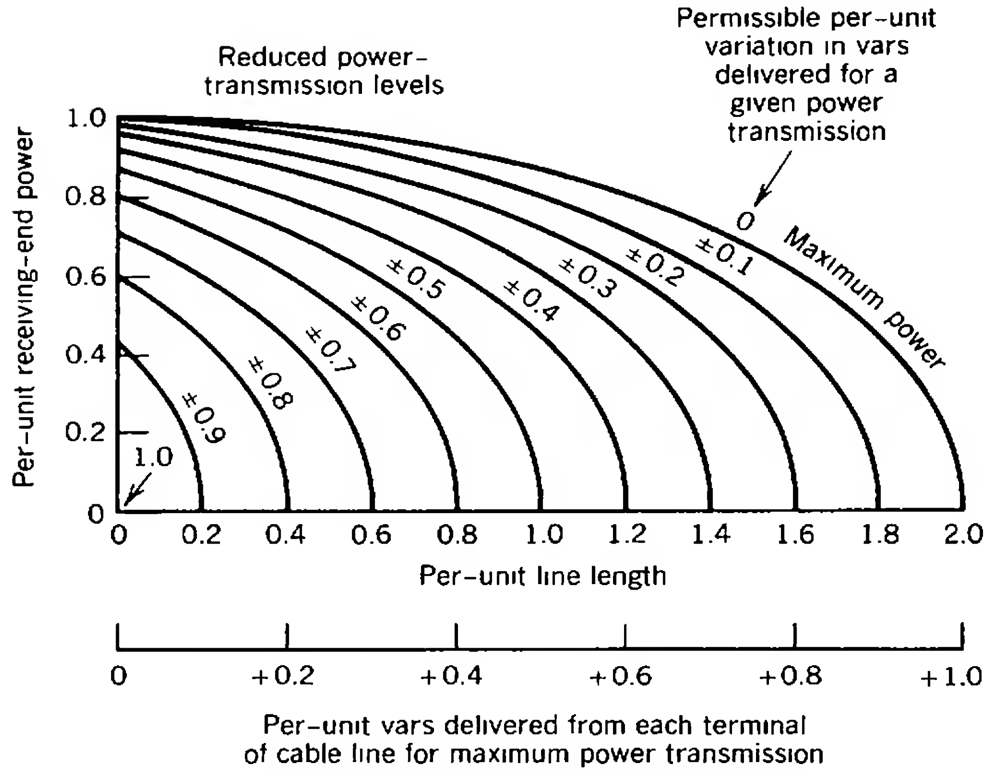


Figure 2.52. Permissible variations in per-unit vars delivered to electric system at each terminal of ac cable line for given power transmission. (From Ref. 12. Used by permission. © 1956 IEEE.)

$$l_0 \cong \frac{I_{l0}}{V_S b}$$

Solution

The long-transmission-line equations can be expressed as

$$\mathbf{V}_S = \mathbf{V}_R \cosh \gamma l + \mathbf{I}_R \mathbf{Z}_C \sinh \gamma l \tag{2.378}$$

and

$$\mathbf{I}_S = \mathbf{I}_R \cosh \gamma l + \mathbf{V}_R \mathbf{Y}_c \sinh \gamma l \tag{2.379}$$

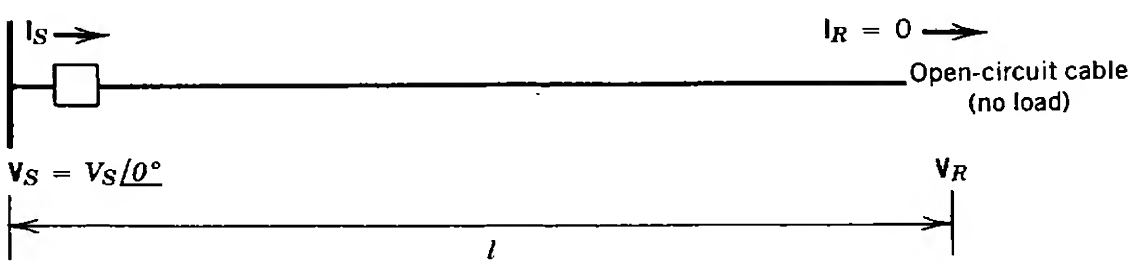


Figure 2.53

Since at critical length, $l = l_0$ and

$$\mathbf{I}_R = 0 \quad \text{and} \quad \mathbf{I}_S = \mathbf{I}_{l0}$$

from equation (2.379), the sending-end current can be expressed as

$$\mathbf{I}_{l0} = \mathbf{V}_R \mathbf{Y}_c \sinh \gamma l_0 \tag{2.380}$$

or

$$\mathbf{I}_{l0} = \mathbf{V}_R \mathbf{Y}_c \frac{e^{\gamma l_0} - e^{-\gamma l_0}}{2} \tag{2.381}$$

$$\mathbf{I}_{l0} = \mathbf{V}_R \mathbf{Y}_c \frac{[1 + \gamma l_0 + (\gamma l_0)^2/2! + \cdots] - [1 - \gamma l_0 + (\gamma l_0)^2/2! - \cdots]}{2}$$

or

$$\mathbf{I}_{l0} = \mathbf{V}_R \mathbf{Y}_c \left[\gamma l_0 + \frac{(\gamma l_0)^3}{3!} + \cdots \right] \tag{2.382}$$

Neglecting $(\gamma l_0)^3/3!$ and higher powers of γl_0,

$$\mathbf{I}_{l0} = \mathbf{V}_R \mathbf{Y}_c \gamma l_0 \tag{2.383}$$

Similarly, from equation (2.378), the sending-end voltage for the critical length can be expressed as

$$\mathbf{V}_S = \mathbf{V}_R \cosh \gamma l_0 \tag{2.384}$$

or

$$\mathbf{V}_S = \mathbf{V}_R \frac{e^{\gamma l_0} + e^{-\gamma l_0}}{2} \tag{2.385}$$

or

$$\mathbf{V}_S = \mathbf{V}_R \frac{[1 + \gamma l_0 + (\gamma l_0)^2/2! + \cdots] + [1 - \gamma l_0 + (\gamma l_0)^2/2! - \cdots]}{2}$$

or

$$\mathbf{V}_S = \mathbf{V}_R \left(1 + \frac{(\gamma l_0)^2}{2!} + \cdots \right) \tag{2.386}$$

Neglecting higher powers of γl_0,

$$\mathbf{V}_S \cong \mathbf{V}_R \left(1 + \frac{(\gamma l_0)^2}{2!} \right) \tag{2.387}$$

Therefore,

$$\mathbf{V}_R = \frac{\mathbf{V}_S}{1 + (\gamma l_0)^2/2!} \tag{2.388}$$

Substituting equation (2.388) into equation (2.383),

$$\mathbf{I}_{l0} = \frac{\mathbf{V}_S}{1 + (\gamma l_0)^2/2!} \mathbf{Y}_c \gamma l_0 \tag{2.389}$$

or

$$\begin{aligned}\mathbf{I}_{l0} &= \mathbf{V}_S \mathbf{Y}_c \gamma l_0 \left[1 + \frac{(\gamma l_0)^2}{2!}\right]^{-1} \\ &= \mathbf{V}_S \mathbf{Y}_c \gamma l_0 \left[1 - \frac{(\gamma l_0)^2}{2!} + \cdots\right]\end{aligned} \tag{2.390}$$

or

$$\mathbf{I}_{l0} = \mathbf{V}_s \mathbf{Y}_c \gamma l_0 - \frac{\mathbf{V}_S \mathbf{Y}_c (\gamma l_0)^3}{2!} \tag{2.391}$$

Neglecting the second term,

$$\mathbf{I}_{l0} \cong \mathbf{V}_S \mathbf{Y}_c \gamma l_0 \tag{2.392}$$

Therefore, the critical length can be expressed as

$$l_0 \cong \frac{I_{l_0}}{\mathbf{V}_S \mathbf{Y}_c \gamma} \tag{2.393}$$

where

$$\mathbf{Y}_c = \sqrt{\frac{\mathbf{y}}{\mathbf{z}}}$$

$$\gamma = \sqrt{\mathbf{z} \times \mathbf{y}}$$

Thus,

$$\mathbf{y} = \mathbf{Y}_c \gamma \tag{2.394}$$

or

$$\mathbf{y} = g + jb \tag{2.395}$$

Therefore, the critical length can be expressed as

$$l_0 \cong \frac{\mathbf{I}_{l_0}}{\mathbf{V}_S \times \mathbf{y}} \tag{2.396}$$

or

$$l_0 \cong \frac{\mathbf{I}_{l_0}}{\mathbf{V}_S (g + jb)} \tag{2.397}$$

Since, for cables, $g \ll b$,

$$\mathbf{y} \cong b \underline{/90^\circ}$$

and assuming

$$\mathbf{I}_{l0} \cong I_{l0} \underline{/90^\circ}$$

from equation (2.397), the critical length can be expressed as

$$l_0 \cong \frac{\mathbf{I}_{l_0}}{\mathbf{V}_S b} \tag{2.398}$$

EXAMPLE 2.20

Figure 2.54(a) shows an open-circuit high-voltage insulated ac underground cable circuit. The critical length of uncompensated cable is l_0 for which $\mathbf{I}_S = \mathbf{I}_0$ is equal to cable ampacity rating. Note that $Q_0 = 3V_S I_0$, where the sending-end voltage $\mathbf{V}_S$ is regulated and the receiving-end voltage $\mathbf{V}_R$ floats. Here, the $|\mathbf{V}_R|$ differs little from $|\mathbf{V}_S|$ because of the low series inductive reactance of cables. Based on the given information, investigate the performances with $\mathbf{I}_R = 0$ (i.e., zero load).

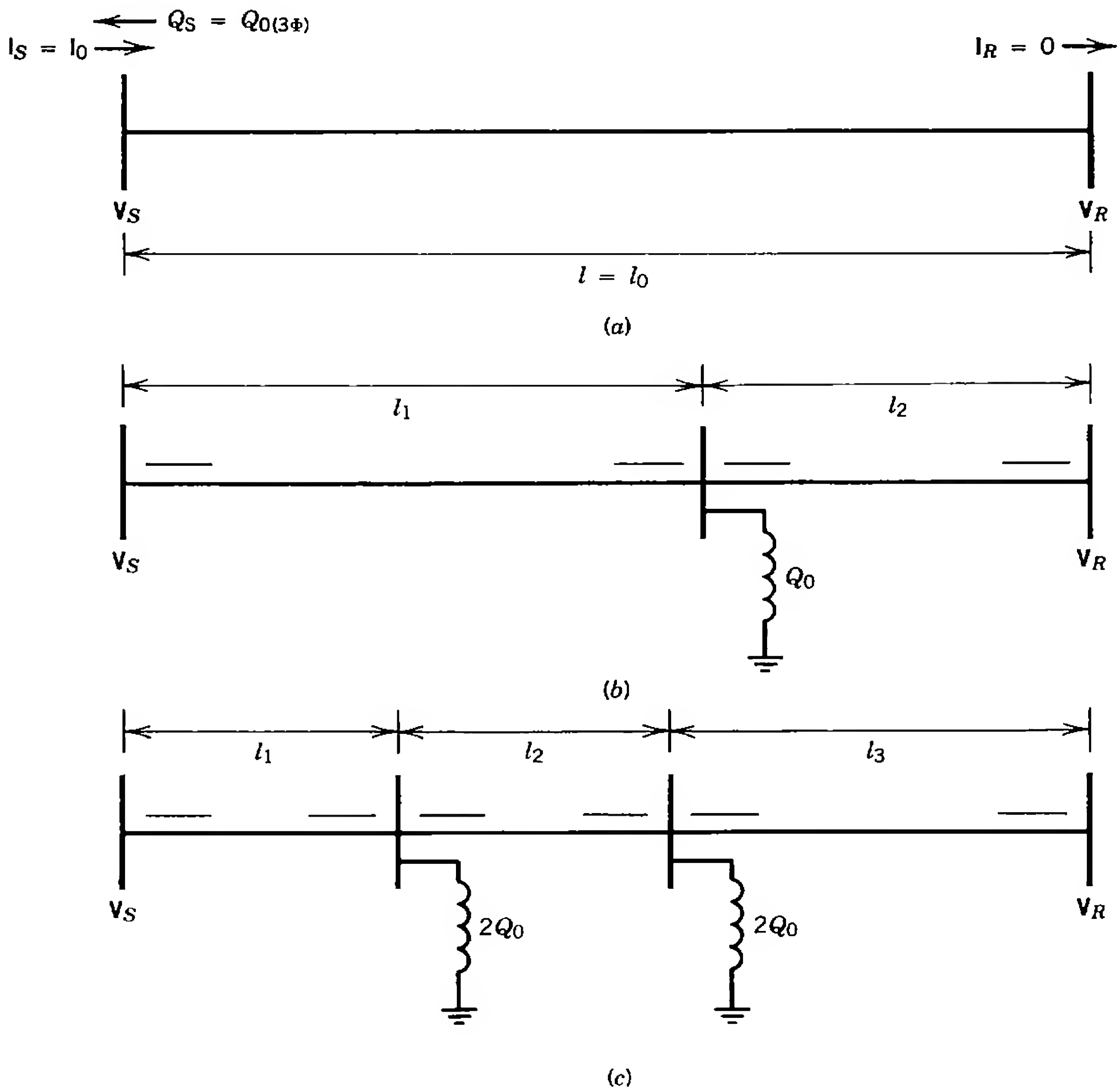
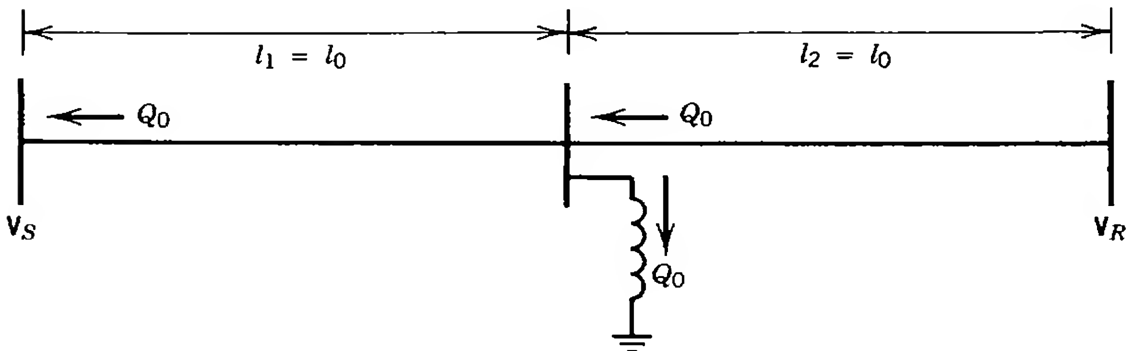


Figure 2.54

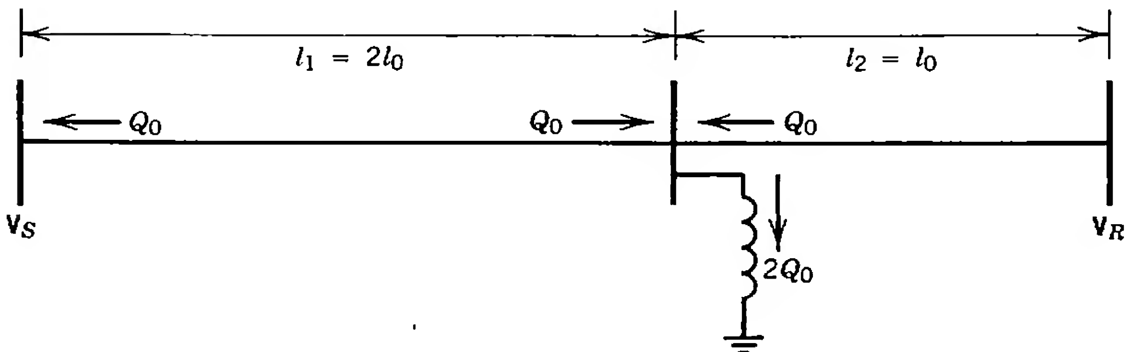
(a) Assume that one shunt inductive reactor sized to absorb Q_0 magnetizing vars is to be purchased and installed as shown in Figure 2.54(*b*). Locate the reactor by specifying l_1 and l_2 in terms of l_0. Place arrowheads on the four short lines, indicated by a solid line, to show the directions of magnetizing var flows. Also show on each line the amounts of var flow, expressed in terms of Q_0.

(b) Assume that one reactor size $2Q_0$ can be afforded and repeat part (a) on a new diagram.

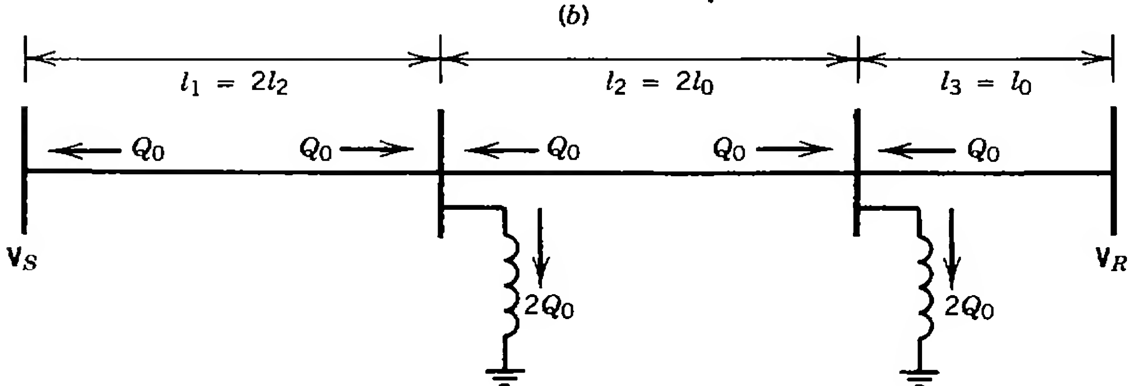
(c) Assume that two shunt reactors, each of size $2Q_0$, are to be installed, as shown in Figure 2.54(*c*), hoping, as usual, to extend the feasible length of cable. Repeat part (a).



(*a*)



(*b*)



(*c*)

Figure 2.55

Solution

The answers for parts (a), (b), and (c) are given in Figures 2.55(*a*), (*b*) and (*c*), respectively.

2.22 BUNDLED CONDUCTORS

Bundled conductors are used at or above 345 kV. Instead of one large conductor per phase, two or more conductors of approximately the same total cross section are suspended from each insulator string. Therefore, by having two or more conductors per phase in close proximity compared with the spacing between phases, the voltage gradient at the conductor surface is significantly reduced. The bundles used at the extra-high-voltage range usually have two, three, or four *subconductors*, as shown in Figure 2.56. Whereas the bundles used at the ultrahigh-voltage range may also have 8, 12, and even 16 conductors. Bundle conductors are also called *duplex*, *triplex*, and so on, conductors, referring to the number of subconductors, and are sometimes referred to as grouped or multiple conductors. The advantages derived from the use of *bundled* conductors instead of single conductors per phase are (1) reduced line inductive reactance; (2) reduced voltage gradient; (3) increased corona critical voltage and, therefore, less corona power loss, audible noise, and radio interference; (4) more power

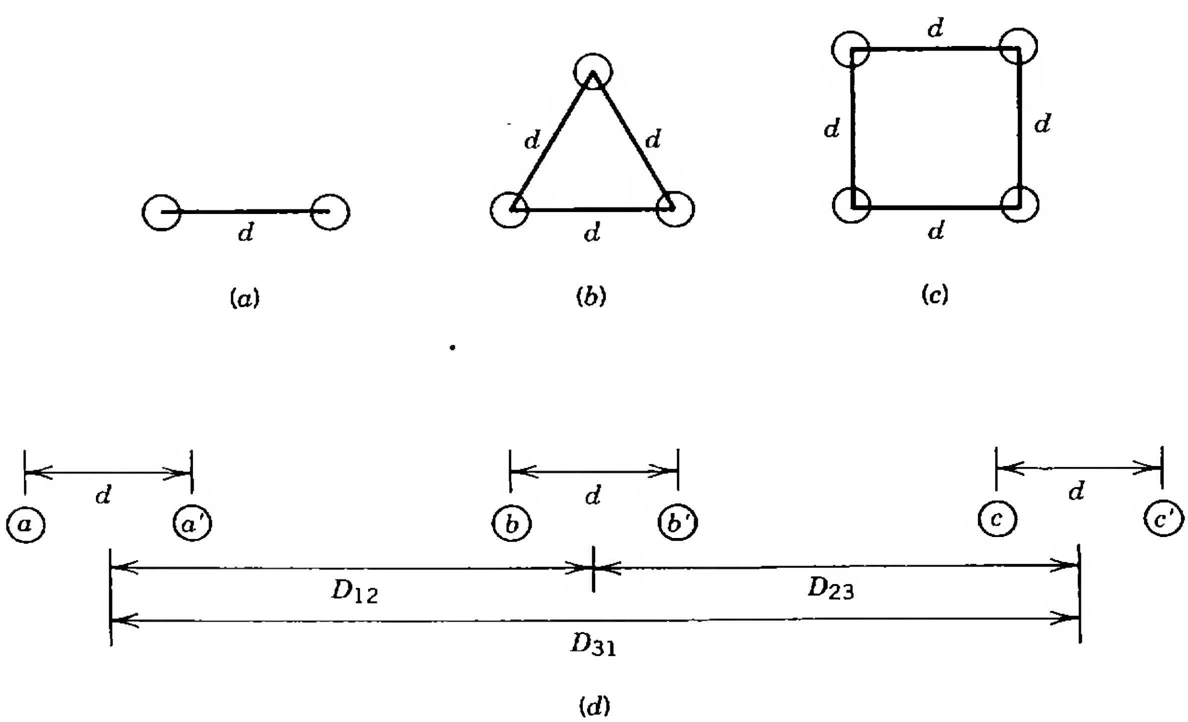


Figure 2.56. Bundle arrangements: (*a*) two-conductor bundle; (*b*) three-conductor bundle; (c) four-conductor bundle; (*d*) cross section of bundled-conductor three-phase line with horizontal tower configuration.

may be carried per unit mass of conductor; and (5) the amplitude and duration of high-frequency vibrations may be reduced. The disadvantages of bundled conductors include (1) increased wind and ice loading; (2) suspension is more complicated and duplex or quadruple insulator strings may be required; (3) the tendency to gallop is increased; (4) increased cost; (5) increased clearance requirements at structures; and (6) increased charging kilovoltamperes.

If the subconductors of a bundle are transposed, the current will be divided exactly between the conductors of the bundle. The GMRs of bundled conductors made up of two, three, and four subconductors can be expressed, respectively, as

$$D_s^b = (D_s \times d)^{1/2} \tag{2.399}$$

$$D_s^b = (D_s \times d^2)^{1/3} \tag{2.400}$$

$$D_s^b = 1.09(D_s \times d^3)^{1/4} \tag{2.401}$$

where D_s^b = GMR of bundled conductor
D_s = GMR of subconductors
d = distance between two subconductors

Therefore, the *average* inductance per phase is

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s^b} \quad \text{H/m} \tag{2.402}$$

and the inductive reactance is

$$X_L = 0.1213 \ln \frac{D_{eq}}{D_s^b} \quad \Omega/\text{mi} \tag{2.403}$$

where

$$D_{eq} \triangleq D_m = (D_{12}D_{23}D_{31})^{1/3} \tag{2.404}$$

The modified GMRs (to be used in capacitance calculations) of bundled conductors made up of two, three, and four subconductors can be expressed, respectively, as

$$D_{sC}^b = (rd)^{1/2} \tag{2.405}$$

$$D_{sC}^b = (r^2 d)^{1/3} \tag{2.406}$$

$$D_{sC}^b = 1.09(rd^3)^{1/4} \tag{2.407}$$

where D_{sC}^b = modified GMR of bundled conductor
r = outside radius of subconductors
d = distance between two subconductors

Therefore, the line-to-neutral capacitance can be expressed as

$$C_n = \frac{2\pi \times 8.8538 \times 10^{-12}}{\ln(D_{eq}/D_{sC}^b)} \quad \text{F/m} \tag{2.408}$$

or

$$C_n = \frac{55.63 \times 10^{-12}}{\ln(D_{eq}/D_{sC}^b)} \quad \text{F/m} \tag{2.409}$$

For a two-conductor bundle, the maximum voltage gradient at the surface of a subconductor can be expressed as

$$E_0 = \frac{V_0(1 + 2r/d)}{2r \ln(D/\sqrt{rd})} \tag{2.410}$$

EXAMPLE 2.21

Consider the bundled-conductor three-phase 200-km line shown in Figure 2.56(d). Assume that the power base is 100 MVA and the voltage base is 345 kV. The conductor used is 1113 kcmil ACSR, and the distance between two subconductors is 12 in. Assume that the distances D_{12}, D_{23}, and D_{31} are 26, 26, and 52 ft, respectively and determine the following:

(a) Average inductance per phase in henries per meter.
(b) Inductive reactance per phase in ohms per kilometer and ohms per mile.
(c) Series reactance of line in per units.
(d) Line-to-neutral capacitance of line in farads per meter.
(e) Capacitive reactance to neutral of line in ohm per kilometers and ohm per miles.

Solution

(a) From Table A.3 in Appendix A, D_s is 0.0435 ft; therefore,

$$\begin{aligned} D_s^b &= (D_s d)^{1/2} \\ &= (0.0435 \times 0.3048 \times 12 \times 0.0254)^{1/2} = 0.0636 \text{ m} \end{aligned}$$

$$\begin{aligned} D_{eq} &= (D_{12} D_{23} D_{31})^{1/3} \\ &= (26 \times 26 \times 52 \times 0.3048^3)^{1/3} = 9.9846 \text{ m} \end{aligned}$$

Thus, from equation (2.402),

$$L_a = 2 \times 10^{-7} \ln \frac{D_{eq}}{D_s^b}$$

$$= 2 \times 10^{-7} \ln \frac{9.9846}{0.0636} = 1.0112\ \mu\text{H/m}$$

(b)

$$X_L = 2\pi f L_a$$

$$= 2\pi 60 \times 1.0112 \times 10^{-6} \times 10^3 = 0.3812\ \Omega/\text{km}$$

and

$$X_L = 0.3812 \times 1.609 = 0.6134\ \Omega/\text{mi}$$

(c)

$$Z_B = \frac{345^2}{100} = 1190.25\ \Omega$$

$$X_L = \frac{0.3812 \times 200}{1190.25} = 0.0641\ \text{pu}$$

(d) From Table A.3 the outside diameter of the subconductor is 1.293 in.; therefore, its radius is

$$r = \frac{1.293 \times 0.3048}{2 \times 12} = 0.0164\ \text{m}$$

$$D_{sC}^b = (rd)^{1/2}$$

$$= (0.0164 \times 12 \times 0.0254)^{1/2} = 0.0707\ \text{m}$$

Thus, the line-to-neutral capacitance of the line is

$$C_n = \frac{55.63 \times 10^{-12}}{\ln(D_{eq}/D_{sC}^b)}$$

$$= \frac{55.63 \times 10^{-12}}{\ln(9.9846/0.0707)} = 11.238 \times 10^{-12}\ \text{F/m}$$

(e) The capacitive reactance to the neutral of the line is

$$X_c = \frac{1}{2\pi f C_n}$$

$$= \frac{10^{12} \times 10^{-3}}{2\pi 60 \times 11.238} = 0.236 \times 10^6\ \Omega\text{-km}$$

and

$$X_c = \frac{0.236 \times 10^6}{1.609} = 0.147 \times 10^6\ \Omega\text{-mi}$$

2.23 EFFECT OF GROUND ON CAPACITANCE OF THREE-PHASE LINES

Consider three-phase line conductors and their images below the surface of the ground, as shown in Figure 2.57. Assume that the line is transposed and that conductors a, b, and c have the charges q_a, q_b, and q_c, respectively, and their images have the charges $-q_a$, $-q_b$, and $-q_c$. The line-to-neutral capacitance can be expressed as [3]

$$C_n = \frac{2\pi \times 8.8538 \times 10^{-12}}{\ln(D_{\text{eq}}/r) - \ln(l_{12}l_{23}l_{31}/h_{11}h_{22}h_{33})^{1/3}} \quad \text{F/m} \qquad (2.411)$$

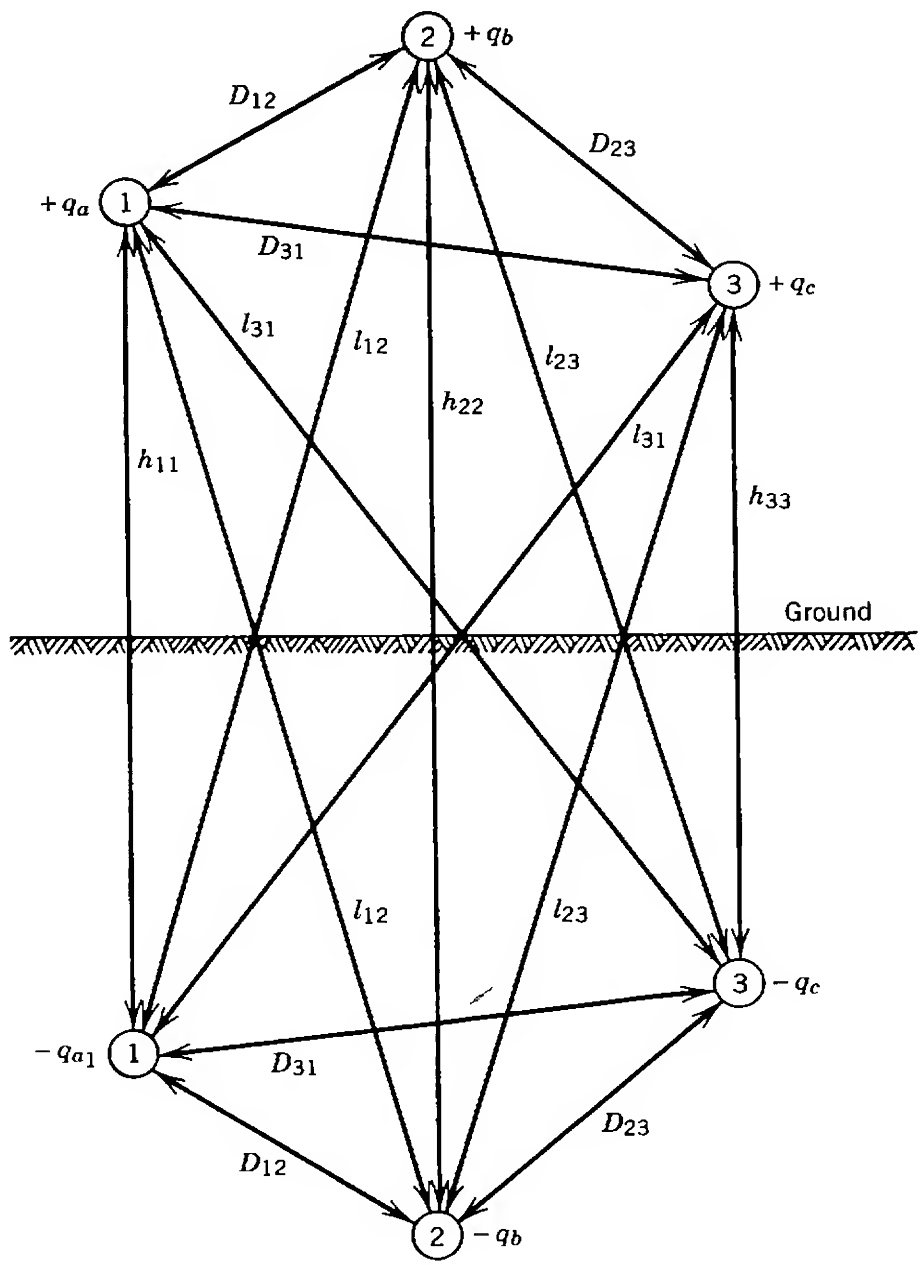


Figure 2.57. Three-phase line conductors and their images.

Whereas, if the effect of the ground is not taken into account, the line-to-neutral capacitance is

$$C_n = \frac{2\pi \times 8.8538 \times 10^{-12}}{\ln(D_{\text{eq}}/r)} \quad \text{F/m} \tag{2.412}$$

Therefore, the effect of the ground is to increase the capacitance of the line. However, since the conductor heights are much larger than the distances between them, the effect of the ground is usually ignored for three-phase lines.†

2.24 ENVIRONMENTAL EFFECTS OF OVERHEAD TRANSMISSION LINES

Recently, the importance of minimizing the environmental effects of overhead transmission lines has increased substantially due to increasing use of greater extra-high- and ultrahigh-voltage levels. Therefore, the magnitude and effect of radio noise, television interference, audible noise, electric field, and magnetic fields must not only be predicted and analyzed in the line design stage but also measured directly. Measurements of corona-related phenomena must include radio and television station signal strengths and radio, television, and audible-noise levels. To determine the effects of transmission line of these quantities, measurements should be taken at three different times: (1) before construction of the line; (2) after construction, but before energization; and (3) after energization of the line. Noise measurements should be made at several locations along a transmission line. Also, at each location, measurements may be made at several points that might be of particular interest. Such points may include the point of maximum moise, the edge of the right of way, and the point 50 ft from the outermost conductor.

Overhead transmission lines and stations also produce electric and magnetic fields, which have to be taken into account in the design process. The study of field effects (e.g., induced voltages and currents in conducting bodies) is becoming especially crucial as the operating voltage levels of transmission lines have been increasing due to the economics and operational benefits involved. Today, for example, such study at ultrahigh-voltage level involves the following:

1. Calculation and measurement techniques for electric and magnetic fields.
2. Calculation and measurement of induced currents and voltages on objects of various shapes for all line voltages and design configurations.

† For further information on the effect of the ground, see Anderson [4, Chapter 3].

3. Calculation and measurement of currents and voltages induced in people as result of various induction mechanisms.
4. Investigation of sensitivity of people to various field effects.
5. Study of conditions resulting in fuel ignition, corona from grounded objects, and other possible field effects [14].

Measurements of the transmission line electric field must be made laterally at midspan and must extend at least to the edges of the right of way to determine the profile of the field. Further, related electric field effects such as currents and voltages induced in vehicles and fences should also be considered. Magnetic field effects are of much less concern than electric field effects for extra-high- and ultrahigh-voltage transmission due to the fact that magnetic field levels for normal values of load current are low. The quantity and character of currents induced in the body by magnetic effects have considerably less impact than those arising from electric induction. For example, the induced current densities in the human body are less than one-tenth those caused by electric field induction. Furthermore, most environmental measurements are highly affected by prevaling weather conditions and transmission line geometry. The weather conditions include temperature, humidity, barometric pressure, precipitation levels, and wind velocity.

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PROBLEMS

2.1. Assume that the impedance of a line connecting buses 1 and 2 is $50\,\underline{/90^\circ}\ \Omega$ and that the bus voltages are $7560\,\underline{/10^\circ}$ and $7200\,\underline{/0^\circ}$ V per phase, respectively. Determine the following:

(a) Real power per phase that is being transmitted from bus 1 to bus 2.

(b) Reactive power per phase that is being transmitted from bus 1 to bus 2.

(c) Complex power per phase that is being transmitted.

2.2. Solve Problem 2.1 assuming that the line impedance is $50\,\underline{/26^\circ}\ \Omega/$phase.

2.3. Verify the following equations:

(a) $V_{\text{pu(L-N)}} = V_{\text{pu(L-L)}}$

(b) $VA_{\text{pu}(1\Phi)} = VA_{\text{pu}(3\Phi)}$

(c) $Z_{\text{pu(Y)}} = Z_{\text{pu}(\Delta)}$

2.4. Verify the following equations:

(a) Equation (2.24) for single-phase system.

(b) Equation (2.80) for three-phase system.

2.5. Show that $Z_{B(\Delta)} = 3Z_{B(\text{Y})}$.

2.6. Consider two three-phase transmission lines with different voltage levels that are located side by side in a close proximity. Assume that the bases of VA_B, $V_{B(1)}$, and $I_{B(1)}$ and the bases of VA_B, $V_{B(2)}$, and $I_{B(2)}$ are designated for the first and second lines, respectively. If the mutual reactance between the lines is $X_m\ \Omega$, show that this mutual reactance in per unit can be expressed as

$$X_{\text{pu}(m)} = (\text{physical } X_m)\,\frac{MVA_B}{[kV_{B(1)}][kV_{B(2)}]}$$

2.7. Consider Example 2.3 and assume that the transformer is connected in wye–wye. Determine the following:

(a) New line-to-line voltage of low-voltage side.

(b) New low-voltage side base impedance.

(c) Turns ratio of windings.

(d) Transformer reactance referred to low-voltage side in ohms.

(e) Transformer reactance referred to low-voltage side in per units.

2.8. Verify the following equations:

(a) Equation (2.92).

(b) Equation (2.93).

(c) Equation (2.94).
(d) Equation (2.96).

2.9. Verify the following equations:
(a) Equation (2.100)
(b) Equation (2.102).

2.10. Consider the one-line diagram given in Figure P2.10. Assume that the three-phase transformer T_1 has the nameplate ratings of 15,000 kVA, 7.97/13.8Y–69Δ kV with leakage impedance of $0.01 + j0.08$ pu based on its ratings and that the three-phase transformer T_2 has the nameplate ratings of 1500 kVA, 7.97Δ kV–277/480Y V with leakage impedance of $0.01 + j0.05$ pu based on its ratings. Assume that the three-phase generator G_1 is rated 10/12.5 MW/MVA, 7.97/13.8Y kV with an impedance of $0 + j1.10$ pu based on its ratings and that three-phase generator G_2 is rated 4/5 MW/MVA, 7.62/13.2Y kV with an impedance of $0 + j0.90$ pu based on its ratings. Transmission line TL_{23} has a length of 50 mi and is composed of 4/0 ACSR conductors with an equivalent spacing (D_m) of 8 ft and has an impedance of $0.445 + j0.976\ \Omega$/mi. Its shunt susceptance is given as 5.78 μS/mi. The line connects buses 2 and 3. Bus 3 is assumed to be an infinite bus, that is, the magnitude of its voltage remains constant at a given values and its phase position is unchanged regardless of the power and power factor demands that may be put on it. Furthermore, it is assumed to have a constant frequency equal to the nominal frequency of the system studied. Transmission line TL_{14} connects buses 1 and 4. It has a line length of 2 mi and an impedance of $0.80 + j0.80\ \Omega$/mi.

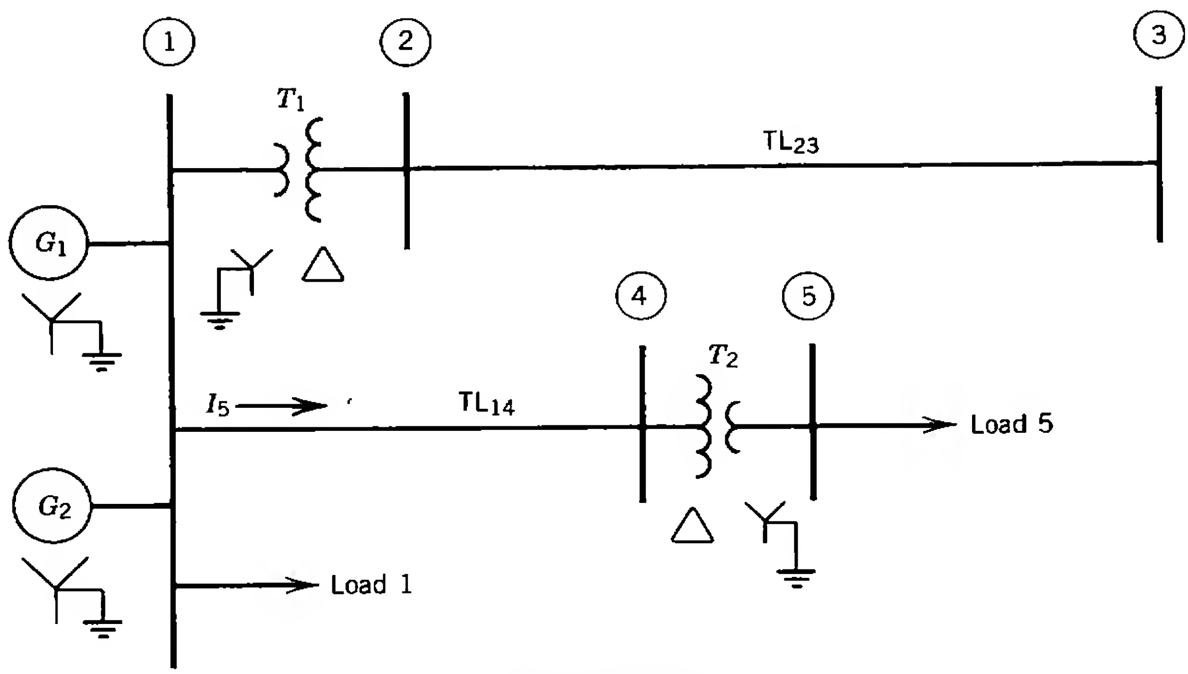


Figure P2.10

Because of the line length, its shunt susceptance is assumed to be negligible. The load that is connected to bus 1 has a current magnitude $|\mathbf{I}_1|$ of 52.3 A and a lagging power factor of 0.707. The load that is connected to bus 5 is given as $8000 + j6000$ kVA. Use the arbitrarily selected 5000 kVA as the three-phase kilovoltampere base and 39.84/69.00 kV as the line-to-neutral and line-to-line voltage base and determine the following:

(a) Complete Table P2.10 for the indicated values. Note the I_L means line current and I_Φ means phase currents in delta-connected apparatus.

TABLE P2.10 TABLE for Problem 2.10

Quantity	Nominally 69-kV Circuits	Nominally 13-kV Circuits	Nominally 480-V Circuits
$kVA_{B(3\Phi)}$	5000 kVA	5000 kVA	5000 kVA
$kV_{B(\text{L-L})}$	69 kV		
$kV_{B(\text{L-N})}$	39.84 kV		
$I_{B(\text{L})}$			
$I_{B(\Phi)}$			
Z_B			
Y_B			

(b) Draw a single-line positive-sequence network of this simple power system. Use the nominal-Π circuit to represent the 69-kV line. Show the values of all impedances and susceptances in per units on the chosen bases. Show all loads in per unit $P + jQ$.

2.11. Assume that a $500 + j200$-kVA load is connected to a load bus that has a voltage of $1.0\,\underline{/0^\circ}$ pu. If the power base is 1000 kVA, determine the per-unit R and X of the load:

(a) When load is represented by parallel connection.

(b) When load is represented by series connection.

2.12. Redraw the phasor diagram shown in Figure 2.24 by using $\mathbf{I}$ as the reference vector and derive formulas to calculate:

(a) Sending-end phase voltage, V_S.

(b) Sending-end power-factor angle, Φ_S.

2.13. A three-phase, 60-Hz, 15-mi-long transmission line provides 10 MW at a power factor of 0.9 lagging at a line-to-line voltage of 34.5 kV. The line conductors are made of 26-strand 300-kcmil ACSR conductors that operate at 25 °C and are equilaterally spaced 3 ft apart. Calculate the following:

(a) Source voltage.
(b) Sending-end power factor.
(c) Transmission efficiency.
(d) Regulation of line.

2.14. Repeat Problem 2.13 assuming the receiving-end power factor of 0.8 lagging.

2.15. Repeat Problem 2.13 assuming the receiving-end power factor of 0.8 leading.

2.16. A single-phase load is supplied by a 34.5-kV feeder whose impedance is $95 + j340\ \Omega$ and a 34.5/2.4 kV transformer whose equivalent impedance is $0.24 + j0.99\ \Omega$ referred to its low-voltage side. The load is 200 kW at a leading power factor of 0.85 and 2.25 kV.

(a) Calculate sending-end voltage of feeder.
(b) Calculate primary-terminal voltage of transformer.
(c) Calculate real and reactive-power input at sending end of feeder.

2.17. A short three-phase line has the series reactance of $15\ \Omega$ per phase. Neglect its series resistance. The load at the receiving end of the transmission line is 15 MW per phase and 12 Mvar lagging per phase. Assume that the receiving-end voltage is given as $115 + j0$ kV per phase and calculate:

(a) Sending-end voltage.
(b) Sending-end current.

2.18. A short 40-mi-long three-phase transmission line has a series line impedance of $0.6 + j0.95\ \Omega/\text{mi}$ per phase. The receiving-end line-to-line voltage is 69 kV. It has a full-load receiving-end current of $300\,\underline{/-30^\circ}$ A.

(a) Calculate the percentage of voltage regulation.
(b) Calculate the **ABCD** constants of the line.
(c) Draw the phasor diagram of V_S, V_R, and I.

2.19. Repeat Problem 2.18 assuming the receiving-end current of $300\,\underline{/-45^\circ}$ A.

2.20. A three-phase, 60-Hz, 12-MW load at a lagging power factor of 0.85 is supplied by a three-phase, 138-kV transmission line of 40 mi. Each line conductor has a resistance of $4\ \Omega/\text{mi}$ and an inductance of 14 mH/mi. Calculate:

(a) Sending-end line-to-line voltage.

(b) Loss of power in transmission line.

(c) Amount of reduction in line power loss if load-power factor were improved to unity.

2.21. A three-phase, 60-Hz transmission line has sending-end voltage of 39 kV and receiving-end voltage of 34.5 kV. If the line impedance per phase is $18 + j57\ \Omega$, compute the maximum power receivable at the receiving end of the line.

2.22. A three-phase, 60-Hz, 45-mi-long short line provides 20 MVA at a lagging power factor of 0.85 at a line-to-line voltage of 161 kV. The line conductors are made of 19-strand 4/0 copper conductors that operate at 50 °C. The conductors are equilaterally spaced with 4 ft spacing between them.

(a) Determine the percentage of voltage regulation of the line.

(b) Determine the sending-end power factor.

(c) Determine the transmission line efficiency if the line is single phase, assuming the use of the same conductors.

(d) Repeat Part (c) if the line is three phase.

2.23. A three-phase, 60-Hz, 15-MW load at a lagging power factor of 0.9 is supplied by two parallel connected transmission lines. The sending-end voltage is 71 kV, and the receiving-end voltage on a full load is 69 kV. Assume that the total transmission line efficiency is 98 percent. If the line length is 10 mi and the impedance of one of the lines is $0.7 + j1.2\ \Omega/\text{mi}$, compute the total impedance per phase of the second line.

2.24. Verify that $(\cosh \gamma l - 1)/\sinh\gamma l = \tanh(1/2)\gamma l$.

2.25. Derive equations (2.204) and (2.205) from equations (2.202) and (2.207).

2.26. Find the general circuit parameters for the network shown in Figure P2.26.

2.27. Find a T equivalent of the circuit shown in Figure P2.26.

2.28. Assume that the line is a 200-mi-long transmission line and repeat Example 2.8.

2.29. Assume that the line is a 200-mi-long transmission line and repeat Example 2.9.

2.30. Assume that the line is a short transmission line and repeat Example 2.8.

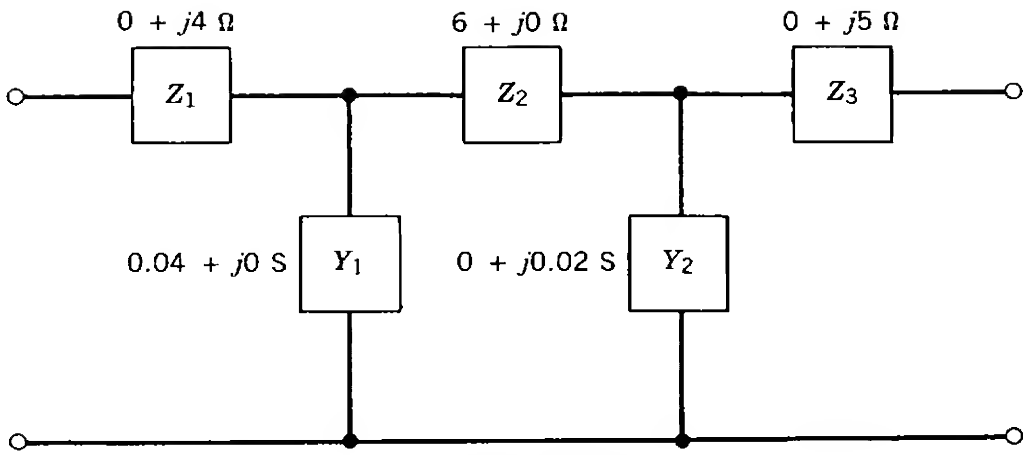


Figure P2.26

2.31. Assume that the line is a short transmission line and repeat Example 2.9.

2.32. Assume that the line in Example 2.10 is 75 mi long and the load is 100 MVA, and repeat the example.

2.33. Develop the equivalent transfer matrix for the network shown in Figure P2.33 matrix manipulation.

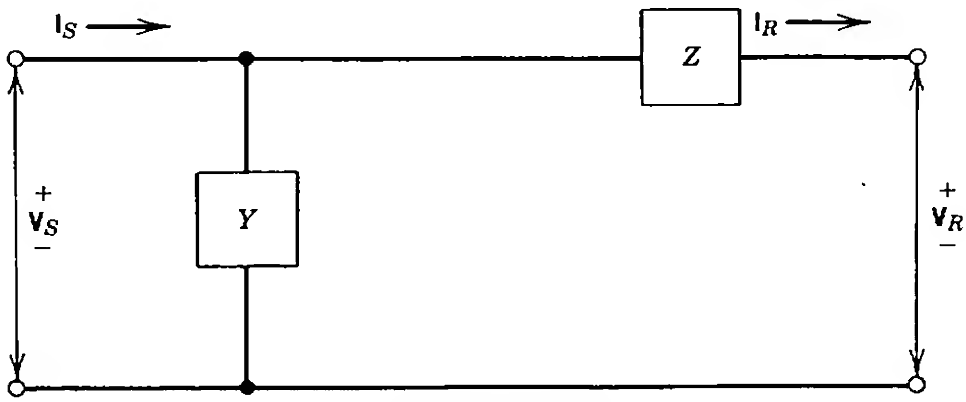


Figure P2.33

2.34. Develop the equivalent transfer matrix for the network shown in Figure P2.34 matrix manipulation.

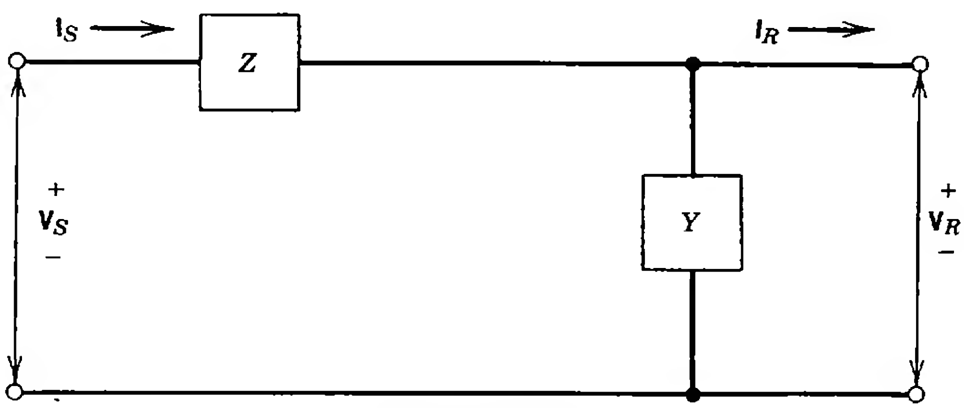


Figure P2.34

2.35. Verify equations (2.291)–(2.294) without using matrix methods.

2.36. Verify equations (2.298)–(2.301) without using matrix methods.

2.37. Assume that the line given in Example 2.8 is a 200-mi-long transmission line. Use the other data given in Example 2.8 accordingly and repeat Example 2.12.

2.38. Use the data from Problem 2.37 and repeat Example 2.13.

2.39. Assume that the shunt compensation of Example 2.18 is to be retained and now 60 percent series compensation is to be used, that is, the X_c is equal to 60 percent of the total series inductive reactance per phase of the transmission line. Determine the following:

(a) Total three-phase SIL of line in megavoltamperes.

(b) Maximum three-phase theoretical steady-state power flow limit in megawatts.

2.40. Assume that the line given in Problem 2.39 is designed to carry a contingency peak load of $2 \times$ SIL and that each phase of the series capacitor bank is to be of series and parallel groups of two-bushing, 12-kV, 150-kvar shunt power factor correction capacitors.

(a) Specify the necessary series–parallel arrangements of capacitors for each phase.

(b) Such capacitors may cost about \$1.50/kvar. Estimate the cost of the capacitors in the entire three-phase series capacitor bank. (Take note that the structure and the switching and protective equipment associated with the capacitor bank will add a great deal more cost.)

2.41. Use Table 2.6 for a 345-kV, pipe-type, three-phase, 1000-kcmil cable. Assume that the percent power factor (PF) cable is 0.5 and maximum electric stress is 300 V/mil and that

$$\mathbf{V}_S = \frac{345{,}000}{\sqrt{3}} \angle 0^\circ \text{ V}$$

Calculate the following:

(a) Susceptance b of cable.

(b) Critical length of cable and compare to value given in Table 2.6.

2.42. Consider the cable given in Problem 2.41 and use Table 2.6 for the relevant data; determine the value of

$$\mathbf{I}_{l0} = \frac{\mathbf{V}_S}{\mathbf{Z}_c} \tanh \gamma l_0$$

accurately and compare it with the given value of cable ampacity in Table 2.6. (*Hint*: Use the exponential form of the tan $h\gamma l_0$ function.)

2.43. Consider equation (2.178) and verify that the maximum power obtainable (i.e., the steady-state power limit) at the receiving end can be expressed as

$$P_{R,\max} = \frac{|\mathbf{V}_S||\mathbf{V}_R|}{X} \sin \gamma$$

2.44. Repeat Problem 2.17 assuming that the given power is the sending-end power instead of the receiving-end power.

2.45. Assume that a three-phase transmission line is constructed of 700 kcmil, 37-strand copper conductors, and the line length is 100 mi. The conductors are spaced horizontally with $D_{ab} = 10$ ft, $D_{bc} = 8$ ft, and $D_{ca} = 18$ ft. Use 60 Hz and 25 °C, and determine the following line constants from tables in terms of:

(a) Inductive reactance in ohms per mile.
(b) Capacitive reactance in ohms per mile.
(c) Total line resistance in ohms.
(d) Total inductive reactance in ohms.
(e) Total capacitive reactance in ohms.

2.46. A 60-Hz, single-circuit, three-phase transmission line is 150 mi long. The line is connected to a load of 50 MVA at a logging power factor of 0.85 at 138 kV. The line impedance and admittance are $\mathbf{z} = 0.7688\,\underline{/77.4^\circ}\ \Omega/\text{mi}$ and $\mathbf{y} = 4.5239 \times 10^{-6}\,\underline{/90^\circ}\ \text{S/mi}$, respectively. Determine the following:

(a) Propagation constant of line.
(b) Attenuation constant and phase-change constant, per mile, of line.
(c) Characteristic impedance of line.
(d) SIL of line.
(e) Receiving-end current.
(f) Incident voltage at sending end.
(g) Reflected voltage at sending end.

2.47. Consider a three-phase transmission and assume that the following values are given:

$$\mathbf{V}_{R(\text{L-N})} = 79{,}674.34\,\underline{/0^\circ}\ \text{V}$$

$$\mathbf{I}_R = 209.18\,\underline{/-31.8^\circ}\ \text{A}$$

$$\mathbf{Z}_c = 469.62 \underline{/-5.37^\circ}\ \Omega$$

$$\gamma l = 0.0301 + j0.3202$$

Determine the following:

(a) Incident and reflected voltages at receiving end of line.
(b) Incident and reflected voltages at sending end of line.
(c) Line voltage at sending end of line.

2.48. Repeat Example 2.21 assuming that the conductor used is 1431-kcmil ACSR and that the distance between two subconductors is 18 in. Also assume that the distances D_{12}, D_{23}, and D_{31} are 25, 25, and 50 ft, respectively.

3

SYMMETRICAL COMPONENTS AND FAULT ANALYSIS

3.1 INTRODUCTION

In general, it can be said that truly balanced three-phase systems exist only in theory. In reality, many systems are very nearly balanced and for practical purposes can be analyzed as if they were truly balanced systems. However, there are also emergency conditions (e.g., unsymmetrical faults, unbalanced loads, open conductors, or unsymmetrical conditions arising in rotating machines) where the degree of unbalance cannot be neglected. To protect the system against such contingencies, it is necessary to size protective devices, such as fuses and circuit breakers, and set the protective relays. Therefore, to achieve this, currents and voltages in the system under such unbalanced operating conditions have to be known (and therefore calculated) in advance.

In 1918, Fortescue [1] proposed a method for resolving an unbalanced set of n related phasors into n sets of balanced phasors called the *symmetrical components* of the original unbalanced set. The phasors of each set are of equal magnitude and spaced 120° or 0° apart. The method is applicable to systems with any number of phases, but in this book only three-phase systems will be discussed.

Today, the symmetrical component theory is widely used in studying unbalanced systems. Furthermore, many electrical devices have been developed and are operating based on the concept of symmetrical components. The examples include (1) the negative-sequence relay to detect system faults, (2) the positive-sequence filter to make generator voltage regulators respond to voltage changes in all three phases rather than in one phase alone, and (3) the Westinghouse type HCB pilot wire relay using positive- and zero-sequence filters to detect faults.

3.2 SYMMETRICAL COMPONENTS

Any unbalanced three-phase system of phasors can be resolved into three balanced systems of phasors: (1) positive-sequence system, (2) negative-sequence system, and (3) zero-sequence system, as illustrated in Figure 3.1.

The *positive-sequence system* is represented by a balanced system of phasors having the same phase sequence (and therefore positive phase rotation) as the original unbalanced system. The phasors of the positive-sequence system are equal in magnitude and displaced from each other by 120°, as shown in Figure 3.1(*b*).

The *negative-sequence system* is represented by a balanced system of phasors having the *opposite* phase sequence (and therefore negative phase rotation) to the original system. The phasors of the negative-sequence system are also equal in magnitude and displaced from each other by 120°, as shown in Figure 3.1(*c*).

Whereas the *zero-sequence system* is represented by three single phasors that are equal in magnitude and angular displacements, as shown in Figure 3.1(*d*). Note that, in the book, the subscripts 0, 1, and 2 denote the zero sequence, positive sequence, and negative sequence, respectively. Therefore, three voltage phasors $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$ of an unbalanced set, as shown in Figure 3.1(*a*) can be expressed in terms of their symmetrical components as

$$\mathbf{V}_a = \mathbf{V}_{a1} + \mathbf{V}_{a2} + \mathbf{V}_{a0} \tag{3.1}$$

$$\mathbf{V}_b = \mathbf{V}_{b1} + \mathbf{V}_{b2} + \mathbf{V}_{b0} \tag{3.2}$$

$$\mathbf{V}_c = \mathbf{V}_{c1} + \mathbf{V}_{c2} + \mathbf{V}_{c0} \tag{3.3}$$

Figure 3.1(*e*) shows the graphical additions of the symmetrical components of Figures 3.1(*b*)–(*d*) to obtain the original three unbalanced phasors shown in Figure 3.1(*a*)

3.3 THE OPERATOR a

Because of the application of the symmetrical components theory to three-phase systems, there is a need for a *unit phasor* (or *operator*) that will rotate another phasor by 120° in the counterclockwise direction (i.e., it will add 120° to the phase angle of the phasor) but leave its magnitude unchanged when it is multiplied by the phasor (see Figure 3 2). Such an operator is a complex number of unit magnitude with an angle of 120° and is defined by

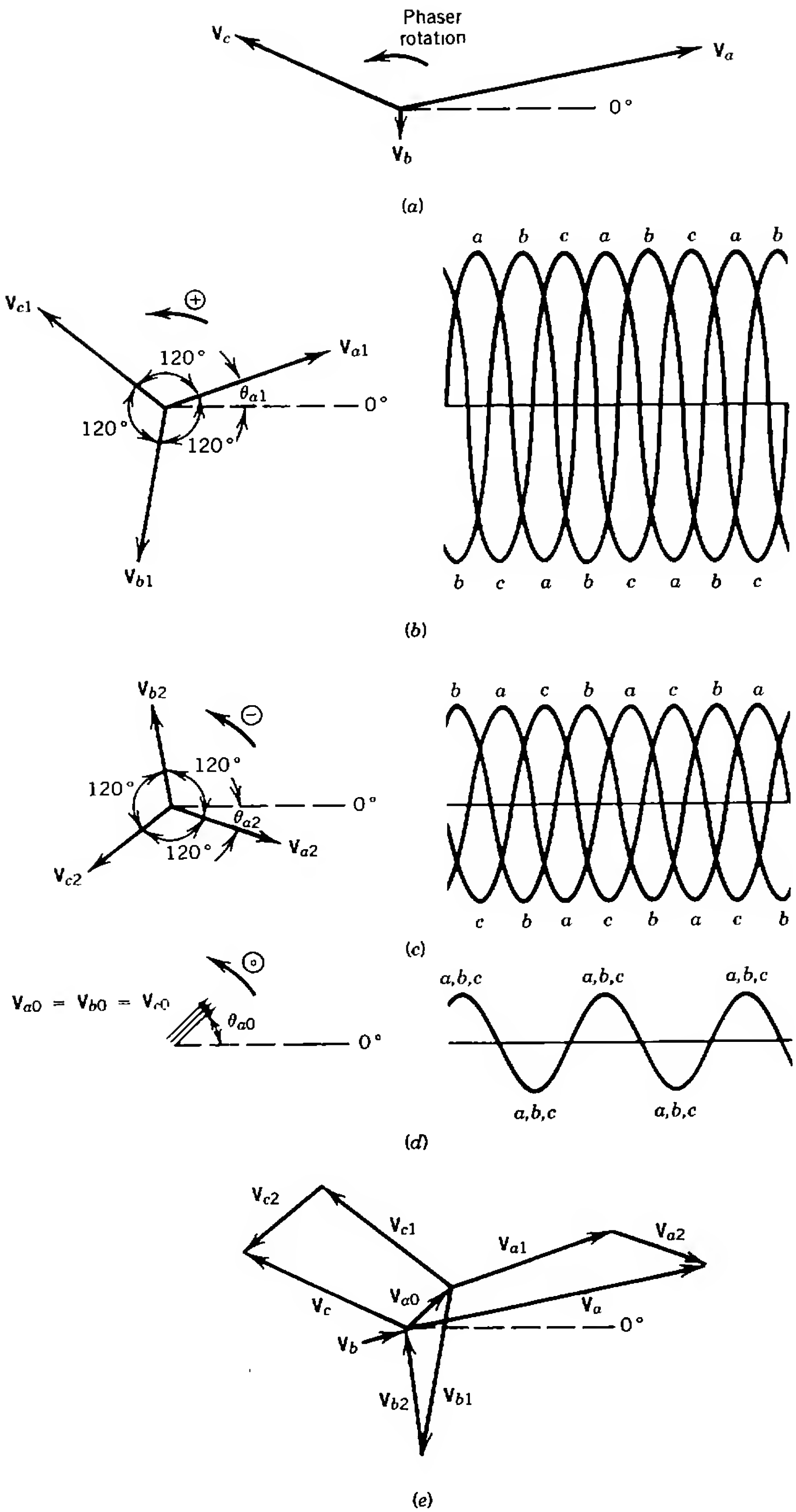


Figure 3.1. Analysis and synthesis of set of three unbalanced voltage phasors: (*a*) original system of unbalanced phasors; (*b*) positive-sequence components, (*c*) negative-sequence components; (*d*) zero-sequence components; (*e*) graphical addition of phasors to gèt original unbalanced phasors.

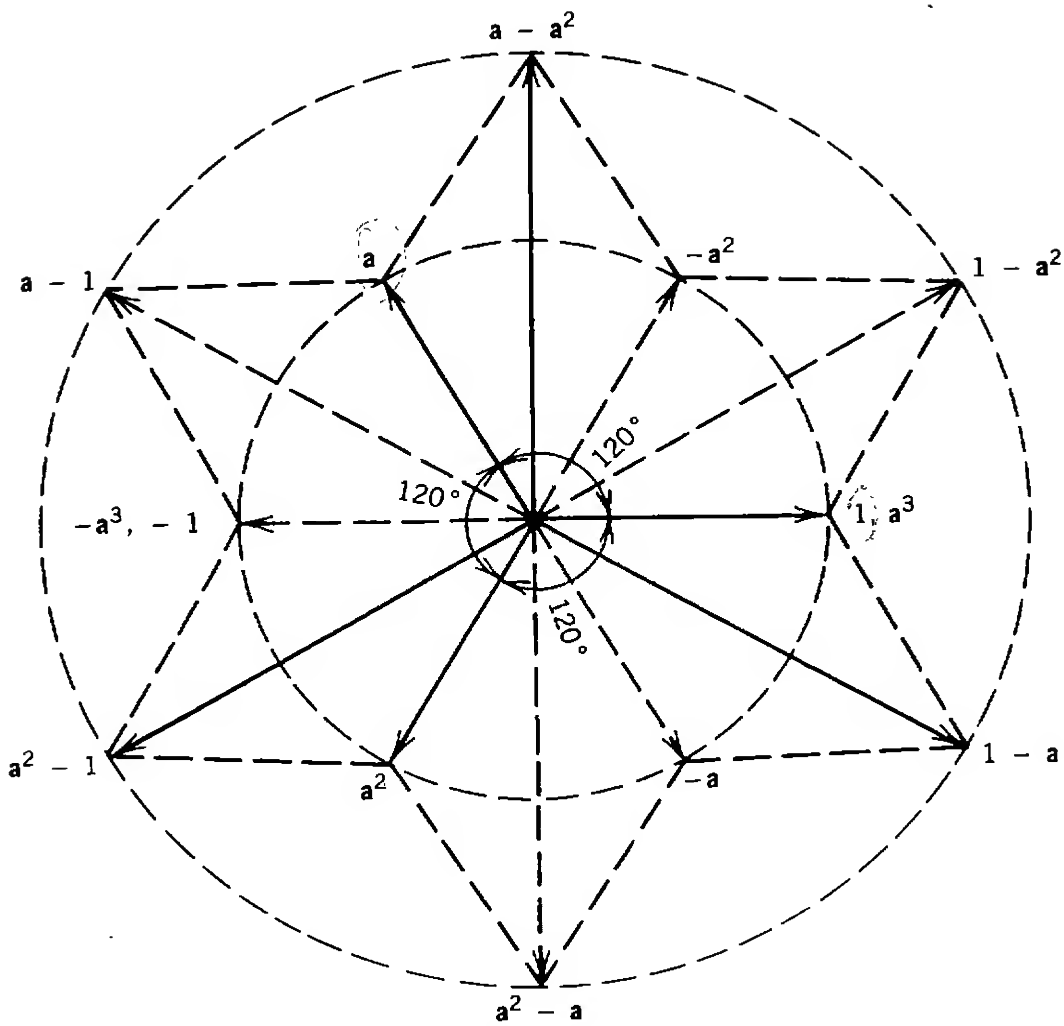


Figure 3.2. Phasor diagram of various powers and functions of operator a.

$$
\begin{aligned}
\mathbf{a} &= 1 \underline{/120^\circ} \\
&= 1e^{j(2\pi/3)} \\
&= 1(\cos 120^\circ + j \sin 120^\circ) \\
&= -0.5 + j0.866
\end{aligned}
$$

where

$$j = \sqrt{-1}$$

It is clear that if the operator a is designated as

$$a = 1 \underline{/120^\circ}$$

then

$$\mathbf{a}^2 = \mathbf{a} \times \mathbf{a} = (1 \underline{/120^\circ})(1 \underline{/120^\circ}) = 1 \underline{/240^\circ} = 1 \underline{/-120^\circ}$$

TABLE 3.1 Powers and Functions of Operator a

Power or Function	In Polar Form	In Rectangular Form
$\mathbf{a}$	$1\angle 120^\circ$	$-0.5 + j0.866$
$\mathbf{a}^2$	$1\angle 240^\circ = 1\angle -120^\circ$	$-0.5 - j0.866$
$\mathbf{a}^3$	$1\angle 360^\circ = 1\angle 0^\circ$	$1.0 + j0.0$
$\mathbf{a}^4$	$1\angle 120^\circ$	$-0.5 + j0.866$
$1 + \mathbf{a} = -\mathbf{a}^2$	$1\angle 60^\circ$	$0.5 + j0.866$
$1 - \mathbf{a}$	$\sqrt{3}\angle -30^\circ$	$1.5 - j0.866$
$1 + \mathbf{a}^2 = -\mathbf{a}$	$1\angle -60^\circ$	$0.5 - j0.866$
$1 - \mathbf{a}^2$	$\sqrt{3}\angle 30^\circ$	$1.5 + j0.866$
$\mathbf{a} - 1$	$\sqrt{3}\angle 150^\circ$	$-1.5 + j0.866$
$\mathbf{a} + \mathbf{a}^2$	$1\angle 180^\circ$	$-1.0 + j0.0$
$\mathbf{a} - \mathbf{a}^2$	$\sqrt{3}\angle 90^\circ$	$0.0 + j1.732$
$\mathbf{a}^2 - \mathbf{a}$	$\sqrt{3}\angle -90^\circ$	$0.0 - j1.732$
$\mathbf{a}^2 - 1$	$\sqrt{3}\angle -150$	$-1.5 - j0.866$
$1 + \mathbf{a} + \mathbf{a}^2$	$0\angle 0^\circ$	$0.0 + j0.0$

$$\mathbf{a}^3 = \mathbf{a}^2 \times \mathbf{a} = (1\angle 240^\circ)(1\angle 120^\circ) = 1\angle 360^\circ = 1\angle 0^\circ$$

$$\mathbf{a}^4 = \mathbf{a}^3 \times \mathbf{a}^2 = (1\angle 0^\circ)(1\angle 120^\circ) = 1\angle 120^\circ = \mathbf{a}$$

$$\mathbf{a}^5 = \mathbf{a}^3 \times \mathbf{a}^2 = (1\angle 240^\circ)(1\angle 240^\circ) = 1\angle 240^\circ = \mathbf{a}^2$$

$$\mathbf{a}^6 = \mathbf{a}^3 \times \mathbf{a}^3 = (1\angle 0^\circ)(1\angle 0^\circ) = 1\angle 0^\circ = \mathbf{a}^3$$

$$\vdots$$

$$\mathbf{a}^{n+3} = \mathbf{a}^3 \times \mathbf{a}^n = \mathbf{a}^n$$

Figure 3.2 shows a phasor diagram of the various powers and functions of the operator **a**. Various combinations of the operator **a** are given in Table 3.1. In manipulating quantities involving the operator **a**, it is useful to remember that

$$1 + \mathbf{a} + \mathbf{a}^2 = 0 \tag{3.4}$$

3.4 RESOLUTION OF THREE-PHASE UNBALANCED SYSTEM OF PHASORS INTO ITS SYMMETRICAL COMPONENTS

In the application of the symmetrical component, it is customary to let the phase a be the reference phase. Therefore, using the operator **a**, the symmetrical components of the positive-, negative-, and zero-sequence components can be expressed as

$$\mathbf{V}_{b1} = \mathbf{a}^2\mathbf{V}_{a1} \tag{3.5}$$

$$\mathbf{V}_{c1} = \mathbf{a}\mathbf{V}_{a1} \tag{3.6}$$

$$\mathbf{V}_{b2} = \mathbf{a}\mathbf{V}_{a2} \tag{3.7}$$

$$\mathbf{V}_{c2} = \mathbf{a}^2\mathbf{V}_{a2} \tag{3.8}$$

$$\mathbf{V}_{b0} = \mathbf{V}_{c0} = \mathbf{V}_{a0} \tag{3.9}$$

Substituting the above equations into equations (3.2) and (3.3), as appropriate, the phase voltages can be expressed in terms of the sequence voltages as

$$\mathbf{V}_a = \mathbf{V}_{a1} + \mathbf{V}_{a2} + \mathbf{V}_{a0} \tag{3.10}$$

$$\mathbf{V}_b = \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} + \mathbf{V}_{a0} \tag{3.11}$$

$$\mathbf{V}_c = \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} + \mathbf{V}_{a0} \tag{3.12}$$

Equations (3.10)–(3.12) are known as the *synthesis equations*. Therefore, it can be shown that the sequence voltages can be expressed in terms of phase voltages as

$$\mathbf{V}_{a0} = \tfrac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c) \tag{3.13}$$

$$\mathbf{V}_{a1} = \tfrac{1}{3}(\mathbf{V}_a + \mathbf{a}\mathbf{V}_b + \mathbf{a}^2\mathbf{V}_c) \tag{3.14}$$

$$\mathbf{V}_{a2} = \tfrac{1}{3}(\mathbf{V}_a + \mathbf{a}^2\mathbf{V}_b + \mathbf{a}\mathbf{V}_c) \tag{3.15}$$

which are known as the *analysis equations*. Alternatively, the synthesis and analysis equations can be written, respectively, in matrix form as

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \tag{3.16}$$

and

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \tag{3.17}$$

or

$$[\mathbf{V}_{abc}] = [\mathbf{A}][\mathbf{V}_{012}] \tag{3.18}$$

and

$$[\mathbf{V}_{012}] = [\mathbf{A}]^{-1}[\mathbf{V}_{abc}] \tag{3.19}$$

where

$$[\mathbf{A}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \tag{3.20}$$

$$[\mathbf{A}]^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \tag{3.21}$$

$$[\mathbf{V}_{abc}] = \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \tag{3.22}$$

$$[\mathbf{V}_{012}] = \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \tag{3.23}$$

Of course, the synthesis and analysis equations in terms of phase and sequence currents can be expressed as

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \tag{3.24}$$

and

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} \tag{3.25}$$

or

$$[\mathbf{I}_{abc}] = [\mathbf{A}][\mathbf{I}_{012}] \tag{3.26}$$

and

$$[\mathbf{I}_{012}] = [\mathbf{A}]^{-1}[\mathbf{I}_{abc}] \tag{3.27}$$

EXAMPLE 3.1

Determine the symmetrical components for the phase voltages of $\mathbf{V}_a = 7.3\,\underline{/12.5^\circ}$, $\mathbf{V}_b = 0.4\,\underline{/-100^\circ}$, and $\mathbf{V}_c = 4.4\,\underline{/154^\circ}$ V.

Solution

$$\begin{aligned}
\mathbf{V}_{a0} &= \tfrac{1}{3}(\mathbf{V}_a + \mathbf{V}_b + \mathbf{V}_c) \\
&= \tfrac{1}{3}(7.3\,\underline{/12.5^\circ} + 0.4\,\underline{/-100^\circ} + 4.4\,\underline{/154^\circ}) = 1.47\,\underline{/45.1^\circ}\text{ V} \\
\mathbf{V}_{a1} &= \tfrac{1}{3}(\mathbf{V}_a + \mathbf{a}\mathbf{V}_b + \mathbf{a}^2\mathbf{V}_c) \\
&= \tfrac{1}{3}[7.3\,\underline{/12.5^\circ} + (1\,\underline{/120^\circ})(0.4\,\underline{/-100^\circ}) + (1\,\underline{/240^\circ})(4.4\,\underline{/154^\circ}] \\
&= 3.97\,\underline{/20.5^\circ}\text{ V}
\end{aligned}$$

$$
\begin{aligned}
\mathbf{V}_{a2} &= \tfrac{1}{3}(\mathbf{V}_a + \mathbf{a}^2\mathbf{V}_b + \mathbf{a}\mathbf{V}_c)\\
&= \tfrac{1}{3}[7.3\,\underline{/12.5^\circ} + (1\,\underline{/240^\circ})(0.4\,\underline{/-100^\circ}) + (1\,\underline{/20^\circ})(4.4\,\underline{/154^\circ})]\\
&= 2.52\,\underline{/-19.7^\circ}\text{ V}\\
\mathbf{V}_{b0} &= \mathbf{V}_{a0} = 1.47\,\underline{/45.1^\circ}\text{ V}\\
\mathbf{V}_{b1} &= \mathbf{a}^2\mathbf{V}_{a1} = (1\,\underline{/240^\circ})(3.97\,\underline{/20.5^\circ}) = 3.97\,\underline{/260.5^\circ}\text{ V}\\
\mathbf{V}_{b2} &= \mathbf{a}\mathbf{V}_{a2} = (1\,\underline{/120^\circ})(2.52\,\underline{/-19.7^\circ}) = 2.52\,\underline{/100.3}\text{ V}\\
\mathbf{V}_{c0} &= \mathbf{V}_{a0} = 1.47\,\underline{/45.1^\circ}\text{ V}\\
\mathbf{V}_{c1} &= \mathbf{a}\mathbf{V}_{a1} = (1\,\underline{/120^\circ})(3.97\,\underline{/20.5^\circ}) = 3.97\,\underline{/140.5^\circ}\text{ V}\\
\mathbf{V}_{c2} &= \mathbf{a}^2\mathbf{V}_{a2} = (1\,\underline{/240^\circ})(2.52\,\underline{/-19.7^\circ}) = 2.52\,\underline{/220.3^\circ}\text{ V}
\end{aligned}
$$

Note that the resulting values for the symmetrical components can be checked numerically [e.g., using equation (3.11)] or graphically, as shown in Figure 3.1(*e*).

3.5 POWER IN SYMMETRICAL COMPONENTS

The three-phase complex power at any point of a three-phase system can be expressed as the sum of the individual complex powers of each phase so that

$$
\begin{aligned}
\mathbf{S}_{3\Phi} &= P_{3\Phi} + jQ_{3\Phi}\\
&= \mathbf{S}_a + \mathbf{S}_b + \mathbf{S}_c\\
&= \mathbf{V}_a\mathbf{I}_a^* + \mathbf{V}_b\mathbf{I}_b^* + \mathbf{V}_c\mathbf{I}_c^*
\end{aligned}
\tag{3.28}
$$

or, in matrix notation,

$$
\mathbf{S}_{3\Phi} = [\mathbf{V}_a \quad \mathbf{V}_b \quad \mathbf{V}_c]\begin{bmatrix}\mathbf{I}_a\\ \mathbf{I}_b\\ \mathbf{I}_c\end{bmatrix}^* = \begin{bmatrix}\mathbf{V}_a\\ \mathbf{V}_b\\ \mathbf{V}_c\end{bmatrix}^{\mathrm{t}}\begin{bmatrix}\mathbf{I}_a\\ \mathbf{I}_b\\ \mathbf{I}_c\end{bmatrix}^* \tag{3.29}
$$

or

$$
\mathbf{S}_{3\Phi} = [\mathbf{V}_{abc}]^{\mathrm{t}}[\mathbf{I}_{abc}]^* \tag{3.30}
$$

where

$$
\begin{aligned}
[\mathbf{V}_{abc}] &= [\mathbf{A}][\mathbf{V}_{012}]\\
[\mathbf{I}_{abc}] &= [\mathbf{A}][\mathbf{I}_{012}]
\end{aligned}
$$

and therefore,

$$[\mathbf{V}_{abc}]^t = [\mathbf{V}_{012}]^t[\mathbf{A}]^t \tag{3.31}$$

$$[\mathbf{I}_{abc}]^* = [\mathbf{A}]^*[\mathbf{I}_{012}]^* \tag{3.32}$$

Substituting equations (3.31) and (3.32) into equation (3.30),

$$\mathbf{S}_{3\Phi} = [\mathbf{V}_{012}]^t[\mathbf{A}]^t[\mathbf{A}]^*[\mathbf{I}_{012}]^* \tag{3.33}$$

where

$$[\mathbf{A}]^t[\mathbf{A}]^* = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{bmatrix} = 3\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Therefore,

$$\mathbf{S}_{3\Phi} = 3[\mathbf{V}_{012}]^t[\mathbf{I}_{012}]^* = 3[\mathbf{V}_{a0} \quad \mathbf{V}_{a1} \quad \mathbf{V}_{a2}]\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix}^* \tag{3.34a}$$

or

$$\mathbf{S}_{3\Phi} = 3[\mathbf{V}_{a0}\mathbf{I}_{a0}^* + \mathbf{V}_{a1}\mathbf{I}_{a1}^* + \mathbf{V}_{a2}\mathbf{I}_{a2}^*] \tag{3.34b}$$

Note that there are no *cross* terms (e.g., $\mathbf{V}_{a0}\mathbf{I}_{a1}^*$ or $\mathbf{V}_{a1}\mathbf{I}_{a0}^*$) in this equation, which indicates that there is no *coupling* of power among the three sequences. Also note that the symmetrical components of voltage and current belong to the same phase.

EXAMPLE 3.2

Assume that the phase voltages and currents of a three-phase system are given as

$$[\mathbf{V}_{abc}] = \begin{bmatrix} 0 \\ 50 \\ -50 \end{bmatrix} \quad \text{and} \quad [\mathbf{I}_{abc}] = \begin{bmatrix} -5 \\ j5 \\ -5 \end{bmatrix}$$

and determine the following:

(a) Three-phase complex power using equation (3.30).
(b) Sequence voltage and current matrices, that is, $[\mathbf{V}_{012}]$ and $[\mathbf{I}_{012}]$.
(c) Three-phase complex power using equation (3.34).

Solution

(a) $\mathbf{S}_{3\Phi} = [\mathbf{V}_{abc}]^t[\mathbf{I}_{abc}]^*$

$$= [0 \quad 50 \quad -50]\begin{bmatrix} -5 \\ -j5 \\ -5 \end{bmatrix} = 250 - j250 = 353.5534\,\underline{/-45^\circ}\text{ VA}$$

(b) $[\mathbf{V}_{012}] = [\mathbf{A}]^{-1}[\mathbf{V}_{abc}]$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} 0 \\ 50 \\ -50 \end{bmatrix} = \begin{bmatrix} 0.0\,\underline{/0^\circ} \\ 28.8675\,\underline{/90^\circ} \\ 28.8675\,\underline{/-90^\circ} \end{bmatrix}\text{ V}$$

$[\mathbf{I}_{012}] = [\mathbf{A}]^{-1}[\mathbf{I}_{abc}]$

$$= \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} -5 \\ j5 \\ -5 \end{bmatrix} = \begin{bmatrix} 3.7268\,\underline{/153.4^\circ} \\ 2.3570\,\underline{/165^\circ} \\ 2.3570\,\underline{/-75^\circ} \end{bmatrix}$$

(c) $\mathbf{S}_{3\Phi} = 3[\mathbf{V}_{a0}\mathbf{I}_{a0}^* + \mathbf{V}_{a1}\mathbf{I}_{a1}^* + \mathbf{V}_{a2}\mathbf{I}_{a2}^*] = 353.5534\,\underline{/-45^\circ}\text{ VA}$

3.6 SEQUENCE IMPEDANCES OF TRANSMISSION LINES

3.6.1 Sequence Impedances of Untransposed Lines

Figure 3.3(*a*) shows a circuit representation of an untransposed transmission line with unequal self-impedances and unequal mutual impedances. Here

$$[\mathbf{V}_{abc}] = [\mathbf{Z}_{abc}][\mathbf{I}_{abc}] \tag{3.35}$$

where

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} \end{bmatrix} \tag{3.36}$$

in which the self-impedances are

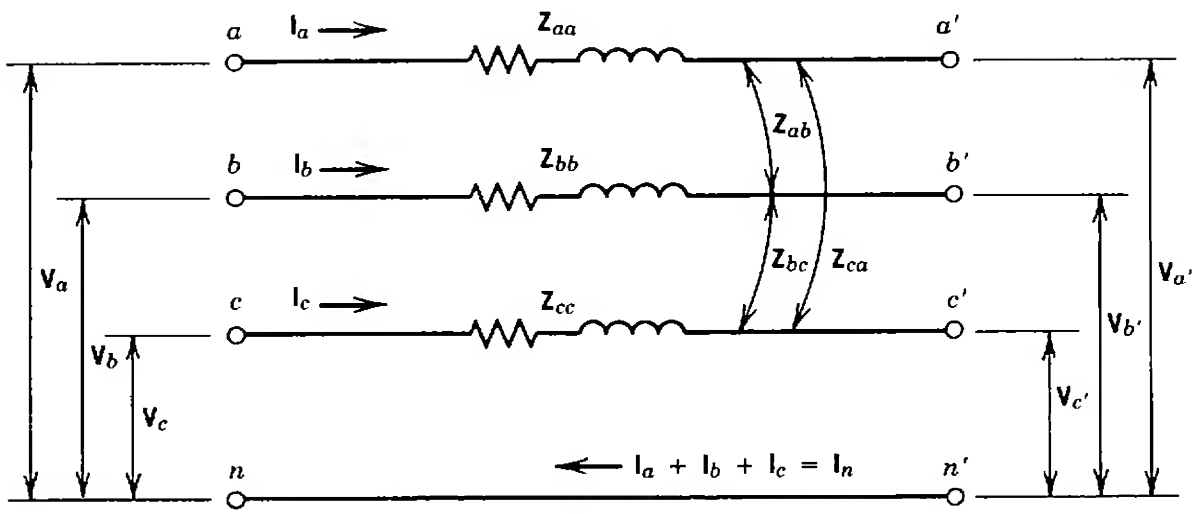
$$\mathbf{Z}_{aa} \neq \mathbf{Z}_{bb} \neq \mathbf{Z}_{cc}$$

and the mutual impedances are

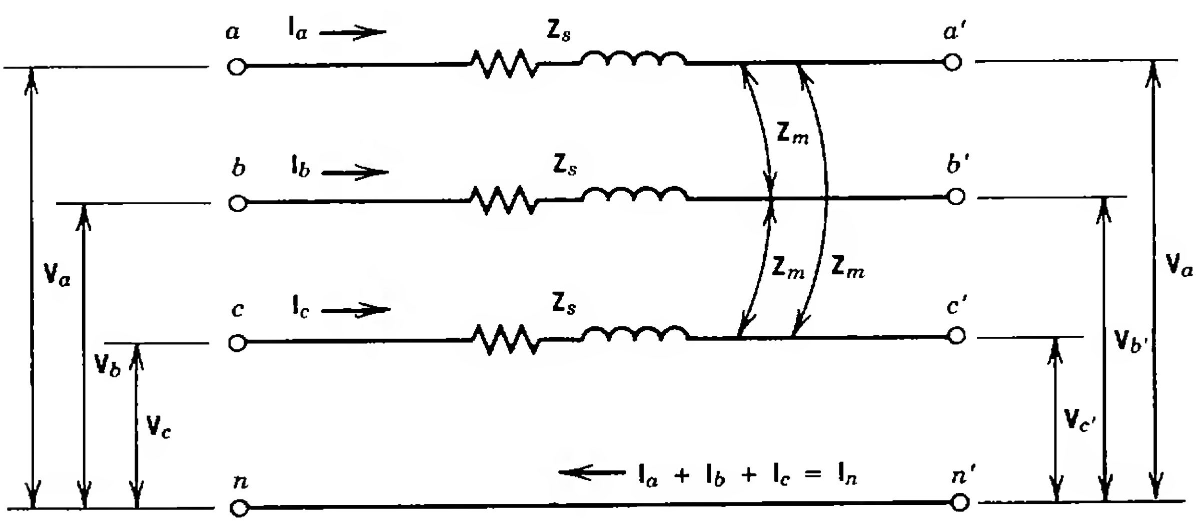
$$\mathbf{Z}_{ab} \neq \mathbf{Z}_{ba} \neq \mathbf{Z}_{ca}$$

Multiplying both sides of equation (3.35) by $[\mathbf{A}]^{-1}$ and also substituting equation (3.26) into equation (3.35),

$$[\mathbf{A}]^{-1}[\mathbf{V}_{abc}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}][\mathbf{I}_{012}] \tag{3.37}$$



(a)



(b)

Figure 3.3. Transmission line circuit diagrams: (*a*) with unequal series and unequal impedances; (*b*) with equal series and equal mutual impedances.

where the similarity transformation is defined as

$$[\mathbf{Z}_{012}] \triangleq [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] \tag{3.38}$$

Therefore, the sequence impedance matrix of an untransposed transmission line can be calculated using equation (3.38) and can be expressed as

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_{00} & \mathbf{Z}_{01} & \mathbf{Z}_{02} \\ \mathbf{Z}_{10} & \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{20} & \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \tag{3.39}$$

or

$$[\mathbf{Z}_{012}] = \begin{bmatrix} (\mathbf{Z}_{s0} + 2\mathbf{Z}_{m0}) & (\mathbf{Z}_{s2} - \mathbf{Z}_{m2}) & (\mathbf{Z}_{s1} - \mathbf{Z}_{m1}) \\ (\mathbf{Z}_{s1} - \mathbf{Z}_{m1}) & (\mathbf{Z}_{s0} - \mathbf{Z}_{m0}) & (\mathbf{Z}_{s2} + 2\mathbf{Z}_{m2}) \\ (\mathbf{Z}_{s2} - \mathbf{Z}_{m2}) & (\mathbf{Z}_{s1} + 2\mathbf{Z}_{m1}) & (\mathbf{Z}_{s0} - \mathbf{Z}_{m0}) \end{bmatrix} \tag{3.40}$$

where, by definition,

$$\mathbf{Z}_{s0} = \text{zero-sequence self-impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{aa} + \mathbf{Z}_{bb} + \mathbf{Z}_{cc}) \tag{3.41}$$

$$\mathbf{Z}_{s1} = \text{positive-sequence self-impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{aa} + \mathbf{a}\mathbf{Z}_{bb} + \mathbf{a}^2\mathbf{Z}_{cc}) \tag{3.42}$$

$$\mathbf{Z}_{s2} = \text{negative-sequence self-impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{aa} + \mathbf{a}^2\mathbf{Z}_{bb} + \mathbf{a}\mathbf{Z}_{cc}) \tag{3.43}$$

$$\mathbf{Z}_{m0} = \text{zero-sequence mutual impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{bc} + \mathbf{Z}_{ca} + \mathbf{Z}_{ab}) \tag{3.44}$$

$$\mathbf{Z}_{m1} = \text{positive-sequence mutual impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{bc} + \mathbf{a}\mathbf{Z}_{ca} + \mathbf{a}^2\mathbf{Z}_{ab}) \tag{3.45}$$

$$\mathbf{Z}_{m2} = \text{negative-sequence mutual impedance}$$

$$\triangleq \tfrac{1}{3}(\mathbf{Z}_{bc} + \mathbf{a}^2\mathbf{Z}_{ca} + \mathbf{a}\mathbf{Z}_{ab}) \tag{3.46}$$

Therefore,

$$[\mathbf{V}_{012}] = [\mathbf{Z}_{012}][\mathbf{I}_{012}] \tag{3.47}$$

Note that the matrix in equation (3.40) is not a symmetrical matrix, and therefore, the application of equation (3.47) will show that there is a mutual coupling among the three sequences, which is not a desirable result.

3.6.2 Sequence Impedances of Transposed Lines

The remedy is either to completely transpose the line or to place the conductors with equilateral spacing among them so that the resulting mutual impedances† are equal to each other, that is, $\mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca} = \mathbf{Z}_m$, as shown in Figure 3.3(b). Furthermore, if the self-impedances of conductors are equal to each other, that is, $\mathbf{Z}_{aa} = \mathbf{Z}_{bb} = \mathbf{Z}_{cc} = \mathbf{Z}_s$, equation (3.36) can be expressed as

† In passive networks $\mathbf{Z}_{ab} = \mathbf{Z}_{ba}$, $\mathbf{Z}_{bc} = \mathbf{Z}_{cb}$, etc.

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} \mathbf{Z}_s & \mathbf{Z}_m & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_s & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_m & \mathbf{Z}_s \end{bmatrix} \tag{3.48}$$

where

$$\mathbf{Z}_s = \left[(r_a + r_e) + j0.1213 \ln \frac{D_e}{D_s} \right] l \quad \Omega \tag{3.49}$$

$$\mathbf{Z}_m = \left[r_e + j0.1213 \ln \frac{D_e}{D_{eq}} \right] l \quad \Omega \tag{3.50}$$

$$D_{\text{eq}} \triangleq D_m = (D_{ab} \times D_{bc} \times D_{ca})^{1/3}$$

r_a = resistance of a single conductor a

The r_e is the resistance of Carson's [2] equivalent (and fictitious) earth return conductor. It is a function of frequency and can be expressed as

$$r_e = 1.588 \times 10^{-3} f \quad \Omega/\text{mi} \tag{3.51}$$

or

$$r_e = 9.869 \times 10^{-4} f \quad \Omega/\text{km} \tag{3.52}$$

At 60 Hz, $r_e = 0.09528\ \Omega/\text{mi}$. The quantity D_e is a function of both the earth resistivity ρ and the frequency f and can be expressed as

$$D_e = 2160 \left(\frac{\rho}{f} \right)^{1/2} \quad \text{ft} \tag{3.53}$$

where ρ is the earth resistivity and is given in Table 3.2 for various earth types. If the actual earth resistivity is unknown, it is customary to use an average value of 100 Ω-m for ρ. Therefore, at 60 Hz, $D_e = 2788.55$ ft. The D_s is the GMR of the phase conductor as before. Therefore, by applying equation (3.38),

TABLE 3.2 Resistivity of Different Soils

Ground Type	Resistivity, ρ (Ω-m)
Seawater	0.01–1.0
Wet organic soil	10
Moist soil (average earth)	100
Dry soil	1000
Bedrock	10^4
Pure slate	10^7
Sandstone	10^9
Crushed rock	1.5×10^8

$$[\mathbf{Z}_{012}] = \begin{bmatrix} (\mathbf{Z}_s + 2\mathbf{Z}_m) & 0 & 0 \\ 0 & (\mathbf{Z}_s - \mathbf{Z}_m) & 0 \\ 0 & 0 & (\mathbf{Z}_s - \mathbf{Z}_m) \end{bmatrix} \tag{3.54}$$

where, by definition,

$$\mathbf{Z}_0 = \text{zero-sequence impedance}$$

$$\triangleq \mathbf{Z}_{00} = \mathbf{Z}_s + 2\mathbf{Z}_m \tag{3.55a}$$

$$= \left[(r_a + 3r_e) + j0.364 \ln \frac{D_e^3}{D_s^{1/3} D_{\text{eq}}^{2/3}} \right] l \tag{3.55b}$$

$$\mathbf{Z}_1 = \text{positive-sequence impedance}$$

$$\triangleq \mathbf{Z}_{11} = \mathbf{Z}_s - \mathbf{Z}_m \tag{3.56a}$$

$$= \left(r_a + j0.1213 \ln \frac{D_{\text{eq}}}{D_s} \right) l \tag{3.56b}$$

$$\mathbf{Z}_2 = \text{negative-sequence impedance}$$

$$\triangleq \mathbf{Z}_{22} = \mathbf{Z}_s - \mathbf{Z}_m \tag{3.57a}$$

$$= \left(r_a + j0.1213 \ln \frac{D_{\text{eq}}}{D_s} \right) l \tag{3.57b}$$

Thus, equation (3.54) can be expressed as

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_0 & 0 & 0 \\ 0 & \mathbf{Z}_1 & 0 \\ 0 & 0 & \mathbf{Z}_2 \end{bmatrix} \tag{3.58}$$

Note that both equations (3.54) and (3.58) indicate that there is no mutual coupling among the three sequences, which is the desirable result. Therefore, the zero-, positive-, and negative-sequence currents cause voltage drops only in the zero-, positive-, and negative-sequence networks, respectively, of the transmission line. Also note in equation (3.54) that the positive- and negative-sequence impedances of the transmission line are equal to each other but they are far less than the zero-sequence impedance of the line. Figure 3.4 shows the sequence networks of a transmission line.

3.6.3 Electromagnetic Unbalances Due to Untransposed Lines

If the line is neither transposed nor its conductors equilaterally spaced, equation (3.48) cannot be used. Instead, use the following equation:

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} \end{bmatrix} \tag{3.59}$$

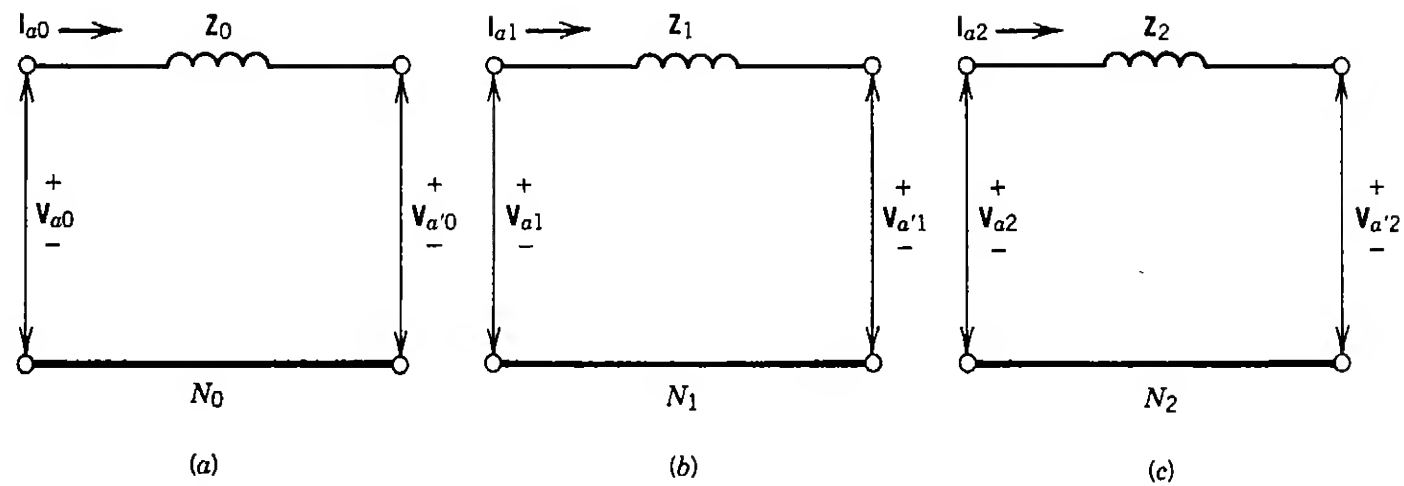


Figure 3.4. Sequence networks of transmission line: (*a*) zero-sequence network; (*b*) positive-sequence network; (*c*) negative-sequence network.

where

$$\mathbf{Z}_{aa} = \mathbf{Z}_{bb} = \mathbf{Z}_{cc} = \left[(r_a + r_e) + j0.1213 \ln \frac{D_e}{D_s} \right] \tag{3.60}$$

$$\mathbf{Z}_{ab} = \mathbf{Z}_{ba} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{ab}} \right] \tag{3.61}$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_{ca} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{ac}} \right] \tag{3.62}$$

$$\mathbf{Z}_{bc} = \mathbf{Z}_{cb} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{bc}} \right] \tag{3.63}$$

The corresponding sequence impedance matrix can be found from equation (3.38) as before. Therefore, the associated sequence admittance matrix can be found as

$$[\mathbf{Y}_{012}] = [\mathbf{Z}_{012}]^{-1} \tag{3.64a}$$

$$= \begin{bmatrix} \mathbf{Y}_{00} & \mathbf{Y}_{01} & \mathbf{Y}_{02} \\ \mathbf{Y}_{10} & \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{20} & \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \tag{3.64b}$$

Therefore,

$$[\mathbf{I}_{012}] = [\mathbf{Y}_{012}][\mathbf{V}_{012}] \tag{3.65}$$

Since the line is neither transposed nor its conductors equilaterally spaced, there is an electromagnetic unbalance in the system. Such unbalance is determined from equation (3.65) with only positive-sequence voltage applied. Therefore,

$$\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} = \begin{bmatrix} \mathbf{Y}_{00} & \mathbf{Y}_{01} & \mathbf{Y}_{02} \\ \mathbf{Y}_{10} & \mathbf{Y}_{11} & \mathbf{Y}_{12} \\ \mathbf{Y}_{20} & \mathbf{Y}_{21} & \mathbf{Y}_{22} \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{V}_{a1} \\ 0 \end{bmatrix} \tag{3.66a}$$

$$= \begin{bmatrix} \mathbf{Y}_{01} \\ \mathbf{Y}_{11} \\ \mathbf{Y}_{21} \end{bmatrix} \mathbf{V}_{a1} \tag{3.66b}$$

According to Gross and Hesse [3], the per-unit unbalances for zero sequence and negative sequence can be expressed, respectively, as

$$\mathbf{m}_0 \triangleq \frac{\mathbf{I}_{a0}}{\mathbf{I}_{a1}} \quad \text{pu} \tag{3.67a}$$

$$= \frac{\mathbf{Y}_{01}}{\mathbf{Y}_{11}} \quad \text{pu} \tag{3.67b}$$

and

$$\mathbf{m}_2 \triangleq \frac{\mathbf{I}_{a2}}{\mathbf{I}_{a1}} \quad \text{pu} \tag{3.68a}$$

$$= \frac{\mathbf{Y}_{21}}{\mathbf{Y}_{11}} \quad \text{pu} \tag{3.68b}$$

Since, in physical systems [3],

$$\mathbf{Z}_{22} \gg \mathbf{Z}_{02} \text{ or } \mathbf{Z}_{21}$$

and

$$\mathbf{Z}_{00} \gg \mathbf{Z}_{20} \text{ or } \mathbf{Z}_{01}$$

the approximate values of the per-unit unbalances for zero and negative sequences can be expressed respectively, as

$$\mathbf{m}_0 \cong -\frac{\mathbf{Z}_{01}}{\mathbf{Z}_{00}} \quad \text{pu} \tag{3.69a}$$

and

$$\mathbf{m}_2 \cong -\frac{\mathbf{Z}_{21}}{\mathbf{Z}_{22}} \quad \text{pu} \tag{3.69b}$$

EXAMPLE 3.3

Consider the compact-line configuration shown in Figure 3.5. The phase conductors used are made up of 500-kcmil, 30/7-stand ACSR. The line length is 40 mi and the line is not transposed. Ignore the overhead ground wire. If the earth has an average resistivity, determine the following:

(a) Line impedance matrix.
(b) Sequence impedance matrix of line.

Solution

(a) The conductor parameters can be found from Table A.3 (Appendix A) as

$$r_a = r_b = r_c = 0.206\ \Omega/\text{mi}$$

$$D_s = D_{sa} = D_{sb} = D_{sc} = 0.0311 \text{ ft}$$

$$D_{ab} = (2^2 + 8^2)^{1/2} = 8.2462 \text{ ft}$$

$$D_{bc} = (3^2 + 13^2)^{12} = 13.3417 \text{ ft}$$

$$D_{ac} = (5^2 + 11^2)^{1/2} = 12.0830 \text{ ft}$$

Since the earth has an average resistivity, $D_e = 2788.5$ ft. At 60 Hz, $r_e = 0.09528\ \Omega/\text{mi}$. From equation (3.60), the self-impedances of the line conductors are

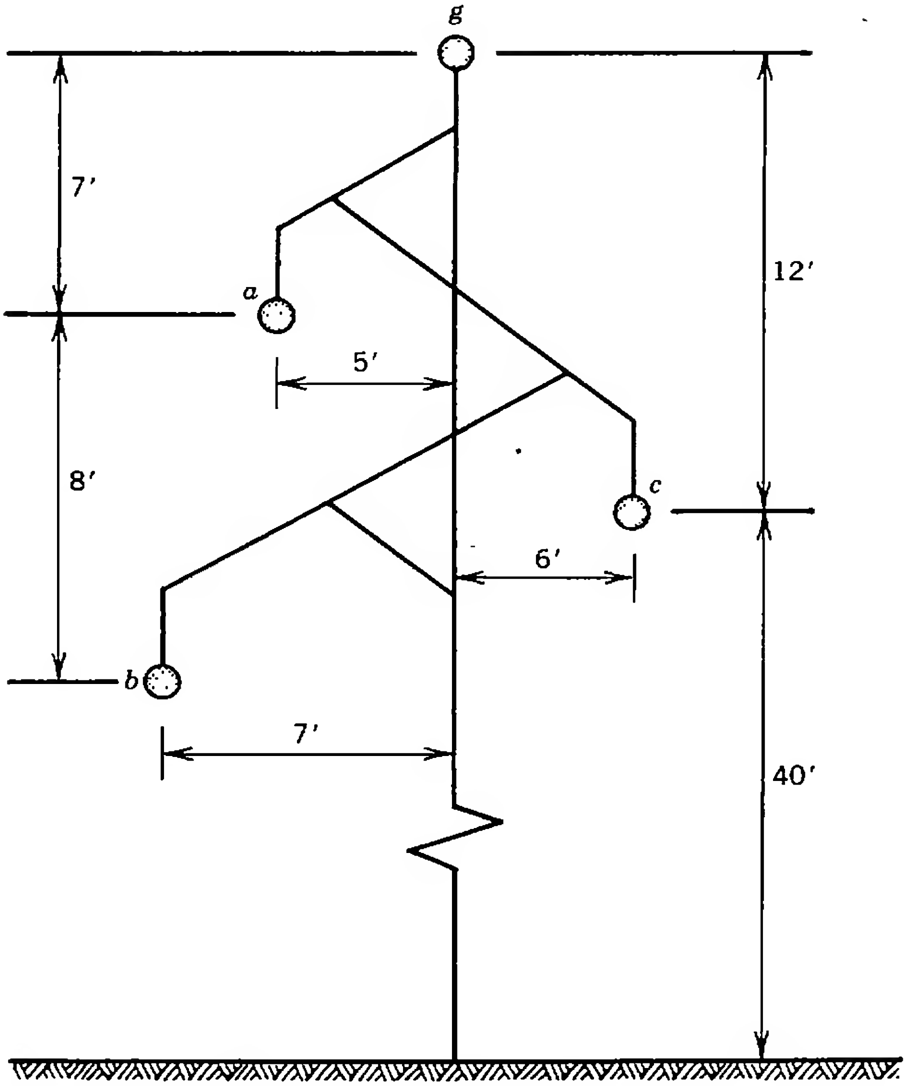


Figure 3.5

$$\mathbf{Z}_{aa} = \mathbf{Z}_{bb} = \mathbf{Z}_{cc} = \left[(r_a + r_e) + j0.1213 \ln \frac{D_e}{D_s}\right] l$$

$$= \left[(0.206 + 0.09528) + j0.1213 \ln \frac{2788.5}{0.0311}\right] \times 40$$

$$= 12.0512 + j55.3495\ \Omega$$

The mutual impedances calculated from equations (3.61)–(3.63) are

$$\mathbf{Z}_{ab} = \mathbf{Z}_{ba} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{ab}}\right] l$$

$$= \left[0.09528 + j0.1213 \ln \frac{2788.5}{8.2462}\right] \times 40$$

$$= 3.8112 + j28.2650\ \Omega$$

$$\mathbf{Z}_{bc} = \mathbf{Z}_{cb} = \left[0.09528 + j0.1213 \ln \frac{2788.5}{13.3417}\right] \times 40$$

$$= 3.8112 + j25.9297\ \Omega$$

$$\mathbf{Z}_{ac} = \mathbf{Z}_{ca} = \left[0.09528 + j0.01213 \ln \frac{2788.5}{12.0830}\right] \times 40$$
$$= 3.8112 + j26.4107\ \Omega$$

Therefore,

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} (12.0512 + j55.3495) & (3.8112 + j28.2650) & (3.8112 + j26.4107) \\ (3.8112 + j28.2650) & (12.0512 + j55.3495) & (3.8112 + j25.9297) \\ (3.8112 + j26.4107) & (3.8112 + j25.9297) & (12.0512 + j55.3495) \end{bmatrix}$$

(b) Thus, the sequence impedance matrix of the line can be found from equation (3.38) as

$$[\mathbf{Z}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] = \begin{bmatrix} (19.67 + j109.09) & (0.54 + j0.47) & (-0.54 + j0.47) \\ (-0.54 + j0.47) & (8.24 + j28.48) & (-1.07 - j0.94) \\ (0.54 + j0.47) & (1.07 - j0.94) & (8.24 + j28.48) \end{bmatrix}$$

EXAMPLE 3.4

Repeat Example 3.3 assuming that the line is completely tansposed.

Solution

(a) From equation (3.49),

$$\mathbf{Z}_s = \left[(r_a + r_e) + j0.1213 \ln \frac{D_e}{D_s}\right] l$$
$$= 12.0512 + j53.3495\ \Omega \quad \text{as before}$$

From equation (3.50),

$$\mathbf{Z}_m = \left[r_e + j0.1213 \ln \frac{D_e}{D_{\text{eq}}}\right] l$$

where $D_{\text{eq}} = (8.2462 \times 13.3417 \times 12.0830)^{1/3} = 11$ ft. Thus,

$$\mathbf{Z}_m = \left[0.09528 + j0.1213 \ln \frac{2788.5}{11}\right] \times 40$$
$$= 3.8112 + 26.8684\ \Omega$$

Therefore,

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} \mathbf{Z}_s & \mathbf{Z}_m & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_s & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_m & \mathbf{Z}_s \end{bmatrix}$$
$$= \begin{bmatrix} (12.0512 + j55.3495) & (3.8112 + j26.8684) & (3.8112 + j26.8684) \\ (3.8112 + j26.8684) & (12.0512 + j55.3495) & (3.8112 + j26.8684) \\ (3.8112 + j26.8684) & (3.8112 + j26.8684) & (12.0512 + j55.3495) \end{bmatrix}$$

(b) From equation (3.54)

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_s + 2\mathbf{Z}_m & 0 & 0 \\ 0 & \mathbf{Z}_s - \mathbf{Z}_m & 0 \\ 0 & 0 & \mathbf{Z}_s - \mathbf{Z}_m \end{bmatrix}$$
$$= \begin{bmatrix} 19.6736 + j109.086 & 0 & 0 \\ 0 & 8.2400 + j28.4811 & 0 \\ 0 & 0 & 8.2400 + j28.4811 \end{bmatrix}$$

or, by substituting equations (3.55b) and (3.56b) into equation (3.58),

$$[\mathbf{Z}_{012}] = \begin{bmatrix} (19.6736 + j109.0824) & 0 & 0 \\ 0 & (8.2400 + j28.4831) & 0 \\ 0 & 0 & (8.2400 + j28.4831) \end{bmatrix}$$

Example 3.5

Consider the results of Example 3.3 and determine the following:

(a) Per-unit electromagnetic unbalance for zero sequence.

(b) Approximate value of per-unit electromagnetic unbalance for negative sequence.

(c) Per-unit electromagnetic unbalance for negative sequence.

(d) Approximate value of per-unit electromagnetic unbalance for negative sequence.

Solution

The sequence admittance of the line can be found as

$$[\mathbf{Y}_{012}] = [\mathbf{Z}_{012}]^{-1}$$

$$= \begin{bmatrix} 1.60 \times 10^{-3} - j8.88 \times 10^{-3} & 7.57 \times 10^{-5} + j1.93 \times 10^{-4} & -2.01 \times 10^{-4} + j6.15 \times 10^{-5} \\ -2.01 \times 10^{-4} + j6.15 \times 10^{-5} & 9.44 \times 10^{-3} - j3.25 \times 10^{-2} & -4.55 \times 10^{-4} - j1.55 \times 10^{-3} \\ 7.57 \times 10^{-5} + j1.93 \times 10^{-4} & 1.60 \times 10^{-3} - j2.54 \times 10^{-4} & 9.44 \times 10^{-3} - j3.25 \times 10^{-2} \end{bmatrix}$$

(a) From equation (3.67b),

$$\mathbf{m}_0 = \frac{\mathbf{Y}_{01}}{\mathbf{Y}_{11}} = \frac{7.57 \times 10^{-5} + j1.93 \times 10^{-4}}{9.44 \times 10^{-3} - j3.25 \times 10^{-2}} = 0.61\ \underline{/142.4^\circ}\ \%$$

(b) From equation (3.69a),

$$\mathbf{m}_0 \cong -\frac{\mathbf{Z}_{01}}{\mathbf{Z}_{00}} = -\frac{0.54 + j0.47}{19.67 + j109.09} = 0.64\ \underline{/141.3^\circ}\ \%$$

(c) From equation (3.68b),

$$\mathbf{m}_2 = \frac{\mathbf{Y}_{21}}{\mathbf{Y}_{11}} = \frac{1.60 \times 10^{-3} - j2.54 \times 10^{-4}}{9.44 \times 10^{-3} - j3.25 \times 10^{-2}} = 4.79\ \underline{/64.8^\circ}\ \%$$

(d) From equation (3.69b),

$$\mathbf{m}_2 \cong -\frac{\mathbf{Z}_{21}}{\mathbf{Z}_{22}} = -\frac{1.07 - j0.94}{8.24 + j28.48} = 4.8\ \underline{/64.8^\circ}\ \%$$

3.6.4 Sequence Impedances of Untransposed Line with Overhead Ground Wire

Assume that the untransposed line shown in Figure 3.5 is *shielded* against direct lightning strikes by the overhead ground wire *u* (used instead of *g*).

Therefore,

$$[\mathbf{V}_{abcu}] = [\mathbf{Z}_{abcu}][\mathbf{I}_{abcu}] \tag{3.70}$$

but since for the ground wire $V_u = 0$,

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} & \mathbf{Z}_{au} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} & \mathbf{Z}_{bu} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} & \mathbf{Z}_{cu} \\ \hline \mathbf{Z}_{ua} & \mathbf{Z}_{ub} & \mathbf{Z}_{uc} & \mathbf{Z}_{uu} \end{array}\right] \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \\ \hline \mathbf{I}_u \end{bmatrix} \tag{3.71}$$

The matrix $[\mathbf{Z}_{abcu}]$ can be determined using equations (3.59)–(3.63), as before, and also using the following equations:

$$\mathbf{Z}_{au} = \mathbf{Z}_{ua} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{au}} \right] l \tag{3.72}$$

$$\mathbf{Z}_{bu} = \mathbf{Z}_{ub} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{bu}} \right] l \tag{3.73}$$

$$\mathbf{Z}_{cu} = \mathbf{Z}_{uc} = \left[r_e + j0.1213 \ln \frac{D_e}{D_{cu}} \right] l \tag{3.74}$$

$$\mathbf{Z}_{uu} = \left[(r_u + r_e) + j0.1213 \ln \frac{D_e}{D_{uu}} \right] l \tag{3.75}$$

where r_u and D_{uu} are the resistance and GMR of the overhead ground wire, respectively.

The matrix $[\mathbf{Z}_{abcu}]$ given in equation (3.71) can be reduced to $[\mathbf{Z}_{abc}]$ by using the Kron reduction technique. Therefore, equation (3.71) can be reexpressed as

$$\begin{bmatrix} \mathbf{V}_{abc} \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{Z}_1 & \mathbf{Z}_2 \\ \hline \mathbf{Z}_3 & \mathbf{Z}_4 \end{array}\right] \begin{bmatrix} \mathbf{I}_{abc} \\ \hline \mathbf{I}_u \end{bmatrix} \tag{3.76}$$

where the submatrices $[\mathbf{Z}_1]$, $[\mathbf{Z}_2]$, $[\mathbf{Z}_3]$, and $[\mathbf{Z}_4]$ are specified in the partitioned matrix $[\mathbf{Z}_{abcu}]$ in equation (3.71). Therefore, after the reduction,

$$[\mathbf{V}_{abc}] = [\mathbf{Z}_{abc}][\mathbf{I}_{abc}] \tag{3.77}$$

where

$$[\mathbf{Z}_{abc}] \triangleq [\mathbf{Z}_1] - [\mathbf{Z}_2][\mathbf{Z}_4]^{-1}[\mathbf{Z}_3] \tag{3.78}$$

Therefore, the sequence impedance matrix can be found from

$$[\mathbf{Z}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] \tag{3.79}$$

Thus, the sequence admittance matrix becomes

$$[\mathbf{Y}_{012}] = [\mathbf{Z}_{012}]^{-1} \tag{3.80}$$

3.7 SEQUENCE CAPACITANCES OF TRANSMISSION LINE

3.7.1 Three-Phase Transmission Line without Overhead Ground Wire

Consider Figure 2.47 and assume that the three-phase conductors are charged. Therefore, for sinusoidal steady-state analysis, both voltage and charge density can be represented by phasors. Thus,

$$[\mathbf{V}_{abc}] = [P_{abc}][Q_{abc}] \tag{3.81}$$

or

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} p_{aa} & P_{ab} & p_{ac} \\ p_{ba} & p_{bb} & p_{bc} \\ p_{ca} & p_{cb} & p_{cc} \end{bmatrix} \begin{bmatrix} q_a \\ q_b \\ q_c \end{bmatrix} \tag{3.82}$$

where

$[P_{abc}]$ = matrix of potential coefficients

$$p_{aa} = \frac{1}{2\pi\epsilon} \ln \frac{h_{11}}{r_a} \quad \mathrm{F^{-1}m} \tag{3.83}$$

$$p_{bb} = \frac{1}{2\pi\epsilon} \ln \frac{h_{22}}{r_b} \quad \mathrm{F^{-1}m} \tag{3.84}$$

$$p_{cc} = \frac{1}{2\pi\epsilon} \ln \frac{h_{33}}{r_c} \quad \mathrm{F^{-1}m} \tag{3.85}$$

$$p_{ab} = p_{ba} = \frac{1}{2\pi\epsilon} \ln \frac{l_{12}}{D_{12}} \quad \mathrm{F^{-1}m} \tag{3.86}$$

$$p_{bc} = p_{cb} = \frac{1}{2\pi\epsilon} \ln \frac{l_{23}}{D_{23}} \quad \mathrm{F^{-1}m} \tag{3.87}$$

$$p_{ac} = p_{ca} = \frac{1}{2\pi\epsilon} \ln \frac{l_{31}}{D_{31}} \quad \mathrm{F^{-1}m} \tag{3.88}$$

Therefore, from equation (3.81),

$$[Q_{abc}] = [P_{abc}]^{-1}[\mathbf{V}_{abc}] \quad \mathrm{C/m} \tag{3.89a}$$

$$= [C_{abc}][\mathbf{V}_{abc}] \quad \mathrm{C/m} \tag{3.89b}$$

since

$$[C_{abc}] = [P_{abc}]^{-1} \quad \text{F/m} \tag{3.90}$$

or

$$[C_{abc}] = \begin{bmatrix} C_{aa} & -C_{ab} & -C_{ac} \\ -C_{ba} & C_{bb} & -C_{bc} \\ -C_{ca} & -C_{cb} & C_{cc} \end{bmatrix} \quad \text{F/m} \tag{3.91}$$

where $[C_{abc}]$ is the matrix of Maxwell's coefficients, the diagonal terms are Maxwell's (or capacitance) coefficients, and the off-diagonal terms are electrostatic induction coefficients.

Therefore, the sequence capacitances can be found by using the similarity transformation as

$$[C_{012}] \triangleq [\mathbf{A}]^{-1}[C_{abc}][\mathbf{A}] \quad \text{F/m} \tag{3.92a}$$

$$= \begin{bmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{bmatrix} \quad \text{F/m} \tag{3.92b}$$

Note that if the line is *transposed*, the matrix of potential coefficients can be expressed in terms of self and mutual potential coefficients as

$$[P_{abc}] = \begin{bmatrix} p_s & p_m & p_m \\ p_m & p_s & p_m \\ p_m & p_m & p_s \end{bmatrix} \tag{3.93}$$

Therefore, using the similarity transformation,

$$[P_{012}] = [\mathbf{A}]^{-1}[P_{abc}][\mathbf{A}] \tag{3.94a}$$

$$= \begin{bmatrix} p_0 & 0 & 0 \\ 0 & p_1 & 0 \\ 0 & 0 & p_2 \end{bmatrix} \tag{3.94b}$$

Thus,

$$[C_{012}] = [P_{012}]^{-1} \tag{3.95a}$$

$$= \begin{bmatrix} 1/p_0 & 0 & 0 \\ 0 & 1/p_1 & 0 \\ 0 & 0 & 1/p_2 \end{bmatrix} \tag{3.95b}$$

$$= \begin{bmatrix} C_0 & 0 & 0 \\ 0 & C_1 & 0 \\ 0 & 0 & C_2 \end{bmatrix} \tag{3.95c}$$

Alternatively, the sequence capacitances can approximately be calculated without using matrix algebra. For example, the zero-sequence capacitance can be calculated [4] from

$$C_0 = \frac{29.842}{\ln(H_{aa}/D_{aa})} \quad \text{nF/mi} \tag{3.96}$$

where

$$\begin{aligned} H_{aa} &= \text{GMD between three conductors and their images} \\ &= [h_{11}h_{22}h_{33}(l_{12}l_{23}l_{31})^2]^{1/9} \end{aligned} \tag{3.97}$$

$$\begin{aligned} D_{aa} &= \text{self-GMD of overhead conductors as composite group but with } D_s \text{ of each conductor taken as its radius} \\ &= [r_a r_b r_c (D_{12}D_{23}D_{31})^2]^{1/9} \end{aligned} \tag{3.98}$$

Note that D_s has been replaced by the conductor radius since all charge on a conductor resides on its surface. The positive- and negative-sequence capacitances of a line are the same owing to the fact that the physical parameters do not vary with a change in sequence of the applied voltage. Therefore, they are the same as the line-to-neutral capacitance C_n and can be calculated from equations (2.411) or (2.412).

Note that the mutual capacitances of the line can be found from equation (3.91). The capacitances to ground can be expressed as

$$\begin{bmatrix} C_{ag} \\ C_{bg} \\ C_{cg} \end{bmatrix} = \begin{bmatrix} C_{aa} & C_{ab} & C_{ac} \\ -C_{ba} & -C_{bb} & C_{bc} \\ -C_{ac} & C_{bc} & -C_{cc} \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix} \tag{3.99}$$

If the line is transposed, the capacitance to ground is an average value that can be determined from

$$C_{g,\text{avg}} = \tfrac{1}{3}(C_{ag} + C_{bg} + C_{cg}) \tag{3.100}$$

Also note that the shunt admittance matrix of the line is

$$[\mathbf{Y}_{abc}] = jw[C_{abc}] \tag{3.101}$$

Therefore,

$$[\mathbf{Y}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Y}_{abc}][\mathbf{A}] \tag{3.102}$$

Thus,

$$[C_{012}] = \frac{[\mathbf{Y}_{012}]}{jw} \tag{3.103}$$

Of course,

$$[\mathbf{I}_{012}] = jw[C_{012}][\mathbf{V}_{012}] = j[B_{012}][\mathbf{V}_{012}] \tag{3.104}$$

and

$$[\mathbf{I}_{abc}] = [\mathbf{A}][\mathbf{I}_{012}] \tag{3.105}$$

or

$$[\mathbf{I}_{abc}] = jw[C_{abc}][\mathbf{V}_{abc}] = j[B_{abc}][\mathbf{V}_{abc}] \tag{3.106}$$

3.7.2 Three-Phase Transmission Line with Overhead Ground Wire

Consider Figure 3.6(a) and assume that the line is transposed and that the overhead ground wire is denoted by u and that there are nine capacitances involved. The voltages and charge densities involved can be represented by phasors. Therefore,

$$[\mathbf{V}_{abcu}] = [P_{abcu}][Q_{abcu}] \tag{3.107}$$

but since, for the ground wire, $V_u = 0$,

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \\ \hline 0 \end{bmatrix} = \left[\begin{array}{ccc|c} p_{aa} & p_{ab} & p_{ac} & p_{au} \\ p_{ba} & p_{bb} & p_{bc} & p_{bu} \\ p_{ca} & p_{cb} & p_{cc} & p_{cu} \\ \hline p_{ua} & p_{ub} & p_{uc} & p_{uu} \end{array}\right] \begin{bmatrix} q_a \\ q_b \\ q_c \\ \hline q_u \end{bmatrix} \tag{3.108}$$

The matrix $[P_{abcu}]$ can be calculated as before. The corresponding matrix of the Maxwell coefficients can be found as

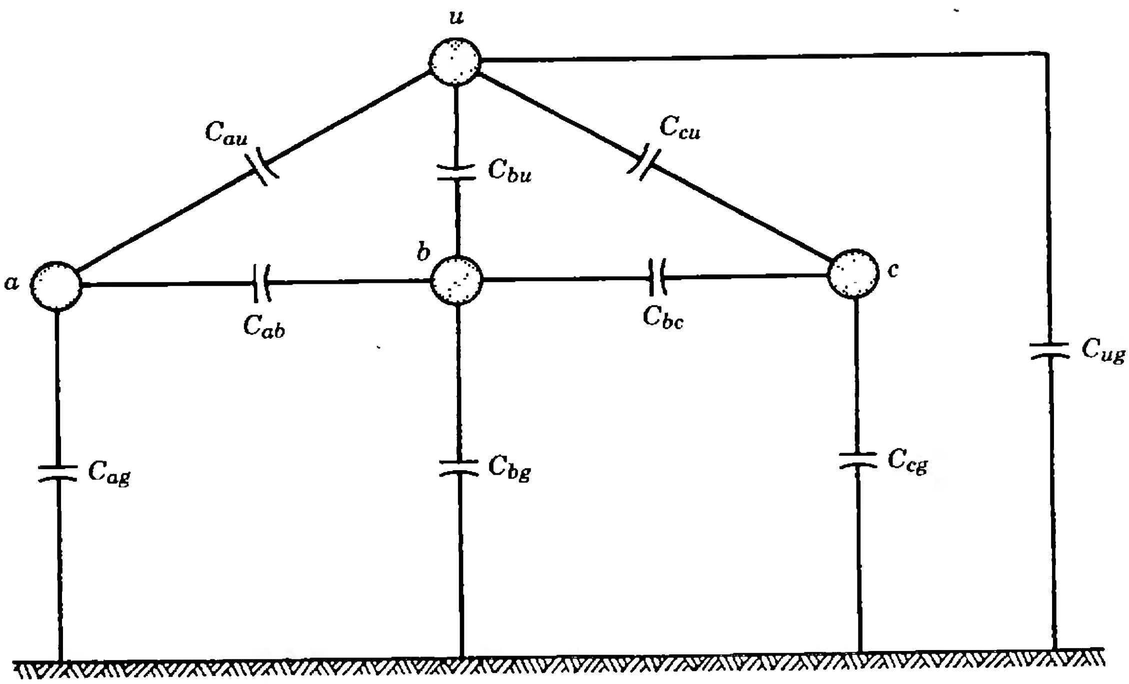
$$[C_{abcu}] = [P_{abcu}]^{-1} \tag{3.109}$$

The corresponding equivalent circuit is shown in Figure 3.6(a). Such equivalent circuit representation is convenient to study switching transients, traveling waves, overvoltages, etc.

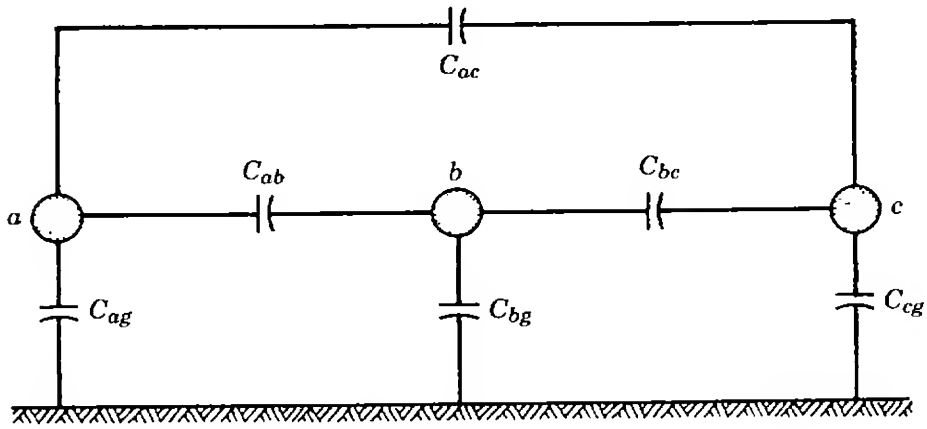
The matrix $[P_{abcu}]$ given in equation (3.108) can be reduced to $[P_{abc}]$ by using the Kron reduction technique. Therefore, equation (3.97) can be reexpressed as

$$\begin{bmatrix} \mathbf{V}_{abc} \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} P_1 & P_2 \\ \hline P_3 & P_4 \end{array}\right] \begin{bmatrix} Q_{abc} \\ \hline Q_u \end{bmatrix} \tag{3.110}$$

Where the submatrices $[P_1]$, $[P_2]$, $[P_3]$, and $[P_4]$ are specified in the



(a)



(b)

Figure 3.6. Three-phase line with one overhead ground wire u: (a) equivalent circuit showing ground wire; (b) equivalent circuit without showing ground wire.

partitioned matrix $[P_{abcu}]$ in equation (3.108). Thus, after the reduction,

$$[\mathbf{V}_{abc}] = [P_{abc}][Q_{abc}] \tag{3.111}$$

where

$$[P_{abc}] \triangleq [P_1] - [P_2][P_4]^{-1}[P_3] \tag{3.112}$$

Thus, the corresponding matrix of the Maxwell coefficients can be found as

$$[C_{abc}] = [P_{abc}]^{-1} \tag{3.113}$$

as before. The corresponding equivalent circuit is shown in Figure 3.6(b), and such representation is convenient to study a load-flow problem. Of course, the average capacitances to ground can be found as before.

Alternatively, the sequence capacitances can approximately be calculated without using the matrix algebra. For example, the zero-sequence capacitance can be calculated [4] from

$$C_0 = \frac{29.842 \ln(h_{gg}/D_{gg})}{\ln(H_{aa}/D_{aa}) \ln(h_{gg}/D_{gg}) - [\ln(H_{ag}/D_{ag})]^2} \quad \text{nF/mi} \tag{3.114}$$

where H_{aa} = given by equation (3.97)
D_{aa} = given by equation (3.98)
h_{gg} = GMD between ground wires and their images
D_{gg} = self-GMD of ground wires with $D_s = r_g$
H_{ag} = GMD between phase conductors and images of ground wires
D_{ag} = GMD between phase conductors and ground wires

If the transmission line is untransposed, both electrostatic and electromagnetic unbalances† exist in the system. If the system neutral is (solidly) grounded, in the event of an electrostatic unbalance, there will be a neutral *residual* current flow in the system due to the unbalance in the charging currents of the line. Such residual current flow is continuous and independent of the load. Since the neutral is grounded, $\mathbf{V}_n = \mathbf{V}_{a0} = 0$, and the *zero-sequence displacement* or *unbalance* is

$$\mathbf{d}_0 \triangleq \frac{\mathbf{C}_{01}}{\mathbf{C}_{11}} \tag{3.115}$$

and the *negative-sequence unbalance* is

$$\mathbf{d}_2 \triangleq -\frac{\mathbf{C}_{21}}{\mathbf{C}_{11}} \tag{3.116}$$

Whereas if the system neutral is not grounded, there will be the neutral voltage $\mathbf{V}_n \neq 0$, and therefore, the neutral point will be shifted. Such zero-sequence *neutral displacement* or *unbalance* is defined as

$$\mathbf{d}_0 \triangleq -\frac{\mathbf{C}_{01}}{\mathbf{C}_{00}} \tag{3.117}$$

† For further information, see Anderson [4].

(d) (From equation (3.115),

$$\mathbf{d}_0 = \frac{\mathbf{C}_{01}}{\mathbf{C}_{11}} = \frac{-1.59 \times 10^{-4} + j1.2 \times 10^{-4}}{1.55 \times 10^{-2}} = 0.0129 \underline{/143^\circ} \text{ or } 1.29\%$$

and from equation (3.116),

$$\mathbf{d}_2 = -\frac{\mathbf{C}_{21}}{\mathbf{C}_{11}} = -\frac{6.44 \times 10^{-4} + j5.29 \times 10^{-4}}{1.55 \times 10^{-2}} = 0.049 \underline{/219.4^\circ} \text{ or } 4.9\%$$

3.8 SEQUENCE IMPEDANCES OF SYNCHRONOUS MACHINES

In general, the impedances to positive-, negative-, and zero-sequence currents in synchronous machines (as well as other rotating machines) have different values. The positive-sequence impedance of the synchronous machine can be selected to be its *subtransient* (X''_d), *transient* (X'_d), or *synchronous*† (X_d) *reactance* depending on the time assumed to elapse from the instant of fault initiation to the instant at which values are desired (e.g., for relay response, breaker opening, or sustained fault conditions). Usually, however, in fault studies, the subtransient reactance is taken as the positive-sequence reactance of the synchronous machine.

The negative-sequence impedance of a synchronous machine is usually determined from

$$\mathbf{Z}_2 = jX_2 = j\left(\frac{X''_d + X''_q}{2}\right) \tag{3.118}$$

In a cylindrical-rotor synchronous machine, the subtransient and negative-sequence reactances are the same, as shown in Table 3.3.

The zero-sequence impedance of a synchronous machine varies widely and depends on the pitch of the armature coils. It is much smaller than the corresponding positive- and negative-sequence reactances. It can be measured by connecting the three armature windings in series and applying a single-phase voltage. The ratio of the terminal voltage of one phase winding to the current is the zero-sequence reactance. It is approximately equal to the zero-sequence reactance. Table 3.3 [5] gives typical reactance values of three-phase synchronous machines. Note that in the above discussion, the resistance values are ignored because they are much smaller than the corresponding reactance values.

Figure 3.7 shows the equivalent circuit of a cylindrical-rotor synchronous machine with constant field current. Since the coil groups of the three-phase stator armature windings are displaced frome each other by 120 electrical degrees, balanced three-phase sinuoidal voltages are induced in the stator

† It is also called the *direct-axis synchronous reactance*. It is also denoted by X_s.

TABLE 3.3 Typical Reactances of Three-Phase Synchronous Machines [5]

	Turbine Generators						Salient-Pole Generators						Synchronous Condensers					
	Two Pole			Four Pole			With Dampers			Without Dampers			Air Cooled			Hydrogen Cooled		
	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High	Low	Avg.	High
X_d	0.95	1.2	1.45	1.00	1.2	1.45	0.6	1.25	1.5	0.6	1.25	1.5	1.25	1.85	2.2	1.5	2.2	2.65
X_d'	0.12	0.15	0.21	0.2	0.23	0.28	0.2	0.3	0.5	0.2	0.3	0.5	0.3	0.4	0.5	0.36	0.48	0.6
X_d''	0.07	0.09	0.14	0.12	0.14	0.17	0.13	0.2	0.32	0.2	0.3	0.5	0.19	0.27	0.3	0.23	0.32	0.36
X_q	0.92	1.16	1.42	0.92	1.16	1.42	0.4	0.7	0.8	0.4	0.7	0.8	0.95	1.15	1.3	1.1	1.35	1.55
X_2	0.07	0.09	0.14	0.12	0.14	0.17	0.13	0.2	0.32	0.35	0.48	0.65	0.18	0.26	0.4	0.22	0.31	0.48
X_0	0.01	0.03	0.08	0.015	0.08	0.14	0.03	0.18	0.23	0.03	0.19	0.24	0.025	0.12	0.15	0.03	0.14	0.18

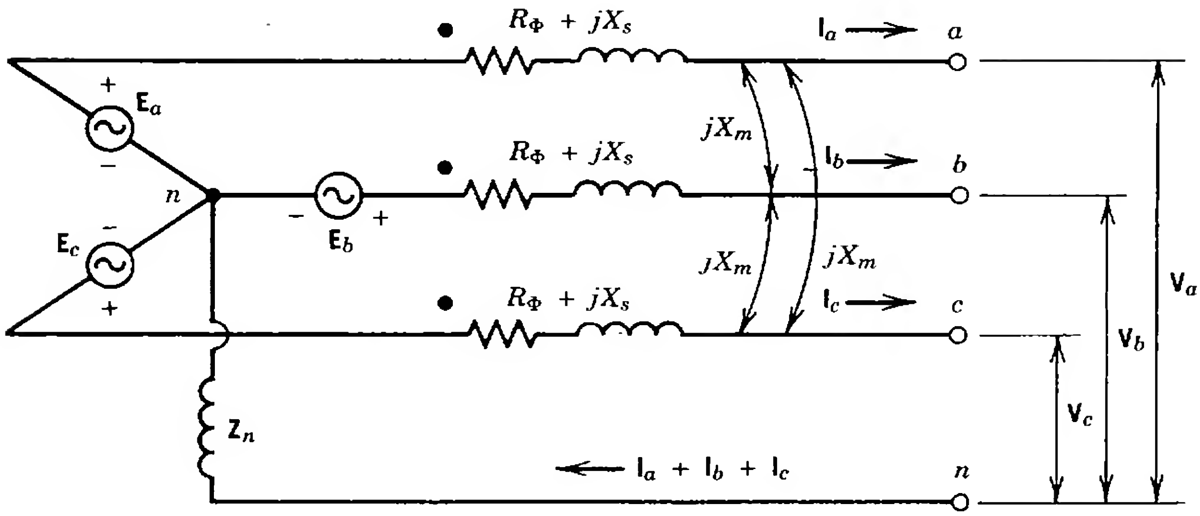


Figure 3.7. Equivalent circuit of cylindrical-rotor synchronous machine.

windings. Furthermore, each of the three self-impedances and mutual impedances are equal to each other, respectively, owing to the machine symmetry. Therefore, taking into account the neutral impedance $\mathbf{Z}_n$ and applying Kirchhoffs voltage law (KVL) it can be shown that

$$\mathbf{E}_a = (R_\Phi + jX_s + \mathbf{Z}_n)\mathbf{I}_a + (jX_m + \mathbf{Z}_n)\mathbf{I}_b + (jX_m + \mathbf{Z}_n)\mathbf{I}_c + \mathbf{V}_a \tag{3.119}$$

$$\mathbf{E}_b = (jX_m + \mathbf{Z}_n)\mathbf{I}_a + (R_\Phi + jX_s + \mathbf{Z}_n)\mathbf{I}_b + (jX_m + \mathbf{Z}_n)\mathbf{I}_c + \mathbf{V}_b \tag{3.120}$$

$$\mathbf{E}_c = (jX_m + \mathbf{Z}_n)\mathbf{I}_a + (jX_m + \mathbf{Z}_n)\mathbf{I}_b + (R_\Phi + jX_s + \mathbf{Z}_n)\mathbf{I}_c + \mathbf{V}_c \tag{3.121}$$

or, in matrix form,

$$\begin{bmatrix} \mathbf{E}_a \\ \mathbf{E}_b \\ \mathbf{E}_c \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_s & \mathbf{Z}_m & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_s & \mathbf{Z}_m \\ \mathbf{Z}_m & \mathbf{Z}_m & \mathbf{Z}_s \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} + \begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \tag{3.122}$$

where

$$\mathbf{Z}_s = R_\Phi + jX_s + \mathbf{Z}_n \tag{3.123}$$

$$\mathbf{Z}_m = jX_m + \mathbf{Z}_n \tag{3.124}$$

$$\mathbf{E}_a = \mathbf{E}_a \tag{3.125}$$

$$\mathbf{E}_b = \mathbf{a}^2\mathbf{E}_a \tag{3.126}$$

$$\mathbf{E}_c = \mathbf{a}\mathbf{E}_a \tag{3.127}$$

Alternatively, equation (3.122) can be written in shorthand matrix notation as

$$[\mathbf{E}_{abc}] = [\mathbf{Z}_{abc}][\mathbf{I}_{abc}] + [\mathbf{V}_{abc}] \tag{3.128}$$

Multiplying both sides of this equation by $[\mathbf{A}]^{-1}$ and also substituting equation (3.26) into it,

$$[\mathbf{A}]^{-1}[\mathbf{E}_{abc}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}][\mathbf{I}_{012}] + [\mathbf{A}]^{-1}[\mathbf{V}_{abc}] \tag{3.129}$$

where

$$[\mathbf{A}]^{-1}[\mathbf{E}_{abc}] = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{E} \\ \mathbf{a}^2\mathbf{E} \\ \mathbf{aE} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{E} \\ 0 \end{bmatrix} \tag{3.130}$$

$$[\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] \triangleq [\mathbf{Z}_{012}] \tag{3.38}$$

$$[\mathbf{A}]^{-1}[\mathbf{V}_{abc}] = [\mathbf{V}_{012}] \tag{3.19}$$

Also, due to the symmetry of the machine,

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_s + 2\mathbf{Z}_m & 0 & 0 \\ 0 & \mathbf{Z}_s - \mathbf{Z}_m & 0 \\ 0 & 0 & \mathbf{Z}_s - \mathbf{Z}_m \end{bmatrix} \tag{3.131}$$

or

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_{00} & 0 & 0 \\ 0 & \mathbf{Z}_{11} & 0 \\ 0 & 0 & \mathbf{Z}_{22} \end{bmatrix} \tag{3.132}$$

where

$$\mathbf{Z}_{00} = \mathbf{Z}_s + 2\mathbf{Z}_m = R_\Phi + j(X_s + 2X_m) + 3\mathbf{Z}_n \tag{3.133}$$

$$\mathbf{Z}_{11} = \mathbf{Z}_s - \mathbf{Z}_m = R_\Phi + j(X_s - X_m) \tag{3.134}$$

$$\mathbf{Z}_{22} = \mathbf{Z}_m - \mathbf{Z}_m = R_\Phi + j(X_s - X_m) \tag{3.135}$$

Therefore, equation (3.128) in terms of the symmetrical components can be expressed as

$$\begin{bmatrix} 0 \\ \mathbf{E}_a \\ 0 \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{00} & 0 & 0 \\ 0 & \mathbf{Z}_{11} & 0 \\ 0 & 0 & \mathbf{Z}_{22} \end{bmatrix}\begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} + \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \tag{3.136}$$

or, in shorthand matrix notation,

$$[\mathbf{E}] = [\mathbf{Z}_{012}][\mathbf{I}_{012}] + [\mathbf{V}_{012}] \tag{3.137}$$

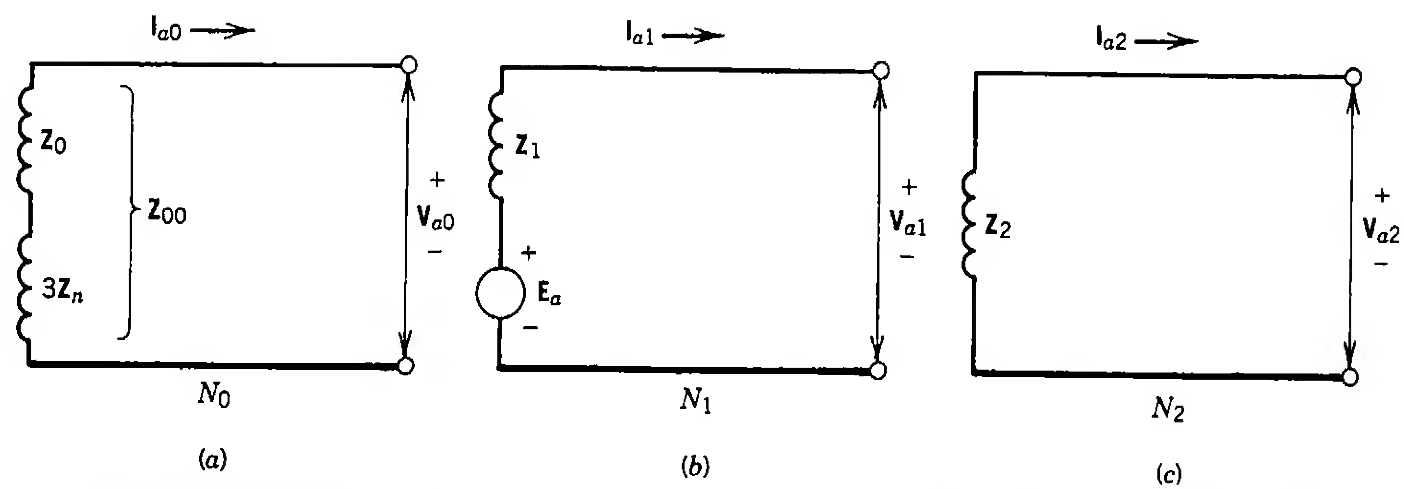


Figure 3.8. Sequence networks of synchronous machine: (*a*) zero-sequence network; (*b*) positive-sequence network; (*c*) negative-sequence network.

Similarly,

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ \mathbf{E}_a \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_{00} & 0 & 0 \\ 0 & \mathbf{Z}_{11} & 0 \\ 0 & 0 & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \tag{3.138}$$

or

$$[\mathbf{V}_{012}] = [\mathbf{E}] - [\mathbf{Z}_{012}][\mathbf{I}_{012}] \tag{3.139}$$

Note that the machine sequence impedances in the above equations are

$$\mathbf{Z}_0 \triangleq \mathbf{Z}_{00} - 3\mathbf{Z}_n \tag{3.140}$$

$$\mathbf{Z}_1 \triangleq \mathbf{Z}_{11} \tag{3.141}$$

$$\mathbf{Z}_2 \triangleq \mathbf{Z}_{22} \tag{3.142}$$

The expression given in equation (3.140) is due to the fact that the impedance $\mathbf{Z}_n$ is external to the machine. Figure 3.8 shows the sequence networks of a synchronous machine.

3.9 ZERO-SEQUENCE NETWORKS

It is important to note that the zero-sequence system, in a sense, is not a three-phase system but a single-phase system. This is because the zero-sequence currents and voltages are equal in magnitude and in phase at any point in all the phases of the system. However, the zero-sequence currents can only exist in a circuit if there is a complete path for their flow. Therefore, if there is no complete path for zero-sequence currents in a circuit, the zero-sequence impedance is infinite. In a zero-sequence network drawing, this infinite impedance is indicated by an open circuit.

Figure 3.9 shows zero-sequence networks for wye- and delta-connected three-phase loads. Note that a wye-connected load with an ungrounded neutral has infinite impedance to zero-sequence currents since there is no return path through the ground or a neutral conductor, as shown in Figure 3.9(*a*). Whereas a wye-connected load with solidly grounded neutral, as shown in Figure 3.9(*b*), provides a return path for the zero-sequence currents flowing through the three phases and their sum, $3\mathbf{I}_{a0}$, flowing through the ground. If the neutral is grounded through some impedance $\mathbf{Z}_n$, as shown in Figure 3.9(*c*), an impedance of $3\mathbf{Z}_n$ should be inserted between the neutral point n and the zero-potential bus N_0 in the zero-sequence network. The reason for this is that a current of $3\mathbf{I}_{a0}$ produces a zero-sequence voltage drop of $3\mathbf{I}_{a0}\mathbf{Z}_n$ between the neutral point n and the ground. Therefore, in order to reflect this voltage drop in the zero-sequence

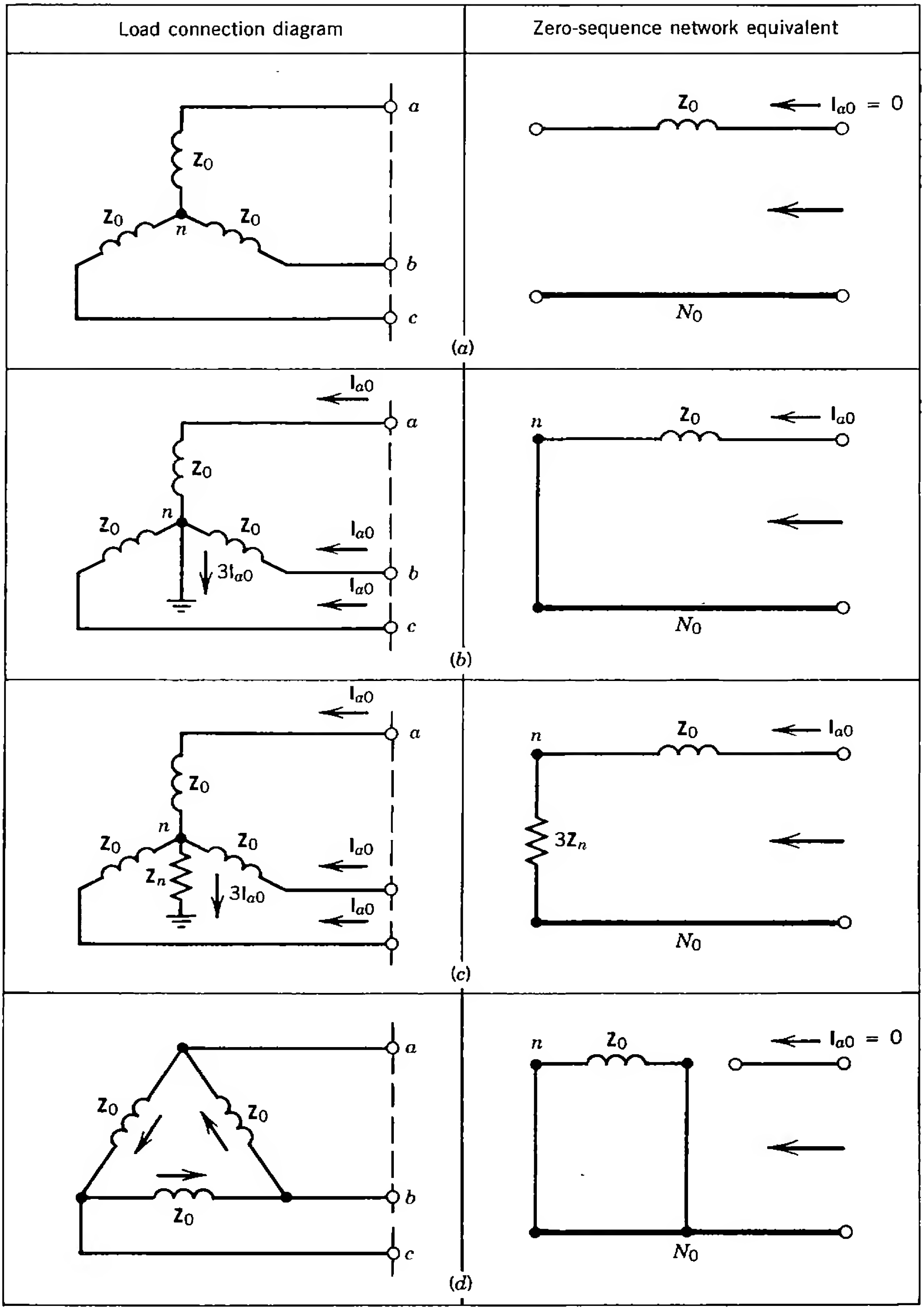


Figure 3.9. Zero-sequence network for wye- and delta-connected three-phase loads: (*a*) wye-connected load with undergrounded neutral; (*b*) wye-connected load with grounded neutral; (*c*) wye-connected load grounded through neutral impedance; (*d*) delta-connected load.

network where the zero-sequence current $\mathbf{I}_{a0}$ flows, the neutral impedance should be $3\mathbf{Z}_n$. A delta-connected load, as shown in Figure 3.9(d), provides no path for zero-sequence currents flowing in the line. Therefore, its zero-sequence impedance, as seen from its terminals, is infinite. Yet it is possible to have zero-sequence currents circulating within the delta circuit. However, they have to be produced in the delta by zero-sequence voltages or by induction from an outside source.

3.10 SEQUENCE IMPEDANCES OF TRANSFORMERS

A three-phase transformer may be made up of three identical single-phase transformers. If this is the case, it is called a three-phase *transformer bank*. Alternatively, it may be built as a three-phase transformer having a single common core (either with shell-type or core-type design) and a tank. For the sake of simplicity, here only the three-phase transformer banks will be reviewed.† The impedance of a transformer to both positive- and negative-sequence currents is the same. Even though the zero-sequence series impedances of three-phase units are little different than the positive- and negative-sequence series impedances, it is often assumed in practice that series impedances of all sequences are the same without paying attention to the transformer type:

$$\mathbf{Z}_0 = \mathbf{Z}_1 = \mathbf{Z}_2 = \mathbf{Z}_{\text{trf}} \tag{3.143}$$

Of course, if the flow of zero-sequence current is prevented by the transformer connection, $\mathbf{Z}_0$ is infinite.

Figure 3.10 shows zero-sequence network equivalents of three-phase transformer banks made up of three identical single-phase transformers having two windings with excitation currents neglected. The possible paths for the flow of zero-sequence current are indicated on the connection diagrams, as shown in Figures 3.10(a), 3.10(c), and 3.10(e). If there is no path shown on the connection diagram, this means that the transformer connection prevents the flow of the zero-sequence current by not providing path for it, as indicated in Figures 3.10(b), 3.10(d), and 3.10(f). Note that even though the delta–delta bank can have zero-sequence currents circulating within its delta windings, it also prevents the flow of the zero-sequence current outside the delta windings by not providing a return path for it, as shown in Figure 3.10(e). Also note that if the neutral point n of the wye winding [shown in Figure 3.10(a) or (c)] is grounded through $\mathbf{Z}_n$, the corresponding zero-sequence impedance $\mathbf{Z}_0$ should be replaced by $\mathbf{Z}_0 + 3\mathbf{Z}_n$. Of course, if the wye winding is *solidly grounded*, the $\mathbf{Z}_n$ is zero, and therefore $3\mathbf{Z}_n$ should be replaced by a short circuit. On the other hand, if

† For those readers interested in the three-phase transformers, see Clarke [6] and Anderson [4].

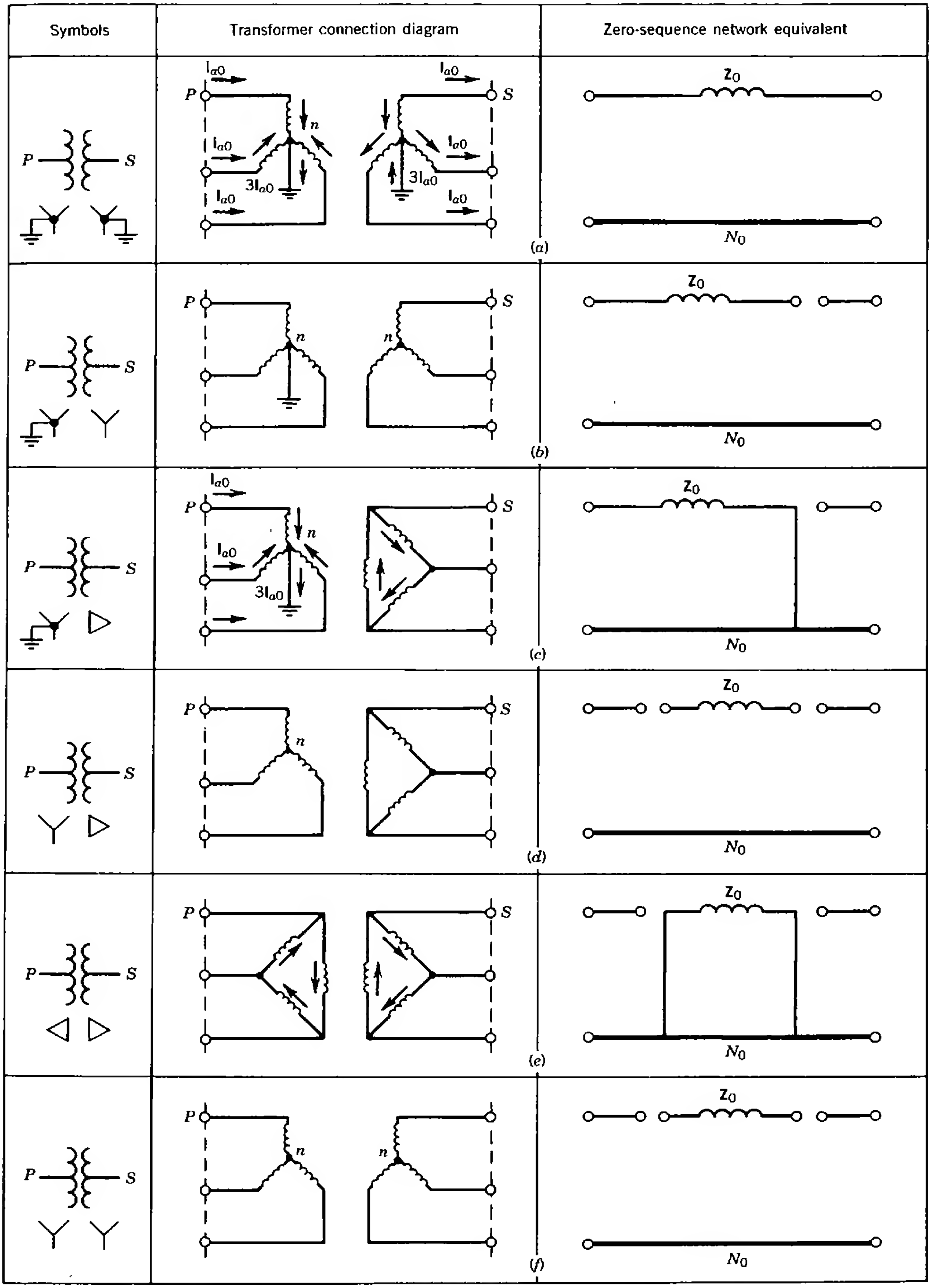


Figure 3.10. Zero-sequence network equivalents of three-phase transformer banks made of three identical single-phase transformers with two windings.

the connection is *ungrounded*, the $\mathbf{Z}_n$ is infinite, and therefore $3\mathbf{Z}_n$ should be replaced with an open circuit. It is interesting to observe that the type of grounding only affects the zero-sequence network, not the positive- and negative-sequence networks.

It is interesting to note that there is no path for the flow of zero-sequence current in a wye-grounded–wye-connected three-phase transformer bank, as shown in Figure 3.10(*b*). This is because there is no zero-sequence current in any given winding on the wye side of the transformer bank since it has an ungrounded wye connection. Therefore, because of the lack of equal and

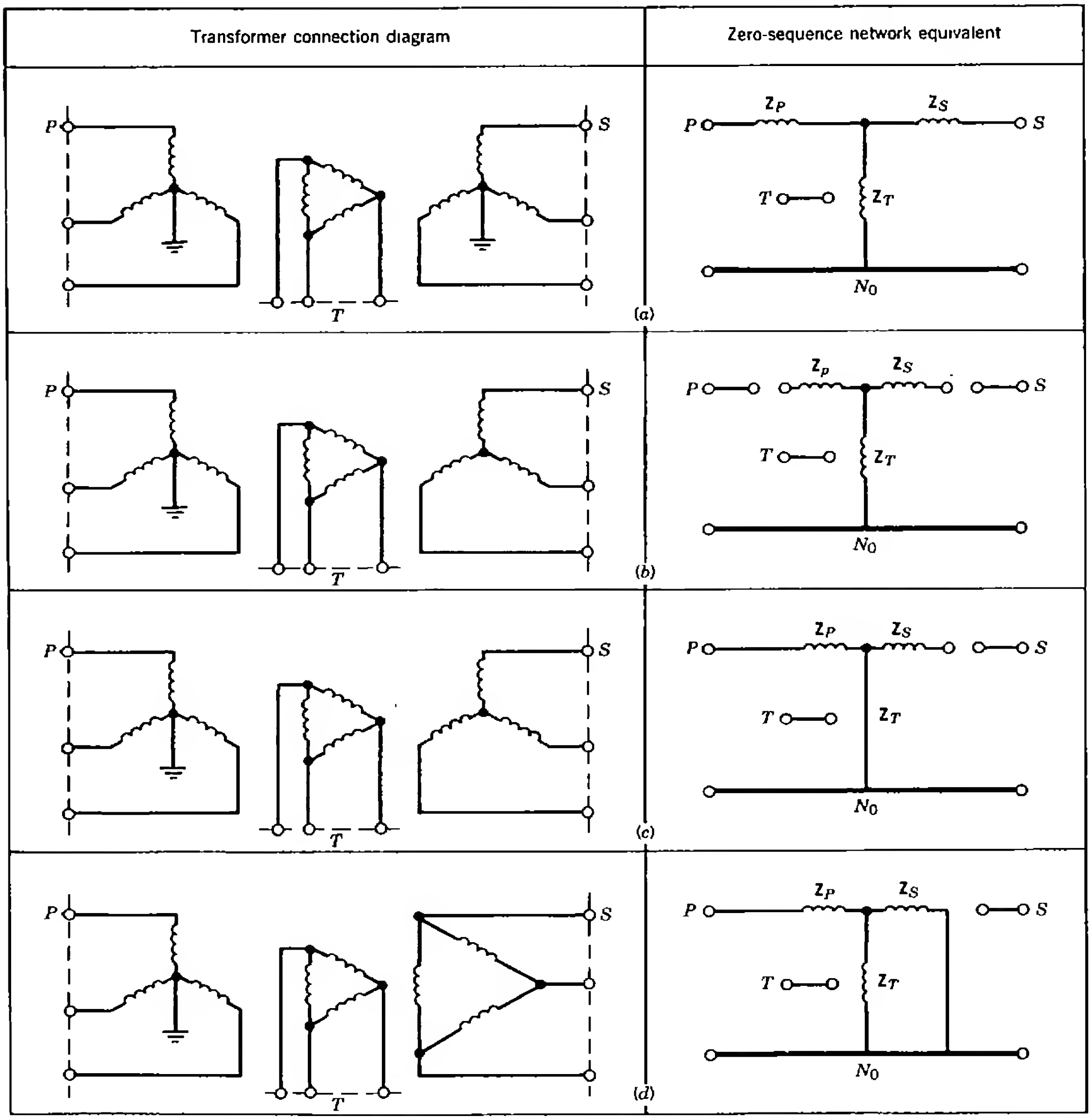


Figure 3.11. Zero-sequence network equivalents of three-phase transformer banks made of three identical single-phase transformers with three windings.

TABLE 3.4 System Data for Example 3.7

Network Component	MVA Rating	Voltage Rating (kV)	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	200	20	0.2	0.14	0.06
G_2	200	13.2	0.2	0.14	0.06
T_1	200	20/230	0.2	0.2	0.2
T_2	200	13.2/230	0.3	0.3	0.3
T_3	200	20/230	0.25	0.25	0.25
T_4	200	13.2/230	0.35	0.35	0.35
TL_{23}	200	230	0.15	0.15	0.3
TL_{56}	200	230	0.22	0.22	0.5

opposite ampere turns in the wye side of the transformer bank, there cannot be any zero-sequence current in the corresponding winding on the wye-grounded side of the transformer, with the exception of a negligible small magnetizing current.

Figure 3.11 shows zero-sequence network equivalents of three-phase transformer banks made of three identical single-phase transformers with three windings. The impedances of the three-winding transformer between primary, secondary, and tertiary terminals, indicated by *P*, *S*, and *T*, respectively, taken two at a time with the other winding open, are $\mathbf{Z}_{PS}$, $\mathbf{Z}_{PT}$, and $\mathbf{Z}_{ST}$, the subscripts indicating the terminals between which the impedances are measured. Note that only the wye–wye connection with delta tertiary, shown in Figure 3.11(*a*), permits zero-sequence current to flow in from either wye line (as long as the neutrals are grounded).

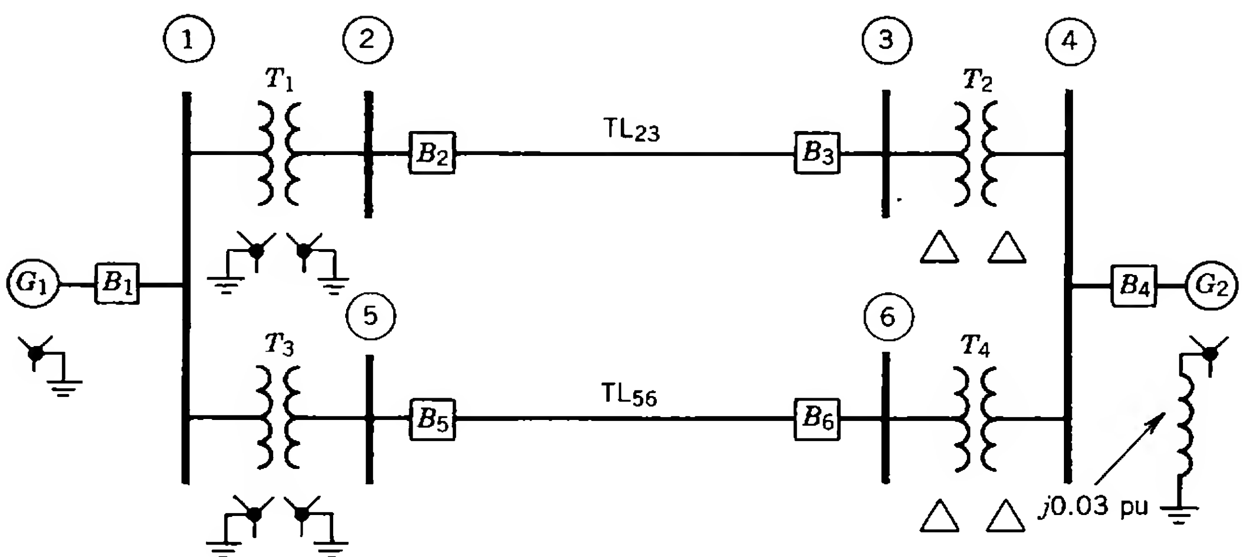
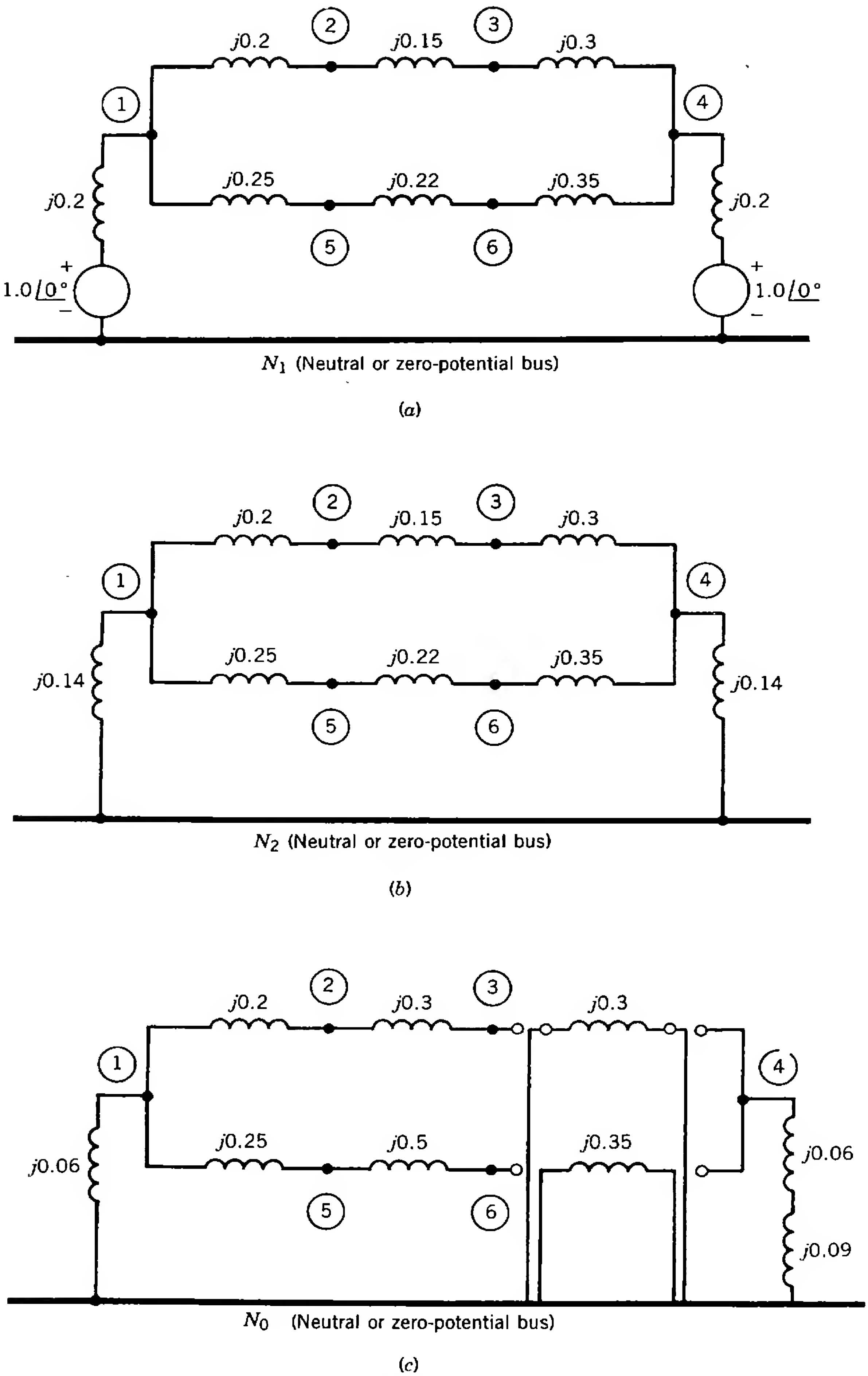


Figure 3.12



j0.2
j0.15
j0.3
j0.25
j0.22
j0.35
j0.2
j0.2
1.0/0°
1.0/0°
N1 (Neutral or zero-potential bus)
(a)
j0.2
j0.15
j0.3
j0.25
j0.22
j0.35
j0.14
j0.14
N2 (Neutral or zero-potential bus)
(b)
j0.2
j0.3
j0.3
j0.25
j0.5
j0.35
j0.06
j0.06
j0.09
N0 (Neutral or zero-potential bus)
(c)

Figure 3.13

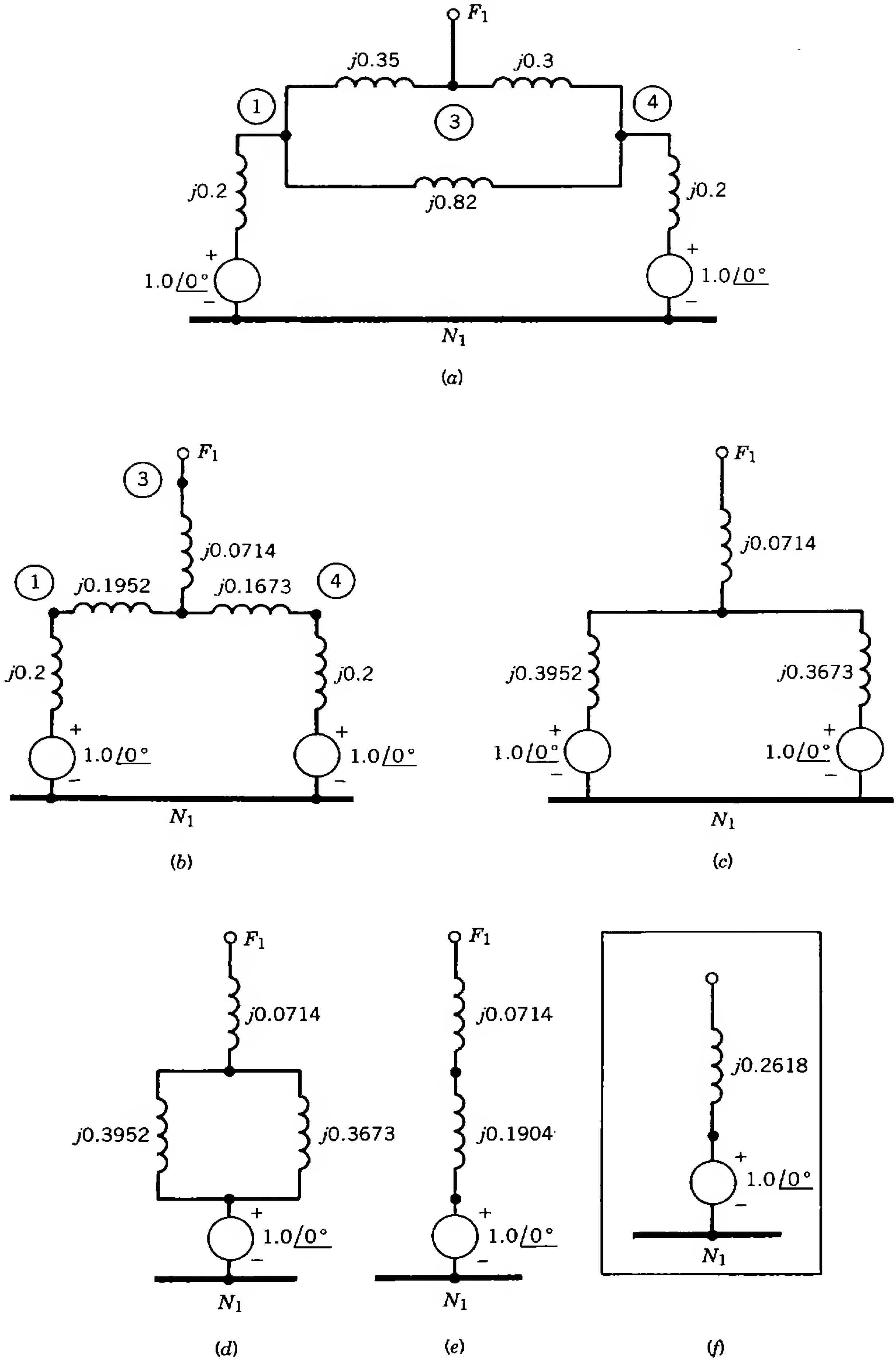

(a)
F1
j0.35
j0.3
1
3
4
j0.82
j0.2
j0.2
1.0/0°
1.0/0°
N1
(b)
F1
3
j0.0714
1
j0.1952
j0.1673
4
j0.2
j0.2
1.0/0°
1.0/0°
N1
(c)
F1
j0.0714
j0.3952
j0.3673
1.0/0°
1.0/0°
N1
(d)
F1
j0.0714
j0.3952
j0.3673
1.0/0°
N1
(e)
F1
j0.0714
j0.1904
1.0/0°
N1
(f)
j0.2618
1.0/0°
N1

Figure 3.14

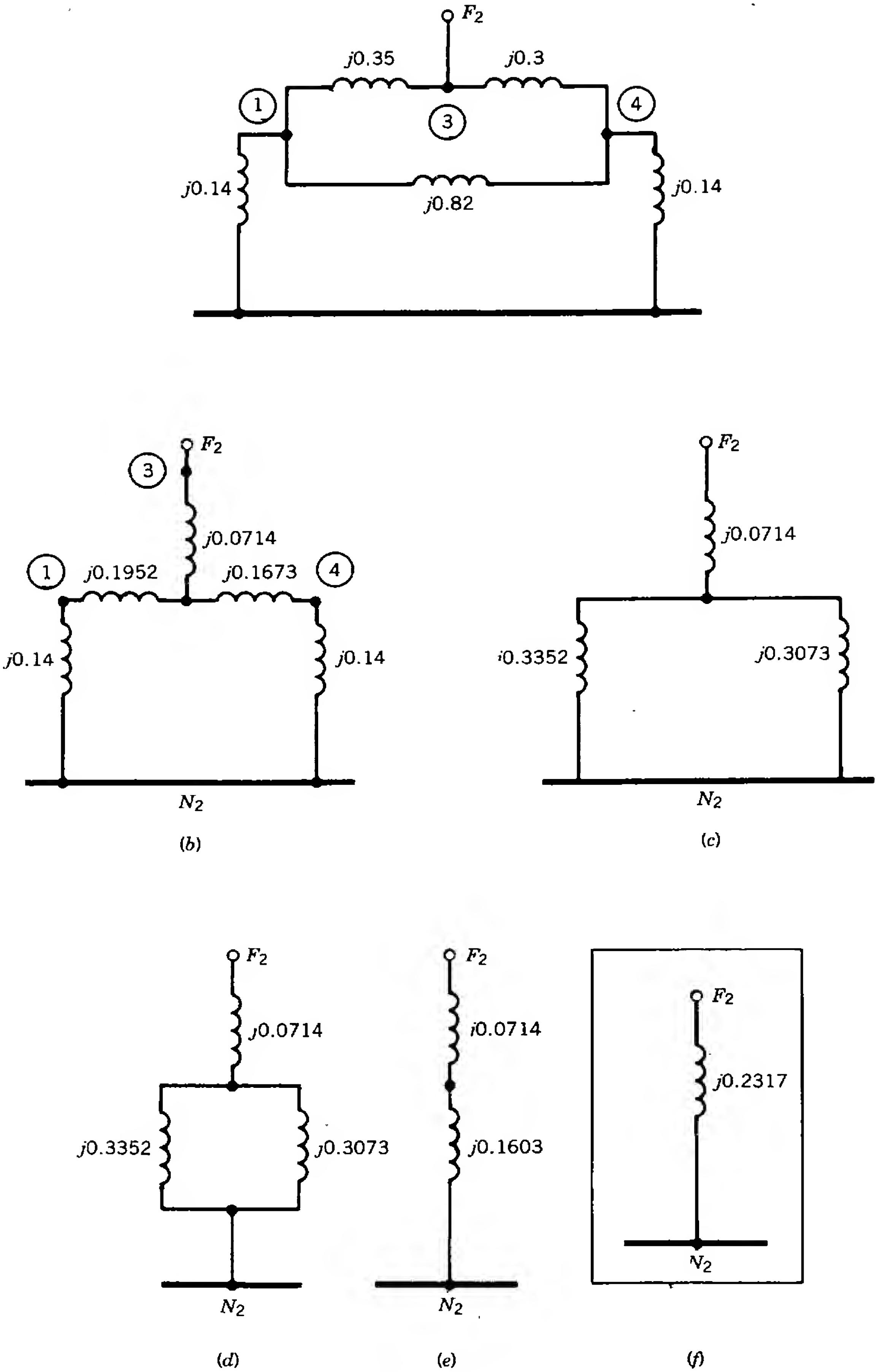

F2
j0.35
j0.3
1
3
4
j0.14
j0.82
j0.14
F2
3
j0.0714
1
j0.1952
j0.1673
4
j0.14
j0.14
N2
(b)
F2
j0.0714
i0.3352
j0.3073
N2
(c)
F2
j0.0714
j0.3352
j0.3073
N2
(d)
F2
i0.0714
j0.1603
N2
(e)
F2
j0.2317
N2
(f)

Figure 3.15

EXAMPLE 3.7

Consider the power system shown in Figure 3.12 and the associated data given in Table 3.4. Assume that each three-phase transformer bank is made of three single-phase transformers. Do the following:

(a) Draw the corresponding positive-sequence network.

(b) Draw the corresponding negative-sequence network.

(c) Draw the corresponding zero-sequence network.

Solution

(a) The positive-sequence network is shown in Figure 3.13(*a*).

(b) The negative-sequence network is shown in Figure 3.13(*b*).

(c) The zero-sequence network is shown in Figure 3.13(*c*).

EXAMPLE 3.8

Consider the power system given in Example 3.7 and assume that there is a fault on bus 3. Reduce the sequence networks drawn in Example 3.7 to their Thévenin equivalents "looking in" at bus 3.

(a) Show the steps of the positive-sequence network reduction.

(b) Show the steps of the negative-sequence network reduction.

(c) Show the steps of the zero-sequence network reduction.

Solution

(a) Figure 3.14 shows the steps of the positive-sequence network reduction.

(b) Figure 3.15 shows the steps of the negative-sequence network reduction.

(c) Figure 3.16 shows the steps of the zero-sequence network reduction.

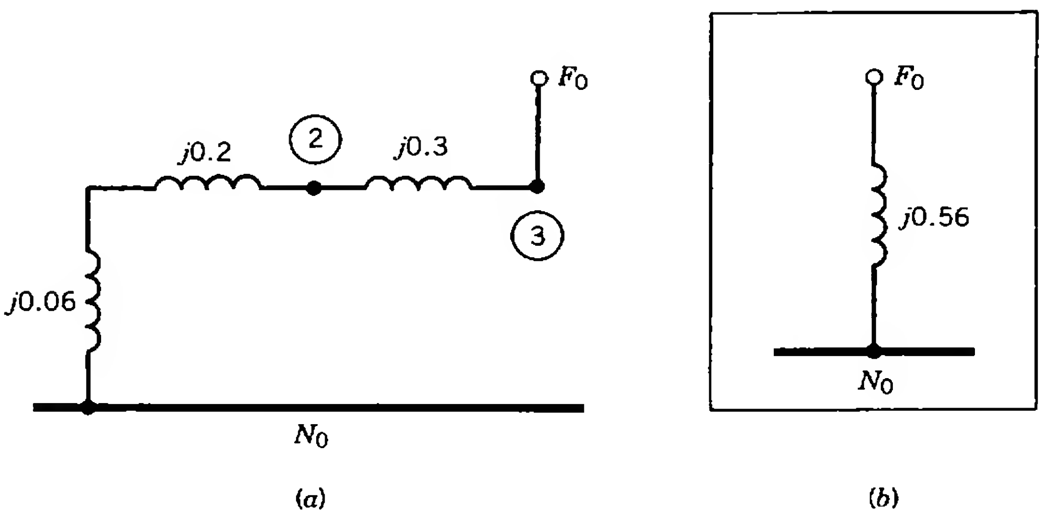


Figure 3.16

3.11 ANALYSIS OF UNBALANCED FAULTS

Most of the faults that occur on power systems are not the balanced (i.e., symmetrical) three-phase faults but the unbalanced (i.e., unsymmetrical) faults, specifically the single line-to-ground faults. For example, reference 5 gives the typical frequency of occurrence for the three-phase, single line-to-ground, line-to-line, and double line-to-ground faults as 5, 70, 15, and 10 percent, respectively.

In general, the three-phase fault is considered to be the most severe one. However, it is possible that the single line-to-ground fault may be more severe than the three-phase fault under two circumstances: (1) the generators involved in the fault have solidly grounded neutrals or low-impedance neutral impedances and (2) it occurs on the wye-grounded side of delta–wye-grounded transformer banks. The line-to-line fault current is about 86.6 percent of the three-phase fault current.

The faults can be categorized as the shunt faults (short circuits), series faults (open conductor), and simultaneous faults (having more than one fault occurring at the same time). The unbalanced faults can be easily solved by using the symmetrical components of an unbalanced system of currents or voltages. Therefore, an unbalanced system can be converted to three fictitious networks: the positive-sequence (the only one that has a driving voltage), the negative-sequence, and the zero-sequence networks interconnected to each other in a particular fashion depending on the fault type involved. In this book only shunt faults are reviewed.†

3.12 SHUNT FAULTS

The voltage to ground of phase a at the fault point F before the fault occurred is $\mathbf{V}_F$, and it is usually selected as $1.0\,\underline{/0^\circ}$ pu. However, it is possible to have a $\mathbf{V}_F$ value that is not $1.0\,\underline{/0^\circ}$ pu. If so, Table 3.5 [8] gives formulas to calculate the fault currents and voltages at the fault point F and their corresponding symmetrical components for various types of faults. Note that the positive-, negative-, and zero-sequence impedances are viewed from the fault point as $\mathbf{Z}_1$, $\mathbf{Z}_2$, and $\mathbf{Z}_0$, respectively. In the table, $\mathbf{Z}_f$ is the fault impedance and $\mathbf{Z}_{\text{eq}}$ is the equivalent impedance to replace the fault in the positive-sequence network. Also, note that the value of the impedance $\mathbf{Z}_g$ is zero in Table 3.5.

3.12.1 Single Line-to-Ground Fault

In general, the single line-to-ground (SLG) fault on a transmission system occurs when one conductor falls to ground or contacts the neutral wire.

† For other fault types, see Gönen [7].

TABLE 3.5 Fault Currents and Voltages at Fault Point F and their Corresponding Symmetrical Components for Various Types of Faults

	Three-Phase Fault through Three-Phase Fault Impedance, $\mathbf{Z}_f$	Line-to-Line, Phases b and c Shorted through Fault Impedance, $\mathbf{Z}_f$	Line-to-Ground Fault, Phase a Grounded through Fault Impedance, $\mathbf{Z}_f$	Double Line-to-Ground Fault, Phases b and c Shorted, Then Grounded through Fault Impedance, $\mathbf{Z}_f$
$\mathbf{I}_{a1}$	$\mathbf{I}_{a1}=\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$\mathbf{I}_{a1}=-\mathbf{I}_{a2}=\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$\mathbf{I}_{a1}=\mathbf{I}_{a2}=\mathbf{I}_{a0}$ $=\dfrac{\mathbf{V}_f}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$\mathbf{I}_{a1}=-(\mathbf{I}_{a2}+\mathbf{I}_{a0})$ $=\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\dfrac{\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}}$
$\mathbf{I}_{a2}$	$\mathbf{I}_{a2}=0$	$\mathbf{I}_{a2}=-\mathbf{I}_{a1}$	$\mathbf{I}_{a2}=\mathbf{I}_{a1}$	$\mathbf{I}_{a2}=-\mathbf{I}_{a1}\dfrac{\mathbf{Z}_0+3\mathbf{Z}_f}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{I}_{a0}$	$\mathbf{I}_{a0}=0$	$\mathbf{I}_{a0}=0$	$\mathbf{I}_{a0}=\mathbf{I}_{a1}$	$\mathbf{I}_{a0}=-\mathbf{I}_{a1}\dfrac{\mathbf{Z}_2}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{V}_{a1}$	$\mathbf{V}_{a1}=\mathbf{I}_{a1}\mathbf{Z}_f$	$\mathbf{V}_{a1}=\mathbf{V}_{a2}+\mathbf{I}_{a1}\mathbf{Z}_f$ $=\mathbf{I}_{a1}(\mathbf{Z}_2+\mathbf{Z}_f)$	$\mathbf{V}_{a1}=-(\mathbf{V}_{a2}+\mathbf{V}_{a2}+\mathbf{V}_{a0})+\mathbf{I}_{a1}(3\mathbf{Z}_f)$ $=\mathbf{I}_{a1}(\mathbf{Z}_0+\mathbf{Z}_2+3\mathbf{Z}_f)$	$\mathbf{V}_{a1}=\mathbf{V}_{a2}=\mathbf{V}_{a0}-3\mathbf{I}_{a0}\mathbf{Z}_f$ $=\mathbf{I}_{a1}\dfrac{\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{V}_{a2}$	$\mathbf{V}_{a2}=0$	$\mathbf{V}_{a2}=-\mathbf{I}_{a2}\mathbf{Z}_2=\mathbf{I}_{a1}\mathbf{Z}_2$	$\mathbf{V}_{a2}=-\mathbf{I}_{a2}\mathbf{Z}_2=-\mathbf{I}_{a1}\mathbf{Z}_2$	$\mathbf{V}_{a2}=-\mathbf{I}_{a2}\mathbf{Z}_2$ $=\mathbf{I}_{a1}\dfrac{\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{V}_{a0}$	$\mathbf{V}_{a0}=0$	$\mathbf{V}_{a0}=0$	$\mathbf{V}_{a0}=-\mathbf{I}_{a0}Z0$ $=-\mathbf{I}_{a1}\mathbf{Z}_0$	$\mathbf{V}_{a0}=-\mathbf{I}_{a0}\mathbf{Z}_0$ $=\mathbf{I}_{a1}\dfrac{\mathbf{Z}_0\mathbf{Z}_2}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{Z}_{eq}$	$\mathbf{Z}_{eq}=\mathbf{Z}_f$	$\mathbf{Z}_{eq}=\mathbf{Z}_2+\mathbf{Z}_f$	$\mathbf{Z}_{eq}=\mathbf{Z}_0+\mathbf{Z}_2+3\mathbf{Z}_f$	$\mathbf{Z}_{eq}=\dfrac{\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_2+\mathbf{Z}_0+3\mathbf{Z}_f}$
$\mathbf{I}_{af}$	$\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	0	$\dfrac{3\mathbf{V}_f}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	0
$\mathbf{I}_{bf}$	$\dfrac{a^2\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$-j\sqrt{3}\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	0	$-j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{Z}_0+3\mathbf{Z}_f-\mathbf{a}\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{I}_{cf}$	$\dfrac{\mathbf{a}\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$j\sqrt{3}\dfrac{\mathbf{V}_f}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	0	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{Z}_0+3\mathbf{Z}_f-\mathbf{a}^2\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{V}_{af}$	$\mathbf{V}_f\dfrac{\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{2\mathbf{Z}_2+\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{3\mathbf{Z}_f}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{3\mathbf{Z}_2(\mathbf{Z}_0+2\mathbf{Z}_f)}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{V}_{bf}$	$\mathbf{V}_f\dfrac{\mathbf{a}^2\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{\mathbf{a}^2\mathbf{Z}_f-\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{3\mathbf{a}^2\mathbf{Z}_f-j\sqrt{3}(\mathbf{Z}_2-\mathbf{a}\mathbf{Z}_0)}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{-3\mathbf{Z}_f\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{V}_{cf}$	$\mathbf{V}_f\dfrac{\mathbf{a}\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{\mathbf{a}\mathbf{Z}_f-\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{3\mathbf{a}\mathbf{Z}f+j\sqrt{3}(\mathbf{Z}_2-a^2\mathbf{Z}_0)}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$\mathbf{V}_f\dfrac{-3\mathbf{Z}_f\mathbf{Z}_2}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{V}_{bc}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{3\mathbf{Z}_f+\mathbf{Z}_0+2\mathbf{Z}_2}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	0
$\mathbf{V}_{ca}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}^2\mathbf{Z}_f}{\mathbf{Z}_1+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}^2\mathbf{Z}_f-j\sqrt{3}\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}^2(3\mathbf{Z}_f+\mathbf{Z}_0)-\mathbf{Z}_2}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$\sqrt{3}\mathbf{V}_f\dfrac{\sqrt{3}\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$
$\mathbf{V}_{ab}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}\mathbf{Z}}{\mathbf{Z}_1+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}\mathbf{Z}_f+j\sqrt{3}\mathbf{Z}_2}{\mathbf{Z}_1+\mathbf{Z}_2+\mathbf{Z}_f}$	$j\sqrt{3}\mathbf{V}_f\dfrac{\mathbf{a}(3\mathbf{Z}_f+\mathbf{Z}_0)-\mathbf{Z}_2}{\mathbf{Z}_0+\mathbf{Z}_1+\mathbf{Z}_2+3\mathbf{Z}_f}$	$-\sqrt{3}\mathbf{V}_f\dfrac{\sqrt{3}\mathbf{Z}_2(\mathbf{Z}_0+3\mathbf{Z}_f)}{\mathbf{Z}_1\mathbf{Z}_2+(\mathbf{Z}_1+\mathbf{Z}_2)(\mathbf{Z}_0+3\mathbf{Z}_f)}$

Source: From Clarke [2].

Figure 3.17(a) shows the general representation of a SLG fault at a fault point F with a fault impedance $\mathbf{Z}_f$.[†] Usually, the fault impedance $\mathbf{Z}_f$ is ignored in fault studies. Figure 3.17(b) shows the interconnection of the resulting sequence networks. For the sake of simplicity in fault calculations, the faulted phase is usually assumed to be phase a, as shown in Figure 3.17(b). However, if the faulted phase in reality is other than phase a (e.g., phase b), the phases of the system can simply be relabeled (i.e., a, b, c becomes c, a, b) [4]. A second method involves the use of the "generalized fault diagram" of Atabekov [9] further developed by Anderson [4]. From Figure 3.17(b), it can be observed that the zero-, positive-, and negative-sequence currents are equal to each other. Therefore,

$$\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_0 + \mathbf{Z}_1 + \mathbf{Z}_2 + 3\mathbf{Z}_f} \tag{3.144}$$

Since

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \tag{3.145}$$

the fault current for phase a can be found as

$$\mathbf{I}_{af} = \mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2}$$

or

$$\mathbf{I}_{af} = 3\mathbf{I}_{a0} = 3\mathbf{I}_{a1} = 3\mathbf{I}_{a2} \tag{3.146}$$

From Figure 3.17(a),

$$\mathbf{V}_{af} = \mathbf{Z}_f \mathbf{I}_{af} \tag{3.147}$$

Substituting equation (3.145) into equation (3.147), the voltage at faulted phase a can be expressed as

$$\mathbf{V}_{af} = 3\mathbf{Z}_f \mathbf{I}_{a1} \tag{3.148}$$

but

$$\mathbf{V}_{af} = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} \tag{3.149}$$

[†] The fault impedance $\mathbf{Z}_f$ may be thought of as the impedances in the arc (in the event of having a flashover between the line and a tower), the tower, and the tower footing.

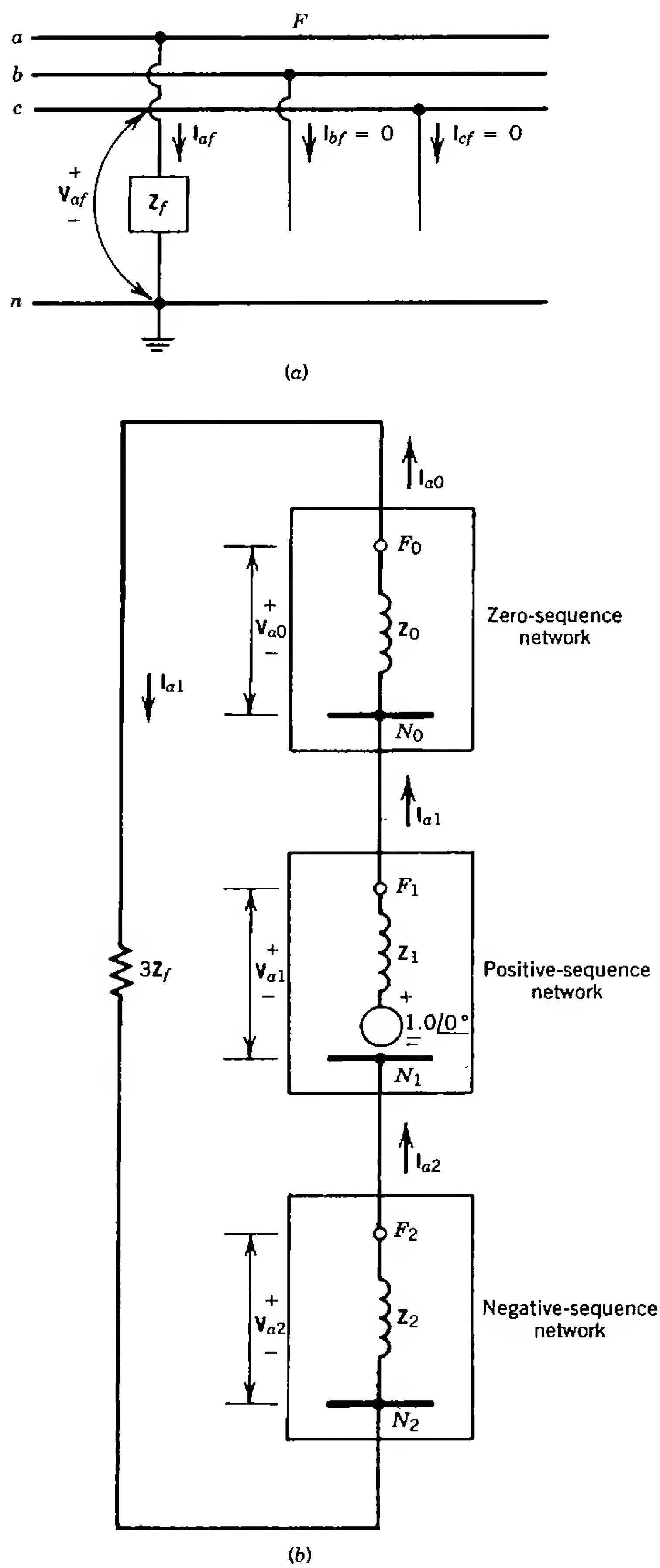


Figure 3.17. Single line-to-ground fault: (*a*) general representation; (*b*) interconnection of sequence networks.

Therefore,

$$\mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} = 3\mathbf{Z}_f \mathbf{I}_{a1} \tag{3.150}$$

which justifies the interconnection of sequence networks in series, as shown in Figure 3.17(*b*).

Once the sequence currents are found, the zero-, positive-, and negative-sequence voltages can be found from

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0\,\underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} \mathbf{Z}_0 & 0 & 0 \\ 0 & \mathbf{Z}_1 & 0 \\ 0 & 0 & \mathbf{Z}_2 \end{bmatrix} \begin{bmatrix} \mathbf{I}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{I}_{a2} \end{bmatrix} \tag{3.151}$$

as

$$\mathbf{V}_{a0} = -\mathbf{Z}_0 \mathbf{I}_{a0} \tag{3.152}$$

$$\mathbf{V}_{a1} = 1.0 - \mathbf{Z}_1 \mathbf{I}_{a1} \tag{3.153}$$

$$\mathbf{V}_{a2} = -\mathbf{Z}_2 \mathbf{I}_{a2} \tag{3.154}$$

In the event of having a SLG fault on phase *b* or *c*, the voltages related to the known phase *a* voltage components can be found from

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} \tag{3.155}$$

as

$$\mathbf{V}_{bf} = \mathbf{V}_{a0} + \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} \tag{3.156}$$

and

$$\mathbf{V}_{cf} = \mathbf{V}_{a0} + \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} \tag{3.157}$$

EXAMPLE 3.9

Consider the system described in Examples 3.7 and 3.8 and assume that there is a SLG fault, involving phase *a*, and that the fault impedance is $5 + j0\ \Omega$. Also assume that $\mathbf{Z}_0$ and $\mathbf{Z}_2$ are $j0.56$ and $j0.3619$ pu, respectively.

(a) Show the interconnection of the corresponding equivalent sequence networks.
(b) Determine the sequence and phase currents.
(c) Determine the sequence and phase voltages.
(d) Determine the line-to-line voltages.

Solution

(a) Figure 3.18 shows the interconnection of the resulting equivalent sequence networks.

(b) The impedance base on the 230-kV line is

$$Z_B = \frac{230^2}{200} = 264.5\ \Omega$$

Therefore,

$$\mathbf{Z}_f = \frac{5\ \Omega}{264.5\ \Omega} = 0.0189\ \text{pu}$$

Thus, the sequence currents and the phase currents are

$$\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \frac{1.0\angle 0^\circ}{\mathbf{Z}_0 + \mathbf{Z}_1 + \mathbf{Z}_2 + 3\mathbf{Z}_f}$$

$$= \frac{1.0\angle 0^\circ}{j0.56 + j0.2618 + j0.3619 + 0.0567} = 0.8438\angle -87.3^\circ\ \text{pu}$$

and

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0.8438\angle -87.3^\circ \\ 0.8438\angle -87.3^\circ \\ 0.8438\angle -87.3^\circ \end{bmatrix} = \begin{bmatrix} 2.5314\angle -87.3^\circ \\ 0 \\ 0 \end{bmatrix} \text{pu}$$

(c) The sequence and phase voltages are

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{I}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.56 & 0 & 0 \\ 0 & j0.2618 & 0 \\ 0 & 0 & j0.3619 \end{bmatrix} \begin{bmatrix} 0.8438\angle -87.3^\circ \\ 0.8438\angle -87.3^\circ \\ 0.8438\angle -87.3^\circ \end{bmatrix}$$

$$= \begin{bmatrix} 0.4725\angle -177.3^\circ \\ 0.7794\angle -0.8^\circ \\ 0.3054\angle -177.3^\circ \end{bmatrix} \quad \text{pu}$$

and

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0.4725\angle -177.3^\circ \\ 0.7794\angle -0.8^\circ \\ 0.3054\angle -177.3^\circ \end{bmatrix} = \begin{bmatrix} 0.0478\angle -87.3^\circ \\ 0.7514\angle -160^\circ \\ 0.7621\angle 159.1^\circ \end{bmatrix} \quad \text{pu}$$

(d) The line-to-line voltages at the fault point are

$$\mathbf{V}_{abf} = \mathbf{V}_{af} - \mathbf{V}_{bf}$$

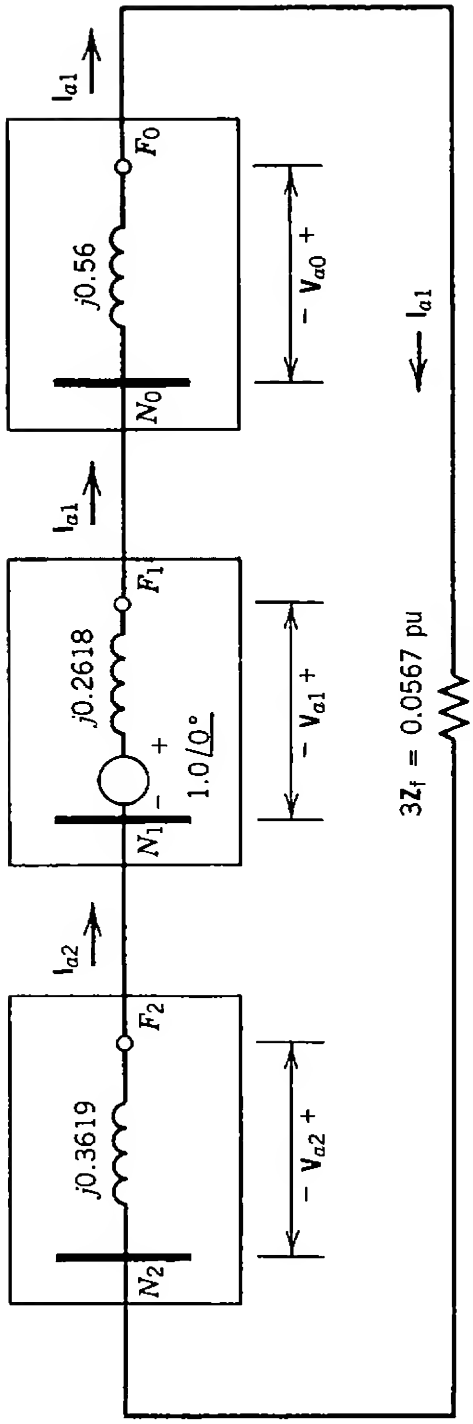
$$= 0.0478\angle -87.3^\circ - 0.7514\angle -160^\circ = 0.7084 + j0.2093$$

$$= 07387\angle 16.5^\circ\ \text{pu}$$

$$\mathbf{V}_{bcf} = \mathbf{V}_{bf} - \mathbf{V}_{cf}$$

$$= 0.7514\angle -160^\circ - 0.7621\angle 159.1^\circ = 0.0059 - j0.0149$$

$$= 0.7825\angle 155.9^\circ\ \text{pu}$$



I_{a1}
F_0
$j0.56$
N_0
$- V_{a0} +$
I_{a1}
I_{a1}
F_1
$j0.2618$
$1.0\angle 0°$
N_1
$- V_{a1} +$
$3Z_f = 0.0567$ pu
I_{a2}
F_2
$j0.3619$
N_2
$- V_{a2} +$

Figure 3.18

$$\mathbf{V}_{caf} = \mathbf{V}_{cf} - \mathbf{V}_{af}$$
$$= 0.7621\,\underline{/159.1^\circ} - 0.0478\,\underline{/-87.3^\circ} = -0.7143 + j0.3196$$
$$= 0.7443\,\underline{/162.5^\circ}\text{ pu}$$

EXAMPLE 3.10

Consider the system given in Figure 3.19(*a*) and assume that the given impedance values are based on the same megavoltampere value. The two three-phase transformer banks are made of three single-phase transformers. Assume that there is a SLG fault, involving phase *a*, at the middle of the transmission line TL_{23}, as shown in the figure.

(a) Draw the corresponding positive-, negative-, and zero-sequence networks, without reducing them, and their corresponding interconnections.
(b) Determine the sequence currents at fault point *F*.
(c) Determine the sequence currents at the terminals of generator G_1.
(d) Determine the phase currents at the terminals of generator G_1.
(e) Determine the sequence voltages at the terminals of generator G_1.
(f) Determine the phase voltages at the terminals of generator G_1.
(g) Repeat parts (c)–(f) for generator G_2.

Solution

(a) Figure 3.19(*b*) shows the corresponding sequence networks.
(b) The sequence currents at fault point *F* are

$$\mathbf{I}_{a0} = \mathbf{I}_{a1} = \mathbf{I}_{a2} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_0 + \mathbf{Z}_1 + \mathbf{Z}_3}$$
$$= \frac{1.0\,\underline{/0^\circ}}{j0.2619 + j0.25 + j0.25} = -j1.3125\text{ pu}$$

(c) Therefore, the sequence current contributions of generator G_1 can be found by symmetry as

$$\mathbf{I}_{a1,G_1} = \tfrac{1}{2}\mathbf{I}_{a1} = -j0.6563\text{ pu}$$

and

$$\mathbf{I}_{a2,G1} = \tfrac{1}{2}\mathbf{I}_{a2} = -j0.6563\text{ pu}$$

and by current division,

$$\mathbf{I}_{a0,G_1} = \frac{0.5}{0.55 + 0.5}(\mathbf{I}_{a0}) = -j0.6250\text{ pu}$$

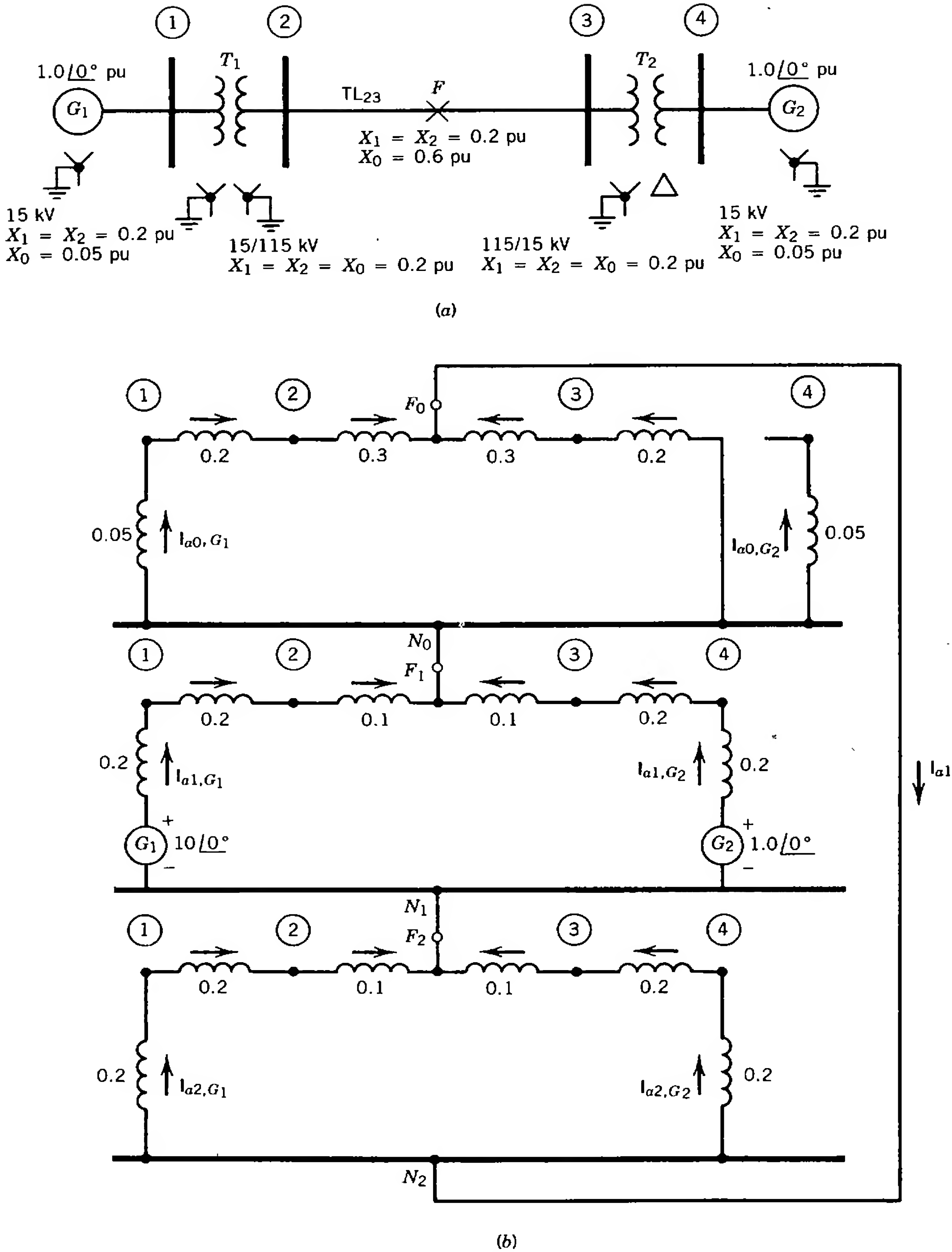

1.0/0° pu
G_1
T_1
TL23
F
$X_1 = X_2 = 0.2$ pu
$X_0 = 0.6$ pu
T_2
1.0/0° pu
G_2
15 kV
$X_1 = X_2 = 0.2$ pu
$X_0 = 0.05$ pu
15/115 kV
$X_1 = X_2 = X_0 = 0.2$ pu
115/15 kV
$X_1 = X_2 = X_0 = 0.2$ pu
15 kV
$X_1 = X_2 = 0.2$ pu
$X_0 = 0.05$ pu
(a)
F_0
I_{a0,G_1}
I_{a0,G_2}
N_0
F_1
I_{a1,G_1}
I_{a1,G_2}
10/0°
1.0/0°
I_{a1}
N_1
F_2
I_{a2,G_1}
I_{a2,G_2}
N_2
(b)

Figure 3.19

(d) The phase currents at the terminals of generator G_1 are

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0.6250 \underline{/-90^\circ} \\ 0.6565 \underline{/-90^\circ} \\ 0.6563 \underline{/-90^\circ} \end{bmatrix} = \begin{bmatrix} 1.9376 \underline{/-90^\circ} \\ 0.0313 \underline{/90^\circ} \\ 0.0313 \underline{/90^\circ} \end{bmatrix} \text{ pu}$$

(e) The sequence voltages at the terminals of generator G_1 are

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1.0 \underline{/0^\circ} \\ 0 \end{bmatrix} - \begin{bmatrix} j0.2619 & 0 & 0 \\ 0 & j0.25 & 0 \\ 0 & 0 & j0.25 \end{bmatrix} \begin{bmatrix} 0.6563 \underline{/-90^\circ} \\ 0.6563 \underline{/-90^\circ} \\ 0.6563 \underline{/-90^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 0.1637 \underline{/180^\circ} \\ 0.8434 \underline{/0^\circ} \\ 0.1641 \underline{/180^\circ} \end{bmatrix}$$

(f) Therefore, the phase voltages are

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0.1637 \underline{/180^\circ} \\ 0.8434 \underline{/0^\circ} \\ 0.1641 \underline{/180^\circ} \end{bmatrix} = \begin{bmatrix} 0.5156 \underline{/0^\circ} \\ 1.0073 \underline{/240^\circ} \\ 1.0073 \underline{/-60^\circ} \end{bmatrix} \text{ pu}$$

(g) Similarly, for generator G_2, by symmetry,

$$\mathbf{I}_{a1,G_2} = \tfrac{1}{2}\mathbf{I}_{a1} = -j0.6563 \text{ pu}$$

and

$$\mathbf{I}_{a2,G_2} = \tfrac{1}{2}\mathbf{I}_{a2} = -j0.6563 \text{ pu}$$

and by inspection

$$\mathbf{I}_{a0,G_2} = 0$$

However, since transformer T_2 has wye–delta connections and the U.S. Standard terminal markings provide that $\mathbf{V}_{a1(\mathrm{HV})}$ leads $\mathbf{V}_{a1(\mathrm{LV})}$ by 30° and $\mathbf{V}_{a2(\mathrm{HV})}$ lags $\mathbf{V}_{a2(\mathrm{LV})}$ by 30°, regardless of which side has the delta-connected windings, taking into account the 30° phase shifts,

$$\mathbf{I}_{a1,G_2} = 0.6563 \underline{/-90^\circ - 30^\circ} = 0.6563 \underline{/-120^\circ} \text{ pu}$$

and

$$\mathbf{I}_{a2,G_2} = 0.6563 \underline{/-90^\circ + 30^\circ} = 0.6563 \underline{/-60^\circ} \text{ pu}$$

This is because generator G_2 is on the low-voltage side of the transformer. Therefore,

$$\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.6563 \underline{/-120^\circ} \\ 0.6563 \underline{/-60^\circ} \end{bmatrix} = \begin{bmatrix} 1.1368 \underline{/-90^\circ} \\ 1.1368 \underline{/90^\circ} \\ 0 \end{bmatrix} \text{ pu}$$

The positive- and negative-sequence voltages on the G_2 side are the same as on the G_1 side. Thus,

$$\mathbf{V}_{a1} = 0.8434 \angle 0° \text{ pu}$$

$$\mathbf{V}_{a2} = 0.1641 \angle 180° \text{ pu}$$

Again, taking into account the 30° phase shifts,

$$\mathbf{V}_{a1} = 0.8434 \angle 0° - 30° = 0.8434 \angle -30° \text{ pu}$$

$$\mathbf{V}_{a2} = 0.1641 \angle 180° + 30° = 0.1641 \angle 210° \text{ pu}$$

Obviously

$$\mathbf{V}_{a0} = 0$$

Therefore, the phase voltages at the terminals of generator G_2 are

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.8434 \angle -30° \\ 0.1641 \angle 210° \end{bmatrix} = \begin{bmatrix} 0.7745 \angle -40.6° \\ 0.7745 \angle 220.6° \\ 1.0775 \angle 90° \end{bmatrix} \text{ pu}$$

3.12.2 Line-to-Line Fault

In general, a line-to-line (L-L) fault on a transmission system occurs when two conductors are short-circuited.† Figure 3.20(a) shows the general representation of a line-to-line fault at fault point F with a fault impedance $\mathbf{Z}_f$. Figure 3.20(b) shows the interconnection of resulting sequence networks. It is assumed, for the sake of symmetry, that the line-to-line fault is between phases b and c. It can be observed from Figure 3.20(a) that

$$\mathbf{I}_{af} = 0 \tag{3.158}$$

$$\mathbf{I}_{bf} = -\mathbf{I}_{cf} \tag{3.159}$$

$$\mathbf{V}_{bc} = \mathbf{V}_b - \mathbf{V}_c = \mathbf{Z}_f \mathbf{I}_{bf} \tag{3.160}$$

From Figure 3.20(b), the sequence currents can be found as

$$\mathbf{I}_{a0} = 0 \tag{3.161}$$

$$\mathbf{I}_{a1} = -\mathbf{I}_{a2} = \frac{1.0 \angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_f} \tag{3.162}$$

If $\mathbf{Z}_f = 0$,

$$\mathbf{I}_{a1} = -\mathbf{I}_{a2} = \frac{1.0 \angle 0°}{\mathbf{Z}_1 + \mathbf{Z}_2} \tag{3.163}$$

† Note that $|\mathbf{I}_{f,\text{L-L}}| = 0.866\,|\mathbf{I}_{f\,3\Phi}|$. Therefore, if the magnitude of the three-phase fault current is known, the magnitude of the line-to-line fault current can readily be found.

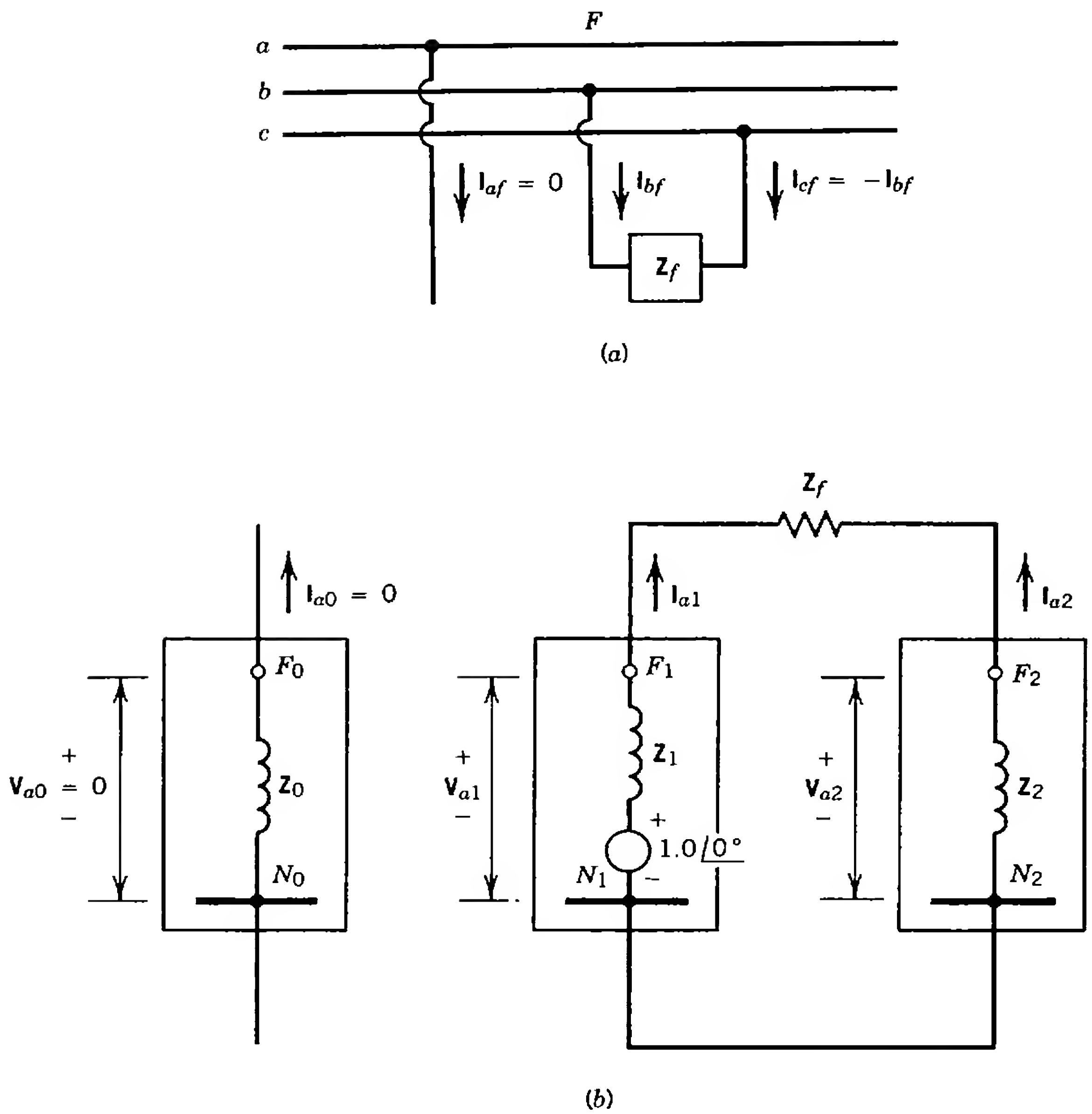


Figure 3.20. Line-to-line fault: (*a*) general representation; (*b*) interconnection of sequence networks.

Substituting equations (3.161) and (3.162) into equation (3.145), the fault currents for phases *a* and *b* can be found as

$$\mathbf{I}_{bf} = -\mathbf{I}_{cf} = \sqrt{3}\mathbf{I}_{a1} \angle -90° \tag{3.164}$$

Similarly, substituting equations (3.161) and (3.162) into equation (3.151), the sequence voltages can be found as

$$\mathbf{V}_{a0} = 0 \tag{3.165}$$

$$\mathbf{V}_{a1} = 1.0 - \mathbf{Z}_1\mathbf{I}_{a1} \tag{3.166}$$

$$\mathbf{V}_{a2} = -\mathbf{Z}_2\mathbf{I}_{a2} = \mathbf{Z}_2\mathbf{I}_{a1} \tag{3.167}$$

Also, substituting equations (3.165)–(3.167) into equation (3.155),

$$\mathbf{V}_{af} = \mathbf{V}_{a1} + \mathbf{V}_{a2} \tag{3.168}$$

or

$$\mathbf{V}_{af} = 1.0 + \mathbf{I}_{a1}(\mathbf{Z}_2 - \mathbf{Z}_1) \tag{3.169}$$

and

$$\mathbf{V}_{bf} = \mathbf{a}^2\mathbf{V}_{a1} + \mathbf{a}\mathbf{V}_{a2} \tag{3.170}$$

or

$$\mathbf{V}_{bf} = \mathbf{a}^2 + \mathbf{I}_{a1}(\mathbf{a}\mathbf{Z}_2 - \mathbf{a}^2\mathbf{Z}_1) \tag{3.171}$$

and

$$\mathbf{V}_{cf} = \mathbf{a}\mathbf{V}_{a1} + \mathbf{a}^2\mathbf{V}_{a2} \tag{3.172}$$

or

$$\mathbf{V}_{cf} = \mathbf{a} + \mathbf{I}_{a1}(\mathbf{a}^2\mathbf{Z}_2 - \mathbf{a}\mathbf{Z}_1) \tag{3.173}$$

Therefore, the line-to-line voltages can be expressed as

$$\mathbf{V}_{ab} = \mathbf{V}_{af} - \mathbf{V}_{bf} \tag{3.174}$$

or

$$\mathbf{V}_{ab} = \sqrt{3}(\mathbf{V}_{a1}\,\underline{/30^\circ} + \mathbf{V}_{a2}\,\underline{/-30^\circ}) \tag{3.175}$$

and

$$\mathbf{V}_{bc} = \mathbf{V}_{bf} - \mathbf{V}_{cf} \tag{3.176}$$

or

$$\mathbf{V}_{bc} = \sqrt{3}(\mathbf{V}_{a1}\,\underline{/-90^\circ} + \mathbf{V}_{a2}\,\underline{/90^\circ}) \tag{3.177}$$

and

$$\mathbf{V}_{ca} = \mathbf{V}_{cf} - \mathbf{V}_{a} \tag{3.178}$$

or

$$\mathbf{V}_{ca} = \sqrt{3}(\mathbf{V}_{a1}\,\underline{/150^\circ} + \mathbf{V}_{a2}\,\underline{/-150^\circ}) \tag{3.179}$$

EXAMPLE 3.11

Repeat Example 3.9 assuming that there is a line-to-line fault, involving phases b and c, at bus 3.

Solution

(a) Figure 3.21 shows the interconnection of the resulting equivalent sequence networks.

(b) The sequence and the phase currents are

$$\mathbf{I}_{a0} = 0$$

$$\mathbf{I}_{a1} = -\mathbf{I}_{a2} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_2 + \mathbf{Z}_f}$$

$$= \frac{1.0\,\underline{/0^\circ}}{j0.2618 + j0.3619 + 0.0189} = 1.6026\,\underline{/-88.3^\circ}\text{ pu}$$

and

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 1.6026\,\underline{/-88.3^\circ} \\ 1.6026\,\underline{/91.7^\circ} \end{bmatrix} = \begin{bmatrix} 0 \\ 2.7758\,\underline{/-178.3^\circ} \\ 2.7758\,\underline{/1.7^\circ} \end{bmatrix} \text{ pu}$$

(c) The sequence and phase voltages are

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.5 & 0 & 0 \\ 0 & j0.2618 & 0 \\ 0 & 0 & j0.3619 \end{bmatrix} \begin{bmatrix} 0 \\ 1.6026\,\underline{/88.3^\circ} \\ 1.6026\,\underline{/91.7^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.5808\,\underline{/-1.2^\circ} \\ 0.5808\,\underline{/1.7^\circ} \end{bmatrix} \text{ pu}$$

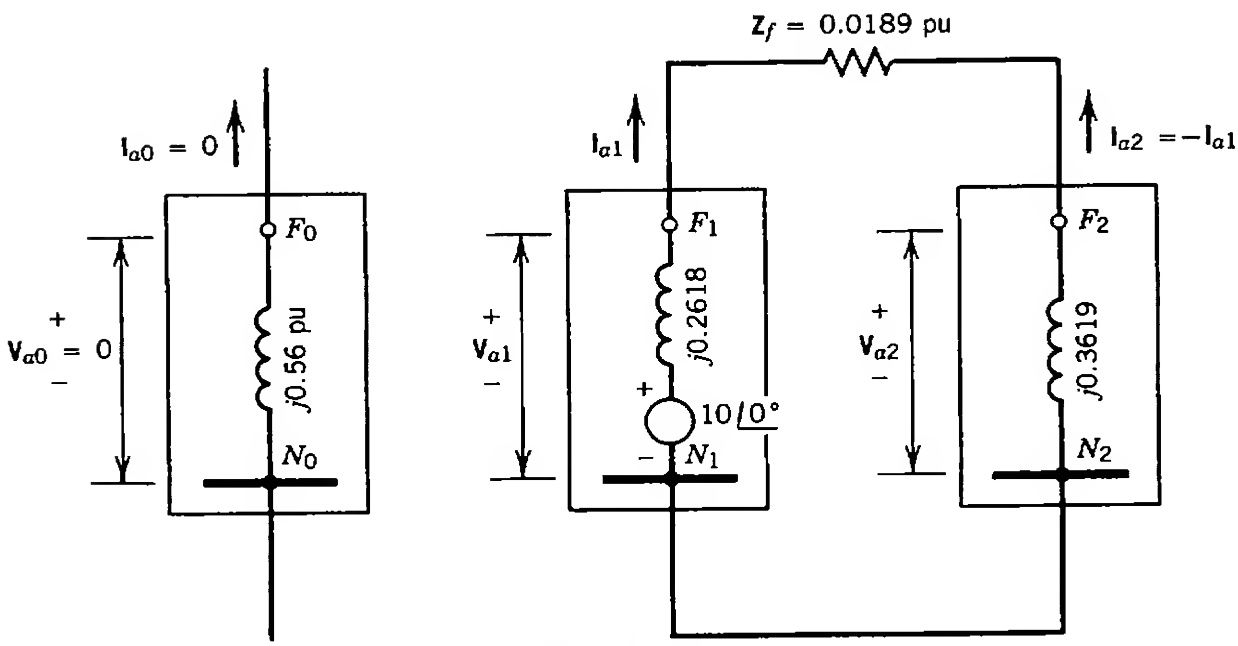


Figure 3.21

and

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.5808 \underline{/-1.2^\circ} \\ 0.5808 \underline{/1.7^\circ} \end{bmatrix} = \begin{bmatrix} 1.1603 \underline{/0.2^\circ} \\ 0.6057 \underline{/-0.2^\circ} \\ 0.5548 \underline{/-0.2^\circ} \end{bmatrix} \text{ pu}$$

(d) The line-to-line voltages at the fault point are

$$\mathbf{V}_{abf} = \mathbf{V}_{af} - \mathbf{V}_{bf} = 0.5546 + j0.0073 = 0.5546 \underline{/0.8^\circ} \text{ pu}$$

$$\mathbf{V}_{bcf} = \mathbf{V}_{bf} - \mathbf{V}_{cf} = 0.0509 - j0.0013 = 0.0509 \underline{/-1.5^\circ} \text{ pu}$$

$$\mathbf{V}_{caf} = \mathbf{V}_{cf} - \mathbf{V}_{af} = -0.6055 - j0.006 = 0.6055 \underline{/180.6^\circ} \text{ pu}$$

3.12.3 Double Line-to-Ground Fault

In general, the double line-to-ground (DLG) fault on a transmission system occurs when two conductors fall and are connected through ground or when two conductors contact the neutral of a three-phase grounded system. Figure 3.22(a) shows the general representation of a DLG fault at a fault point F with a fault impedance $\mathbf{Z}_f$ and the impedance from line to ground $\mathbf{Z}_g$ (which can be equal to zero or infinity). Figure 3.22(b) shows the interconnection of resultant sequence networks. As before, it is assumed, for the sake of symmetry, that the DLG fault is between phases b and c. It can be observed from Figure 3.22(a) that

$$\mathbf{I}_{af} = 0 \tag{3.180}$$

$$\mathbf{V}_{bf} = (\mathbf{Z}_f + \mathbf{Z}_g)\mathbf{I}_{bf} + \mathbf{Z}_g \mathbf{I}_{cf} \tag{3.181}$$

$$\mathbf{V}_{cf} = (\mathbf{Z}_f + \mathbf{Z}_g)\mathbf{I}_{cf} + \mathbf{Z}_g \mathbf{I}_{bf} \tag{3.182}$$

From Figure 3.22(b), the positive-sequence currents can be found as

$$\mathbf{I}_{a1} = \frac{1.0 \underline{/0^\circ}}{(\mathbf{Z}_1 + \mathbf{Z}_f) + \dfrac{(\mathbf{Z}_2 + \mathbf{Z}_f)(\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g)}{\mathbf{Z}_0 + \mathbf{Z}_2 + 2\mathbf{Z}_f + 3\mathbf{Z}_g}} \tag{3.183}$$

The negative- and zero-sequence currents can be found, by using current division, as

$$\mathbf{I}_{a2} = -\left[\frac{(\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g)}{(\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g) + (\mathbf{Z}_2 + \mathbf{Z}_f)}\right]\mathbf{I}_{a1} \tag{3.184}$$

and

$$\mathbf{I}_{a0} = -\left[\frac{(\mathbf{Z}_2 + \mathbf{Z}_f)}{(\mathbf{Z}_2 + \mathbf{Z}_f) + (\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g)}\right]\mathbf{I}_{a1} \tag{3.185}$$

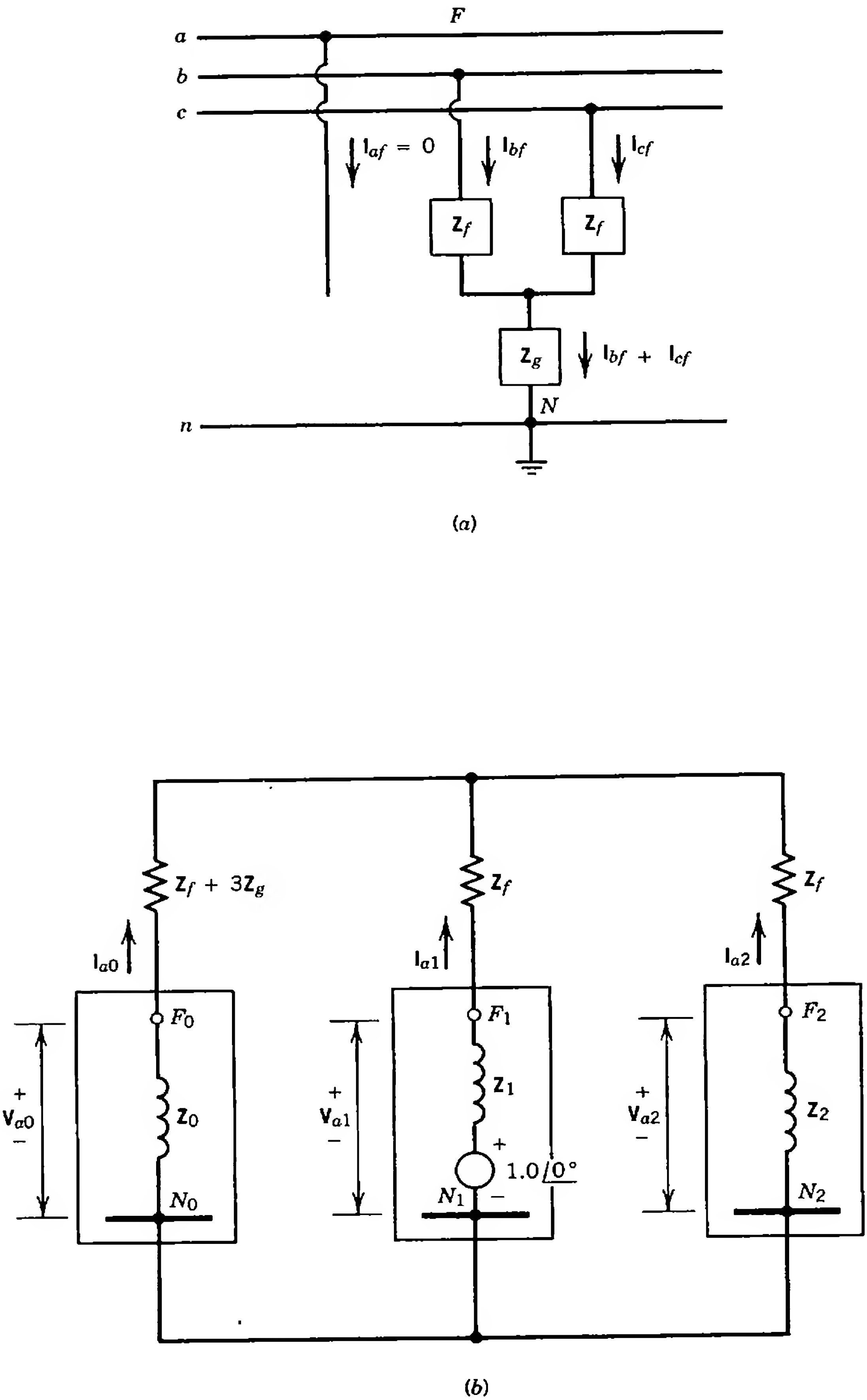


Figure 3.22. Double line-to-ground fault: (*a*) general representation; (*b*) interconnection of sequence networks.

or as an alternative method, since

$$\mathbf{I}_{af} = 0 = \mathbf{I}_{a0} + \mathbf{I}_{a1} + \mathbf{I}_{a2}$$

then if $\mathbf{I}_{a1}$ and $\mathbf{I}_{a2}$ are known,

$$\mathbf{I}_{a0} = -(\mathbf{I}_{a1} + \mathbf{I}_{a2}) \tag{3.186}$$

Note that in the event of having $\mathbf{Z}_f = 0$ and $\mathbf{Z}_g = 0$, the positive-, negative-, and zero-sequences can be expressed as

$$\mathbf{I}_{a1} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_1 + (\mathbf{Z}_0 \times \mathbf{Z}_2)/(\mathbf{Z}_0 + \mathbf{Z}_2)} \tag{3.187}$$

and

$$\mathbf{I}_{a2} = -\left[\frac{\mathbf{Z}_0}{\mathbf{Z}_0 + \mathbf{Z}_2}\right]\mathbf{I}_{a1} \tag{3.188}$$

$$\mathbf{I}_{a0} = -\left[\frac{\mathbf{Z}_2}{\mathbf{Z}_0 + \mathbf{Z}_2}\right]\mathbf{I}_{a1} \tag{3.189}$$

Note that the fault current for phase a is already known to be

$$\mathbf{I}_{af} = 0$$

the fault currents for phases a and b can be found by substituting equations (3.183)–(3.185) into equation (3.145) so that

$$\mathbf{I}_{bf} = \mathbf{I}_{a0} + \mathbf{a}^2\mathbf{I}_{a1} + \mathbf{a}\mathbf{I}_{a2} \tag{3.190}$$

and

$$\mathbf{I}_{cf} = \mathbf{I}_{a0} + \mathbf{a}\mathbf{I}_{a1} + \mathbf{a}^2\mathbf{I}_{a2} \tag{3.191}$$

It can be shown that the total fault current flowing into the neutral is

$$\mathbf{I}_n = \mathbf{I}_{bf} + \mathbf{I}_{cf} = 3\mathbf{I}_{a0} \tag{3.192}$$

The sequence voltages can be found from equation (3.151) as

$$\mathbf{V}_{a0} = -\mathbf{Z}_0\mathbf{I}_{a0} \tag{3.193}$$

$$\mathbf{V}_{a1} = 1.0 - \mathbf{Z}_1\mathbf{I}_{a1} \tag{3.194}$$

$$\mathbf{V}_{a2} = -\mathbf{Z}_2\mathbf{I}_{a2} \tag{3.195}$$

Similarly, the phase voltages can be found from equation (3.155) as

$$\mathbf{V}_{af} = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a2} \tag{3.196}$$

$$\mathbf{V}_{bf} = \mathbf{V}_{a0} + a^2\mathbf{V}_{a1} + a\mathbf{V}_{a2} \tag{3.197}$$

$$\mathbf{V}_{cf} = \mathbf{V}_{a0} + a\mathbf{V}_{a1} + a^2\mathbf{V}_{a2} \tag{3.198}$$

or, alternatively, the phase voltages $\mathbf{V}_{bf}$ and $\mathbf{V}_{cf}$ can be determined from equations (3.181) and (3.182). As before, the line-to-line voltages can be found from

$$\mathbf{V}_{ab} = \mathbf{V}_{af} - \mathbf{V}_{bf} \tag{3.199}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bf} - \mathbf{V}_{cf} \tag{3.200}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cf} - \mathbf{V}_{af} \tag{3.201}$$

Note that in the event of having $\mathbf{Z}_f = 0$ and $\mathbf{Z}_g = 0$, the sequence voltages become

$$\mathbf{V}_{a0} = \mathbf{V}_{a1} = \mathbf{V}_{a2} = 1.0 - \mathbf{Z}_1\mathbf{I}_{a1} \tag{3.202}$$

where the positive-sequence current is found by using equation (3.187). Once the sequence voltages are determined from equation (3.202), the negative- and zero-sequence currents can be determined from

$$\mathbf{I}_{a2} = -\frac{\mathbf{V}_{a2}}{\mathbf{Z}_2} \tag{3.203}$$

and

$$\mathbf{I}_{a0} = -\frac{\mathbf{V}_{a0}}{\mathbf{Z}_0} \tag{3.204}$$

Using the relationship given in equation (3.202) the resultant phase voltages can be expressed as

$$\mathbf{V}_{af} = \mathbf{V}_{a0} + \mathbf{V}_{a1} + \mathbf{V}_{a3} = 3\mathbf{V}_{a1} \tag{3.205}$$

$$\mathbf{V}_{bf} = \mathbf{V}_{cf} = 0 \tag{3.206}$$

Therefore, the line-to-line voltages become

$$\mathbf{V}_{ab} = \mathbf{V}_{af} - \mathbf{V}_{bf} = \mathbf{V}_{af} \tag{3.207}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bf} - \mathbf{V}_{cf} = 0 \tag{3.208}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cf} - \mathbf{V}_{af} = -\mathbf{V}_{af} \tag{3.209}$$

EXAMPLE 3.12

Repeat Example 3.9 assuming that there is a DLG fault with $\mathbf{Z}_f = 5\ \Omega$ and $\mathbf{Z}_g = 10\ \Omega$, involving phases b and c, at bus 3.

Solution

(a) Figure 3.23 shows the interconnection of the resulting equivalent sequence networks.

(b) Since

$$\mathbf{Z}_f + 3\mathbf{Z}_g = \frac{5+30}{264.5} = 0.1323 \text{ pu}$$

the sequence currents are

$$\mathbf{I}_{a1} = \frac{1.0\,\underline{/0^\circ}}{(\mathbf{Z}_1 + \mathbf{Z}_f) + \dfrac{(\mathbf{Z}_2 + \mathbf{Z}_f)(\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g)}{\mathbf{Z}_0 + \mathbf{Z}_2 + 2\mathbf{Z}_f + 3\mathbf{Z}_g}}$$

$$= \frac{1.0\,\underline{/0^\circ}}{(j0.2618 + 0.0189) + \dfrac{(j0.3619 + 0.0189)(j0.56 + 0.1323)}{j0.3619 + 0.0189 + j0.56 + 0.1323}}$$

$$= 2.0597\,\underline{/-84.5^\circ} \text{ pu}$$

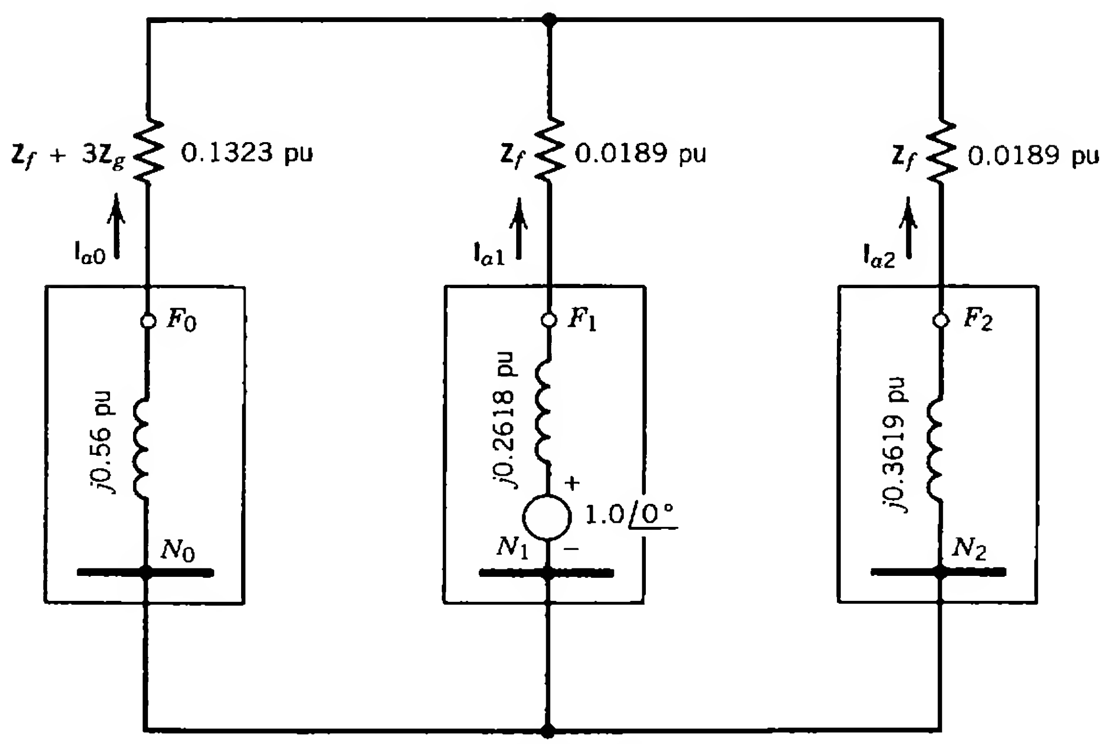


Figure 3.23

$$\mathbf{I}_{a2} = -\left[\frac{\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g}{(\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g) + (\mathbf{Z}_2 + \mathbf{Z}_f)}\right]\mathbf{I}_{a1}$$

$$= -\left[\frac{0.5754\ \underline{/76.7^\circ}}{0.9342\ \underline{/80.7^\circ}}\right](2.0597\ \underline{/-84.5^\circ})$$

$$= -1.2686\ \underline{/-88.5^\circ}\ \text{pu}$$

$$\mathbf{I}_{a0} = -\left[\frac{\mathbf{Z}_2 + \mathbf{Z}_f}{(\mathbf{Z}_2 + \mathbf{Z}_f) + (\mathbf{Z}_0 + \mathbf{Z}_f + 3\mathbf{Z}_g)}\right]\mathbf{I}_{a1}$$

$$= -\left[\frac{0.3624\ \underline{/87^\circ}}{0.9342\ \underline{/80.7^\circ}}\right](2.0597\ \underline{/-84.5^\circ})$$

$$= -0.799\ \underline{/-78.2^\circ}\ \text{pu}$$

and the phase currents are

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} -0.799\ \underline{/-78.2^\circ} \\ 2.0597\ \underline{/-84.5^\circ} \\ -1.2686\ \underline{/-88.5^\circ} \end{bmatrix} = \begin{bmatrix} 0 \\ 3.2677\ \underline{/162.7^\circ} \\ 2.9653\ \underline{/27.6^\circ} \end{bmatrix} \text{pu}$$

(c) The sequence and phase voltages are

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.56 & 0 & 0 \\ 0 & j0.2618 & 0 \\ 0 & 0 & j0.3619 \end{bmatrix} \begin{bmatrix} -0.799\ \underline{/-78.2^\circ} \\ 2.0597\ \underline{/-84.5^\circ} \\ -1.2686\ \underline{/-88.5^\circ} \end{bmatrix}$$

$$= \begin{bmatrix} 0.4474\ \underline{/11.8^\circ} \\ 0.4662\underline{/-6.4^\circ} \\ 0.4591\ \underline{/1.5^\circ} \end{bmatrix} \text{pu}$$

and

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0.4474\ \underline{/11.8^\circ} \\ 0.4662\ \underline{/-6.4^\circ} \\ 0.4591\ \underline{/1.5^\circ} \end{bmatrix} = \begin{bmatrix} 1.3611\ \underline{/2.2^\circ} \\ 0.1333\ \underline{/126.1^\circ} \\ 0.1198\ \underline{/74.4^\circ} \end{bmatrix} \text{pu}$$

(d) The line-to-line voltages at the fault point are

$$\mathbf{V}_{abf} = \mathbf{V}_{af} - \mathbf{V}_{bf} = 1.4386 - j0.555 = 1.4397\ \underline{/-2.2^\circ}\ \text{pu}$$

$$\mathbf{V}_{bcf} = \mathbf{V}_{bf} - \mathbf{V}_{cf} = -0.1107 - j0.0077 = 0.111\ \underline{/184}\ \text{pu}$$

$$\mathbf{V}_{caf} = \mathbf{V}_{cf} - \mathbf{V}_{af} = -1.3279 + j0.0632 = 1.3294\ \underline{/177.3^\circ}\ \text{pu}$$

3.12.4 Three-Phase Fault

In general, the three-phase (3Φ) fault is not an unbalanced (i.e., unsymmetrical) fault. Instead, the three-phase fault is a balanced (i.e., symmetrical) fault that could also be analyzed using symmetrical components. Figure 3.24(*a*) shows the general representation of a balanced three-phase fault at a fault point *F* with impedances $\mathbf{Z}_f$ and $\mathbf{Z}_g$. Figure 3.24(*b*) shows the lack of

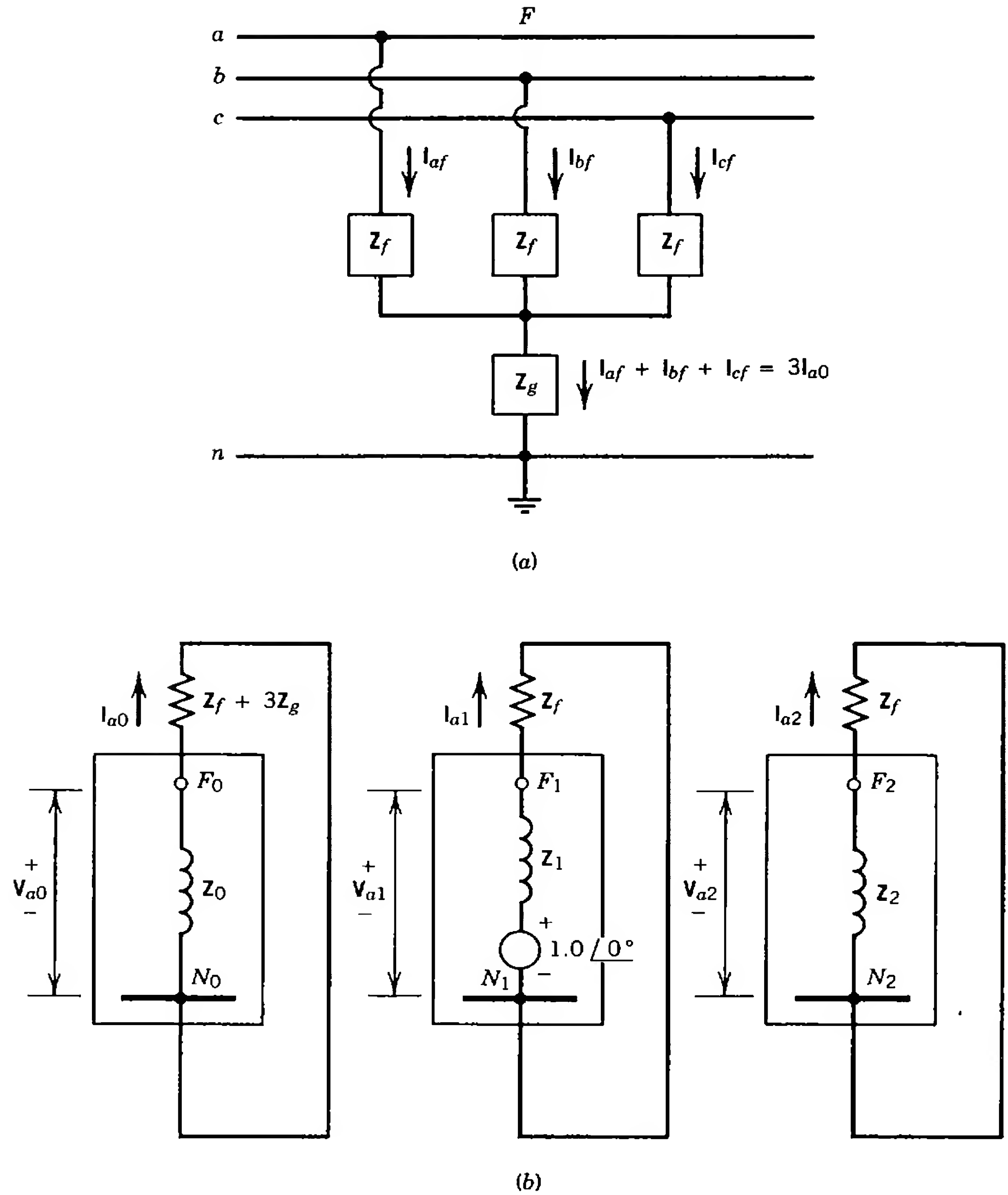


Figure 3.24. Three-phase fault: (*a*) general representation; (*b*) interconnection of sequence networks.

interconnection of resulting sequence networks. Instead, the sequence networks are short-circuited over their own fault impedances and are therefore isolated from each other. Since only the positive-sequence network is considered to have internal voltage source, the positive-, negative-, and zero-sequence currents can be expressed as

$$\mathbf{I}_{a0} = 0 \tag{3.210}$$

$$\mathbf{I}_{a2} = 0 \tag{3.211}$$

$$\mathbf{I}_{a1} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_f} \tag{3.212}$$

If the fault impedance $\mathbf{Z}_f$ is zero,

$$\mathbf{I}_{a1} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_1} \tag{3.213}$$

Substituting equations (3.210)–(3.212) into equation (3.145),

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{I}_{a1} \\ 0 \end{bmatrix} \tag{3.214}$$

from which

$$\mathbf{I}_{af} = \mathbf{I}_{a1} = \frac{1.0\,\underline{/0^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_f} \tag{3.215}$$

$$\mathbf{I}_{bf} = \mathbf{a}^2\mathbf{I}_{a1} = \frac{1.0\,\underline{/240^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_f} \tag{3.216}$$

$$\mathbf{I}_{cf} = \mathbf{a}\mathbf{I}_{a1} = \frac{1.0\,\underline{/120^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_f} \tag{3.217}$$

Since the sequence networks are short-circuited over their own fault impedances,

$$\mathbf{V}_{a0} = 0 \tag{3.218}$$

$$\mathbf{V}_{a1} = \mathbf{Z}_f\mathbf{I}_{a1} \tag{3.219}$$

$$\mathbf{V}_{a2} = 0 \tag{3.220}$$

Therefore, substituting equations (3.218)–(3.220) into equation (3.155),

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ \mathbf{V}_{a1} \\ 0 \end{bmatrix} \tag{3.221}$$

Thus,

$$\mathbf{V}_{af} = \mathbf{V}_{a1} = \mathbf{Z}_f\mathbf{I}_{a1} \tag{3.222}$$

$$\mathbf{V}_{bf} = \mathbf{a}^2\mathbf{V}_{a1} = \mathbf{Z}_f\mathbf{I}_{a1}\,\underline{/240^\circ} \tag{3.223}$$

$$\mathbf{V}_{cf} = \mathbf{a}\mathbf{V}_{a1} = \mathbf{Z}_f\mathbf{I}_{a1}\,\underline{/120^\circ} \tag{3.224}$$

Hence, the line-to-line voltages become

$$\mathbf{V}_{ab} = \mathbf{V}_{af} - \mathbf{V}_{bf} = \mathbf{V}_{a1}(1 - a^2) = \sqrt{3}\mathbf{Z}_f\mathbf{I}_{a1} \underline{/30^\circ} \tag{3.225}$$

$$\mathbf{V}_{bc} = \mathbf{V}_{bf} - \mathbf{V}_{cf} = \mathbf{V}_{a1}(a^2 - a) = \sqrt{3}\mathbf{Z}_f\mathbf{I}_{a1} \underline{/-90^\circ} \tag{3.226}$$

$$\mathbf{V}_{ca} = \mathbf{V}_{cf} - \mathbf{V}_{af} = \mathbf{V}_{a1}(a - 1) = \sqrt{3}\mathbf{Z}_f\mathbf{I}_{a1} \underline{/150^\circ} \tag{3.227}$$

Note that in the event of having $\mathbf{Z}_f = 0$,

$$\mathbf{I}_{af} = \frac{1.0 \underline{/0^\circ}}{\mathbf{Z}_1} \tag{3.228}$$

$$\mathbf{I}_{bf} = \frac{1.0 \underline{/240^\circ}}{\mathbf{Z}_1} \tag{3.229}$$

$$\mathbf{I}_{cf} = \frac{1.0 \underline{/120^\circ}}{\mathbf{Z}_1} \tag{3.230}$$

and

$$\mathbf{V}_{af} = 0 \tag{3.231}$$

$$\mathbf{V}_{bf} = 0 \tag{3.232}$$

$$\mathbf{V}_{cf} = 0 \tag{3.233}$$

and, of course,

$$\mathbf{V}_{a0} = 0 \tag{3.234}$$

$$\mathbf{V}_{a1} = 0 \tag{3.235}$$

$$\mathbf{V}_{a2} = 0 \tag{3.236}$$

EXAMPLE 3.13

Repeat Example 3.9 assuming that there is a symmetrical three-phase fault with $\mathbf{Z}_f = 5\ \Omega$ and $\mathbf{Z}_g = 10\ \Omega$ at bus 3.

Solution

(a) Figure 3.25 shows the interconnection of the resulting equivalent sequence networks.

(b) The sequence and phase currents are

$$\mathbf{I}_{a0} = \mathbf{I}_{a2} = 0$$

$$\mathbf{I}_{a1} = \frac{1.0 \underline{/0^\circ}}{\mathbf{Z}_1 + \mathbf{Z}_F} = \frac{1.0 \underline{/0^\circ}}{j0.2618 + 0.0189} = 3.8098 \underline{/-85.9^\circ} \text{ pu}$$

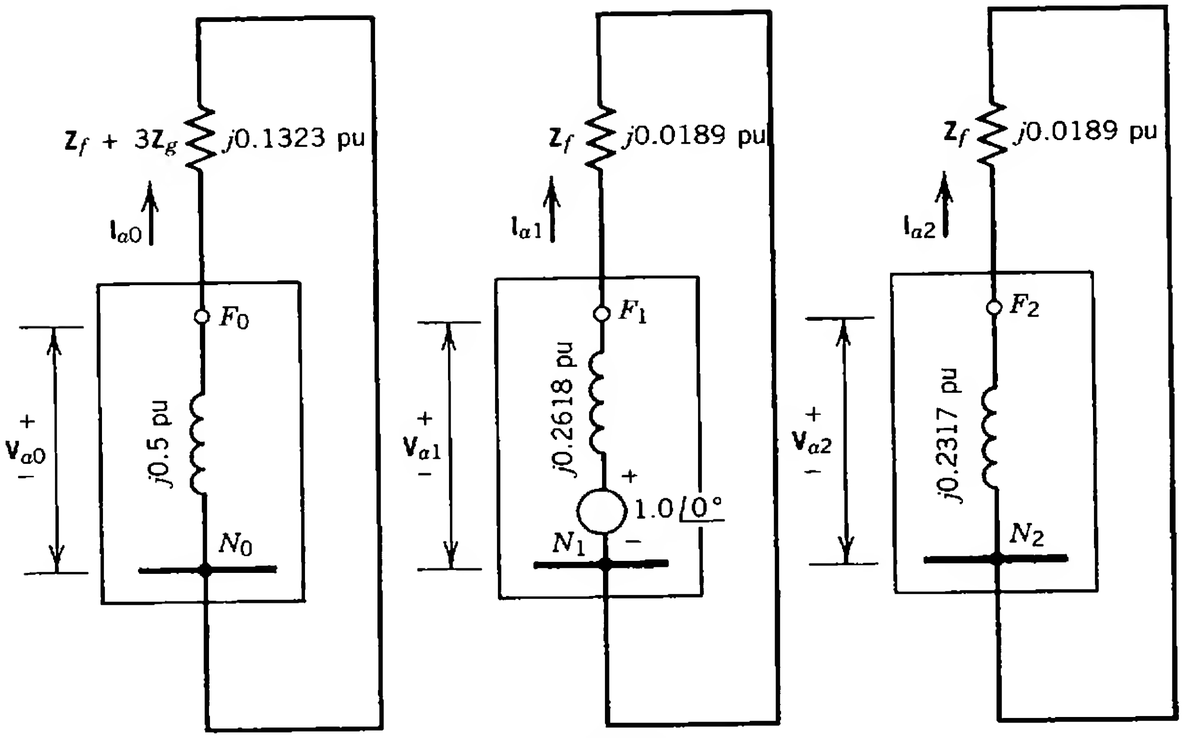


Figure 3.25

and

$$\begin{bmatrix} \mathbf{I}_{af} \\ \mathbf{I}_{bf} \\ \mathbf{I}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 3.8098 \underline{/-85.9°} \\ 0 \end{bmatrix} = \begin{bmatrix} 3.8098 \underline{/-85.9°} \\ 3.8098 \underline{/154.1°} \\ 3.8098 \underline{/34.1°} \end{bmatrix} \text{ pu}$$

(c) The sequence and phase voltages are

$$\begin{bmatrix} \mathbf{V}_{a0} \\ \mathbf{V}_{a1} \\ \mathbf{V}_{a2} \end{bmatrix} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} - \begin{bmatrix} j0.5 & 0 & 0 \\ 0 & j0.2618 & 0 \\ 0 & 0 & j0.2317 \end{bmatrix} \begin{bmatrix} 0 \\ 3.8098 \underline{/-85.9°} \\ 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 \\ 0.0720 \underline{/-85.9°} \\ 0 \end{bmatrix} \text{ pu}$$

and

$$\begin{bmatrix} \mathbf{V}_{af} \\ \mathbf{V}_{bf} \\ \mathbf{V}_{cf} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \begin{bmatrix} 0 \\ 0.0720 \underline{/-85.9°} \\ 0 \end{bmatrix} = \begin{bmatrix} 0.0720 \underline{/-85.9°} \\ 0.0720 \underline{/154.9°} \\ 0.0720 \underline{/34.1°} \end{bmatrix} \text{ pu}$$

(d) The line-to-line voltages at the fault point are

$$\mathbf{V}_{abf} = \mathbf{V}_{af} - \mathbf{V}_{bf} = 0.0601 - j0.1023 = 0.1186 \underline{/-79.6°} \text{ pu}$$

$$\mathbf{V}_{bcf} = \mathbf{V}_{bf} - \mathbf{V}_{cf} = -0.1248 - j0.0099 = 0.1252 \underline{/184.5°} \text{ pu}$$

$$\mathbf{V}_{caf} = \mathbf{V}_{cf} - \mathbf{V}_{af} = 0.0545 + j0.1122 = 0.1247 \underline{/64°} \text{ pu}$$

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PROBLEMS

3.1. Determine the symmetrical components for the phase currents of $\mathbf{I}_a = 125\,\underline{/20^\circ}$, $\mathbf{I}_b = 175\,\underline{/-100^\circ}$, and $\mathbf{I}_c = 95\,\underline{/155^\circ}$.

3.2. Assume that the unbalanced phase currents are $\mathbf{I}_a = 100\,\underline{/180^\circ}$, $\mathbf{I}_b = 100\,\underline{/0^\circ}$, and $\mathbf{I}_c = 10\,\underline{/20^\circ}$ A.

(a) Determine the symmetrical components.

(b) Draw a phasor diagram showing $\mathbf{I}_{a0}$, $\mathbf{I}_{a1}$, $\mathbf{I}_{a2}$, $\mathbf{I}_{b0}$, $\mathbf{I}_{b1}$, $\mathbf{I}_{b2}$, $\mathbf{I}_{c0}$, $\mathbf{I}_{c1}$, and $\mathbf{I}_{c2}$ (i.e., the positive-, negative-, and zero-sequence currents for each phase).

(c) Draw the unbalanced phase current phasors in the phasor diagram of part (b).

3.3. Assume that $\mathbf{V}_{a1} = 180\,\underline{/0^\circ}$, $\mathbf{V}_{a2} = 100\,\underline{/100^\circ}$, and $\mathbf{V}_{a0} = 250\,\underline{/-40^\circ}$ V.

(a) Draw a phasor diagram showing all the nine symmetrical components.

(b) Find the phase voltages $[\mathbf{V}_{abc}]$ using the equation

$$[\mathbf{V}_{abc}] = [\mathbf{A}][\mathbf{V}_{012}]$$

(c) Find the phase voltages $[\mathbf{V}_{abc}]$ graphically and check the results against the ones found in part (b).

3.4. Repeat Example 3.2 assuming that the phase voltages and currents are given as

$$[\mathbf{V}_{abc}] = \begin{bmatrix} 100\,\angle 0^\circ \\ 100\,\angle 60^\circ \\ 100\,\angle -60^\circ \end{bmatrix} \quad \text{and} \quad [\mathbf{I}_{abc}] = \begin{bmatrix} 10\,\angle -30^\circ \\ 10\,\angle 30^\circ \\ 10\,\angle -90^\circ \end{bmatrix}.$$

3.5. Determine the symmetrical components for the phase currents of $\mathbf{I}_a = 100\,\angle 20^\circ$, $\mathbf{I}_b = 50\,\angle -20^\circ$, and $\mathbf{I}_c = 150\,\angle 180^\circ$ A. Draw a phasor diagram showing all the nine symmetrical components.

3.6. Assume that $\mathbf{I}_{a0} = 50 - j86.6$, $\mathbf{I}_{a1} = 200\,\angle 0^\circ$, and $\mathbf{I}_a = 400\,\angle 0^\circ$ A. Determine the following:

(a) $\mathbf{I}_{a2}$.

(b) $\mathbf{I}_b$.

(c) $\mathbf{I}_c$.

3.7. Determine the symmetrical components for the phase currents of $\mathbf{I}_a = 200\,\angle 0^\circ$, $\mathbf{I}_b = 175\,\angle -90^\circ$, and $\mathbf{I}_c = 100\,\angle 90^\circ$.

3.8. Use the symmetrical components for the phase voltages and verify the following line-to-line voltage equations:

(a) $\mathbf{V}_{ab} = \sqrt{3}(\mathbf{V}_{a1}\,\angle 30^\circ + \mathbf{V}_{a2}\,\angle -30^\circ)$

(b) $\mathbf{V}_{bc} = \sqrt{3}(\mathbf{V}_{a1}\,\angle -90^\circ + \mathbf{V}_{a2}\,\angle 90^\circ)$

(c) $\mathbf{V}_{ca} = \sqrt{3}(\mathbf{V}_{a1}\,\angle 150^\circ + \mathbf{V}_{a2}\,\angle -150^\circ)$

3.9. Consider Example 3.3 and assume that the voltage applied at the sending end of the line is $69\,\angle 0^\circ$ kV. Determine the phase current matrix from equation (3.35).

3.10. Consider a three-phase horizontal line configuration and assume that the phase spacings are $D_{ab} = 30$ ft, $D_{bc} = 30$ ft, and $D_{ac} = 60$ ft. The line conductors are made of 500 kcmil, 37-strand copper conductors. Assume that the 100-mi-long untransposed transmission line operates at 50 °C, 60 Hz. If the earth has an average resistivity, determine the following:

(a) Self-impedances of line conductors in ohms per mile.

(b) Mutual impedances of line conductors in ohms per mile.

(c) Phase impedance matrix of line in ohms.

3.11. Consider a 50-mi-long completely transposed transmission line operating at 25 °C, 50 Hz, and having 500-kcmil ACSR conductors. The three-phase conductors have a triangular configuration with spacings of $D_{ab} = 6$ ft, $D_{bc} = 10$ ft, and $D_{ac} = 8$ ft. If the earth is considered to be dry earth, determine the following:

(a) Zero-sequence impedance of line.

(b) Positive-sequence impedance of line.

(c) Negative-sequence impedance of line.

3.12. Consider the three-phase vertical conductor configuration shown in Figure 2.1(*c*). Assume that the phase spacings are $D_{ab} = 72$ in., $D_{bc} = 72$ in., and $D_{ca} = 144$ in. The line conductors are made of 795-kcmil, 30/19-strand ACSR. If the line is 100 mi long and not transposed, determine the following:

(a) Phase impedance matrix of line.

(b) Phase admittance matrix of line.

(c) Sequence impedance matrix of line.

(d) Sequence admittance matrix of line.

3.13. Repeat Problem 3.12 assuming that the phase spacings are $D_{ab} = 144$ in., $D_{bc} = 144$ in., and $D_{ac} = 288$ in.

3.14. Repeat Problem 3.12 assuming that the conductor is 795-kcmil, 61 percent conductivity, 37-strand, hard-drawn aluminum.

3.15. Repeat Problem 3.13 assuming that the conductor is 750-kcmil, 97.3 percent conductivity, 37-strand, hard-drawn copper conductor.

3.16. Consider the line configuration shown in Figure 3.5. Assume that the 115-kV line is transposed and its conductors are made up of 500-kcmil, 30/7-strand ACSR conductors. Ignore the overhead ground wire but consider the heights of the conductors and determine the zero-sequence capacitance of the line in nanofarads per mile and nanofarads per kilometer.

3.17. Solve Problem 3.16 taking into account the overhead ground wire. Assume that the overhead ground wire is made of $\frac{3}{8}$-in. E.B.B. steel conductor.

3.18. Repeat Example 3.6 without ignoring the overhead ground wire. Assume that the overhead ground wire is made of $\frac{3}{8}$-in. E.B.B. steel conductor.

3.19. Consider the line configuration shown in Figure 3.5. Assume that the 115-kV line is transposed and its conductors are made of 500-kcmil, 30/7-strand ACSR conductors. Ignore the effects of conductor heights and overhead ground wire and determine the following:

(a) Positive- and negative-sequence capacitances to ground of line in nanofarads per mile.

(b) The 60-Hz susceptance of line in microsiemens per mile.

(c) Charging kilovoltampers per phase per mile of line.

(d) Three-phase charging kilovoltamperes per mile of line.

3.20. Repeat Problem 3.19 without ignoring the effects of conductor heights.

3.21. Consider the untransposed line shown in Figure P3.21. Assume that the 50-mi-long line has an overhead ground wire of 3/0 ACSR and that the phase conductors are of 556.5-kcmil. 30/7 strand, ACSR. Use a frequency of 60 Hz, an ambient temperature of 50 °C, and average earth resistivity and determine the following:

(a) Phase impedance matrix of line.

(b) Sequence impedance matrix of line.

(c) Sequence admittance matrix of line.

(d) Electrostatic zero- and negative-sequence unbalance factors of line.

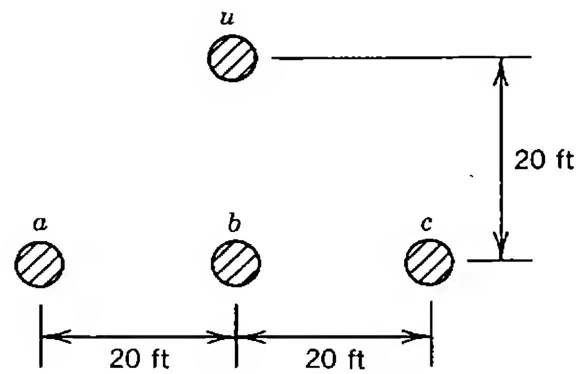


Figure P3.21

3.22. Repeat Problem 3.21 assuming that there are two overhead ground wires, as shown in Figure P3.22.

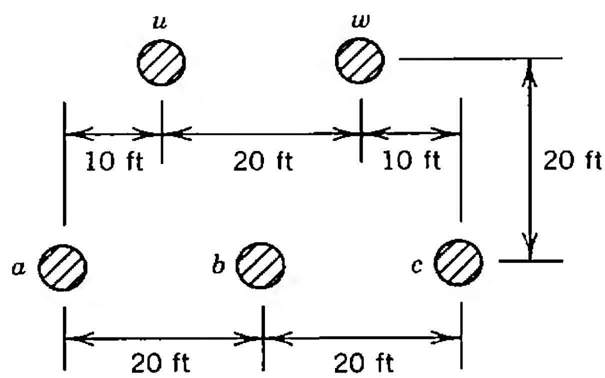


Figure P3.22

3.23. Consider the power system given in Example 3.7 and assume that transformers T_1 and T_2, T_3, and T_4 are connected as wye-grounded/delta, delta/wye-grounded, and wye-grounded/delta, respectively. Assume that there is a fault on bus 3 and do the following:

(a) Draw the corresponding zero-sequence network.

(b) Reduce the zero-sequence network to its Thévenin equivalent looking in at bus 3.

3.24. Consider the power system given in Example 3.7 and assume that all four transformers are connected as wye-grounded/wye-grounded. Assume there is a fault on bus 3 and do the following:

(a) Draw the corresponding zero-sequence network.

(b) Reduce the zero-sequence network to its Thévenin equivalent looking in at bus 3.

3.25. Consider the power system given in Problem 3.36. Use 25 MVA as the megavoltampere base and draw the positive-, negative-, and zero-sequence networks (but do not reduce them). Assume that the two three-phase transformer bank connections are:

(a) Both wye-grounded.

(b) Delta/wye-grounded for transformer T_1 and wye-grounded/delta for transformer T_2.

(c) Wye-grounded/wye for transformer T_1 and delta/wye for transformer T_2.

3.26. Assume that a three-phase, 45-MVA, 34.5/115-kV transformer bank of three single-phase transformers, with nameplate impedances of 7.5 percent, is connected wye/delta with the high-voltage side delta. Determine the zero-sequence equivalent circuit (in per-unit values) under the following conditions:

(a) If neutral is ungrounded.

(b) If neutral is solidly grounded.

(c) If neutral is grounded through 10-Ω resistor.

(d) If neutral is grounded through 4000-μF capacitor.

3.27. Consider the system shown in Figure P3.27. Assume that the following data are given based on 20 MVA and the line-to-line base voltages as shown in Figure P3.27.

Generator G_1: $X_1 = 0.25$ pu, $X_2 = 0.15$ pu, $X_0 = 0.05$ pu.
Generator G_2: $X_1 = 0.90$ pu, $X_2 = 0.60$ pu, $X_0 = 0.05$ pu.
Transformer T_1: $X_1 = X_2 = X_0 = 0.10$ pu.
Transformer T_2: $X_1 = X_2 = 0.10$ pu, $X_0 = \infty$.
Transformer T_3: $X_1 = X_2 = X_0 = 0.50$ pu.
Transformer T_4: $X_1 = X_2 = 0.30$ pu, $X_0 = \infty$.
Transmission line TL_{23}: $X_1 = X_2 = 0.15$ pu, $X_0 = 0.50$ pu.
Transmission line TL_{35}: $X_1 = X_2 = 0.30$ pu, $X_0 = 1.00$ pu.
Transmission line TL_{57}: $X_1 = X_2 = 0.30$ pu, $X_0 = 1.00$ pu.

(a) Draw the corresponding positive-sequence network.

(b) Draw the corresponding negative-sequence network.

(c) Draw the corresponding zero-sequence network.

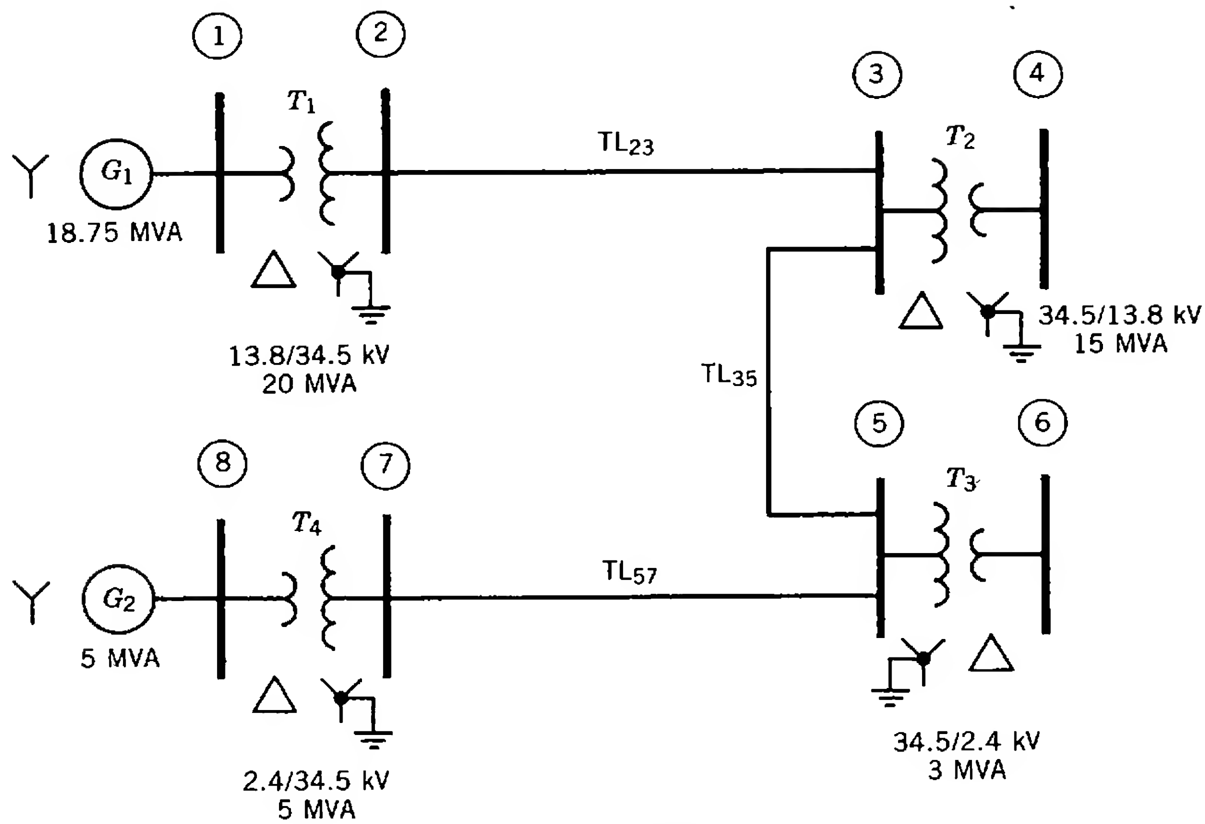


Figure P3.27

3.28. Consider the system showing in Figure P3.28 and the following data:

Generator G_1: 15 kV, 50 MVA, $X_1 = X_2 = 0.10$ pu and $X_0 = 0.05$ pu based on its own ratings.

Generator G_1: 15 kV, 20 MVA, $X_1 = X_2 = 0.20$ pu and $X_0 = 0.07$ pu based on its own ratings.

Transformer T_1: 15/115 kV, 30 MVA, $X_1 = X_2 = X_0 = 0.06$ pu based on its own ratings.

Transformer T_2: 115/15 kV, 25 MVA, $X_1 = X_2 = X_0 = 0.07$ pu based on its own ratings.

Transmission line TL_{23}: $X_1 = X_2 = 0.03$ pu and $X_0 = 0.10$ pu based on its own ratings.

Assume a single line-to-ground fault at bus 4 and determine the fault current in per units and amperes. Use 50 MVA as the megavoltampere base and assume that $\mathbf{Z}_f$ is $j0.1$ pu based on 50 MVA.

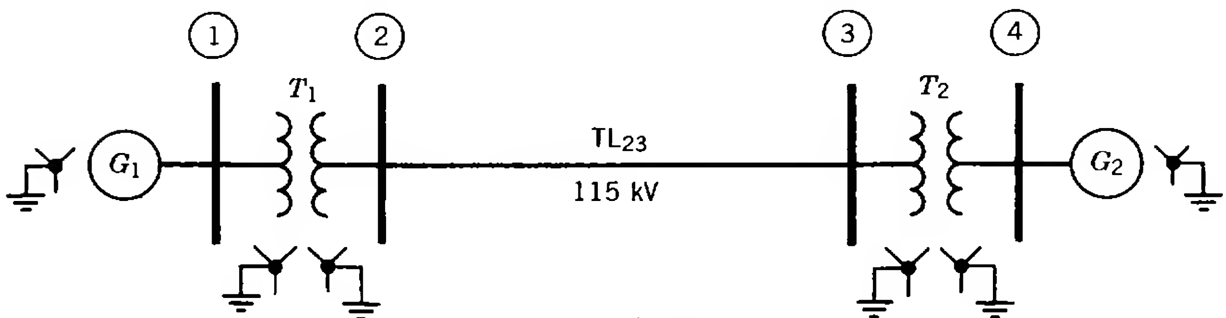


Figure P3.28

3.29. Consider the system given in Problem 3.28 and assume that there is a line-to-line fault at bus 3 involving phases b and c. Determine the fault currents for both phases in per units and amperes.

3.30. Consider the system given in Problem 3.28 and assume that there is a DLG fault at bus 2, involving phases b and c. Assume that $\mathbf{Z}_f$ is $j0.1$ pu and $\mathbf{Z}_g$ is $j0.2$ pu (where $\mathbf{Z}_g$ is the neutral-to-ground impedance) both based on 50 VA.

3.31. Consider the system shown in Figure P3.31 and assume that the generator is loaded and running at the rated voltage with the circuit breaker open at bus 3. Assume that the reactance values of the generator are given as $X''_d = X_1 = X_2 = 0.14$ pu and $X_0 = 0.08$ pu based on its ratings. The transformer impedances are $Z_1 = Z_2 = Z_0 = j0.05$ pu based on its ratings. The transmission line TL_{23} has $Z_1 = Z_2 = j0.04$ pu and $Z_0 = j0.10$ pu. Assume that the fault point is located on bus 1. Select 25 MVA as the megavoltampere base, and 8.5 and 138 kV as the low-voltage and high-voltage voltage bases, respectively, and determine the following:

(a) Subtransient fault current for three-phase fault in per units and amperes.

(b) Line-to-ground fault. [Also find the ratio of this line-to-ground fault current to the three-phase fault current found in part (a)].

(c) Line-to-line fault. (Also find the ratio of this line-to-line fault current to previously calculated three-phase fault current.)

(d) Double line-to-ground fault.

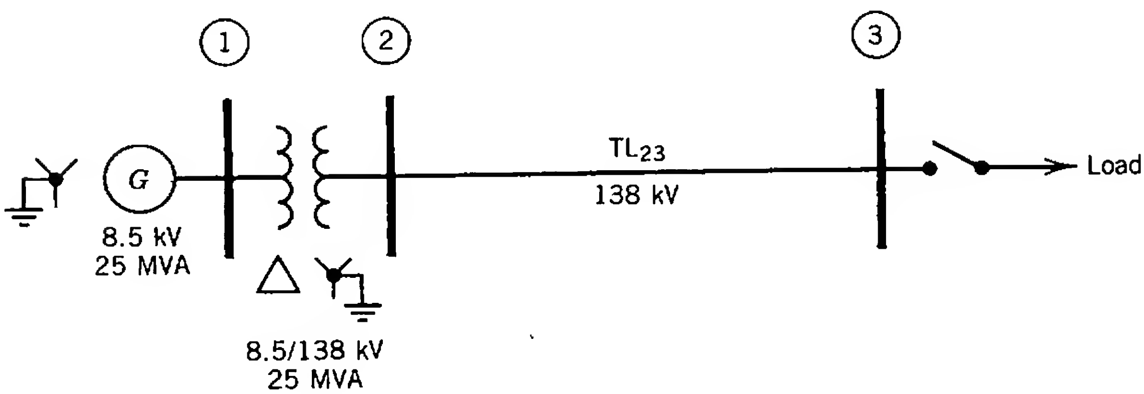


Figure P3.31

3.32. Repeat Problem 3.31 assuming that the fault is located on bus 2.

3.33. Repeat Problem 3.31 assuming that the fault is located on bus 3.

3.34. Consider the system shown in Figure P3.34(a) Assume that loads, line capacitance, and transformer-magnetizing currents are neglected and that the following data is given based on 20 MVA and the line-to-line voltages as shown in Figure P3.34(a). Do not neglect the resistance of the transmission line TL_{23}. The prefault positive-sequence voltage at bus 3 is $\mathbf{V}_{an} = 1.0\angle 0°$ pu, as shown in Figure P3.34(b).

Generator: $X_1 = 0.20$ pu, $X_2 = 0.10$ pu, $X_0 = 0.05$ pu.

Transformer T_1: $X_1 = X_2 = 0.05$ pu, $X_0 = X_1$ (looking into high-voltage side).

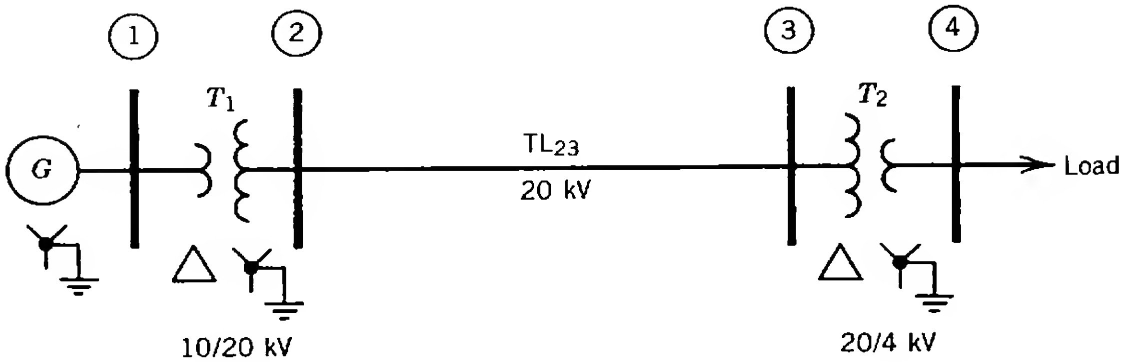
Transformer T_2: $X_1 = X_2 = 0.05$ pu, $X_0 = \infty$ (looking into high-voltage side).

Transmission line: $\mathbf{Z}_1 = \mathbf{Z}_2 = 0.2 + j0.2$ pu, $\mathbf{Z}_0 = 0.6 + j0.6$ pu.

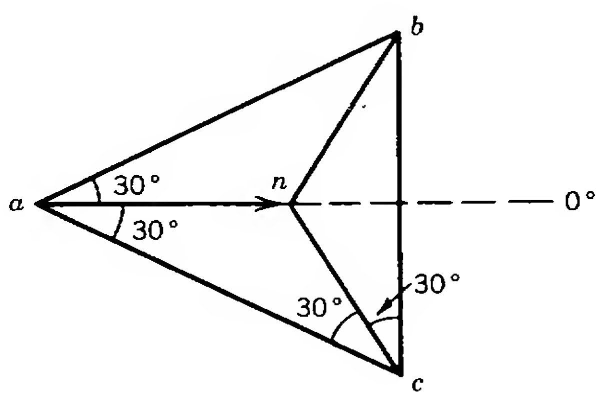
Assume that there is a bolted (i.e., with zero fault impedance) line-to-line fault on phases b and c at bus 3 and determine the following:

(a) Fault current $\mathbf{I}_{bf}$ in per units and amperes.

(b) Phase voltages $\mathbf{V}_a$, $\mathbf{V}_b$, and $\mathbf{V}_c$ at bus 2 in per units and kilovolts.



(a)



(b)

Figure P3.34

(c) Line-to-line voltages $\mathbf{V}_{ab}$, $\mathbf{V}_{bc}$, and $\mathbf{V}_{ca}$ at bus 2 in kilovolts.

(d) Generator line currents $\mathbf{I}_a$, $\mathbf{I}_b$, and $\mathbf{I}_c$.
Given: per-unit positive-sequence currents on the low-voltage side of the transformer bank lag positive-sequence currents on the high-voltage side by 30° and similarly for negative-sequence currents excepting that the low-voltage currents lead the high-voltage by 30°.

3.35. Consider Figure P3.35 and assume that the generator ratings are 2.40/4.16Y kV, 15 MW (3Φ), 18.75 MVA (3Φ), 80 percent power factor, two poles, 3600 rpm. Generator reactances are $X_1 = X_2 = 0.10$ pu and $X_0 = 0.05$ pu, all based on generator ratings. Note that the given value of X_1 is subtransient reactance X'', one of several different positive-sequence reactances of a synchronous machine. The subtransient reactance corresponds to the initial symmetrical fault current (the transient dc component not included) that occurs before demagnetizing armature magnetomotive force begins to weaken the net field excitation. If manufactured in accordance with U.S. standards, the coils of a synchronous generator will withstand the mechanical forces that accompany a three-phase fault current, but not more. Assume that this generator is to supply a four-wire, wye-connected distribution. Therefore, the neutral grounding reactor X_n should have the smallest possible reactance. Consider both SLG and DLG faults. Assume the prefault positive-sequence internal voltage of phase a is $2500\,\underline{/0^\circ}$ or $1.042\,\underline{/0^\circ}$ pu and determine the following:

(a) Specify X_n in ohms and in per units.

(b) Specify the minimum allowable momentary symmetrical current rating of the reactor in amperes.

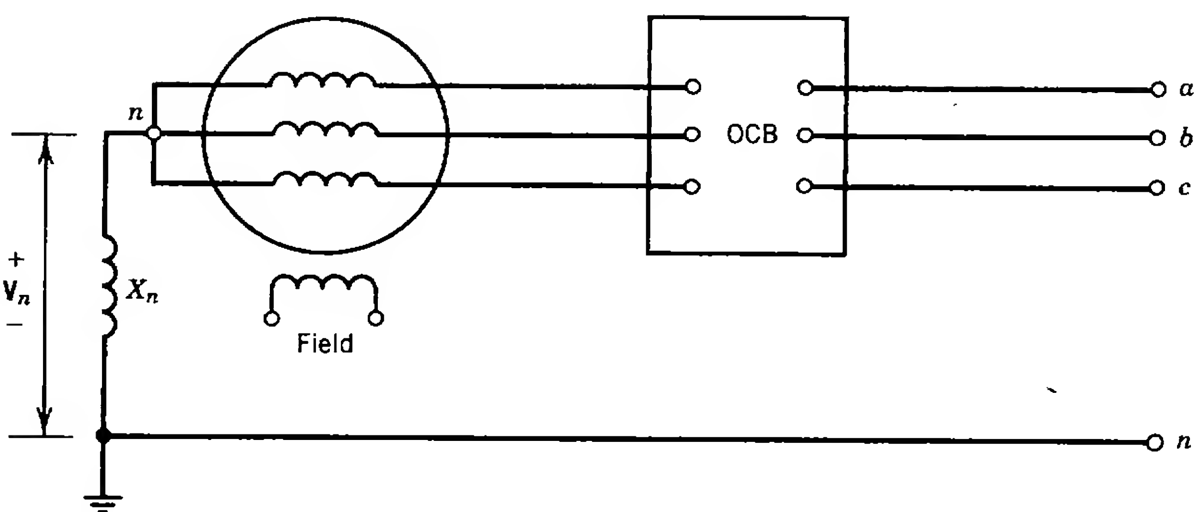


Figure P3.35

(c) Find the initial symmetrical voltage across the reactor, $\mathbf{V}_n$, when a bolted SLG fault occurs on the oil circuit breaker (OCB) terminal in volts.

3.36. Consider the system shown in Figure P3.36 and the following data:

Generator G: $X_1 = X_2 = 0.10$ pu and $X_0 = 0.05$ pu based on its ratings.
Motor: $X_1 = X_2 = 0.10$ pu and $X_0 = 0.05$ pu based on its ratings.
Transformer T_1: $X_1 = X_2 = X_0 = 0.05$ pu based on its ratings.
Transformer T_2: $X_1 = X_2 = X_0 = 0.10$ pu based on its ratings.
Transmission line TL_{23}: $X_1 = X_2 = X_0 = 0.09$ pu based on 25 MVA.

Assume that bus 2 is faulted and determine the faulted phase currents.

(a) Determine the three-phase fault.
(b) Determine the line-to-ground fault involving phase a.
(c) Use the results of part (a) and calculate the line-to-neutral phase voltages at the fault point.

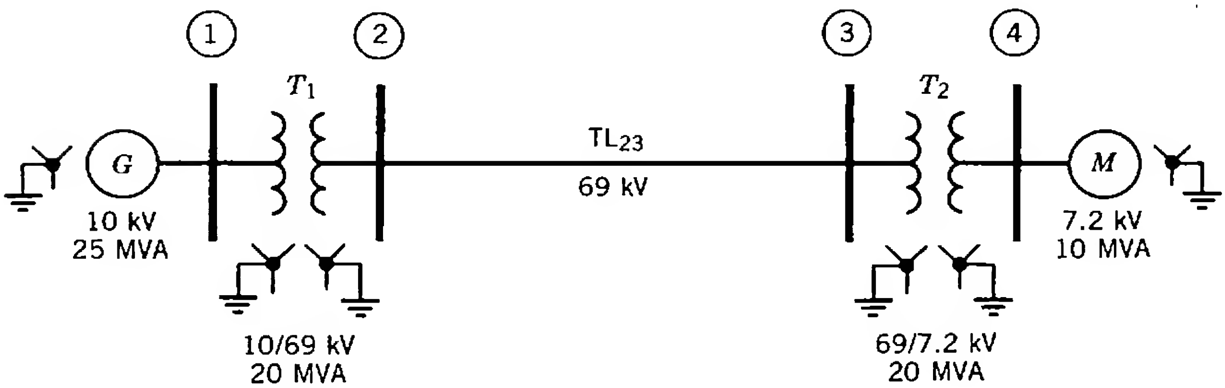


Figure P3.36

3.37. Consider the system given in Problem 3.36 and assume a line-to-line fault, involving phases b and c, at bus 2 and determine the faulted phase currents.

3.38. Consider the system shown in Figure P3.38 and assume that the associated data is given in Table P3.38 and is based on a 100-MVA base and referred to nominal system voltages.

Assume that there is a three-phase fault at bus 6. Ignore the prefault currents and determine the following.

(a) Fault current in per units at faulted bus 6.
(b) Fault current in per units in transmission line TL_{25}.

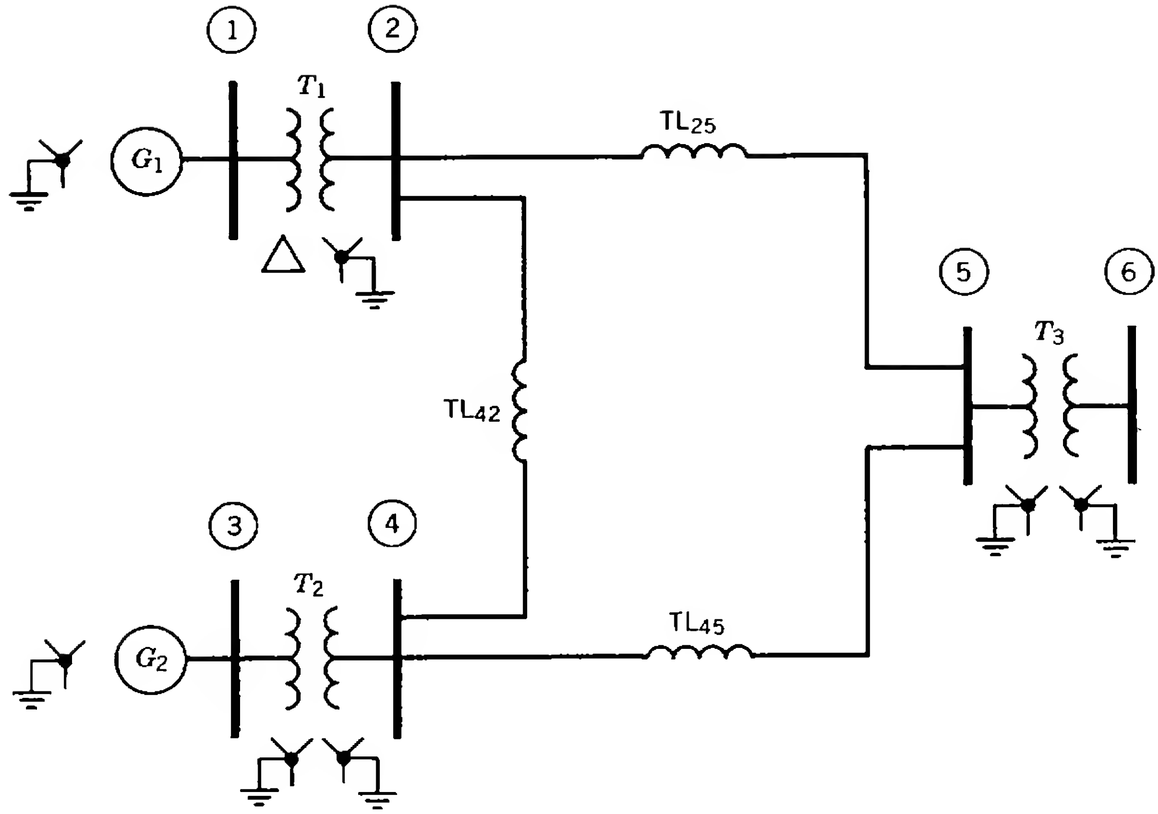


Figure P3.38

TABLE P3.38 Data for Problem 3.38

Network Component	X_1 (pu)	X_2 (pu)	X_3 (pu)
G_1	0.35	0.35	0.09
G_2	0.35	0.35	0.09
T_1	0.10	0.10	0.10
T_2	0.10	0.10	0.10
T_3	0.05	0.05	0.05
TL_{42}	0.45	0.45	1.80
TL_{25}	0.35	0.35	1.15
TL_{45}	0.35	0.35	1.15

3.39. Use the results of Problem 3.38 and calculate the line-to-neutral phase voltages at the faulted bus 6.

3.40. Repeat Problem 3.38 assuming a line-to-ground fault, with $\mathbf{Z}_f = 0$ pu, at bus 6.

3.41. Use results of Problem 3.40 and calculate the line-to-neutral phase voltages at the following buses.

(a) Bus 6.

(b) Bus 2.

3.42. Repeat Problem 3.38 assuming a line-to-line fault at bus 6.

3.43. Repeat Problem 3.38 assuming a double line-to-ground fault, with $\mathbf{Z}_f = 0$ and $\mathbf{Z}_g = 0$, at bus 6.

3.44. Consider the system described in Example 3.7 and assume that there is a SLG fault, involving phase a, at the indicated bus. Show the interconnection of the resulting reduced equivalent sequence networks. Determine sequence and phase currents and sequence and phase voltages.

(a) Fault is at bus 1.
(b) Fault is at bus 2.
(c) Fault is at bus 4.
(d) Fault is at bus 5.
(e) Fault is at bus 6.

3.45. Repeat Problem 3.44 assuming a fault impedance of 5 Ω.

3.46. Repeat Problem 3.45 assuming a line-to-line fault involving phases b and c.

3.47. Repeat Problem 3.46 assuming a fault impedance of 5 Ω.

3.48. Repeat Problem 3.44 assuming a DLG fault involving phases b and c.

3.49. Repeat Problem 3.48 assuming a fault through a fault impedance $\mathbf{Z}_f$ of 5 Ω on each phase and then to ground G through impedance $\mathbf{Z}_g$ of 10 Ω.

3.50. Repeat Problem 3.44 assuming a three-phase fault.

3.51. Repeat Problem 3.50 assuming a fault impedance $\mathbf{Z}_f$ of 5 Ω on each phase.

3.52. Consider the system shown in Figure P3.52 and data given in Table P3.52. Assume there is a fault at bus 2. After drawing the corresponding sequence networks, reduce them to their Thévenin equivalents looking in at bus 2.

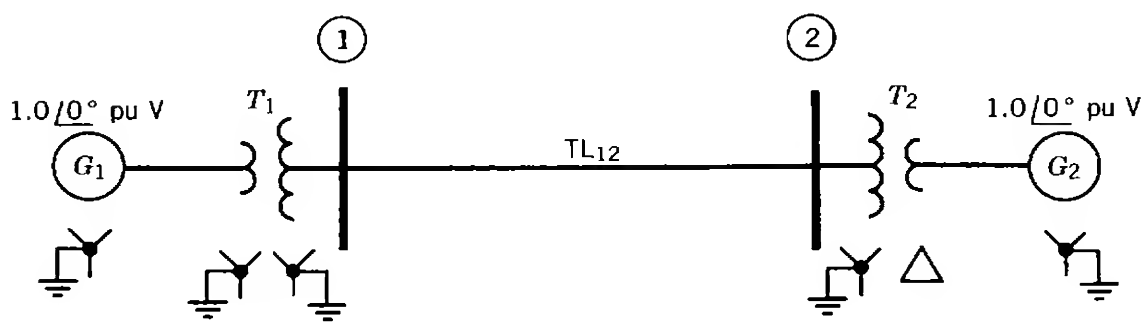


Figure P3.52

(a) Positive-sequence network.
(b) Negative-sequence network.
(c) Zero-sequence network.

TABLE P3.52 Table for Problem 3.52

Network Component	Base MVA	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	100	0.2	0.15	0.05
G_2	100	0.3	0.2	0.05
T_1	100	0.2	0.2	0.2
T_2	100	0.15	0.15	0.15
TL_{12}	100	0.6	0.6	0.9

3.53. Use the solution of Problem 3.52 and calculate the fault currents for the following faults and draw the corresponding interconnected sequence networks.

(a) Single line-to-ground fault at bus 2 assuming faulted phase is phase a.
(b) Double line-to-ground fault at bus 2 involving phases b and c.
(c) Three-phase fault at bus 2.

3.54. Consider the system shown in Figure P3.54 and data given in Table P3.54. Assume that there is a SLG fault at bus 3. Determine the following:

(a) Thévenin equivalent positive-sequence impedance.
(b) Thévenin equivalent negative-sequence impedance.
(c) Thévenin equivalent zero-sequence impedance.

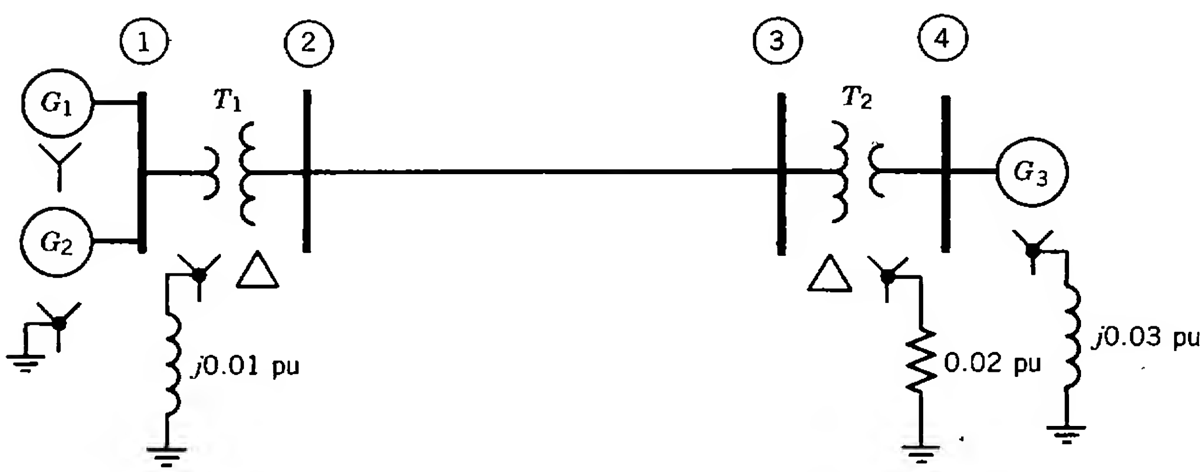


Figure P3.54

TABLE P3.54 Table for Problem 3.54

Network Component	MVA Rating	Voltage Rating (kV)	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	100	13.8	0.15	0.15	0.05
G_2	100	13.8	0.15	0.15	0.05
G_3	100	13.8	0.15	0.15	0.05
T_1	100	13.8/115	0.20	0.20	0.20
T_2	100	115/13.8	0.18	0.18	0.18
TL_{23}	100	115	0.30	0.30	0.90

(d) Positive-, negative-, and zero-sequence currents.

(e) Phase currents in per units and amperes.

(f) Positive-, negative-, and zero-sequence voltages.

(g) Phase voltages in per units and kilovolts.

(h) Line-to-line voltages in per units and kilovolts.

(i) Draw a voltage phasor diagram using before-the-fault line-to-neutral and line-to-line voltage values.

(j) Draw a voltage phasor diagram using the resultant after-the-fault line-to-neutral and line-to-line voltage values.

3.55. Consider the system shown in Figure P3.55 and assume that the following data on the same base are given:

Generator G_1: $X_1 = 0.15$ pu, $X_2 = 0.10$ pu, $X_0 = 0.05$ pu.
Generator G_2: $X_1 = 0.30$ pu, $X_2 = 0.20$ pu, $X_0 = 0.10$ pu.
Transformer T_1: $X_1 = X_2 = X_0 = 0.10$ pu.
Transformer T_2: $X_1 = X_2 = X_0 = 0.15$ pu.
Transmission line TL_{12}: $X_1 = X_2 = 0.30$ pu, $X_0 = 0.60$ pu.
Transmission line TL'_{12}: $X_1 = X_2 = 0.30$ pu, $X_0 = 0.60$ pu.

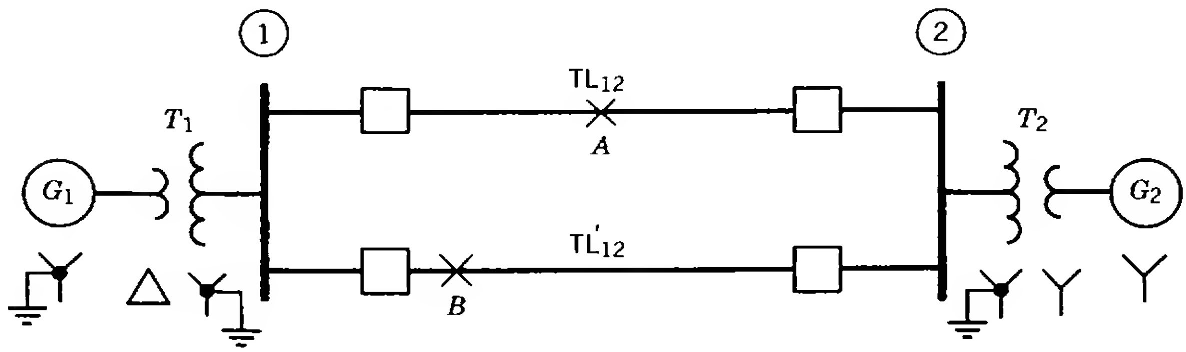


Figure P3.55

Assume that fault point A is located at the middle of the top transmission line, as shown in the figure, and determine the fault current(s) in per units for the following faults:

(a) Single line-to-ground fault (involving phase a).

(b) Double line-to-ground fault (involving phases b and c).

(c) Three-phase fault.

3.56 Repeat Problem 3.55 assuming that the fault point is B and is located at the beginning of the bottom line.

3.57 Consider the system shown in Figure P3.57 and assume that the following data on the same base are given:

Generator G_1: $X_1 = 0.15$ pu, $X_2 = 0.10$ pu, $X_0 = 0.05$ pu.
Generator G_2: $X_1 = 0.15$ pu, $X_2 = 0.10$ pu, $X_0 = 0.05$ pu.
Transformer T_1: $X_1 = X_2 = X_0 = 0.10$ pu.
Transformer T_2: $X_1 = X_2 = X_0 = 0.15$ pu.
Transmission lines: $X_1 = X_2 = 0.30$ pu, $X_0 = 0.60$ (all three are identical).

Assume that fault point A is located at the middle of the bottom line, as shown in the figure, and determine the fault current(s) in per units for the following faults:

(a) Single line-to-ground fault (involving phase a).

(b) Single line-to-ground fault (involving phases b and c).

(c) Double line-to-ground fault (involving phases b and c).

(d) Three-phase fault.

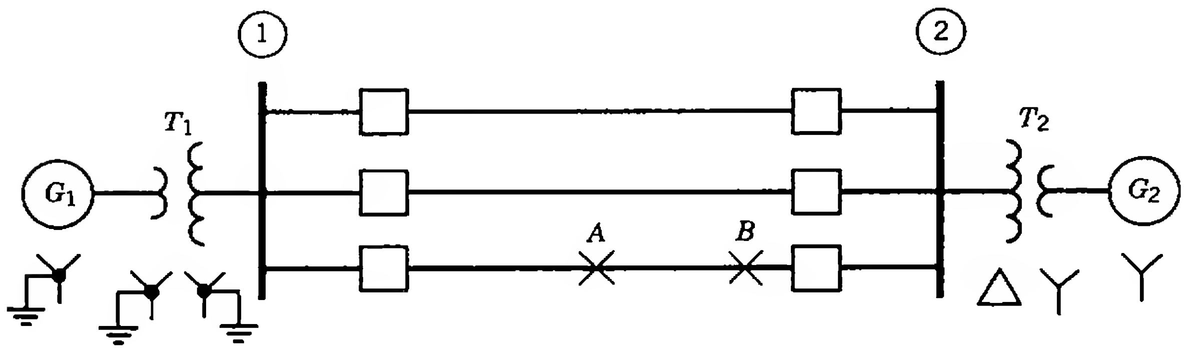


Figure P3.57

3.58. Repeat Problem 3.57 assuming that the faulted point is B and is located at the end of the bottom line.

3.59 Consider the system shown in Figure P3.59 and its data given in Table P3.59. Assume that there is a SLG fault involving phase a at fault point F.

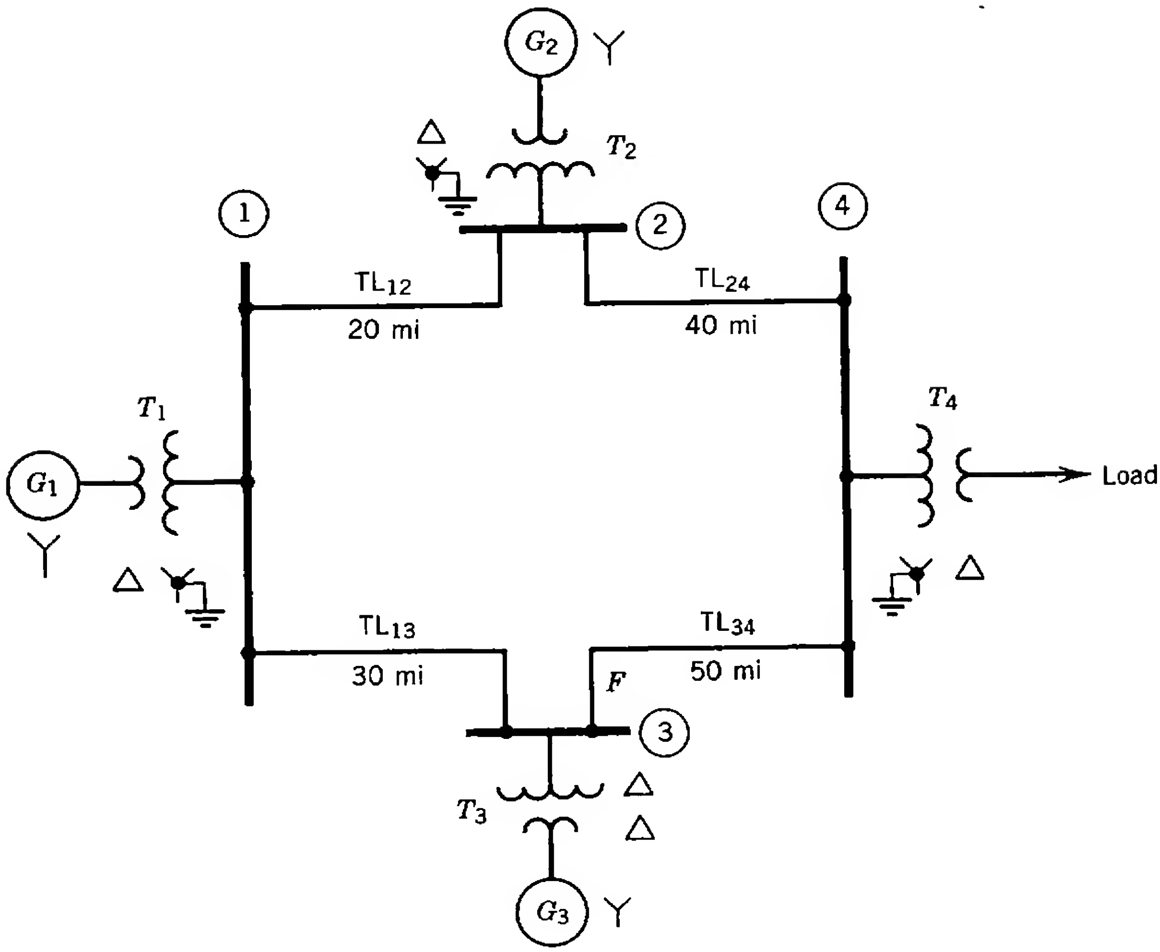


Figure P3.59

(a) Draw the corresponding equivalent positive-sequence network.
(b) Draw the corresponding equivalent negative-sequence network.
(c) Draw the coresponding equivalent zero-sequence network.

TABLE P3.59 Table for Problem 3.59

Network Component	Base MVA	Base $kV_{(L\text{-}L)}$	X_1 (pu)	X_2 (pu)	X_0 (pu)
G_1	100	230	0.15	0.15	
G_2	100	230	0.20	0.20	
G_3	100	230	0.25	0.25	
T_1	100	230	0.10	0.10	0.10
T_2	100	230	0.09	0.09	0.09
T_3	100	230	0.08	0.08	0.08
T_4	100	230	0.11	0.11	0.11
TL_{12}	100	230	0.10	0.10	0.36
TL_{13}	100	230	0.20	0.20	0.60
TL_{24}	100	230	0.35	0.35	1.05
TL_{34}	100	230	0.40	0.40	1.20

3.60. Use the results of Problem 3.59 and determine the interior sequence currents flowing in each of the four transmission lines.

(a) Positive-sequence currents.

(b) Negative-sequence currents.

(c) Zero-sequence currents.

3.61. Use the results of Problem 3.60 and determine the interior phase currents in each of the four transmission lines.

(a) Phase a currents.

(b) Phase b currents.

(c) Phase c currents.

3.62. Use the results of Problems 3.60 and 3.61 and draw a three-line diagram of the given system. Show the phase and sequence currents on it.

(a) Determine the SLG fault current.

(b) Is the fault current equal to the sum of the zero-sequence currents (i.e., $\mathbf{I}_{f(\mathrm{SLG})} = \Sigma\, 3\mathbf{I}_{a0}$)?

4

UNDERGROUND CABLES

4.1 INTRODUCTION

Underground cables may have one or more conductors within a protective sheath. The protective sheath is an impervious covering over insulation, and it usually is lead. The conductors are separated from each other and from the sheath by insulating materials. The insulation materials used are

1. rubber and rubberlike compounds,
2. varnished cambric, and
3. oil-impregnated paper.

Rubber is used in cables rated 600 V–35 kV, whereas polyethylene (PE), propylene (PP), and polyvinyl chloride (PVC) are used in cables rated 600 V–138 kV. The high-moisture resistance of rubber makes it ideal for submarine cables. Varnished cambric is used in cables rated 600 V–28 kV. Oil-impregnated paper is used in solid-type cables up to 69 kV and in pressurized cables up to 345 kV. In the solid-type cables, the pressure within the oil-impregnated cable is not raised above atmospheric pressure. In the pressurized cables, the pressure is kept above atmospheric pressure either by gas in gas pressure cables or by oil in oil-filled cables. Impregnated paper is used for higher voltages because of its low dielectric losses and lower cost.

Cables used for 59 kV and below are either (1) low pressure, not over 15 psi, or (2) medium pressure, not over 45 psi. High-pressure cables, up to 200 psi, installed pipes are not economical for voltages of 69 kV and below.

Voids or cavities can appear as the result of faulty product or during the operation of the cable under varying load. Bending the cable in handling and on installation, and also the different thermal expansion coefficient of the insulating paper, the impregnating material and the lead sheath result in voids in the insulation of cable not under pressure. The presence of higher electrical field strength ionization that appears in the voids in the dielectric leads to destruction of the insulation. The presence of ionization can be detected by means of the power factor change as a test voltage is applied. The formation of voids is avoided in the case of the oil-filled cable. With the gas-filled cable, the pressure in the insulation is increased to such a value that existing voids or cavities are ionization free. Ionization increases with temperature and decreases with increasing pressure.

4.2 CONDUCTORS

The conductors used in underground cables can be copper or aluminum. Aluminum dictates larger conductor sizes to carry the same current as copper. The need for mechanical flexibility requires stranded conductors to be used. The equivalent aluminum cable is lighter in weight and larger in diameter in comparison to copper cable. Stranded conductors can be in various configurations, for example, concentric, compressed, compact, and rope.

4.3 UNDERGROUND CABLE TYPES

Cables are classified in numerous ways. For example, they can be classified as (1) underground, (2) submarine, and (3) aerial, depending on location. They can be classified according to the type of insulation, such as (1) rubber and rubberlike compounds, (2) varnished cambric, and (3) oil-impregnated paper. They can be classified as (1) single conductor, (2) two conductor duplex, three conductor, etc., depending on the number of conductors in a given cable. They can be classified as shielded (as in the Höchstadter or type H cable) or nonshielded (belted), depending on the presence or absence of metallic shields over the insulation. Shielded cables can be solid, oil filled, or gas filled. They can be classified by their protective finish such as (1) metallic (e.g., a lead sheath) or (2) nonmetallic (e.g., plastic).

Insulation shields help to (1) confine the electric field within the cable; (2) protect cable better from induced potentials; (3) limit electromagnetic or electrostatic interference; (4) equalize voltage stress within the insulation, minimizing surface discharges; and (5) reduce shock hazard (when properly grounded) [1].

In general, shielding should be considered for nonmetallic covered cables operating at a circuit voltage over 2 kV and where any of the following conditions exist [2]:

1. Transition from conducting to nonconducting conduit.
2. Transition from moist to dry earth.
3. In dry soil, such as in the desert.
4. In damp conduits.
5. Connections to aerial lines.
6. Where conducting pulling compounds are used.
7. Where surface of cable collects conducting materials, such as soot, salt, or cement deposits.
8. Where electrostatic discharges are of low enough intensity not to damage cable but are sufficient in magnitude to interface with radio or television reception.

In general, cables are pulled into underground ducts. However, if they have to be buried directly in the ground, the lead sheath (i.e., the covering over insulation) has to be protected mechanically by armor. The armor is made of two steel tapes wound overlapping each other or heavy steel wires.

Where heavy loads are to be handled, the usage of single-conductor cables are advantageous since they can be made in conductor sizes up to 3.5 kcmil or larger. They are also used where phase isolation is required or where balanced single-phase transformer loads are supplied. They are often used to terminate three-conductor cables in single-conductor potheads, such as at pole risers, to provide training in small manholes. They can be supplied triplexed or wound three in parallel on a reel, permitting installation of three single-conductor cables in a single duct. Figure 4.1 shows a single-conductor, paper-insulated power cable.

The belted cable construction is generally used for three-phase low-voltage operation, up to 5 kV, or with the addition of conductor and belt shielding, in the 10–15 kV voltage range. It receives its name from the fact that a portion of the total insulation is applied over partially insulated conductors in the form of an insulating belt, which provides a smooth "cushion" for the lead sheath.

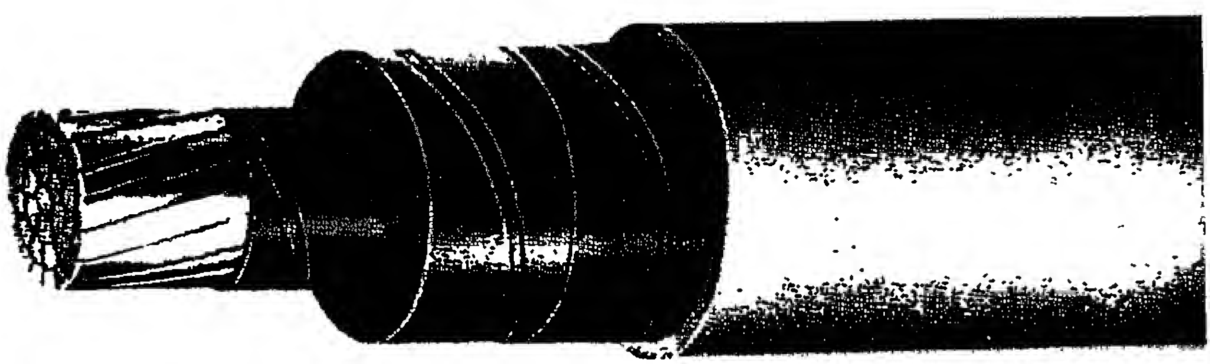


Figure 4.1. Single-conductor, paper-insulated power cable. (Courtesy of Okonite Company.)

Even though this design is generally more economical than the shielded (or Type H) construction, the electrical field produced by three-phase ac voltage is asymmetrical, and the fillers are also under electric stress. These disadvantages restrict the usage of this cable to voltages below 15 kV. Figure 4.2 shows a three-conductor, belted, compact-sector, paper-insulated cable. They can have concentric round, compact round, or compact-sector conductors.

The three-conductor shielded, or type H, construction with compact sector conductors is the design most commonly and universally used for three-phase applications at the 5–46-kV voltage range. Three-conductor cables in sizes up to 1 kcmil are standard, but for larger sizes, if overall size and weights are important factors, single-conductor cables should be preferred.

It confines the electric stress to the primary insulation, which causes the voltage rating (radial stress) to be increased and the dielectric losses to be reduced. The shielded paper–oil dielectric has the greatest economy for power cables at high voltages where reliability and performance are of prime importance. Figure 4.3 shows a three-conductor, shielded (type H), compact-sector, paper-insulated cable.

Figure 4.4 presents various protective outer coverings for solid-type cables, depending on installation requirements. Figure 4.5 shows the recommended voltage ranges for various types of paper-insulated power cables.

Most cable insulations are susceptible to deterioration by moisture to varying degrees. Paper and oil, which have had all the moisture completely extracted in the manufacture of a paper cable, will reabsorb moisture when exposed to the atmosphere, and prolonged exposure will degrade the exceptionally high electrical qualities. Because of this, it is mandatory in all paper cable splices and terminations to reduce exposure of the insulation to moisture and to construct and seal the accessories to ensure the complete exclusion of moisture for long and satisfactory service life.

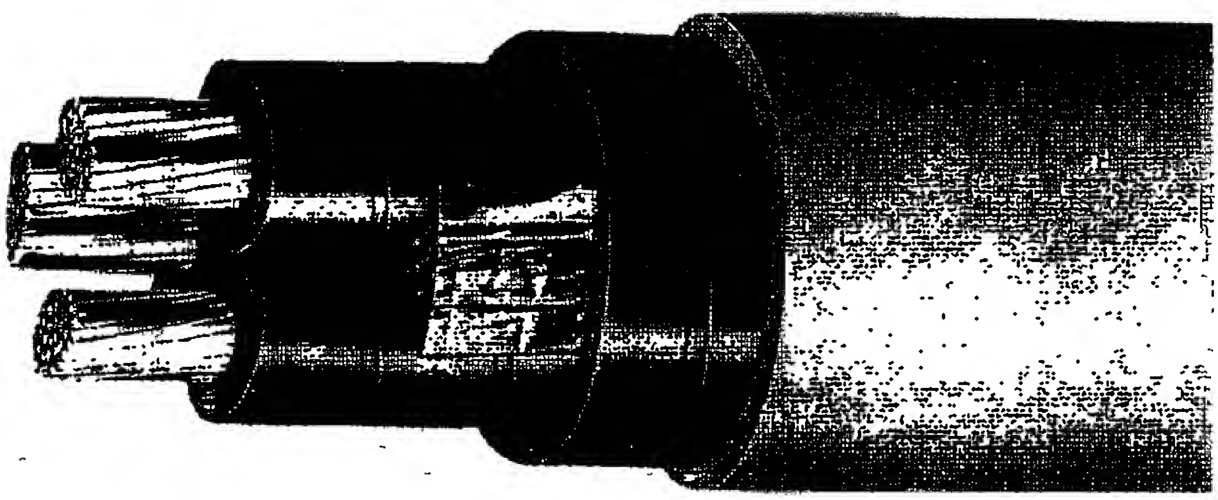


Figure 4.2. Three-conductor, belted, compact-sector, paper-insulated cable. (Courtesy of Okonite Company.)

Figure 4.3. Three-conductor, shielded (type-H), compact sector, paper-insulated cable. (Courtesy of Okonite Company.)

Therefore, it is important that all cable ends are tested for moisture before splicing or potheading. The most reliable procedure is to remove rings of insulating paper from the section cut for the connector at the sheath, at the midpoint, and nearest the conductor and immerse the tape "loops" in clean oil or flushing compound heated to 280–300°F. If any traces of moisture are present, minute bubbles will exclude from the tape and form "froth" in the oil.

The shields and metallic sheaths of power cables must be grounded for safety and reliable operation. Without such grounding, shields would operate at a potential considerably above the ground potential. Therefore, they would be hazardous to touch and would incur rapid degradation of the jacket or other material that is between shield and ground. The grounding conductor and its attachment to the shield or metallic sheath, normally at a termination or splice, needs to have an ampacity no lower than that of the shield. In the case of a lead sheath, the ampacity must be large enough to carry the available fault current and duration without overheating. Usually,

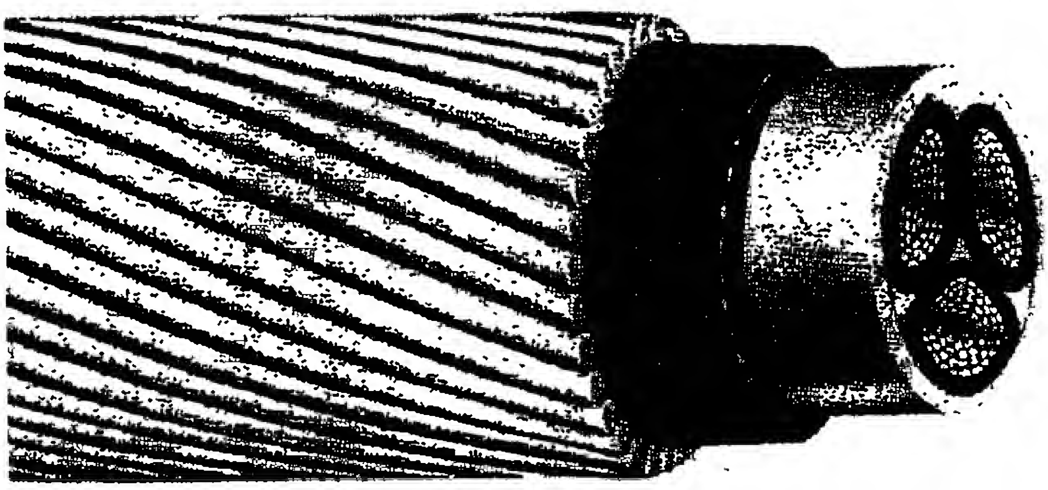


Figure 4.4. Various protective outer coverings for solid-type paper-insulated cables. (Courtesy of Okonite Company.)

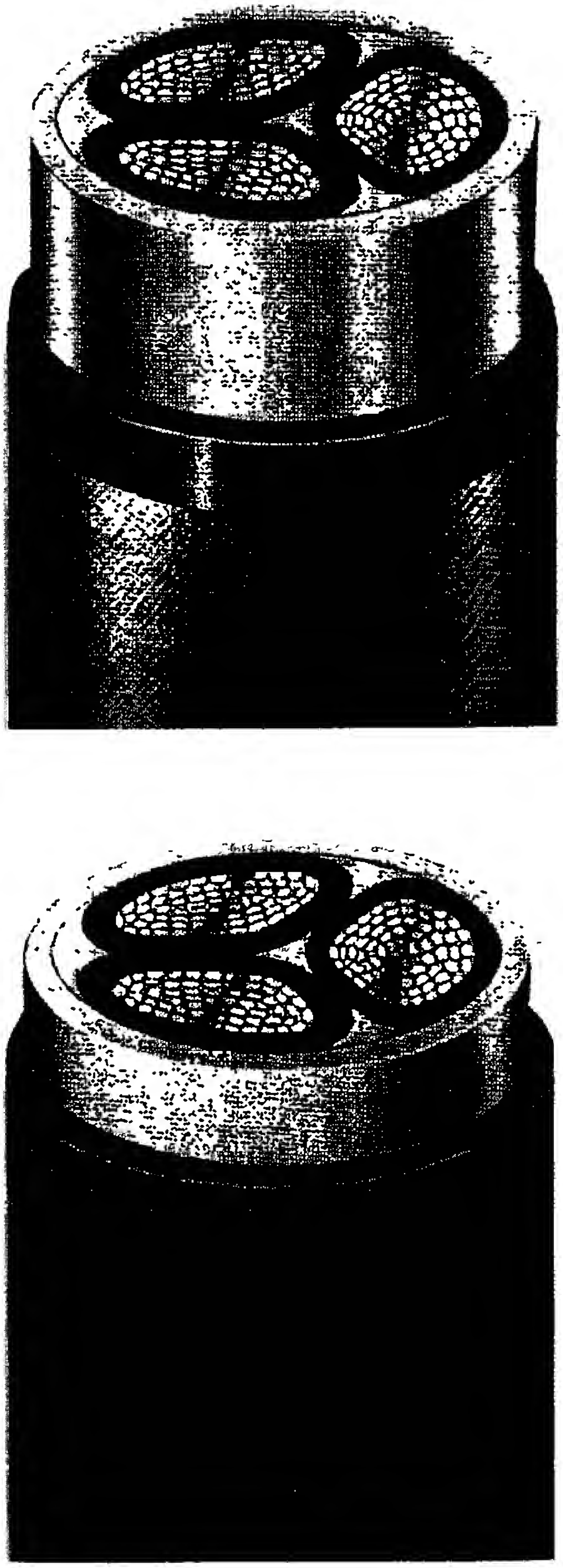


Figure 4.4 (*Continued*).

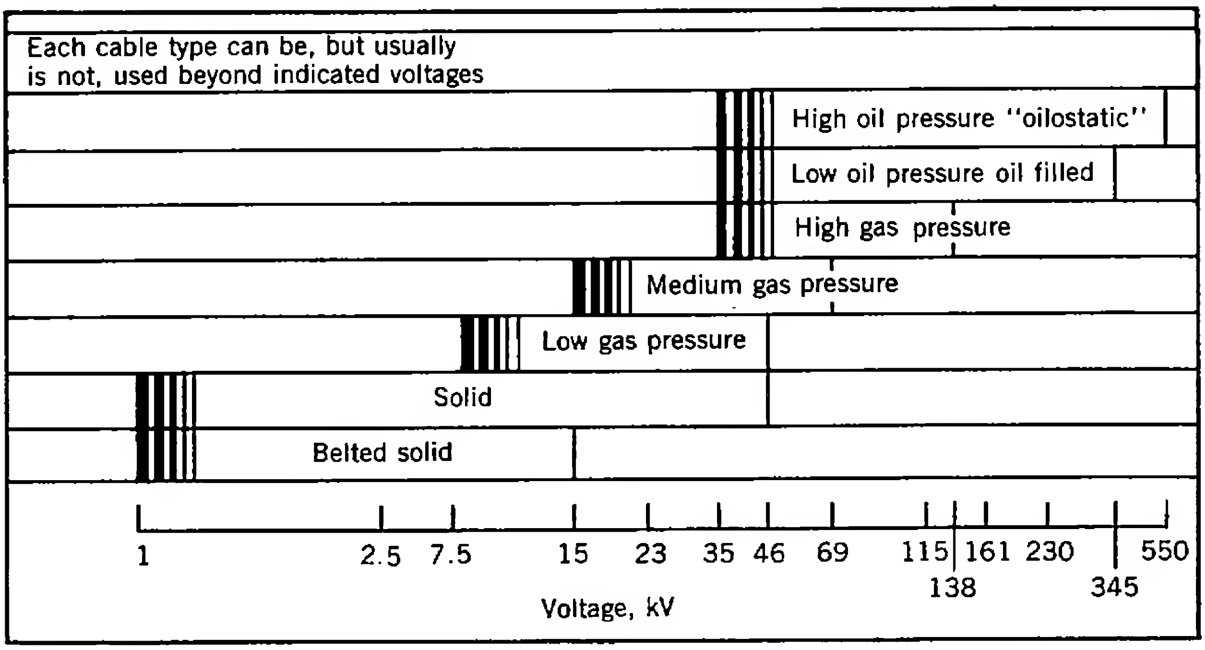


Figure 4.5. Recommended voltage ranges for various of paper-insulated paper cables. (Courtesy of Okonite Company.)

the cable shield lengths are grounded at both ends such that the fault current would divide and flow to both ends, reducing the duty on the shield and therefore the chance of damage.

The capacitive charging current of the cable insulation, which is on the order of 1 mA/ft of conductor length, normally flows, at power frequency, between the conductor and the earth electrode of the cable, normally the shield. Of course, the shield, or metallic sheath, provides the fault return path in the event of insulation failure, permitting rapid operation of the protection devices [1].

4.4 CABLE INSTALLATION TECHNIQUES

There are a number of ways to install the underground cables; for example:

1. Direct burial in the soil, as shown in Figure 4.6. The cable is laid in a trench that is usually dug by machine.
2. In ducts or pipes with concrete sheath, as shown in Figure 4.7. For secondary network systems, duct lines may have 6–12 ducts.
3. Wherever possible, in tunnels built for other purposes, for example, sewer lines, water mains, gas pipes, and duct lines for telephone and telegraph cables.

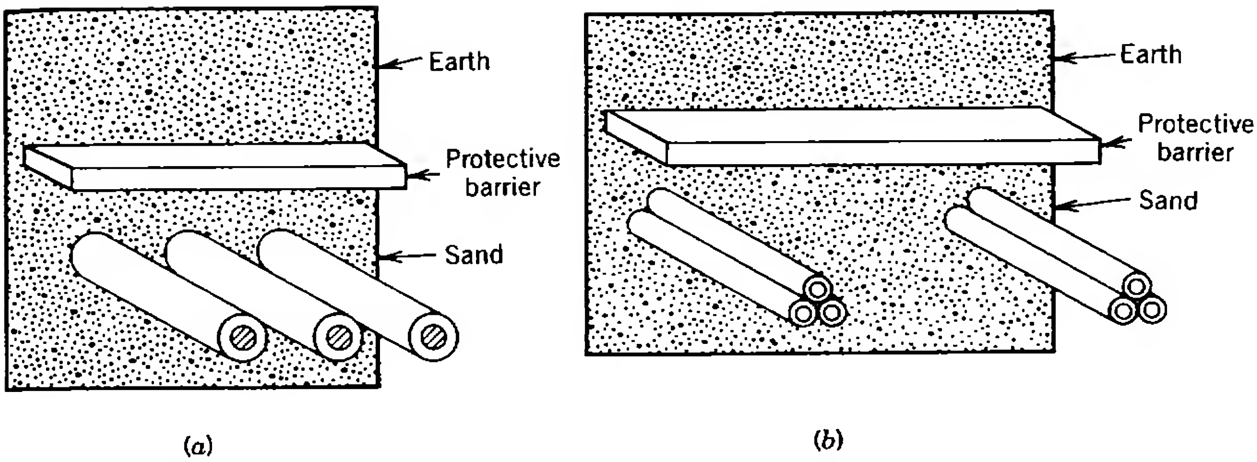


Figure 4.6. Direct burial: (*a*) for single-conductor cables; (*b*) for triplexed cables.

In general, manholes are built at every junction point and corner. The spacing of manholes is affected by the types of circuits installed, allowable cable-pulling tensions, and utility company's standards and practice. Manholes give easily accesible and protected space in which cables and associated apparatus can be operated properly. For example, they should provide enough space for required switching equipment, transformers, and splices and terminations. Figure 4.8 shows a straight-type manhole. Figure 4.9 shows a typical street cable manhole, which is usually used to route cables at street intersections or other locations where cable terminations are required.

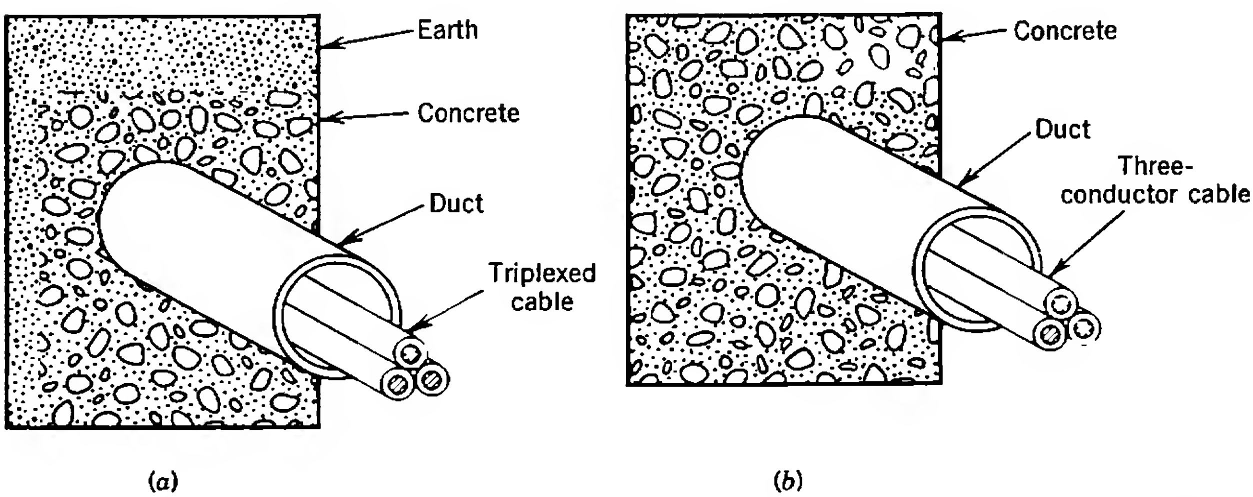


Figure 4.7. Burial in underground cuts (or duct bank): (*a*) for three single-conductor or triplexed, cables, (*b*) for three-conductor cable.

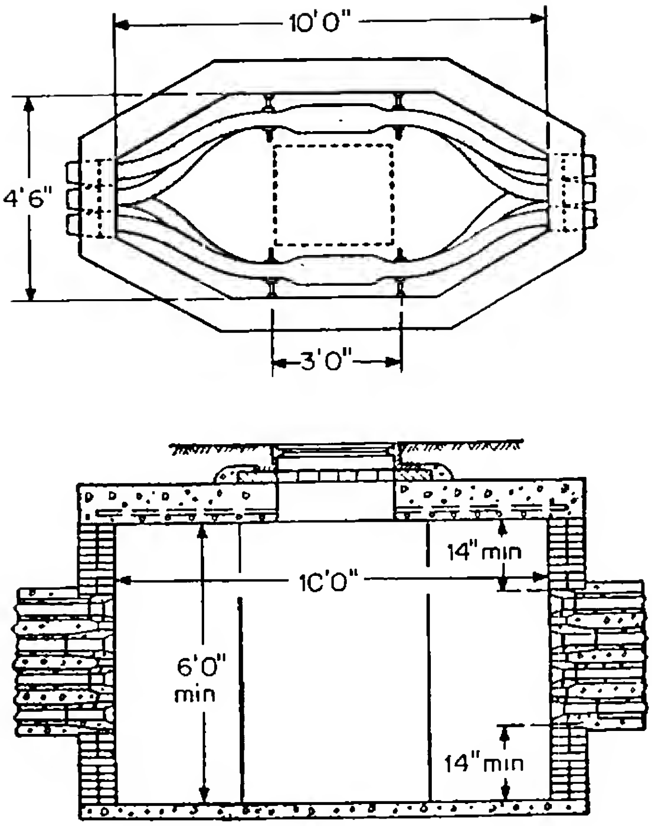


Figure 4.8. Straight-type manhole [4].

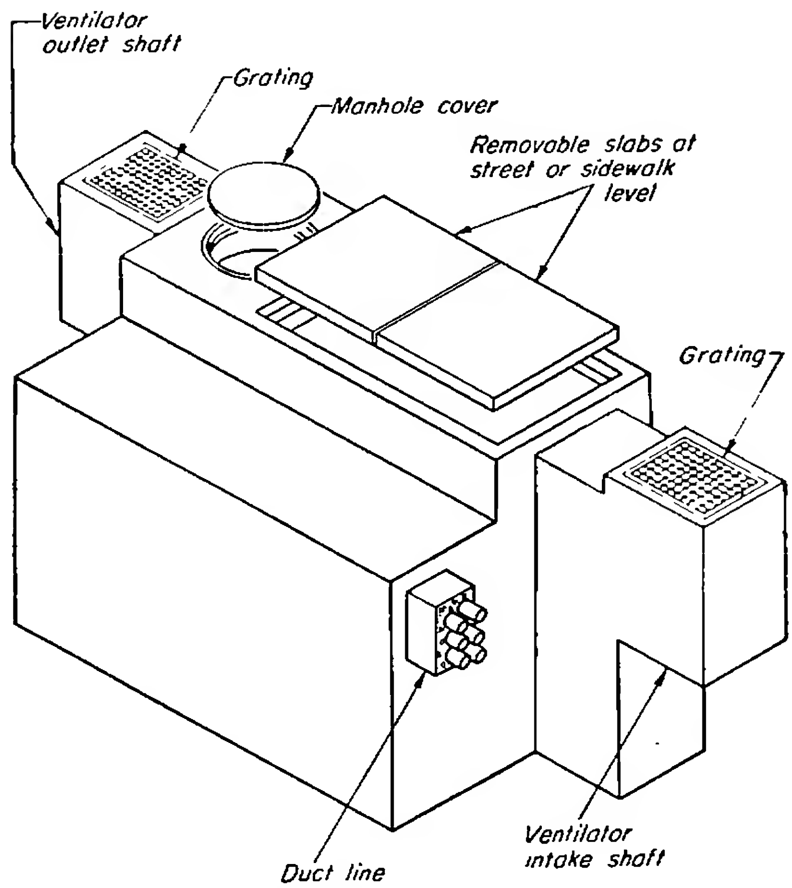


Figure 4.9. Street cable manhole [8].

4.5 ELECTRICAL CHARACTERISTICS OF INSULATED CABLES

4.5.1 Electric Stress in Single-Conductor Cable

Figure 4.10 shows a cross section of a single-conductor cable. Assume that the length of the cable is 1 m.

Let the charge on the conductor surface be q coulomb per meter of length. Assume that the cable has a perfectly homogeneous dielectric and perfect symmetry between conductor and insulation. Therefore, according to Coulomb's law, the electric flux density at a radius of x is

$$D = \frac{q}{2\pi x} \quad \mathrm{C/m^2} \tag{4.1}$$

where D = electric flux density at radius x in Coulombs per square meter
q = charge on conductor surface in Coulombs per square meter
x = distance from center of conductor in meters, where $r < x < R$

Since the absolute permittivity of the insulation is

$$\epsilon = \frac{D}{E} \tag{4.2}$$

the electric field or potential gradient or electric stress or so-called dielectric stress E at radius x is

$$E = \frac{q}{2\pi\epsilon x} \quad \mathrm{V/m} \tag{4.3}$$

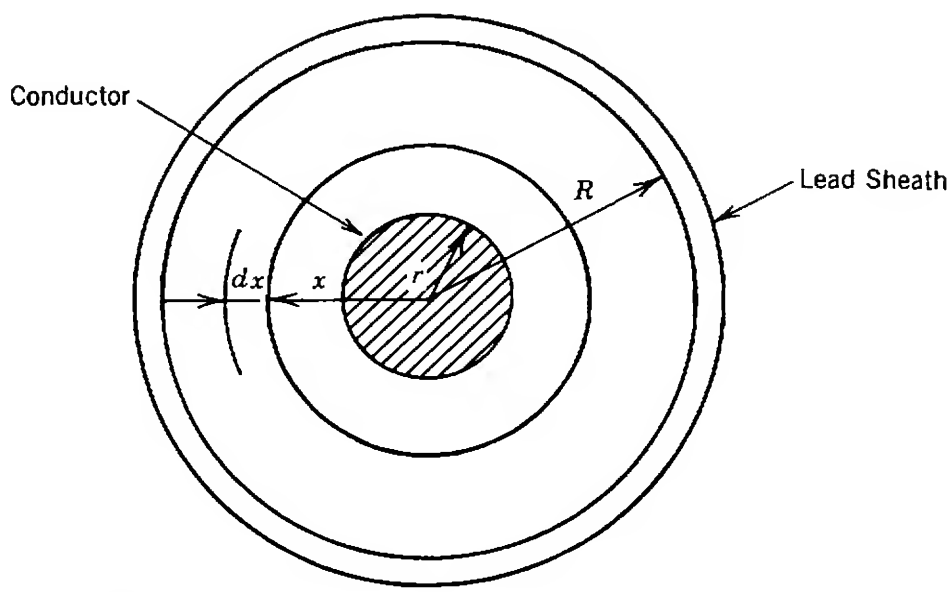


Figure 4.10. Cross section of single-conductor cable.

If the potential gradient at radius x is dV/dx, the potential difference V between conductor and lead sheath is

$$V = \int_r^R E\, dx \tag{4.4}$$

or

$$V = \int_r^R \frac{q}{2\pi\epsilon\chi}\, d\chi \tag{4.5}$$

or

$$V = \frac{q}{2\pi\epsilon} \ln \frac{R}{r} \quad \text{V} \tag{4.6}$$

From equation (4.3),

$$\frac{q}{2\pi\epsilon} = Ex \tag{4.7}$$

substituting it into equation (4.6),

$$V = Ex \ln \frac{R}{r} \quad \text{V} \tag{4.8}$$

Therefore,

$$E = \frac{V}{x \ln \dfrac{R}{r}} \quad \text{V/m} \tag{4.9}$$

where E = electric stress of cable in volts per meter
V = potential difference between conductor and lead sheath in volts
x = distance from center of conductor in meters
R = outside radius of insulation or inside radius of lead sheath in meters
r = radius of conductor in meters

Dielectric strength is the maximum voltage that a dielectric can stand in a uniform field before it breaks down. It represents the permissible voltage gradient through the dielectric.

Average stress is the amount of voltage across the insulation material divided by the thickness of the insulation.

Maximum stress in a cable usually occurs at the surface of the conductor, while the minimum stress occurs at the outer surface of the insulation. Average stress is the amount of voltage across the insulation material divided by the thickness of the insulation.

Therefore, the maximum electric stress in the cable shown in Figure 4.10 occurs at $x = r$; thus,

$$E_{\max} = \frac{V}{r \ln \dfrac{R}{r}} \quad \text{V/m} \tag{4.10}$$

and the minimum electric stress occurs at $x = R$; hence,

$$E_{\min} = \frac{V}{R \ln \dfrac{R}{r}} \quad \text{V/m} \tag{4.11}$$

Thus, for a given V and R, there is one particular radius that gives the minimum stress at the conductor surface. In order to get the smallest value of $E_{\max}$, let

$$\frac{dE_{\max}}{dr} = 0 \tag{4.12}$$

from which

$$\ln \frac{R}{r} = 1 \tag{4.13}$$

or

$$\frac{R}{r} = e \tag{4.14}$$

Thus,

$$R = 2.718r \tag{4.15}$$

and the insulation thickness is

$$R - r = 1.718r \tag{4.16}$$

and the actual stress at the conductor stress is

$$E_{\max} = \frac{V}{r} \tag{4.17}$$

where r is the optimum conductor radius that satisfies equation (4.15).

EXAMPLE 4.1

A single-conductor belted cable of 5 km long has a conductor diameter of 2 cm and an inside diameter of lead sheath of 5 cm. The cable is used at

24.9 kV line-to-neutral voltage and 60 Hz frequency. Calculate the following:

(a) Maximum and minimum values of electric stress.

(b) Optimum value of conductor radius that results in smallest (minimum) value of maximum stress.

Solution

(a) From equation (4.10),

$$E_{max} = \frac{V}{r \ln \frac{R}{r}} = \frac{24.9}{1 \ln 2.5} = 27.17 \text{ kV/cm}$$

and from equation (4.11),

$$E_{min} = \frac{V}{R \ln \frac{R}{r}} = \frac{24.9}{2.5 \ln 2.5} = 10.87 \text{ kV/cm}$$

(b) From equation (4.15), the optimum conductor radius is

$$r = \frac{R}{2.718} = \frac{2.5}{2.718} = 0.92 \text{ cm}$$

Therefore, the minimum value of the maximum stress is

$$E_{max} = \frac{24.9}{0.92 \ln(2.5/0.92)} = 27.07 \text{ kV/cm}$$

EXAMPLE 4.2

Assume that a single-conductor belted cable has a conductor diameter of 2 cm and has insulation of two layers of different materials each 2cm thick, as shown in Figure 4.11. The dielectric constants for the inner and the outer layers are 4 and 3, respectively. If the potential difference between the conductor and the outer lead sheath is 19.94 kV, calculate the potential gradient at the surface of the conductor.

Solution

$$r = 1 \text{ cm}$$

$$r_1 = r + t_1 = 3 \text{ cm}$$

$$R = r_1 + r_2 = 5 \text{ cm}$$

Since

$$E_1 = \frac{2q}{rK_1} \quad \text{and} \quad E_2 = \frac{2q}{r_1 K_2}$$

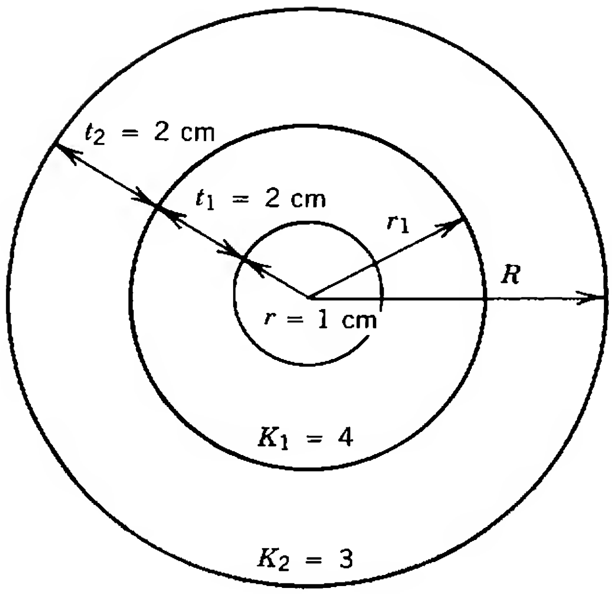


Figure 4.11

their division gives

$$\frac{E_1}{E_2} = \frac{r_1 t_2}{r t_1}$$

$$= \frac{3 \times 3}{1 \times 4} = 2.25$$

In addition,

$$E_1 = \frac{V_1}{r \ln(r_1/r)}$$

$$= \frac{V_1}{1 \ln(3/1)}$$

and

$$E_2 = \frac{V_2}{r_1 \ln(R/r_1)}$$

$$= \frac{19.94 - V_1}{3 \ln(5/3)}$$

or

$$\frac{E_1}{E_2} = \frac{V_1}{1 \ln(3/1)} \; \frac{3 \ln(5/3)}{19.94 - V_1}$$

or

$$\frac{E_1}{E_2} = \frac{1.532V_1}{21.906 - 1.099V_1}$$

but it was found previously that

$$\frac{E_1}{E_2} = 2.25$$

Therefore,

$$\frac{1.532V_1}{21.906 - 1.099V_1} = 2.25$$

from which

$$V_1 = 12.308 \text{ kV}$$

Hence,

$$\begin{aligned} E_1 &= \frac{V_1}{1 \ln 3} \\ &= \frac{12.308}{1 \ln 3} \\ &= 11.20 \text{ kV/cm} \end{aligned}$$

4.5.2. Capacitance of Single-Conductor Cable

Assume that the potential difference is V V between the conductor and the lead sheath of the single-conductor cable shown in Figure 4.10. Let the charges on the conductor and sheath be $+q$ and $-q$ C/m of length. From equation (4.6),

$$V = \frac{q}{2\pi\epsilon} \ln \frac{R}{r} \quad \text{V} \tag{4.6}$$

where V = potential difference between conductor and lead sheath in volts
ϵ = absolute permittivity of insulation
R = outside radius of insulation in meters
r = radius of conductor in meters

Therefore, the capacitance between conductor and sheath is

$$C = \frac{q}{V} \tag{4.18}$$

or

$$C = \frac{2\pi\epsilon}{\ln(R/r)} \quad \text{F/m} \tag{4.19}$$

Since

$$\epsilon = \epsilon_0 K \tag{4.20}$$

Therefore,

$$C = \frac{2\pi\epsilon_0 K}{\ln(R/r)} \quad \text{F/m} \tag{4.21}$$

where

$$\epsilon_0 = \frac{1}{36\pi \times 10^9} \quad \text{F/m for air} \tag{4.22}$$

or

$$\epsilon_0 = 8.85 \times 10^{-12} \quad \text{F/m} \tag{4.23}$$

and

K = dielectric constant of cable insulation[†]

Substituting equation (4.22) into equation (4.21),

$$C = \frac{10^{-9}K}{18 \ln(R/r)} \quad \text{F/m} \tag{4.24}$$

or

$$C = \frac{K}{18 \ln(R/r)} \quad \mu\text{F/km} \tag{4.25}$$

or

$$C = \frac{0.0345K}{\ln(R/r)} \quad \mu\text{F/mi} \tag{4.26}$$

or

$$C = \frac{0.0065K}{10^6 \ln(R/r)} \quad \text{F/1000 ft} \tag{4.27}$$

or

$$C = \frac{0.0241K}{\log_{10}(R/r)} \quad \mu\text{F/km} \tag{4.28}$$

or

$$C = \frac{0.0388K}{\log_{10}(R/r)} \quad \mu\text{F/mi} \tag{4.29}$$

or

$$C = \frac{0.0073K}{10^6 \log_{10}(R/r)} \quad \text{F/1000 ft} \tag{4.30}$$

[†] Note that there has been a shift in notation and K stands for dielectric constant.

TABLE 4.1 Typical Values of Various Dielectric Materials

Dielectric Material	K
Air	1
Impregnated paper	3.3
Polyvinyl chloride (PVC)	3.5–8.0
Ethylene propylene insulation	2.8–3.5
Polyethylene insulation	2.3
Cross-Linked polyethylene	2.3–6.0

Sources: The Okonite Company Bulletin EHB-78 and Ref. 3.

4.5.3 Dielectric Constant of Cable Insulation

The dielectric constant of any material is defined as the ratio of the capacitance of a condenser with the material as a dielectric to the capacitance of a similar condenser with air as the dielectric. It is also called the *relative permittivity* or *specific inductive capacity*. It is usually denoted by K. (It is also represented by ϵ_r or SIC.) Table 4.1 gives the typical values of the dielectric constants for various dielectric materials.

Using the symbol K, for example, in equation (4.30), the formula for calculating the capacitance of a shielded or concentric neutral single-conductor cable becomes

$$C = \frac{0.0073K}{10^6 \log_{10}(D/d)} \quad \text{F}/1000\,\text{ft} \tag{4.31}$$

where C = capacitance in farads per 1000 ft
K = dielectric constant of cable insulation
D = diameter over insulation in unit length
d = diameter over conductor shield in unit length

4.5.4 Charging Current

By definition of susceptance,

$$b = wC \quad \text{S} \tag{4.32}$$

or

$$b = 2\pi fC \quad \text{S} \tag{4.33}$$

Then the admittance $\mathbf{Y}$ corresponding to C is

$$\mathbf{Y} = jb$$

or

$$\mathbf{Y} = j2\pi fC \quad \mathrm{S} \tag{4.34}$$

Therefore, the charging current is

$$\mathbf{I}_c = \mathbf{Y}\mathbf{V}_{(\mathrm{L\text{-}N})} \tag{4.35}$$

or, ignoring j,

$$I_c = 2\pi fCV_{(\mathrm{L\text{-}N})} \tag{4.36}$$

For example, substituting equation (4.31) into equation (4.36), the charging current of a single conductor cable is found as

$$I_c = \frac{2\pi f \times 0.0073KV_{(\mathrm{L\text{-}N})}}{10^6 \log_{10}(D/d)} \tag{4.37}$$

or

$$I_c = \frac{0.0459fKV_{(\mathrm{L\text{-}N})}}{10^3 \log_{10}(D/d)} \quad \mathrm{A/1000\,ft} \tag{4.38}$$

where f = frequency in hertz
D = diameter over insulation in unit length
d = diameter over conductor shield in unit length
K = dielectric constant of cable insulation
V = line-to-neutral voltage in kilovolts

At 60 Hz frequency,

$$I_c = \frac{2.752KV_{(\mathrm{L\text{-}N})}}{10^3 \log_{10}(D/d)} \quad \mathrm{A/1000\,ft} \tag{4.39}$$

The charging current and the capacitance are relatively greater for insulated cables than in overhead circuits because of closer spacing and the higher dielectric constant of the insulation of the cable. In general, the charging current is negligible for overhead circuits at distribution voltages, contrary to high-voltage transmission circuits.

4.5.5 Determination of Insulation Resistance of Single-Conductor Cable

Assume that the cable shown in Figure 4.12 has a length of 1 m.

Then the incremental insulation resistance of the cylindrical element in the radial direction is

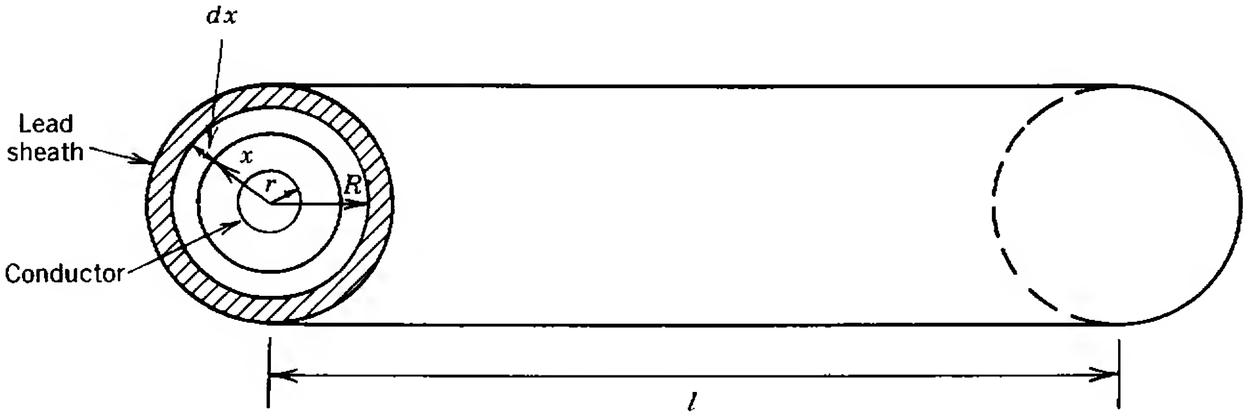


Figure 4.12. Cross section of single-conductor cable.

$$\Delta R_i = \frac{\rho}{2\pi x l}\,dx \tag{4.40}$$

Therefore, the total insulation resistance between the conductor and the lead sheath is

$$R_i = \int_r^R \frac{\rho}{2\pi l}\,\frac{dx}{x}$$

or

$$R_i = \frac{\rho}{2\pi l}\ln\frac{R}{r} \tag{4.41}$$

where R_i = total insulation resistance in ohms
ρ = insulation (dielectric) resistivity in ohm meters
l = total length of cable in meters
R = outside radius of insulation or inside radius of lead sheath in meters
r = radius of conductor in meters

A more practical version of equation (4.41) is given by the Okonite Company† as

$$R_i = r_{si}\log\frac{D}{d} \quad \text{M}\Omega/1000\ \text{ft} \tag{4.42}$$

where R_i = total insulation resistance in megohms per 1000 ft for particular cable construction
r_{si} = specific insulation resistance in megohms per 1000 ft at 60 °F
D = inside diameter of sheath
d = outside diameter of conductor

† Engineering Data for Copper and Aluminum Conductor Electrical Cables, by the Okonite Company. Bulletin EHB-78. (Used with permission.)

Table 4.2 gives typical r_{si} values of various insulation materials.

Equation (4.19) indicates that the insulation resistance is inversely proportional to the length of the insulated cable. An increase in insulation thickness increases the disruptive critical voltage of the insulation but does not give a *proportional* decrease in voltage gradient at the conductor surface. Therefore, it does not permit a *proportional* increase in voltage rating.

EXAMPLE 4.3

A 250-kcmil, single-conductor, synthetic rubber, belted cable has a conductor diameter of 0.575 in. and an inside diameter of sheath of 1.235 in. The cable has a length of 6000 ft and is going to be used at 60 Hz and 115 kV. Calculate the following:

(a) Total insulation resistance in megohms at 60 °F.
(b) Power loss due to leakage current flowing through insulation resistance.

Solution

(a) By using equation (4.42),

$$R_i = r_{si} \log \frac{D}{d}$$

From Table 4.2, specific insulation resistance r_{si} is 2000 MΩ/1000 ft. Therefore, the total insulation resistance is

$$R_i = 6 \times 2000 \log \frac{1.235}{0.575}$$
$$= 3.984\ \text{M}\Omega$$

(b) The power loss due to leakage current is

$$\frac{V^2}{R_i} = \frac{115{,}000^2}{3984 \times 10^6}$$
$$= 3.3195\ \text{W}$$

TABLE 4.2 Typical Values of r_{si}

Insulation Material	r_{si} (MΩ/1000 ft)
Synthetic rubber	2,000
Ethylene propylene insulation	20,000
Polyethylene	50,000
PVC	2,000
Cross-linked polyethylene	20,000

4.5.6 Capacitance of Three-Conductor Belted Cable

As shown in Figure 4.13, two insulation thicknesses are to be considered in belted cables: (1) the conductor insulation of thickness T and (2) the belt insulation of thickness t. The belt insulation is required because with line voltage $\mathbf{V}_L$ between conductors, the conductor insulation is only adequate for $\frac{1}{2}\mathbf{V}_L$ voltage, whereas the voltage between each conductor and ground (or earth) is $\mathbf{V}_L/\sqrt{3}$.

In the three-conductor belted cable, there are capacitances of C_c between conductors and capacitances of C_s between each conductor and the sheath, as shown in Figure 4.14. The arrangement of the capacitors, representing these capacitances per unit length, is equivalent to a delta system connected in parallel with a wye system, as shown in Figure 4.15. Further, the delta system, representing the capacitances C_c, can be represented by an equivalent wye system of capacitance C_1, as shown in Figure 4.16. In the delta system, the capacitance between, say, conductors 1 and 2 is

$$C_c + \tfrac{1}{2}C_c = \tfrac{3}{2}C_c \tag{4.43}$$

In the wye system, it is

$$\tfrac{1}{2}C_1 \tag{4.44}$$

Since the delta and wye systems are equivalent, the capacitance between the conductors must be the same:

$$\tfrac{3}{2}C_c = \tfrac{1}{2}C_1 \tag{4.45}$$

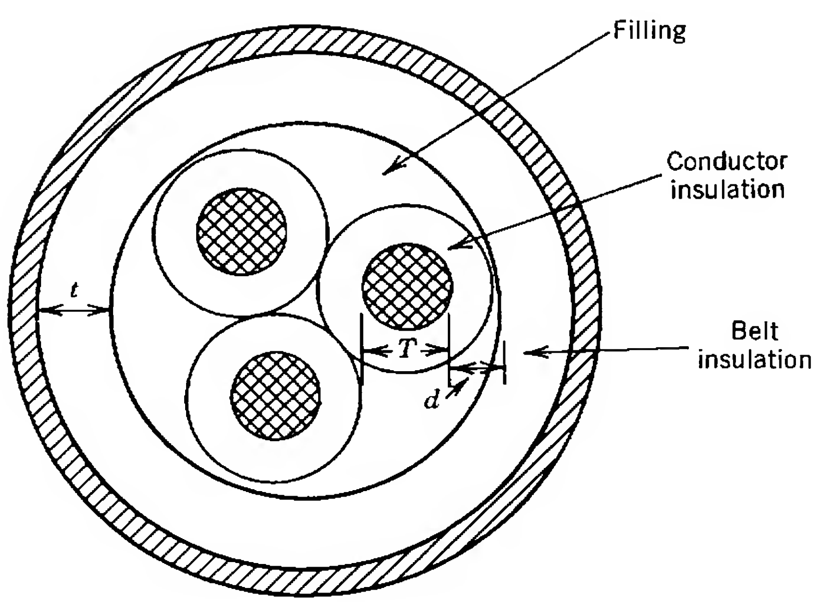


Figure 4.13. Three-conductor belted cable cross section.

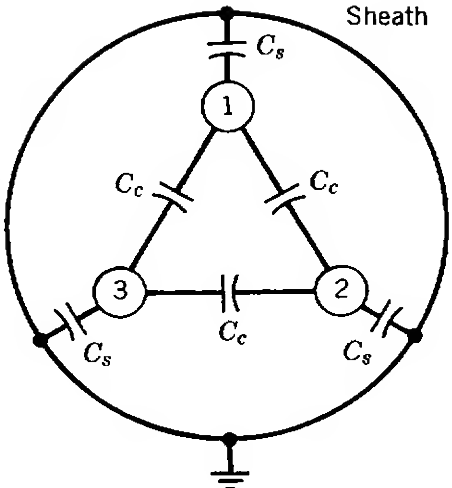


Figure 4.14. Effective capacitances.

or

$$C_1 = 3C_c \tag{4.46}$$

Alternatively, let the voltage across capacitor C_c in the delta system be $\mathbf{V}_{(\text{L-L})}$, the line-to-line voltage. Therefore, the phase current through the capacitor is equal to $wC_c\mathbf{V}_{(\text{L-L})}$, and the line current is

$$\mathbf{I}_L = 3wC_c\mathbf{V}_{(\text{L-L})} \tag{4.47}$$

On the other hand, in the equivalent wye systems, the line-to-neutral voltage is

$$\mathbf{V}_{(\text{L-N})} = \frac{1}{\sqrt{3}}\,\mathbf{V}_{(\text{L-L})} \tag{4.48}$$

and the phase current and the line current are the same. Therefore,

$$\mathbf{I}_L = wC_1\,\frac{\mathbf{V}_{(\text{L-L})}}{\sqrt{3}} \tag{4.49}$$

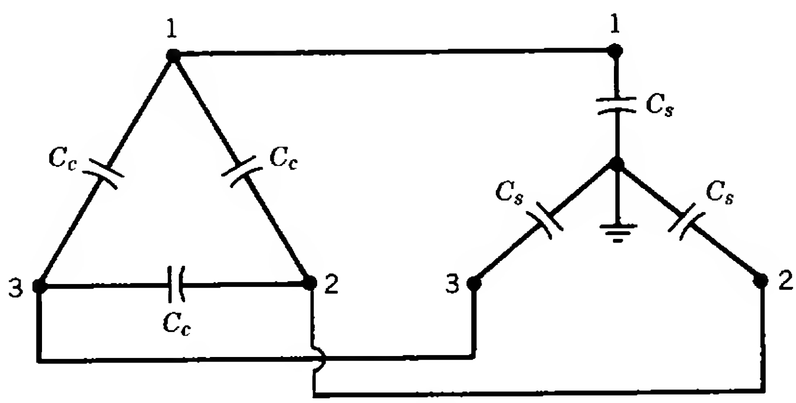


Figure 4.15. Equivalent circuit.

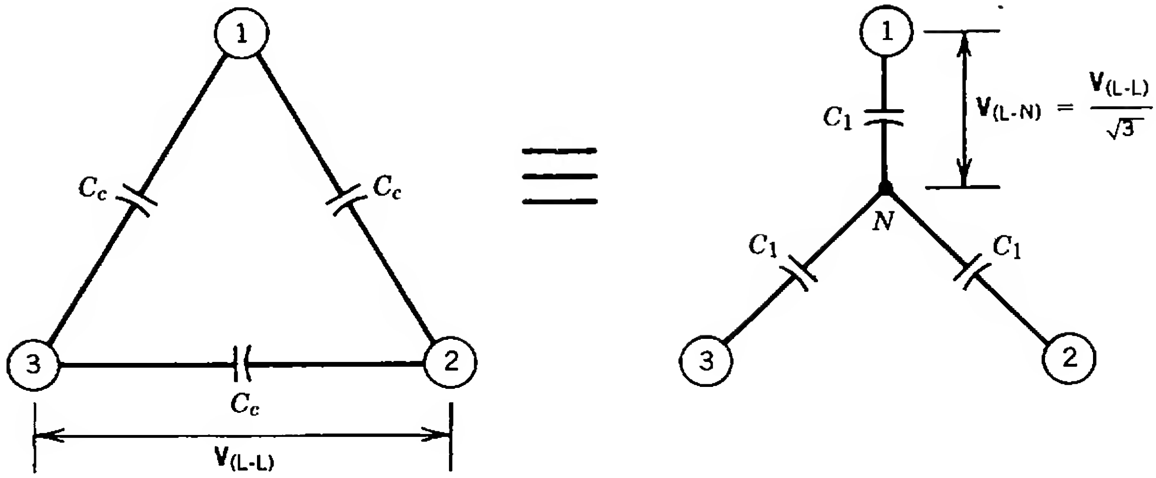


Figure 4.16

Thus, for equivalent delta and wye systems, by equating equations (4.47) and (4.49),

$$3wC_c\mathbf{V}_{(\mathrm{L\text{-}L})} = wC_1 \frac{\mathbf{V}_{(\mathrm{L\text{-}L})}}{3}$$

or

$$C_1 = 3C_c \tag{4.50}$$

which is as same as equation (4.46). Therefore, the delta system is converted to the wye system, as shown in Figure 4.16. All C_s capacitors are in wye connection with respect to the sheath and all C_1 capacitors are in wye connection with respect to the neutral point N and in parallel with the first wye system of capacitors. The effective capacitance of each conductor to the grounded neutral is therefore

$$C_N = C_s + 3C_c \tag{4.51}$$

The value of C_N can be calculated with usually acceptable accuracy by using the formula

$$C_N = \frac{0.048K}{\log_{10}\{1 + [(T + t)/d][3.84 - 1.7(t/T) + 0.52(t^2/T^2)]\}} \quad \mu\mathrm{F/mi} \tag{4.52}$$

where K = dielectric constant of insulation
T = thickness of conductor insulation
t = thickness of belt insulation
d = diameter of conductor

In general, however, since the conductors are not surrounded by isotropic homogeneous insulation of one known permittivity, the C_c and C_s are not easily calculated and are generally obtained by measurements. The tests are performed at the working voltage, frequency, and temperature.

In determining the capacitances of this type of cable, the common tests are:

1. Measure the capacitance C_a between two conductors by means of a Schering bridge connecting the third conductor to the sheath to eliminate one of the C_s's, as shown in Figures 4.17 and 4.18. Therefore,

$$C_a = C_c + \tfrac{1}{2}(C_c + C_s) \tag{4.53}$$

or

$$C_a = \tfrac{1}{2}(C_s + 3C_c) \tag{4.54}$$

Substituting equation (4.51) into equation (4.54),

$$C_a = \tfrac{1}{2}C_N \tag{4.55}$$

or

$$C_N = 2C_a \tag{4.56}$$

2. Measure the capacitance C_b between the sheath and all three conductors joined together to eliminate (or to short out) all three C_c's and to parallel all three C_s's, as shown in Figure 4.19. Therefore,

$$C_b = 3C_s \tag{4.57}$$

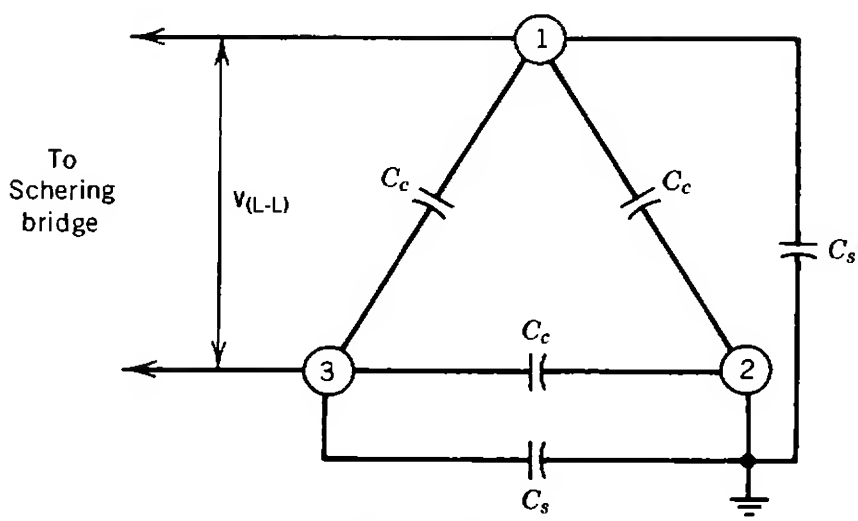


Figure 4.17

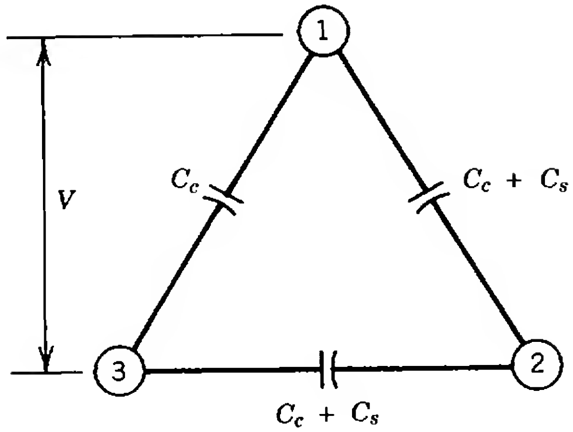


Figure 4.18

or

$$C_s = \tfrac{1}{3} C_b \tag{4.58}$$

3. Connect two conductors to the sheath, as shown in Figure 4.20. Measure the capacitance C_d between the remaining single conductor and the two other conductors and the sheath. Therefore,

$$C_d = C_s + 2C_c \tag{4.59}$$

or

$$2C_c = C_d - C_s \tag{4.60}$$

Substituting equation (4.58) into equation (4.60),

$$C_c = \tfrac{1}{2}(C_d - \tfrac{1}{3} C_b) \tag{4.61}$$

Substituting this equation and equation (4.58) into equation (4.51), the effective capacitance to neutral is

$$C_N = \tfrac{1}{6}(9C_d - C_b) \tag{4.62}$$

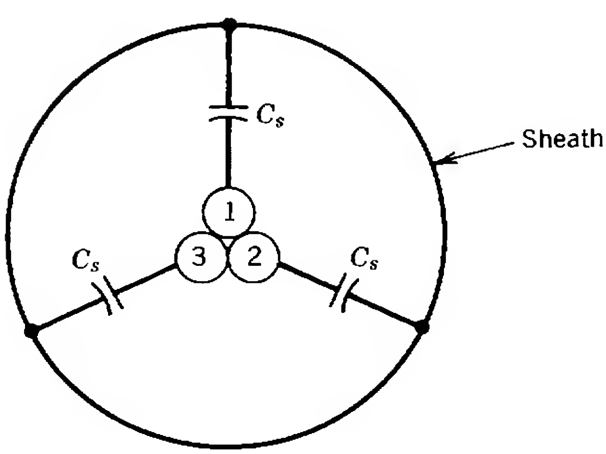


Figure 4.19

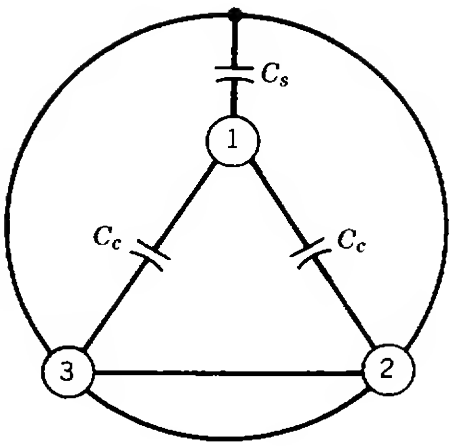


Figure 4.20

EXAMPLE 4.4

A three-conductor three-phase cable has 2 mi of length and is being used at 34.5 kV, three phase, and 60 Hz. The capacitance between a pair of conductors on a single phase is measured to be 2 μF/mi. Calculate the charging current of the cable.

Solution

The capacitance between two conductors is given as

$$C_a = 2\ \mu\text{F/mi}$$

or for total cable length,

$$C_a = (2\ \mu\text{F/mi}) \times (2\ \text{mi})$$
$$= 4\ \mu\text{F}$$

The capacitance of each conductor to neutral can be found by using equation (4.56),

$$C_N = 2C_a$$
$$= 8\ \mu\text{F}$$

Therefore, the charging current is

$$I_c = wC_N V_{(\text{L-N})}$$
$$= 2\pi \times 60 \times 8 \times 10^{-6} \times 19{,}942$$
$$= 60.14\ \text{A}$$

EXAMPLE 4.5

Assume that a three-conductor belted cable 4 mi long is used as a three-phase underground feeder and connected to a 13.8-kV, 60-Hz substation bus. The load, at the receiving end, draws 40 A at 0.85 lagging power factor.

The capacitance between any two conductors is measured to bè 0.45 μF/mi. Ignoring the power loss due to leakage current and also the line voltage drop, calculate the following:

(a) Charging current of feeder.
(b) Sending-end current
(c) Sending-end power factor.

Solution

The current phasor diagram is shown in Figure 4.21.

(a) The capacitance between two conductors is given as

$$C_a = 0.45\ \mu\text{F/mi}$$

or for total feeder length,

$$C_a = 0.45\ \mu\text{F/mi} \times 4\ \text{mi} = 1.80\ \mu\text{F}$$

The capacitance of each conductor to neutral can be found by using equation (4.56),

$$C_N = 2C_a$$
$$= 3.6\ \mu\text{F}$$

Thus, the charging current is

$$I_c = wC_N V_{(\text{L-N})}$$
$$= 2\pi \times 60 \times 3.6 \times 10^{-6} \times 13{,}800 \times \frac{1}{1.73}$$
$$= 10.83\ \text{A}$$

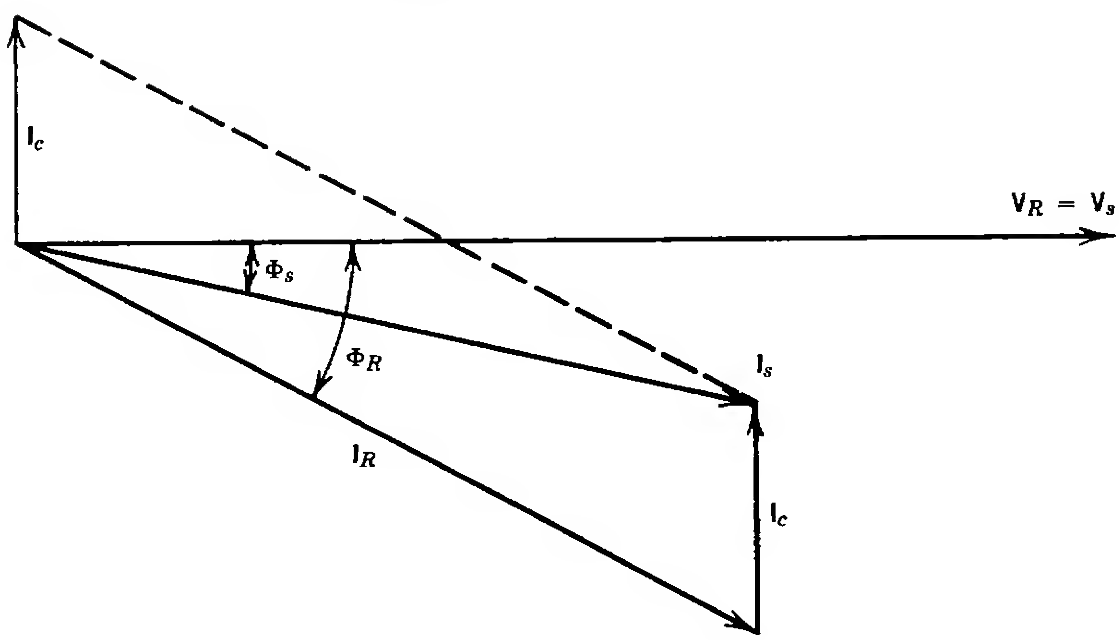


Figure 4.21

or, in complex form,

$$\mathbf{I}_c = +j10.83 \text{ A}$$

(b) The receiving-end current is

$$\begin{aligned}\mathbf{I}_r &= 30(\cos\phi_r - j\sin\phi_r)\\ &= 30(0.85 - j0.5268)\\ &= 25.5 - j15.803 \text{ A}\end{aligned}$$

Therefore, the sending-end current is

$$\begin{aligned}\mathbf{I}_s &= \mathbf{I}_r + \mathbf{I}_c\\ &= 25.5 - j15.803 + j10.83\\ &= 25.5 - j4.973\\ &= 25.98 \underline{/-11.04^\circ} \text{ A}\end{aligned}$$

(c) Hence, the sending-end power factor is

$$\begin{aligned}\cos\phi_s &= \cos 11.04^\circ\\ &= 0.98\end{aligned}$$

and it is a lagging power factor.

4.5.7 Cable Dimensions

Overall diameter of a cable may be found from the following equations. They apply to conductors of circular cross section. For a single-conductor cable,

$$D = d + 2T + 2S \tag{4.63}$$

For a two-conductor cable,

$$D = 2(d + 2T + t + S) \tag{4.64}$$

For a three-conductor cable,

$$D = 2.155(d + 2T) + 2(t + S) \tag{4.65}$$

For a four-conductor cable,

$$D = 2.414(d + 2T) + 2(t + S) \tag{4.66}$$

For a sector-type three-conductor cable,

$$D_{3s} = D - 0.35d \tag{4.67}$$

where D = overall diameter of cable with circular cross-sectional conductors
D_{3s} = overall diameter of cable with sector-type three conductors
d = diameter of conductor
S = lead sheath thickness of cable
t = belt insulation thickness of cable
T = thickness of conductor insulation in inches

4.5.8 Geometric Factors

The geometric factor is defined as the relation in space between the cylinders formed by sheath internal surface and conductor external surface in a single-conductor belted cable. For a three-conductor belted cable, this relation (i.e., geometric factor) is sector shaped, and by relative thicknesses of conductor insulation T and belt insulation t. For a single-conductor cable, the geometric factor G is given by

$$G = 2.303 \log_{10} \frac{D}{d} \tag{4.68}$$

where D = inside diameter of sheath
d = outside diameter of conductor

Table 4.3 presents geometric factors for single-conductor and three-conductor belted cables. In this table, G indicates the geometric factor for a single-conductor cable, G_0 indicates the zero-sequence geometric factor, and G_1 indicates the positive-sequence geometric factor for three-conductor belted cables. Also, Figures 4.22 and 4.23 give geometric factors for single-conductor and three-conductor belted cables. In Figure 4.24, G_0 indicates the zero-sequence geometric factor and G_1 indicates the positive-sequence geometric factor.

In Table 4.3 and Figures 4.22 and 4.23,

T = thickness of conductor insulation in inches

t = thickness of belt insulation in inches

d = outside diameter of conductor in inches

For single-conductor cables,

$$t = 0$$

TABLE 4.3 Table of Geometric Factors of Cables

Ratio $\frac{T+t}{d}$	Single conductor, G	Sector Factor	Three-Conductor cables					
			G_0 at ratio t/T			G_1 at ratio t/T		
			0	0.5	1.0	0	0.5	1.0
0.2	0.34		0.85	0.85	0.85	1.2	1.28	1.4
0.3	0.47	0.690	1.07	1.075	1.03	1.5	1.65	1.85
0.4	0.59	0.770	1.24	1.27	1.29	1.85	2.00	2.25
0.5	0.69	0.815	1.39	1.43	1.46	2.10	2.30	2.60
0.6	0.79	0.845	1.51	1.57	1.61	2.32	2.55	2.95
0.7	0.88	0.865	1.62	1.69	1.74	2.35	2.80	3.20
0.8	0.96	0.880	1.72	1.80	1.86	2.75	3.05	3.45
0.9	1.03	0.895	1.80	1.89	1.97	2.96	3.25	3.70
1.0	1.10	0.905	1.88	1.98	2.07	3.13	3.44	3.87
1.1	1.16	0.915	1.95	2.06	2.15	3.30	3.60	4.03
1.2	1.22	0.921	2.02	2.13	2.23	3.45	3.80	4.25
1.3	1.28	0.928	2.08	2.19	2.29	3.60	3.95	4.40
1.4	1.33	0.935	2.14	2.26	2.36	3.75	4.10	4.60
1.5	1.39	0.938	2.20	2.32	2.43	3.90	4.25	4.75
1.6	1.44	0.941	2.26	2.38	2.49	4.05	4.40	4.90
1.7	1.48	0.944	2.30	2.43	2.55	4.17	4.52	5.05
1.8	1.52	0.946	2.35	2.49	2.61	4.29	4.65	5.17
1.9	1.57	0.949	2.40	2.54	2.67	4.40	4.76	5.30
2.0	1.61	0.952	2.45	2.59	2.72	4.53	4.88	5.42

Source: From Fink and Beaty [4].

Therefore,

$$\frac{T+t}{d} = \frac{T}{d} \tag{4.69}$$

which is used to find the value of geometric factor G for a single-conductor cable.

The geometric factor can be useful to calculate various cable characteristics such as capacitance, charging current, dielectric loss, leakage current, and heat transfer. For example, the general capacitance equation is given as [1]

$$C = \frac{0.0169nK}{G} \quad \mu\text{F}/1000\ \text{ft} \tag{4.70}$$

where K = dielectric constant of insulation
n = number of conductors
G = geometric factor

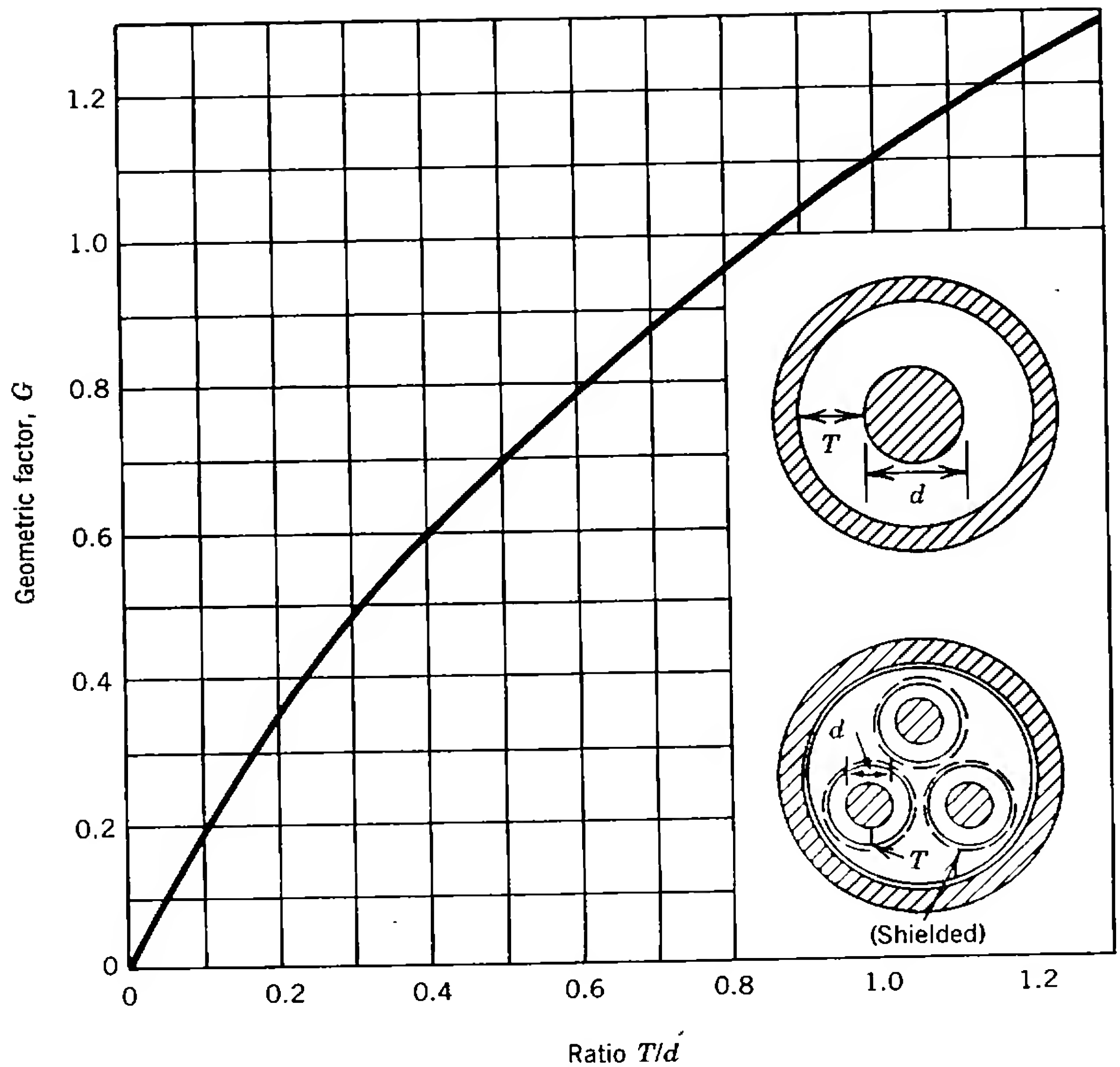


Figure 4.22. Geometric factor for single-conductor cables, or three-conductor shielded cables having round conductors [2].

Also, the charging current of a three-conductor three-phase cable is given as [5]

$$I = \frac{3 \times 0.106 fK\mathrm{V}_{(\mathrm{L\text{-}N})}}{1000 G_1} \quad \mathrm{A/1000\,ft} \tag{4.71}$$

where f = frequency in hertz
K = dielectric constant of insulation
$\mathrm{V}_{(\mathrm{L\text{-}N})}$ = line-to-neutral voltage in kilovolts
G_1 = geometric factor for three-conductor cable from Table 4.3

Example 4.6

A 60-Hz, 138-kV, three-conductor, paper-insulated, belted cable is going to be installed at 138 kV and used as a three-phase underground feeder. The

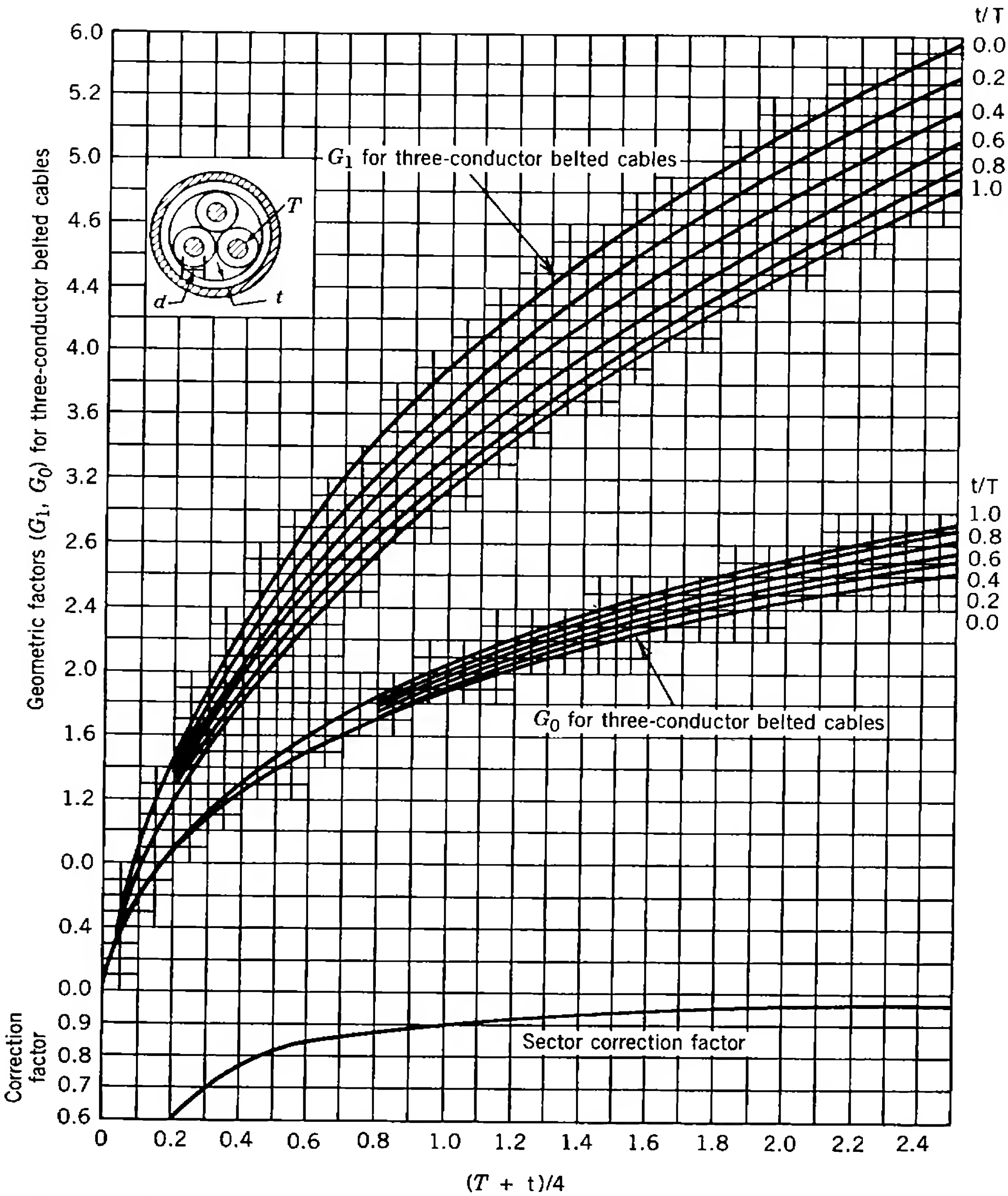


Figure 4.23. Geometric factor for three-conductor belted cables having round or sector conductors.

cable has three 250-kcmil sector-type conductors each with $\frac{11}{64}$ in. of conductor insulation and $\frac{5}{64}$ in. of belt insulation. Calculate the following:

(a) Geometric factor of cable using Table 4.3.

(b) Charging current in amperes per 1000 ft.

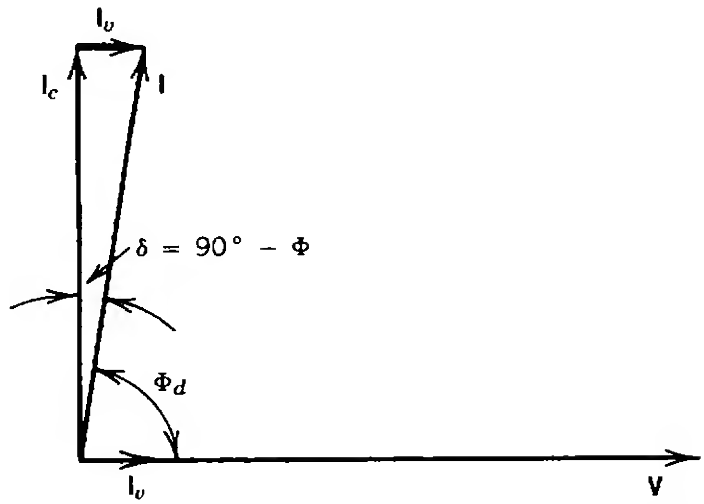


Figure 4.24. Phasor diagram for cable dielectric.

Solution

(a) $T = 0.172$ in., $t = 0.078$, $d = 0.575$, $t/T \cong 0.454$:

$$\frac{t}{T} \simeq 0.454 \quad \text{and} \quad \frac{T+t}{d} = \frac{0.172 + 0.078}{0.575} \simeq 0.435$$

From Table 4.3, by interpolation,

$$G_1 = 2.09$$

Since the cable has sector-type conductors, to find the real geometric factor G_1', G_1 has to be multiplied by the sector factor obtained for $(T + t)/d = 0.435$ from Table 4.3, by interpolation,

$$\begin{aligned} G_1' &= G_1 \times (\text{sector factor}) \\ &= 2.09 \times 0.7858 \\ &= 1.642 \end{aligned}$$

(b) $$\begin{aligned} V_{(\text{L-N})} &= \frac{V_{(\text{L-L})}}{\sqrt{3}} \\ &= \frac{138 \text{ kV}}{\sqrt{3}} = 79.6743 \text{ kV} \end{aligned}$$

For impregnated-paper cable K is 3.3. Therefore, using equation (4.71), the charging current is

$$\begin{aligned} I_c &= \frac{3 \times 0.106 f K V_{(\text{L-N})}}{1000 G_1} \\ &= \frac{3 \times 0.106 \times 60 \times 3.3 \times 79.6743}{1000 \times 1.642} \\ &= 3.055 \text{ A}/1000 \text{ ft} \end{aligned}$$

4.5.9 Dielectric Power Factor and Dielectric Loss

When a voltage is applied across a perfect dielectric, there is no dielectric loss because of the existence of an induced capacitance current I_c, located 90° ahead of the voltage V. However, in practice, since a perfect dielectric cannot be achieved, there is a small current component I_V that is in phase with voltage V. Therefore, the summation of these two current vectors gives the current vector **I** that leads the voltage V by less than 90°, as shown in Figure 4.24. The cosine of the angle Φ_d is the power factor of the dielectric, which provides a useful measure of the quality of the cable dielectric. The power factor of a dielectric is

$$\cos \Phi_d = \frac{\text{losses in dielectric (W)}}{\text{apparent power (VA)}} \tag{4.72}$$

The power factor of an impregnated-paper dielectric is very small, approximately 0.003. The dielectric power factor should not be confused with the regular (supply) power factor. The dielectric power factor represents loss and therefore, an attempt to reduce it should be made. Whereas an attempt should be made to increase the supply power factor toward unity.

Since for a good dielectric insulation, Φ is close to 90°, δ is sometimes called the *dielectric loss angle*. Therefore, δ is, in radians,

$$\delta \simeq \tan \delta \simeq \sin \delta = \cos \Phi_d \tag{4.73}$$

since $\delta = 90 - \Phi_d$ and $\delta < 0.5°$ for most cables.

Here $\cos \Phi_d$ should be held very small under all operating conditions. If it is large, the power loss is large, and the insulation temperature T rises considerably. The rise in temperature causes a rise in power loss in the dielectric, which again results in additional temperature rise. If the cable is to operate under conditions where $\partial(\cos \Phi_d)/\partial T$ is significantly large, the temperature continues to increase until the insulation of the cable is damaged.

When an ac $V_{(\text{L-N})}$ voltage is applied across the effective cable capacitance C, the power loss in the dielectric, P_{dl}, is

$$P_{dl} = \omega C V^2_{(\text{L-N})} \cos \Phi_d \tag{4.74}$$

This is larger than the dielectric power loss if the applied voltage is dc. The increase in the power loss is due to dielectric hysteresis, and it usually is much greater than leakage loss. The dielectric hysteresis loss cannot be measured separately. The total dielectric loss, consisting of dielectric hysteresis loss and the power loss due to leakage current flowing through the insulation resistance, can be measured by means of the Schering bridge. These losses depend on voltage, frequency, and the state of the cable dielectric. Therefore, the test has to be made at rated voltage and frequency for a given cable.

For a balanced three-phase circuit, the dielectric loss at rated voltage and temperature is

$$P_{dl} = 3wCV^2_{(\text{L-N})} \cos \Phi_d \quad \text{W/1000 ft} \tag{4.75}$$

where P_{dl} = cable dielectric loss in watts per 1000 ft
$w = 2\pi f$
C = positive-sequence capacitance to neutral in farads per 1000 ft
$V_{(\text{L-N})}$ = line-to-neutral voltage in kilovolts
$\cos \Phi_d$ = power factor of dielectric (insulation) at given temperature

EXAMPLE 4.7

A single-conductor belted cable has a conductor diameter of 0.814 in., inside diameter of sheath of 2.442 in., and a length of 3.5 mi. The cable is to be operated at 60 Hz and 7.2 kV. The dielectric constant is 3.5, the power factor of the dielectric on open circuit at a rated frequency and temperature is 0.03, and the dielectric resistivity of the insulation is 1.3×10^7 MΩ-cm. Calculate the following:

(a) Maximum electric stress occurring in cable dielectric.
(b) Capacitance of cable.
(c) Charging current of cable.
(d) Insulation resistance.
(e) Power loss due to leakage current flowing through insulation resistance.
(f) Total dielectric loss.
(g) Dielectric hysteresis loss.

Solution

(a) By using equation (4.10),

$$E_{\max} = \frac{V}{r \ln(R/r)} \quad \text{V/m}$$

$$E_{\max} = \frac{7.2}{0.407 \times 2.54 \ln 3}$$

$$= 6.34 \text{ kV/cm}$$

(b) From equation (4.29),

$$C = \frac{0.0388K}{\log_{10}(R/r)} \quad \mu\text{F/mi}$$

$$= \frac{0.0388 \times 3.5}{\log_{10} 3}$$

$$= 0.2846 \ \mu\text{F/mi}$$

or the capacitance of the cable is

$$0.2846\ \mu\text{F/mi} \times 3.5\ \text{mi} = 0.9961\ \mu\text{F}$$

(c) By using equation (4.36),

$$\begin{aligned} I_c &= 2\pi f C V_{(\text{L-N})} \quad \text{A} \\ &= \frac{2\pi \times 60 \times 0.9961 \times 7.2}{10^3} \\ &= 2.704\ \text{A} \end{aligned}$$

(d) From equation (4.41),

$$\begin{aligned} R_i &= \frac{\rho}{2\pi l} \ln \frac{R}{r} \\ &= \frac{1.3 \times 10^7}{2\pi \times 3.5 \times 5280 \times 12 \times 2.54} \ln 3 \\ &= 4\ \text{M}\Omega \end{aligned}$$

(e) The power loss due to leakage current flowing through the insulation is

$$P_{lc} = \frac{V^2}{R_i} = \frac{7200^2}{4 \times 10^6} = 12.85\ \text{W}$$

(f) The total dielectric loss is

$$P_{dl} = V_{(\text{L-N})} I \cos \Phi_d$$

or

$$P_{dl} = V_{(\text{L-N})} I \sin \delta$$

or

$$P_{dl} = wCV^2_{(\text{L-N})} \sin \delta$$

or

$$\begin{aligned} P_{dl} &= wCV^2_{(\text{L-N})} \delta \\ &= 2\pi \times 60 \times 0.9961 \times 7200^2 \times 0.03 \\ &= 584.01\ \text{W} \end{aligned}$$

(g) The dielectric hysteresis loss is

$$\begin{aligned} P_{dh} &= P_{dl} - P_{lc} \\ &= 584.01 - 12.85 \\ &= 571.16\ \text{W} \end{aligned}$$

4.5.10 Effective Conductor Resistance

The factors that determine effective ac resistance R_{eff} of each conductor of a cable are

1. dc resistance,
2. skin effect,
3. proximity effect,
4. sheath losses, and
5. armor losses if there is any armor.

Therefore, the effective resistance R_{eff} in ac resistance can be given as

$$R_{eff} = (\lambda_1 + \lambda_2 + \lambda_3 + \lambda_4)R_{dc} \tag{4.76}$$

where R_{dc} = dc resistance of conductor
λ_1 = constant (or resistance increment) due to skin effect
λ_2 = constant (or resistance increment) due to proximity effect
λ_3 = constant (or resistance increment) due to sheath losses
λ_4 = constant (or resistance increment) due to armor losses

For example, λ_3 constant can be calculated as follows, since

$$\text{Sheath loss} = \lambda_3 \times (\text{conductor loss})$$

$$\lambda_3 = \frac{\text{sheath loss}}{\text{conductor loss}}$$

Similarly, since

$$\text{Armor loss} = \lambda_4 \times (\text{conductor loss})$$

then

$$\lambda_4 = \frac{\text{armor loss}}{\text{conductor loss}}$$

4.5.11 Direct-Current Resistance

Direct-current resistance R_{dc} of a conductor is

$$R_{dc} = \frac{\rho l}{A}$$

where ρ = resistivity of conductor
l = conductor length
A = cross-sectional area

The units used must be of a consistent set. In practice, several different sets of units are used in the calculation of resistance. For example, in the International System of Units (SI units), l is in meters, A is in square meters and ρ is in ohms per meter. Whereas in power systems in the United States, ρ is in ohm-circular mils per foot (Ω-cmil/ft), or ohms per circular mil-foot, l is usually in feet, and A is in circular mils (cmil). Resistivity ρ is 10.66 Ω-cmil/ft, or 1.77×10^{-8} Ω-m, at 20 °C for hard-drawn copper and 10.37 Ω-cmil/ft at 20 °C for standard annealed copper. For hard-drawn aluminum at 20 °C ρ is 17.00 Ω-cmil/ft, or 2.83×10^{-8} Ω-m.

The dc resistance of a conductor in terms of temperature is given by

$$\frac{R_2}{R_1} = \frac{T_0 + t_2}{T_0 + t_1}$$

where R_1 = conductor resistance at temperature t_1
R_2 = conductor resistance at temperature t_2
t_1, t_2 = conductor temperatures in degrees Celsius
T_0 = constant varying with conductor material
= 234.5 for annealed copper
= 241 for hard-drawn copper
= 228 for hard-drawn aluminum

The maximum allowable conductor temperatures are given by the Insulated Power Cable Engineers Association (IPCEA) for polyethylene and cross-linked-polyethylene-insulated cables as follows:

Under Normal Operation

Polyethylene insulated cables:	75 °C
Cross-linked polyethylene insulated cables:	90 °C

Under Emergency Operation

Polyethylene insulated cables:	90 °C
Cross-linked polyethylene insulted cables:	130 °C

The maximum conductor temperatures for impregnated paper-insulated cables are given in Table 4.4.

4.5.12 Skin Effect

For dc currents, a uniform current distribution is assumed throughout the cross section of a conductor. This is not true for alternating current. As the frequency of ac current increases, the nonuniformity of current becomes greater. The current tends to flow more densely near the outer surface of

TABLE 4.4 Maximum Conductor Temperatures for Impregnated Paper-Insulated Cable

Rated voltage kV	Conductor Temperature, °C		
	Normal Operation	Emergency Operation	
Solid-Type Multiple Conductor Belted			
1	85	105	
2–9	80	100	
10–15	75	95	
Solid-Type Multiple Conductor Shielded and Single Conductor			
1–9	85	105	
10–17	80	100	
18–29	75	95	
30–39	70	90	
40–49	65	85	
50–59	60	75	
60–69	55	70	
Low-Pressure Gas-Filled			
8–17	80	100	
18–29	75	95	
30–39	70	90	
40–46	65	85	
Low-Pressure Oil-Filled and High-Pressure Pipe Type			
		100 h	300 h
15–17	85	105	100
18–39	80	100	95
40–162	75	95	90
163–230	70	90	85

Source: From Refs. 4 and 6.

the conductor than near the center. The phenomenon responsible for this nonuniform distribution is called *skin effect.*

Skin effect is present because the magnetic flux linkages of current near the center of the conductor are relatively greater than the linkages of current flowing near the surface of the conductor. Since the inductance of any element is proportional to the flux linkages per ampere, the inner areas of the conductor offer greater reactance to current flow. Therefore, the current follows the outer paths of lower reactance, which in turn reduces the effective path area and increases the effective resistance of the cable.

Skin effect is a function of conductor size, frequency, and the relative resistance of the conductor material. It increases as the conductor size and the frequency increase. It decreases as the material's relative resistance decreases. For example, for the same size conductors, the skin effect is larger for copper than for aluminum.

The effective resistance of a conductor is a function of power loss and the current in the conductor. Therefore,

$$R_{\text{eff}} = \frac{P_{\text{loss in conductor}}}{|I|^2}$$

where P_{loss} = power loss in conductor in watts
I = current in conductor in amperes

Skin effect increases this effective resistance. Also, it can decrease reactance as internal flux linkages decrease. Stranding the conductor considerably reduces the skin effect. In an underground cable, the central conductor strands are sometimes omitted since they carry small current. For example, some large cables are sometimes built over a central core of nonconducting material.

4.5.13 Proximity Effect

The proximity effect is quite similar in nature to the skin effect. An increase in resistance is present due to nonuniformity in current density over the conductor section caused by the magnetic flux linkages of current in the other conductors. The result, as in the case of skin effect, is a crowding of the current in both conductors toward the portions of the cross sections that are immediately adjacent to each other. It can cause a significant change in the effective ac resistance of multiconductor cables or cables located in the same duct. This phenomenon is called *proximity effect.* It is greater for a given size conductor in single-conductor cables than in three-conductor belted cables. Table 4.5 gives the dc resistance and skin effect and proximity effect multipliers for copper and aluminum conductors at 25 °C. Additional tables of electrical characteristics are supplied by the manufacturers for their cables.

TABLE 4.5 dc Resistance and Correction Factors for ac Resistance

Conductor Size, AWG or kcmil	dc Resistance, Ω/1000 ft at 25 °C[a]		ac Resistance multiplier: Single-Conductor Cables[b]		ac Resistance multiplier: Multiconductor Cables[c]	
	Copper	Aluminum	Copper	Aluminum	Copper	Aluminum
8	0.6532	1.071	1.000	1.000	1.00	1.00
6	0.4110	0.6741	1.000	1.000	1.00	1.00
4	0.2584	0.4239	1.000	1.000	1.00	1.00
2	0.1626	0.2666	1.000	1.000	1.01	1.00
1	0.1289	0.2114	1.000	1.000	1.01	1.00
10	0.1022	0.1676	1.000	1.000	1.02	1.00
20	0.08105	0.1329	1.000	1.001	1.03	1.00
30	0.06429	0.1054	1.000	1.001	1.04	1.01
40	0.05098	0.08361	1.000	1.001	1.05	1.01
250	0.04315	0.07077	1.005	1.002	1.06	1.02
300	0.03595	0.05897	1.006	1.003	1.07	1.02
350	0.03082	0.05055	1.009	1.004	1.08	1.03
500	0.02157	0.03538	1.018	1.007	1.13	1.06
750	0.01438	0.02359	1.039	1.015	1.21	1.12
1000	0.01079	0.01796	1.067	1.026	1.30	1.19
1500	0.00719	0.01179	1.142	1.058	1.53	1.36
2000	0.00539	0.00885	1.233	1.100	1.82	1.56

Source: From Fink and Beaty [4].

[a] To correct to other temperatures, use the following:

For copper:
$$R_T = R_{25} \times \frac{234.5 + T}{259.5}$$

For aluminum:
$$R_T = R_{25} \times \frac{228 + T}{253}$$

where R_r is the new resistance at temperature T and R_{25} is the tabulated resistance.

[b] Includes only skin effect (use for cables in separate ducts).

[c] Includes skin effect and proximity effect (use for triplex, multiconductor, or cables in the same duct).

EXAMPLE 4.8

A single-conductor, paper-insulated, belted cable will be used as an underground feeder of 3 mi. The cable has a 2000-MCM (2000-kcmil) copper conductor.

(a) Calculate the total dc resistance of the conductor at 25 °C.
(b) Using Table 4.5, determine the effective resistance and the skin effect on the effective resistance in percent if the conductor is used at 60-Hz alternating current.
(c) Calculate the percentage of reduction in cable ampacity in part (b).

Solution

(a) From Table 4.5, the dc resistance of the cable is

$$R_{dc} = 0.00539\ \Omega/1000\ \text{ft}$$

or the total dc resistance is

$$R_{dc} = 0.00539 \times 5280 \times 3 = 0.0854\ \Omega$$

(b) From Table 4.5, the skin effect coefficient is 1.233; therefore, the effective resistance at 60 Hz is

$$\begin{aligned} R_{eff} &= (\text{skin effect coefficient}) \times R_{dc} \\ &= (1.233) \times 0.0854 \\ &= 0.1053\ \Omega \end{aligned}$$

or it is 23.3 percent greater than for direct current.

(c) The reduction in the cable ampacity is also 23.3 percent.

4.6 SHEATH CURRENTS IN CABLES

The flow of ac current in the conductors of single-conductor cables induces ac voltages in the cable sheaths. When the cable sheaths are bonded together at their ends, the voltages induced give rise to sheath (eddy) currents, and therefore, additional I^2R losses occur in the sheath. These losses are taken into account by increasing the resistance of the relevant conductor. For a single-conductor cable with bonded sheaths operating in three phase and arranged in equilateral triangular formation, the increase in conductor resistance is

$$\Delta r = r_s \frac{X_m^2}{r_s^2 + X_m^2} \tag{4.77}$$

where X_m = mutual reactance between conductors and sheath per phase in ohms per mile
r_s = sheath resistance per phase in ohms per mile

The mutual reactance between conductors and sheath can be calculated from

$$X_m = 0.2794 \frac{f}{60} \log_{10} \frac{2S}{r_0 + r_i} \tag{4.78}$$

and the sheath resistance of a lead sheath cable can be determined from

$$r_s = \frac{0.2}{(r_0 + r_i)(r_0 - r_i)} \tag{4.79}$$

where f = frequency in hertz
S = spacing between conductor centers in inches
r_0 = outer radius of lead sheath in inches
r_i = inner radius of lead sheath in inches

In equation (4.78),

$$\text{GMR} = D_s = \tfrac{1}{2}(r_0 + r_i) \tag{4.80}$$

and

$$\text{GMD} = D_m = S$$

Therefore, for other conductor arrangements, that is, other than equilateral triangular formation,

$$X_m = 0.2794 \frac{f}{60} \log_{10} \frac{D_m}{D_s} \quad \Omega/\text{mi/phase} \tag{4.81}$$

and if the frequency used is 60 Hz,

$$X_m = 0.2794 \log_{10} \frac{D_m}{D_s} \tag{4.82}$$

or

$$X_m = 0.1213 \ln \frac{D_m}{D_s} \tag{4.83}$$

Hence, in single-conductor cables, the total resistance to positive- or negative-sequence current flow, including the effect of sheath current, is

$$r_a = r_c + \frac{r_s X_m^2}{r_s^2 + X_m^2} \quad \Omega/\text{mi/phase} \tag{4.84}$$

where r_a = total positive- or negative-sequence resistance, including sheath current effects
r_c = ac resistance of conductor, including skin effect

The sheath loss due to sheath currents is

$$P_s = I^2 \, \Delta r \tag{4.85}$$

or

$$P_s = I^2 \frac{r_s X_m^2}{r_s^2 + X_m^2} \tag{4.86}$$

or

$$P_s = r_s \left(\frac{I^2 X_m^2}{r_s^2 + X_m^2} \right) \quad \text{W/mi/phase} \tag{4.87}$$

where r_s = sheath resistance per phase in ohms per mile
I = current in one conductor in amperes
X_m = mutual reactance between conductors and sheath per phase in ohms per mile

For a three-conductor cable with round conductors, the increase in conductor resistance due to sheath currents is

$$\Delta r = 0.04416 \frac{S^2}{r_s (r_0 + r_i)^2} \quad \Omega/\text{mi/phase} \tag{4.88}$$

where

$$S = \frac{d + 2T}{\sqrt{3}} \tag{4.89}$$

and r_s = sheath resistance, from equation (4.79)
r_0 = outer radius of lead sheath in inches
r_i = inner radius of lead sheath in inches
S = distance between conductor center and sheath center for three-conductor cable made of round conductors
d = conductor diameter in inches
T = conductor insulation thickness in inches

For sector-shaped conductors, use equations (4.88) and (4.82) but conductor diameter is

d = 82–86% of diameter of round conductor having same cross-sectional area

Sheaths of single-conductor cables may be operated short-circuited or open-circuited. If the sheaths are short-circuited, they are usually bonded and grounded at every manhole. This decreases the sheath voltages to zero but allows the flow of sheath currents. There are various techniques of operating with the sheaths open-circulated:

1. When a ground wire is used, one terminal of each sheath section is bound to the ground wire. The other terminal is left open so that no current can flow in the sheath.
2. With cross bonding, at each section, connections are made between the sheaths of cables a, b, and c, as shown in Figure 4.25, so that only the sheaths are transposed electrically. The sheaths are bonded together and grounded at the end of each complete transposition. Thus, the sum of sheath voltages induced by the positive-sequence currents becomes zero.
3. With impedance bonding, impedances are added in each cable sheath to limit sheath currents to predetermined values without eliminating any sequence currents.
4. With bonding transformers [5].

Example 4.9

Assume that three 35-kV, 350-kcmil, single-conductor belted cables are located in touching equilateral formation with respect to each other and the sheaths are bounded to ground at several points. The cables are operated at 34.5 kV and 60 Hz. The cable has a conductor diameter of 0.681 in., insulation thickness of 345 cmil, lead sheath thickness of 105 cmil, and a length of 10 mi. Conductor ac resistance is 0.190 Ω/mi per phase at 50 °C. Calculate the following:

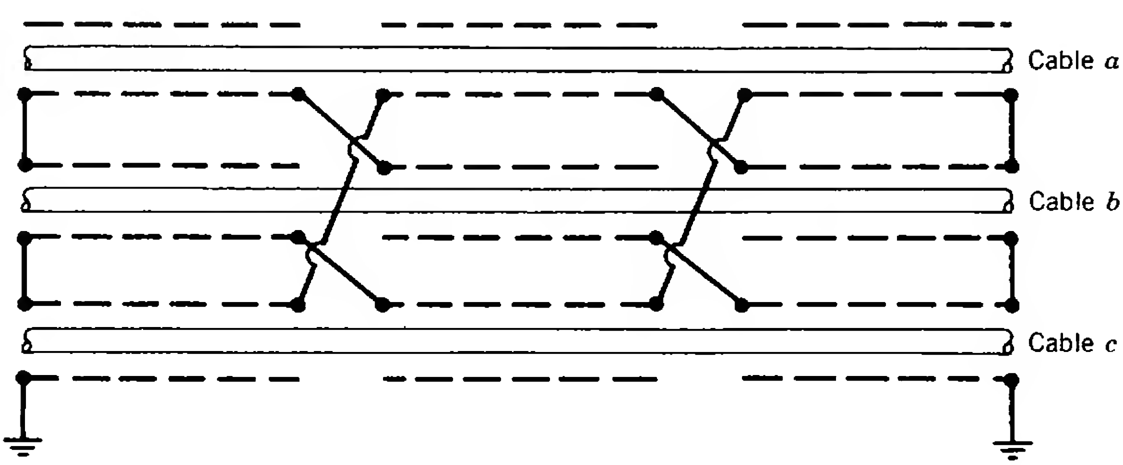


Figure 4.25. Cross bonding of single-conductor cables.

(a) Mutual reactance between conductors and sheath.
(b) Sheath resistance of cable.
(c) Increase in conductor resistance due to sheath currents.
(d) Total resistance of conductor including sheath loss.
(e) Ratio of sheath loss to conductor loss.
(f) Total sheath losses of feeder in watts if current in conductor is 400 A.

Solution

(a) By using equation (4.78),

$$X_m = 0.2794 \log_{10} \frac{2S}{r_0 + r_t} \quad \Omega/\text{mi/phase}$$

where

$$r_t = \frac{0.681}{2} + 0.345 = 0.686 \text{ in.}$$

$$r_0 = r_t + 0.105 = 0.791 \text{ in.}$$

$$S = 1.582 \text{ in.}$$

Therefore,

$$X_m = 0.2794 \log_{10} \frac{2 \times 1.582}{0.791 + 0.686}$$

$$= 0.09244 \ \Omega/\text{mi}$$

or

$$X_m = 0.9244 \ \Omega/\text{phase}$$

(b) By using equation (4.79),

$$r_s = \frac{0.2}{(r_0 + r_t)(r_o - r_t)}$$

$$= \frac{0.2}{(0.791 + 0.686)(0.791 - 0.686)}$$

$$= 1.2896 \ \Omega/\text{mi}$$

or

$$r_s = 12.896 \ \Omega/\text{phase}$$

(c) Using equation (4.77),

$$\Delta r = r_s \frac{X_m^2}{r_s^2 + X_m^2}$$

$$= 1.2896 \frac{0.09244^2}{1.2896^2 + 0.09244^2}$$

$$= 0.00659 \ \Omega/\text{mi}$$

or

$$\Delta r = 0.0659\ \Omega/\text{phase}$$

(d) Using equation (4.84),

$$r_a = r_c + \frac{r_s X_m^2}{r_s^2 + X_m^2}$$
$$= 0.190 + 0.00659$$
$$= 0.19659\ \Omega/\text{mi}$$

or

$$r_a = 1.9659\ \Omega/\text{phase}$$

(e) $$\frac{\text{Sheath loss}}{\text{conductor loss}} = \frac{I^2 r_s X_m^2}{r_s^2 + X_m^2} \frac{1}{I^2 r_c}$$
$$= \frac{r_s X_m^2}{r_s^2 + X_m^2} \frac{1}{r_c}$$
$$= \frac{0.00659}{0.190}$$
$$= 0.0347$$

That is,

$$\text{Sheath loss} = 3.47\% \times \text{conductor } I^2R \text{ loss}$$

(f) Using equation (4.87),

$$P_s = r_s \frac{I^2 X_m^2}{r_s^2 + X_m^2} \quad \text{W/mi}$$

or, for three-phase loss

$$P_s = 3I^2 \frac{r_s X_m^2}{r_s^2 + X_m^2}$$
$$= 3 \times 400^2 \times 0.00659$$
$$= 3163.2\ \text{W/mi}$$

or, for total feeder length,

$$P_s = 31{,}632\ \text{W}$$

4.7 POSITIVE- AND NEGATIVE-SEQUENCE REACTANCES

4.7.1 Single-Conductor Cables

The positive- and negative-sequence reactances for single-conductor cables when sheath currents are present can be determined as

$$X_1 = X_2 = 0.1213 \frac{f}{60} \ln \frac{D_m}{D_s} - \frac{X_m^3}{X_m^2 + r_s^2} \quad \Omega/\text{mi} \tag{4.90}$$

or

$$X_1 = X_2 = 0.2794 \frac{f}{60} \log_{10} \frac{D_m}{D_s} - \frac{X_m^3}{X_m^2 + r_s^2} \tag{4.91}$$

or

$$X_1 = X_2 = 0.2794 \frac{f}{60} \log_{10} \frac{D_m}{0.7788r} - \frac{X_m^3}{X_m^2 + r_s^2} \tag{4.92}$$

where r is the outside of the radius of the conductors. For cables, it is convenient to express D_m, D_s, and r in inches. In equation (4.91),

X_1 = positive-sequence reactance per phase in ohms per mile

X_2 = negative-sequence reactance per phase in ohms per mile

f = frequency in hertz

D_m = geometric mean distance (GMD) among conductors

D_s = geometric mean radius (GMR), or self-GMD, of one conductor

X_m = mutual reactance between conductors and sheath per phase in ohms per mile

r_s = sheath resistance per phase in ohms per mile

Equation (4.91) can be also expressed as

$$X_1 = X_2 = X_a + X_d - \frac{X_m^3}{X_m^2 + r_s^2} \quad \Omega/\text{mi} \tag{4.93}$$

where

$$X_a = 0.2794 \frac{f}{60} \log_{10} \frac{12}{D_s} \tag{4.94}$$

and

$$X_d = 0.2794 \frac{f}{60} \log_{10} \frac{D_m}{12} \tag{4.95}$$

Here, X_a and X_d are called *conductor component of reactance* and *separation component of reactance*, respectively. If the frequency is 60 Hz, equations (4.94) and (4.95) may be written as

$$X_a = 0.2794 \log_{10} \frac{12}{D_s} \quad \Omega/\text{mi} \tag{4.96}$$

or

$$X_a = 0.1213 \ln \frac{12}{D_s} \tag{4.97}$$

and

$$X_d = 0.2794 \log_{10} \frac{D_m}{12} \tag{4.98}$$

or

$$X_d = 0.1213 \ln \frac{D_m}{12} \tag{4.99}$$

In equation (4.93), the last term symbolizes the correction for the existence of sheath currents. The negative sign is there because the current in the sheath is in a direction opposite to that in the conductor, therefore inclining to restrict the flux to the region between the conductor and the sheath. The last term is taken from equation (4.77), with X_m substituted for r_s, and is derived by considering the current in the sheath and the component of voltage it induces in the conductor in quadrature to the conductor current.

4.7.2 Three-Conductor Cables

The positive- and negative-sequence reactances for three-conductor cables can be determined as

$$X_1 = X_2 = 0.2794 \frac{f}{60} \log_{10} \frac{D_m}{D_s} \quad \Omega/\text{mi} \tag{4.100}$$

or

$$X_1 = X_2 = X_a + X_d \tag{4.101}$$

where D_m is the GMD among the three conductors. If the frequency is 60 Hz, X_a and X_d can be calculated from equations (4.96) or (4.97) and (4.98) or (4.99), respectively. Equations (4.100) and (4.101) can be used for both shielded and non-shielded cables because of negligible sheet current effects.

4.8 ZERO-SEQUENCE RESISTANCE AND REACTANCE

The return of the zero-sequence currents flowing along the phase conductors of a three-phase cable is in either the ground, or the sheaths, or in the parallel combination of both ground and sheaths.

4.8.1 Three-Conductor Cables

Figure 4.26 shows an actual circuit of a single-circuit three-conductor cable with solidly bonded and grounded sheath. It can be observed that

$$(\mathbf{I}_{a0} + \mathbf{I}_{b0} + \mathbf{I}_{c0}) + (\mathbf{I}_{0(s)} + \mathbf{I}_{0(g)}) = 0 \tag{4.102}$$

Figure 4.27 shows the equivalent circuit of this actual circuit in which $\mathbf{Z}_c$ represents the impedance of a composite conductor consisting of three single conductors. The zero-sequence current $\mathbf{I}_{0(a)}$ in the composite conductor can be expressed as [7]

$$\mathbf{I}_{0(a)} = 3\mathbf{I}_{a0} \tag{4.103}$$

First assume that there is no return (zero-sequence) current flowing in the sheath, and therefore it is totally in the ground. Hence, the zero-sequence impedance of the composite conductor can be written as

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.36396 \frac{f}{60} \ln \frac{D_e}{D_{aa}} \quad \Omega/\text{mi/phase} \tag{4.104}$$

or

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.8382 \frac{f}{60} \log_{10} \frac{D_e}{D_{aa}} \tag{4.105}$$

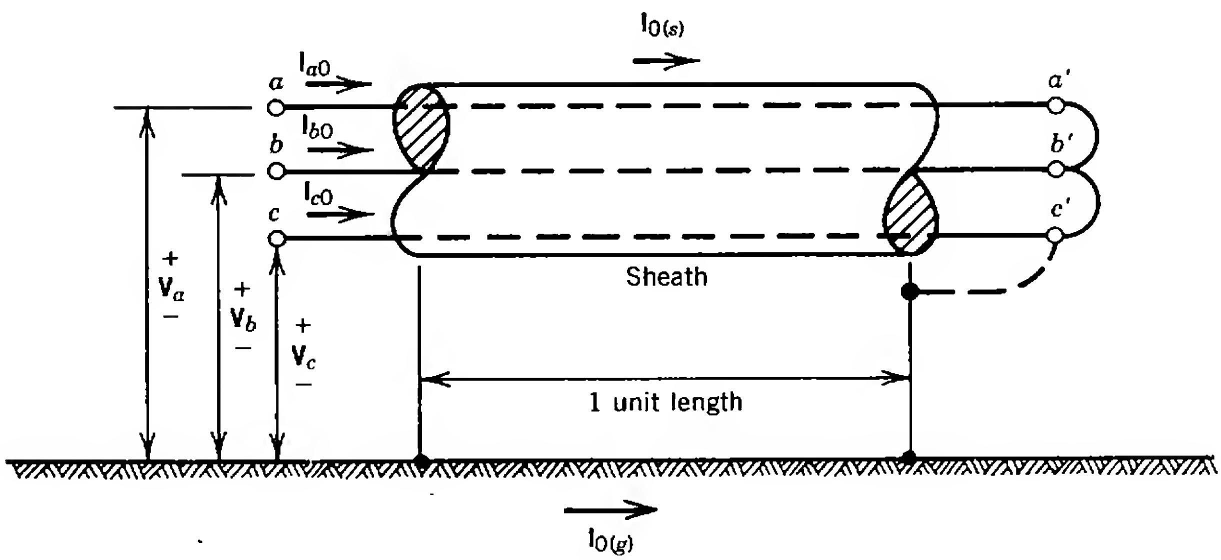


Figure 4.26. Actual circuit of three-conductor lead-sheathed cable.

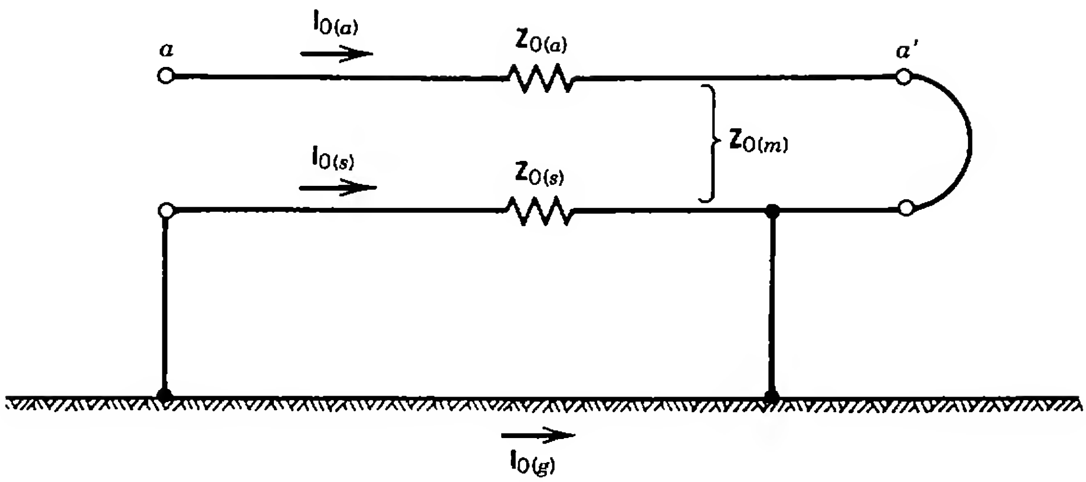


Figure 4.27. Equivalent circuit of three-conductor cable.

since the GMR, or the self-GMD, of this composite conductor is

$$D_{aa} = D_s^{1/3} D_{\mathrm{eq}}^{2/3} \tag{4.106}$$

and since for three-conductor cables made of round conductors,

$$D_{\mathrm{eq}} = D_m = d + 2T \tag{4.107}$$

where d = conductor diameter in inches
T = conductor insulation thickness in inches

equations (4.104) and (4.105) for 60 Hz frequency can be expressed as

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.36396 \ln \frac{D_e}{D_s^{1/3} D_{\mathrm{eq}}^{2/3}} \quad \Omega/\mathrm{mi/phase} \tag{4.108}$$

and

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.8382 \frac{f}{60} \log_{10} \frac{D_e}{D_s^{1/3} D_{\mathrm{eq}}^{2/3}} \tag{4.109}$$

Equations (4.108) and (4.109) are sometimes written as

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.1213 \ln \frac{D_e^3}{D_s D_{\mathrm{eq}}^2} \tag{4.110}$$

and

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.2794 \log_{10} \frac{D_e^3}{D_s D_{\mathrm{eq}}^2} \tag{4.111}$$

or

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j(X_a + X_e - 2X_d) \tag{4.112}$$

where

$$X_a = 0.1213 \ln \frac{12}{D_s} \quad \Omega/\text{mi} \tag{4.113}$$

$$X_e = 3 \times 0.1213 \ln \frac{D_e}{12} \quad \Omega/\text{mi} \tag{4.114}$$

$$X_d = 0.1213 \ln \frac{D_{\text{eq}}}{12} \quad \Omega/\text{mi} \tag{4.115}$$

where r_a = ac resistance of one conductor in ohms per mile
r_e = ac resistance of earth return,
$= 0.00476f\ \Omega/\text{mi}$
D_e = equivalent depth of earth return path,
$= 25920\sqrt{\rho/f}$ in.,
D_{eq} = equivalent, or geometric, mean distance among conductor centers in inches
D_s = GMR, or self-GMD, of one conductor in inches
X_a = reactance of individual phase conductor at 12 in. spacing in ohms per mile

Second, consider only ground return path and sheath return path but not the composite conductor. Therefore, the zero-sequence impedance of the sheath to zero-sequence currents is

$$\mathbf{Z}_{0(s)} = (3r_s + r_e) + j0.36396 \frac{f}{60} \ln \frac{2D_e}{r_0 + r_i} \tag{4.116}$$

or

$$\mathbf{Z}_{0(s)} = (3r_s + r_e) + j0.8382 \frac{f}{60} \log_{10} \frac{2D_e}{r_0 + r_i} \quad \Omega/\text{mi/phase} \tag{4.117}$$

or, at 60 Hz frequency,

$$\mathbf{Z}_{0(s)} = (3r_s + r_e) + j0.36396 \ln \frac{2D_e}{r_0 + r_i} \tag{4.118}$$

or

$$\mathbf{Z}_{0(s)} = (3r_s + r_e) + j0.8382 \log_{10} \frac{2D_e}{r_0 + r_i} \tag{4.119}$$

or

$$\mathbf{Z}_{0(s)} = (3r_s + r_e) + j(3X_s + X_e) \quad \Omega/\text{mi/phase} \tag{4.120}$$

where

$$r_s = \frac{0.2}{(r_0 + r_i)(r_0 - r_i)} \quad \text{for lead sheaths, } \Omega/\text{mi} \tag{4.121}$$

$$X_s = 0.1213 \ln \frac{24}{r_0 + r_i} \quad \Omega/\text{mi} \tag{4.122}$$

$$X_e = 3 \times 0.1213 \ln \frac{D_e}{12} \quad \Omega/\text{mi} \tag{4.123}$$

where r_s = sheath resistance of lead sheath cable in ohms per mile
r_e = ac resistence of earth return in ohms per mile
r_0 = outer radius of lead sheath in inches
r_i = inner radius of lead sheath in inches
X_s = reactance of sheath in ohms per mile

The zero-sequence mutual impedance between the composite conductor and sheath can be expressed as

$$\mathbf{Z}_{0(m)} = r_e + j0.36396 \frac{f}{60} \ln \frac{2D_e}{r_0 + r_i} \tag{4.124}$$

or

$$\mathbf{Z}_{0(m)} = r_e + j0.8382 \frac{f}{60} \log_{10} \frac{2D_e}{r_0 + r_i} \quad \Omega/\text{mi/phase} \tag{4.125}$$

or, at 60 Hz frequency,

$$\mathbf{Z}_{0(m)} = r_e + j0.36396 \ln \frac{2D_e}{r_0 + r_i} \tag{4.126}$$

or

$$\mathbf{Z}_{0(m)} = r_e + j0.8382 \log_{10} \frac{2D_e}{r_0 + r_i} \tag{4.127}$$

or

$$\mathbf{Z}_{0(m)} = r_e + j(3X_s + X_e) \tag{4.128}$$

The equivalent circuit shown in Figure 4.27 can be modified as shown in Figure 4.28.

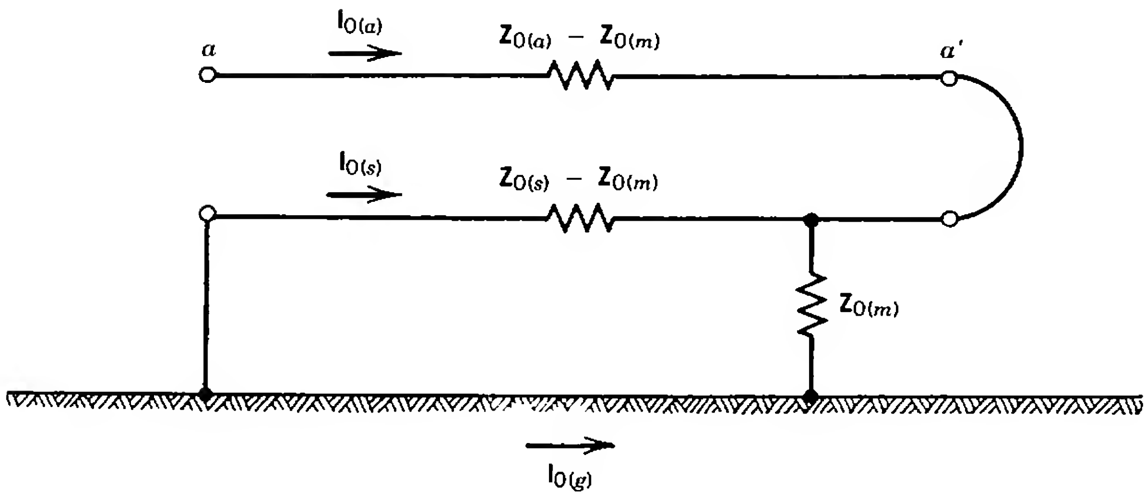


Figure 4.28. Modified equivalent circuit.

Total zero-sequence impedance can be calculated for three different cases as:

1. When both ground and sheath return paths are present,

$$\mathbf{Z}_{00} = \mathbf{Z}_0 = (\mathbf{Z}_{0(a)} - \mathbf{Z}_{0(m)}) + \frac{(\mathbf{Z}_{0(s)} - \mathbf{Z}_{0(m)})\mathbf{Z}_{0(m)}}{\mathbf{Z}_{0(s)}} \tag{4.129}$$

or

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)} - \frac{\mathbf{Z}_{0(m)}^2}{\mathbf{Z}_{0(s)}} \quad \Omega/\text{mi/phase} \tag{4.130}$$

or

$$\mathbf{Z}_{00} = [(r_a + r_e) + j(X_a + X_e - 2X_d)] - \frac{[r_e + j(3X_s + X_e)]^2}{[(3r_s + r_e) + j(3X_s + X_e)]} \tag{4.131}$$

2. When there is only sheath return path,

$$\mathbf{Z}_{00} = (\mathbf{Z}_{0(a)} - \mathbf{Z}_{0(m)}) + (\mathbf{Z}_{0(s)} - \mathbf{Z}_{0(m)}) \tag{4.132}$$

or

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)} + \mathbf{Z}_{0(s)} - 2\mathbf{Z}_{0(m)} \tag{4.133}$$

or

$$\mathbf{Z}_{00} = [(r_a + r_e) + j(X_a + X_e - 2X_d)] + [(3r_s + r_e) + j(3X_s + X_e)] - 2[r_e + j(3X_s + X_e)] \tag{4.134}$$

TABLE 4.6 D_e, r_e, and X_e for Various Earth Resistivities at 60 Hz [2]

Earth Resistivity (Ω-m)	Equivalent Depth of Earth Return, D_e		Equivalent Earth Resistance, r_e (Ω/mi)	Equivalent Earth Reactance, r_e (Ω/mi)
	in.	ft		
1	3.36×10^3	280	0.286	2.05
5	7.44×10^3	620	0.286	2.34
10	1.06×10^4	880	0.286	2.47
50	2.40×10^4	2,000	0.286	2.76
100	3.36×10^4	2,800	0.286	2.89
500	7.44×10^4	6,200	0.286	3.18
1,000	1.06×10^5	8,800	0.286	3.31
5,000	2.40×10^5	20,000	0.286	3.60
10,000	3.36×10^5	28,000	0.286	3.73

or

$$\mathbf{Z}_{00} = (r_a + 3r_s) + j(X_a - 2X_d - 3X_s) \quad \Omega/\text{mi/phase} \tag{4.135}$$

3. When there is only ground return path (e.g., nonsheathed cables),

$$\mathbf{Z}_{00} = (\mathbf{Z}_{0(a)} - \mathbf{Z}_{0(m)}) + \mathbf{Z}_{0(m)} \tag{4.136}$$

or

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)} \tag{4.137}$$

or

$$\mathbf{Z}_{00} = (r_a + r_e) + j(X_a + X_e - 2X_d) \quad \Omega/\text{mi/phase} \tag{4.138}$$

In the case of shielded cables, the zero-sequence impedance can be computed as if the shielding tapes were not present, with very small error. In general, calculating only the zero-sequence impedance for all return current in the sheath and none in the ground is sufficient. Table 4.6 offers the values of D_e, r_e, and X_e for various earth resistivities.

4.8.2 Single-Conductor Cables

The actual circuit of three single-conductor cables with solidly bonded and grounded sheath in a perfectly transposed three-phase circuit is shown in Figure 4.29. It can be observed from the figure that

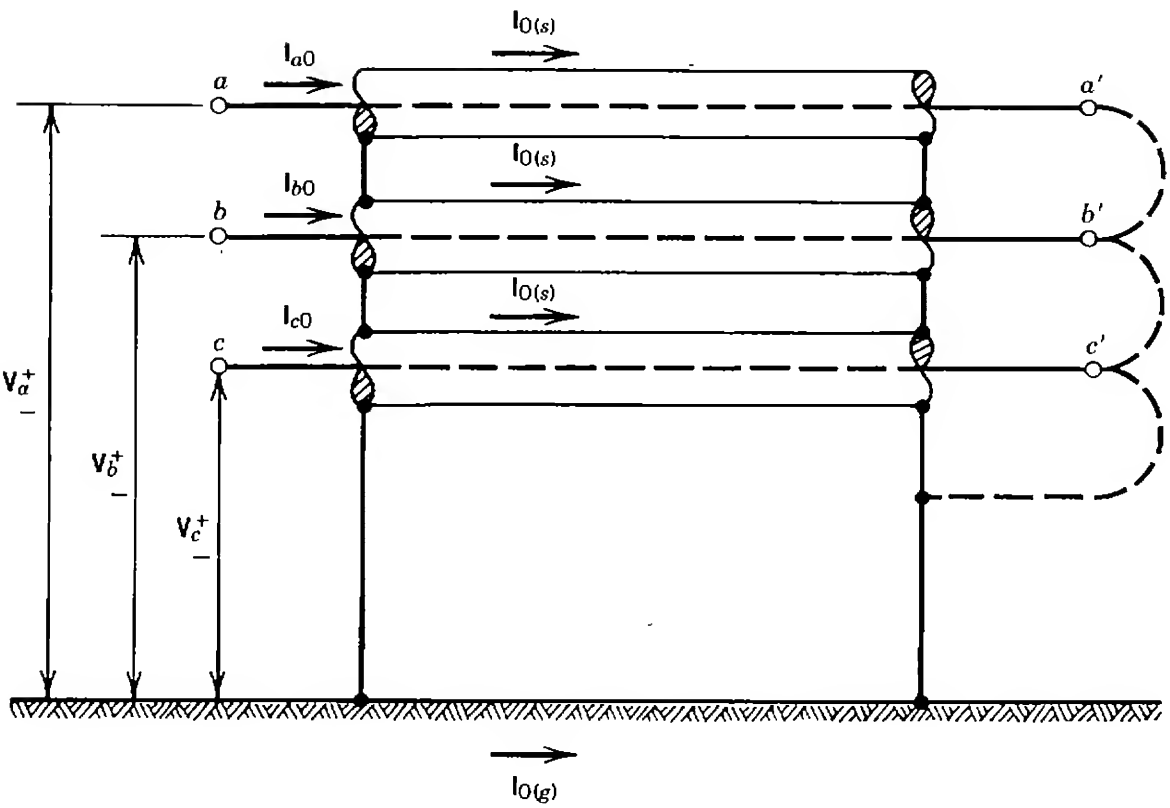


Figure 4.29. Actual circuit of three single-conductor lead-sheathed cables.

$$(\mathbf{I}_{a0} + \mathbf{I}_{b0} + \mathbf{I}_{c0}) + (3\mathbf{I}_{0(s)} + \mathbf{I}_{0(g)}) = 0 \tag{4.139}$$

In general, the equivalent circuits shown in Figures 4.27 and 4.28 are still applicable. The zero-sequence impedance of the composite conductor at 60 Hz frequency can be expressed as before,

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.36396 \ln \frac{D_e}{D_s^{1/3} D_{\text{eq}}^{2/3}} \tag{4.140}$$

or

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.8382 \log_{10} \frac{D_e}{D_s^{1/3} D_{\text{eq}}^{2/3}} \tag{4.141}$$

or

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j(X_a + X_e - 2X_d) \tag{4.142}$$

where

$$D_{\text{eq}} = (D_{ab} D_{bc} D_{ca})^{1/3} = D_m \quad \text{in.}$$
$$= \text{GMD among conductor centers}$$

The zero-sequence impedance of the sheath to zero-sequence currents is

$$\mathbf{Z}_{0(s)} = (r_s + r_e) + j0.36396 \ln \frac{D_e}{D_{s(3s)}} \tag{4.143}$$

or

$$\mathbf{Z}_{0(s)} = (r_s + r_e) + j0.8382 \log_{10} \frac{D_e}{D_{s(3s)}} \tag{4.144}$$

or

$$\mathbf{Z}_{0(s)} = (r_s + r_e) + j(X_s + X_e - 2X_d) \tag{4.145}$$

where

$$D_{s(3s)} = \left(D_m^2 \frac{r_0 + r_t}{2}\right)^{1/3} \quad \text{in.}$$

= GMR, or self-GMD, of conducting path composed of three sheaths in parallel (4.146)

The zero-sequence mutual impedance between conductors and the sheaths can be expressed as

$$\mathbf{Z}_{0(m)} = r_e + j0.36396 \ln \frac{D_e}{D_{m(3c\text{-}3s)}} \tag{4.147}$$

or

$$\mathbf{Z}_{0(m)} = r_e + j0.8382 \log_{10} \frac{D_e}{D_{m(3c\text{-}3s)}} \tag{4.148}$$

or

$$\mathbf{Z}_{0(m)} = r_e + j(X_e + X_s - 2X_d) \tag{4.149}$$

where

$$D_{m(3c\text{-}3s)} = \left[D_m^6 \left(\frac{r_0 + r_t}{2}\right)^3\right]^{1/9} \tag{4.150}$$

or

$$D_{m(3c\text{-}3s)} = \left[D_m^2 \frac{r_0 + r_t}{2}\right]^{1/3} \quad \text{in.} \tag{4.151}$$

= GMD of all distances between conductors and sheaths

Total zero-sequence impedance can be calculated, from Figure 4.29, for three different cases:

1. When both ground and sheath return paths are present,

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)} - \frac{\mathbf{Z}_{0(m)}^2}{\mathbf{Z}_{0(s)}} \tag{4.152}$$

or

$$\mathbf{Z}_{00} = [(r_a + r_e) + J(X_a + X_e - 2X_d)] - \frac{[r_e + j(X_e + X_s - 2X_d]^2}{(r_s + r_e) + j(X_e + X_s - 2X_d)} \tag{4.153}$$

2. When there is only a sheath return path,

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)} + \mathbf{Z}_{0(s)} - 2\mathbf{Z}_{0(m)} \tag{4.154}$$

or

$$\begin{aligned}\mathbf{Z}_{00} = {} & [(r_a + r_e) + j(X_e + X_s - 2X_d)] + [(r_s + r_e) + j(X_s + X_e - 2X_d)] \\ & - 2[r_e + j(X_e + X_s - 2X_d)]\end{aligned} \tag{4.155}$$

or

$$\mathbf{Z}_{00} = (r_a + r_s) + j(X_a - X_s) \tag{4.156}$$

or

$$\mathbf{Z}_{00} = (r_a + r_s) + j0.36396 \ln \frac{D_{s(3s)}}{D_s^{1/3} D_{\text{eq}}^{2/3}} \tag{4.157}$$

or

$$\mathbf{Z}_{00} = (r_a + r_s) + j0.8382 \log_{10} \frac{D_{s(3s)}}{D_s^{1/3} D_{\text{eq}}^{2/3}} \tag{4.158}$$

3. When there is only a ground return path,

$$\mathbf{Z}_{00} = (\mathbf{Z}_{0(a)} - \mathbf{Z}_{0(m)}) + \mathbf{Z}_{0(m)} \tag{4.159}$$

or

$$\mathbf{Z}_{00} = \mathbf{Z}_{0(a)}$$

or

$$\mathbf{Z}_{00} = (r_a + r_e) + j(X_a + X_e - 2X_d) \tag{4.160}$$

EXAMPLE 4.10

A three-phase, 60-Hz, 23-kV cable of three 250-kcmil, concentric-strand, paper-insulated single-conductor cables with solidly bonded and grounded lead sheath connected between a sending bus and receiving bus, as shown in Figures 4.29 and 4.30. The conductor diameter is 0.575 in., insulation thickness is 245 mils, and lead sheath thickness is 95 mils. The conductor resistance is 0.263 Ω/mi per phase and the earth resistivity is 100 Ω-m. The GMR of one conductor is 0.221 in. Sheath resistance is 1.72 Ω/mi. Calculate the total zero-sequence impedance:

(a) When both ground and return paths are present.
(b) When there is only sheath return path.
(c) When there is only ground return path.

Solution

$$T = \text{insulation thickness}$$
$$= \frac{245 \text{ mils}}{1000} = 0.245 \text{ in.}$$

$$\text{Lead sheath thickness} = \frac{95 \text{ mils}}{1000} = 0.095 \text{ in.}$$

Therefore,

$$r_t = \frac{\text{conductor diameter}}{2} + \text{insulation thickness}$$
$$= \frac{0.575}{2} + 0.245 = 0.5325 \text{ in.}$$

and

$$r_0 = r_t + \text{lead sheath thickness}$$
$$= 0.5325 + 0.095 = 0.6275 \text{ in.}$$

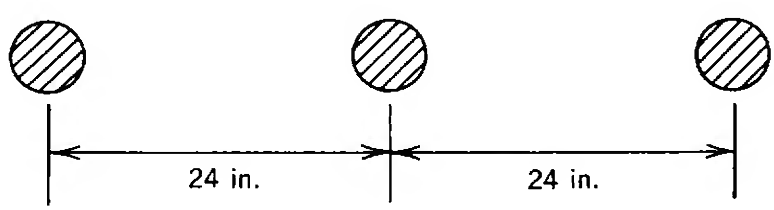


Figure 4.30

By using equation (4.140),

$$\mathbf{Z}_{0(a)} = (r_a + r_e) + j0.36396 \ln \frac{D_e}{D_s^{1/3} D_{\text{eq}}^{2/3}}$$

where

$$r_a = 0.263 \ \Omega/\text{mi}$$

$$r_e = 0.00476f = 0.2856 \ \Omega/\text{mi}$$

$$D_e = 25{,}920\sqrt{\frac{\rho}{60}} = 33{,}462.6 \text{ in.}$$

$$D_s = 0.221 \text{ in.}$$

$$D_{\text{eq}} = D_m = (D_{ab}D_{bc}D_{ca})^{1/3} = 30.24 \text{ in.}$$

Therefore,

$$\mathbf{Z}_{0(a)} = (0.263 + 0.2856) + j0.36396 \ln \frac{33{,}462.6}{\sqrt{0.221 \times 30.24^2}}$$

$$= 0.5486 + j3.1478 = 3.1952 \underline{/80.1^\circ} \ \Omega/\text{mi}$$

By using equation (4.143),

$$\mathbf{Z}_{0(s)} = (r_s + r_e) + j0.36396 \ln \frac{D_e}{D_{s(3s)}}$$

where

$$r_s = 1.72 \ \Omega/\text{mi}$$

$$r_e = 0.2856 \ \Omega/\text{mi}$$

$$D_e = 33{,}462.6 \text{ in.}$$

$$D_{s(3s)} = \left(D_{\text{eq}}^2 \frac{r_0 + r_i}{2}\right)^{1/3}$$

$$= \left(30.24^2 \times \frac{0.5325 + 0.6275}{2}\right)^{1/3} = 8.09 \text{ in.}$$

Therefore,

$$\mathbf{Z}_{0(s)} = (1.72 + 0.2856) + j0.36396 \ln \frac{33{,}462.6}{8.09}$$

$$= 2.006 + \text{j}3.03 = 3.634 \underline{/56.5^\circ} \ \Omega/\text{mi}$$

By using equation (4.147)

$$\mathbf{Z}_{0(m)} = r_e + j0.36396 \ln \frac{D_e}{D_{m(\text{3c-3s})}}$$

where

$$D_{m(3c\text{-}3s)} = D_{s(3s)} = 8.09 \text{ in.}$$

Hence,

$$\mathbf{Z}_{0(m)} = 0.2856 + j0.36396 \ln \frac{33{,}462.6}{8.09}$$
$$= 0.2856 + j3.03 = 3.04 \,\underline{/84.6^\circ}\; \Omega/\text{mi}$$

Therefore, the total zero-sequence impedances are:

(a) When both ground and return paths are present,

$$\mathbf{Z}_0 = \mathbf{Z}_{0(a)} - \frac{\mathbf{Z}_{0(m^2)}}{\mathbf{Z}_{0(s)}}$$
$$= 0.5486 + j3.1478 - \frac{(3.04\,\underline{/84.6^\circ})^2}{3.634\,\underline{/56.5^\circ}}$$
$$= 1.534 + j0.796 = 1.728\,\underline{/27.4^\circ}\; \Omega/\text{mi}$$

(b) When there is only sheath return path,

$$\mathbf{Z}_0 = \mathbf{Z}_{0(a)} + \mathbf{Z}_{0(s)} - 2\mathbf{Z}_{0(m)}$$
$$= (0.5486 + 2.006 - 2 \times 0.2856)$$
$$+ j(3.1478 + 3.03 - 2 \times 3.03)$$
$$= 1.983 + j0.117 = 1.987\,\underline{/3.4^\circ}\; \Omega/\text{mi}$$

(c) When there is only ground return path,

$$\mathbf{Z}_0 = \mathbf{Z}_{0(a)}$$
$$= 0.5486 + j3.1478 = 3.1952\,\underline{/80.1^\circ}\; \Omega/\text{mi}$$

4.9 SHUNT CAPACITIVE REACTANCE

Tables A.12–A.19 of Appendix A give shunt capacitive reactances directly in ohms per mile. Also, the following formulas give the shunt capacitance, shunt capacitive reactance, and charging current [2].

1. For single-conductor and three-conductor shielded cables:

$$C_0 = C_1 = C_2 = \frac{0.0892K}{G} \quad \mu\text{F/mi/phase} \tag{4.161}$$

$$X_0 = X_1 = X_2 = \frac{1.79G}{fK} \quad \text{M}\Omega/\text{mi/phase} \tag{4.162}$$

$$I_0 = I_1 = I_2 = \frac{0.323 f K V_{(\text{L-N})}}{1000 G} \quad \text{A/mi/phase} \tag{4.163}$$

2. For three-conductor belted cables with no conductor shielding:

$$C_0 = \frac{0.0892\text{K}}{\text{G}_0} \quad \mu\text{F/mi/phase} \tag{4.164}$$

$$C_1 = C_2 = \frac{0.267 K}{G_1} \quad \mu\text{F/mi/phase} \tag{4.165}$$

$$X_0 = \frac{1.79 G_0}{f K} \quad \text{M}\Omega\text{/mi/phase} \tag{4.166}$$

$$X_1 = X_2 = \frac{0.597 G_1}{f K} \quad \text{M}\Omega\text{/mi/phase} \tag{4.167}$$

$$I_0 = \frac{0.323 f K V_{(\text{L-N})}}{1000 G_0} \quad \text{A/mi/phase} \tag{4.168}$$

$$I_1 = I_2 = \frac{0.97 f K V_{(\text{L-N})}}{1000 G_1} \quad \text{A/mi/phase} \tag{4.169}$$

where C_0 = zero-sequence capacitance in microfarads per mile per phase
C_1 = positive-sequence capacitance in microfarads per mile per phase
C_2 = negative-sequence capacitance in microfarads per mile
K = dielectric constant of insulation, from Table 4.1
G = geometric factor, from Figure 4.23
G_1 = geometric factor, from Figure 4.24
f = frequency in hertz
$V_{(\text{L-N})}$ = line-to-neutral voltage in kilovolts

Here,

$$X = \frac{X,\ \Omega\text{/phase/mi}}{l,\ \text{mi}} \quad \Omega\text{/phase} \tag{4.170}$$

Example 4.11

A 60-Hz, 15-kV, three-conductor, paper-insulated, shielded cable will be used at 13.8 kV as a three-phase underground feeder of 10 mi. The cable has three 350-kcmil compact sector-type conductors with a diameter of 0.539 in. and a dielectric constant of 3.7. The insulation thickness is 175 mils. Calculate the following:

(a) Shunt capacitance for zero, positive, and negative sequences.
(b) Shunt capacitive reactance for zero, positive, and negative sequences.
(c) Charging current for zero, positive, and negative sequences.

Solution

$$
\begin{aligned}
T &= \text{insulation thickness} \\
&= \frac{175 \text{ mils}}{1000} = 0.175 \text{ in.} \\
d &= \text{conductor diameter} = 0.539 \text{ in.} \\
D &= d + 2T = 0.539 + 2 \times 0.175 = 0.889 \text{ in.} \\
G &= \text{geometric factor from Figure 4.23} \\
&= 0.5
\end{aligned}
$$

or, by using equation (4.68),

$$
\begin{aligned}
G &= 2.303 \log_{10} \frac{D}{d} \\
&= 2.303 \log_{10} \frac{0.889}{0.539} = 0.5005
\end{aligned}
$$

Since the conductors are compact-sector type, from Table 4.3, for

$$\frac{T + t}{d} = 0.3247$$

the sector factor is found to be 0.710.

(a) By using equation (4.161),

$$
\begin{aligned}
C_0 = C_1 = C_2 &= \frac{0.0892K}{G} \\
&= \frac{0.0892 \times 3.7}{0.5005 \times 0.710} = 0.93 \ \mu\text{F/mi/phase}
\end{aligned}
$$

(b) By using equation (4.162),

$$
\begin{aligned}
X_0 = X_1 = X_2 &= \frac{1.79G}{fK} \\
&= \frac{1.79 \times 0.5005 \times 0.710}{60 \times 3.7} \\
&= 2.86 \text{ k}\Omega\text{/mi/phase}
\end{aligned}
$$

(c) By using equation (4.163),

$$
\begin{aligned}
I_0 = I_1 = I_2 &= \frac{0.323 fKV_{(\text{L-N})}}{1000G} \\
&= \frac{0.323 \times 60 \times 3.7 \times (13.8/\sqrt{3})}{1000 \times 0.5005 \times 0.710} \\
&= 1.609 \text{ A/mi/phase}
\end{aligned}
$$

4.10 CURRENT-CARRYING CAPACITY OF CABLES

Tables A.12–A.19 of Appendix A give current-carrying capacities of paper-insulated cables. The earth temperature is assumed to be uniform and at 20 °C.

In general, the calculation of ampacities of cables is very complex due to the characteristics of the thermal circuit, skin and proximity effects, and the nature of the insulation.

4.11 CALCULATION OF IMPEDANCES OF CABLES IN PARALLEL

4.11.1 Single-Conductor Cables

Figure 4.31 shows a three-phase circuit consisting of three single-conductor cables with concentric neutrals. Therefore, there are six circuits, each with ground return, three for the phase conductors and three for the concentric neutrals. Here x, y, and z indicate the concentric neutrals, and a, b, and c indicate the phases.

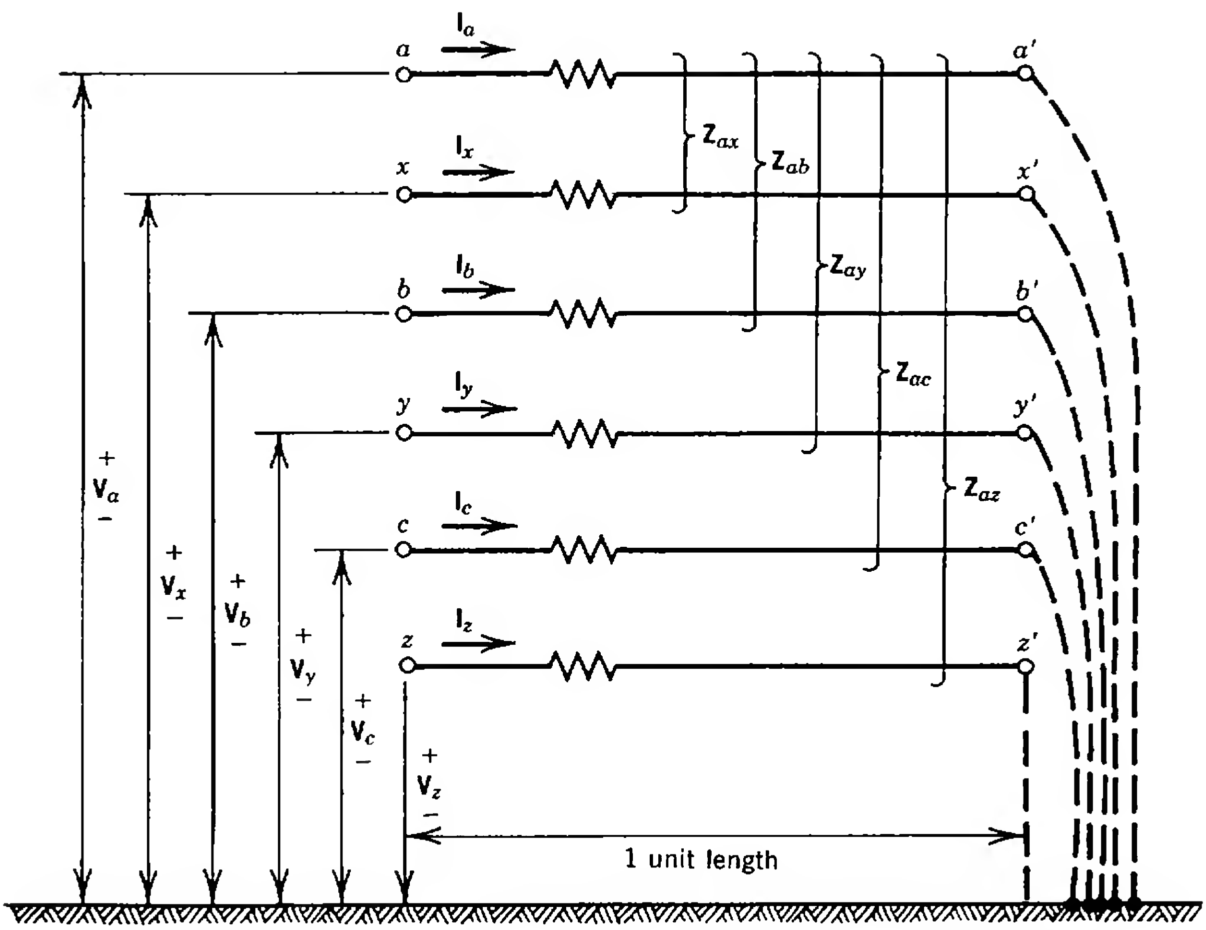


Figure 4.31. Three single-conductor cables with ground return.

Therefore, the voltage drop equations in the direction of current flow can be written as

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \\ \mathbf{V}_x \\ \mathbf{V}_y \\ \mathbf{V}_z \end{bmatrix} = \begin{bmatrix} \mathbf{V}_{aa'} \\ \mathbf{V}_{bb'} \\ \mathbf{V}_{cc'} \\ \mathbf{V}_{xx'} \\ \mathbf{V}_{yy} \\ \mathbf{V}_{zz'} \end{bmatrix} = \begin{bmatrix} \mathbf{V}_a - \mathbf{V}_{a'} \\ \mathbf{V}_b - \mathbf{V}_{b'} \\ \mathbf{V}_c - \mathbf{V}_{c'} \\ \mathbf{V}_x - \mathbf{V}_{x'} \\ \mathbf{V}_y - \mathbf{V}_{y'} \\ \mathbf{V}_z - \mathbf{V}_{z'} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} & \mathbf{Z}_{ax} & \mathbf{Z}_{ay} & \mathbf{Z}_{az} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} & \mathbf{Z}_{bx} & \mathbf{Z}_{by} & \mathbf{Z}_{bz} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} & \mathbf{Z}_{cx} & \mathbf{Z}_{cy} & \mathbf{Z}_{cz} \\ \mathbf{Z}_{xa} & \mathbf{Z}_{xb} & \mathbf{Z}_{xc} & \mathbf{Z}_{xx} & \mathbf{Z}_{xy} & \mathbf{Z}_{xz} \\ \mathbf{Z}_{ya} & \mathbf{Z}_{yb} & \mathbf{Z}_{yc} & \mathbf{Z}_{yx} & \mathbf{Z}_{yy} & \mathbf{Z}_{yz} \\ \mathbf{Z}_{za} & \mathbf{Z}_{zb} & \mathbf{Z}_{zc} & \mathbf{Z}_{zx} & \mathbf{Z}_{zy} & \mathbf{Z}_{zz} \end{bmatrix} \times \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \\ \mathbf{I}_x \\ \mathbf{I}_y \\ \mathbf{I}_z \end{bmatrix} \tag{4.171}$$

By taking advantage of symmetry, the voltage matrix equation (4.171) can be written in partitioned form as

$$\begin{bmatrix} \mathbf{V}_{abc} \\ \hline \mathbf{V}_{xyz} \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{Z}_s & \mathbf{Z}_M \\ \hline \mathbf{Z}_M^{\mathrm{t}} & \mathbf{Z}_N \end{array}\right] \begin{bmatrix} \mathbf{I}_{abc} \\ \hline \mathbf{I}_{xyz} \end{bmatrix} \tag{4.172}$$

where

$$\mathbf{Z}_s = \begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} \end{bmatrix} \tag{4.173}$$

$$\mathbf{Z}_N = \begin{bmatrix} \mathbf{Z}_{xx} & \mathbf{Z}_{xy} & \mathbf{Z}_{xz} \\ \mathbf{Z}_{yx} & \mathbf{Z}_{yy} & \mathbf{Z}_{yz} \\ \mathbf{Z}_{zx} & \mathbf{Z}_{zy} & \mathbf{Z}_{zz} \end{bmatrix} \tag{4.174}$$

$$\mathbf{Z}_M = \begin{bmatrix} \mathbf{Z}_{ax} & \mathbf{Z}_{ay} & \mathbf{Z}_{az} \\ \mathbf{Z}_{bx} & \mathbf{Z}_{by} & \mathbf{Z}_{bz} \\ \mathbf{Z}_{cx} & \mathbf{Z}_{cy} & \mathbf{Z}_{xz} \end{bmatrix} \tag{4.175}$$

and

$$\mathbf{Z}_M^{\mathrm{t}} = \begin{bmatrix} \mathbf{Z}_{xa} & \mathbf{Z}_{xb} & \mathbf{Z}_{xc} \\ \mathbf{Z}_{ya} & \mathbf{Z}_{yb} & \mathbf{Z}_{yc} \\ \mathbf{Z}_{za} & \mathbf{Z}_{zb} & \mathbf{Z}_{zc} \end{bmatrix} \tag{4.176}$$

Since the $[\mathbf{V}_{xyz}]$ submatrix for the voltage drops in the neutral conductors to ground will be a zero matrix due to having all its terms zero, equation (4.172) can be rewritten as

$$\begin{bmatrix} \mathbf{V}_{abc} \\ \hline 0 \end{bmatrix} = \left[\begin{array}{c|c} \mathbf{Z}_s & \mathbf{Z}_M \\ \hline \mathbf{Z}_M^{\mathrm{t}} & \mathbf{Z}_N \end{array}\right] \begin{bmatrix} \mathbf{I}_{abc} \\ \hline \mathbf{I}_{xyz} \end{bmatrix} \tag{4.177}$$

By Kron reduction,

$$[\mathbf{V}_{abc}] = [\mathbf{Z}_{\text{new}}][\mathbf{I}_{abc}] \tag{4.178}$$

where

$$[\mathbf{Z}_{\text{new}}] = [\mathbf{Z}_s] - [\mathbf{Z}_M][\mathbf{Z}_N]^{-1}[\mathbf{Z}_M^t] \tag{4.179}$$

Once the impedance matrix for the three-phase cable configuration is known, the sequence impedances can be computed by similarity transformation as

$$[\mathbf{Z}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{\text{new}}][\mathbf{A}] \tag{4.180}$$

where

$$[\mathbf{A}]^{-1} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix} \tag{4.181}$$

and

$$[\mathbf{A}] = \begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a}^2 & \mathbf{a} \\ 1 & \mathbf{a} & \mathbf{a}^2 \end{bmatrix} \tag{4.182}$$

and

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_{00} & \mathbf{Z}_{01} & \mathbf{Z}_{02} \\ \mathbf{Z}_{10} & \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{20} & \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \tag{4.183}$$

where the diagonal elements ($\mathbf{Z}_{00}$, $\mathbf{Z}_{11}$, and $\mathbf{Z}_{22}$) are self-impedances, or simply the sequence impendances, and the off-diagonal elements ($\mathbf{Z}_{01}$, $\mathbf{Z}_{02}$, or $\mathbf{Z}_{12}$) are mutual impedances. For completely symmetrical or transposed circuits, the mutual terms are all zero.

Therefore, the sequence voltage drops can be computed from

$$[\mathbf{V}_{012}] = [\mathbf{Z}_{012}][\mathbf{I}_{012}] \tag{4.184}$$

or

$$\begin{bmatrix} \mathbf{V}_{0(a)} \\ \mathbf{V}_{1(a)} \\ \mathbf{V}_{2(a)} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{00} & \mathbf{Z}_{01} & \mathbf{Z}_{02} \\ \mathbf{Z}_{10} & \mathbf{Z}_{11} & \mathbf{Z}_{12} \\ \mathbf{Z}_{20} & \mathbf{Z}_{21} & \mathbf{Z}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{I}_{0(a)} \\ \mathbf{I}_{1(a)} \\ \mathbf{I}_{2(a)} \end{bmatrix} \tag{4.185}$$

where

$$[\mathbf{V}_{012}] = [\mathbf{A}]^{-1}[\mathbf{V}_{abc}] \tag{4.186}$$

or

$$\begin{bmatrix} \mathbf{V}_{0(a)} \\ \mathbf{V}_{1(a)} \\ \mathbf{V}_{2(a)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \end{bmatrix} \tag{4.187}$$

and

$$[\mathbf{I}_{012}] = [\mathbf{A}]^{-1}[\mathbf{I}_{abc}] \tag{4.188}$$

or

$$\begin{bmatrix} \mathbf{I}_{0(a)} \\ \mathbf{I}_{1(a)} \\ \mathbf{I}_{2(a)} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & \mathbf{a} & \mathbf{a}^2 \\ 1 & \mathbf{a}^2 & \mathbf{a} \end{bmatrix}\begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \end{bmatrix} \tag{4.189}$$

However, when the three conductors involved are identical and the circuit is completely symmetrical or transposed so that the mutual impedances between phases are identical, that is,

$$\mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{ca}$$

the expressions to calculate the sequence impedances directly are

$$\mathbf{Z}_{00} = \mathbf{Z}_{aa} + 2\mathbf{Z}_{ab} - \frac{(\mathbf{Z}_{ax} + 2\mathbf{Z}_{ab})^2}{\mathbf{Z}_{xx} + 2\mathbf{Z}_{ab}} \tag{4.190}$$

and

$$\mathbf{Z}_{11} = \mathbf{Z}_{22} = \mathbf{Z}_{aa} - \mathbf{Z}_{ab} - \frac{(\mathbf{Z}_{ax} - \mathbf{Z}_{ab})^2}{\mathbf{Z}_{xx} - \mathbf{Z}_{ab}} \tag{4.191}$$

in which case equation (4.183) becomes

$$[\mathbf{Z}_{012}] = \begin{bmatrix} \mathbf{Z}_{00} & 0 & 0 \\ 0 & \mathbf{Z}_{11} & 0 \\ 0 & 0 & \mathbf{Z}_{22} \end{bmatrix} \tag{4.192}$$

In case of distribution cables, equations (4.190)–(4.191) are also valid for the asymmetrical case if the average value $\mathbf{Z}_{ab}$ in the equations is set equal to

$$\mathbf{Z}_{ab(\text{avg})} = \mathbf{Z} = \tfrac{1}{3}(\mathbf{Z}_{ab} + \mathbf{Z}_{bc} + \mathbf{Z}_{ca}) \tag{4.193}$$

When the concentric neutral conductors are not present or open-circuited, the neutral currents are zero,

$$[\mathbf{I}_{xyz}] = [0]$$

and therefore, the sequence impedance can be calculated from

$$\mathbf{Z}_{00} = \mathbf{Z}_{aa} + 2\mathbf{Z}_{ab} \tag{4.194}$$

$$\mathbf{Z}_{11} = \mathbf{Z}_{22} = \mathbf{Z}_{aa} - \mathbf{Z}_{ab} \tag{4.195}$$

When the neutral conductors are connected together but not grounded, if the circuit is completely symmetrical or transposed, the positive- and negative-sequence impedances are not affected due to lack of ground return current. However, in the case of zero sequence, due to symmetry,

$$\begin{aligned}\mathbf{I}_{0(x)} = \mathbf{I}_{0(y)} = \mathbf{I}_{0(z)} &= -\mathbf{I}_{0(a)} = -\mathbf{I}_{0(b)} \\ &= -\mathbf{I}_{0(c)} = -1\end{aligned} \tag{4.196}$$

Thus, in equation (4.7), the matrix $[\mathbf{V}_{xyz}]$ is not a null matrix, that is,

$$[\mathbf{V}_{xyz}] \neq [0]$$

Therefore,

$$\mathbf{Z}_{00} = \mathbf{Z}_{aa} + \mathbf{Z}_{xx} - 2\mathbf{Z}_{ax} \tag{4.197}$$

which is the same as the impedance of a single conductor with its own neutral.

If the conductor arrangement is not a symmetrical one, the usage of equation (4.197) to calculate zero-sequence impedance would still be valid.

4.11.2 Bundled Single-Conductor Cables

At times it might be necessary to use two three-phase cable circuits to connect a sending bus to a receiving one, as shown in Figure 4.32. Each phase has two paralleled and unsheathed (or with open-circuited sheaths) single conductors. There is no ground return current.

Therefore, before "bundling," that is, connecting two conductors per phase, the voltage drop equations can be expressed,

$$\begin{bmatrix} \mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \\ \mathbf{V}_x \\ \mathbf{V}_y \\ \mathbf{V}_z \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} & \mathbf{Z}_{ax} & \mathbf{Z}_{ay} & \mathbf{Z}_{az} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} & \mathbf{Z}_{bx} & \mathbf{Z}_{by} & \mathbf{Z}_{bz} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} & \mathbf{Z}_{cx} & \mathbf{Z}_{cy} & \mathbf{Z}_{cz} \\ \mathbf{Z}_{xa} & \mathbf{Z}_{xb} & \mathbf{Z}_{xc} & \mathbf{Z}_{xx} & \mathbf{Z}_{xy} & \mathbf{Z}_{xz} \\ \mathbf{Z}_{ya} & \mathbf{Z}_{yb} & \mathbf{Z}_{yc} & \mathbf{Z}_{yx} & \mathbf{Z}_{yy} & \mathbf{Z}_{yz} \\ \mathbf{Z}_{za} & \mathbf{Z}_{zb} & \mathbf{Z}_{zc} & \mathbf{Z}_{zx} & \mathbf{Z}_{zy} & \mathbf{Z}_{zz} \end{bmatrix} \begin{bmatrix} \mathbf{I}_a \\ \mathbf{I}_b \\ \mathbf{I}_c \\ \mathbf{I}_x \\ \mathbf{I}_y \\ \mathbf{I}_z \end{bmatrix} \quad \text{V/unit length} \tag{4.198}$$

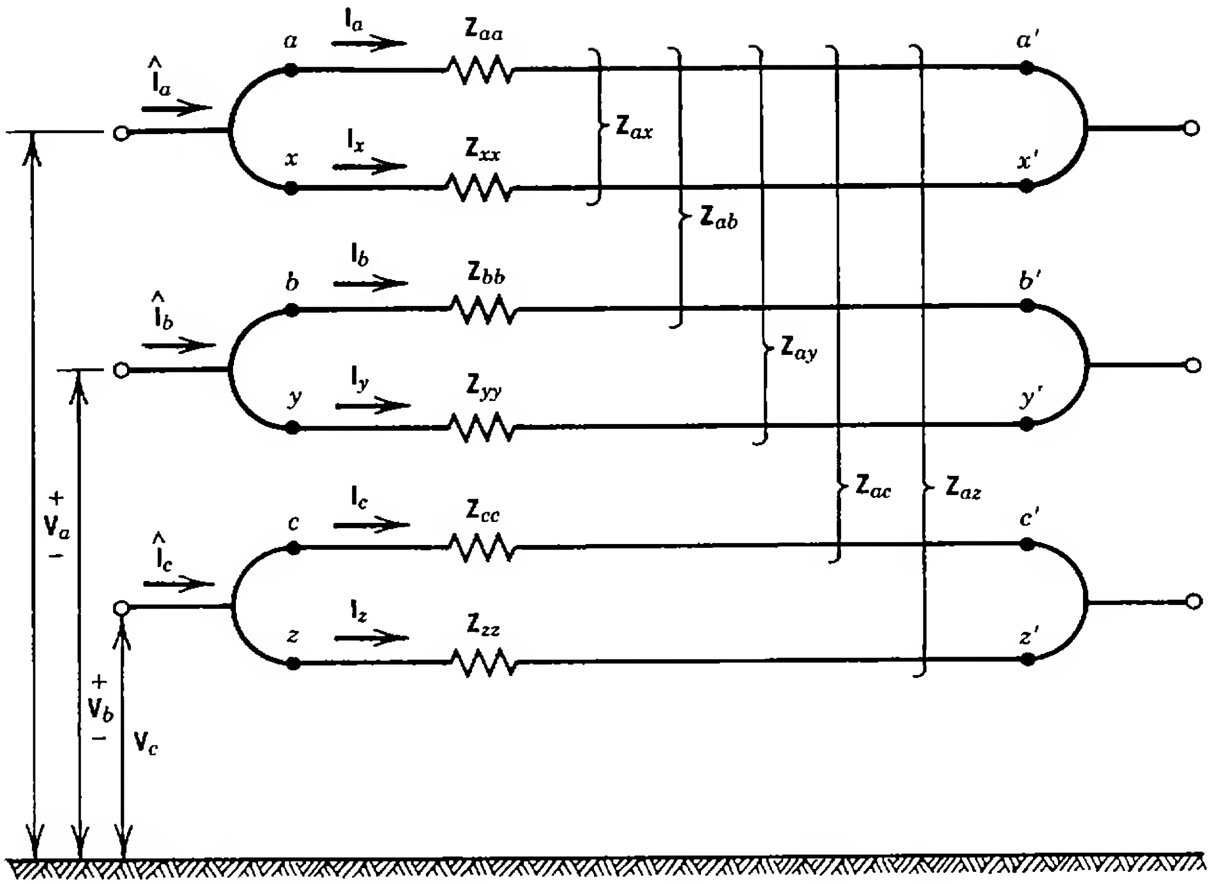


Figure 4.32. Equivalent circuit for *bundled* parallel cables.

The assumption of not having ground return current simplifies the calculation of the self-impedance and mutual impedance elements of the impedance matrix.

Self-impedances:

$$\mathbf{Z}_{aa} = l(r_a + jX_a) \quad \Omega \tag{4.199}$$

where

$$X_a = j0.1213 \ln \frac{12}{D_s} \quad \Omega/\text{mi} \tag{4.200}$$

where r_a = ac resistance of conductor a in ohms per mile
X_a = reactance of individual phase conductor at 12 in. spacing in ohms per mile
D_s = GMR or self-GMD of conductor a in inches

Based on the respective conductor characteristics, the self-impedances for conductors b, c, r, x, y, and z can be computed in a similar manner.

Mutual impedances:

$$\begin{aligned}\mathbf{Z}_{ab} &= l\left(j0.1213 \ln \frac{12}{D_{\text{eq}}}\right) \\ &= jlX_d \quad \Omega/\text{mi}\end{aligned} \tag{4.201}$$

where D_{eq} = equivalent, or geometric, mean distance among conductor centers in inches; based on the D_{eq}, other mutual impedances can be calculated similarly

After bundling, the constraining equations can be written as

$$\begin{aligned}\mathbf{V}_x - \mathbf{V}_a &= 0 \\ \mathbf{V}_y - \mathbf{V}_b &= 0 \\ \mathbf{V}_z - \mathbf{V}_c &= 0\end{aligned} \tag{4.202}$$

and it is also possible to define

$$\begin{aligned}\hat{\mathbf{I}}_a &= \mathbf{I}_a + \mathbf{I}_x \\ \hat{\mathbf{I}}_b &= \mathbf{I}_b + \mathbf{I}_y \\ \hat{\mathbf{I}}_c &= \mathbf{I}_c + \mathbf{I}_z\end{aligned} \tag{4.203}$$

By taking advantage of symmetry, the voltage matrix equation (4.198) can be expressed in partioned form as

$$\left[\begin{array}{c}\mathbf{V}_{abc} \\ \hline \mathbf{V}_{xyz}\end{array}\right]\left[\begin{array}{c:c}\mathbf{Z}_s & \mathbf{Z}_M \\ \hline \mathbf{Z}_M^{\mathrm{t}} & \mathbf{Z}_s\end{array}\right]\left[\begin{array}{c}\mathbf{I}_{abc} \\ \hline \mathbf{I}_{xyz}\end{array}\right] \tag{4.204}$$

If the bundled cabled did not consist of two identical cables, the two $[\mathbf{Z}_s]$ matrices would be different. When the $\mathbf{V}_x$, $\mathbf{V}_y$, and $\mathbf{V}_z$ equations in (4.25) are replaced by a new equation calculated from equation set (4.202) and the $\mathbf{I}_{abc}$ submatrix is replaced by $\hat{\mathbf{I}}_{abc}$,

$$\left[\begin{array}{c}\mathbf{V}_a \\ \mathbf{V}_b \\ \mathbf{V}_c \\ \hline \hat{0} \\ \hat{0} \\ \hat{0}\end{array}\right] = \left[\begin{array}{ccc:ccc}\mathbf{Z}_{aa} & \mathbf{Z}_{ab} & \mathbf{Z}_{ac} & \mathbf{Z}_{ax}-\mathbf{Z}_{aa} & \mathbf{Z}_{ay}-\mathbf{Z}_{ab} & \mathbf{Z}_{az}-\mathbf{Z}_{ac} \\ \mathbf{Z}_{ba} & \mathbf{Z}_{bb} & \mathbf{Z}_{bc} & \mathbf{Z}_{bx}-\mathbf{Z}_{ba} & \mathbf{Z}_{by}-\mathbf{Z}_{bb} & \mathbf{Z}_{bz}-\mathbf{Z}_{bc} \\ \mathbf{Z}_{ca} & \mathbf{Z}_{cb} & \mathbf{Z}_{cc} & \mathbf{Z}_{cx}-\mathbf{Z}_{ca} & \mathbf{Z}_{cy}-\mathbf{Z}_{cb} & \mathbf{Z}_{cz}-\mathbf{Z}_{cc} \\ \hline (\mathbf{Z}_{xa}-\mathbf{Z}_{aa} & \mathbf{Z}_{xb}-\mathbf{Z}_{ab} & \mathbf{Z}_{xc}-\mathbf{Z}_{ac} & \hat{\mathbf{Z}}_{xx} & \hat{\mathbf{Z}}_{xy} & \hat{\mathbf{Z}}_{xz} \\ \mathbf{Z}_{ya}-\mathbf{Z}_{ba} & \mathbf{Z}_{yb}-\mathbf{Z}_{bb} & \mathbf{Z}_{yc}-\mathbf{Z}_{bc} & \hat{\mathbf{Z}}_{yx} & \hat{\mathbf{Z}}_{yy} & \hat{\mathbf{Z}}_{yz} \\ \mathbf{Z}_{za}-\mathbf{Z}_{ca} & \mathbf{Z}_{ab}-\mathbf{Z}_{cb} & \mathbf{Z}_{zc}-\mathbf{Z}_{cc} & \hat{\mathbf{Z}}_{zx} & \hat{\mathbf{Z}}_{zy} & \hat{\mathbf{Z}}_{zz}\end{array}\right]\left[\begin{array}{c}\mathbf{I}_a+\mathbf{I}_x \\ \mathbf{I}_b+\mathbf{I}_y \\ \mathbf{I}_c+\mathbf{I}_z \\ \hline \hat{\mathbf{I}}_x \\ \hat{\mathbf{I}}_y \\ \hat{\mathbf{I}}_z\end{array}\right] \tag{4.205}$$

where all elements in the lower right position can be computed from

$$\hat{\mathbf{Z}}_{pq} = \mathbf{Z}_{pq} - \mathbf{Z}_{iq} - \mathbf{Z}_{pk} + \mathbf{Z}_{ik} \quad \text{where } i, k = a, b, c, \text{ and } p, q = x, y, z \tag{4.206}$$

or, in matrix notation,

$$\left[\begin{array}{c} \mathbf{V}_{abc} \\ \hline 0 \end{array}\right] = \left[\begin{array}{c|c} \mathbf{Z}_s & \mathbf{Z}_m - \mathbf{Z}_s \\ \hline \mathbf{Z}_m^t - \mathbf{Z}_s & \mathbf{Z}_k \end{array}\right] \left[\begin{array}{c} \hat{\mathbf{I}}_{abc} \\ \hline \mathbf{I}_{xyz} \end{array}\right] \tag{4.207}$$

where

$$[\mathbf{Z}_k] = [\mathbf{Z}_s] - [\mathbf{Z}_m] - \{[\mathbf{Z}_m]^t - [\mathbf{Z}_s]\} \tag{4.208}$$

By Kron reduction,

$$[\mathbf{V}_{abc}] = [\mathbf{Z}_{\text{new}}][\mathbf{I}_{abc}]$$

where

$$[\mathbf{Z}_{\text{new}}] = [\mathbf{Z}_s] - \{[\mathbf{Z}_m] - [\mathbf{Z}_s]\}[\mathbf{Z}_k]^{-1}\{[\mathbf{Z}_m]^t - [\mathbf{Z}_s]\} \tag{4.209}$$

Therefore, the sequence impedances and sequence voltage drops can be computed from

$$[\mathbf{Z}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] \tag{4.210}$$

and

$$[\mathbf{V}_{012}] = [\mathbf{Z}_{012}][\mathbf{I}_{012}] \tag{4.211}$$

respectively.

Example 4.12

A three-phase, 60-Hz bundled cable circuit is connected between a sending bus and a receiving bus using six single-conductor unsheathed cables as shown in Figures 4.32 and 4.33. Each phase has two paralleled single conductors. There is no ground return current. The cable circuit operates at 35 kV and 60 Hz. All of the single-conductor cables are of 350-kcmil copper, concentric strand, paper insulated, 10 mi long, and spaced 18 in. apart from each other, as shown in Figure 4.33. Assume that the conductor resistance is 0.19 Ω/mi and the GMR of one conductor is 0.262 in. and calculate:

(a) Phase impedance matrix $[\mathbf{Z}_{abc}]$.
(b) Sequence-impedance matrix $[\mathbf{Z}_{012}]$.

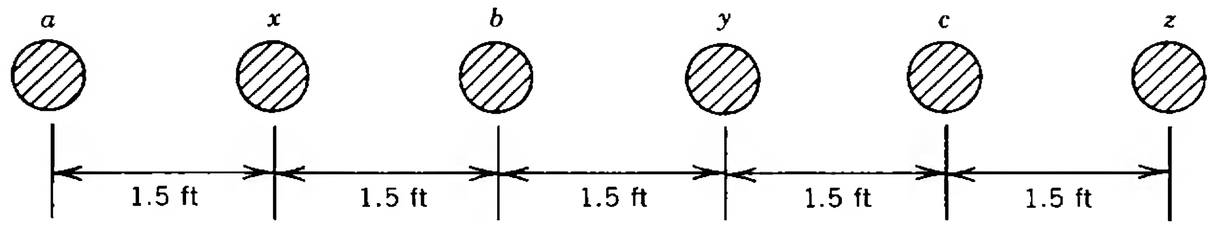


Figure 4.33

Solution

(a) $\mathbf{Z}_{aa} = \mathbf{Z}_{bb} = \mathbf{Z}_{cc} = \cdots = \mathbf{Z}_{zz} = l(r_a + jX_a)$

where

$$r_a = 0.19\ \Omega/\text{mi}$$

$$X_a = j0.1213 \ln \frac{12}{D_s}$$

$$= j0.1213 \ln \frac{12}{0.262} = 0.464\ \Omega/\text{mi}$$

Hence,

$$\mathbf{Z}_{aa} = \mathbf{Z}_{bb} = \cdots = \mathbf{Z}_{zz} = 10(0.19 + j0.464)$$

$$= 1.9 + j4.64\ \Omega$$

$$\mathbf{Z}_{ab} = \mathbf{Z}_{bc} = \mathbf{Z}_{xy} = \mathbf{Z}_{yz} = -l\left(j0.1213 \ln \frac{12}{D_{\text{eq}}}\right)$$

$$= -10\left(j0.1213 \ln \frac{12}{36}\right)$$

$$= -j1.3326\ \Omega$$

$$\mathbf{Z}_{bz} = \mathbf{Z}_{ay} = \mathbf{Z}_{cx} = -10 \times j0.1213 \ln \frac{12}{54}$$

$$= -j1.8214\ \Omega$$

$$\mathbf{Z}_{ac} = -10 \times j0.1213 \ln \frac{12}{72}$$

$$= -j2.1734\ \Omega$$

$$\mathbf{Z}_{ax} = \mathbf{Z}_{bx} = \mathbf{Z}_{by} = \mathbf{Z}_{cy} = \mathbf{Z}_{cz} = -10 \times j0.1213 \ln \frac{12}{18}$$

$$= -j0.4918\ \Omega$$

$$\mathbf{Z}_{az} = -10 \times j0.1213 \ln \frac{12}{90}$$

$$= -j2.4441\ \Omega$$

Therefore,

$$[\mathbf{Z}_{abcxyz}] =$$

$$\begin{bmatrix} 1.9+j4.64 & -j1.3326 & -j2.1734 & -j0.4918 & -j1.8244 & -j2.4441 \\ -j1.3326 & 1.9+j4.64 & -j1.3326 & -j0.4918 & -j0.4918 & -j1.8244 \\ -j2.1734 & -j1.3326 & 1.9+j4.64 & -j1.8244 & -j0.4918 & -j0.4918 \\ -j0.4918 & -j0.4918 & -j1.8244 & 1.9+j4.64 & -j1.3326 & -j2.1734 \\ -j1.8244 & -j0.4918 & -j0.4918 & -j1.3326 & 1.9+j4.64 & -j1.3326 \\ -j2.4441 & -j1.8244 & -j0.4918 & -j2.1734 & -j1.3326 & 1.9+j4.64 \end{bmatrix}$$

By Kron reduction,

$$[\mathbf{Z}_{\text{new}}] = [\mathbf{Z}_s] - \{[\mathbf{Z}_m] - [\mathbf{Z}_s]\}[\mathbf{Z}_k]^{-1}\{[\mathbf{Z}_m]^t - [\mathbf{Z}_s]\}$$

or

$$[\mathbf{Z}_{abc}] = \begin{bmatrix} 4.521+j2.497 & 1.594-j2.604 & 1.495-j3.090 \\ 1.594-j2.604 & 2.897+j3.802 & 0.930-j1.929 \\ 1.495-j3.090 & 0.930-j1.929 & 2.897+j3.802 \end{bmatrix}$$

(b) By doing the similarity transformation,

$$[\mathbf{Z}_{012}] = [\mathbf{A}]^{-1}[\mathbf{Z}_{abc}][\mathbf{A}] \quad \Omega$$

or

$$[\mathbf{Z}_{012}] = \begin{bmatrix} 6.118-j1.716 & 0.887-j0.770 & 0.606-j0.713 \\ 0.606-j0.713 & 2.099+j5.908 & -0.149+j0.235 \\ 0.887-j0.770 & 0.412+j0.121 & 2.099+j5.908 \end{bmatrix}.$$

4.12 LOCATION OF FAULTS IN UNDERGROUND CABLES

There are various methods for locating faults in underground cables. The method used for locating any particular fault depends on the nature of the fault and the extent of the experience of the testing engineer. Cable faults can be categorized as

1. Conductor failures or
2. Insulation failures.

In general, conductor failures are located by comparing the capacity of the insulated conductors. On the other hand, insulation failures are located by fault tests that compare the resistance of the conductors.

In short cables, the fault is usually located by inspection, that is, looking for smoking manholes or listening for cracking sound when the kenetron† is applied to the faulty cable.

The location of ground faults on cables of known length can be determined by means of the balanced-bridge principle.

† It is a two-electrode high-vacuum tube. They are used as power rectifiers for applications requiring low currents at high dc voltages, such as for electronic dust precipitation and high-voltage test equipment.

4.12.1 Fault Location by Using Murray Loop Test

It is the simplest of the bridge methods for locating cable failures between conductors and ground in any cable where there is a second conductor of the same size as the one with the fault. It is one of the best methods of locating high-resistance faults in low-conductor-resistance circuits. Figure 4.34 shows a Murray loop.

The faulty conductor is looped to an unfaulted conductor of the same cross-sectional area, and a slide-wire resistance box with two sets of coils is connected across the open ends of the loop. Obviously, the Murray loop cannot be established if the faulty conductor is broken at any point. Therefore, the continuity of the loop should be tested before applying the bridge principle. In order to avoid the effects of earth currents, the galvanometer is connected as shown in the figure. A battery energizes the bridge between the sliding contact or resistance box center and the point at which the faulty line is grounded. Balance is obtained by adjustment of the sliding contact or resistance. If the nongrounded (unfaulted) line and the grounded (faulted) line have the same resistance per unit length and if the slide wire is of uniform cross-sectional area,

$$\frac{A}{B} = \frac{2L - X}{X}$$

or

$$X = \frac{2L}{1 + \dfrac{A}{B}} \quad \text{units of length} \tag{4.212}$$

or

$$X = \frac{2LB}{A + B} \quad \text{units of length} \tag{4.213}$$

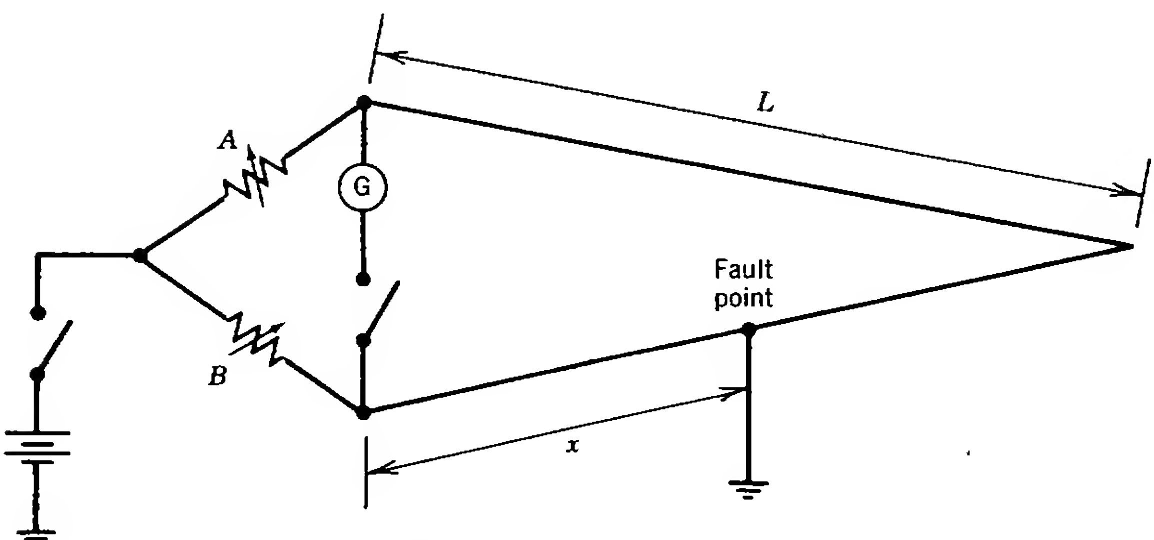


Figure 4.34. Murray loop.

where X = distance from measuring end to fault point
L = length of each looped conductor
A = resistance of top left-hand side bridge arm in balance
B = resistance of bottom left-hand side bridge arm in balance

Therefore, the distance X from the measuring end to the fault can be found directly in terms of the units used to measure the distance L.

4.12.2 Fault Location by Using Varley Loop Test

It can be used for faults to ground where there is a second conductor of the same size as the one with the fault. It is particularly applicable in locating faults in relatively high-resistance circuits. Figure 4.35 shows a Varley loop.

The resistance per unit length of the unfaulted conductor and the faulted conductor must be known. Therefore, if the conductors have equal resistances per unit length (e.g., r_c Ω), the resistance $(2L - X)r_c$ constitutes one arm of the bridge and the resistance

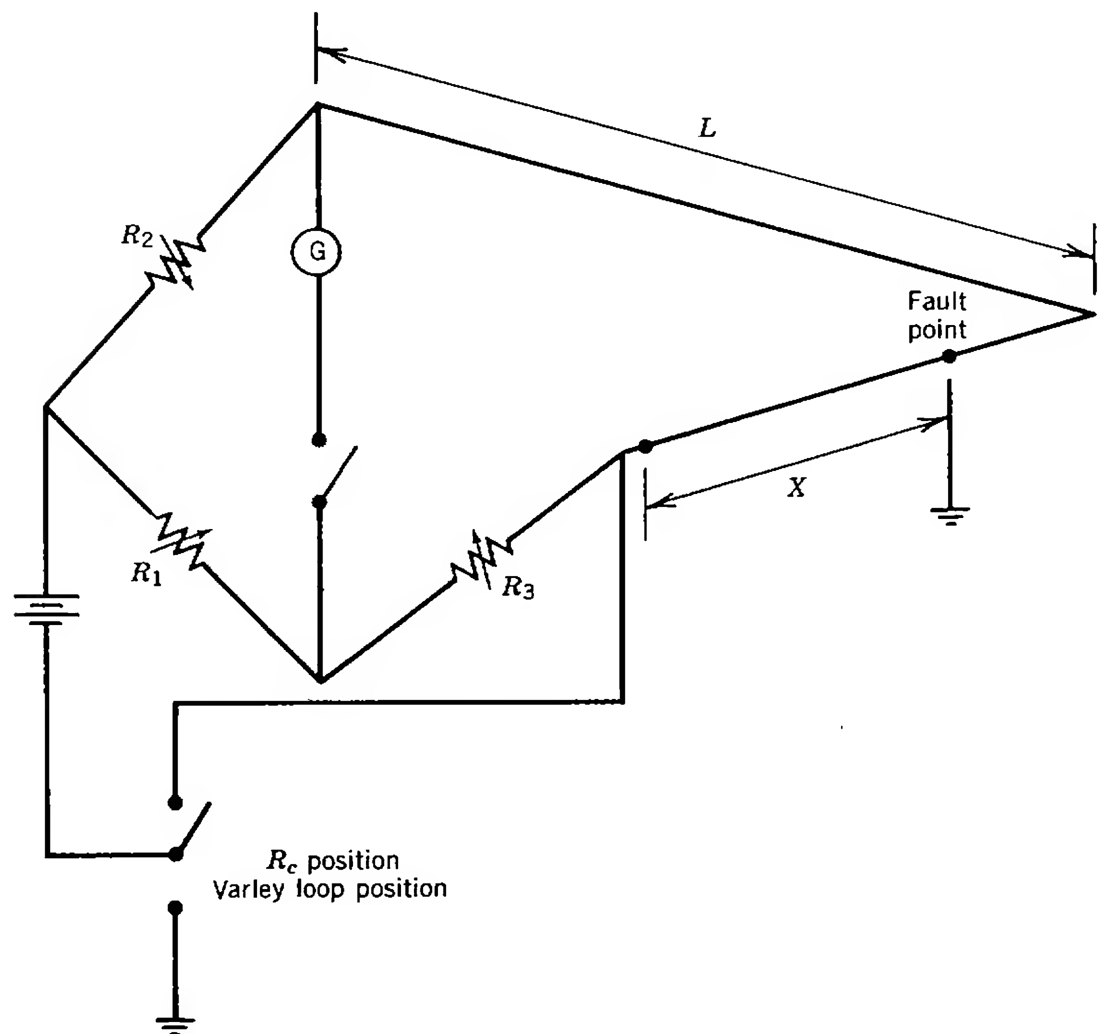


Figure 4.35. Varley loop.

$$\frac{R_1}{R_2} = \frac{R_3 + Xr_c}{(2L - X)r_c}$$

or

$$X = \frac{2L(R_1/R_2) - R_3/r_c}{1 + R_1/R_2} \quad \text{units of length} \tag{4.214}$$

or

$$X = \frac{R_2}{R_1 + R_2}\left(2L\,\frac{R_1}{R_2} - \frac{R_3}{r_c}\right) \quad \text{units of length} \tag{4.215}$$

where X = distance from measuring end to fault point
L = length of each looped conductor
R_1 = resistance of bottom left-hand side bridge arm in balance
R_2 = resistance of top left-hand bridge arm in balance
R_3 = adjustable resistance of known magnitude
r_c = conductor resistance in ohms per unit length.

If the conductor resistance is not known, it can easily be found by changing the switch to the r_c position and measuring the resistance of the conductor $2L$ by using the Wheatstone bridge method.

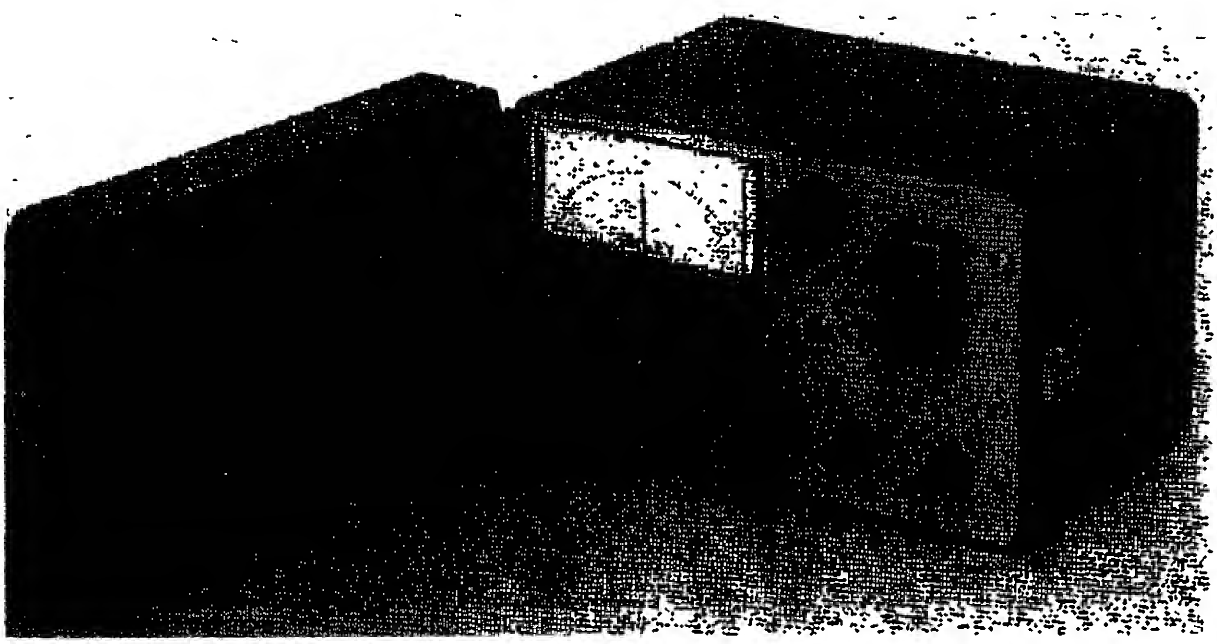


Figure 4.36. Portable Murray loop resistance bridge for cable fault-locating work (Courtesy of James G. Biddle Company.)

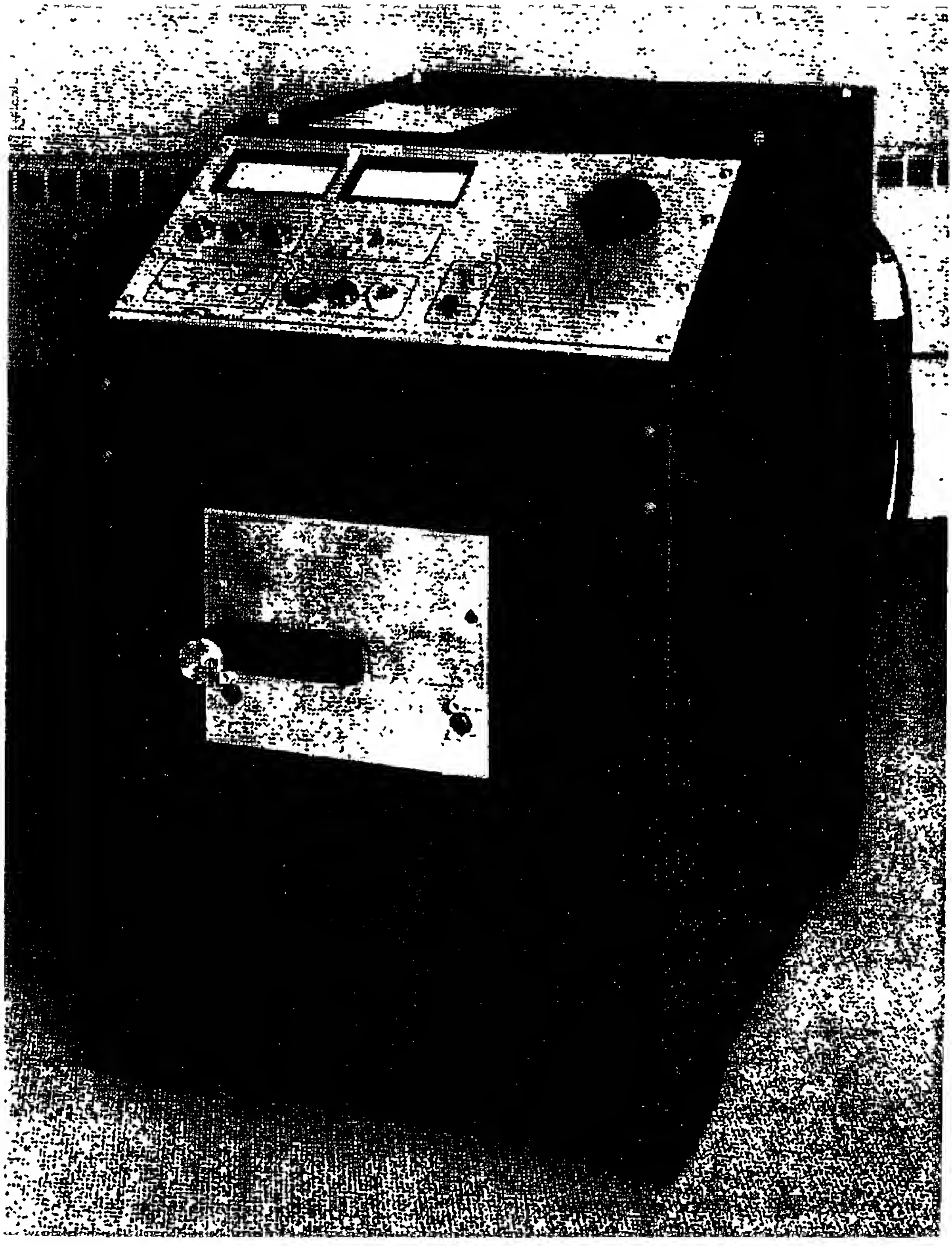


Figure 4.37. Heavy-duty cable test and fault-locating system. (Courtesy of James G. Biddle Company.)

4.12.3 Distribution Cable Checks

Newly installed cables should be subjected to a nondestructive test at higher than normal use values. Megger testing is a common practice. The word *Megger* is the trade name of a line of ohmmeters manufactured by the James G. Biddle Company. Certain important information regarding the quality

Figure 4.38. Lightweight battery-operated cable route tracer. (Courtesy of James G. Biddle Company.)

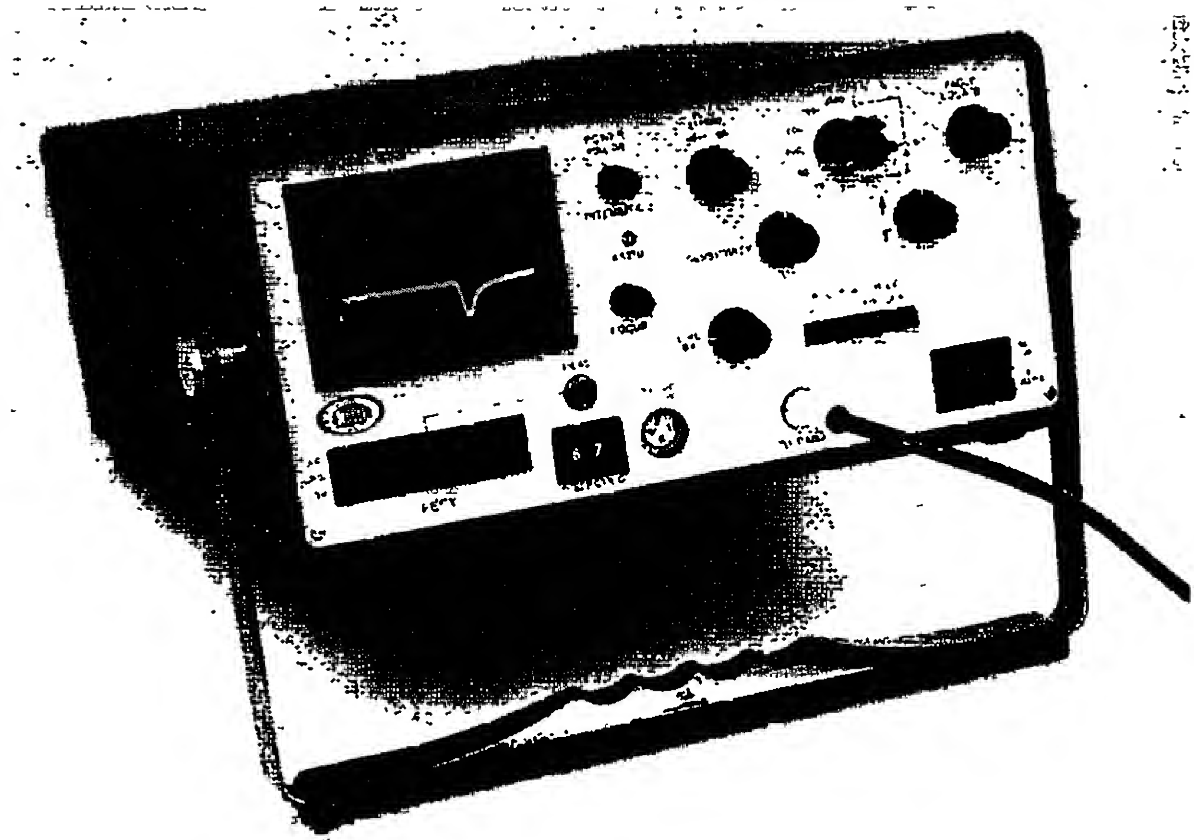


Figure 4.39. Automatic digital radar cable test set. (Courtesy of James G Biddle Company.)

and condition of insulation can be determined from regular Megger readings that is a form of preventive maintenance.

For example, Figure 4.36 shows a portable high-resistance bridge for cable-fault-locating work. Faults can be between two conductors or between a conductor and its conducting sheath, concentric neutral, or ground. Figure 4.37 shows a heavy-duty cable test and fault-locating system, which can be used for either grounded or ungrounded neutral 15-kV cables. The full 100 mA output current allows rapid reduction of high-resistance faults on cables rated 35 kV ac or higher to the level of 25 kV or lower for fault-locating purposes. Figure 4.38 shows a lightweight battery-operated cable route trace that can be used to locate, trace, and measure the depth of buried energized power cables. Figure 4.39 shows an automatic digital radar cable test set that requires no distance calculations, insulation calibrations, or zero pulse alignments.

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PROBLEMS

4.1. Assume that a 7.2-kV, 60-Hz, single-conductor, belted cable has a conductor diameter of 2 cm and a lead sheath with inside diameter of 4 cm. The resistivity of the insulation is 1.2×10^8 MΩ-cm, and the length of the cable is 3.5 mi. Calculate the following:

(a) Total insulation resistance in megohms

(b) Power loss due to leakage current flowing through insulation resistance.

4.2. Assume that a 2-mi-long, three-conductor, belted cable is connected to a 24.9-kV, three-phase, 60-Hz bus. A test result shows that the capacitance between bunch conductors and sheath is 0.8 μF/mi, and the capacitance between two conductors bunched together with the sheath and third conductor is 0.60 μF/mi. Find the charging current per conductor.

4.3. Assume that a single-conductor belted cable has a conductor diameter of 0.681 in., inside diameter of sheath of 1.7025 in., and a length of

8000 ft. The cable is to be operated at 12.47 kV. The dielectric constant is 4.5, and the power factor of the dielectric at a rated frequency and temperature is 0.05. Calculate the following:

(a) Capacitance of cable.

(b) Charging current.

(c) Dielectric loss of cable.

(d) Equivalent resistance of insulation.

4.4. Assume that a single-conductor belted cable has a conductor diameter of 2 cm and an inside diameter of sheath of 5 cm. Its insulation resistance is given as 275 MΩ/mi. Find the dielectric resistivity of the insulation.

4.5. Assume that a test has been conducted by means of a Schering bridge on a three-conductor belted cable at a rated voltage and frequency of 12.47 kV and 60 Hz, respectively. The capacitance between the two conductors when the third one is connected to the lead sheath was found to be 1.2 μF. Also, the capacitance between the three conductors connected together and the lead sheath was measured as 1.4 μF. Calculate the following:

(a) Effective capacitance to neutral.

(b) Charging current per conductor.

(c) Total charging current of cable.

(d) Capacitance between each conductor and sheath.

(e) Capacitance between each pair of conductors.

4.6. Assume that a single-phase concentric cable is 3 mi long and is connected to 60-Hz, 7.2-kV bus bars. The conductor diameter is 0.630 in., and the radial thickness of uniform insulation is 0.425 in. The relative permittivity of the dielectric is 4. Find the charging kilovoltamperes.

4.7. Assume that a single-phase voltage of 7.97 kV at 60 Hz frequency is applied between two of the conductors of a three-phase belted cable. The capacitances between conductors and between a conductor and a sheath are measured as 0.30 and 0.2 μF, respectively. Calculate the following:

(a) Potential difference between third conductor and sheath.

(b) Total charging current of cable.

4.8. Assume that a three-conductor, paper-insulted, belted cable is used as a three-phase underground feeder of 18 mi. It is operated at 60 Hz and 33 kV. The cable has three 350-kcmil sector-type conductors each with $\frac{10}{32}$ in. of conductor insulation and $\frac{5}{32}$ in. of belt insulation. Calculate the following:

(a) Geometric factor of cable using Table 4.3.

(b) Total charging current of line.

(c) Total charging kilovoltampere of line.

4.9. Assume that a three-conductor, paper-insulted, belted cable is used as a three-phase underground feeder of 5000 ft. The cable is operated at 15 kV, 60 Hz, and 75 °C. The cable has a 350-kcmil copper conductor. Calculate the effective resistance of the cable.

4.10. Repeat Problem 4.9 assuming the conductor is aluminum.

4.11. Repeat Problem 4.9 assuming three single-conductor cables are located in the separate ducts.

4.12. Repeat Example 4.9 assuming the spacing between conductor centers is 4.125 in. and the cables are located in the same horizontal plane.

4.13. Consider Example 4.12 and assume that the phase voltages are balanced and have a magnitude of 34.5 kV. Calculate the sequence voltage drop matrix $[\mathbf{V}_{012}]$.

4.14. A 60-Hz, 15-kV three-conductor, paper-insulated cable is used at 13.8 kV as a three-phase underground feeder of 10 mi. The cable has three 350-kcmil *compact sector-type* conductors with a diameter of 0.539 in. and a dielectric constant of 3.78. Calculate, for the zero, positive, and negative sequences, the shunt capacitance, shunt capacitive reactance, and charging current by using the formulas given in Section 4.9:

(a) For three-conductor shielded cables (insulation thickness 175 mils).

(b) For three-conductor belted cables with no conductor shielding (conductor insulation thickness 155 mils, belt insulation thickness 75 mils).

5

DIRECT-CURRENT POWER TRANSMISSION

5.1 BASIC DEFINITIONS

CONVERTER. A machine, device, or system for changing ac power to dc power or vice versa.

RECTIFIER. A converter for changing alternating current to direct current.

INVERTER. A converter for changing direct current to alternating current.

ARCBACK. A mulfunctioning phenomenon in which a valve conducts in the reverse direction.

PULSE NUMBER (*p*). The number of pulsations (i.e., cycles of ripple) of the direct voltage per cycle of alternating voltage (e.g., pulse numbers for three-phase one-way and three-phase two-way rectifier bridges are 3 and 6, respectively).

RIPPLE. The ac component from dc power supply arising from sources within the power supply. It is expressed in peak, peak-to-peak, root-mean-square (rms) volts, or as percent root-mean-square. Since HVDC converters have large dc smoothing reactors, approximately 1 H, the resultant direct current is constant (i.e., free from ripple). However, the direct voltage on the valve side of the smoothing reactor has ripple.

RIPPLE AMPLITUDE. The maximum value of the instantaneous difference between the average and instantaneous value of a pulsating unidirectional wave.

REACTOR. An inductive reactor between the dc output of the converter and the load. It is used to smooth the ripple in the direct current adequately, to reduce harmonic voltages and currents in the dc line, and to limit the magnitude of fault current. It is also called a smoothing reactor.

COMMUTATION. The transfer of current from one valve to another in the same row.

DELAY ANGLE (α). The time, expressed in electrical degrees, by which the starting point of commutation is delayed. It cannot exceed 180°. It is also called ignition angle or firing angle.

OVERLAP ANGLE (u). The time, expressed in degrees, during which the current is commutated between two rectifying elements. It is also called commutation time. In normal operation, it is less than 60° and is usually somewhere between 20° and 25° at full load.

EXTINCTION ANGLE (δ). The sum of the delay angle α and the overlap angle u of a rectifier and is expressed in degrees.

IGNITION ANGLE (β). The delay angle of an inverter and is equal to $\pi - \alpha$ electrical degrees.

EXTINCTION (ADVANCE) ANGLE (γ). The extinction angle of an inverter and is equal to $\pi - \gamma$ electrical degrees. It is defined as the time angle between the end of conduction and the reversal of the sign of the sinusoidal commutation voltage of the source.

COMMUTATION MARGIN ANGLE (ζ). The time angle between the end of conduction and the reversal of the sign of the nonsinusoidal voltage across the outgoing valve of an inverter. Under normal operating conditions, the commutation margin angle is equal to the extinction advance angle.

EQUIVALENT COMMUTATING RESISTANCE (R_c). The ratio of drop of direct voltage to direct current. However, it does not consume any power.

THYRISTOR (SCR). A thyristor (silicon-controlled rectifier) is a semiconductor device with an anode, a cathode terminal, and a gate for the control of the firing.

5.2 OVERHEAD HIGH-VOLTAGE dc TRANSMISSION

Figure 5.1 shows some of the typical circuit arrangements (links) for high-voltage dc transmissions. In the monopolar arrangement, shown in Figure 5.1(a), there is only one insulated transmission conductor (pole) installed and ground return is used. It is the least expensive arrangement but has certain disadvantages. For example, it causes the corrosion of buried pipes, cable sheaths, ground electrodes, etc. due to the electrolysis

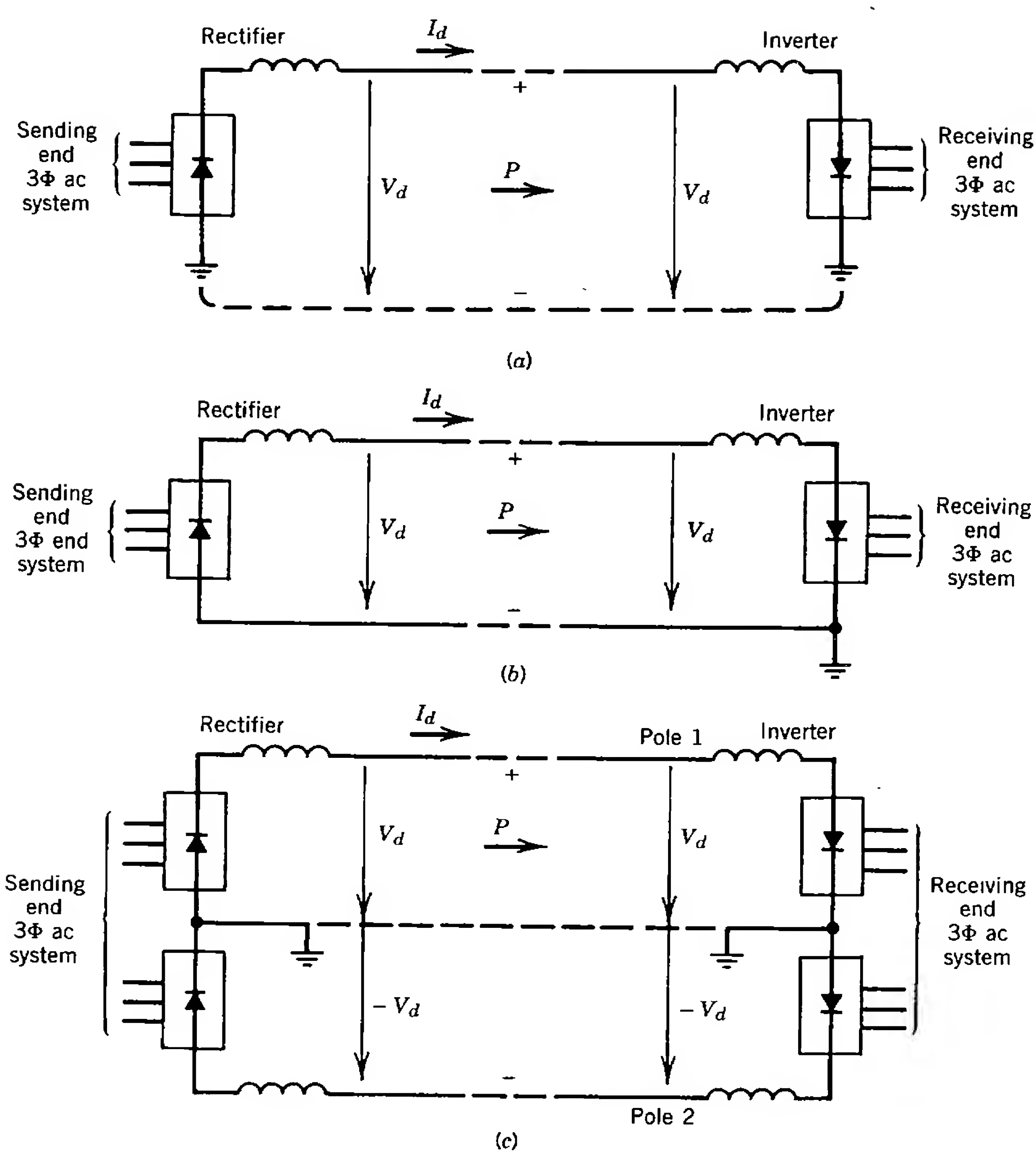


Figure 5.1. Typical circuit arrangements for high-voltage dc transmissions: (*a*) monopolar arrangement with ground return; (*b*) monopolar arrangement with metallic return grounded at one end; (*c*) bipolar arrangement.

phenomenon caused by the ground return current. It is used in dc systems that have low power ratings, primarily with cable transmission. In order to eliminate the aforementioned electrolysis phenomenon, a metallic return (conductor) can be used, as shown in Figure 5.1(*b*).

The bipolar circuit arrangement has two insulated conductors used as plus and minus poles. The two poles can be used independently if both neutrals are grounded. Under normal operation, the currents flowing in each pole

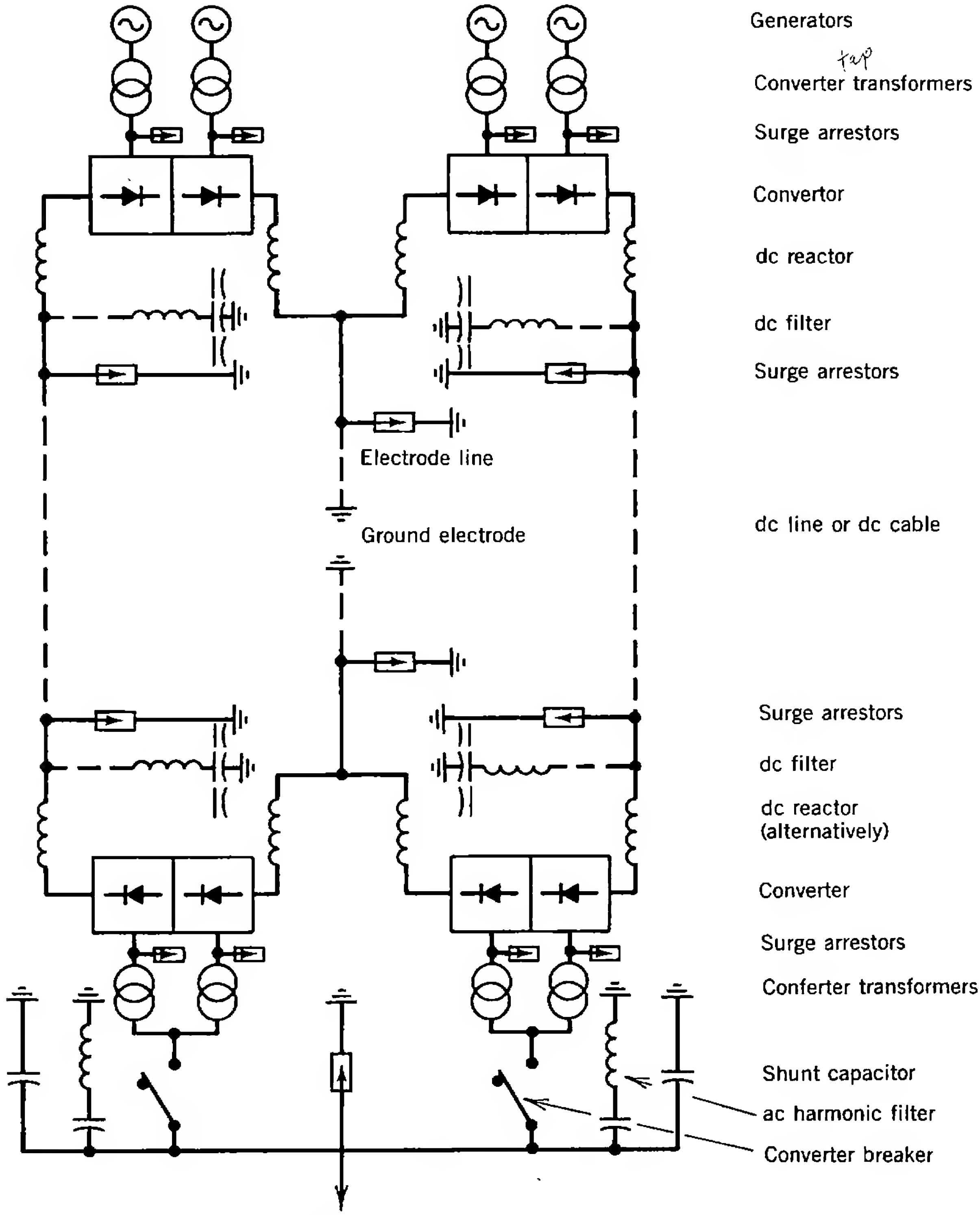


Figure 5.2. A dc transmission system operating in bipolar mode [2].

are equal, and therefore, there is no ground current. Under emergency operation, the ground return can be used to provide for increased transmission capacity. For example, if one of the two poles is out of order, the other conductor with ground return can carry up to the total power of the link. In that case, the transmission line losses are doubled. As shown in Figure 5.1(c), the rated voltage of a bipolar arrangement is given as $\pm V_d$ (e.g.,

±500 kV, which is read as "plus and minus 500 kV"). Figure 5.2 shows a dc transmission system operating in the bipolar mode.

It is possible to have two or more poles all having the same polarity and always having a ground return. This arrangement is known as the *homopolar arrangement* and is used to transmit power in dc systems that have very large ratings. The dc tower normally carries only two insulated conductors, and the ground return can be used as the additional conductor.

5.3 COMPARISON OF POWER TRANSMISSION CAPACITY OF HIGH-VOLTAGE dc AND ac

Assume that there are two comparable transmission lines; one is the ac and the other the dc line. Assume that both lines have the same length and are made of the same conductor sizes and that the loading of both lines is thermally limited so that current I_d equals the rms ac current I_L. Also assume that the ac line has three phase and three wires and has a power factor of 0.945, and the dc line is a bipolar circuit arrangement with two conductors. Furthermore, assume that the ac and dc insulators withstand the same crest voltage to ground so that the voltage V_d is equal to $\sqrt{2}$ times the rms ac voltage. Therefore, it can be shown that the dc power per conductor is

$$p_{(\mathrm{dc})} = V_d I_d \quad \text{W/conductor} \tag{5.1}$$

and the ac power per conductor is

$$P_{(\mathrm{ac})} = V_{(\mathrm{L\text{-}N})} I_L \cos\Phi \quad \text{W/conductor} \tag{5.2}$$

where V_d = line-to-ground dc voltage in volts
$V_{(\mathrm{L\text{-}N})}$ = line-to-neutral ac voltage in volts
I_d = dc line current in amperes
I_L = ac line current in amperes

Therefore, the ratio of the dc power per conductor to the ac power per conductor (phase) can be expressed as

$$\frac{p_{(\mathrm{dc})}}{p_{(\mathrm{ac})}} = \frac{V_d I_d}{V_{(\mathrm{L\text{-}N})} I_L \cos\Phi} \tag{5.3}$$

or

$$\frac{p_{(\mathrm{dc})}}{p_{(\mathrm{ac})}} = \frac{\sqrt{2}}{\cos\Phi} \tag{5.4}$$

but since

$$\cos \Phi = 0.945$$

then

$$\frac{p_{(dc)}}{p_{(ac)}} = 1.5 \tag{5.5}$$

or

$$p_{(dc)} = 1.5 p_{(ac)} \quad \text{W/conductor} \tag{5.6}$$

Furthermore, the total power transmission capabilities for the dc and ac lines can be expressed as

$$P_{(dc)} = 2 p_{(dc)} \quad \text{W} \tag{5.7}$$

and

$$P_{(ac)} = 3 p_{(ac)} \quad \text{W} \tag{5.8}$$

Therefore, their ratio can be expressed as

$$\frac{P_{(dc)}}{P_{(ac)}} = \frac{2}{3} \frac{p_{(dc)}}{p_{(ac)}} \tag{5.9}$$

Substituting equation (5.5) into equation (5.9),

$$\frac{P_{(dc)}}{P_{(ac)}} = \frac{2}{3} \frac{3}{2} = 1$$

or

$$P_{(dc)} = P_{(ac)} \quad \text{W} \tag{5.10}$$

Thus, both lines have the same transmission capability and can transmit the same amount of power. However, the dc line has two conductors rather than three and thus requires only two-thirds as many insulators. Therefore, the required towers and rights-of-way are narrower in the dc line than the ac line. Even though the power loss per conductor is the same for both lines, the total power loss of the dc line is only two thirds that of the ac line. Thus, studies indicate that a dc line generally costs about 33 percent less than an ac line of the same capacity. Furthermore, if a two-pole (homopolar) dc line is compared with a double-circuit three-phase ac line, the dc line costs would be about 45 percent less than the ac line. In general, the cost advantage of the dc line increases at higher voltages. The power losses due to the corona phenomena are smaller for dc than for ac lines.

The reactive powers generated and absorbed by a high-voltage ac transmission line can be expressed as

$$Q_c = X_c V^2 \quad \text{vars/unit length} \tag{5.11}$$

or

$$Q_c = wcV^2 \quad \text{vars/unit length} \tag{5.12}$$

and

$$Q_L = X_L I^2 \quad \text{vars/unit length} \tag{5.13}$$

or

$$Q_L = wLI^2 \quad \text{vars/unit length} \tag{5.14}$$

where X_c = capacitive reactance of line in ohms per unit length
X_L = inductive reactance of line in ohms per unit length
C = shunt capacitance of line in farads per unit length
L = series inductance of line in farads per unit length
V = line-to-line operating voltage in volts
I = line current in amperes

If the reactive powers generated and absorbed by the line are equal to each other,

$$Q_c = Q_L$$

or

$$wcV^2 = wLI^2$$

from which the surge impedance of the line can be found as

$$Z_c = \frac{V}{I}$$

$$= \sqrt{\frac{L}{C}} \quad \Omega \tag{5.15}$$

Therefore, the power transmitted by the line at the surge impedance can be expressed as

$$\text{SIL} = \frac{V^2_{(\text{L-L})}}{Z_c} \quad \text{W} \tag{5.16}$$

Note that this surge impedance loading (or natural load) is a function of the voltage and line inductance and capacitance. However, it is not a function of the line length. In general, the economical load of a given overhead transmission line is larger than its SIL. In which case, the net reactive power absorbed by the line must be provided from one or both ends of the line and from intermediate series capacitors. Therefore, the costs of necessary series capacitor and shunt reactor compensation should be taken into account in the comparison of ac versus dc lines. The dc line itself does not require any reactive power. However, the converters at both ends of the line require reactive power from the ac systems.

Underground cables used for ac transmission can also be used for dc, and they can normally carry more dc power than ac due to the absence of capacitive charging current and better utilization of insulation and less dielectric wear. However, a high-voltage dc transmission cable is designed somewhat differently than that of an ac transmission cable. Since a power cable employed for dc power transmission does not have capacitive leakage currents, the power transmission is restricted by the I^2R losses only. Furthermore, submarine or underground ac cables are always operated at a load that is far less than the surge impedance load in order to prevent overheating. As a result of this practice, the reactive power generated by charging the shunt capacitance is greater than that absorbed by the series inductance. Therefore, compensating shunt reactors are to be provided at regular intervals (approximately 20 mi). Whereas dc cables do not have such restrictions. Thus, the power transmission using dc cable is much cheaper than ac cable.

The major advantages of the dc transmission can be summarized as:

1. If the high cost of converter stations is excluded, the dc overhead lines and cables are less expensive than ac overhead lines and cables. The break-even distance is about 500 mi for the overhead lines, somewhere between 15 and 30 mi for submarine cables, and 30 and 60 mi for underground cables. Therefore, in the event that the transmission distance is less than the break-even distance, the ac transmission is less expensive than dc; otherwise, the dc transmission is less expensive. The exact break-even distance depends on local conditions, line performance requirements, and connecting ac system characteristics.
2. A dc link is asynchronous; that is, it has no stability problem in itself. Therefore, the two ac systems connected at each end of the dc link do not have to be operating in synchronism with respect to each other or even necessarily at the same frequency.
3. The corona loss and radio interference conditions are better in the dc than the ac lines.

4. The power factor of the dc line is always unity, and therefore no reactive compensation is needed.
5. Since the synchronous operation is not demanded, the line length is not restricted by stability.
6. The interconnection of two separate ac systems via a dc link does not increase the short-circuit capacity, and thus the circuit breaker ratings, of either system.
7. The dc line loss is smaller than for the comparable ac line.

The major disadvantages of the dc transmission can be summarized as follows:

1. The converters generate harmonic voltages and currents on both ac and dc sides, and therefore filters are required.
2. The converters consume reactive power.
3. The dc converter stations are expensive.
4. The dc circuit breakers have disadvantages with respect to the ac circuit breakers because the dc current does not decrease to zero twice a cycle, contrary to the ac current.

5.4 HIGH-VOLTAGE dc TRANSMISSION LINE INSULATION

The factors that affect the insulation of the high-voltage dc overhead transmission lines are (1) steady-state operating voltages, (2) switching surge overvoltages, and (3) lightning overvoltages. There factors must be restricted to values that cannot cause puncture or flashover of the insulation. The steady-state operating voltage affects the selection of leakage distance, particularly when there is considerable pollution in the environment. Whereas the switching surge and lightning overvoltages influence the required insulator chain length and strike distance.

Consider the transmission line conductor configurations shown in Figure 5.3 for the comparable ac and dc systems. For the steady-state operating voltages, it can be shown that the factor K given by the following equation relates the dc and ac voltages to ground that may be applied to a given insulation:

$$K = \frac{V_d/2}{E_p} \tag{5.17}$$

or

$$\tfrac{1}{2}V_d = KE_p \quad \text{V} \tag{5.18}$$

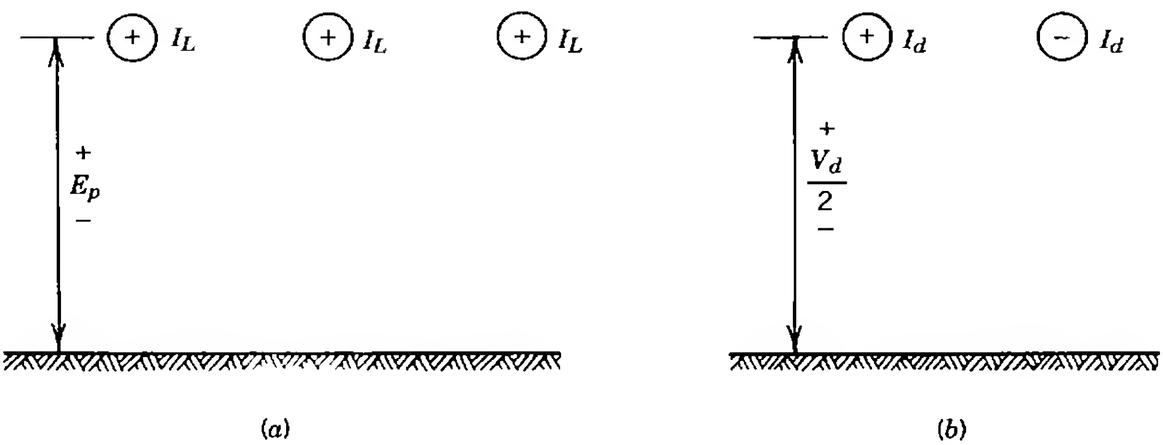


Figure 5.3. Transmission line conductor configuration for high voltage: (*a*) ac system; (*b*) dc system.

where V_d = line-to-line dc voltage in volts
$\frac{1}{2}V_d$ = line-to-ground dc voltage in volts
E_p = rms line-to-neutral ac voltage in volts

Typical values of factor K are

$K = \sqrt{2}$	for indoor porcelain
$K = 1$	for outdoor porcelain
$2 \leq K \leq 6$	for insulated power cables

The given data for factor K imply that conventional insulators have inferior wet flashover performance when used in the high-voltage service. Therefore, the high-voltage dc lines require special insulators.

Typically, the following approximate insulation levels are required to withstand switching surges on overhead lines:

For high-voltage ac lines,

$$\text{ac insulation level} = K_1 E_p \cong 2.5 E_p \tag{5.19}$$

and for high-voltage dc lines,

$$\text{dc insulation level} = K_2(\tfrac{1}{2}V_d) \cong 1.7(\tfrac{1}{2}V_d) \tag{5.20}$$

Therefore, for a fixed value of insulation level, the following operating voltages can be used from the standpoint of switching surge performance:

$$K_1 E_p = K_2(\tfrac{1}{2}V_d) \tag{5.21}$$

If $K_1 = 2.5$ and $K_2 = 1.7$,

$$2.5E_p = 1.7(\tfrac{1}{2}V_d)$$

or

$$(\tfrac{1}{2}V_d) = 1.47E_p \quad \text{V} \tag{5.22}$$

On overhead lines, the maximum steady-state operating voltage or the minimum conductor size is also restricted by the power losses and radio interference due to corona. Whereas in cables, the restricting factor is usually the normal steady-state operating voltage.

EXAMPLE 5.1

Assume that the overhead ac and dc lines shown in Figure 5.3 have the same line length and are made of the same size conductors that transmit the same amount of power and have the same total I^2R losses. Assume that the ac line is three phase, has three wires, and has a unity power factor and that the dc line has two wires plus ground return. Furthermore, assume that the factors K_1 and K_2 are 2.5 and 1.7, respectively, and determine the following:

(a) Line-to-line dc voltage of V_d in terms of line-to-neutral voltage E_p.
(b) The dc line current I_d in terms of ac line current I_L.
(c) Ratio of dc insulation level to ac insulation level.

Solution

(a) Since the power losses are the same in either system,

$$P_{\text{loss(dc)}} = P_{\text{loss(ac)}} \quad \text{W}$$

or

$$2I_d^2R_{\text{(dc)}} = 3I_L^2R_{\text{(ac)}} \quad \text{W} \tag{5.23}$$

where, ignoring the skin effects,

$$R_{\text{(dc)}} = R_{\text{(ac)}} \quad \Omega$$

so that

$$I_d = \sqrt{\tfrac{3}{2}}I_L \quad \text{A}$$

or

$$I_d = 1.225I_L \quad \text{A} \tag{5.24}$$

(b) Since

$$P_{\text{(dc)}} = P_{\text{(ac)}}$$

or

$$V_dI_d = 3E_pI_L \tag{5.25}$$

by substituting equation (5.24) into equation (5.25),

$$V_d = 2.45E_p \tag{5.26}$$

(c) The ratio is

$$\frac{\text{dc insulation level}}{\text{ac insulation level}} = \frac{K_2(V_d/2)}{K_1E_p} \tag{5.27}$$

where

$$K_1 = 2.5 \quad K_2 = 1.7$$

Therefore, substituting equation (5.26) into equation (5.27),

$$\frac{\text{dc insulation level}}{\text{ac insulation level}} = 8.8328 \tag{5.28}$$

or

$$\text{dc insulation level} = 0.8328 \text{ (ac insulation level)} \tag{5.29}$$

EXAMPLE 5.2

Consider an existing three-phase high-voltage cable circuit made of three single-conductor insulated power cables. The loading of the circuit is thermally limited at the cable rms ampacity current I_L. Assume that the normal ac operating voltage is E_p. Investigate the merits of converting the cable circuit to high-voltage dc operation wherein one of the three existing cables is used either as a spare or as the grounded neutral conductor. Assume that factor K is 3 and determine the following:

(a) Maximum operating V_d in terms of voltage E_p.
(b) Maximum power transmission capability ratio, that is, ratio of $P_{(dc)}$ to $P_{(ac)}$.
(c) Ratio of total I^2R losses, that is, ratio of $P_{loss(dc)}$ to $P_{loss(ac)}$, that accompany maximum power flow. (Assume that the power factor for the ac operation is unity and that the skin effect is negligible.)

Solution

(a) From equation (5.18),

$$\tfrac{1}{2}V_d = KE_p$$
$$= 3E_p$$

Therefore,

$$V_d = bE_p \tag{5.30}$$

(b) The maximum power transmission capability ratio is

$$\frac{P_{(dc)}}{P_{(ac)}} = \frac{V_dI_d}{3E_pI_L} \tag{5.31}$$

Since the circuit is thermally limited,

$$I_d = I_L \tag{5.32}$$

Therefore, substituting equations (5.30) and (5.32) into equation (5.31),

$$\frac{P_{(\mathrm{dc})}}{P_{(\mathrm{ac})}} = 2 \tag{5.33}$$

or

$$P_{(\mathrm{dc})} = 2P_{(\mathrm{ac})} \tag{5.34}$$

(c) The ratio of total I^2R losses is

$$\frac{P_{\mathrm{loss(dc)}}}{P_{\mathrm{loss(ac)}}} = \frac{2I_d^2R}{3I_L^2R} \tag{5.35}$$

Since

$$R_{(\mathrm{dc})} \cong R_{(\mathrm{ac})} \tag{5.36}$$

Substituting equations (5.32) and (10.36) into equation (5.35) yields

$$\frac{P_{\mathrm{loss(dc)}}}{P_{\mathrm{loss(ac)}}} = \frac{2}{3} \tag{5.37}$$

or

$$P_{\mathrm{loss(dc)}} = \tfrac{2}{3}P_{\mathrm{loss(ac)}} \tag{5.38}$$

5.5 THREE-PHASE BRIDGE CONVERTER

The energy conversion from ac to dc is called *rectification* and the conversion from dc to ac is called *inversion*. A converter can operate as a rectifier or as an inverter provided that it has grid control. A valve, whether it is a mercury arc valve or a solid-state (thyristor) valve, can conduct in only one direction (the forward direction), from anode to cathode. The resultant arc voltage drop is less than 50 V. The valve can endure a considerably high voltage in the negative (inverse) direction without conducting. Any arcback in mercury arc rectifiers can be stopped by grid control and by a bypass valve.

Presently, the thyristors have converter current ratings up to 2000 A. Their typical voltage rating is 3000 V. A solid-state valve has a large number of thyristors connected in series to provide proper voltage division among the thyristors. The thyristors are also connected in parallel, depending on the valve current rating. The thyristors are grouped in modules, each having 2–10 thyristors with all auxiliary circuits. Some of the advantages of thyristors are as follows:

1. There is no possibility of arcback.
2. They have lower maintenance requirements.
3. They have less space requirements.
4. They have shorter deionization time.
5. There is no need for degassing facilities.
6. There is no need for bypass valves.

In this chapter, the term *valve* includes the solid-state devices as well as the mercury arc valves.

5.6 RECTIFICATION

In a given bridge rectifier, the transfer of current from one valve to another in the same row is called *commutation*. The time during which the current is commutated between two rectifying elements is known as the *overlap angle* or *commutation time*. Therefore, if two valves conduct simultaneously, there is no overlap, that is, commutation delay. The time during which the starting point of commutation is delayed is called the delay angle. The delay angle is governed by the grid control setting.

Neglecting overlap angle, the average direct voltage for a given delay angle α can be exressed as

$$V_d = \frac{3\sqrt{3}}{\pi} E_m \cos \alpha \tag{5.39}$$

or

$$V_d = V_{d0} \cos \alpha \tag{5.40}$$

since

$$V_{d0} = \frac{3\sqrt{3}}{\pi} E_m \tag{5.41}$$

where V_{d0} = ideal no-load direct voltage
E_m = maximum value of phase-neutral alternating voltage
α = delay angle

However, if there is no delay, that is, $\alpha = 0$, the average direct voltage can be expressed as

$$V_{d0} = \frac{3\sqrt{3}}{\pi} E_m \tag{5.42}$$

or

$$V_{d0} = \frac{3\sqrt{6}}{\pi} E_{(\text{L-N})}$$
$$= 2.34 E_{(\text{L-N})} \tag{5.43}$$

or

$$V_{d0} = \frac{3\sqrt{2}}{\pi} E_{(\text{L-L})}$$
$$= 1.35 E_{(\text{L-L})} \tag{5.44}$$

where $E_{(\text{L-N})}$ = rms line-to-neutral alternating voltage
$E_{(\text{L-L})}$ = rms line-to-line alternating voltage

From equation (5.40), one can observe that the delay angle α can change the average direct voltage by the factor $\cos\alpha$. Since α can take values from 0 to almost 180°, the average direct voltage can take values from positive V_{d0} to negative V_{d0}. However, the negative direct voltage V_d with positive current I_d causes the power to flow in the opposite direction. Therefore, the converter operates as an inverter rather than as a rectifier. Note that since the current can only flow from anode to cathode, the direction of current I_d remains the same.

It can be shown that the rms value of the fundamental-frequency component of alternating current is

$$I_{L1} = \frac{\sqrt{6}}{\pi} I_d \tag{5.45}$$

or

$$I_{L1} = 0.780 I_d \tag{5.46}$$

When losses are disregarded, the active ac power can be set equal to the dc power, that is,

$$P_{(\text{ac})} = P_{(\text{dc})} \tag{5.47}$$

where

$$P_{(\text{ac})} = 3E_{(\text{L-N})} I_{L1} \cos\Phi \tag{5.48}$$

$$P_{(\text{dc})} = V_d I_d \tag{5.49}$$

Substituting equation (5.45) into equation (5.48),

$$P_{(\text{ac})} = \frac{3\sqrt{6}}{\pi} E_{(\text{L-N})} I_d \cos\Phi \tag{5.50}$$

Also substituting equations (5.40) and (5.43) simultaneously into equation (5.49),

$$P_{(\mathrm{dc})} = \frac{3\sqrt{6}}{\pi} E_{(\mathrm{L\text{-}N})} I_d \cos \alpha \tag{5.51}$$

Therefore, by substituting equations (5.50) and (5.51) into equation (5.47), it can be shown that

$$\cos \Phi = \cos \alpha \tag{5.52}$$

where $\cos \Phi$ = displacement factor (or vector power factor)
Φ = angle by which fundamental-frequency component of alternating line current lags line-to-neutral source voltage
α = delay angle

Thus, the delay angle α displaces the fundamental component of the current by an angle Φ. Therefore, the converter draws reactive power from the ac system: "The rectifier is said to take lagging current from the ac system, and the inverter is said either to take lagging current or to deliver leading current to the ac system" [1].

When there is an overlap angle (u), it causes the alternating current in each phase to lag behind its voltage. Therefore, the corresponding decrease in direct voltage due to the commutation delay can be expressed as

$$\Delta V_d = \frac{V_{d0}}{2} [\cos \alpha - \cos(\alpha + u)] \tag{5.53}$$

Thus, the associated average direct voltage can be expressed as

$$V_d = V_{d0} \cos \alpha - \Delta V_d \tag{5.54}$$

or

$$V_d = \tfrac{1}{2} V_{d0} [\cos \alpha + \cos(\alpha + u)] \tag{5.55}$$

Note that the extinction angle δ is

$$\delta \triangleq \alpha + u \tag{5.56}$$

Thus, substituting equation (5.56) into equations (5.53) and (5.55),

$$\Delta V_d = \tfrac{1}{2} V_{d0} (\cos \alpha - \cos \delta) \tag{5.57}$$

and

$$V_d = \tfrac{1}{2} V_{d0} (\cos \alpha + \cos \delta) \tag{5.58}$$

The overlap angle u is due to the fact that the ac supply source has inductance. Thus, the currents in it cannot change instantaneously. Therefore, the current transfer from one phase to another takes a certain time, which is known as the commutation time or overlap time (u/w). In normal operation, the overlap angle is $0° < u < 60°$. Whereas in the abnormal operation mode, it is $60° < u < 120°$. The commutation delay takes place when two phases of the supplying ac source are short-circuited. Therefore, it can be shown that at the end of the commutation,

$$I_d = I_{s2}[\cos\alpha - \cos(\alpha + u)] \tag{5.59}$$

but

$$I_{s2} = \frac{\sqrt{3}E_m}{2wL_c} \tag{5.60}$$

Substituting equation (5.60) into equation (5.59),

$$I_d = \frac{\sqrt{3}E_m}{2wL_c}[\cos\alpha - \cos(\alpha + u)] \tag{5.61}$$

where I_{s2} = maximum value of current in line-to-line short circuit on ac source
L_c = series inductance per phase of ac source

By dividing equations (5.53) and (5.59) side by side,

$$\frac{\Delta V_d}{I_d} = \frac{V_{d0}[\cos\alpha - \cos(\alpha + u)]}{2I_{s2}[\cos\alpha - \cos(\alpha + u)]}$$

or

$$\frac{\Delta V_d}{I_d} = \frac{V_{d0}}{2I_{s2}} \tag{5.62}$$

so that

$$\Delta V_d = \frac{I_d}{2I_{s2}} V_{d0} \tag{5.63}$$

substituting equation (5.63) into equation (5.54),

$$V_d = V_{d0}\left(\cos\alpha - \frac{I_d}{2I_{s2}}\right) \tag{5.64}$$

or

$$V_d = V_{d0}\cos\alpha - R_c I_d \tag{5.65}$$

where

R_c = equivalent commutation resistance per phase (it does not consume any power and represents voltage drop due to commutation)

$$= \frac{3}{\pi} X_c \tag{5.66}$$

or

$$R_c = \frac{3}{\pi} wL_c \tag{5.67}$$

or

$$R_c = 6fL_c \tag{5.68}$$

Figure 5.4 shows two different representations of the equivalent circuit of a bridge rectifier based on equation (5.65). The direct voltage V_d can be controlled by changing the delay angle α or by varying the no-load direct voltage using a transformer tap changer.

EXAMPLE 5.3

Figure 5.5 shows that a rectifier transformer with a tap changer under load is connected to a large ac network. Assume that the Thévenin equivalent voltage of the ac network is given as 92.95/161Y kV and that the impedance of the rectifier transformer is 0.10 pu Ω based on transformer ratings. The subtransient Thévenin impedances of the ac system are to be computed from the three-phase short-circuit data given in the figure for the faults occurring at the bus. Assume zero power factor faults in circuits of this size. Assume that the bridge rectifier ratings are given as 125 kV and 1600 A for the maximum continuous no-load direct voltage (i.e., V_{dr0}) and maximum continuous current (i.e., I_d), respectively. Use the given data and specify the rectifier transformer in terms of:

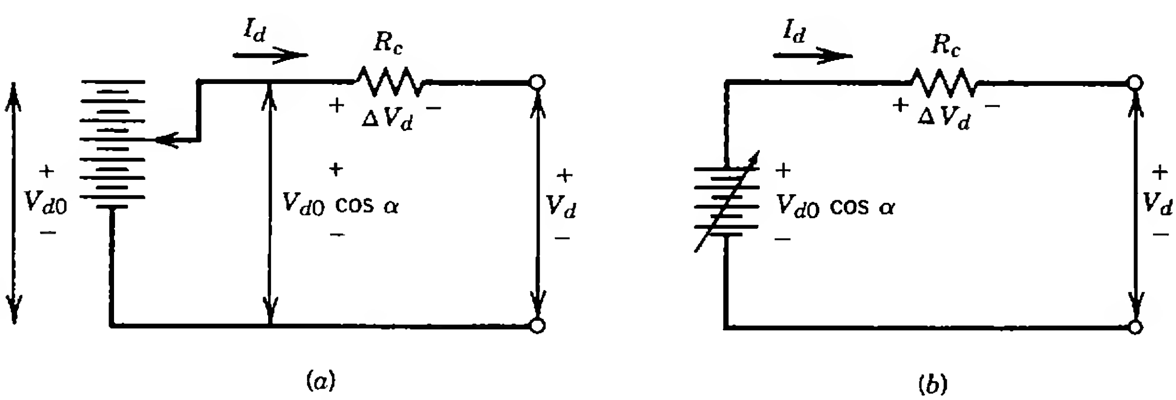
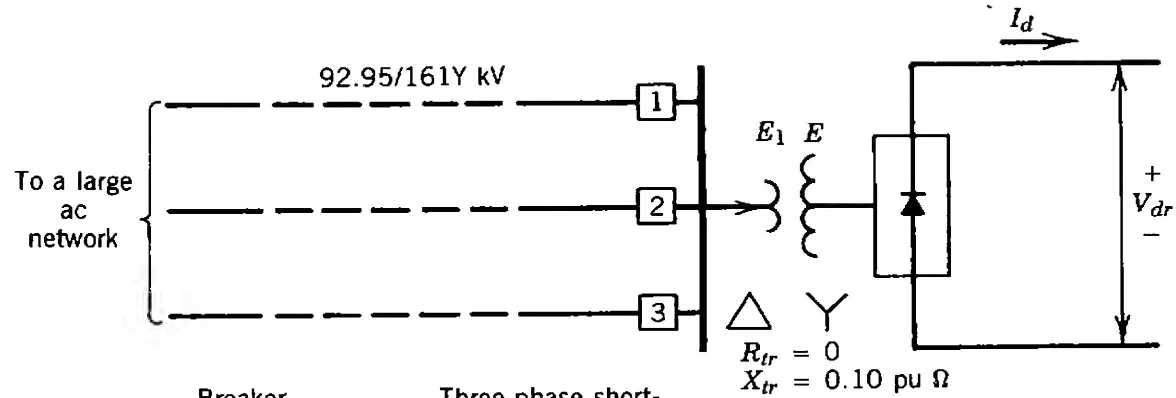


Figure 5.4. Equivalent circuit representations of bridge rectifier.



Breaker 1	Breaker 2	Breaker 3	Three-phase short-circuit fault MVA
C	C	C	4000
O	C	C	2500
O	O	C	1000

Note:
C = Closed
O = Open

Figure 5.5

(a) Three-phase kilovoltampere rating.
(b) Wye-side kilovolt rating.

Solution

(a) The three-phase kilovoltampere rating of a rectifier transformer can be determined from

$$S_{(B)} = 1.047 V_{d0} I_d \tag{5.69}$$

where

$$V_{d0} = V_{dr0} = 125 \text{ kV}$$

Therefore,

$$S_{(B)} = 1.047(125 \text{ kV})(1600 \text{ A})$$
$$= 209{,}400 \text{ kVA}$$

(b) Since

$$V_{d0} = 2.34 E_{(\text{L-N})}$$

then

$$E_{(\text{L-N})} = \frac{V_{d0}}{2.34}$$
$$= \frac{125 \text{ kV}}{2.34} = 53.4188 \text{ kV}$$

EXAMPLE 5.4

Use the results of Example 5.3 and determine the commutating reactance X_c, in ohms, referred to the wye side for all three possible values of ac system reactance.

Solution

(a) *When all three breakers are closed*:
The system reactance can be calculated from

$$X_{sys} = \frac{[\text{nominal kV}_{(\text{L-L})}]^2}{\text{short-circuit MVA}}$$

$$= \frac{[\sqrt{3}E_{(\text{L-N})}]^2}{4000}$$

$$= \frac{[\sqrt{3} \times 53.4188 \text{ kV}]^2}{4000 \text{ MVA}} = 2.14\ \Omega \tag{5.70}$$

The rms value of the wye-side phase current is

$$I_{(1\Phi)} \cong \sqrt{\tfrac{2}{3}} I_d$$

$$= 0.816(1600 \text{ A}) = 1305.6 \text{ A} \tag{5.71}$$

Also, the associated reactance base is

$$X_{(B)} = \frac{E_{(\text{L-N})}}{I_{(1\Phi)}} \tag{5.72}$$

or

$$X_{(B)} = \frac{E_{(\text{L-N})}}{\sqrt{\tfrac{2}{3}} I_d}$$

$$= \frac{53{,}418.8 \text{ V}}{1305.6 \text{ A}} = 40.915\ \Omega \tag{5.73}$$

Therefore, the reactance of the rectifier transformer is

$$X_{tr} = X_{tr} X_{(B)}$$

$$= (0.10 \text{ pu } \Omega)(40.915\ \Omega) = 4.0915\ \Omega$$

Thus, the commutating reactance is

$$X_c = X_{sys} + X_{tr}$$

$$= 2.14 + 4.0915 = 6.2315\ \Omega \tag{5.74}$$

(b) *When breaker 1 is open*:
The system reactance is

$$X_{sys} = [\frac{\sqrt{3}E_{(L\text{-}N)}]^2}{2500\text{ MVA}}$$

$$= \frac{[\sqrt{3} \times 53.4188\text{ kV}]^2}{2500\text{ MVA}} = 3.4243\ \Omega$$

Thus, the commutating reactance is

$$X_c = X_{sys} + X_{tr}$$
$$= 3.4243 + 4.0915 = 7.5158\ \Omega$$

(c) *When breakers 1 and 2 are open*:
The system reactance is

$$X_{sys} = \frac{\sqrt{3}E_{(L\text{-}N)}]^2}{1000\text{ MVA}}$$
$$= 8.5607\ \Omega$$

Therefore, the commutating reactance is

$$X_c = X_{sys} + X_{tr}$$
$$= 8.5607 + 4.0915 = 12.6522\ \Omega$$

EXAMPLE 5.5

Use the results of Example 5.4 and assume that all three breakers are closed, the load tap changer (LTC) is on neutral, the delay angle α is zero, and the maximum continuous current I_d is 1600 A. Determine the following:

(a) Overlap angle u of rectifier.
(b) The dc voltage V_{dr} of rectifier.
(c) Displacement (i.e., power) factor of rectifier.
(d) Magnetizing var input to rectifier.

Solution

(a) Since the delay angle is zero, the overlap angle u can be expressed as

$$u = \delta$$
$$= \cos^{-1}\left(1 - \frac{2X_cI_d}{\sqrt{3}E_m}\right) \tag{5.75}$$

where

$$E_m = \sqrt{2}E_{(L\text{-}N)}$$

Therefore,

$$u = \cos^{-1}\left(1 - \frac{2X_cI_d}{\sqrt{6}E_{(L\text{-}N)}}\right)$$

$$= \cos^{-1}\left(1 - \frac{2(6.2315\ \Omega)(1600\text{ A})}{\sqrt{6}(53{,}418.8\text{ V})}\right) = 32.1° \tag{5.76}$$

(b) The dc voltage of the rectifier can be expressed as

$$V_d = V_{dr}$$
$$= V_{s0}\cos\alpha - R_c I_d$$

where

$$R_c = \frac{3}{\pi} X_c$$

Thus,

$$V_d = V_{d0}\cos\alpha - \frac{3}{\pi} X_c I_d$$
$$= (125{,}000\text{ V})(1.0) - \frac{3}{\pi}(6.2315\ \Omega)(1600\text{ A})$$
$$= 115{,}479\text{ V}$$

(c) The displacement or power factor of the rectifier can be expressed as

$$\cos\Phi \cong \frac{V_d}{V_{d0}}$$
$$= \frac{115{,}479\text{ V}}{125{,}000\text{ V}} = 0.924 \tag{5.77}$$

and

$$\Phi = 22.5°$$

(d) The magnetizing var input can be expressed as

$$Q_r = P_{r(\text{dc})}\tan\Phi \tag{5.78}$$

or

$$Q_r = V_d I_d \tan\Phi$$
$$= (115{,}479\text{ V})(1600\text{ A})(0.414) \cong 76.532\text{ Mvar} \tag{5.79}$$

EXAMPLE 5.6

Assume that all three breakers given in Example 5.3 are closed, the LTC is on neutral, the dc voltage of the rectifier is 100 kV, and the maximum continuous dc current of the rectifier is 1600 A. Determine the following:

(a) Firing angle α.
(b) Overlap angle u.
(c) Power factor.
(d) Magnetizing var input.

Solution

(a) The firing angle α can be determined from

$$V_d = V_{d0}\cos\alpha - R_c I_d$$

or

$$\cos\alpha = \frac{V_d + R_c I_d}{V_{d0}} \tag{5.80}$$

where

$$R_c = \frac{3}{\pi} X_c$$

Therefore,

$$\cos\alpha = \frac{V_d + (3/\pi)X_c I_d}{V_{d0}}$$

$$= \frac{100{,}000 + (3/\pi)(6.2315)(1600)}{125{,}000}$$

$$= 0.876 \tag{5.81}$$

and

$$\alpha = 28.817°$$

(b) The overlap angle u can be determined from

$$V_d = \tfrac{1}{2}V_{d0}(\cos\alpha + \cos\delta)$$

or

$$\cos\delta = \frac{2V_d}{V_{d0}} - \cos\alpha$$

$$= \frac{2\times 100{,}000}{125{,}000} - 0.876 = 0.724 \tag{5.82}$$

and

$$\delta = 43.627°$$

Since

$$\delta = \alpha + u$$

the overlap angle u is

$$u = \delta - \alpha$$

$$= 43.627° - 28.817° = 14.81°$$

(c) The associated power factor is

$$\cos\Phi \cong \frac{V_d}{V_{d0}}$$
$$= \frac{100\text{ kV}}{125\text{ kV}} = 0.8$$

and

$$\Phi = 36.87°$$

(d) The magnetizing var input is

$$Q_r = V_d I_d \tan\Phi$$
$$= 100{,}000 \times 1600 \times 0.75 = 120\text{ Mvar}$$

EXAMPLE 5.7

Use the setup and data of Example 5.3 and assume that the worst possible second-contingency outage in the ac system has occurred, that is, two of the ac breakers are open.

(a) Determine whether or not a dc current of 1600 A causes the rectifier to operate at the second mode.
(b) If so, at what I_d does the first mode operation cease? If not, what is V_{dr} when the dc current is 1600 A?

Solution

(a) From Example 5.4, when two breakers are open, the commutating reactance is

$$X_c = 12.6522$$

and

$$\alpha = 0$$

so that

$$u = \delta$$
$$= \cos^{-1}\left(1 - \frac{2X_c I_d}{\sqrt{6}E_{(\text{L-N})}}\right)$$
$$= \cos^{-1}\left(1 - \frac{2(12.6522)(1600)}{\sqrt{6}(53{,}440)}\right) = 46.3°$$

Since

$$u < 60°$$

the rectifier operates at the first mode, the normal operating mode.

(b) Since the rectifier does not operate at the second mode,

$$V_{dr} = V_{d0}\cos\alpha - \frac{3}{\pi} X_c I_d$$

$$= (125{,}000)(1.0) - \frac{3}{\pi}(12.6522)(1600)$$

$$= 105{,}668.9 \text{ V}$$

Note that the commutating reactance causes a dc voltage drop.

5.7 PER-UNIT SYSTEMS AND NORMALIZING

The per-unit value of any quantity is its ratio to the chosen base quantity of the same dimensions. Therefore, a per-unit quantity is a "normalized" quantity with respect to a selected base value. Figure 5.6 shows a one-line diagram of a single-bridge converter system connected to a transformer with a tap changer under load. The figure also shows the fundamental ac and dc system quantities. The base quantities are indicated by the subscript B.

All ac voltages indicated in Figure 5.6 are line-to-neutral voltages. Therefore,

$$E = E_{(\text{L-N})}$$

$$= \frac{E_m}{\sqrt{2}} \quad \text{V} \tag{5.83}$$

The ratio of base ac voltages is a fixed value and is defined as

$$a \triangleq \frac{E_{1(B)}}{E_{(B)}} = \frac{I_{(B)}}{I_{1(B)}} \tag{5.84}$$

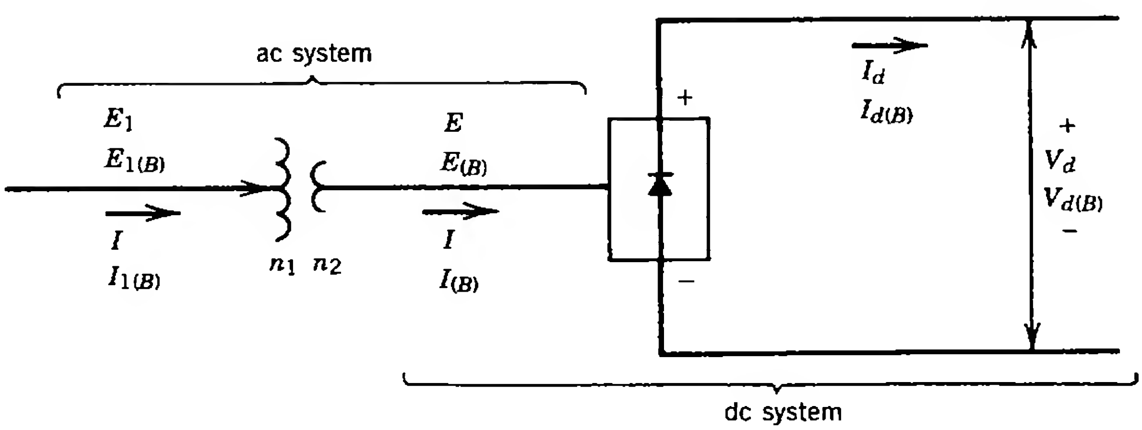


Figure 5.6

On the other hand, the turns ratio in use is a variable that is changeable by the LTC position and is defined as

$$n \triangleq \frac{n_1}{n_2} = \frac{E_1}{E} = \frac{I}{I_1} \tag{5.85}$$

Therefore, the per-unit voltage on the ac side of the converter transformer is

$$E_{1(\text{pu})} = \frac{E_1}{E_{1(B)}} \quad \text{pu V} \tag{5.86}$$

Whereas the per-unit voltage on the dc side of the converter transformer is

$$E_{(\text{pu})} = \frac{E}{E_{(B)}} \quad \text{pu V} \tag{5.87}$$

where

$$E = \frac{E_1}{n} \quad \text{V} \tag{5.88}$$

$$E_{(B)} = \frac{E_{1(B)}}{a} \quad \text{V} \tag{5.89}$$

thus, equation (5.87) can be expressed as

$$E_{(\text{pu})} = \left(\frac{a}{n}\right) E_{1(\text{pu})} \quad \text{pu V} \tag{5.90}$$

Note that when the LTC is on neutral,

$$n = a$$

and

$$E_{\text{pu}} < E_{1(\text{pu})} \qquad n > a$$

Thus, when the voltage on the dc side of the converter transformer is lowered with respect to the voltage on the ac side of the converter transformer by using the LTC of the transformer, the dc voltage V_d decreases.

5.7.1 Alternating-Current System Per-Unit Bases

The perunit bases for the quantities that are located on the ac side of the converter transformer are

$E_{1(B)}$ = arbitrarily chosen voltage value

$S_{1(B)}$ = arbitrarily chosen voltampere value

$$I_{1(B)} = \frac{S_{(B)}}{3E_{1(B)}} \quad \text{A} \tag{5.91}$$

$$Z_{1(B)} = \frac{E_{1(B)}}{I_{1(B)}} \quad \Omega \tag{5.92}$$

$$L_{1(B)} = \frac{Z_{1(B)}}{w_{(B)}} \quad \text{H} \tag{5.93}$$

where

$$w_{(B)} = w = 377 \quad \text{if } f = 60 \text{ Hz}$$

On the other hand, the per-unit bases for the quantities that are located on the dc side of the converter transformer are

$$E_{(B)} = \frac{E_{1(B)}}{a} \quad \text{V} \tag{5.94}$$

$S_{(B)}$ = arbitrarily chosen voltampere value (same as before)

$= S_{1(B)}$

$$I_{(B)} = aI_{1(B)} \quad \text{A} \tag{5.95}$$

$$Z_{(B)} = \frac{Z_{1(B)}}{a^2} \quad \Omega \tag{5.96}$$

$$L_{(B)} = \frac{L_{1(B)}}{a^2} \quad \text{H} \tag{5.97}$$

5.7.2 Direct-Current System Per-Unit Bases

The dc system per-unit bases are constrained by the bridge circuit steady-state equations to be related to the chosen ac system per unit bases. Note the analogy to the selection of ac bases on the two sides of a transformer, wherein such bases must be related by the transformer turns ratio.

When the firing angle α, overlap angle u, and dc current I_d are zero, the dc voltage base is

$$V_{d(B)} \triangleq V_{d0}$$

or

$$V_{d(B)} = \frac{3\sqrt{6}}{\pi} E_{(B)} \tag{5.98}$$

or substituting equation (5.94) into equation (5.98), the dc voltage base can be expressed as

$$V_{d(B)} = \frac{3\sqrt{6}}{\pi} \frac{E_{1(B)}}{a} \tag{5.99}$$

The dc current base can be expressed as

$$I_{d(B)} = \sqrt{\tfrac{3}{2}} I_{(B)} \tag{5.100}$$

or substituting equation (5.95) into equation (5.100),

$$I_{d(B)} = \sqrt{\tfrac{3}{2}} (a I_{1(B)}) \tag{5.101}$$

which is exact only if $u = 0$. However, it is an approximate relation with a maximum error of 4.3 percent at $u = 60°$ and only 1.1 percent at $u \leq 30°$ (i.e., the normal operating range).

The dc resistance base in the dc system can be expressed as

$$R_{d(B)} = \frac{V_{d(B)}}{I_{d(B)}} \tag{5.102}$$

or substituting equations (5.99) and (5.101) into equation (5.102),

$$R_{d(B)} = \frac{6}{\pi} \frac{Z_{1(B)}}{a^2} \tag{5.103}$$

or

$$R_{d(B)} = \frac{6}{\pi} Z_{(B)} \tag{5.104}$$

Similarly, the dc inductance base in the dc system can be expressed as

$$L_{d(B)} = \frac{6}{\pi} L_{(B)} \quad \text{H} \tag{5.105}$$

or

$$L_{d(B)} = \frac{6}{\pi} \frac{L_{1(B)}}{a^2} \quad \text{H} \tag{5.106}$$

EXAMPLE 5.8

Normalize the steady-state rectifier equation of

$$V_d = V_{d0} \cos \alpha - \frac{3}{\pi} w L_c I_d \tag{5.107}$$

Solution

Since

$$V_{d0} = \frac{3\sqrt{6}}{\pi} E_{(\text{L-N})}$$

the steady-state rectifier equation can be expressed as

$$V_d = \frac{3\sqrt{6}}{\pi} E_{(\text{L-N})} \cos \alpha - \frac{3}{\pi} wL_c I_d \qquad (5.108)$$

Dividing both sides of the equation by $V_{d(B)}$,

$$\frac{V_d}{V_{d(B)}} = \frac{3\sqrt{6}}{\pi} \frac{E}{V_{d(B)}} \cos \alpha - \frac{3}{\pi} \frac{wL_c I_d}{V_{d(B)}} \qquad (5.109)$$

Substituting equation (5.98) into equation (5.108) and simplifying the resultant,

$$V_{d(\text{pu})} = \frac{a}{n} E_{1(\text{pu})} \cos \alpha - \frac{1}{2} L_{c(\text{pu})} I_{d(\text{pu})} \quad \text{pu V} \qquad (5.110)$$

EXAMPLE 5.9

Assume that the steady-state rectifier equation given in Example 5.8 can be modified by the substitution of

$$R_c = \frac{3}{\pi} wL_c$$

(a) Determine the normalized form of the modified steady-state rectifier equation.
(b) Find $\cos \alpha$ from the normalized equation derived in part (a).

Solution

(a) From equation (5.107),

$$V_d = V_{d0} \cos \alpha - \frac{3}{\pi} wL_c I_d$$

or

$$V_d = V_{d0} \cos \alpha - R_c I_d \quad \text{V} \qquad (5.65)$$

Since, in per units,

$$R_{c(\text{pu})} = \frac{R_c}{R_{d(B)}} \quad \text{pu} \qquad (5.111)$$

or

$$R_c = R_{c(\text{pu})} R_{d(B)} \quad \Omega \qquad (5.112)$$

and

$$I_{d(\text{pu})} = \frac{I_d}{I_{d(B)}} \quad \text{pu A} \tag{5.113}$$

or

$$I_d = I_{d(\text{pu})} I_{d(B)} \quad \text{A} \tag{5.114}$$

substituting equations (5.112) and (5.114) into equation (5.65) and dividing both sides of the resultant equation by $V_{d(B)}$ yields

$$V_{d(\text{pu})} = E_{(\text{pu})} \cos \alpha - R_{c(\text{pu})} I_{d(\text{pu})} \quad \text{pu V} \tag{5.115}$$

or, alternatively,

$$V_{d(\text{pu})} = \frac{a}{n} E_{1(\text{pu})} \cos \alpha - R_{c(\text{pu})} I_{d(\text{pu})} \quad \text{pu V} \tag{5.116}$$

(b) From equation (5.116),

$$\cos \alpha = \frac{V_{d(\text{pu})} + R_{c(\text{pu})} I_{d(\text{pu})}}{(a/n) E_{1(\text{pu})}} \tag{5.117}$$

EXAMPLE 5.10

Assume that all three breakers given in Example 5.3 are closed, the dc voltage of the rectifier is 100 kV, the maximum continuous dc current of the rectifier is 1600 A, and the firing angle is zero. Assume that the LTC of the rectifier transformer is used to reduce the dc voltage to 100 kV. Employ the per-unit quantities and relations and determine the following:

(a) Open-circuit dc voltage.
(b) Open-circuit ac voltage on wye side of transformer.
(c) Overlap angle u.
(d) Power factor of rectifier.
(e) Magnetizing var input to rectifier.
(f) Number of 0.625 percent steps of buck required on load tap changer of transformer.

Solution

(a) Since, in per units,

$$V_{d(\text{pu})} = E_{(\text{pu})} \cos \alpha - R_{c(\text{pu})} I_{d(\text{pu})} \quad \text{pu V}$$

from which

$$E_{(\text{pu})} = \frac{V_{d(\text{pu})} + R_{c(\text{pu})} I_{d(\text{pu})}}{\cos \alpha}$$

where

$$R_c = \frac{3}{\pi} X_c = \frac{3}{\pi} (6.2315) = 5.95\ \Omega$$

the dc current and voltage bases are

$$I_{d(B)} = 1600 \text{ A}$$

and

$$V_{d(B)} = 125{,}000 \text{ V}$$

The resistance base can be determined as

$$R_{d(B)} = \frac{V_{d(B)}}{I_{d(B)}} = \frac{125{,}000 \text{ V}}{1600 \text{ A}} = 78.125 \ \Omega$$

Therefore,

$$V_{d(\text{pu})} = \frac{V_d}{V_{d(B)}} = \frac{100{,}000 \text{ V}}{125{,}000 \text{ V}} = 0.8 \text{ pu V}$$

$$I_{d(\text{pu})} = \frac{I_d}{I_{d(B)}} = \frac{1600 \text{ A}}{1600 \text{ A}} = 1.0 \text{ pu A}$$

$$R_{c(\text{pu})} = \frac{R_c}{R_{d(B)}} = \frac{5.95 \ \Omega}{78.125 \ \Omega} = 0.0762 \text{ pu}$$

Thus,

$$E_{(\text{pu})} = \frac{0.8 + (0.0762)(1.0)}{1.0} = 0.8762 \text{ pu V}$$

However,

$$E_{(\text{pu})} = \frac{V_{d0}}{V_{d(B)}}$$

from which

$$V_{d0} = E_{(\text{pu})} V_{d(B)} = (0.8762)(125{,}000) = 109{,}520 \text{ V}$$

(b) The open-circuit ac voltage on the wye side can be found from

$$V_{d0} = 2.34E$$

or

$$E = \frac{V_{d0}}{2.34} = \frac{109{,}520 \text{ V}}{2.34} = 46{,}803 \text{ V}$$

(c) Since

$$\cos\delta = \cos\alpha - \frac{X_{c(\text{pu})} I_{d(\text{pu})}}{(a/n) E_{1(\text{pu})}}$$

where

$$a = n = \frac{E_{1(\text{L-N})}}{E_{(\text{L-N})}} = \frac{92.95 \text{ kV}}{53.44 \text{ kV}} = 1.74$$

$$X_{c(\mathrm{pu})} = 2R_{c(\mathrm{pu})} = 2(0.0762) = 0.1524\ \mathrm{pu}$$

$$E_{1(\mathrm{pu})} = \frac{E_1}{E_{1(B)}} = \frac{92.95\ \mathrm{kV}}{92.95\ \mathrm{kV}} = 1.0\ \mathrm{pu}$$

then

$$\cos\delta = 1.0 - \frac{(0.1524)(1.0)}{(1.74/1.74)(1.0)} = 0.8476$$

and

$$\delta = 32.04°$$

where

$$\delta = u + \alpha \qquad \alpha = 0$$

so that the overlap angle is

$$u = \delta = 32.04°$$

(d) The power factor of the rectifier is

$$\cos\Phi \cong \frac{V_d}{V_{d0}} = \frac{100{,}000\ \mathrm{V}}{109{,}520\ \mathrm{V}} = 0.913$$

and

$$\Phi = 24.07°$$

(e) The magnetizing var input to the rectifier is

$$\begin{aligned} Q_r &= V_d I_d \tan\Phi \\ &= (100{,}000\ \mathrm{V})(1600\ \mathrm{A})(0.4466) = 71.458\ \mathrm{Mvar} \end{aligned}$$

(f) Since the necessary change in voltage is

$$\Delta V = 53{,}440 - 46{,}803 = 6637\ \mathrm{V}$$

and one buck step can change

$$\begin{aligned} (5/8\%)E_{(\mathrm{L\text{-}N})} &= 0.00625(53{,}440\ \mathrm{V}) \\ &= 334\ \mathrm{V/step} \end{aligned}$$

the number of 0.625 percent steps of buck required is

$$\begin{aligned} \text{Number of bucks} &= \frac{6637\ \mathrm{V}}{334\ \mathrm{V/step}} \\ &= 20\ \text{steps} \end{aligned}$$

5.8 INVERSION

In a given converter, the current flow is always from anode to cathode, that is, the unidirectional inside a rectifying valve or thyristor, so that the cathode remains the positive terminal. Therefore, the current direction in the converter cannot be reversed. When it is required to operate the converter as an inverter in order to reverse the direction of power flow, the direction of the average direct voltage must be reversed. This can be obtained by using the grid control to change the delay angle α until the average direct voltage V_d becomes negative. If there is no overlap, the voltage V_d decreases as the delay angle α is advanced, and it becomes zero when α is 90°. With further increase in the delay angle α, the average direct voltage becomes negative. Therefore, it can be said that the rectification and inversion processes occur when $0° < \alpha < 90°$ and $90° < \alpha < 180°$, respectively. If there is an overlap, the inversion process may start at a value of the delay angle that is less than 90°. Therefore,

$$\alpha = \pi - \delta \tag{5.118}$$

or

$$\alpha = \tfrac{1}{2}(\pi - u) \tag{5.119}$$

where α = delay angle in electrical degrees
δ = extinction angle in electrical degrees
u = overlap angle in electrical degrees

Figure 5.7 shows relations among angles used in converter theory and the reason the curvature of the front of a current pulse of an inverter differs from that of a rectifier. Kimbark [1] gives the relations among the various inverter angles as

$$\beta = \pi - \alpha \tag{5.120}$$

$$\gamma = \pi - \delta \tag{5.121}$$

$$u = \delta - \alpha \tag{5.122}$$

$$u = \beta - \gamma \tag{5.123}$$

where β = inverter ignition angle in electrical degrees
γ = inverter extinction angle in electrical degrees

In order to provide adequate time for the deionization of the arc for the appropriate valve, the minimum value of the inverter extinction angle γ_0 must be in the range of 1°–8°. If the value of γ_0 is not adequate, the valve starts to conduct again. This is called *commutation failure.*

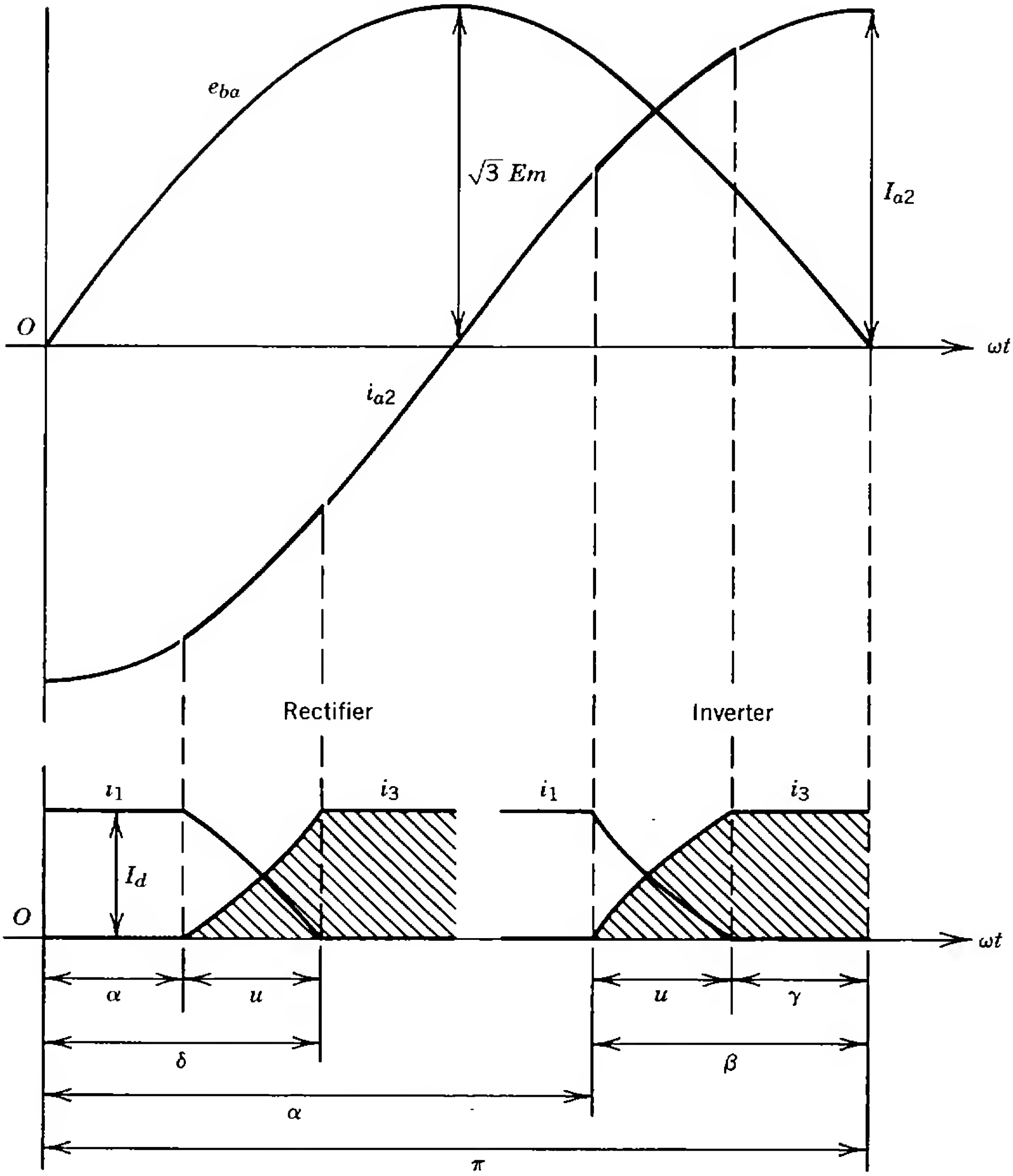


Figure 5.7. Relations among angles used in converter theory [1].

The rectifier equations can be used to describe the inverter operation by substituting α and δ by $\pi - \beta$ and $\pi - \gamma$, respectively. In order to differentiate the inverter equations from the rectifier equations, it is customary to use the subscripts i and r to signify the inverter and rectifier operations, respectively. Therefore, it can be expressed that

$$I_{di} = I_{s2}(\cos\gamma - \cos\beta) \tag{5.124}$$

or substituting equation (5.60) into equation (5.124),

$$I_{d_t} = \frac{\sqrt{3}E_m}{2wL_c}(\cos\gamma - \cos\beta) \tag{5.125}$$

In general, it is customary to express the inverter voltage as negative when it is used in conjunction with a rectifier voltage in a given equation. Otherwise, when it is used alone, it is customary to express it as positive. Therefore, it can be expressed that

$$V_{d_t} = \tfrac{1}{2}V_{d0_t}(\cos\gamma + \cos\beta) \tag{5.126}$$

Furthermore, for inverters with constant-ignition-angle (CIA) control,

$$V_{d_t} = V_{d0_t}\cos\beta + R_c I_d \tag{5.127}$$

or

$$V_{d_t} = V_{d0_t}\cos\beta + \frac{3}{\pi}X_c I_d \tag{5.128}$$

and for inverters with constant-extinction-angle (CEA) control,

$$V_{d_t} = V_{d0_t}\cos\gamma - R_c I_d \tag{5.129}$$

or

$$V_{d_t} = V_{d0_t}\cos\gamma - \frac{3}{\pi}X_c I_d \tag{5.130}$$

Note that it is preferable to operate inverters with CEA control rather than with CIA control. Figure 5.8 shows the corresponding equivalent inverter circuit representations.

It can be said that an inverter has a leading power factor, contrary to a rectifier, which has a lagging power factor. This is due to the fact that the lagging reactive power is provided to the inverter by the ac system into which the inverter is feeding active power. Therefore, this is equivalent to the inverter feeding current to the ac system at a leading power factor. The required additional reactive power by the inverter is provided by the synchronous capacitors or by static shunt capacitors. Furthermore, harmonic filters are needed on both ac and dc sides of converters in order to prevent the harmonics generated by the rectifier and inverter from entering into the ac and dc systems. The order of harmonics in the direct voltage are expressed by

$$N = pq$$

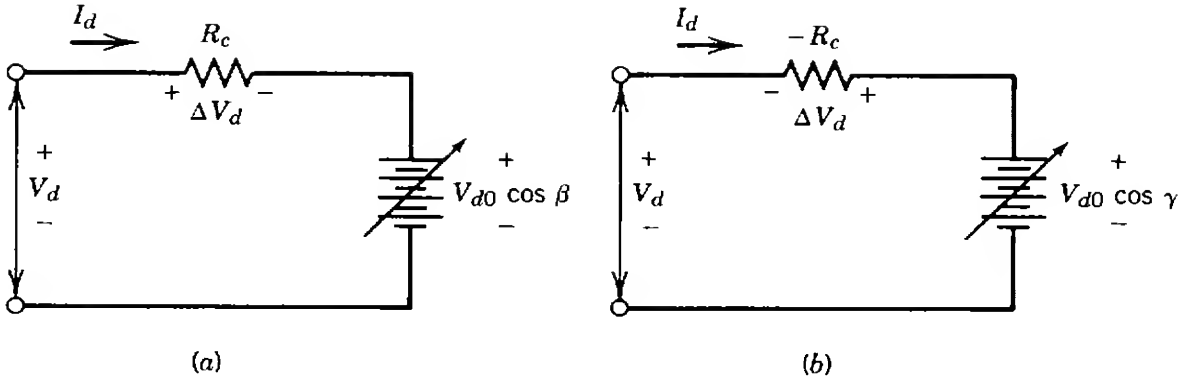


Figure 5.8. Equivalent circuits of inverter: (*a*) with constant β; (*b*) with constant γ.

and the order of harmonics in the alternating current are given by

$$N = pq \pm 1$$

where p = pulse number
q = integer number

EXAMPLE 5.11

Consider a single-bridge inverter and do the following:

(a) Verify that the power factor of the fundamental component of the inverter ac line current can be expressed as

$$\cos \Phi_{1i} \cong \tfrac{1}{2}(\cos \beta + \cos \gamma)$$

(b) Explain the approximation involved in part (a).
(c) Explain the effect of increasing the ignition advance angle β on the power factor in part (a).
(d) Explain the effect of increasing the extinction angle γ on the power factor in part (a).
(e) Explain whether or not the power factor that can be found using the equation given in part (a) is greater or less than the power factor that would be measured in terms of the readings of the switchboard wattmeter, voltmeter, and ammeter?

Solution

(a) From equation (5.126), inverter voltage can be expressed as

$$V_{di} = V_{d0i}[\tfrac{1}{2}(\cos \beta + \cos \gamma)]$$

where

$$V_{d0i} = \left(\frac{3\sqrt{6}}{\pi}\right)E_{(\text{L-N})} \qquad (5.43)$$

Therefore,

$$V_{d_t} = \frac{3\sqrt{6}}{\pi}\,[\tfrac{1}{2}(\cos\beta + \cos\gamma)]E_{(\text{L-N})} \tag{5.131}$$

When losses are disregarded, the active ac power can be set equal to the dc power,

$$P_{(\text{ac})} = P_{(\text{dc})}$$

where

$$P_{(\text{ac})} = 3E_{(\text{L-N})}I_{L1}\cos\Phi_{1_t} \tag{5.132}$$

and

$$P_{(\text{dc})} = V_{di}I_d$$

or

$$P_{(\text{dc})} = \frac{3\sqrt{6}}{\pi}\,[\tfrac{1}{2}(\cos\beta + \cos\gamma)]E_{(\text{L-N})}I_d \tag{5.133}$$

Thus, from equations (5.132) and (5.133),

$$3E_{(\text{L-N})}I_{L1}\cos\Phi_{1_t} = \frac{3\sqrt{6}}{\pi}\,[\tfrac{1}{2}(\cos\beta + \cos\gamma)]E_{(\text{L-N})}I_d \tag{5.134}$$

or

$$I_{L1}\cos\Phi_{1_t} = \frac{\sqrt{6}}{\pi}\,[\tfrac{1}{2}(\cos\beta + \cos\gamma)]I_d \tag{5.135}$$

However,

$$I_{L1} \cong \frac{\sqrt{6}}{\pi}\,I_d \tag{5.136}$$

which is an approximation. Therefore,

$$\cos\Phi_{1_t} \cong \tfrac{1}{2}(\cos\beta + \cos\gamma) \tag{5.137}$$

which is also an approximation.

(b) Equations (5.136) and (5.137) would be exact only if $u = 0$. Otherwise, they will be some approximate values with a maximum error of 4.3 percent at $u = 60°$. At normal operating range (i.e., $u \leq 30°$), the error involved is less than 1.1 percent.

(c) An increase in the ignition advance angle β causes the power factor to decrease.

(d) An increase in the extinction angle γ causes the power factor to decrease.

(e) The power factor determined based on the readings can be expressed as

$$\cos \Phi = \frac{W_{\text{reading}}}{\sqrt{3}(V_{(\text{L-L})\text{reading}})(I_{\text{reading}})}$$

Therefore, the power factor calculated from equation (5.137) is greater than the power factor determined from the readings because of the harmonics involved.

EXAMPLE 5.12

Consider the single-bridge inverter in Example 5.11 and verify that the power factor of the fundamental component of the inverter ac line current can be expressed as

$$\cos \Phi_{1_i} \cong \frac{V_{di}}{V_{d0_i}}$$

Solution

The inverter's direct voltage is

$$V_{d_i} = V_{d0_i} \cos \beta - \Delta V_{di} \tag{5.138}$$

where

$$\Delta V_{d_i} = \tfrac{1}{2} V_{d0i}(\cos \beta - \cos \gamma) \tag{5.139}$$

Therefore,

$$V_{d_i} = V_{d0_i} \cos \beta - \tfrac{1}{2} V_{d0i}(\cos \beta - \cos \gamma)$$

or

$$V_{d_i} = \tfrac{1}{2} V_{d0i}(\cos \beta + \cos \gamma) \tag{5.140}$$

Thus,

$$\frac{\cos \beta + \cos \gamma}{2} = \frac{V_{di}}{V_{d0_i}} \tag{5.141}$$

but

$$\cos \Phi_{1_i} \cong \frac{\cos \beta + \cos \gamma}{2} \tag{5.142}$$

Therefore,

$$\cos \Phi_{1i} \cong \frac{V_{di}}{V_{d0i}} \tag{5.143}$$

Alternatively, from equation (5.126),

$$V_{di} = V_{d0i} \frac{\cos \beta + \cos \gamma}{2}$$

from which

$$\frac{\cos\beta + \cos\gamma}{2} = \frac{V_{di}}{V_{d0i}}$$

Therefore,

$$\cos\Phi_{1i} \cong \frac{V_{di}}{V_{d0i}}$$

EXAMPLE 5.13

Consider the single-bridge inverter in Example 5.11 and verify that the magnetizing var input to the inverter can be expressed as

$$Q_i \cong P_{(\text{dc})}\left[\left(\frac{V_{d0i}}{V_{di}}\right)^2 - 1\right]^{1/2} \quad \text{var}$$

Solution

From Figure 5.9(*b*),

$$Q_i \cong P_{(\text{dc})}\tan\Phi_{1i} \tag{5.144}$$

From Figure 5.9(*a*),

$$\tan\Phi_{1i} = \frac{(V_{d0i}^2 - V_{di}^2)^{1/2}}{V_{di}} \tag{5.145}$$

or

$$\tan\Phi_{1i} = \left[\frac{V_{d0i}^2}{V_{di}^2} - 1\right]^{1/2} \tag{5.146}$$

Therefore,

$$Q_i \cong P_{(\text{dc})}\left[\left(\frac{V_{d0i}}{V_{di}}\right)^2 - 1\right]^{1/2} \quad \text{var} \tag{5.147}$$

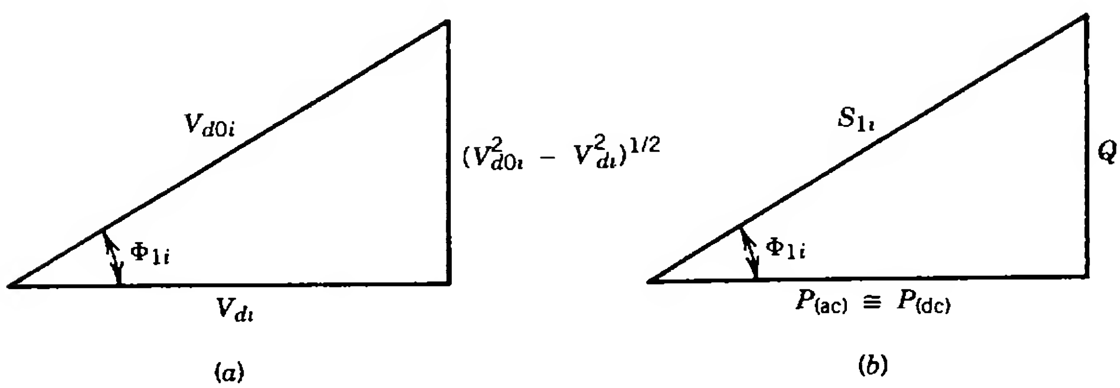


Figure 5.9

EXAMPLE 5.14

Consider the single-bridge inverter of Example 5.13 and do the following:

(a) Verify that the voltampere load due to the fundamental component of inverter ac line current can be expressed as

$$S_{1_i} \cong P_{(\mathrm{dc})} \frac{V_{di0}}{V_{di}} \quad \mathrm{VA}$$

(b) Explain whether or not the total voltampere load is larger or smaller than the S_{1_i} in part (a).

Solution

(a) From Figure 5.4(b),

$$\cos \Phi_{1_i} \cong \frac{P_{(\mathrm{dc})}}{S_{1_i}} \tag{5.148}$$

from which

$$S_{1_i} \cong P_{(\mathrm{dc})} \frac{1}{\cos \Phi_{1_i}} \tag{5.149}$$

But, from Example 5.12,

$$\cos \Phi_{1_i} \cong \frac{V_{di}}{V_{d0_i}} \tag{5.143}$$

Therefore, substituting equation (5.143) into equation (5.149),

$$S_{1_i} \cong P_{(\mathrm{dc})} \frac{V_{d0_i}}{V_{di}} \quad \mathrm{VA} \tag{5.150}$$

(b) The total voltampere load is larger than the one found in part (a) because in part (a) the effects of harmonics are ignored and only the fundamental component of the inverter ac line current is considered.

5.9 MULTIBRIDGE (B-BRIDGE) CONVERTER STATIONS

Figure 5.10 shows a typical converter station layout. For such a station, the general arrangement of a converter station with 12-pulse converters is shown in Figure 5.11. Figure 5.12 shows a one-line diagram for a B-bridge converter (rectifier) station and the supplying ac network. The ac network system is represented by Thévenin equivalent E_1 voltage and $X_{1(\mathrm{sys})}$ reactance. The E_1 voltage can be assumed to be sinusoidal due to the ac filter connected at the ac bus. The converter bank is made of two or more three-phase bridges, and each bridge contains up to six mercury arc valves

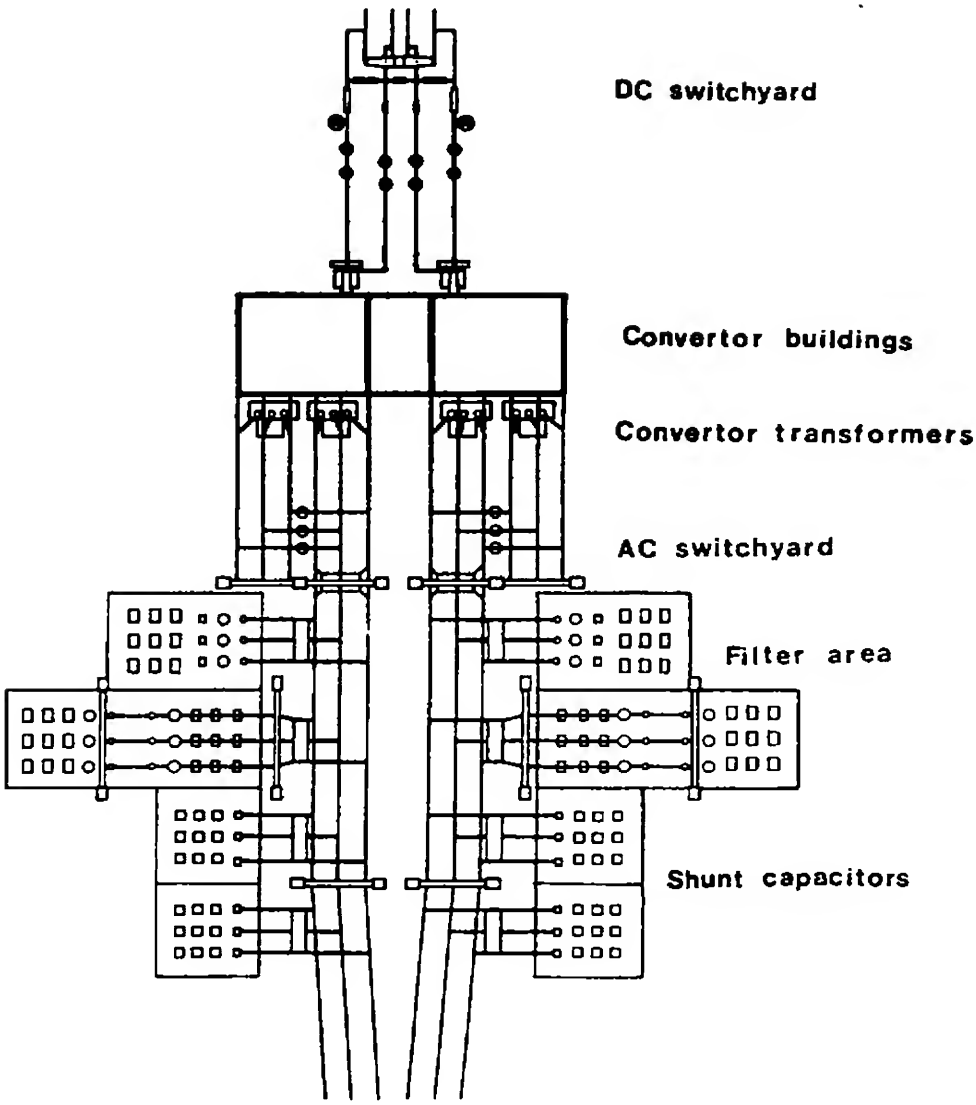


Figure 5.10. Typical converter station layout [2].

or thyristors. Note that there are B bridges in the figure. The number of bridges required is dictated by the direct voltage level selected for economical transmission.

In order to eliminate certain harmonics, the transformer connections are arranged in a certain way so that one-half of the bridge transformers has 0° phase shift and the other half has 30° phase shift. This arrangement gives 12-pulse operation. The two sets of transformer banks are connected either one set in wye–wye and the other set in wye–delta with 30° phase shift or one set in delta–delta with 0° phase shift and the other in wye–delta with 30° phase shift (or from one three-winding bank connected wye–wye–delta). As a result of this arrangement, the two halves of the bridges do not commutate simultaneously. The current on the dc side of the converter is almost completely smoothed due to the dc reactors (L_d) connected. As can be seen from Figure 5.12, the B bridges are connected in series on the dc side and in parallel on the ac side. Therefore, the direct voltage can be expressed as

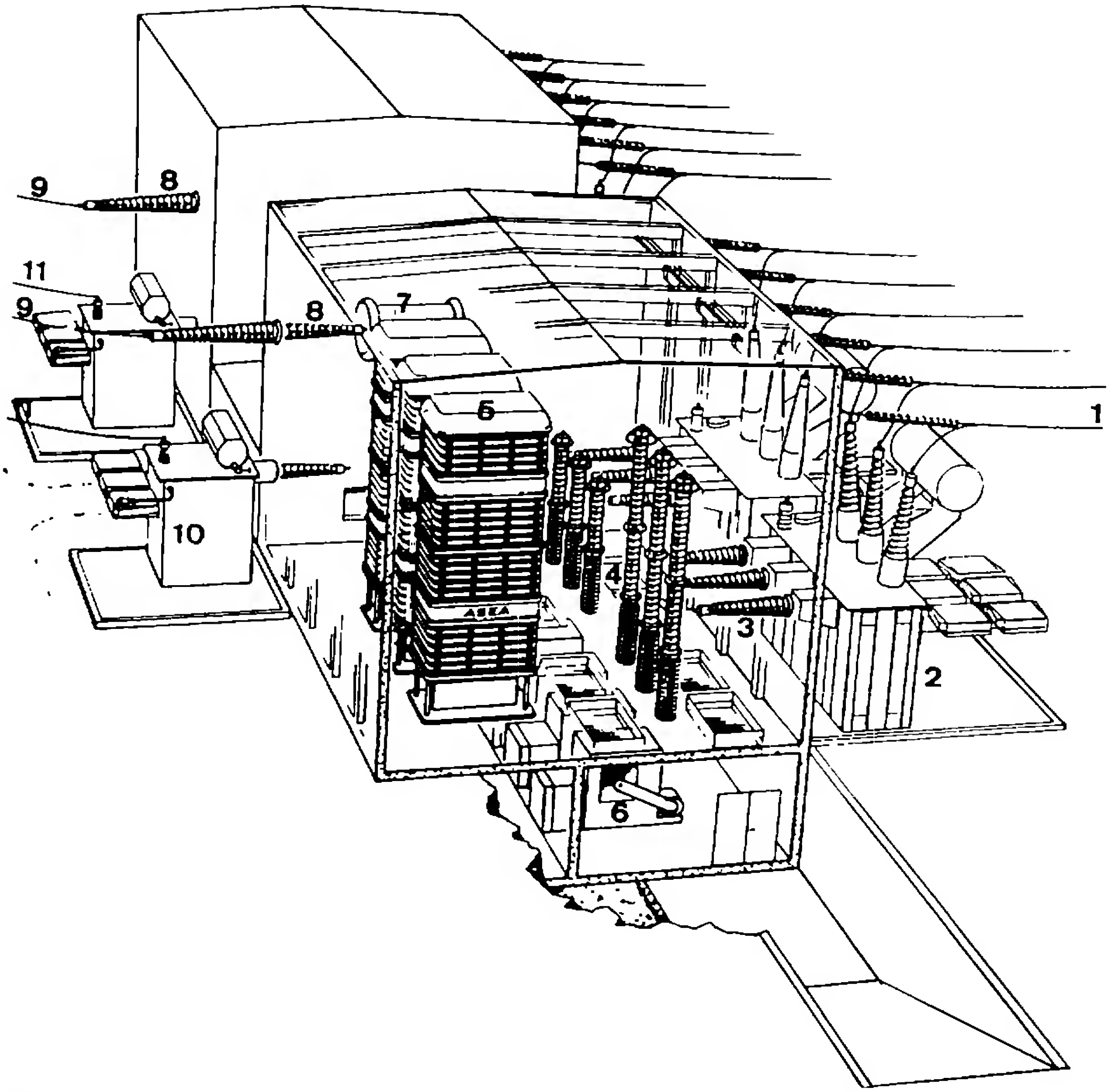


Figure 5.11. General arrangement of converter station with 12-pulse converters: (1) ac busbar; (2) converter transformer; (3) valve-side bushing of converter transformer; (4) surge arresters; (5) quadruple valves; (6) valve-cooling fans; (7) air core reactor; (8) wall bushing; (9) outgoing dc buswork; (10) smoothing reactor; (11) outgoing electrode line connection [2].

$$V_d = \left(\frac{3\sqrt{6}}{\pi}\right) BnE_{(\text{L-N})} \cos \Phi \quad \text{V} \tag{5.151}$$

or

$$V_d = 2.34 BnE_{(\text{L-N})} \cos \Phi \quad \text{V} \tag{5.152}$$

or

$$V_d = 1.35 BnE_{(\text{L-L})} \cos \Phi \quad \text{V} \tag{5.153}$$

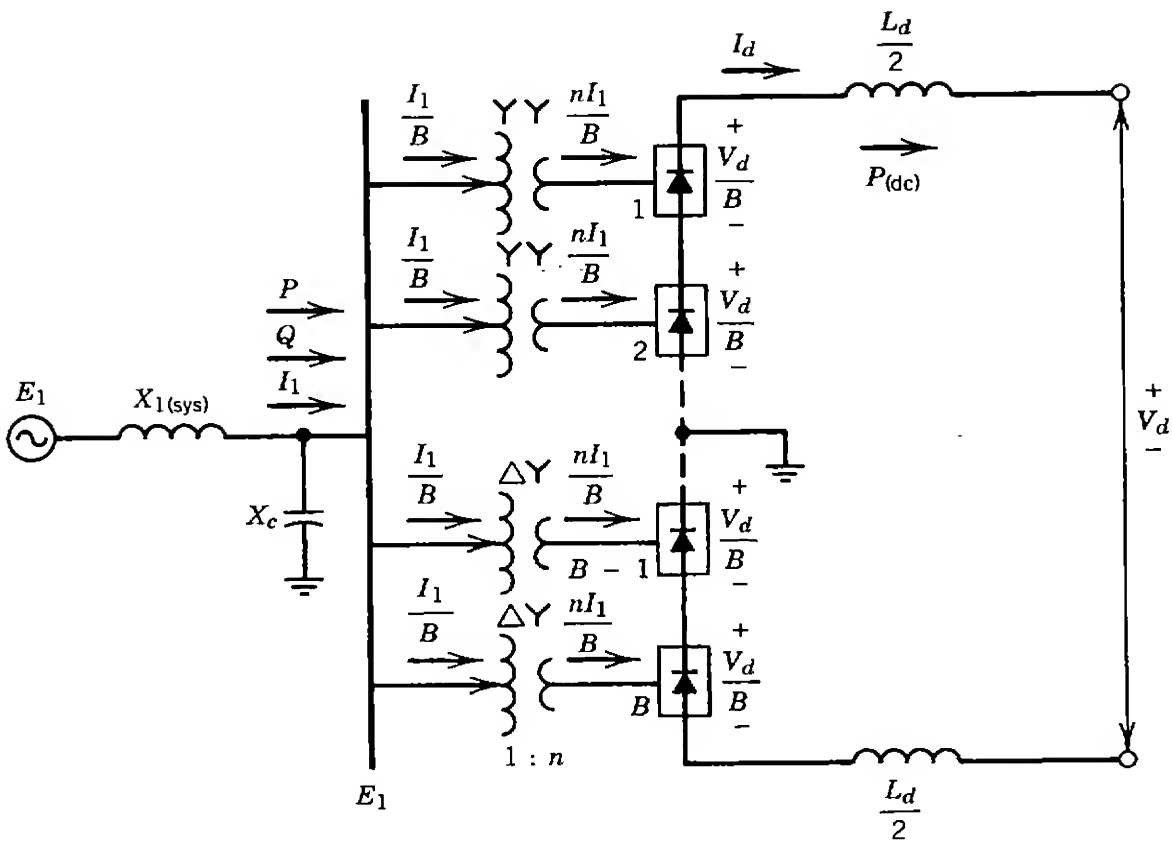


Figure 5.12. One-line diagram for B-bridge converter (rectifier) station.

where B = number of bridges
n = turns ratio in use
$E_{(L\text{-}N)}$ = line-to-neutral voltage in volts
$E_{(L\text{-}L)}$ = line-to-line voltage in volts
$\cos\Phi$ = power factor of fundamental component of ac line current

The fundamental component of the ac line current can be expressed as

$$I_{1(a)} = \frac{\sqrt{6}}{\pi} BnI_d \quad \text{A} \tag{5.154}$$

or

$$I_{1(a)} = 0.78BnI_d \quad \text{A} \tag{5.155}$$

The active ac power is equal to dc power, ignoring losses:

$$P_{(ac)} = P_{(dc)} \quad \text{W}$$

where

$$P_{(dc)} = V_d I_d \quad \text{W} \tag{5.156}$$

and

$$P_{(\mathrm{ac})} = 3E_{(\mathrm{L\text{-}N})}I_{1(a)}\cos\Phi \quad \mathrm{W} \tag{5.157}$$

5.10 PER-UNIT REPRESENTATION OF B-BRIDGE CONVERTER STATIONS

Figure 5.13 shows a one-line diagram representation of two ac systems connected by a dc transmission link. The system has two B-bridge converter stations; the one on the left operates as a rectifier and the one on the right operates as an inverter. Of course, it is possible to reverse the direction of power flow by interchanging the functions of the converter stations.

In this section, only the first-mode operation is to be reviewed; that is, the overlap angle u is less than 30° so that the one-half of the bridges with 0° phase and the other half of the bridges with 30° phase shift do not commutate simultaneously.

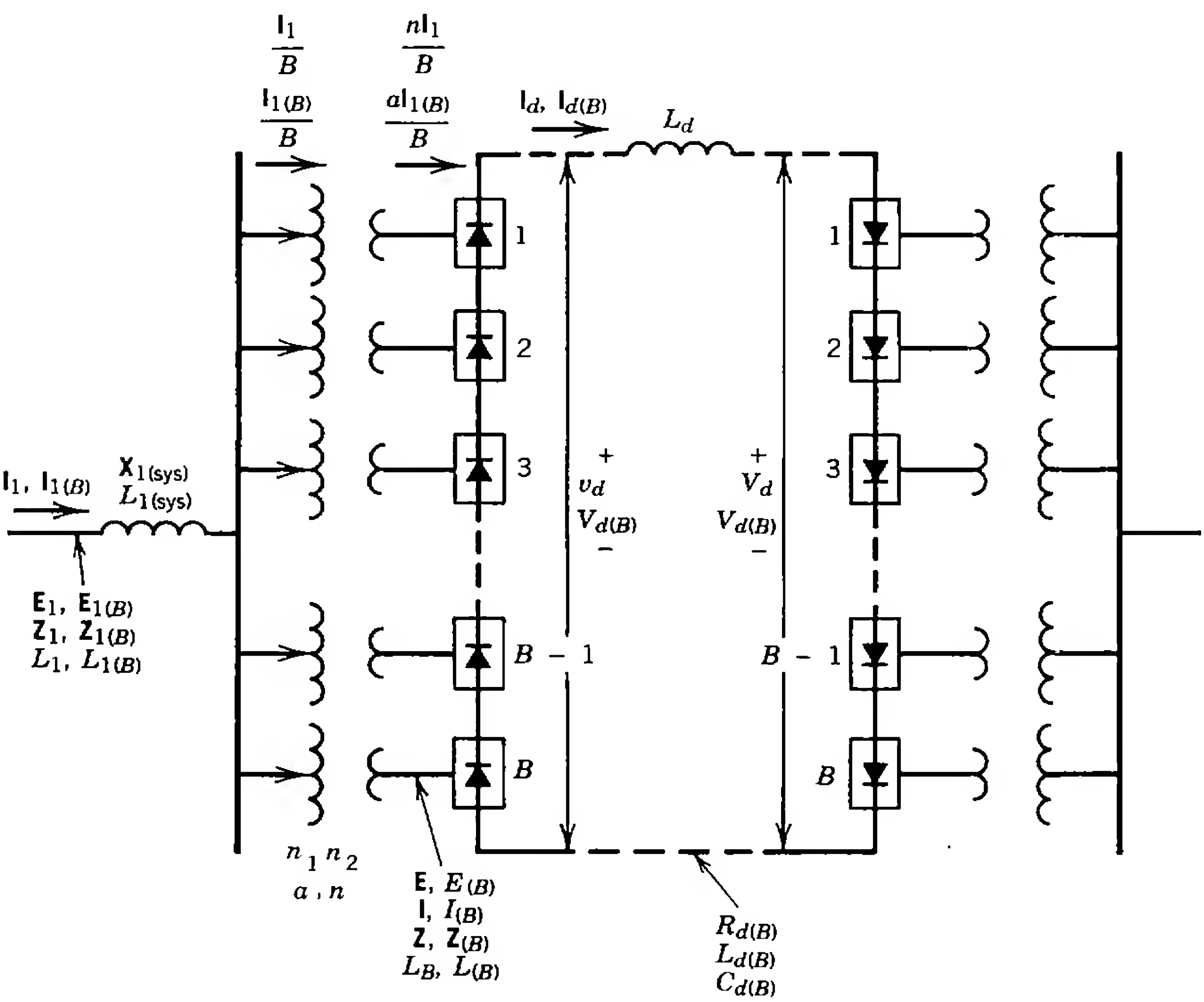


Figure 5.13. One-line diagram representation of two B-bridge converter stations connecting two ac systems over a dc transmission link.

The notation that will be used is largely defined in the illustration. As before, the subscript B designates the base value. An additional subscript in terms of r or i may be added to define the rectifier or inverter operation involved, respectively. Assume that all ac voltages given are line-to-neutral voltages. The following notation is applicable for each transformer:

S_{tr} = transformer nameplate rated in three-phase voltamperes
a = ratio of base ac voltages

$$= \frac{E_{1(B)}}{E_{(B)}} = \frac{BI_{(B)}}{I_{1(B)}} \tag{5.158}$$

n = turns ratio in use (variable changeable with LTC position)

$$= \frac{n_1}{n_2} = \frac{E_1}{E} = \frac{I}{I_1} \tag{5.159}$$

X_{tr} = leakage reactance referred to dc side in ohms
$X_{1(tr)}$ = leakage reactance referred to ac side in ohms

$$= n^2 X_{tr} \tag{5.160}$$

L_{tr} = leakage inductance referred to dc side in henries
$L_{1(tr)}$ = leakage inductance referred to ac side in henries

$$L_{1(tr)} = n^2 L_{(tr)} \tag{5.161}$$

$X_{tr(pu)}$ = per-unit leakage reactance (when LTC is on neutral)

$$= \frac{X_{(tr)}}{Z_{(B)}} = \frac{a^2 X_{tr}}{a^2 Z_{(B)}} = \frac{X_{1(tr)}}{Z_{1(B)}} \tag{5.162}$$

$L_{tr(pu)}$ = per-unit leakage inductance
$= X_{tr(pu)}$

If there is a significant amount of ac system impedance, the following notation is applicable:

$X_{1(sys)}$ = ac system reactance referred to ac side in ohms
X_{sys} = ac system reactance referred to dc side in ohms

$$= \frac{X_{1(sys)}}{n^2} \tag{5.163}$$

$L_{1(sys)}$ = ac system inductance referred to ac side in henries
L_{sys} = ac system inductance referred to dc side in henries

$$= \frac{L_{1(sys)}}{n^2} \tag{5.164}$$

5.10.1 Alternating-Current System Per-Unit Bases

The per-unit bases for the quantities located on the ac side of the converter transformer are

$E_{1(B)}$ = arbitrarily chosen voltage value
$S_{1(B)}$ = arbitrarily chosen voltampere value

$$I_{1(B)} = \frac{S_{1(B)}}{3E_{1(B)}} \quad \text{A} \tag{5.165}$$

$$Z_{1(B)} = \frac{E_{1(B)}}{I_{1(B)}} \quad \Omega \tag{5.166}$$

$$L_{1(B)} = \frac{Z_{1(B)}}{w_{(B)}} \quad \text{H} \tag{5.167}$$

$$C_{1(B)} = \frac{1}{w_{(B)} Z_{1(B)}} \quad \text{F} \tag{5.168}$$

where

$$w_{(B)} = w = 377 \quad \text{if } f = 60 \text{ Hz}$$

On the other hand, the per-unit bases for the quantities located on the dc side of the converter transformer are

$$E_{(B)} = \frac{E_{1(B)}}{a} \quad \text{V} \tag{5.169}$$

$$S_{(B)} = S_{1(B)} \tag{5.170}$$

$$I_{(B)} = aI_{1(B)} \quad \text{A} \tag{5.171}$$

$$Z_{(B)} = \frac{Z_{1(B)}}{a^2} \quad \Omega \tag{5.172}$$

$$L_{(B)} = \frac{L_{1(B)}}{a^2} \quad \text{H} \tag{5.173}$$

$$C_{(B)} = a^2 C_{1(B)} \quad \text{F} \tag{5.174}$$

Note that the per-unit size of each transformer is

$$S_{\text{tr(pu)}} = \frac{1}{B} \quad \text{pu VA} \tag{5.175}$$

provided that

$$S_{(B)} = BS_{\text{tr}} \quad \text{VA} \tag{5.176}$$

is selected. For example, the per-unit size of each transformer of a four-bridge converter station is

$$\begin{aligned} S_{\text{tr(pu)}} &= \frac{S_{\text{tr}}}{S_{(\text{B})}} \\ &= \frac{S_{\text{tr}}}{4S_{\text{tr}}} \\ &= 0.25 \text{ pu VA} \end{aligned}$$

5.10.2 Direct-Current System Per-Unit Bases

The dc system per-unit bases for a B-bridge converter are selected somewhat differently than the previous bases used for a single-bridge converter in Section 5.7.

When the firing angle α, overlap angle u, and dc current I_d are zero, the ratio of base ac voltages a and turns ratio in use n are equal, and

$$E_{(\mathrm{pu})} = E_{1(\mathrm{pu})} = 1.0 \quad \mathrm{pu\ V}$$

the dc voltage base is

$$V_{d(B)} \triangleq V_{d0}$$

or

$$V_{d(B)} = \frac{3\sqrt{6}}{\pi} BE_{(B)} \quad \mathrm{V} \tag{5.177}$$

or

$$V_{d(B)} = \frac{3\sqrt{6}}{\pi} \frac{BE_{1(B)}}{a} \quad \mathrm{V} \tag{5.178}$$

By forcing the ac and dc power bases to be exactly equal,

$$3E_{(B)}I_{(B)} = V_{d(B)}I_{d(B)} \quad \mathrm{VA} \tag{5.179}$$

so that

$$I_{d(B)} = \frac{3E_{(B)}I_{(B)}}{V_{d(B)}} \quad \mathrm{A} \tag{5.180}$$

Substituting equations (5.177) and (5.178) into equation (5.180) separately,

$$I_{d(B)} = \frac{\pi}{\sqrt{6}} \frac{I_{(B)}}{B} \quad \mathrm{A} \tag{5.181}$$

and

$$I_{d(B)} = \frac{\pi}{\sqrt{6}} \frac{aI_{1(B)}}{B} \quad \mathrm{A} \tag{5.182}$$

The fundamental component of the ac current per line to a bridge having no overlap is

$$I_{1(a)} = \frac{\sqrt{6}}{\pi} I_d \quad \mathrm{A} \tag{5.183}$$

or

$$I_{1(a)} = 0.78 I_d \quad \text{A} \tag{5.184}$$

whereas the total rms ac current is

$$I_{(a)} = \sqrt{\tfrac{2}{3}} I_d \quad \text{A} \tag{5.185}$$

or

$$I_{(a)} = 0.8165 I_d \quad \text{A} \tag{5.186}$$

Note that, in the bases discussed in Section 5.7, the constant $\sqrt{3/2}$ was in the definition of $I_{d(B)}$ and that $P_{d(B)}$ was not exactly equal to the ac power base. However, in the present bases, the constant $\sqrt{6}/\pi$ is in the definition of $I_{d(B)}$, and ac and dc power bases are exactly equal.

The dc resistance base in the dc system can be expressed as

$$R_{d(B)} = \frac{V_{d(B)}}{I_{d(B)}} \quad \Omega \tag{5.187}$$

or

$$R_{d(B)} = \frac{18 B^2 Z_{(B)}}{\pi^2} \quad \Omega \tag{5.188}$$

or

$$R_{d(B)} = \frac{18 B^2 Z_{1(B)}}{\pi^2 a^2} \quad \Omega \tag{5.189}$$

The dc inductance base in the dc system can be expressed as

$$L_{d(B)} = \frac{V_{d(B)} t_{(B)}}{I_{d(B)}} \quad \text{H} \tag{5.190}$$

or

$$L_{d(B)} = \frac{R_{d(B)}}{w_{(B)}} \quad \text{H} \tag{5.191}$$

where

$$t_b = \frac{1}{w_{(B)}} \quad \text{S} \tag{5.192}$$

Similarly, the dc capacitance base in the dc system can be expressed as

$$C_{d(B)} = \frac{1}{R_{d(B)}w_{(B)}}, \quad \text{F} \tag{5.193}$$

5.11 OPERATION OF DIRECT-CURRENT TRANSMISSION LINK

Figure 5.14 shows the equivalent circuit for a simple dc transmission link. Here, the dc link may be a transmission line, a cable, or a link with negligible length. The subscripts r and i signify rectifier and inverter, respectively.

The direct current I_d that flows from the rectifier to the inverter can be expressed as

$$I_d = \frac{V_{dr} - V_{di}}{R_L} \tag{5.194}$$

and the sending-end power can be expressed as

$$P_{(\text{dc})} = V_{dr}I_d \tag{5.195}$$

Since it is possible for a converter to become a rectifier or an inverter by grid control, the direction of the power flow can be reversible. This can be accomplished by reversing the direct voltage, as previously explained. Figure 5.15 illustrates this reversion in power flow direction. It can be shown, from Kirchhoff's voltage law, that

$$V_{ab} = I_dR_L + V_{cd} \tag{5.196}$$

Therefore, when the V_{ab} *and* V_{cd} voltages represent the average direct voltages of a rectifier and an inverter, respectively, equation (5.196) can be expressed as

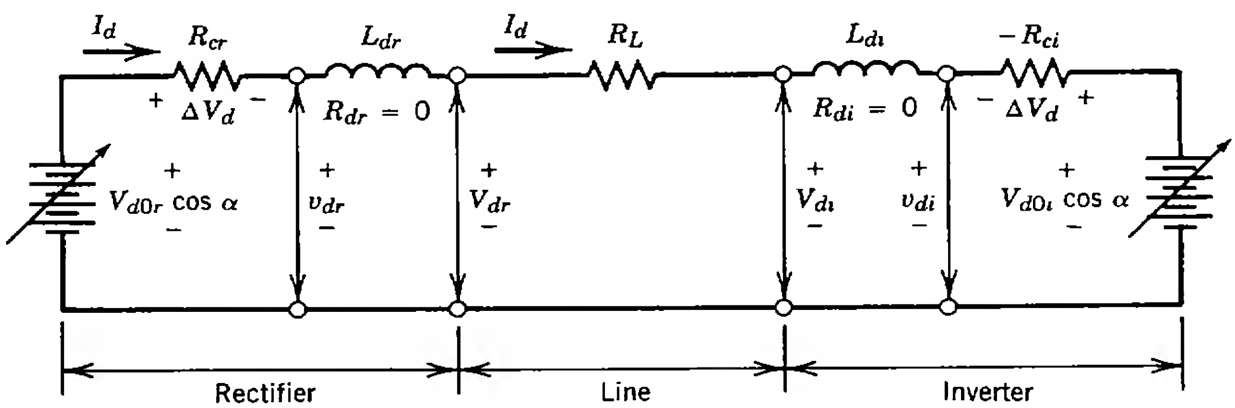


Figure 5.14. Equivalent circuit of dc link.

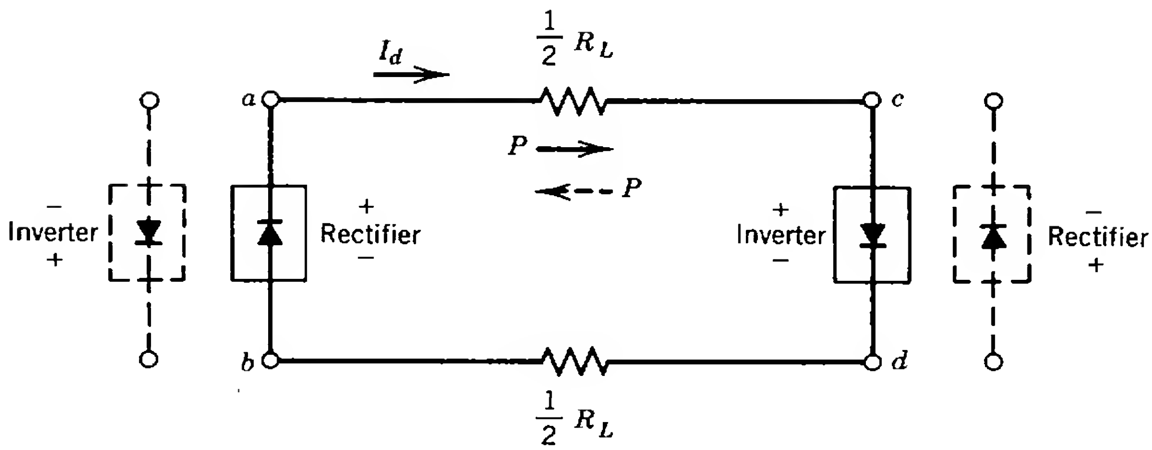


Figure 5.15. Illustration of reversion in power flow direction.

$$V_{dr} = I_d R_L - V_{di} \tag{5.197}$$

Similarly, when the V_{ab} and V_{cd} voltages represent the average direct voltages of an inverter and a rectifier, respectively, equation (5.196) can be expressed as

$$V_{di} = I_d R_L - V_{dr} \tag{5.198}$$

Therefore, it can be shown in either case that

$$I_d R_L = V_{dr} + V_{di} \tag{5.199}$$

where

$$V_{dr} = V_{d0r}\cos\alpha - \frac{3}{\pi} wL_{cr} I_d \tag{5.200}$$

$$V_{di} = -V_{d0i}\cos\alpha + \frac{3}{\pi} wL_{ci} I_d \tag{5.201}$$

Thus,

$$I_d = \frac{V_{d0r}\cos\alpha - V_{d0i}\cos\gamma}{R_L + \frac{3}{\pi} wL_{cr} - \frac{3}{\pi} wL_{ci}} \tag{5.202}$$

or

$$I_d = \frac{V_{d0r}\cos\alpha - V_{d0i}\cos\gamma}{R_L + R_{cr} - R_{ci}} \tag{5.203}$$

where

$$R_{cr} = \frac{3}{\pi} wL_{cr} \tag{5.204}$$

$$R_{ci} = \frac{3}{\pi} wL_{ci} \tag{5.205}$$

The value of the direct current I_d can be controlled by changing either V_{d0r} and V_{d0i} values or delay angle α or extinction angle γ. The value of V_{d0r} and V_{d0i} can be governed by using the LTCs of the supply transformers to change the ratio between the dc and ac voltages. Unfortunately, this method is very slow to be practical. Whereas the delay angle α can be controlled very fast by using the grid control system. However, this method causes the converter to consume an excessively large amount of reactive power. Therefore, it is usual to operate the rectifier with minimum delay angle and the inverter with minimum extinction angle in order to achieve the control with minimum amount of reactive power consumption. Therefore, it is a better practice to operate the rectifier with a constant-current characteristic and the inverter with a constant-voltage characteristic, as shown in Figure 5.16. As succinctly put by Kimbark [1], "the rectifier controls the direct current and the inverter controls the direct voltage." In practice, the values of the delay angle α and the extinction angle γ are usually selected in the ranges of 12°–18° and 15°–18°, respectively. Since a converter can be

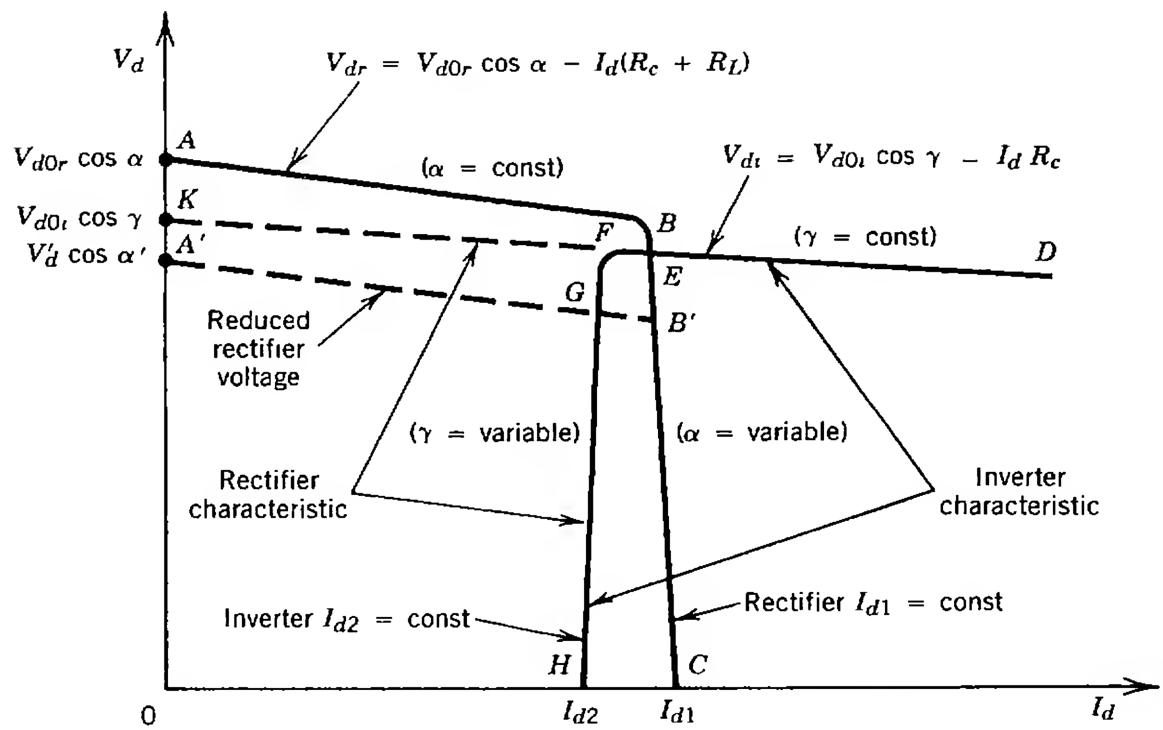


Figure 5.16. Inverter and rectifier operation characteristics with constant-current compounding.

operating as a rectifier or inverter depending on the direction of power flow, it is necessary that each converter have dual-control systems, as shown in Figure 5.17.

Note that, in Figure 5.16, the normal operation point is E, where the characteristics of rectifier and inverter intersect. The rectifier characteristic has two line segments: the AB segment, at which the minimum delay angle α is constant, and the BC segment, at which the rectifier current I_{d1} is constant. Similarly, the inverter characteristic has two segments: the DF segment, at which the minimum extinction angle γ is constant, and the FH segment, at which the inverter current I_{d2} is constant. Normally, the current regulator of the inverter is set at a lower current value than the one of the rectifier. Therefore, the difference in currents is

$$\Delta I_d = I_{d1} - I_{d2} \tag{5.206}$$

where ΔI_d = current margin (usually 10–15 percent of rated current)
I_{d1} = constant current of rectifier
I_{d2} = constant current of inverter

As aforementioned, under normal operating conditions, the operation point is E, at which the rectifier controls the direct current and the inverter controls the direct voltage.

However, under emergency conditions, the operation point may change. For example, if the rectifier voltage characteristic is shifted down due to a large dip in rectifier voltage, it intersects the FH constant-current segment of the inverter characteristic at a new operation point G, where the inverter controls the direct current and the rectifier controls the direct voltage. Note that if the inverter were not equipped with the constant-current (control) regulator, it would have the characteristic DFK, as shown in Figure 5.16. Therefore, it can be seen that the shifted rectifier characteristic $A'B'$ would not have intersected the inverter characteristic. Consequently, the current and power would have decreased to zero.

Under normal operation, if the current is required to increase, the current setting is increased first at the rectifier and second at the inverter. Whereas if the current is required to decrease, the order of the operation is reversed; the current setting is decreased first at the inverter and second at the rectifier. This keeps the sign of the current margin the same and thus prevents any unexpected reversing in the direction of power flow.

Furthermore, the current margin between such points s E and G shown in Figure 5.16 has to be adequate in order to prevent both inverter and rectifier current regulators operating at the same time. Note also that the voltage margin indicated by BE is due to a trade-off between minimum reactive compensation by holding the delay angle α as low as possible and preventing inverter control, which affects current.

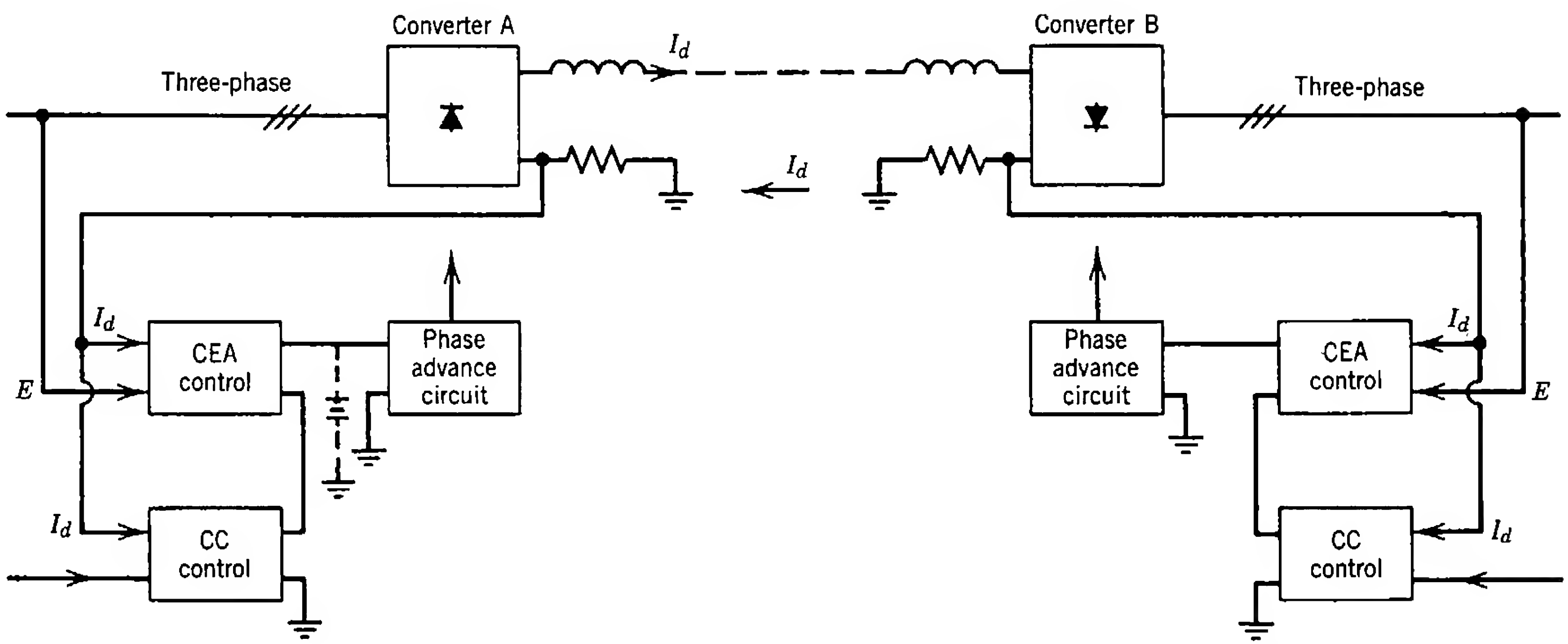


Figure 5.17. Schematic control diagram of high-voltage dc system with constant extinction angle (CEA) and constant-current (CC) control [3].

5.12 STABILITY OF CONTROL†

As aforementioned, the control system is made of constant-current control of the rectifier and constant extinction angle control of the inverter. An inappropriate control system can make oscillations by various disturbances, that is, converter faults and line-to-ground faults, and can cause *instability*. Kimbark [1] gives the following approximate method to study this phenomenon. Figure 5.18 shows an equivalent circuit of a dc link for analysis of stability of control. Note that the rectifier on a constant-current control shows a large resistance $K + R_{c1}$ (where K is the gain of the constant-current regulator). Whereas the inverter on a constant-extinction-angle control shows a low negative resistance, $-R_{c2}$.

Therefore, it can be written in the s domain that

$$E_1(s) = Z_1(s)I_1(s) - Z_m(s)I_2(s) \tag{5.207}$$

$$E_2(s) = Z_m(s)I_1(s) - Z_2(s)I_2(s) \tag{5.208}$$

or, in matrix notation,

$$\begin{bmatrix} Z_1(s) & -Z_m(s) \\ Z_m(s) & -Z_2(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = \begin{bmatrix} E_1(s) \\ E_2(s) \end{bmatrix} \tag{5.209}$$

Since the transient response is being studied,

$$\begin{bmatrix} Z_1(s) & -Z_m(s) \\ Z_m(s) & -Z_2(s) \end{bmatrix} \begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} = 0$$

But

$$\begin{bmatrix} I_1(s) \\ I_2(s) \end{bmatrix} \neq 0$$

Therefore,

$$\begin{bmatrix} Z_1(s) & -Z_m(s) \\ Z_m(s) & -Z_2(s) \end{bmatrix} = 0$$

Thus, the characteristic equation of the circuit can be expressed as

$$-Z_1(s)Z_2(s) + Z_m^2(s) = 0 \tag{5.210}$$

where

$$Z_1(s) = R_1 + L_1 s + \frac{1}{Cs} \tag{5.211}$$

† This section is based on *Direct Current Transmissions* by E. W. Kimbark.

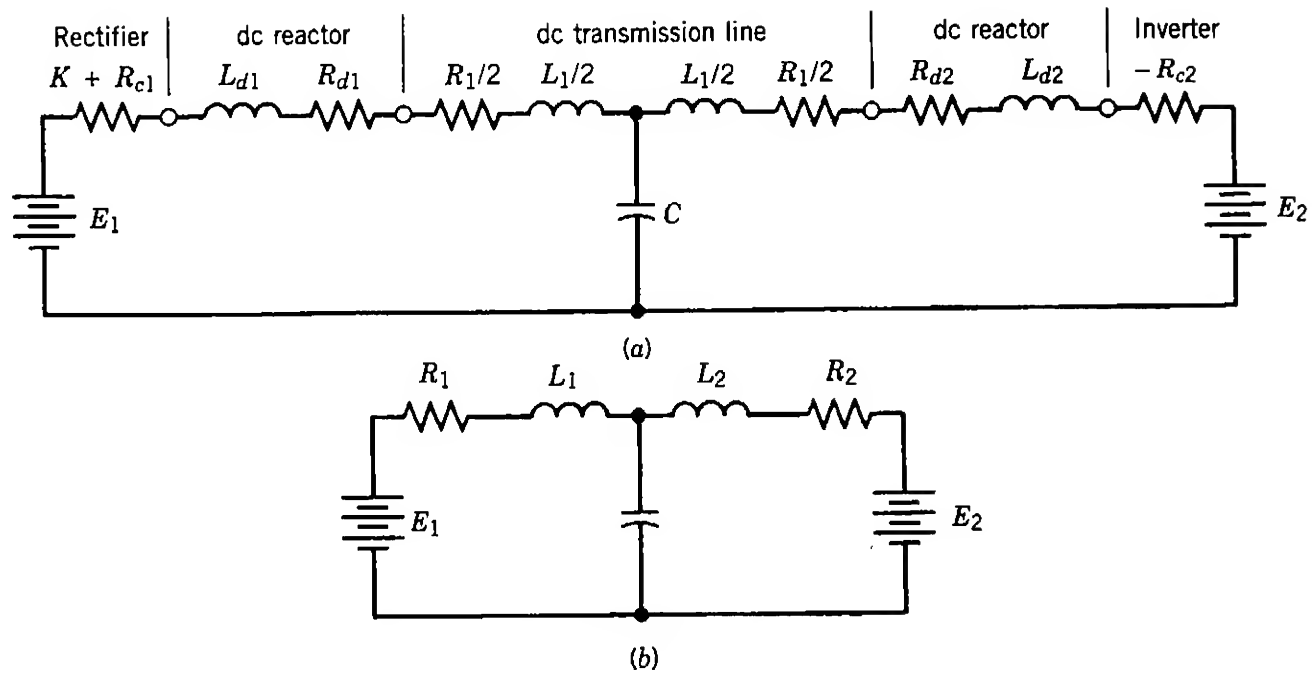


Figure 5.18. Equivalent circuit of dc link for analysis of stability of control: (*a*) before combination of line with terminal equipment; (*b*) after combination [1].

$$Z_2(s) = R_2 + L_2 s + \frac{1}{Cs} \tag{5.212}$$

$$Z_m(s) = \frac{1}{Cs} \tag{5.213}$$

$$s = \text{complex frequency}$$
$$= \sigma + jw$$

Substituting equations (5.211)–(5.213) into equation (5.210),

$$-\left(R_1 + L_1 s + \frac{1}{Cs}\right)\left(R_2 + L_2 s + \frac{1}{Cs}\right) + \left(\frac{1}{Cs}\right)^2 = 0 \tag{5.214}$$

or

$$CL_1L_2s^3 + C(R_1L_2 + R_2L_1)s^2 + (L_1 + L_2 + R_1D_2C)s + (R_1 + R_2) = 0 \tag{5.215}$$

Assume that $R_1 \gg L_1 s$ and that $L_1 = 0$. Thus, equation (5.125) becomes

$$CR_1L_2s^2 + (L_2 + R_1R_2C)s + (R_1 + R_2) = 0 \tag{5.216}$$

or

$$s^2 + \left(\frac{1}{R_1C} + \frac{R_2}{L_2}\right)s + \frac{R_1 + R_2}{CR_1L_2} = 0 \tag{5.217}$$

However, since $R_1 \ll R_2$,

$$R_1 + R_2 \cong R_1 \tag{5.218}$$

substituting equation (5.128) into equation (5.217),

$$s^2 + \left(\frac{1}{R_1C} + \frac{R_2}{L_2}\right)s + \frac{1}{CL_2} = 0 \tag{5.219}$$

Compring equation (5.219) with the standard equation of

$$s^2 + 2\zeta w_n s + w_n^2 = 0 \tag{5.220}$$

the undamped natural frequency can be expressed as

$$w_n \cong \frac{1}{\sqrt{CL_2}} \tag{5.221}$$

Since

$$2\zeta w_n = \frac{1}{R_1 C} + \frac{R_2}{L_2} \tag{5.222}$$

the damping coefficient can be expressed as

$$\begin{aligned} \sigma &= \zeta w_n \\ &= \frac{1}{2}\left(\frac{1}{R_1 C} + \frac{R_2}{L_2}\right) \end{aligned} \tag{5.223}$$

from which the damping ratio can be found as

$$\zeta = \frac{1}{2R_1}\sqrt{\frac{L_2}{C}} + \frac{R_2}{2}\sqrt{\frac{C}{L_2}} \tag{5.224}$$

From equation (5.224),

$$R_1 \cong \frac{L_2}{2\zeta\sqrt{L_2 C} - R_2 C} \tag{5.225}$$

In general, a positive, but less than critical, damping (i.e., $\zeta = 1$) is required. For example, when $\zeta = 0.7$,

$$R_1 \cong \frac{L_2}{1.4\sqrt{L_2 C} - R_2 C}$$

Note that at critical damping,

$$R_1 \cong \frac{L_2}{2\sqrt{L_2 C} - R_2 C} \tag{5.226}$$

Applying Routh's criterion to the characteristic equation of the high-voltage dc link given by equation (5.215) and assuming equal dc smoothing reactors (i.e., $L_{d1} = L_{d2} = L_d$), it can be found that

$$R_1 = -\frac{L_d}{R_2 C} \tag{5.227}$$

or

$$R_1 = \frac{L_d}{(-R_2)C} \tag{5.228}$$

or since

$$L_2 \cong L_d$$

then

$$R_1 = \frac{L_2}{(-R_2)C} \tag{5.229}$$

Therefore, in order to have a stable system or oscillations to be damped, the maximum value of R_1 must be

$$R_{1(\max)} < \frac{L_2}{|R_2|C} \tag{5.230}$$

But

$$R_{1(\max)} = K_{(\max)} + R_{c1} + R_{d1} + \tfrac{1}{2}R_l \tag{5.231}$$

Therefore, there is a maximum value of K for which the system is stable. However, note that if $R_2 < 0$,

$$R_{1(\max)} > \frac{L_1}{L_2}|R_2| \tag{5.232}$$

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PROBLEMS

5.1. Assume that the following data are given for the overhead ac line discussed in Example 5.1:

$$\text{Steady-state operating voltage} = 200/346\ \text{kV}$$

$$\text{Line current} = 1000\ \text{A}$$

$$\text{Power} = 600\ \text{MVA}$$

$$\text{Insulation level} = 500\ \text{kV}$$

Use the assumptions and results given in the example, and determine the following for the comparable dc line:

(a) Line-to-line dc voltage in kilovolts.
(b) Line-to-ground dc voltage in kilovolts.
(c) The dc line current in amperes
(d) Associated dc power in megavoltamperes
(e) The dc line power loss in kilowatts.
(f) The dc insulation level.

5.2. Assume that factor K is 4 and repeat Example 5.2.

5.3. Assume that factor K is 5 and repeat Example 5.2.

5.4. Assume that a three-conductor dc overhead line with equal conductor sizes is considered to be employed to transmit three-phase, three-conductor ac energy at a 0.92 power factor (see Figure P5.4). If maximum voltages to ground and transmission line efficiencies are the same for both dc and ac and the load is balanced, determine the change in the power transmitted in percent.

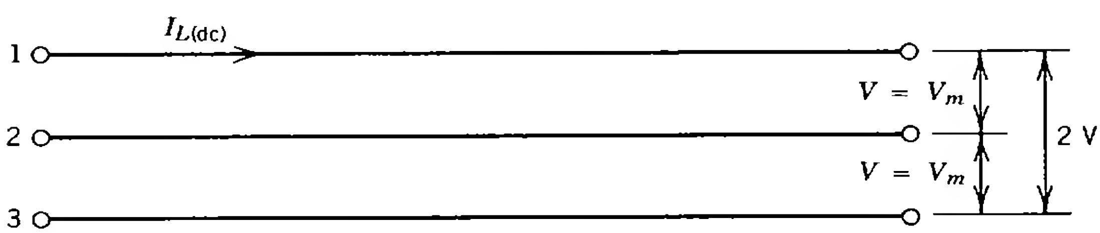


Figure P5.4

5.5. Derive equation (5.110) from equation (5.109).

5.6. Derive equations (5.68) and (5.69).

5.7. Derive equation (5.73).

5.8. Verify the following equations:

(a) $L_{d(B)} = \dfrac{18B^2 Z_{(B)}}{\pi^2 w_{(B)}}$ H

(b) $L_{d(B)} = \dfrac{18B^2 Z_{1(B)}}{\pi^2 a^2 w_{(B)}}$ H

5.9. Verify the following equations:

(a) $C_{d(B)} = \dfrac{\pi^2}{18B^2 Z_{(B)} w_{(B)}}$ F

(b) $C_{d(B)} = \dfrac{\pi^2 a^2}{18B^2 Z_{(B)} w_{(B)}}$ F

5.10. Derive the equation

$$V_{d(\mathrm{pu})} = \frac{a}{n} E_{1(\mathrm{pu})} \cos \alpha - R_{c(\mathrm{pu})} I_{d(\mathrm{pu})}$$

from equation (5.65)

5.11. Verify equation (5.65).

5.12. Consider a B-bridge converter station and use the angles α and δ, which imply rectifier action. Consider only the first-mode operation, ($u \leq 30°$). Apply the equation

$$I_d = \frac{\sqrt{3} E_m}{2wL_c} (\cos \alpha - \cos \delta)$$

which gives the average dc current in any one bridge if E_m is properly interpreted. Also apply the equation

$$V_d = V_{d0} \cos \alpha - \Delta V_d$$

which gives the average dc terminal voltage of one bridge if $B = 1$ bridge is being analyzed.

(a) Redefine V_d, V_{d0}, and ΔV_{d0} to designate total voltages for B bridges in series on the dc sides and show that

$$V_d = V_{d0} \cos \alpha - \frac{3}{\pi} wBL_c I_d$$

is valid for the pole-to-pole voltage of the B-bridge station.

(b) Define V_d, in part (a), in terms of B, E, and E_1.

5.13. Review Problem 5.12 carefully to ensure that the definition and meaning of L_c are clearly in mind. Remember further that only first-mode operation is being studied. Assume that relative to the treatment of the LTC transformers and the following information from General Electric Company's Bulletin GET-1285: "In an actual transformer, tests will show that changing the taps on a specific winding will not materially affect the per unit short-circuit impedance of the transformer, based on the new voltage base, as determined by the tap position."

(a) Determine the L_c inductance of the rectifier referred to the dc side in henries.

(b) Determine the L_{c1} inductance of the rectifier referred to the ac side in henries.

(c) Repeat part (a) in per units.

(d) Repeat part (b) in per units.

5.14. Explain two different circumstances under which L_{s1} (i.e., the system inductance that is common to all bridges) may be negligibly small.

5.15. Normalize the dc average terminal voltage equation for a B-bridge converter station using the per-unit system bases given in Section 5.10. This time, formalize the treatment of radian frequency with

$$w = w_{(\mathrm{pu})} w_{(B)}$$

where, ordinarily, $w_{(\mathrm{pu})} = 1.00$ and $w_{(B)} = 377$ rad/s. Show that the result is

$$V_{d(\mathrm{pu})} = \frac{a}{n} E_{1(\mathrm{pu})} \cos \alpha - \frac{\pi w_{(\mathrm{pu})}}{6B} \left(L_{(\mathrm{tr}(\mathrm{pu}} + \frac{B}{2} L_{(\mathrm{sys})\mathrm{pu}} \right) \left(\frac{a}{n} \right)^2 I_{d(\mathrm{pu})}$$

5.16. Assume that a high-voltage dc transmission link consists of two four-bridge converter stations operating as a rectifier and an inverter, similar to the setup given in Figure 5.13. Assume that the maximum continuous V_{d0r} and V_{d0i} voltages are 125 kV and that the maximum continuous direct current I_d is 1200 A for all eight bridges involved. Each converter transformer has a continuous rating $S_{(\mathrm{tr})}$ of 157.05 MVA and a LTC range of ±20% in 32 steps of 1.25 percent. The rated transformer voltages for the rectifier station have been given as 199.2/345 kV for the ac side and 53.44/92.56 kV for the dc side. Similarly, the rated transformer voltages for the inverter station have been given as 288.67/500 kV for the ac side and 53.44/92.56 kV for the dc side. The leakage reactances have been given as 0.14 pu for the 345-kV transformers and 0.16 pu for the 500-kV transformers. Assume that the voltage ratings and the reactances given are correct when the LTC is in the neutral position. Assume that the arbitrary ac system bases are 500 MVA for three-phase voltampere and 199.2 and 288.67 kV for the $E_{1(B)r}$ and $E_{1(B)i}$ line-to-neutral rectifier and inverter voltages, respectively. Determine the following:

(a) Base voltage ratios of a_r for a_i for rectifier and inverter, respectively.

(b) The ac side rectifier per-unit system bases of $I_{1(B)r}$, $I_{(B)r}$, $Z_{1(B)r}$, $Z_{(B)r}$, $L_{1(B)r}$, and $L_{(B)}$.

(c) The ac side inverter per-unit system bases of $I_{1(B)i}$, $I_{(B)i}$, $Z_{1(B)i}$, $Z_{(B)i}$, $L_{1(B)i}$, and $L_{(B)i}$.

(d) The dc side per-unit system bases of $V_{d(B)}$, $I_{d(B)}$, $R_{d(B)}$, $L_{d(B)}$, and $C_{d(B)}$.

5.17. In Problem 5.16, assume that the three-phase short-circuit fault duties are given as 20,000 MVA on the 500-kV inverter bus and 10,000 MVA on the 345-kV rectifier bus and determine the following:

(a) Commutating inductances of rectifier and inverter in per units.

(b) Commutating inductances of rectifier and inverter referred to dc sides in henries.

(c) Commutating inductances of rectifier and inverter referred to ac sides in per units.

5.18. In Problem 5.17, assume that the dc transmission line has three-conductor bundles of 1590-kcmil ACSR conductor, with 18-in. equilateral triangular configuration of bundles, 32-in. pole-to-pole spacing. Use 50° resistances for the 400-mi transmission line. Assume that the high-voltage dc link is being operated at reduced capacity so that the current I_d is 1000 A and the inverter station power P_{di} is 400 MW and the voltages $E_{1(\mathrm{pu})r}$ and $E_{1(\mathrm{pu})i}$ are 1 pu. Assume that the minimum extinction angle of the inverter $\gamma_{\min}$ is desired as 10° in the interest of minimum var consumption by the inverter station. Use normalized equations and per-unit variables.

(a) Determine if the CEA control and $\gamma_{\min}$ will prevail during this reduced load operation. If so, find the corresponding value of a_i/n_i. How many steps of LTC buck or boost is this? If $\gamma_{\min}$ cannot be used, find the best value of γ and the corresponding a_i/n_i and LTC position.

(b) Find the rectifier station terminal voltage $V_{d(\mathrm{pu})r}$.

5.19. In Problem 5.18, assume the CIA operation of the rectifier station with 1000 A as the set value of the constant current I_d and 15° as the set value of the ignition angle. For this particular problem, assume the rectifier constant-current (CC) characteristic is vertical. The rectifier station LTCs are so positioned that terminal voltage $V_{d(r)}$ is 450 kV when CIA control changes to the CC control. Assume that the rectifier LTCs retain the position described.

(a) Find a_r/n_r for the rectifier.

(b) Find the rectifier LTC positions and specify them in terms of the number of steps of buck or boost.

(c) Find the ignition angle α that prevails during the reduced-load operation being studied.

(d) Find $V_{dr0}\cos\alpha_{\text{set}}$ and V_{d0r} in kilovolts.

5.20. The curves shown in Figure P5.20 are for a simple single-bridge rectifier–inverter link. Assume that the rectifier and inverter stations have identical apparatus and parameters and that the steady-state operating point at the rectifier terminal voltage $V_{d(r)}$ is 100 kV. Assume that the dc line has a total loop resistance of 5.00 Ω, a pole-to-pole capacitance of 0.010 μF/m, and a loop inductance of 3.20 mH/mi. Use dimensional one-bridge equations.

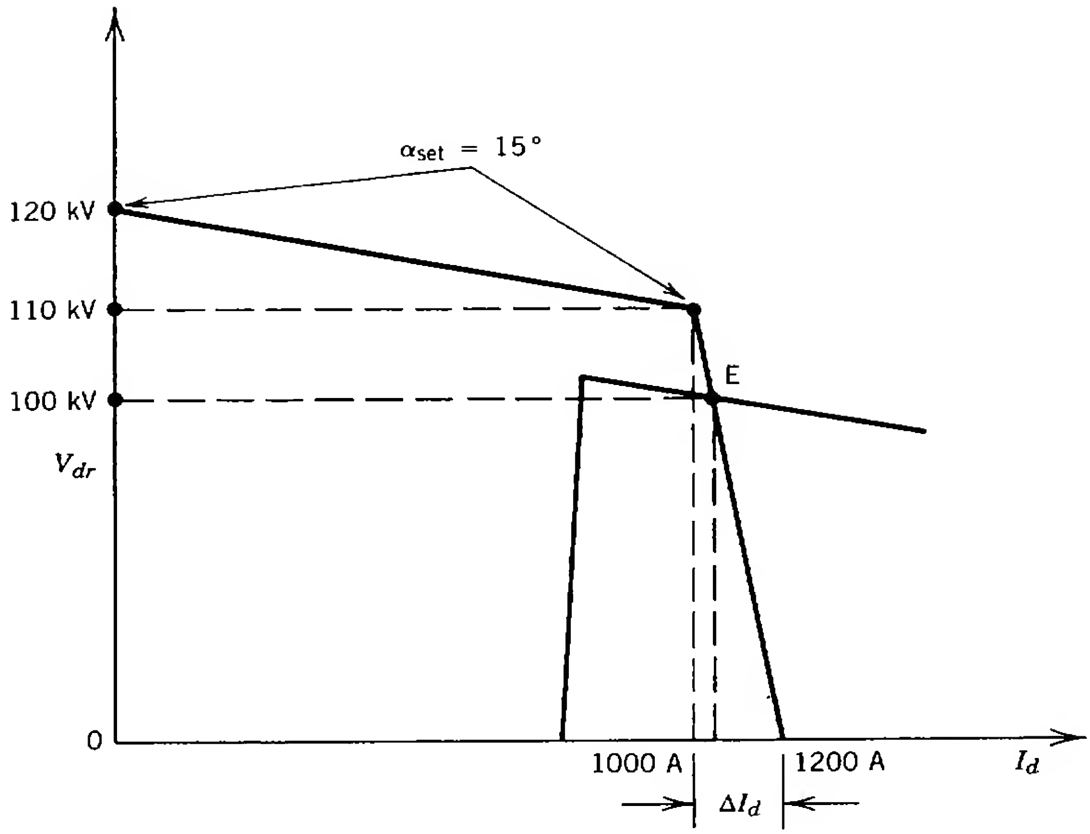


Figure P5.20

(a) Determine the rectifier commutating resistance R_c.

(b) Determine the inverter dc average terminal voltage $V_{d(i)}$.

(c) Determine the rectifier firing angle α at the operating point E.

(d) Assume that the line length is 50 mi, and the current margin ΔI_d is 200 A. Find the necessary inverter station dc smoothing inductance L_{d2} if the damping ratio is desired to be 0.70. Use the necessary simplifying assumptions.

5.21. Consider the rectifier CC control and inverter CEA control. Use the steps and notation of Figure 5.18, but simplify by setting the dc smoothing reactor resistances to zero, that is, $R_{d1} = R_{d2} = 0$. When the dc transmission line is relatively long but $\frac{1}{2}R_L + R_{c2} < 0$, it appears that there may be some difficulty in achieving both a desirably large damping ratio and a desirably small value of ΔI_d. Show how this problem can arise by approximating R_1,

$$R_1 \cong \frac{1}{2\zeta}\sqrt{\frac{L_2}{C}} \quad \Omega$$

and then modifying the approximate R_1 equation to contain system parameters and line length X explicitly. Show the nature of the possible problem described and discuss the possible remedies.

6

TRANSMISSION SYSTEM RELIABILITY

6.1 BASIC DEFINITIONS

The following definitions of terms for reporting and analyzing outages of power system facilities and interruptions are taken from reference 1 and included here by permission of the Institute of Electrical and Electronics Engineers.

OUTAGE. It describes the state of a component when it is not available to perform its intended function due to some event *directly associated* with that component. An outage may or may not cause an interruption of service to consumers depending on system configuration.

FORCED OUTAGE. It is an outage that results from emergency conditions directly associated with a component requiring that component to be taken out of service immediately, either automatically or as soon as switching operations can be performed, or an outage caused by improper operation of equipment or human error.

SCHEDULED OUTAGE. It is an outage that results when a component is deliberately taken out of service at a selected time, usually for purposes of construction, preventive maintenance, or repair. The key test to determine if an outage should be classified as forced or scheduled is as follows. If it is possible to defer the outage when such deferment is desirable, the outage is a scheduled outage; otherwise, the outage is a forced outage. Deferring an outage may be desirable, for example, to prevent overload of facilities or an interruption of service to consumers.

TRANSIENT FORCED OUTAGE. It is a component outage whose cause is immediately self-clearing so that the affected component can be restored to service either automatically or as soon as a switch or a circuit breaker can be reclosed or a fuse replaced. An example of a transient forced outage is a lightning flashover that does not permanently disable the flashed component.

PERSISTENT FORCED OUTAGE. It is a component outage whose cause is *not* immediately-self-clearing but must be corrected by eliminating the hazard or by repairing or replacing the affected component before it can be returned to service. An example of a persistent forced outage is a lightning flashover that shatters an insulator, thereby disabling the component until repair or replacement can be made.

INTERRUPTION. It is the loss of service to one or more consumers or other facilities and is the result of one or more component outages, depending on system configuration.

FORCED INTERRUPTION. It is an interruption caused by a forced outage.

SCHEDULED INTERRUPTION. It is an interruption caused by a scheduled outage.

MOMENTARY INTERRUPTION. It has a duration limited to the period required to restore service by automatic or supervisory-controlled switching operations or by manual switching at locations where an operator is immediately available. Such switching operations are typically completed in a few minutes.

6.2 NATIONAL ELECTRIC RELIABILITY COUNCIL

The National Electric Reliability Council (NERC) was established by the electric utility industry in 1968 and incorporated in 1975. The purpose of the council is to increase the reliability and adequacy of bulk power supply of the electric utility systems in North America. It is a forum of nine regional reliability councils and covers essentially all of the power systems of the U.S. and the Canadian power systems in the provinces of Ontario, British Columbia, Manitoba, and New Brunswick, as shown in Figure 6.1. The figure shows the total number of bulk power outages reported and the ratio of the number of bulk outages to electric sales for each regional electric reliability council area.

Note that the terms *reliability* and *adequacy* define two separate but interdependent concepts. *Reliability* describes the security of the system and the avoidance of the power outages, whereas *adequacy* refers to having sufficient system capacity to supply the electrical energy requirements of the customers.

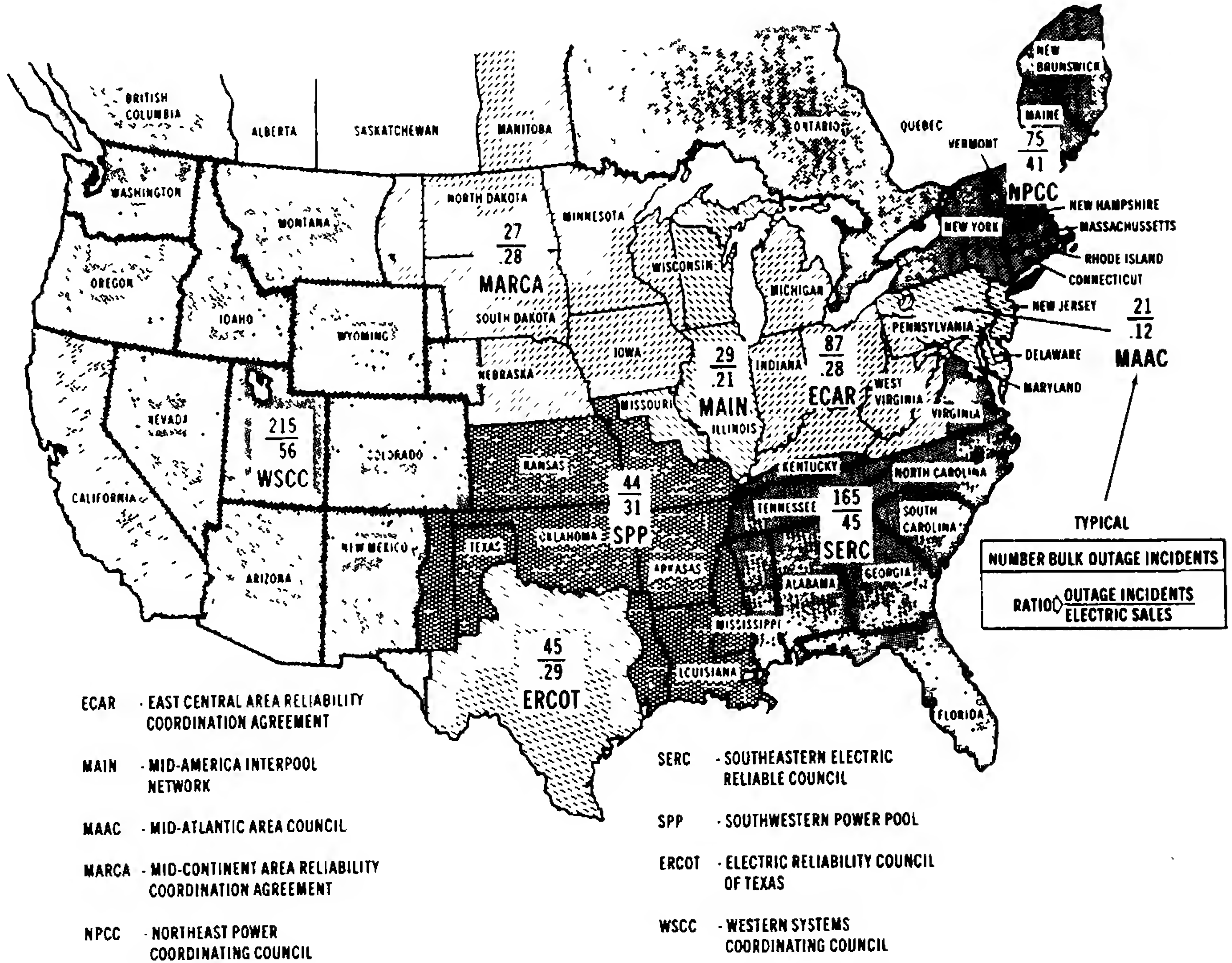


Figure 6.1. Regional electric reliability councils [3].

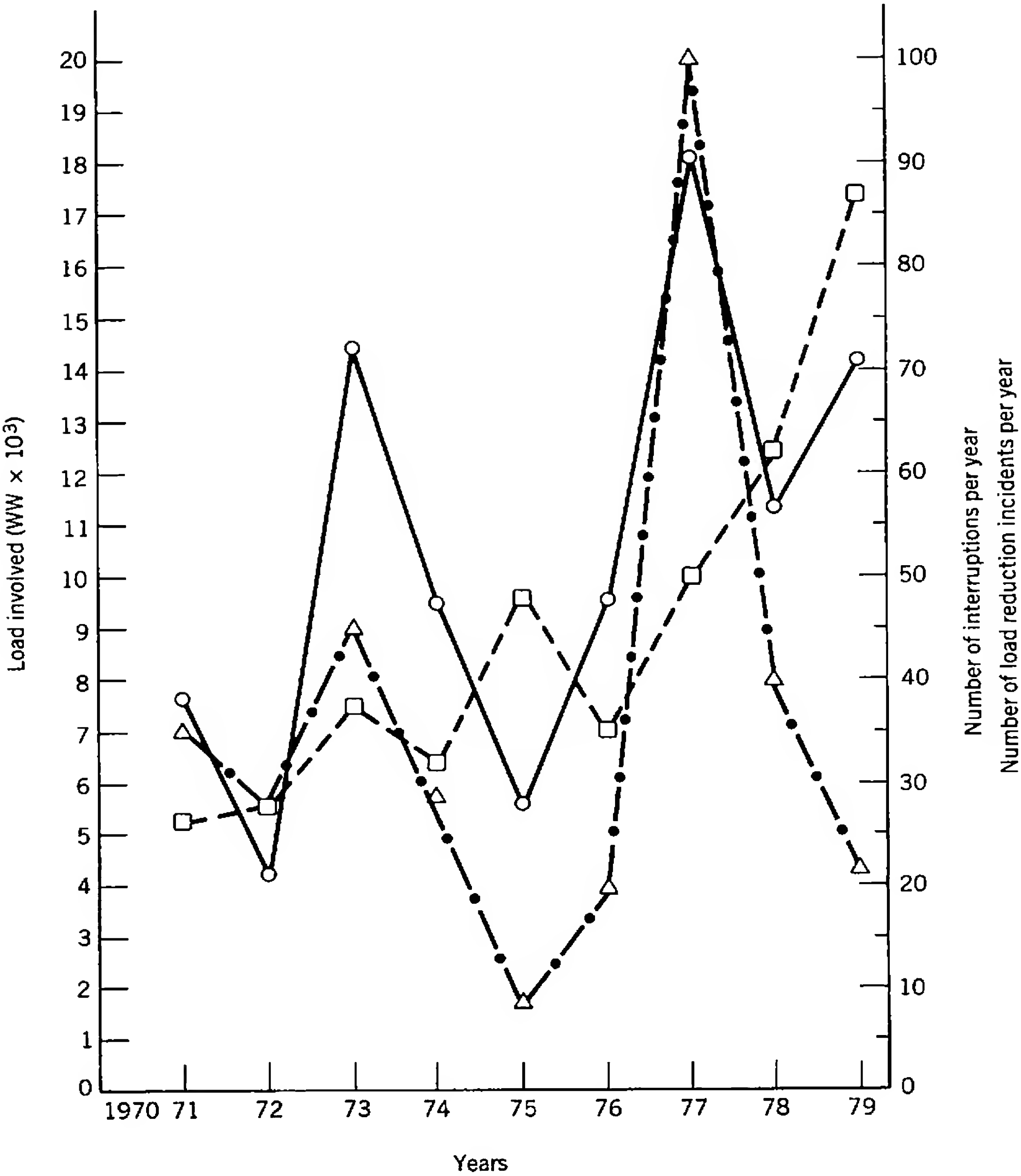


Figure 6.2. Bulk power system interruption data, 1971–1979 [3]: O———O, load involved ($\times 10^3$ MW); □– – – – –□, interruptions per year; △ · · · · · △, load reduction incidents per year.

The term *power pool* defines usually a formal organization established by two or more utilities for the purpose of increased economy, security, or reliability in power system planning or operations. Each pool arrangement is unique due to the different needs and system design of the individual member utilities that are included in the pool. The level of joint planning and operations in power pools can vary from very flexible arrangements for bulk power transfers to coordinated planning and operations to completely

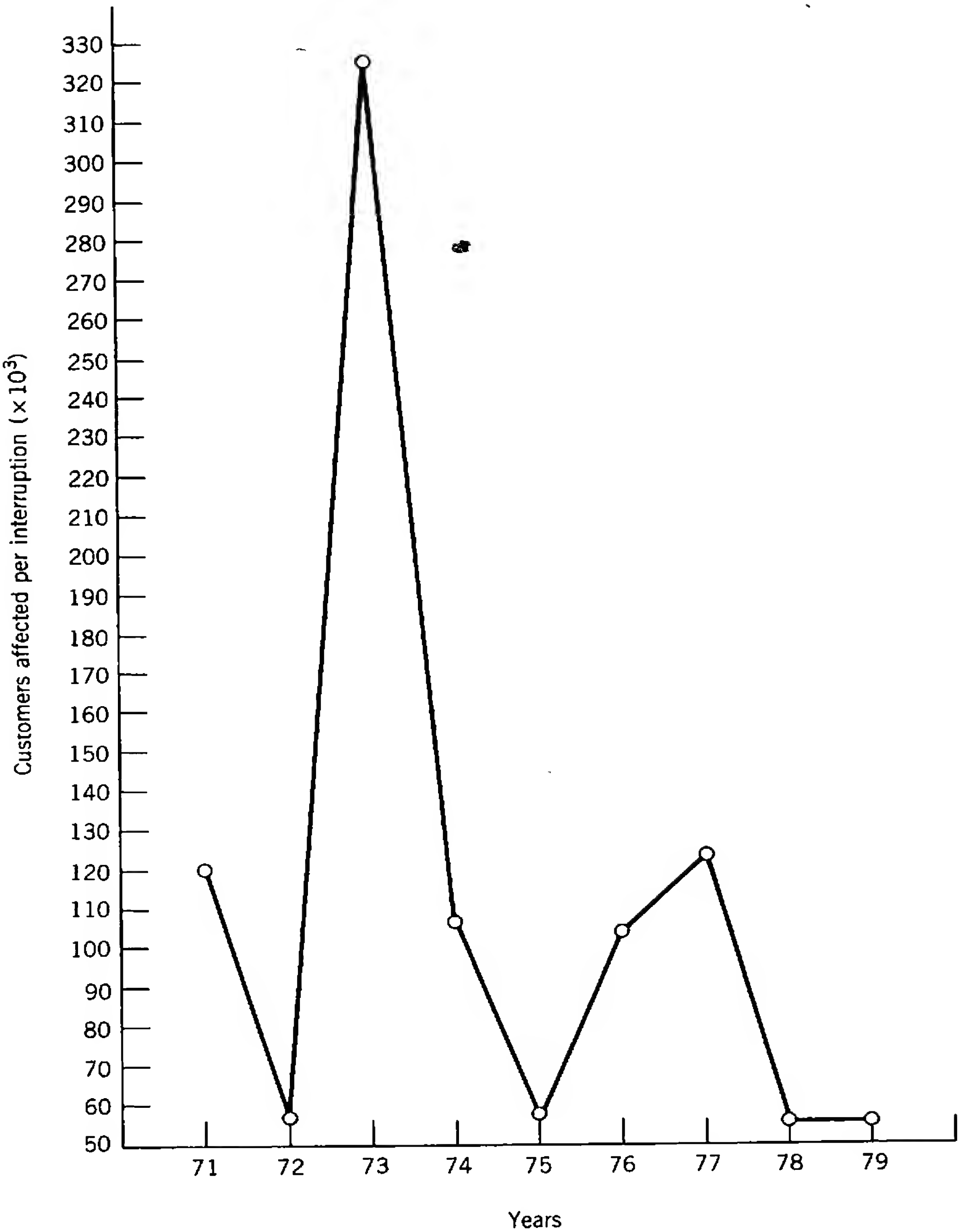


Figure 6.3. Number of customers, in thousands, affected per bulk power interruption, 1971–1979 [3].

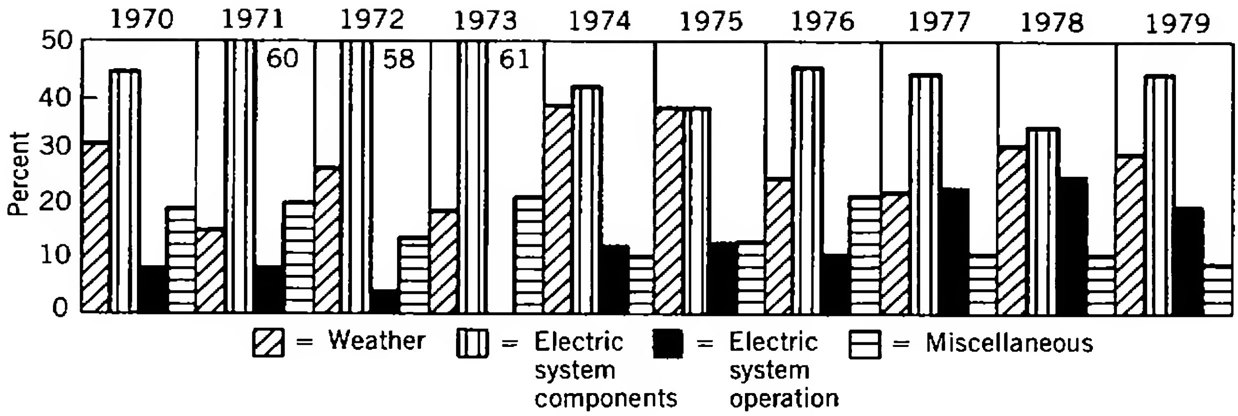


Figure 6.4. Bulk power outages by cause, 1970–1979 [3].

integrated operations. In the integrated-operations-type pools, decisions are made centrally by the pool, and the benefits are allocated to the member utilities. There are approximately 30 power pools at the present time, both formal and informal [2].

Figure 6.2 shows the number of interruptions, load reduction incidents, and load involved per year for the years from 1971 to 1979 (based on U.S. Department of Energy reports; as are data for Figures 6.2–6.6). The number of customers affected per bulk power outage per year is shown in Figure 6.3. The generic causes of bulk power outages can be classified as weather, electric system components, electric system operation, and miscellaneous factors. Figure 6.4 shows bulk power outages by generic cause by year for the years from 1970 to 1979. The generic cause of bulk power outages for each utility subsystem, that is, generation, transmission, and distribution, is shown in Figure 6.5, whereas Figure 6.6 shows bulk power outages by utility subsystem. Note that the transmission system is involved in 60–80 percent of all bulk power outages.

6.3 INDEX OF RELIABILITY

The index of reliability is a convenient performance measure that has been used in the past to provide an indication of positive system performance. It is defined as the ratio of the total customer hours per year minus the total customer hours interrupted per year to total customer hours per year. Therefore,

$$\text{Index of reliability} = \frac{(\text{total customer-hours per year}) - \text{total customer-hours interrupted per year}}{(\text{total customer-hours per year})} \tag{6.1}$$

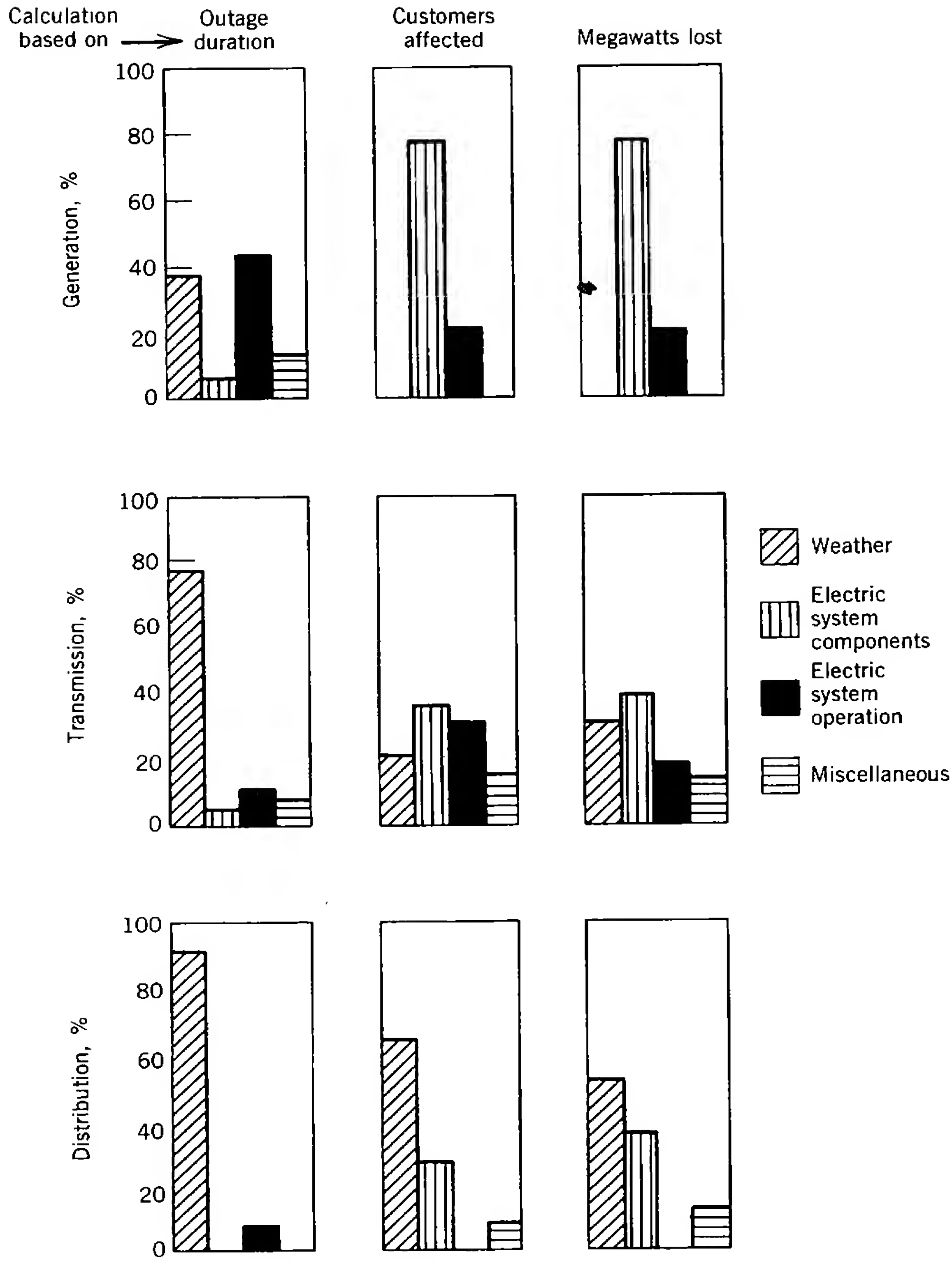


Figure 6.5. Bulk power outages by subsystem by cause [3].

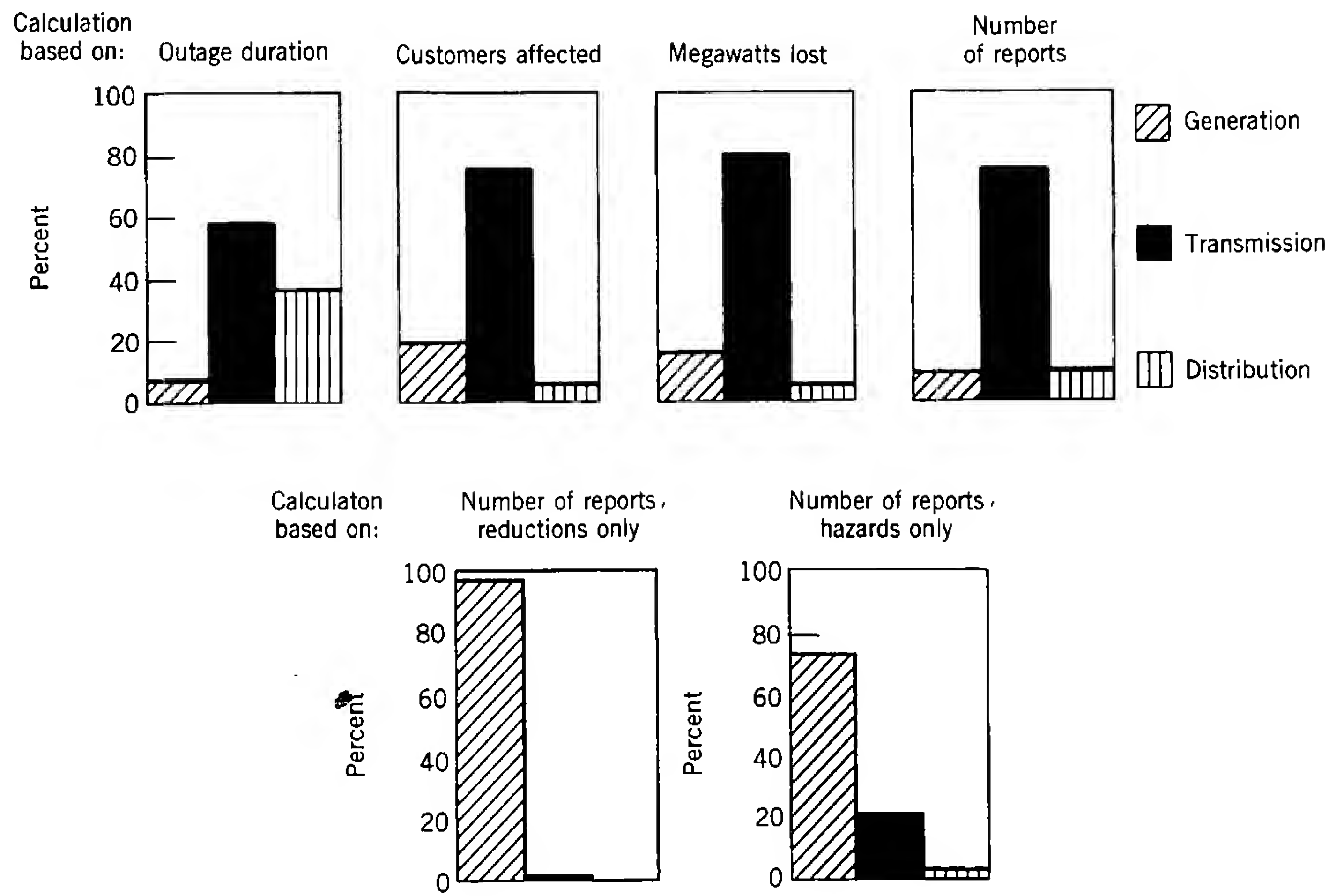


Figure 6.6. Bulk power outages by generic subsystem [3].

TABLE 6.1 Service Reliability in United States

Year	Index of Reliability
1956	99.8801
1957	99.9852
1958	99.9824
1959	99.9849
1960	99.9812
1961	99.9848
1962	99.9858
1963	99.9860
1964	99.9829
1965	99.9754
1966	99.9883
1967	99.9845
1974	99.9984[a]
1975	99.9987[a]
1976	99.9980[a]
1977	99.9968[a]
1978	99.9983[a]

[a] Includes only bulk power system outages.

Table 6.1 gives the index of reliability for all reporting U.S. power utility industries for the years from 1956 to 1967 and from 1974 to 1978. The data for the years from 1956 to 1967 have been taken from a 1968 Edison Electric Institute (EEI) Survey on service reliability. The data include distribution system interruptions as well as bulk power system interruptions. Whereas the data for the years from 1974 to 1978 have been taken from the National Electric Reliability Study [3] and include only bulk power system interruptions. In either case, the reliability performance of the bulk power systems appears consistently high in the United States.

It is usually understood that the bulk power system includes basic generation plants and the transmission system that permits transportation of power from these plants to primary load centers.

6.4 SECTION 209 OF PURPA OF 1978

Figure 6.7 shows Section 209 of the Public Utility Regulatory Policies Act (PURPA) of 1978. The legislation requires study of numerous reliability-related questions, for example, appropriate levels of reliability, procedures to minimize public disruption during an outage, appropriate generation, transmission and distribution mix, or appropriate electric utility reliability standards. The basic objectives of the legislation have been summarized in the 1981 National Electric Reliability Study [3] as:

SEC. 209. RELIABILITY. 16 USC 824a-z.

(a) STUDY.—(1) The Secretary, in consultation with the Commission, shall conduct a study with respect to—

(A) the level of reliability appropriate to adequately serve the needs of electric consumers, taking into account cost effectiveness and the need for energy conservation,

(B) the various methods which could be used in order to achieve such level of reliability and the cost effectiveness of such methods, and

(C) the various procedures that might be used in case of an emergency outage to minimize the public disruption and economic loss that might be caused by such an outage and the cost effectiveness of such procedures.

Such study shall be completed and submitted to the President and the Congress not later than 18 months after the date of the enactment of this Act. Before such submittal the Secretary shall provide an opportunity for public comment on the results of such study. Report to President and Congress.

(2) The study under paragraph (1) shall include consideration of the following:

(A) the cost effectiveness of investments in each of the components involved in providing adequate and reliable electric service, including generation, transmission, and distribution facilities, and devices available to the electric consumer;

(B) the environmental and other effects of the investments considered under subparagraph (A);

(C) various types of electric utility systems in terms of generation, transmission, distribution and customer mix, the extent to which differences in reliability levels may be desirable, and the cost-effectiveness of the various methods which could be used to decrease the number and severity of any outages among the various types of systems;

(D) alternatives to adding new generation facilities to achieve such desired levels of reliability (including conservation);

(E) the cost-effectiveness of adding a number of small, decentralized conventional and nonconventional generating units rather than a small number of large generating units with a similar total megawatt capacity for achieving the desired level of reliability; and

(F) any standards for electric utility reliability used by, or suggested for use by, the electric utility industry in terms of cost-effectiveness in achieving the desired level of reliability, including equipment standards, standards for operating procedures and training of personnel, and standards relating the number and severity of outages to periods of time.

(b) EXAMINATION OF RELIABILITY ISSUES BY RELIABILITY COUNCILS.—The Secretary, in consultation with the Commission, may, from time to time, request the reliability councils established under section 202(a) of the Federal Power Act or other appropriate persons (including Federal agencies) to examine and report to him concerning any electric utility reliability issue. The Secretary shall report to the Congress (in its annual report or in the report required under subsection (a) if appropriate) the results of any examination under the preceding sentence. 16 USC 824a. Report to Congress.

(c) DEPARTMENT OF ENERGY RECOMMENDATIONS.—The Secretary, in consultation with the Commission, and after opportunity for public comment, may recommend industry standards for reliability to the electric utility industry, including standards with respect to equipment, operating procedures and training of personnel, and standards relating to the level or levels of reliability appropriate to adequately and reliably serve the needs of electric consumers. The Secretary shall include in his annual report—

(1) any recommendations made under this subsection or any recommendations respecting electric utility reliability problems under any other provision of law, and

(2) a description of actions taken by electric utilities with respect to such recommendations.

<u>EXHIBIT 1</u>

Figure 6.7

1. Providing answers to the following three issues:
   (a) the level of reliability appropriate to serve adequately the needs of electric consumers, taking into account cost-effectiveness and the need for energy conservation;
   (b) the various methods that could be used in order to achieve such a level of reliability and the cost-effectiveness of such methods; and

   (c) the various procedures that might be used in case of an emergency outage to minimize the public disruption and economic loss that might result from such an outage and the cost of such procedures.
2. Recommending industry and/or government goals or policies needed to maintain or improve reliability if necessary.

The objectives of Section 209 are consistent with the overall objectives stipulated in the National Energy Act to encourage conservation of energy supplied by electric utilities, optimize the efficient use of facilities and resources by electric utilities, and provide equitable rates to consumers of electric power.

However, in general, there is no one level of reliability that is suitable for all utilities. Furthermore, even the level of reliability that is suitable for a single utility can change over a period of time. The National Electric Reliability Study [4, 5] points out:

1. Roughly seventy-five percent of all reported transmission outages causing customer interruptions resulting from problems related to components, maintenance, or operation and coordination, with the remaining twenty-five percent arising from events outside the control of the utility.
2. Roughly twenty percent of the interruptions which could have been avoided were caused by inadequate transmission system operation. High percentage of failures related to operations indicated that there are definite possibilities for improved transmission reliability performance with existing facilities by improving the adequacy of supporting systems such as communications and control.
3. Most major interruptions initiated by inadequate transmission operation are the results of protection and relaying problems which are not adequately addressed by current reliability evaluation techniques.
4. The level of transmission utility may be inadequate from a regional perspective.
5. The size and complexity of the transmission system makes it very difficult to calculate meaningful transmission reliability measures.

6.5 BASIC PROBABILITY THEORY

There are various reliability indices that can be employed in measuring the reliability of a given system and/or comparing the reliabilities of various possible system designs. These reliability indices are defined in Section 6.4. The reliability indices may involve values that are probabilistic in nature (e.g., probability of an event occurring or not occurring), mean time between failures, etc. Thus, these values are random variables that may change randomly in time.

The probability theory can be defined as the theory based on an equally likely set of events or as relative frequencies. The relative frequency of the occurrence of an event in a large number of repetitions of situations where the event may occur is defined as the probability of the event. A set of outcomes is called an event. An event is said to occur if any one of its outcomes occur. A series of events is said to be random if one event has no predictable effect on the next. The probability of an event E_i is a number between 0 and 1;

$$0 \leq P(E_i) \leq 1 \qquad \forall_i \tag{6.2}$$

where $\forall_i$ means "for all i." If the event cannot occur, its probability is 0. On the other hand, if it must occur (its occurrence is certain), its probability is 1. Other wise, its probability is somewhere in between 0 and 1.

Assume that a chance experiment (an equation whose outcome cannot be predicted in advance) is to be performed and that there may be various possible outcomes that can occur when the experiment is performed. If an event A occurs with m of these outcomes, then the probability of the event A occurring is

$$p = P(A) = m/n \tag{6.3}$$

where p = probability of event A (also called its success), $= P(A)$
n = total number of outcomes possible

For example, if the event occurs, on the average, in 4 out of every 10 trials, then the probability of its occurrence is 0.4. Simlarly, the probability of nonoccurrence of the event is

$$q = P(\text{not } A) = P(\bar{A}) = \frac{n-m}{n} = 1 - \frac{m}{n} = 1 - p = 1 - P(A) \tag{6.4}$$

where q = probability of nonoccurrence of event (also called its failure), $= p(\bar{A})$

For example, the probability of nonoccurrence of the event in the previous example is 0.6. Therefore, the probability of an event A is equal to the sum of the probabilities of the sample points in A. Note that

$$p + q = 1 \tag{6.5}$$

or

$$P(A) + P(\bar{A}) = 1 \tag{6.6}$$

or, in a general expression,

$$\sum_{S} P(E_i) = 1 \tag{6.7}$$

where S = sample space
$P(E_i)$ = probability of event E_i

6.5.1 Set Theory

The discussion of probability is greatly facilitated if it can be presented in the terms of set theory. Therefore, some very elementary definitions and operations of set theory will be presented in this section. A *set* is a well-defined collection of distinct elements. An *element* of a set is any one of its members. A set may have a finite or infinite number of elements or no elements at all. If $x_1, x_2, x_3, \ldots, x_n$ are elements of set A, it is denoted as

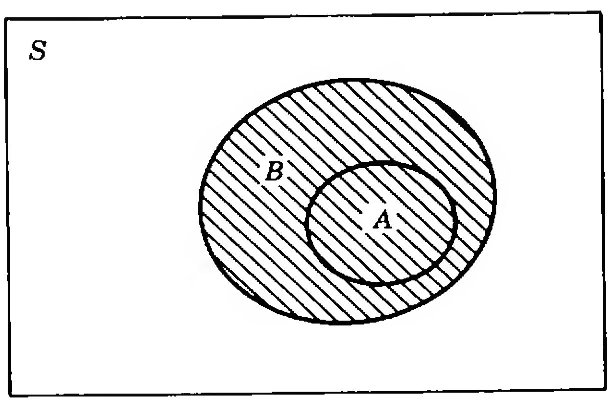
$$A = \{x_1, x_2, x_3, \ldots, x_n\}$$

If set A is a set and x is an element of A, the set membership is indicated as $x \in A$ and is read as "x belongs to A." On the other hand, if x is not an element of A, this fact is indicated as $x \notin A$.

If $x \in A$ implies $x \in B$, then it can be said that A is a *subset* of B (A is contained in or equal to B), and this fact is indicated by writing $A \subseteq B$ which is read as "A is a subset of B." In other words, $A \subseteq B$ if every element of A is also an element of B. An equivalent notation, $B \supseteq A$, is read as "B contains or equals A."

The relationships that exist among sets can be more easily defined if a Venn diagram, that is, a Euler diagram, is utilized. The Venn diagram displays the entire sample space S as a rectangular area. Here, the sample space defines the entirety of the set of elements under consideration. [It is also called the *universal set* (or universe) and is denoted by U.] For example, Figure 6.8 shows a Venn diagram that illustrates the set–subset relationship. Note that $B \subseteq S$, $A \subseteq S$, and $A \subseteq B$.

If every element in A is also an element of B and if every element in B is also an element of A, then A equals B (i.e., if $A \subseteq B$ and $B \subseteq A$, then $A = B$). Alternatively, $A = B$ if $A \subseteq B$ and $A \supseteq B$. If A is a subset of B and $A \neq B$, then A is a *proper* subset of B. If A is any set, then A is a subset itself. A set that contains no elements is said to be *empty* and is called the *null set*. If A is any set, then $\emptyset \subseteq A$.



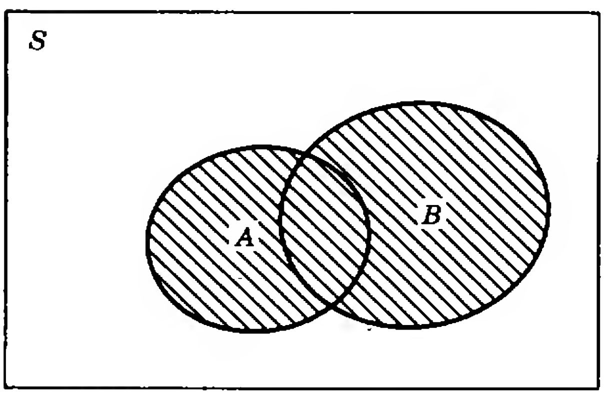
$A \subseteq B$

Figure 6.8. Set–subset relationship.

It is possible to form new sets from given sets. For instance, given two sets A and B, the *union* of A and B is the set of all elements that (1) are in A or (2) are in B or (3) are in both A and B. The word union is symbolized by $\cup$ so that A union B (the union of A and B) is written $A \cup B$. Figure 6.9 illustrates the concept of union in which the shaded area represents the union of A and B, symbolized by $A \cup B$ or, in some applications, by $A + B$. Here, the crucial word to remember about the concept of union is *or*. The criterion for including any element in the union of A and B is whether that element is contained in A *or* B *or* both A and B. Therefore, mathematically, $A \cup B$ is the set $\{x \in A \text{ or } x \in B\}$. For example, if A includes $\{1, 2, 3, 4, 5\}$ and B includes $\{3, 4, 5, 6, 7, 8\}$, then $A \cup B = \{1, 2, 3, 4, 5, 6, 7, 8\}$. Note that numbers contained in both A and B are not represented twice in $A \cup B$, that is, $A \cup B$ is not equal to $\{1, 2, 3, 4, 5, 3, 4, 5, 6, 7, 8\}$. Of course, there can be more than two sets in a given sample space, for example, A, B, and C, whose union can be expressed as $A \cup B \cup C$.

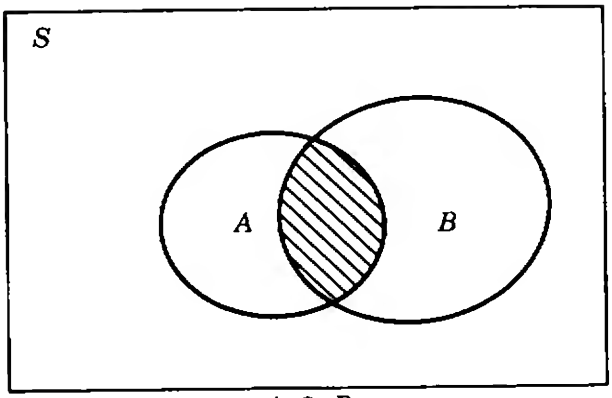
Given two sets A and B, the *intersection* of A and B contains all elements that are in both A and B but not in A or B alone. The symbol for intersection is $\cap$, and A intersection B is written $A \cap B$. Therefore, mathematically speaking, $A \cap B$ is the set $\{x \in S | x \in A \text{ and } x \in B\}$. Here, $A \cap B$ is read "A intersect B." Figure 6.10(a) illustrates the intersection of A and B in which $A \cap B$ is the shaded portion of the diagram. For example, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{3, 4, 5, 6, 7, 8\}$, then $A \cap B = \{3, 4, 5\}$. Here, the emphasis is on the word *and* because in order for an element to be a member of the intersection of A and B ($A \cap B$), it must be contained in both A *and* B. The intersection represents the *common* portion of two sets or the elements shared by two sets. Naturally, there can be more than two sets in a given sample space, for example, three sets, which would be expressed as $A \cap B \cap C$.

Assume that $A \cap B$ contains no elements (i.e., $A \cap B = \emptyset$); then A and B share no common elements, that is, no element in A is also in B and no element in B is also in A. If $A \cap B = \emptyset$, then A and B are called *disjoint sets*,



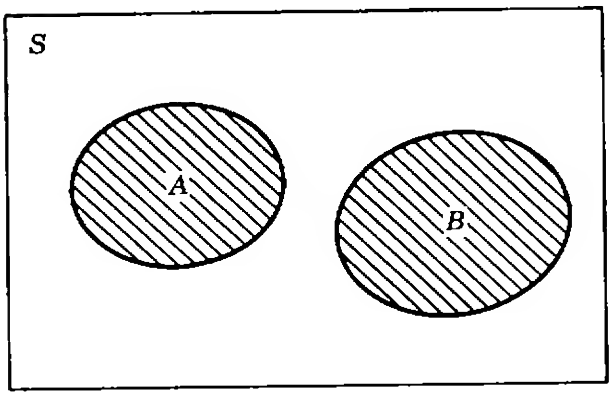
$A \cup B$

Figure 6.9. Union, $A \cup B$.



$A \cap B$

(a)



$A \cap B = \phi$

(b)

Figure 6.10. (*a*) Intersection, $A \cap B$. (*b*) Two disjoint sets, $A \cap B = \emptyset$.

or mutually exclusive sets, due to the fact that there is no common element to "join" them together. Figure 6.10(*b*) illustrates this concept. For instance, if $A = \{1, 2, 3, 4, 5\}$ and $B = \{6, 7, 8, 9, 10\}$, then $A \cap B = \emptyset$ or $A \cap B = \{\ \}$, where $\emptyset$ is said to be empty and is called the null set.

Another example of disjoint sets is A and $\bar{A}$. Here, the set $\bar{A}$ contains all elements that are not in the set A and is called the *complement* of A. Therefore, $\bar{A} = S - \bar{A}$ or $A \cup \bar{A} = S$, and $A \cap \bar{A} = 0$. Of course, for any set A, A and $\bar{A}$ are disjoint. Thus, mathematically speaking, the *complement* of A is the set $\{x \in S | x \not\subseteq A\}$. For example, if $S = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$ and $A = \{1, 2, 3, 4, 5\}$, then $\bar{A} = \{6, 7, 8, 9, 10\}$. Figure 6.11 illustrates the concept of complement.

Figure 6.12 shows the *difference* set $A - B$, which includes only those elements of A that are not also in B. It is crucial to be aware of the fact that $(A - B) + B = A + B$, not just A.†

† Because of this misleading result, the notation + should be avoided as much as possible.

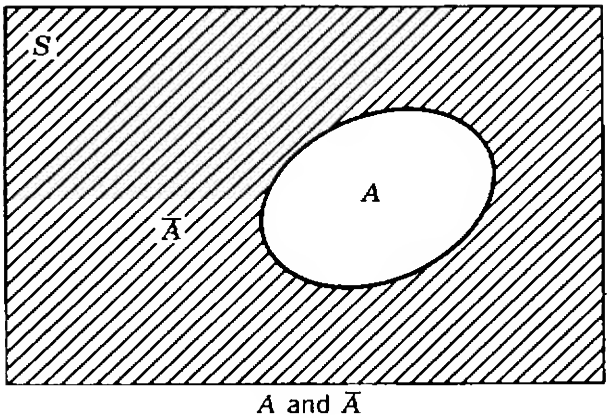


Figure 6.11. Complement set, $\bar{A}$.

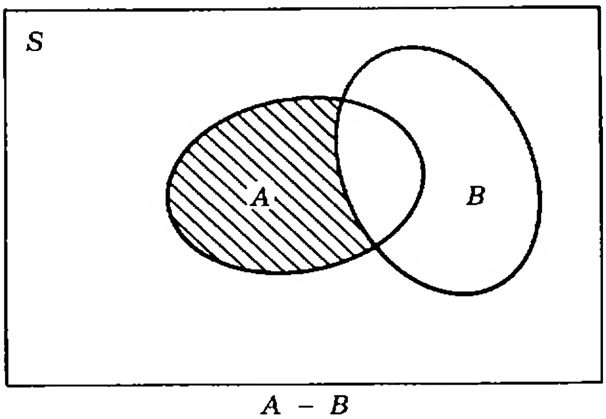


Figure 6.12. Difference, $A - B$.

6.5.2 Probability and Set Theory

It is possible to explain the probability theory in terms of set theory. In probability theory, any operation whose outcome cannot be predicted with certainty is called an *experiment*. The experiments that appear most often in examples to describe the concept of probability are flipping coins, rolling dice, selecting balls from an urn, and dealing cards from a deck. The set of all possible outcomes or results of an experiment is called the *sample* space and is denoted by S. For example, if a single die is rolled one time, then the experiment is the roll of the die. A sample space for this experiment could be

$$S = \{1, 2, 3, 4, 5, 6\}$$

where each of the integers 1–6 is meant to represent the face having that many spots being uppermost when the die stops rolling.

Each individual possible outcome is represented in the sample space by one *sample point* and is denoted by A. Therefore, the totality of sample points is the sample space. An *event* is a subset of the sample space. As event occurs if any one of its elements is the outcome of the experiment.

The sample space used in the aforementioned example was

$$S = \{1, 2, 3, 4, 5, 6\}$$

Therefore, each of the sets

$$A = \{1\}$$
$$B = \{1, 3, 5\}$$
$$C = \{2, 4, 6\}$$
$$D = \{4, 5, 6\}$$
$$E = \{1, 3, 4, 6\}$$

is an event (these are not not the only events since they are not the only subsets of S). Of course, these are all different events since no two of these subsets are equal. If an actual experiment of rolling the die is performed and the resulting outcome is 4, then events C, D, and E are said to have occurred since each of these has 4 as an element. Events A and B did not occur since $4 \not\in A$ and $4 \not\in B$.

The theory of probability is concerned with consistent ways of assigning numbers of events (subsets of the sample space S), which are called the *probabilities of occurrence* of these events. Alternatively, assume that a weight w_i is assigned to each point p_i of the sample space in a way such that

$$w_i \geq 0 \qquad \forall_i$$

and

$$w_1 + w_2 + w_3 + \cdots = \sum_i w_i = 1$$

Then the *probability that event A will occur* is the sum of the weights of the sample points that are in A and is denoted by $P(A)$. Succinctly put, a probability function is a real-valued set function defined on the class of all subsets of the sample space S; the value that is associated with a subset A is denoted by $P(A)$. The probability of an event is always nonnegative and can never exceed 1. The probabilities of the certain event and the impossible events are 1 and 0, respectively. The assignment of probabilities must satisfy the following three rules (which are called the axioms of probability†) so that the set function may be called a probability function:

$$P(S) = 1 \tag{6.8}$$

† Note that they are postulated and cannot be proved. However, the calculus of probability can be based on them.

$$P(A) \geq 0 \quad \text{for all } A \subset S \tag{6.9}$$

$$P(A_1 \cup A_2 \cup \cdots) = P(A_1) + P(A_2) + \cdots \quad \text{if } A_i \cap A_j = \emptyset \qquad \forall_{i \neq j} \tag{6.10}$$

Equation (6.9) can alternatively be represented as

$$0 \leq P(A) \leq 1 \tag{6.11}$$

which implies that the probability of an event is always nonnegative and can never exceed 1. Equation (6.10) implies that if A and B are *disjoint* (*mutually exclusive*) *events* in S, as shown in Figure 6.10, then

$$P(A + B) = P(A) + P(B) \tag{6.12}$$

where $A + B$ is the event A or B, that is, the occurrence of one of them excludes the occurrence of the other. Note that the union of A and B $(A \cup B)$ is defined to be the event containing all sample points in A or B or both. Therefore, it is usually expressed as A or B rather than $A + B$. Here, the events A and B may be said to be disjoint if they cannot both happen at the same time. The sample space represents a single trial of the experiment, and A and B are disjoint if they cannot both occur if one trial. For example, for rolling a pair of dice, the sample space consists of the 36 outcomes. If A is the event that the total is 4, then it includes three outcomes:

$$A = \{2-2, 1-3, 3-1\}$$

Similarly, if B is the event that the total is 3, it contains two outcomes:

$$B = \{1-2, 2-1\}$$

Then A or B is the event that the total is either 4 or 3:

$$A \text{ or } B = \{2-2, 1-3, 3-1, 1-2, 2-1\}$$

Therefore, it can be shown that

$$\begin{aligned} P(A \cup B) &= P(A + B) = P(A \text{ or } B) \\ &= P(A) + P(B) \\ &= \tfrac{3}{36} + \tfrac{2}{36} = \tfrac{5}{36} \end{aligned}$$

Note that the dice cannot total both 4 and 3 at once; but none of this stops them from totaling 4 on on trial and 3 on another trial.

In general, it can be expressed that the probability of a union, as shown in Figure 6.8, is equal to

$$P(A \cup B) = P(A) + P(B) - P(AB) \tag{6.13}$$

or

$$P(A \cup B) = P(A) + P(B) - P(A \cap B) \tag{6.14}$$

If A and B are mutually exclusive,

$$P(AB) = 0 \tag{6.15}$$

and therefore,

$$P(A \cup B) = P(A) + P(B) \tag{6.16}$$

In other words, two events A and B are said to be mutually exclusive if the event AB contains no sample points. Note that equation (6.13) can be reexpressed as

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B) \tag{6.17}$$

Equations (6.13) and (6.16) are referred to as the *additive law of probability*. For example, assume that a card is drawn at random from a deck and that A is the event of drawing a spade and B is the event of drawing a face card. Thus, "A and B" is the event that the card is both a spade and a face card. The number of outcomes in A is 13, the number in B is 12, and the number in "A and B" is 3. Therefore, the probability of drawing a spade or a face card is

$$P(A \text{ or } B) = \tfrac{13}{52} + \tfrac{12}{52} - \tfrac{3}{52} = \tfrac{22}{52}$$

The complement of an event A is the "opposite" event, the one that occurs exactly when A does not. Therefore, the complement of an event A is the collection of all sample points in S and not in A. The complement of A is denoted by the symbol $\bar{A}$. Since

$$\sum_{S} P(E_i) = 1 \tag{6.7}$$

then

$$P(A) + P(\bar{A}) \triangleq 1 \tag{6.6}$$

therefore,

$$P(A) = 1 - P(\bar{A}) \tag{6.18}$$

or

$$P(\bar{A}) = 1 - P(A) \tag{6.19}$$

The *conditional probability* of an event B given another event A is denoted by $P(B|A)$ and is defined by

$$P(B|A) = \frac{P(A \cap B)}{P(A)} \qquad \text{if } P(A) \neq 0 \tag{6.20}$$

or

$$P(B|A) = \frac{P(A \text{ and } B)}{P(A)} \qquad \text{if } P(A) \neq 0 \tag{6.21}$$

or

$$P(B|A) = \frac{P(AB)}{P(A)} \qquad \text{if } P(A) \neq 0 \tag{6.22}$$

Likewise,

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \qquad \text{if } P(B) \neq 0 \tag{6.23}$$

is called the conditionl probability of event A given event B, where $A \cap B$ represents the event consisting of all points in the sample space S common to both A and B. In equation (6.20), should one of the individual probabilities be zero, the corresponding conditional probability is undefined, of course. The vertical bar in the parenthesis of $(B|A)$ is read "given," and the events appearing to the right of the line are the events that are known to have occurred. Note that multiplying both sides of equation (6.20) by $P(A)$ gives

$$P(A \cap B) = P(A)P(B|A) \tag{6.24}$$

or

$$P(A \text{ and } B) = P(A)P(B|A) \tag{6.25}$$

For example, assume that two cards are drawn at random without replacement from a bridge deck and that A is the event of the first card being red and B is the event of the second card being red. Then the probability of both cards being red is

$$\begin{aligned} P(\text{both cards red}) &= P(A)P(B|A) \\ &= \tfrac{1}{2} \times \tfrac{25}{51} = 0.2451 \end{aligned}$$

Two events A and B are said to be *independent* if either

$$P(A|B) = P(A) \tag{6.26}$$

or

$$P(B|A) = P(B) \tag{6.27}$$

Otherwise, the events are said to be *dependent*. If the events A and B are independent, then

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = P(B) \tag{6.28}$$

Note that the probability of the intersection AB is

$$\begin{aligned} P(A \cap B) &= P(AB) \\ &= P(A)P(B|A) && (6.29) \\ &= P(B)P(A|B) && (6.30) \end{aligned}$$

If A and B are independent,

$$\begin{aligned} P(A \cap B) &= P(AB) \\ &= P(A)P(B) && (6.31) \end{aligned}$$

Equations (6.29)–(6.31) are referred to as the *multiplicative law of probability*. An example for the independent events is the rolling of two dice where the outcome of rolling one die is independent of the roll of the second one. For instance, assume that a fair die is tossed twice; since the two tosses are made independently of each other, the probability of getting a pair of 4s is

$$\begin{aligned} P(\text{pair of 4s}) &= P(\text{four spots on first toss})P(\text{four spots on second toss}) \\ &= \tfrac{1}{6} \times \tfrac{1}{6} = \tfrac{1}{36} \end{aligned}$$

Assume that B is an event and $\bar{B}$ is its complement. If A is another event which occurs if and only if B or $\bar{B}$ occur, then the probability is

$$P(B|A) = \frac{P(B)P(A|B)}{P(B)P(A|B) + P(\bar{B})P(A|\bar{B})} \tag{6.32}$$

This formula constitutes *Bayes's rule* or *Bayes's law* or *Bayes's theorem*. Given the prior probability $P(B)$ [and $P(\bar{B}) = 1 - P(B)$] and the respective probabilities $P(A|B)$ and $P(\bar{A}|B)$, one can use Bayes's law to calculate the *posterior* (after-the-fact) probability $P(B|A)$. In a sense, Bayes's law is updating or revising the prior probability $P(B)$ by incorporating the observed information contained within event A into the model.

6.6 COMBINATIONAL ANALYSIS

The probability problems often require the enumeration of the possible ways that events can occur. The combinational analysis methods make this cumbersome operation easier. If one thing can occur in n different ways and another thing can occur in m different ways, then both things can occur together or in succession in $m \times n$ different ways. For instance, if a couple of dice are rolled simultaneously, since one of them rolls in $n = 6$ ways and the other one rolls in $m = 6$ ways, then together they all roll in $n \times m = 36$ ways.

If n is a positive integer, then n factorial, denoted by $n!$, is defined as

$$n! = 1 \times 2 \times 3 \times \cdots \times (n-1) \times n \tag{6.33}$$

For instance,

$$5! = 1 \times 2 \times 3 \times 4 \times 5 = 120$$

where

$$0! \triangleq 1$$

The number of ways of selecting and arranging r objects taken from n distinct objects is called a *permutation of n things taken r at a time*, denoted by $P_{(n,r)}$, ${}_nP_r$, or $P_{n,r}$, and is defined as

$$\begin{aligned} {}_nP_r &\triangleq \frac{n!}{(n-r)!} \\ &= n(n-1)(n-2)\cdots(n-r+1) \end{aligned} \tag{6.34}$$

For example, the total number of possible permutations of the letters A, B, and C taken two at a time can be determined as

$$\begin{aligned} {}_3P_2 &= \frac{3!}{(3-2)!} \\ &= \frac{3!}{1!} = 6 \end{aligned}$$

which are AB, BA, AC, CA, BC, and CB.

The number of *combinations* of n distinct objects taken r at a time (the number of subsets of size r), denoted by $C_{(n,r)}$, ${}_nC_r$, $C_{n,r}$ or $\binom{n}{r}$, is defined as

$$\begin{aligned} {}_nC_r &\triangleq \frac{n!}{r!(n-r)!} \\ &= \frac{{}_nP_r}{r!} \\ &= \frac{n(n-1)\cdots(n-r+1)}{r!} \end{aligned} \tag{6.35}$$

For instance, the number of combinations of 5-card hands that can be dealt from a 52-card deck can be determined as

$$
\begin{aligned}
{}_{52}C_5 &= \binom{52}{5} \\
&= \frac{52!}{5!(52-5)!} = 2{,}598{,}960
\end{aligned}
$$

6.7 PROBABILITY DISTRIBUTIONS†

If a given set has the values of $x_1, x_2, \ldots, x_n$ and we have the probabilities $P(x_1), P(x_2), \ldots, P(x_n)$ that $x_1, x_2, \ldots, x_n$ will occur, then this group of individual probabilities is referred to as a *density distribution*. By cumulating these individual probabilities of a discrete set of values $x_1, x_2, \ldots, x_n$, a *cumulative probability distribution* can be obtained. Because the variable x can assume certain values with given probabilities, it is often called a *discrete random variable* (or sometimes a *stochastic variable*). Random variables plan an extremely important role in probability theory. Therefore, since random variables can be discrete or continuous, the resultant probability distributions can either be discrete or continuous. The following are some examples of discrete and continuous probability distributions.

The *binomial distribution* is a discrete distribution. It is also called the *Bernoulli distribution*. For example, if p is the probability that an event will occur (sometimes it is identified as the probability of *success*) in any single trial and $q = 1 - p$ is the probability that it will fail to occur (probability of *failure*), then the probability that the event will occur exactly x times in n trials is given by the expression

$$
\begin{aligned}
P(x = k) &= {}_nC_k p^k q^{n-k} \\
&= \frac{n!}{k!(n-k)!} p^k q^{n-k} \qquad (6.36)
\end{aligned}
$$

Note that this distribution is a function of the two parameters, p and n. The probability distribution of this random variable x is shown in Figure 6.13. An interesting interpretation of the binomial distribution is obtained when $n = 1$, that is,

$$P(x = 0) = 1 - P \quad \text{and} \quad P(x = 1) = P$$

Such a random variable is said to have a Bernoulli distribution. Thus, if a random variable takes on two values, say, 0 or 1, with probability $1 - p$ or

† This section is based on Gönen [6]. Included with permission of McGraw-Hill Book Company.

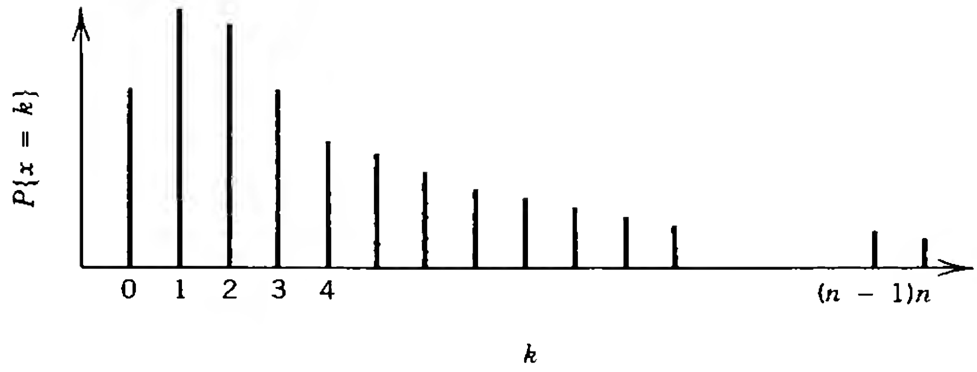


Figure 6.13. Binomial probability distribution with fixed n and p.

p, respectively, it is defined as a Bernoulli random variable. The upturned face of a flipped coin is such an example. A binomial distribution has a mean value of $\mu = np$, a variance value of

$$\sigma^2 = npq \tag{6.37}$$

and a standard deviation of

$$\sigma = (npq)^{1/2} \tag{6.38}$$

The Poisson *distribution* is a discrete probability distribution, and it is a special case of the binomial distribution where n is large and p is small but the mean $\lambda = \mu = np$ is of moderate magnitude. Thus, a random variable x is said to have a Poisson distribution if its probability distribution can be written as

$$P(x = k) = \frac{\lambda^k e^{-\lambda}}{k!} \tag{6.39}$$

where $k = 0, 1, 2, \ldots$
$e = 2.71823$
$\lambda =$ positive constant

An example of the probability distribution of a Poisson random variable is shown in Figure 6.14. A Poisson distribution has a mean value of $\mu = \lambda$, a variance value of $\sigma^2 = \lambda$, and a standard deviation of $\sigma = \sqrt{\lambda}$.

One of the most important and used distributions is the normal distribution. It is a continuous probability distribution, and it is also called *normal curve* or *Gaussian distribution* and is given by

$$Y = \frac{1}{\sum \sqrt{2\pi}} e^{-1/2(x-\mu)^2/\sigma^2} \tag{6.40}$$

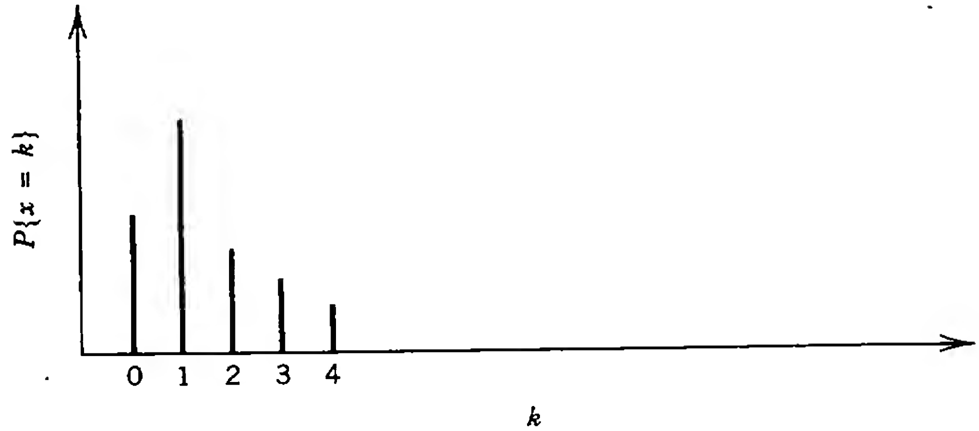


Figure 6.14. Poisson probability distribution.

where μ = mean
σ = standard deviation
$\pi = 3.14159$
Y = ordinate, that is, height of given curve corresponding to assigned value of x

A graph of a typical normal density function is given in Figure 6.15. here, the total area bounded by the curve and x axis is 1. Therefore, the area under the curve between the ordinates $x = a$ and $x = b$ represents the probability that x lies between a and b, denoted by $P(a < x < b)$. Also see Figure 6.16. From Figures 6.15 and 6.16, it is obvious that a normal distribution curve is a symmetric curve. Here, the parameter σ is a measure of the relative width and maximum height of the curve, and the shape of the curve, becomes higher and thinner as σ is reduced. Here σ can be any real positive number. If the variable X is expressed in terms of *standard units*,

$$Z = \frac{X - \mu}{\sigma} \tag{6.41}$$

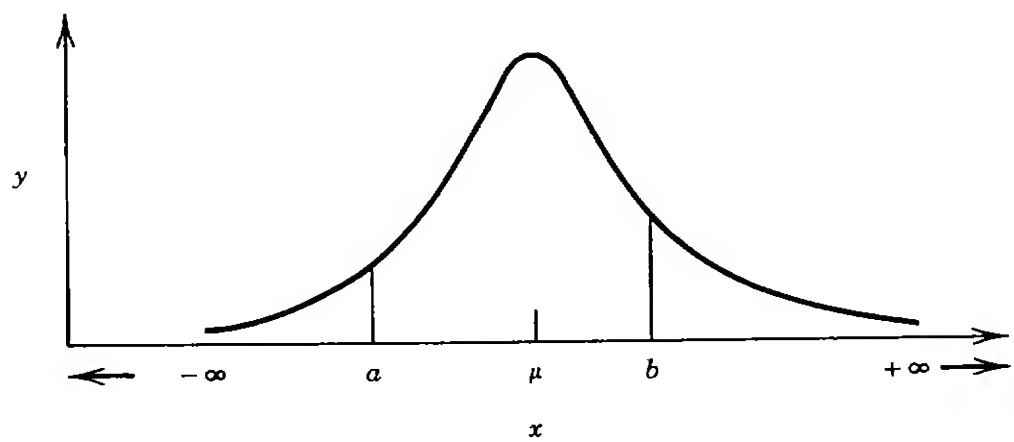


Figure 6.15. Normal probability distribution.

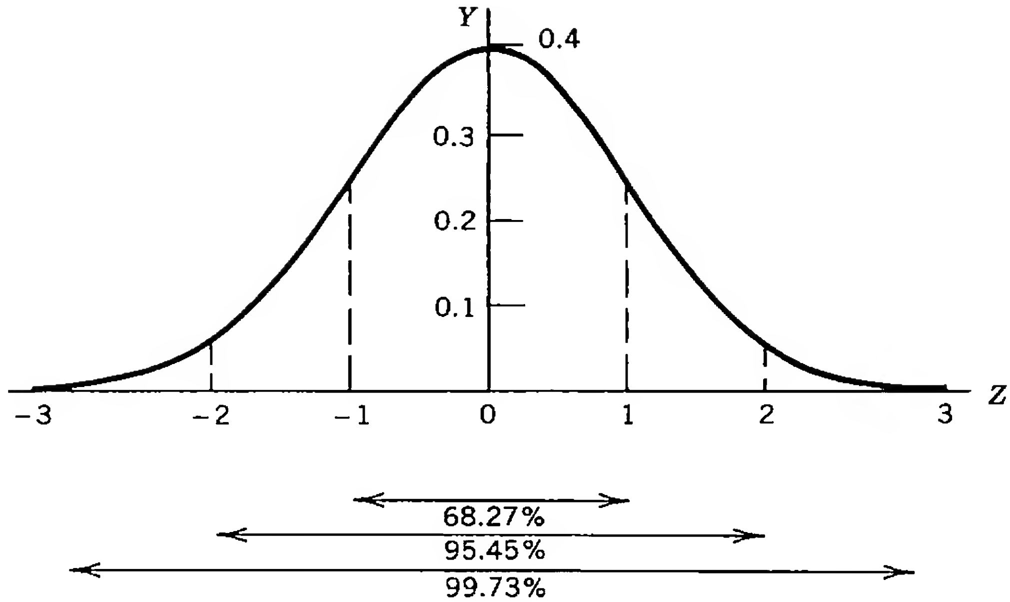


Figure 6.16. Standardized normal curve.

the distribution equation (6.40) becomes

$$Y = \frac{1}{\sqrt{2\pi}} e^{-1/2(Z^2)} \tag{6.42}$$

where Z is normally distributed with a mean of 0 and variance of 1. Figure 6.16 shows a typical standardized normal curve. It shows the areas included between $Z = -1$ and $Z = +1$, $Z = -2$, $Z = +2$, and $Z = -3$ and $Z = +3$, which are equal to 68.27, 95.45, and 99.73 percent, respectively, of the total area, which is 1. A normal distribution has a mean value of μ, a variance value of σ^2, and a standard value of σ.

The *exponential distribution* is also a continuous probability distribution. It is often used in physical reliability problems. For example, the reliability $R(t)$ is usually given as a function of time and gives the number of identical components in a system surviving at time t and divided by the original number of components,

$$R(t) = \frac{N_s(t)}{N_0} \tag{6.43}$$

where $N_S(t)$ = number of components surviving at time t
N_0 = number of original components at time t_0

It can be shown that if the failure rate is constant,

$$R(t) = e^{-\lambda t} \tag{6.44}$$

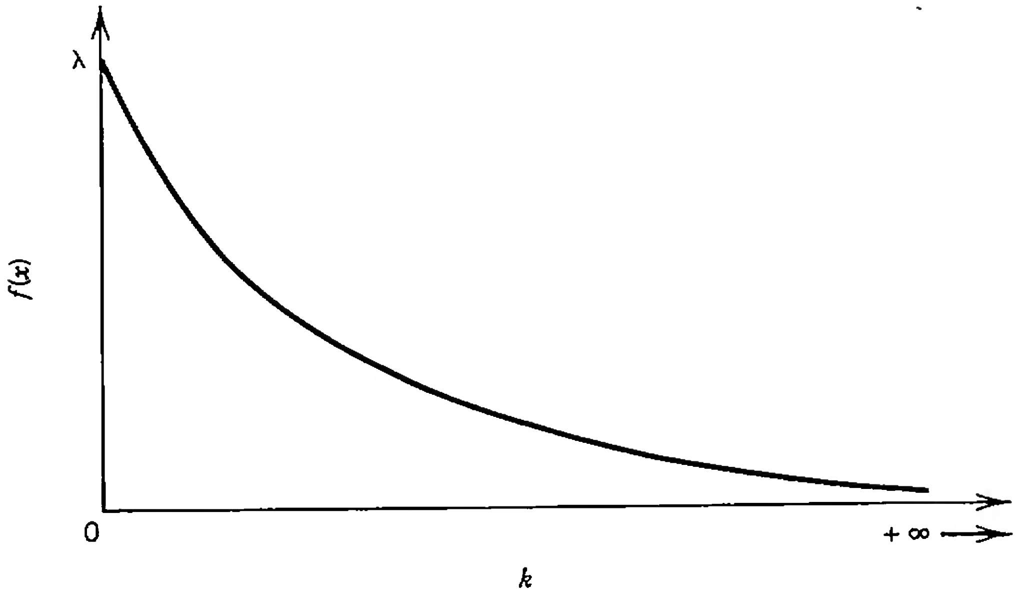


Figure 6.17. Exponential distribution.

Therefore, the exponential distribution is given by

$$f(t) = \lambda e^{-\lambda t} \tag{6.45}$$

The exponential distribution is a special case of the gamma distribution and is shown in Figure 6.17.

6.8 BASIC RELIABILITY CONCEPTS

The probability of failure of a given component (or system) can be expressed as a function of time as

$$P(T \leq t) = F(t) \qquad t \geq 0 \tag{6.46}$$

where T = random variable representing failure time
$F(t)$ = probability that component will fail by time t

Therefore, the failure distribution function, $F(t)$, is also defined as the *unreliability function*. Thus, the reliability function can be expressed as

$$\begin{aligned} R(t) &= 1 - F(t) \\ &= P(T > t) \end{aligned} \tag{6.47}$$

Hence, the probability that the component will survive at time t is defined as the reliability function $R(t)$. Note that

$$
\begin{aligned}
R(t) &= 1 - F(t) \\
&= 1 - \int_0^t f(t)\, dt \\
&= \int_0^\infty f(t)\, dt
\end{aligned}
\tag{6.48}
$$

where

$$F(t) = \int_0^t f(t)\, dt \tag{6.49}$$

provided that the time to failure, random variable T, has a density function $f(t)$. Therefore, it is possible to express the probability of failure of a given system in a specific time interval (t_1, t_2) in terms of either the unreliability function as

$$
\begin{aligned}
\int_{t_1}^{t_2} f(t)\, dt &= \int_{-\infty}^{t_2} f(t)\, dt - \int_{-\infty}^{t_1} f(t)\, dt \\
&= F(T_2) - F(T_1)
\end{aligned}
\tag{6.50}
$$

or in terms of the reliability function as

$$
\begin{aligned}
\int_{t_1}^{t_2} f(t)\, dt &= \int_{t_1}^{\infty} f(t)\, dt - \int_{t_2}^{\infty} f(t)\, dt \\
&= R(t_1) - R(t_2)
\end{aligned}
\tag{6.51}
$$

The *hazard rate* or failure rate is defined as the rate at which failures occur in a given time interval (t_1, t_2). In other words, it is the probability that a failure per unit time occurs in the time interval provided that a failure has not occurred before the time t, that is, at the beginning of the time interval. Thus, the hazard rate can be expressed as

$$h(t) = \frac{R(t_1) - R(t_2)}{(t_2 - t_1)R(t_1)} \tag{6.52}$$

Alternatively, by redefining the time interval as

$$\Delta t = t_2 - t_1 \tag{6.53}$$

so that

$$t_1 = t \qquad t_2 = t + \Delta t$$

the hazard rate can be expressed as

$$h(t) = \lim_{\Delta t \to 0} \frac{P\{\text{component of age } t \text{ will fail in } \Delta t | \text{it has survived up to } t\}}{\Delta t}$$

or

$$\begin{aligned} h(t) &= \lim_{\Delta t \to 0} \frac{R(t) - R(t + \Delta t)}{\Delta t\, R(t)} \\ &= \frac{1}{R(t)} \left[-\frac{d}{dt} R(t) \right] \\ &= \frac{f(t)}{R(t)} \end{aligned} \tag{6.54}$$

where

$$\begin{aligned} f(t) &= \text{probability density function} \\ &= \frac{dR(t)}{dt} \end{aligned} \tag{6.55}$$

By substituting equation (6.48) into equation (6.54),

$$h(t) = \frac{f(t)}{1 - F(t)} \tag{6.56}$$

Thus,

$$h(t)\, dt = \frac{dF(t)}{1 - F(t)} \tag{6.57}$$

or

$$\int_0^t h(t)\, dt = -\ln[1 - F(t)]|_0^t$$

Therefore,

$$\ln \frac{1 - F(t)}{1 - F(0)} = -\int_0^t h(t)\, dt \tag{6.58}$$

or

$$1 - F(t) = \exp\left[-\int_0^t h(t)\, dt \right] \tag{6.59}$$

By substituting equation (6.59) into equation (6.56),

$$f(t) = h(t) \exp\left[-\int_0^t h(t)\, dt \right] \tag{6.60}$$

Furthermore, by substituting equation (6.48) into equation (6.59),

$$R(t) = \exp\left[-\int_0^t h(t)\, dt\right] \tag{6.61}$$

or

$$R(t) = \exp\left(-\int_0^t h(t)\, dt\right) \tag{6.62}$$

Let

$$\lambda(t) = h(t)$$

Therefore,

$$R(t) = \exp\left(-\int_0^t \lambda(t)\, dt\right) \tag{6.63}$$

which is called the general reliability function. Note that the failure $\lambda(t)$ is a transition rate associated with the number of transitions that a component makes between normal operating state and failure state. It is the rate at which failures happen and is a function of the number of failures in a given period of time during which failures can happen and the number of components exposed to failure. Therefore, it can be defined as

$$\lambda(t) = \frac{\text{number of failures per unit exposure time}}{\text{number of components exposed to failure}}$$

Alternatively, if the hazard rate can be assumed to be independent of time, that is,

$$h(t) = \lambda \text{ failures/unit time}$$

the failure density function can be expressed as

$$f(t) = \lambda e^{-\lambda t} \tag{6.64}$$

Thus, the reliability function can be expressed as

$$R(t) = e^{-\lambda t} \tag{6.65}$$

A typical hazard function can be represented as the bathtub curve, which can be segmented into three separate sections, as shown in Figure 6.18. The first segment represents the *break-in period* or *debugging period*, in which the failures occur due to design or manufacturing errors. The second

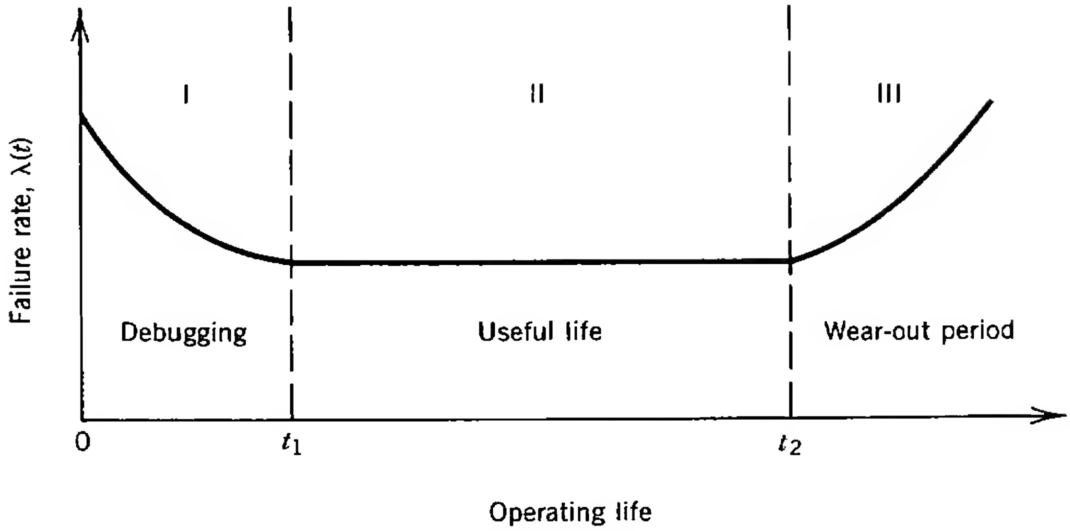


Figure 6.18. *Bathtub* hazard function.

segment represents the *useful life period* or *normal operating period*, in which the failure rates are relatively constant and are called *random failures*. The third represents the *wear-out period*, in which the failure rate increases due to the aging process of the component.

Further, it can be shown that

$$\int_0^t f(t)\,dt + \int_t^\infty f(t)\,dt = \int_0^\infty f(t)\,dt \triangleq 1 \tag{6.66}$$

from which

$$\int_0^t f(t)\,dt = 1 - \int_t^\infty f(t)\,dt \tag{6.67}$$

where

$$R(t) = \int_0^\infty f(t)\,dt$$

and

$$R(t) + Q(t) \triangleq 1 \tag{6.68}$$

Therefore, the unreliability can be defined as

$$\begin{aligned} Q(t) &= 1 - R(t) \\ &= 1 - \int_t^\infty f(t)\,dt \\ &= \int_0^t f(t)\,dt \end{aligned} \tag{6.69}$$

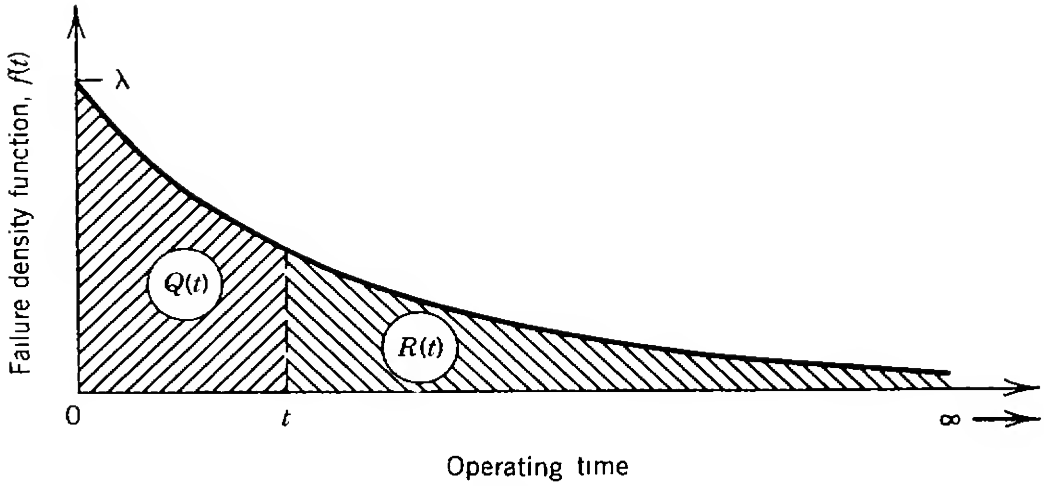


Figure 6.19. Relationship between reliability and unreliability.

The relationship between reliability and unreliability has been illustrated graphically in Figure 6.19.

The expected life of a component is the expected time during which the component will survive and perform successfully. It can be expressed as

$$E(T) = \int_0^\infty R(t)\, dt \tag{6.70}$$

or

$$E(T) = \int_0^\infty \left\{ \exp\left[-\int_0^t \lambda(t)\, dt \right] \right\} dt \tag{6.71}$$

or if the failure rate is constant,

$$E(T) = \int_0^\infty e^{-\lambda t}\, dt$$

$$= \frac{1}{\lambda} \tag{6.72}$$

If the component is not renewed through maintenance and repairs but simply replaced by a good component, the expected life can also be called the *mean time to failure* and is denoted as

$$\text{MTTF} = \bar{m} = \frac{1}{\lambda} \tag{6.73}$$

where λ = constant failure rate

Whereas if the component is renewed, through maintenance and repairs, the expected life can also be called the *mean time between failures* and is denoted as

$$\text{MTBF} = \bar{T} = \bar{m} + \bar{r} \tag{6.74}$$

where $\bar{T}$ = mean cycle time
$\bar{m}$ = mean time to failure
$\bar{r}$ = mean time to repair (MITR), $=1/\mu$
μ = mean repair rate

Assume that a system can be represented by a two state model so that the system is either in the up (or in) state or the down (or out) state at a given time. Thus, the mean time to failure of the system can be estimated as

$$\text{MTTF} = \bar{m} = \frac{\sum_{i=1}^{n} m_i}{n} \tag{6.75}$$

where m_i = observed time to failure for the ith cycle
n = total number of cycles

In the same way, the estimate for the mean time to repair can be expressed as

$$\text{MTTR} = \bar{r} = \frac{\sum_{i=1}^{n} r_i}{n} \tag{6.76}$$

where r_i = observed time to repair for ith cycle
n = total number of cycles

Thus, equation (6.74) can be reexpressed as

$$\text{MTBF} = \text{MTTF} + \text{MTTR} \tag{6.77}$$

Alternatively, equation (6.74) can be reexpressed as

$$\bar{T} = \frac{1}{\lambda} + \frac{1}{\mu} \tag{6.78}$$

or

$$\bar{T} = \frac{\lambda + \mu}{\lambda \mu} \tag{6.79}$$

Here, the average time that is necessary for the component to finish one cycle of operation (i.e., failure, repair, and restart) is called the *mean cycle time*. The reciprocal of the mean cycle time is called the *mean failure frequency* and is denoted

$$\bar{f} = \frac{1}{\bar{T}} \tag{6.80}$$

or

$$\bar{f} = \frac{\lambda\mu}{\lambda + \mu} \tag{6.81}$$

Alternatively, since in the two-state model the component is either "up" (available for service), or "down" (unavailable for service),

$$A + U = 1 \tag{6.82}$$

or

$$A + \bar{A} = 1 \tag{6.83}$$

where A = availability of component, that is, fraction of time component is up
U = unavailability of component, that is, fraction of time component is down, $= \bar{A}$

Thus, as time t goes to infinity, the availability can be expressed as

$$A \triangleq \frac{\bar{m}}{\bar{T}} \tag{6.84}$$

or

$$A = \frac{\text{MTTF}}{\text{MTBT}} \tag{6.85}$$

or substituting equation (6.77) into equation (6.85),

$$A = \frac{\text{MTTF}}{\text{MTTF} + \text{MTTR}} \tag{6.86}$$

or

$$A = \frac{\bar{m}}{\bar{m} + \bar{r}} \tag{6.87}$$

or

$$A = \frac{\mu}{\lambda + \mu} \tag{6.88}$$

Therefore, the unavailability can be expressed as

$$U \triangleq 1 - A \tag{6.89}$$

or

$$U = \frac{\bar{r}}{\bar{T}} \tag{6.90}$$

or

$$U = \frac{\bar{r}}{\bar{r} + \bar{m}} \tag{6.91}$$

or

$$U = \frac{\lambda}{\lambda + \mu} \tag{6.92}$$

6.8.1 Series Systems

The definition of a series system can be given as a set of components that must all operate for system success in terms of reliability or only one requires to fail for system failure. A block diagram for a series system that has two independent components connected in series is shown in Figure 6.20(*a*). Thus, in order to have the system and perform its designated function, both components must perform successfully. Thus, a series system is a nonredundant system. Therefore, the system reliability can be expressed as the probability of system success as

$$R_{sys} = P[E_1 \cap E_2] \tag{6.93}$$

or

$$R_{sys} = P(E_1)P(E_2) \tag{6.94}$$

assuming that the components are independent. Thus,

$$R_{sys} = \mathrm{R}_1 \times \mathrm{R}_2 \tag{6.95}$$

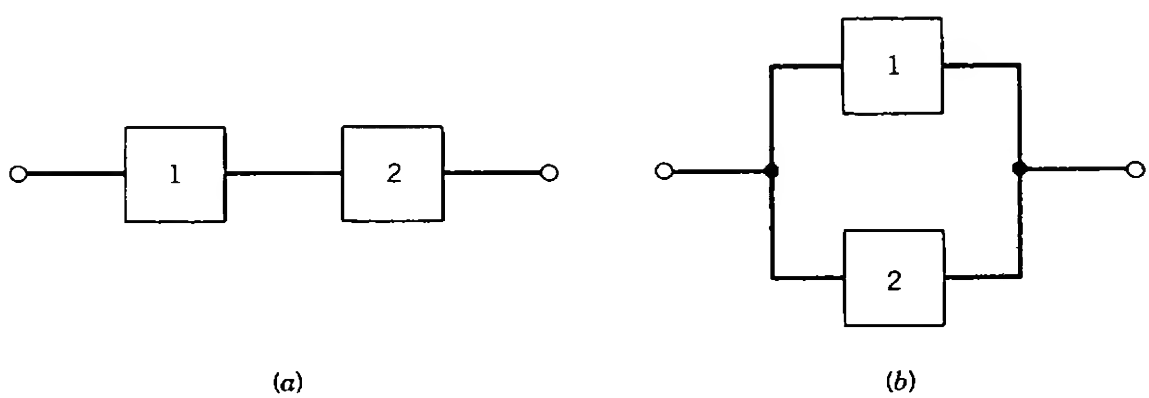


Figure 6.20. Block diagrams of system with two components: (*a*) connected in series; (*b*) connected in parallel.

or

$$R_{\text{sys}} = \prod_{i=1}^{2} R_i \tag{6.96}$$

where E_i = event that component i operates successfully
R_i = reliability of component i, $= P(E_i)$
R_{sys} = reliability of system

Therefore, the system reliability of a series system with n independent component can be expressed as

$$R_{\text{sys}} = P[E_1 \cap E_2 \cap \cdots \cap E_n] \tag{6.97}$$

or

$$R_{\text{sys}} = P(E_1)P(E_2) \cdots P(E_n) \tag{6.98}$$

or

$$R_{\text{sys}} = R_1 \times R_2 \times \cdots \times R_n \tag{6.99}$$

or

$$R_{\text{sys}} = \prod_{i=1}^{n} R_i \tag{6.100}$$

Thus, equation (6.100) is called the *product rule* or the *chain rule* of reliability. Note that the reliability of a series system will always be less than or equal to the least reliable component, that is,

$$R_{\text{sys}} \leq \min_i \{R_i\} \tag{6.101}$$

Therefore, the reliability of a series system decreases as the number of components increases due to the product rule. Therefore, it is the function of the number of series components and the component reliability level. Alternatively, the unreliability (or failure) of the series system can be expressed as

$$Q_{\text{sys}} = 1 - R_{\text{sys}} \tag{6.102}$$

or

$$Q_{\text{sys}} = 1 - \prod_{i=1}^{n} R_i \tag{6.103}$$

Note that the product rule that is given in equation (6.100) is applicable to both time-independent and time-dependent probabilities. In case of time-dependent probabilities, if the component reliability can be represented by a (negative) exponential distribution with a failure rate of λ_i, the system reliability can be expressed as

$$R_{\text{sys}}(t) = \prod_{i=1}^{n} \exp(-\lambda_i t) \tag{6.104}$$

or

$$R_{\text{sys}}(t) = \exp\left(-\sum_{i=1}^{n} \lambda_i t\right) \tag{6.105}$$

or

$$R_{\text{sys}}(t) = \exp(-\lambda_e t) \tag{6.106}$$

where

$$\begin{aligned} \lambda_e &= \text{equivalent failure rate of system} \\ &= \sum_{i=1}^{n} \lambda_i \end{aligned} \tag{6.107}$$

Alternatively, if the probability of a component failure is q and is the same for all n components of a given series system,

$$R_{\text{sys}} = (1-q)^n \tag{6.108}$$

or according to the binomial theorem,

$$R_{\text{sys}} = 1 + n(q)^1 + \frac{n(n-1)}{2}(-q)^2 + \cdots + (-q)^n \tag{6.109}$$

where q = probability of component failure
n = total number of components connected in series

If the probability of component failure (q) is small, the system reliability approximately is

$$R_{\text{sys}} \cong 1 - nq \tag{6.110}$$

whereas if the q's are different for each component, the system reliability approximately is

$$R_{\text{sys}} \cong 1 - \sum_{i=1}^{n} q_i \tag{6.111}$$

6.8.2 Parallel Systems

The definition of a parallel system can be given as a set of components for which only one is required to operate for system success in terms of reliability or all must fail for system failure. Thus, a parallel system is a fully redundant system. A block diagram for a parallel system that has two independent components connected in parallel is shown in Figure 6.20(*b*). Since both components must fail simultaneously to cause system reliability, the system unreliability can be expressed as

$$Q_{sys} = P[\bar{E}_1 \cap \bar{E}_2] \tag{6.112}$$

or

$$Q_{sys} = P(\bar{E}_1)P(\bar{E}_2) \tag{6.113}$$

assuming that the components are independent. Therefore,

$$Q_{sys} = Q_1 Q_2 \tag{6.114}$$

or

$$Q_{sys} = \prod_{i=1}^{2} Q_i \tag{6.115}$$

or

$$Q_{sys} = \prod_{i=1}^{2} (1 - R_i) \tag{6.116}$$

where $\bar{E}_i$ = event that component i fails
Q_i = unreliability of component i, $= P(\bar{E}_i)$
Q_{sys} = unreliability of system

Then, the system reliability can be expressed as

$$R_{sys} = 1 - Q_{sys} \tag{6.117}$$

or

$$R_{sys} = 1 - \prod_{i=1}^{2} (1 - R_i) \tag{6.118}$$

Therefore, the system reliability of a parallel system with n independent component can be expressed as

$$Q_{sys} = P[\bar{E}_1 \cap \bar{E}_2 \cap \cdots \cap \bar{E}_n] \tag{6.119}$$

or

$$Q_{sys} = P(\bar{E}_1)P(\bar{E}_2)\cdots P(\bar{E}_n) \tag{6.120}$$

or

$$Q_{sys} = Q_1 \times Q_2 \times \cdots \times Q_n = \prod_{i=1}^{n} Q_i \tag{6.121}$$

Thus, the system reliability can be expressed as

$$\begin{aligned} R_{sys} &= 1 - Q_{sys} \\ &= 1 - [Q_1 \times Q_2 \times \cdots \times Q_n] \\ &= 1 - [(1 - R_1)(1 - R_2)\cdots(1 - R_n)] \\ &= 1 - \prod_{i=1}^{n} Q_i \\ &= 1 - \prod_{i=1}^{n} (1 - R_i) \end{aligned} \tag{6.122}$$

Note that the unreliability of a partial system decreases as the number of parallel components increases. Alternatively, the reliability of a parallel system increases as the number of parallel components increases. Note that equation (6.122) is applicable to both time-independent and time-dependent probabilities. In the case of time-dependent probabilities, if the component unreliability can be represented by an exponential distribution with a failure rate of λ_i, the system unreliability can be expressed as

$$Q_{sys}(t) = \prod_{i=1}^{n} (1 - e^{-\lambda_i t}) \tag{6.123}$$

6.8.3 Combined Series–Parallel Systems

Simple combinations of series–parallel systems can be analyzed by using a reduction technique (similar to the network reduction technique). The reduction technique is simply sequential reduction of the given mixed configuration by combining proper series and parallel branches until a single equivalent element is left. For example, assume that a mixed series–parallel system has m parallel branches and that each branch involved has n components connected in series. Such a system may also be called a parallel–series system and has a high-level redundancy. The equivalent reliability of the system can be given as

$$R_{sys} = 1 - (1 - R^n)^m \tag{6.124}$$

where R_{sys} = equivalent reliability of system
R^n = equivalent reliability of branch
R = reliability of component
n = total number of components connected in series in branch
m = total number of paths

On the other hand, assume that a mixed series–parallel system has n series units (or banks) with m parallel components in each. Such a system may also be called a series–parallel system. The equivalent reliability of the system can be given as

$$R_{sys} = [1 - (1 - R)^m]^n \tag{6.125}$$

where $1 - (1 - R)^m$ = equivalent reliability of parallel unit (bank)
R = reliability of component
m = total number of components in parallel unit
n = total number of units

Note that the series–parallel configuration gives higher system reliability than the parallel–series configuration.

6.9 SYSTEMS WITH REPAIRABLE COMPONENTS†

The series and parallel systems presented in Section 6.8 are based on the assumption tht the components of the systems are not repairable. However, a more realistic approach would be to assume that the components are independent and repairable.

6.9.1 Repairable Components in Series

Consider a series system with two components, as shown in Figure 6.20(a), and assume that the components are independent and repairable. Thus, the availability or the steady-state probability of success (i.e., operation) of the system is

$$A_{sys} = A_1 A_2 \tag{6.126}$$

where A_{sys} = availability of system
A_1 = availability of component 1
A_2 = availability of component 2

† The technique presented in this section is primarily based on Billinton et al. [7].

Since,

$$A_1 = \frac{\bar{m}_1}{\bar{m}_1 + \bar{r}_1} \tag{6.127}$$

and

$$A_2 = \frac{\bar{m}_2}{\bar{m}_2 + \bar{r}_2} \tag{6.128}$$

the availability of the system can be expressed as

$$A_{\text{sys}} = \frac{\bar{m}_1}{\bar{m}_1 + \bar{r}_1} \frac{\bar{m}_2}{\bar{m}_2 + \bar{r}_2} \tag{6.129}$$

or

$$A_{\text{sys}} = \frac{\bar{m}_{\text{sys}}}{\bar{m}_{\text{sys}} + \bar{r}_{\text{sys}}} \tag{6.130}$$

where $\bar{m}_1$ = mean time to failure of component 1
$\bar{m}_2$ = mean time to failure of component 2
$\bar{m}_{\text{sys}}$ = mean time to failure of system
$\bar{r}_1$ = mean time to repair of component 1
$\bar{r}_2$ = mean time to repair of component 2
$\bar{r}_{\text{sys}}$ = mean time to repair of system

The average frequency of the system failure is the sum of the average frequency of component 1 failing, given that component 2 is operable, plus the average frequency of component 2 failing while component 1 is operable. Thus, the average frequency of the system failure is

$$\bar{f}_{\text{sys}} = A_2 \bar{f}_1 + A_1 \bar{f}_2 \tag{6.131}$$

where $\bar{f}_i$ = average frequency of failure of component i
A_i = availability of component i

However,

$$\bar{f}_i = \frac{1}{\bar{m}_i + \bar{r}_i} \tag{6.132}$$

and

$$A_i = \frac{\bar{m}_i}{\bar{m}_i + \bar{r}_i} \tag{6.133}$$

Therefore,

$$\bar{f}_{\text{sys}} = \frac{1}{\bar{m}_1 + \bar{r}_1} \frac{\bar{m}_2}{\bar{m}_2 + \bar{r}_2} + \frac{1}{\bar{m}_2 + \bar{r}_2} \frac{\bar{m}_1}{\bar{m}_1 + \bar{r}_1} \tag{6.134}$$

From equation (6.130),

$$A_{\text{sys}} = \bar{m}_{\text{sys}} \bar{f}_{\text{sys}} \tag{6.135}$$

Therefore, the mean time to failure for the series system with two components is

$$\bar{m}_{\text{sys}} = \frac{1}{1/\bar{m}_1 + 1/\bar{m}_2} \tag{6.136}$$

Thus, the mean time to failure of a series with n components is

$$\bar{m}_{\text{sys}} = \frac{1}{1/\bar{m}_1 + 1/\bar{m}_2 + \cdots + 1/\bar{m}_n} \tag{6.137}$$

However, the reciprocal of the mean time to failure is defined as the failure rate. Therefore, the failure rate for the two-component series system is

$$\lambda_{\text{sys}} = \lambda_1 + \lambda_2 \tag{6.138}$$

and for the n-component system, it is

$$\lambda_{\text{sys}} = \lambda_1 + \lambda_2 + \cdots + \lambda_n \tag{6.139}$$

Similarly, the mean time to repair for the two-component series system can be expressed as

$$\bar{r}_{\text{sys}} = \frac{\lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2 + (\lambda_1 \bar{r}_1)(\lambda_2 \bar{r}_2)}{\lambda_{\text{sys}}} \tag{6.140}$$

or approximately,

$$\bar{r}_{\text{sys}} = \frac{\lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2}{\lambda_{\text{sys}}} \tag{6.141}$$

Thus, for an n-component series system, it is

$$\bar{r}_{\text{sys}} \cong \frac{\lambda_1 \bar{r}_1 + \lambda \bar{r}_2 + \cdots + \lambda_n \bar{r}_n}{\lambda_{\text{sys}}} \tag{6.142}$$

or

$$\bar{r}_{\text{sys}} \cong \frac{\lambda_1 \bar{r}_1 + \lambda \bar{r}_2 + \cdots + \lambda_n \bar{r}_n}{\lambda_1 + \lambda_2 + \cdots + \lambda_n} \tag{6.143}$$

The average total outage duration of the series system can be found as

$$
\begin{aligned}
U_{\mathrm{sys}} &= \frac{\bar{r}_{\mathrm{sys}}}{\bar{r}_{\mathrm{sys}} + 1/\lambda_{\mathrm{sys}}} \\
&\cong \lambda_{\mathrm{sys}}\bar{r}_{\mathrm{sys}}
\end{aligned} \tag{6.144}
$$

so that for the two-component series system, it is

$$
U_{\mathrm{sys}} \cong \lambda_1\bar{r}_1 + \lambda_2\bar{r}_2 \tag{6.145}
$$

and for an n-component series system, it is

$$
U_{\mathrm{sys}} \cong \sum_{i=1}^{n} \lambda_i\bar{r}_i \tag{6.146}
$$

6.9.2 Repairable Components in Parallel

Consider a parallel system with two components, as shown in Figure 6.20(b), and assume that the components are independent and repairable. Thus, the unavailability or the steady-state probability of failure of the system is

$$
U_{\mathrm{sys}} = U_1 U_2 \tag{6.147}
$$

where U_1 = unavailability of component 1
U_2 = unavailability of component 2

However,

$$
\begin{aligned}
U_1 &= 1 - A_1 \\
&= \frac{\lambda_1\bar{r}_1}{1 + \lambda_1\bar{r}_1}
\end{aligned} \tag{6.148}
$$

and

$$
\begin{aligned}
U_2 &= 1 - A_2 \\
&= \frac{\lambda_2\bar{r}_2}{1 + \lambda_2\bar{r}_2}
\end{aligned} \tag{6.149}
$$

Therefore, the system unreliability is

$$
U_{\mathrm{sys}} = \frac{\lambda_1\bar{r}_1}{1 + \lambda_1\bar{r}_1}\,\frac{\lambda_2\bar{r}_2}{1 + \lambda_2\bar{r}_2} \tag{6.150}
$$

or

$$U_{sys} \cong \lambda_1 \lambda_2 \bar{r}_1 \bar{r}_2 \tag{6.151}$$

which gives the approximate average total outage duration of the parallel system.

The average frequency of the system failure can be expressed as

$$f_{sys} = U_2 \bar{f}_1 + U_1 \bar{f}_2 \tag{6.152}$$

where $\bar{f}_i$ = average frequency of failure of component i
U_1 = unavailability of component i

Since,

$$\bar{f}_1 = \frac{\lambda_1}{1 + \lambda_1 \bar{r}_1} \tag{6.153}$$

and

$$\bar{f}_2 = \frac{\lambda_2}{1 + \lambda_2 \bar{r}_2} \tag{6.154}$$

the average frequency of the system failure is

$$\bar{f}_{sys} = \frac{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)}{(1 + \lambda_1 \bar{r}_1)(1 + \lambda_2 \bar{r}_2)} \tag{6.155}$$

The system unavailability is

$$U_{sys} \triangleq \frac{\bar{r}_{sys}}{\bar{T}_{sys}} \tag{6.156}$$

or

$$U_{sys} = \bar{r}_{sys} \bar{f}_{sys} \tag{6.157}$$

so that

$$\bar{r}_{sys} = \frac{U_{sys}}{\bar{f}_{sys}} \tag{6.158}$$

Then, substituting equations (6.150) and (6.155) into equation (6.158), the average repair time (or downtime) of the two-component parallel system can be found as

$$\bar{r}_{sys} = \frac{\bar{r}_1 \bar{r}_2}{\bar{r}_1 + \bar{r}_2} \tag{6.159}$$

or

$$\frac{1}{\bar{r}_{sys}} = \frac{1}{\bar{r}_1} + \frac{1}{\bar{r}_2} \tag{6.160}$$

Similarly, the system unavailability can be given as

$$U_{sys} \triangleq \frac{\bar{r}_{sys}}{\bar{r}_{sys} + \bar{m}_{sys}} \tag{6.161}$$

then

$$\bar{m}_{sys} = \frac{\bar{r}_{sys}(1 - U_{sys})}{U_{sys}} \tag{6.162}$$

so that the average time to failure (or operation time, or uptime) of the parallel system can be found as

$$\bar{m}_{sys} = \frac{1 + \lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2}{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)} \tag{6.163}$$

Since the failure rate of the parallel system is

$$\lambda_{sys} \triangleq \frac{1}{\bar{m}_{sys}} \tag{6.164}$$

then

$$\lambda_{sys} = \frac{\lambda_1 \lambda_2 (\bar{r}_1 + \bar{r}_2)}{1 + \lambda_1 \bar{r}_1 + \lambda_2 \bar{r}_2} \tag{6.165}$$

Contrary to the series system, the equations derived for the two-component parallel system cannot be easily extended to a general n-component system. In certain parallel systems, it is possible to combine two components at a time. However, it is more recommendable to calculate the probabilities of the system by using the binomial distribution or conditional probabilities.

6.10 RELIABILITY EVALUATION OF COMPLEX SYSTEMS

Many systems cannot be classified as simple series–parallel structures. These non-series–parallel structures exhibit complex system characteristics. They can be evaluated by using the conditional probability method minimal-cut set method.

6.10.1 Conditional Probability Method

In this method [8, 9], a proper component of the given complex system (say, C_i) is first short-circuited (i.e., substituted by a component that never fails) and then open-circuited (i.e., assumed to be a failure). The resulting series–parallel subsystems are reunited based on the conditional probability concept discussed in section 6.5.2. Therefore, the probability of system success (i.e., the system reliability) can be expressed as

$$R_{\text{sys}} = P(\text{system operates}|C_i \text{ operates})P(C_i) + P(\text{system operates}|C_i \text{ fails})P(\bar{C}_i) \quad (6.166)$$

Similarly, the probability of system failure (i.e., system unreliability) can be expressed as

$$Q_{\text{sys}} = P(\text{system fails}|C_i \text{ operates})P(C_i) + P(\text{system fails}|C_i \text{ fails})P(\bar{C}_i) \quad (6.167)$$

As an example, consider the bridge-type network shown in Figure 6.21(a). Here, the system success dictates that at least one of the paths (made up of components C_1C_3, C_2C_4, $C_1C_5C_4$, and $C_2C_5C_3$) is good, and therefore, the system operates. Thus, the best choice for component C_i is component 5 (i.e., C_5). Figures 6.21(b) and 6.21(c) show the modified networks with component 5 short-circuited and open-circuited, respectively. Therefore, from equation (6.162), the system reliability can be expressed as

$$R_{\text{sys}} = P(\text{system operates}|C_5 \text{ operates})P(C_5) + P(\text{system operates}|C_5 \text{ fails})P(\bar{C}_5) \quad (6.168)$$

or, alternatively,

$$R_{\text{sys}} = R_{\text{sys}}(\text{if } C_5 \text{ operates})R_5 + R_{\text{sys}}(\text{if } C_5 \text{ fails})Q_5$$

where

$$R_{\text{sys}}(\text{if } C_5 \text{ operates}) = (1 - Q_1Q_2)(1 - Q_3Q_4)$$
$$R_{\text{sys}}(\text{if } C_5 \text{ fails}) = 1 - (1 - R_1R_3)(1 - R_2R_4)$$

Therefore,

$$R_{\text{sys}} = [(1 - Q_1Q_2)(1 - Q_3Q_4)]R_5 + [1 - (1 - R_1R_3)(1 - R_2R_4)]Q_5 \quad (6.169)$$

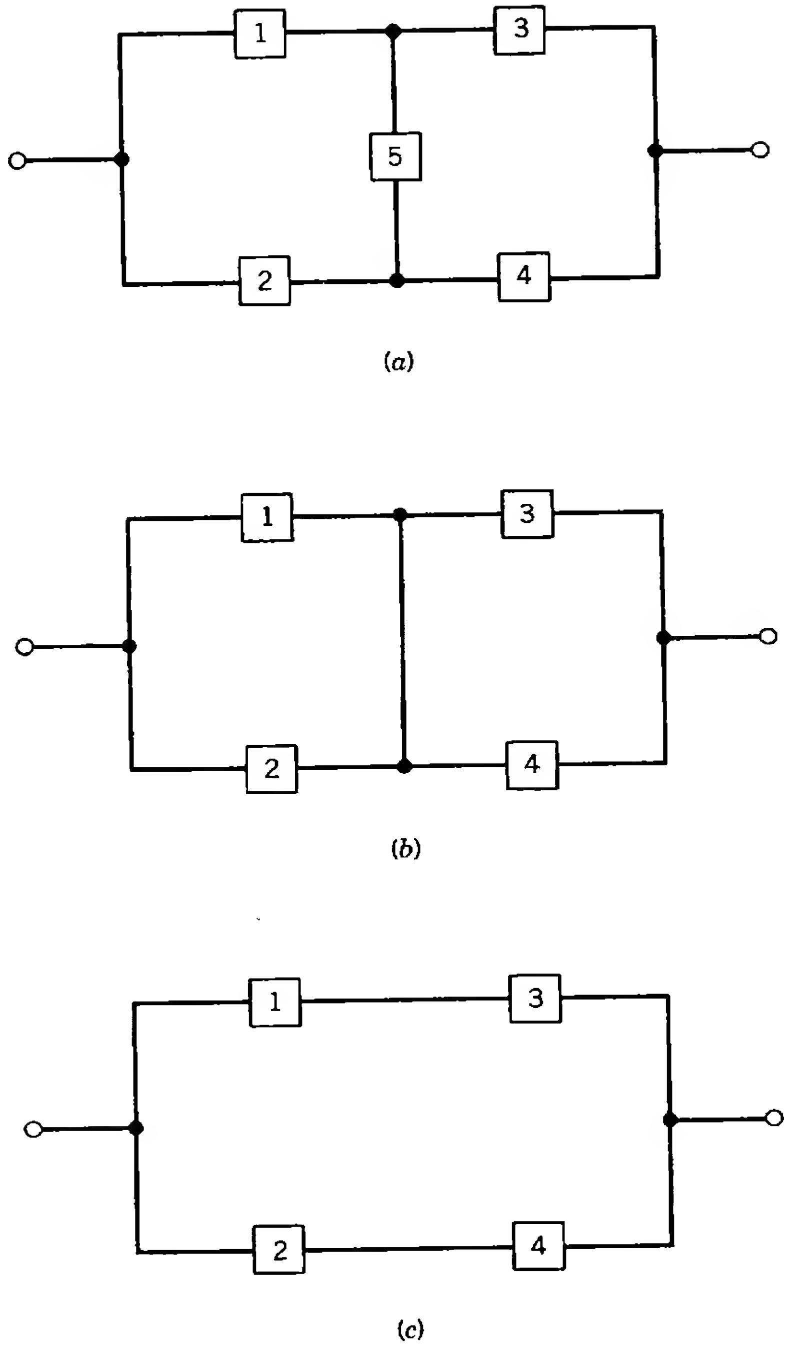


Figure 6.21. Reliability block diagrams: (*a*) bridge-type network; (*b*) modified network with component 5 shorted; (*c*) modified network with component 5 opened.

If the components are identical, the system reliability becomes

$$\begin{aligned} R_{\text{sys}} &= [1-2Q^2+Q^4]R+[1-(1-2R^2+R^4)]Q \\ &= [1-2(1-R)^2+(1-R)^4]R+[1-(1-2R^2+R^4)](1-R) \\ &= 2R^2+2R^3-5R^4+2R^5 \qquad (6.170) \end{aligned}$$

Similarly, the system unreliability can be calculated from equation (6.167).

6.10.2 Minimal-Cut-Set Method

A *tie set* is a set of edges (representing components) that constitute a path from input to output. If the components operate, the system operates properly. If no node is passed through more than once when tracing the tie set, such a tie set is called the *minimal tie set*. In other words, if any one of the components of a given minimal tie set is removed, the remaining set is no longer a tie set. A *cut set* is a set of edges that, when removed, divides the block diagram into the input and output subblocks. In other words, if the components of a given cut set fail, the system fails. If a given cut set cannot be divided into a subset that can be another cut set, it is called the *minimal cut set*. Therefore, if all components of a miniml cut set fails, the system fails.

As an example, consider the bridge-type network given in Figure 6.22(a). The minimal tie sets are made up of components C_1C_3, C_2C_4, $C_1C_5C_4$, and $C_2C_5C_3$, as shown in Figure 6.22(b). Therefore, it can be shown that

$$S = (C_1 \cap C_3) \cup (C_2 \cap C_4) \cup (C_1 \cap C_5 \cap C_4) \cup (C_2 \cap C_5 \cap C_3) \tag{6.171}$$

Similarly, the miniml cut sets are made up of components C_1C_2, C_3C_4, $C_1C_5C_4$, and $C_2C_5C_3$, as shown Figure 6.22(c). Thus, it can be expressed that

$$\bar{S} = (\bar{C}_1 \cap \bar{C}_2) \cup (\bar{C}_3 \cap \bar{C}_4) \cup (\bar{C}_1 \cap \bar{C}_5 \cap \bar{C}_4) \cup (\bar{C}_2 \cap \bar{C}_5 \cap \bar{C}_3) \tag{6.172}$$

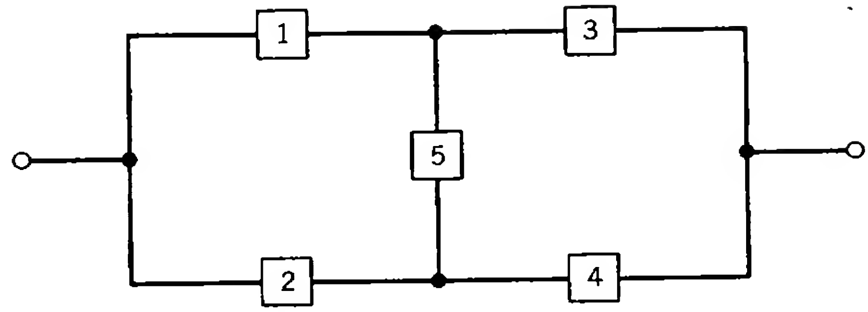
Therefore, as illustrated in Figure 6.22, a given non-series–parallel structures' logic diagram can be converted to series–parallel diagrams using either minimal-tie-set or minimal-cut-set methods [9–12].

Therefore, employing the minimum-cut-set method, the unreliability of the system can be expressed as

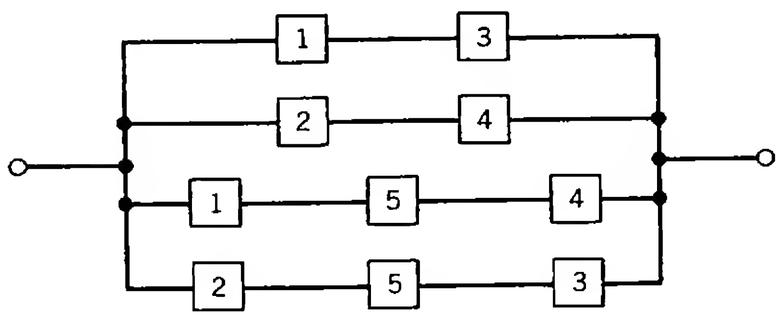
$$\begin{aligned} Q_{\text{sys}} &= P(E_1 \cup E_2 \cup E_3 \cup E_4) \\ &= P(E_1) + P(E_2) + P(E_3) + P(E_4) - P(E_1 \cap E_2) - P(E_1 \cap E_3) \\ &\quad - P(E_1 \cap E_4)P(E_2 \cap E_3) - P(E_2 \cap E_4) - p(E_3 \cap E_4) \\ &\quad + P(E_1 \cap E_2 \cap E_3) + P((E_1 \cap E_2 \cap E_4) + P(E_1 \cap E_3 \cap E_4) \\ &\quad + P(E_2 \cap E_3 \cap E_4) - P(E_1 \cap E_2 \cap E_3 \cap E_4) \end{aligned} \tag{6.173}$$

where

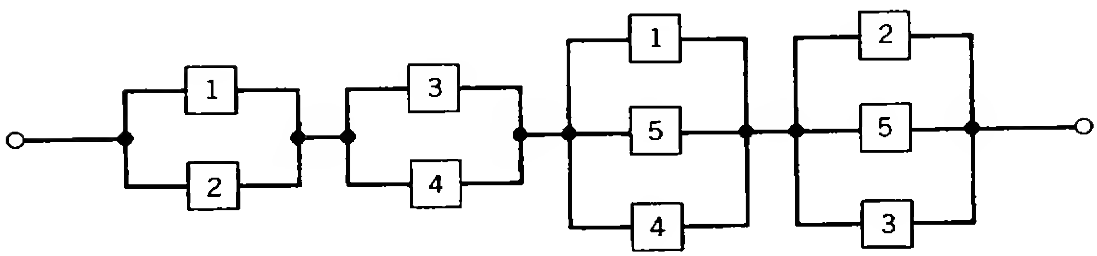
$$P(E_1) = \bar{C}_1 \cap \bar{C}_2$$
$$P(E_2) = \bar{C}_3 \cap \bar{C}_4$$
$$P(E_3) = \bar{C}_1 \cap \bar{C}_5 \cap \bar{C}_4$$
$$P(E_4) = \bar{C}_2 \cap \bar{C}_5 \cap \bar{C}_3$$



(a)



(b)



(c)

Figure 6.22. Reliability block diagrams showing bridge arrangement and its equivalents: (*a*) bridge-type network; (*b*) equivalent minimal-tie diagram; (*c*) equivalent minimal-cut diagram.

Note that equation (6.173) is an exact one. However, usually, an approximation is made, and the system unreliability can be expressed as

$$Q_{\text{sys}} \cong P(E_1) + P(E_2) + P(E_3) + P(E_4) \tag{6.174}$$

which sets the upper limit to system unreliability. It can be reexpressed as

$$Q_{\text{sys}} \cong Q_1Q_2 + Q_3Q_4 + Q_1Q_5Q_4 + Q_2Q_5Q_3 \tag{6.175}$$

If the components are identical, the system unreliability becomes

$$Q_{\text{sys}} = 2Q^2 + 2Q_3^2 \tag{6.176}$$

6.11 MARKOV PROCESSES

A *stochastic process* can be defined as a family of random variables, $\{X(t), t \in T\}$, defined over some index set or parameter space T. Therefore, for each t contained in the index set T, $X(t)$ is a random variable. The T is sometimes also defined as the time range, and $X(t)$ represents the observation at time t. The stochastic process is called a discrete-parameter or continuous-parameter process based on the nature of the time range. For example, if T is an infinite sequence, that is, $T = \{0, \pm 1, \pm 2, \ldots\}$ or $T = \{0, 1, 2, \ldots\}$, the stochastic process $\{X(t), t \in T\}$ is said to be a discrete-parameter process defined on the index set T. On the other hand, if T is an interval or algebraic combination of intervals, that is, $T = \{t\colon -\infty < T < +\infty\}$ or $T = \{t\colon 0 \le t < +\infty\}$, the stochastic process $\{X(t), t \in T\}$ is a continuous-parameter process defined on the index set T.

In reliability studies, the variable t denotes time, and $X(t)$ represents the *state* of the system at time t. The states at a given time t_n actually represent the mutually exclusive outcomes of the system at that time (i.e., operating, failed, in maintenance, etc.). All the possible states of a system are defined as the *state space*. The state space and the transitions between the states are illustrated in a *state space diagram*.

A stochastic process for which the occurrence of a future state depends only on the immediately prior state is defined as the *Markov process*. Therefore, the Markovian process is characterized by a lack of memory. Thus, a discrete-parameter stochastic process, $\{X(t); t = 0, 1, 2, \ldots\}$, or a continuous-parameter stochastic process, $\{X(t); t \ge 0\}$, is a Markov process if it has the following *Markovian property*:

$$\begin{aligned} P\{X(t_n) \le x_n | X(t_1) = x_1, \ldots, X(t_{n-1}) = x_{n-1}\} \\ = P\{X(t_n) \le x_n | X(T_{n-1}) = x_{n-1}\} \end{aligned} \tag{6.177}$$

for any set of n time points $t_1 < t_2 \cdots < t_n$ in the index set of the process and any real numbers $x_1, x_2, \ldots, x_n$. In nonmathematical language, one can say that, given the "present" condition of the process, the "future" is independent of the "past." The probability of

$$p_{x_{n-1}, x_n} = P\{X(t_n) = x_n | X(t_{n-1}) = x_{n-1}\} \tag{6.178}$$

is called the *transition probability* and represents the *conditional probability* of the system being in x_n at t_n, given it was x_{n-1} at t_{n-1}. It is also defined as a *one-step* transition probability as it represents the system between t_{n-1} and t_n.

A *Markov chain* can be defined by a sequence of discrete-valued random variables $\{X(t_n)\}$, where t_n is discrete valued or continuous. Thus, it is possible to define the Markov chain as the Markov process with a discrete state space. Define

$$p_{ij} = P\{X(t_n) = j | X(t_{n-1}) = i\} \tag{6.179}$$

as the *one-step transition probability* of going from state i at t_{n-1} to state j at t_n and assume that these probabilities do not change over time. The term used to describe this assumption is *stationarity*, if the transition probability depends only on the time difference, the Markov chain is defined to be stationary in time. Therefore, a Markov chain is completely defined by its transition probabilities of going from state i to state j, given in matrix form as

$$[P] = \begin{array}{cc} & \overbrace{\begin{array}{cccccc} 0 & 1 & 2 & 3 & \cdots & n \end{array}}^{\text{To state } j} \\ \text{From state } i \left\{ \begin{array}{c} 0 \\ 1 \\ 2 \\ 3 \\ \vdots \\ n \end{array} \right. & \begin{bmatrix} p_{00} & p_{01} & p_{02} & p_{03} & \cdots & p_{0n} \\ p_{10} & p_{11} & p_{12} & p_{13} & \cdots & p_{1n} \\ p_{20} & p_{21} & p_{22} & p_{23} & \cdots & p_{2n} \\ p_{30} & p_{31} & p_{32} & p_{33} & \cdots & p_{3n} \\ \vdots & \vdots & \vdots & \vdots & & \vdots \\ p_{n0} & p_{n1} & p_{n2} & p_{n3} & \cdots & p_{nn} \end{bmatrix} \end{array} \tag{6.180}$$

The matrix P is called a *one-step transition matrix* (or *stochastic matrix*) since all the transition probabilities p_{ij}'s are fixed and independent of time. The matrix P is also called just the *transition matrix* when there is no possibility of confusion. Since the p_{ij}'s are conditional, they must satisfy the conditions

$$\sum_{j}^{n} p_{ij} = 1 \qquad \forall_i \tag{6.181}$$

and

$$p_{ij} \geq 0 \qquad \forall_{ij} \tag{6.182}$$

where $i = 0, 1, 2, \ldots, n$, $j = 0, 1, 2, \ldots, n$. If the number of transitions (or states) are not too large, the information in a given transition matrix P can be represented by a transition diagram. For example, a given system has two states: (1) state 1, which represents the system being up, and (2) state 2, which represents the system being down. Then the associated transition probabilities can be defined as

p_{11} = probability of being in state 1 at time t given that it was in state 1 at time zero

p_{12} = probability of being in state 2 at time t given that it was in state 1 at time zero

p_{21} = probability of being in state 1 at time t given that it was in state 2 at time zero

p_{22} = probability of being in state 2 at time t given that it was in state 2 at time zero

Hence, the transition matrix can be expressed as

$$[P] = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$$

and its transition diagram can be drawn as in Figure 6.23.

By definition, the one-step transition probabilities are

$$p_{ij} = p_{ij}^{(1)} = P\{X(t_1) = j | X(t_0) = i\} \tag{6.183}$$

Therefore, the n-step transition probabilities can be defined by induction as

$$p_{ij}^{(n)} = P\{X(t_n) = j | X(t_0) = i\} \tag{6.184}$$

In other words, $p_{ij}^{(n)}$ is the probability (absolute probability) that the process is in state j at time t_n given that it was in state i at time t_0. Of course, it can be observed from this definition that $p_{ij}^{(0)}$ must be 1 if $i = j$ and 0 otherwise.

The Chapman–Kolmogorov equations provide a method for determining these n-step transition probabilities. In general form, these equations are given as

$$p_{ij}^{(n)} = \sum_k p_{ik}^{(n-m)} p_{kj}^{(m)} \qquad \forall_{ij} \tag{6.185}$$

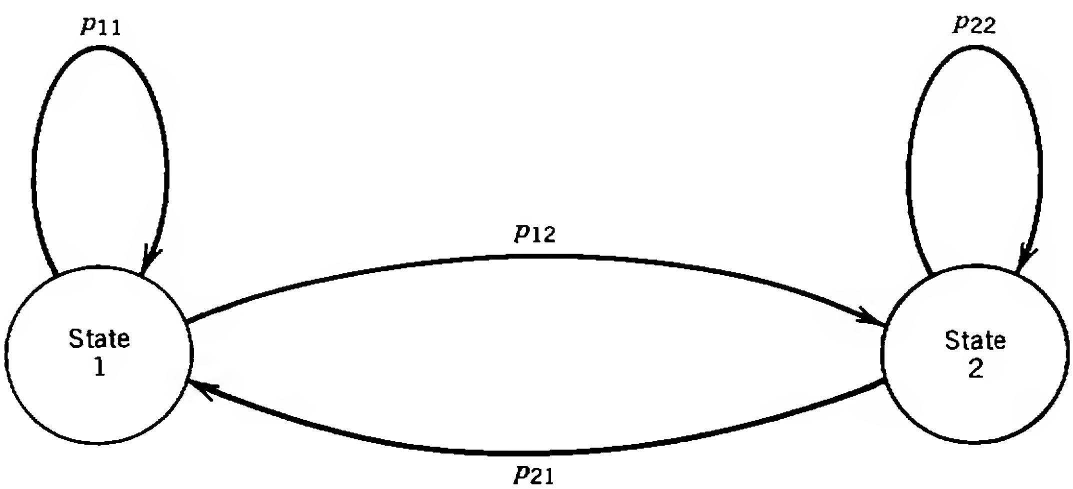


Figure 6.23. Transition diagram for two-state system.

for any m between zero and n. Note that this equation can be represented in matrix form by

$$P^{(n)} = P^{(n-m)}P^{(m)} \tag{6.186}$$

Therefore, the elements of a higher order transition matrix (e.g., $\|p_{ij}^{(n)}\|$) can be obtained directly by matrix multiplication. Hence,

$$\|p_{ij}^{(n)}\| = P^{(n-m)}P^{(m)} = P^{(n)} = P^n \tag{6.187}$$

Note that a special case of equation (6.185) is

$$p_{ij}^{(n)} \sum_k p_{ik}^{(n-1)} p_{kj} \qquad \forall_{ij} \tag{6.188}$$

and therefore, the special cases of equations (6.186) and (6.187) are

$$P^{(n)} = P^{(n-1)}P \tag{6.189}$$

and

$$\|p_{ij}^{(n)}\| = P^{(n-1)}P = P^{(n)} = P^n \tag{6.190}$$

respectively.

The unconditional probabilities such as

$$p_{ij}^{(n)} = P\{X(t_n) = j\} \tag{6.191}$$

are called the *absolute probabilities* or *state probabilities*. In order to determine the state probabilities, the initial conditions must be known. Therefore,

$$\begin{aligned} p_j^{(n)} &= P\{X(T_n) = j\} \\ &= P\sum_i \{X(t_n) = j \mid X(t_0) = i\} P\{X(t_0) = i\} \\ &= \sum_i p_i^{(0)} p_{ij}^{(n)} \end{aligned} \tag{6.192}$$

Note that equation (6.192) can be represented in matrix form by

$$p^{(n)} = p^{(0)}p^{(n)} \tag{6.193}$$

where $p^{(n)}$ = vector of state probabilities at time t_n
$p^{(0)}$ = vector of initial state probabilities at time t_0
$p^{(n)}$ = n-step transition matrix

Of course, the state probabilities or absolute probabilities are defined in vector form as

$$p^{(n)} = [p_1^{(n)} p_2^{(n)} p_3^{(n)} \cdots p_k^{(n)}] \tag{6.194}$$

and

$$p^{(0)} = [p_1^{(0)} p_2^{(0)} p_3^{(0)} \cdots p_k^{(0)}] \tag{6.195}$$

The long-run absolute probabilities are independent of the initial state probabilities, that is, $p^{(0)}$. Therefore, the resulting probabilities are called the *steady-state probabilities* and are defined as the set of π_j, where

$$\pi_j = \lim_{n\to\infty} p^{(n)} = \lim_{n\to\infty} P\{X(t_n) = j\} \tag{6.196}$$

In general, the initial state tends to be less important to the n-step transition probability as n increases, such that

$$\lim_{n\to\infty} P\{X(t_n) = j | X(t_0) = i\} = \lim_{n\to\infty} P\{X(t_n) = j\} = \Pi_j \tag{6.197}$$

so that one can get unconditional steady-state probability distribution from the n-step transition probabilities by taking n to infinity without taking the initial states into account. Therefore,

$$P^{(n)} = P^{(n-1)}P \tag{6.198}$$

or

$$\lim_{n\to\infty} P^{(n)} = \lim_{n\to\infty} P^{(n-1)}P \tag{6.199}$$

and thus,

$$[\Pi] = [\Pi][P] \tag{6.200}$$

where

$$[\Pi] = \begin{bmatrix} \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \\ \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \\ \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \\ \vdots & \vdots & \vdots & & \vdots \\ \Pi_1 & \Pi_2 & \Pi_3 & \cdots & \Pi_k \end{bmatrix} \tag{6.201}$$

Note that the matrix Π has identical rows so that each row is a row vector of

$$[\Pi] = [\Pi_1 \Pi_2 \Pi_3 \cdots \Pi_k] \tag{6.202}$$

Since the transpose of a row vector Π is a column vector Π^t, equation (6.200) can also be expressed as

$$\Pi^t = p^{(t)}\Pi^{(t)} \tag{6.203}$$

which is a set of linear equations.

In order to be able to solve equation sets (6.200) or (6.203) for individual Π_i's, one additional equation is required. This equation is called the *normalizing equation* and can be expressed as

$$\sum_{\text{all } i} \Pi_i = 1 \tag{6.204}$$

6.12 TRANSMISSION SYSTEM RELIABILITY METHODS

In recent years, transmission system reliability has gained considerable attention. Numerous methods have been developed for the quantitative evaluation of transmission system reliability. Some of the developed methods are included in this section.

6.12.1 Average Interruption Rate Method

This method has been introduced by Todd [13] in his paper published in 1964. The method determines the probability of forced outage of a specified minimum duration to calculate the customer interruption rate and the number of interruptions expected to exceed the given duration. here, the forced outage rate† is defined as the ratio of the total component outage time to the total component exposure time. In other words, the forced outage rate p is the probability of component outage presence and can be calculated from

$$p = \frac{\begin{array}{c}\text{Sum of days on which outage of}\\ \text{specified minimum duration occurred}\end{array}}{\text{sum of unit days}} \tag{6.205}$$

The method is simple and can easily be applied to systems with series–parallel connected components. It is based on the assumption of complete redundancy in parallel components and the continuity of supply to the load points. The expected number of days in a year that the specified outage for a given load point will happen is called the *average annual customer interruption rate* (AACIR). The systems that have a combination of series and parallel configurations can be included using the network reduction method.

† The terms *outage rate* and *failure rate* mean the very same thing. Here, they are used interchangeably.

Another alternative is to employ the minimal-cut-set procedure. Todd [13] gives a practical example for the application of the average interruption rate method. It is usually assumed that all outages occur simultaneously.

6.12.2 Frequency and Duration Method

In general, the failure rate of a given transmission line is a function of a fluctuating environment characterized by normal and severe (e.g., storms, ice, or sleet) weather conditions. It is possible that the failure rate during the severe weather conditions may become several orders of magnitude larger than the rate during the normal weather conditions. Also, it is possible for redundant systems to have multiple failures during the severe weather conditions. This phenomenon of overlapping forced outages during the periods of high environmental stress is known as *failure bunching*.

A method has been introduced by Gaver, Montmeat, and Patton [14] in their classical paper to take into account the changing environmental conditions. The proposed two-state weather model assumes that the weather fluctuates between normal and stormy periods. It again deals with series and parallel systems, as the first method. The method is based on the assumption that the component failure times, repair times, storm durations, and normal weather durations can be represented by exponential probability distributions. It is also based on the assumption of complete redundancy in parallel components and the continuity of supply to the load points. Unfortunately, the method results in an approximation as the number of parallel elements that are combined increases. In order to increase the computational efficiency of the method, a computer program has been developed [15]. The method gives two separate sets of equations for series and parallel systems. The parameters used are

λ_i = normal weather component failure rate of component i in failures per year of normal weather

λ_i' = stormy weather component failure rate of component i in failures per year of stormy weather

λ_i'' = component maintenance outage rate of component i in outages per year

r_i = expected repair time for all forced outages of component i in years

r_i'' = component maintenance repair rate of component i (i.e., expected down time for maintenance outages) in years

N = expected duration of normal weather period in years

S = expected duration of stormy weather period in years

Series Systems

The approximate overall (normal and stormy weather) annual forced outage rate λ_f of component i can be expressed as

$$\lambda_{f,i} = \frac{N}{N+S}\lambda_i + \frac{S}{N+S}\lambda'_i \quad \text{outages/yr} \tag{6.206}$$

Therefore, the overall annual forced outage rate (i.e., failure rate) for an n-component series system is

$$\lambda_{f,e} = \sum_{i=1}^{n} \lambda_{f,i} \quad \text{outages/yr} \tag{6.207}$$

Similarly, the annual maintenance outage rate for an n-component series system can be given as

$$\lambda''_e = \sum_{i=1}^{n} \lambda''_i \quad \text{outages/yr} \tag{6.208}$$

In the event that the series system is in parallel with other components, it is required to calculate the normal and stormy weather failure rates for the equivalent component e as

$$\lambda_e = \sum_{i=1}^{n} \lambda_i \quad \text{outages/yr of normal weather} \tag{6.209}$$

and

$$\lambda'_e = \sum_{i=1}^{n} \lambda'_i \quad \text{outages/yr of stormy weather} \tag{6.210}$$

Expected outage duration due to forced outage can be calculated from

$$r_{f,e} = \frac{\sum_{i=1}^{n} \lambda_{f,i} r_i}{\lambda_{f,e}} \quad \text{yr} \tag{6.211}$$

Similarly, the expected outage duration due to maintenance outages can be calculated from

$$r''_e = \frac{\sum_{i-1}^{n} \lambda''_i r''_i}{\lambda''_e} \quad \text{yr} \tag{6.212}$$

Therefore, the total annual outage rate can be expressed as

$$\lambda_{\text{sys}} = \lambda_{f,e} + \lambda''_e \quad \text{outages/yr} \tag{6.213}$$

The expected outage duration (i.e., restoration time) can be expressed as

$$r_{\text{sys}} = \frac{\lambda_{f,e} r_{f,e} + \lambda''_e r''_e}{\lambda_{\text{sys}}} \quad \text{yr} \tag{6.214}$$

or

$$r_{\text{sys}} = 8760 \frac{\lambda_{f,e} r_{f,e} + \lambda''_e r''_e}{\lambda_{\text{sys}}} \quad \text{h} \tag{6.215}$$

The total outage time per year can be expressed as

$$U_{\text{sys}} = \frac{r_{\text{sys}}}{r_{\text{sys}} + 1/\lambda_{\text{sys}}} \quad \text{yr/yr} \tag{6.216}$$

or

$$U_{\text{sys}} \cong \lambda_{\text{sys}} r_{\text{sys}} \quad \text{yr/yr} \tag{6.217}$$

or

$$U_{\text{sys}} \cong 8760 \lambda_{\text{sys}} r_{\text{sys}} \quad \text{h/yr} \tag{6.218}$$

Note that Gaver et al. [14] included maintenance outages as a random parameter as given in the aforementioned equations. However, Billinton [8] suggested to exclude them since it is questionable whether maintenance can be described as a random outage behavior.

Parallel Systems

In this method, the components are considered in pairs (two at a time) and are reduced to an equivalent component for some further combination based on the network reduction technique. However, unfortunately, this procedure increases the approximation involved in calculations as the number of parallel elements increase. Therefore, the approximate overall annual failure rate due to normal and stormy weather forced outages for a two-component parallel system can be expressed as

$$\begin{aligned}\lambda_{\text{sys}} = \frac{N}{N+S} \left[\lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} (\lambda_1 \lambda'_2 r_1 + \lambda_2 \lambda'_1 r_2) \right] \\ + \frac{S}{N+S} [\lambda'_1 \lambda_2 r_1 + \lambda'_2 \lambda_1 r_2 + 2S \lambda'_1 \lambda'_2] \quad \text{outages/yr}\end{aligned} \tag{6.219}$$

In the event that maintenance is to be included in the evaluation, a maintenance failure rate component, given by equation (6.220), should be added to the right side of equation (6.219). Here, it is assumed that the maintenance outages occur at random only during normal weather conditions. The maintenance failure rate can be calculated from

$$\lambda''_e = \lambda''_1 \lambda_2 r''_1 + \lambda''_2 \lambda_1 r''_2 \quad \text{outages/yr} \tag{6.220}$$

where λ''_i = component maintenance outage rate in outages per year
r''_i = component maintenance repair rate in years

In the event that the parallel system operates in parallel with other components, normal and stormy weather outage rates and the expected outage duration for the equivalent component representing the parallel system for further combinations can be expressed as

$$\lambda_e = \lambda_1 \lambda_2 (r_1 + r_2) + \frac{S}{N} (\lambda'_1 \lambda_2 r_1 + \lambda'_2 \lambda_1 r_2) \quad \text{outages/yr of normal weather} \tag{6.221}$$

$$\lambda'_e = \lambda_1 \lambda'_2 r_1 + \lambda_2 \lambda'_1 r_2 + 2S\lambda'_1 \lambda'_2 \quad \text{outages/yr of stormy weather} \tag{6.222}$$

$$r_e = \frac{r_1 r_2}{r_1 + r_2} \quad \text{yr} \tag{6.223}$$

Similarly, the equivalent failure rate of a two-component parallel system as a result of component forced-outage overlapping-component maintenance outage periods can be expressed as

$$\lambda''_{m,e} = \lambda''_1 \lambda_2 r''_1 + \lambda''_2 \lambda_1 r''_2 \quad \text{outages/yr} \tag{6.224}$$

Therefore, the expected downtime of the two-component system due to component forced-outages overlapping-component maintenance outage can be expressed as

$$r''_{m,e} = \frac{\lambda''_1 \lambda_2 r''_1}{\lambda''_1 \lambda_2 r''_1 + \lambda''_2 \lambda_1 r''_2} \, \frac{r_2 r''_1}{r_2 + r''_1} + \frac{\lambda''_2 \lambda_1 r''_2}{\lambda''_1 \lambda_2 r''_1 + \lambda''_2 \lambda_1 r''_2} \, \frac{r_1 r''_2}{r_1 + r''_2} \quad \text{yr} \tag{6.225}$$

or substituting equation (6.224) into equation (6.225),

$$r''_{m,e} = \frac{\lambda''_1 \lambda_2 r''_1}{\lambda''_{m,e}} \, \frac{r_2 r''_1}{r_2 + r''_1} + \frac{\lambda''_2 \lambda_1 r''_2}{\lambda''_{m,e}} \, \frac{r_1 r''_2}{r_1 + r''_2} \quad \text{yr} \tag{6.226}$$

Note that in the event that the stormy–normal weather approach cannot be applied and overall annual failure rates are used, the expected number of failures for a parallel system made up of two components can be expressed as [8]

$$\lambda_{f,e} = \lambda_1 \lambda_2 (r_1 + r_2) \quad \text{outages/yr} \tag{6.227}$$

The approximate value of the expected downtime duration can be expressed as

$$U_{\text{sys}} \cong \lambda_{f,e} r_T \quad \text{yr/yr} \tag{6.228}$$

where

r_T = expected down-time duration due to overlapping forced outages

$$= \frac{r_1 r_2}{r_1 + r_2} \quad \text{yr} \tag{6.229}$$

Therefore, substituting equation (6.229) into equation (6.228), the expected downtime duration can be reexpressed as

$$U_{\text{sys}} \cong \frac{r_1 r_2}{r_1 + r_2} \lambda_{f,e} \quad \text{yr/yr} \tag{6.230}$$

The aforementioned equations give the failure rates due to permanent (or sustained) outages. Billinton and Grover [16] derived equations to take into account the temporary outages. For example, the equivalent temporary failure rate of a two-component parallel system due to component temporary outages overlapping-component permanent outages can be expressed as

$$\lambda_{t,e} = \lambda_{1T} \lambda_2 r_2 + \lambda_{2T} \lambda_1 r_1 \quad \text{outages/yr} \tag{6.231}$$

where λ_{iT} = temporary outage rate of component i in outages per year

In the event that the two components are identical, the system temporary outage rate can be expressed as

$$\lambda_{t,e} = 2\lambda_T \lambda r \quad \text{outages/yr} \tag{6.232}$$

Similarly, the equivalent temporary failure rate of a two-component parallel system due to component temporary outages overlapping-component maintenance outages can be expressed as

$$\lambda''_{t,e} = \lambda_{1T} \lambda''_2 r''_2 + \lambda_{2T} \lambda''_1 r''_1 \quad \text{outages/yr} \tag{6.233}$$

If the two components are identical,

$$\lambda''_{t,e} = 2\lambda_T \lambda'' r'' \quad \text{outages/yr} \tag{6.234}$$

Therefore, the total temporary outage rate due to component temporary outages overlapping-component permanent and maintenance outage can be given as

$$\lambda_{T,e} = \lambda_{t,e} + \lambda''_{t,e} \quad \text{outages/yr} \tag{6.235}$$

The aforementioned equivalent temporary outage equations can be modified to take into account the failure bunching due to adverse weather [16].

6.12.3 Markov Application Method

In 1964, Desieno and Stine [17] have briefly introduced the application of Markov processes. They have mentioned the fact that within the limits of the assumptions made with regard to distributions involved, the Markov approach is theoretically the most accurate approach. Billinton and Bollinger [18] illustrated the application of the Markov method to series–parallel components operating in the two-state fluctuating weather environment. They have effectively demonstrated that the transitional probability matrix has all the probabilities of a transition from any one state to any of the other states. For example, in order to determine the limiting probabilities, the number of simultaneous equations that are necessary to be solved is 2×2^n for an n-component system in a two-state fluctuating weather environment. This number increases rapidly as the number of components increase.

In this method, it is assumed that the weather durations are distributed exponentially and that the system involved is operating in a two-state fluctuating weather environment (i.e., normal and stormy weather). In addition, the system can be represented as being either in the up state (the system is operating) or in the down state (the system has failed). Therefore, a state space diagram can be developed for each component, as shown in Figure 6.24. Thus, the required differential equations can be written directly from the state space diagram and can be expressed in matrix form as

$$\begin{bmatrix} P_0'(t) \\ P_0'(t) \\ P_0'(t) \\ P_3'(t) \end{bmatrix} = \begin{bmatrix} -(\lambda + n) & m & \mu & 0 \\ n & -(m + \lambda') & 0 & \mu' \\ \lambda & 0 & -(\mu + n) & m \\ 0 & \lambda' & n & -(\mu' + m) \end{bmatrix} \begin{bmatrix} P_0(t) \\ P_1(t) \\ P_2(t) \\ P_3(t) \end{bmatrix} \tag{6.236}$$

where λ = normal weather failure rate of component
μ = normal weather repair rate of component
λ' = stormy weather failure rate of component
μ' = stormy weather repair rate of component

Also,

$$n = \frac{1}{N}$$

and

$$m = \frac{1}{S}$$

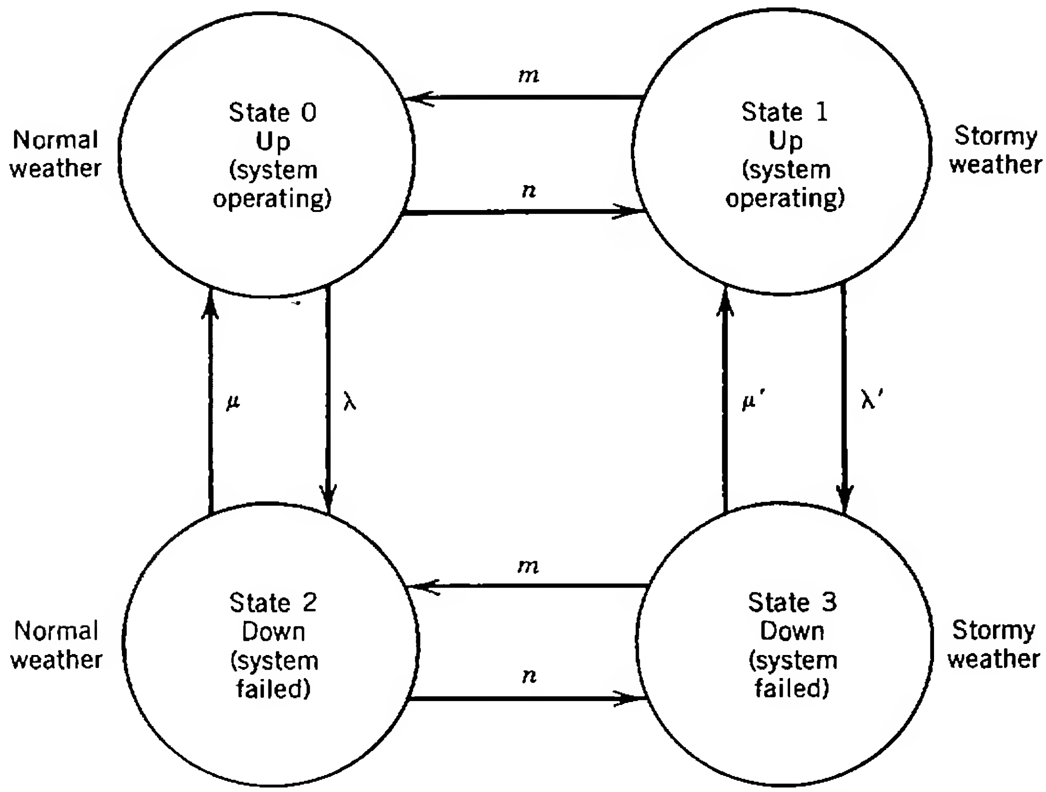


Figure 6.24. State space diagram for single component operating in two-weather environment.

where N = expected duration of normal weather period
S = expected duration of stormy weather period

Therefore, the long-term or steady-state probabilities of being in various states can be found by equating the differential matrix to zero so that

$$-(\lambda + n)P_0 + mP_1 + \mu P_2 = 0 \tag{6.237}$$

$$nP_0 - (m + \lambda')P_1 + \mu' P_3 = 0 \tag{6.238}$$

$$\lambda P_0 - (\mu + n)P_2 + mP_3 = 0 \tag{6.239}$$

$$\lambda' P_1 + nP_2 - (\mu' + m)P_3 = 0 \tag{6.240}$$

Also, of course, the sum of the steady-state probabilities of being in different states is equal to 1, that is,

$$P_0 + P_1 + P_2 + P_3 = 0 \tag{6.241}$$

Thus, from equations (6.237)–(6.241), the steady-state probabilities can be determined. For the given system, the probability of the system being in the up state is the sum of probabilities of being in the states 0 and 1. Therefore,

$$P(\text{up}) = P_0 + P_1 \tag{6.242}$$

Similarly, the probability of the system being in the down state can be found as

$$P(\text{down}) = P_2 + P_3 \tag{6.243}$$

Therefore, based on the assumptions that the repair rate is independent of environment (i.e., $\mu = \mu'$), the probabilities of the system being in the upstate and the down state can be expressed as [8]

$$P(\text{up}) = \frac{\mu}{m+n} \frac{(m+n)^2 + m(\mu+\lambda') + n(\mu+\lambda)}{(\mu+\lambda)(\mu+\lambda') + m(\mu+\lambda) + n(\mu+\lambda')} \tag{6.244}$$

and

$$P(\text{down}) = \frac{1}{m+n} \frac{n\lambda'(n+\mu) + m\lambda(m+\mu) + nm(\lambda+\lambda') + \lambda\lambda'(m+n)}{(\mu+\lambda)(\mu+\lambda') + m(\mu+\lambda) + n(\mu+\lambda')} \tag{6.245}$$

In the event that no repairs are done during the stormy weather (i.e., $\mu' = 0$), then, for example, the probability of being in the down state becomes

$$P(\text{down}) = \frac{1}{m+n} \frac{m(\lambda m + \lambda' n) + n(\lambda'\mu + \lambda m + \lambda' n)}{(\mu+\lambda)(\mu+\lambda') + m(\mu+\lambda) + n(\mu+\lambda')} \tag{6.246}$$

Note that the frequency of being in a given state can be found by multiplying the steady-state probability of being in that state by the rate of departure from that state. The average duration of a state can be found from the reciprocal of the rate of departure from that state [19].

In the event that only normal weather needs to be considered, then $\lambda = 0$, $m = 1$, and $n = 0$. Therefore,

$$P(\text{up}) = \frac{\mu}{\lambda + \mu} \tag{6.247}$$

and

$$P(\text{down}) = \frac{\lambda}{\lambda + \mu} \tag{6.248}$$

which are previously given in equations (6.88) and (6.92), respectively.

Since in this case the absorbing states are states 2 and 3, Billinton [8] shows that the mean time to failure can be expressed as

$$\text{MTTF} = \frac{m + \lambda' + n}{\lambda m + \lambda' n + \lambda\lambda'} \tag{6.249}$$

Of course, in the event that only normal weather needs to be considered, then $\lambda' = 0$, $m = 1$, and $n = 0$. Therefore,

$$\text{MITF} = \frac{1}{\lambda} \tag{6.250}$$

which is previously given as equation (6.73). The average failure rate can be found from equation (6.249) as

$$\lambda_{\text{av}} = \frac{1}{\text{MITF}}$$

$$\lambda_{\text{av}} = \frac{\lambda m + \lambda' n + \lambda\lambda'}{m + \lambda' + n} \tag{6.251}$$

In the event that $\lambda\lambda' \ll \lambda m + n$ and $\lambda' \ll m + n$, from equation (6.251), the approximate average failure rate can be expressed as

$$\lambda_{\text{av}} = \frac{\lambda m}{m+n} + \frac{\lambda' n}{m+n} \tag{6.252}$$

or

$$\lambda_{\text{av}} = \frac{N}{N+S}\,\lambda + \frac{S}{N+S}\,\lambda' \tag{6.253}$$

which is previously given as equation (6.206).

6.12.4 Common-Cause Forced Outages of Transmission Lines

The Task Force of the IEEE-PES Subcommittee on the Application of Probability Methods [20] defines a *common-cause* or *common-mode outage* as an event having a single external cause with multiple failure effects where the effects are not consequences of each other (nor consequences of common protection system response). Most often, the common-cause outages are encountered among the transmission circuits that are located on the same right-of-way. In general, such outages involve common external causes such as storms, hurricanes, lightning, floods, lines broken by planes, and towers hit by cars. Other examples of common-cause failures may include systematic human error, changes in the characteristics of the system, and changes in the environment. The importance of including common-cause outages in the modeling of redundant transmission systems has been recognized recently [21–24]. In all the models developed, it is assumed that the component state residence times are distributed exponentially. Recently, the validity of this assumption has been questioned by Singh and Ebrahimian [25]. They suggest to use a nonexponential distribution (i.e., an Erlangian distribution) for the repair time on the reliability indices. Based on this assumption, they developed a non-Markovian model for common-mode failures in transmission systems.

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PROBLEMS

6.1. A coin is tossed three times. On each result head or tail is observed. Determine the following:

(a) Sample space.

(b) Venn diagram.
(c) Tree diagram.

6.2. A university has two scholarships, in the amounts of \$400 and \$1000, to award. If three freshmen, three sophomores, and three juniors are eligible, determine the sample space in terms of class rank by using:
(a) Venn diagram.
(b) Tree diagram.

6.3. Consider Problem 6.2 and use a Venn diagram and a tree diagram to represent each of the following events:
(a) $A =$ the presentation of the \$400 scholarship to a freshman.
(b) $B =$ the presentation of the \$1000 scholarship to a sophomore.
(c) $C =$ the presentation of both scholarships to a junior.

6.4. If one card is drawn at random from a bridge deck, there are 52 possible outcomes, and each should occur with the same relative frequency. If the events are defined as (*a*) red card, (*b*) spade, (*c*) red card or spade, and (*d*) face card (king, queen, or jack), determine the following probabilities:
(a) $P(A)$.
(b) $P(B)$.
(c) $P(C)$.
(d) $P(D)$.

6.5. Assume that one card is drawn at random from a bridge deck and the following events are defined: A, spade; B, honor card (ace, king, queen, jack, or 10); C, black card; then;

$A \cap B$:	spade honor card
$A \cap C$:	spade
$B \cap C$:	black honor card

Determine the following:
(a) $P(A \cup B) = P(\text{spade or honor card})$.
(b) $P(A \cup C)$.
(c) $P(B \cup C)$.

6.6. When dealing a 13-card hand from a bridge deck, determine the probability that the hand contains at least two spades.

6.7. If A is the event of drawing a king from a deck of cards and B is the event of drawing an ace, determine the probability of drawing either a king or an ace in a single draw.

6.8. If A is the event of drawing a spade from a deck of cards and B is the event of drawing an ace, determine the probability of drawing either an ace or a spade or both.

6.9. Verify, using set theory, that the probability of a null set is zero for a given sample space, that is,

$$P(\emptyset) = 0 \quad \text{for any } S$$

6.10. Verify equation (6.19) using set theory.

6.11. By using set theory, verify that $P(A \cap B) = P(B) - P(A \cap B)$.

6.12. By using set theory, verify that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

6.13. Assume that two balls are selected at random without replacement from an urn that contains four white and eight black balls. Determine the following:

(a) Probability that both balls are white.

(b) Probability that second ball is white.

6.14. Assume that the personnel office of the NP&NL Utility Company has collected the following statistics of its 100 engineers:

Age	Bachelors's Degree only	Master's Degree	Total
Under 30	45	5	50
30–40	10	15	25
Over 40	20	5	25
Total	75	25	100

If one engineer is selected at random from the company, determine:

(a) Probability he has only a bachelor's degree.

(b) Probability he has a master's degree, given that he is over 40.

(c) Probability he is under 30, given that he has only a bachelor's degree.

6.15. Assume that the NL&NP Utility Company has 40 distribution transformers, of which 6 are defective, in its Riverside warehouse. Determine the probability of finding exactly two defective transformers in a group of five chosen randomly.

6.16. Determine the probability of getting exactly three heads out of eight tosses of a fair coin.

6.17. Assume that the expected life of a component can be given as $E(T) = \int_0^\infty t f(t)\, dt$ and derive equation (6.70).

6.18. Verify equation (6.90).

6.19. Consider equation (6.86) and assume that the total number of components involved in the system is very large and that the mean time to failure is very much larger than the mean time to repair. Show that

$$A = 1 \cong \frac{\text{MTTR}}{\text{MTTF}}$$

6.20. Assume that 10 identical components are going to be connected in series in a given system and that the minimum acceptable system reliability is required to be 0.98. Determine the approximate value of the component reliability.

6.21. Assume that five components are going to be connected in series in a given system and that the individual component reliabilities are given as 0.90, 0.92, 0.94, 0.96, and 0.98. Determine the approximate value of the system reliability.

6.22. Assume that two components are connected in parallel and that the probabilities involved are time-dependent exponentials. If λ_1 and λ_2 are the failure rates of components 1 and 2, respectively, verify that the system reliability can be expressed as

$$R_{\text{sys}}(t) = e^{-\lambda_1 t} + e^{-\lambda_2 t} - e^{-(\lambda_1 + \lambda_2)t}$$

6.23. Consider the various combinations of the reliability block diagrams shown in Figure P6.23. Assume that each component has a reliability of 0.95. Determine the equivalent system reliability of each configuration.

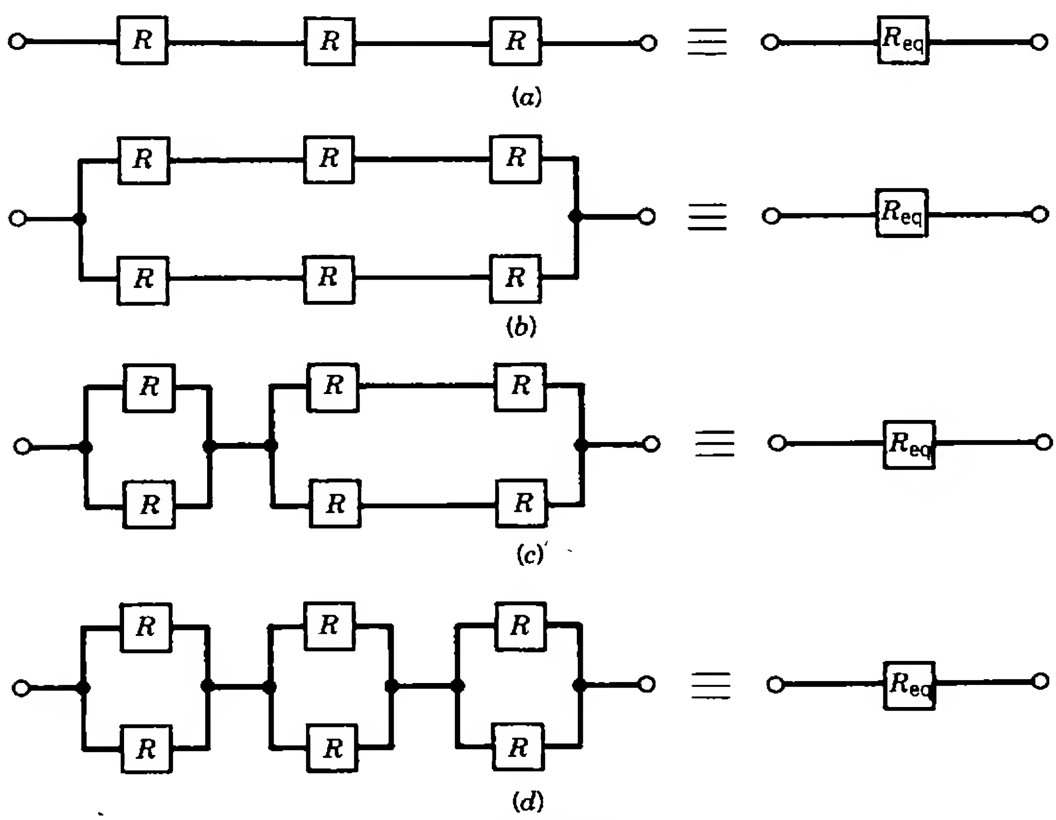


Figure P6.23

6.24. Determine the equivalent reliability of the system shown in Figure P6.24, assume that each component has the indicated reliability.

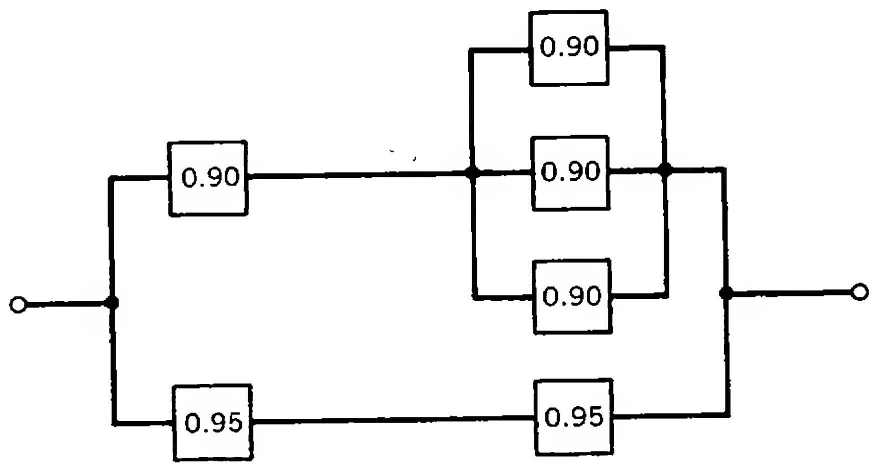
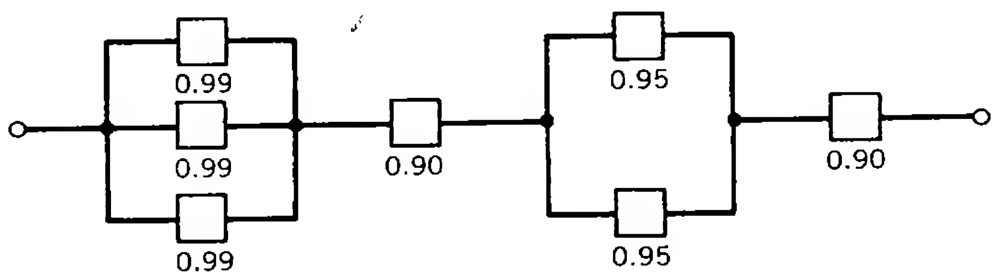
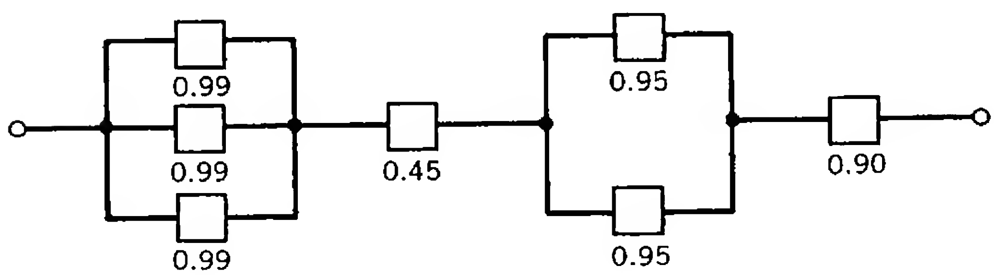


Figure P6.24

6.25. Determine the equivalent reliability of each of the system configurations shown in Figure P6.25, assuming that each component has the indicated reliability.



(*a*)



(*b*)

Figure P6.25

6.26. Assume that the components of the bridge-type network shown in Figure 6.20(*a*) are identical with a component reliability of 0.95. Determine the system reliability.

6.27. Assume that in the block diagram shown in Figure P6.27 two parallel paths (i.e., 14 and 25) operate to assure system supply if at least one of the paths is good. However, since neither 1 nor 2 is sufficiently reliable, a third element, 3, is added to supply either 4 or 5. Use the conditional probability method (or Bayes's theorem) and determine the system reliability.

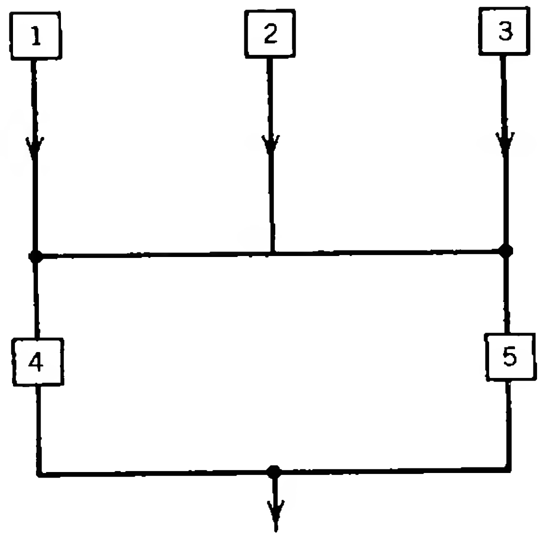


Figure P6.27

6.28. Assume that the block diagram given in Problem 6.27 has been modified, as shown in Figure P6.28. Use the conditional probability method (or Bayes's theorem) and determine the system reliability.

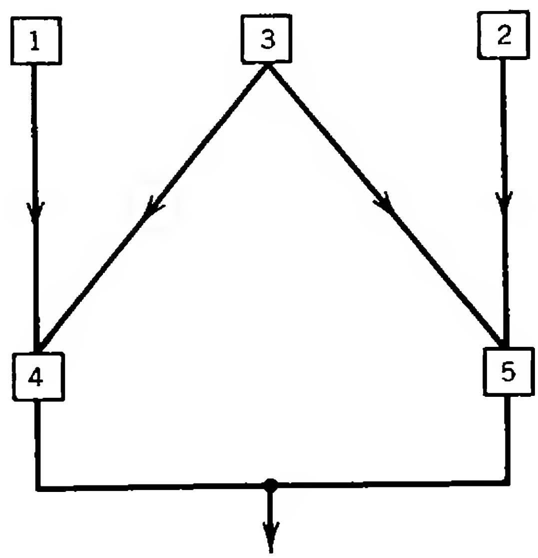


Figure P6.28

6.29. Assume that the components of the bridge-type network given in Problem 6.26 are identical with a component reliability of 0.95. Determine the system unreliability using:

(a) Equation (6.170).

(b) Equation (6.176).

6.30. For the network block diagram shown in Figure P6.30 (which represents four subtransmission systems supplying a load), use the conditional probability method (or Bayes's theorem) and determine system reliability when three out of four subtransmission lines are required for system success.

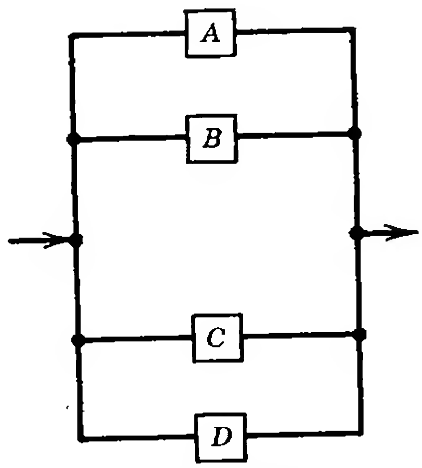


Figure P6.30

6.31. For the network block diagram shown in Figure P6.31, use the conditional probability method (or Bayes's theorem) and determine system reliability. Assume that the successful operation of the system requires that at least one of the paths (i.e., 12, 32, or 45) is good and operating.

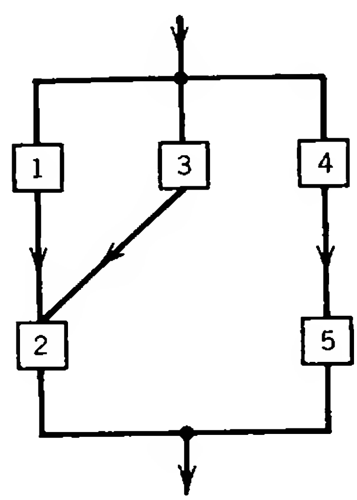


Figure P6.31

6.38. Consider the 50-mi-long, 138-kV line shown in Figure P6.38 and assume that the outage rate for the 138-kV line is 0.0065 failures per mile per year and that the annual failure rates for the 138-kV circuit breakers, 69-kV circuit breakers, 138/69-kV transformer, and 69-kV bus are given as 0.00857, 0.00612, 0.0891, and 0.0111, respectively. Use the average interruption rate method and determine the probability of an outage occurring for the 69-kV load bus and the associated average annual customer interruption rate. Assume that the 138-kV bus is 100 percent reliable.

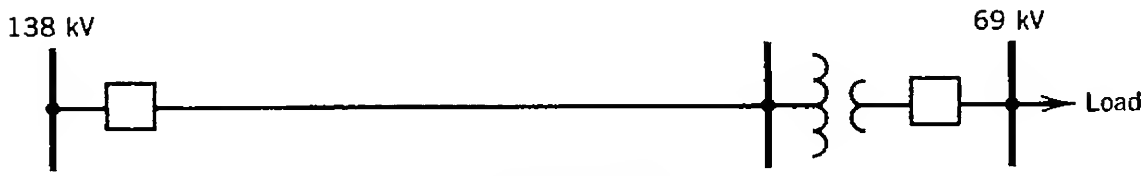


Figure P6.38

6.39. Consider Problem 6.38 and assume that an identical line (with identical component failure rates) has been connected in parallel with the first line, as shown in Figure P6.39. Use the average interruption rate method and determine the following:

(a) Probability of outage occurring for 69-kV load bus.

(b) Associated average annual customer interruption rate.

(c) Number of interruptions observed in past 10 years.

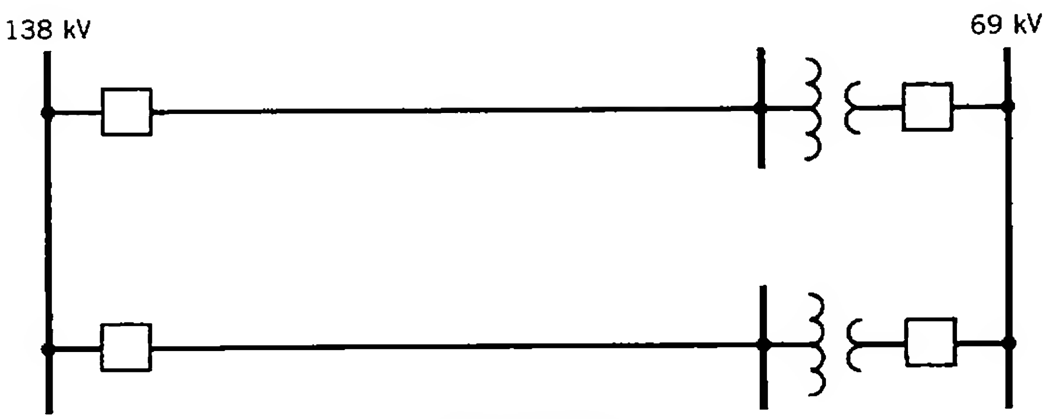


Figure P6.39

6.40. Consider the transmission system shown in Figure P6.40(a). Assume that the annual failure rates for components 1 and 2 are given as 0.5 and 0.6 failures for normal weather and 15 and 18 failures for stormy weather, respectively. Assume that annual component maintenance outage rates and exected repair times for all forced outages for each of

6.36. Consider Problem 6.34 and assume that an additional line has been connected between buses A and C with an annual failure rate of 0.7 failures per year, as shown in Figure P6.36. Use the average interruption rate method.

(a) Determine the probability of an outage occurring on line 4.

(b) Determine the probability of an outage occurring for load bus B and the associated average annual customer interruption rate.

(c) Repeat part (b) for load bus C.

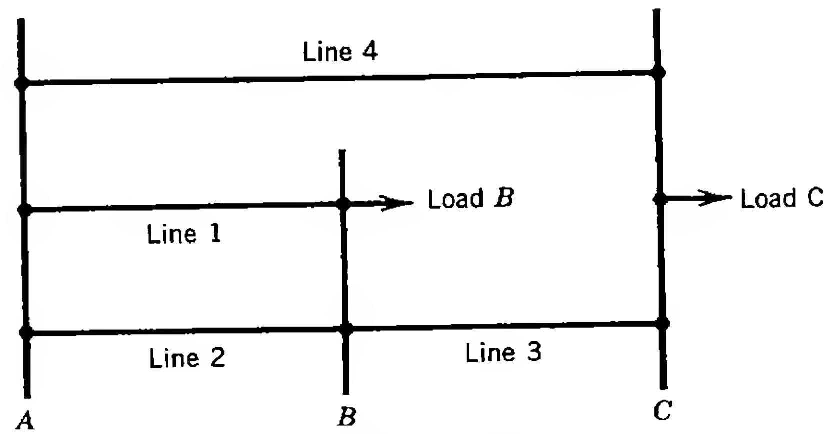


Figure P6.36

6.37. Consider Problem 6.34 and assume that an additional bus, bus D, has been added, as shown in Figure P6.37, and that the annual failure rates of the connecting lines 4 and 5 are given as 0.1 and 0.2, respectively. Use the average interruption rate method.

(a) Determine the probability of an outage occurring for load bus B and the associated average annual customer interruption rate.

(b) Repeat part (a) for load bus C.

(c) Repeat part (a) for load bus D.

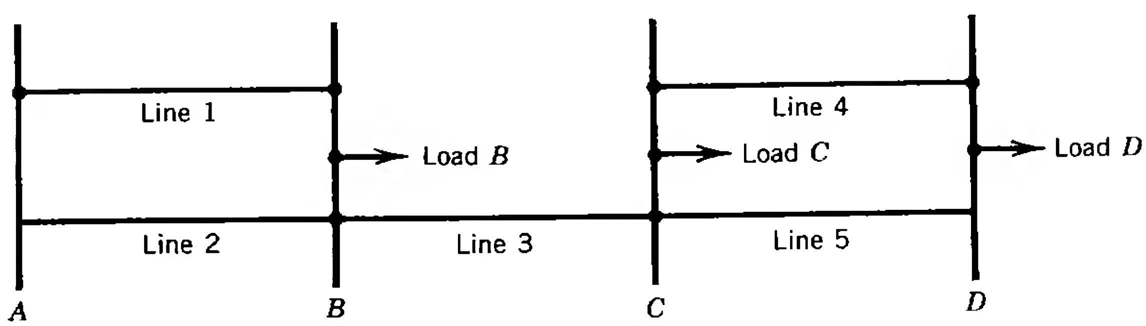


Figure P6.37

6.32. Assume that the components given in Problem 6.31 are all identical.
(a) Determine the system reliability.
(b) Calculate the system reliability if the component reliability is 0.99.

6.33. Repeat Problem 6.30 using the binomial theorem.

6.34. Consider the transmission system network configuration shown in Figure P6.34. Assume that the annual failure rates for line sections 1, 2, and 3 are given as 0.4, 0.3, and 0.6 failures per year, respectively. Use the average interruption rate method.
(a) Determine the probability of an outage occurring on each of the lines 1, 2, and 3.
(b) Determine the probability of an outage occurring for load bus *B* and the associated average annual customer interruption rate.
(c) Repeat part (b) for load bus *C*.

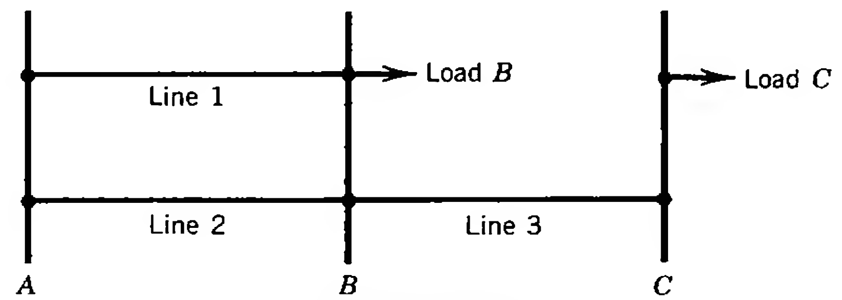


Figure P6.34

6.35. Consider Problem 6.34 and assume that an additional line has been connected between buses *B* and *C* with an annual failure rate of 0.5 failure per year, as shown in the Figure P6.35. Use the average interruption rate method.
(a) Determine the probability of an outage occurring on line 4.
(b) Determine the probability of an outage occurring for load bus *B* and the associated average annual customer interruption rate.
(c) Repeat part (b) for load bus *C*.

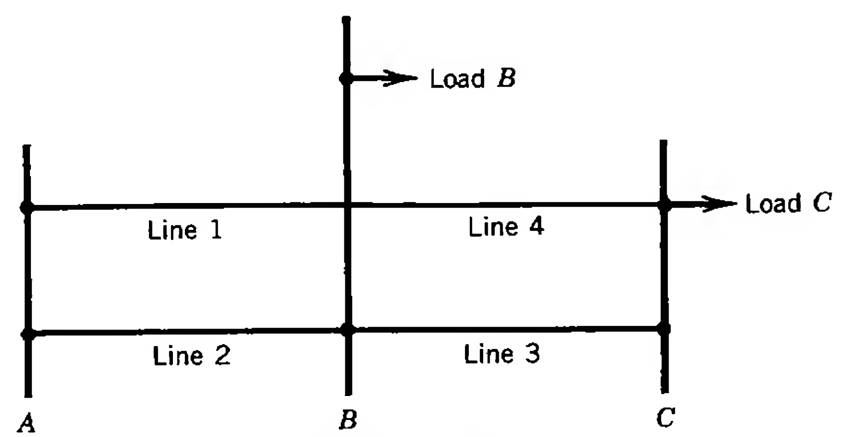


Figure P6.35

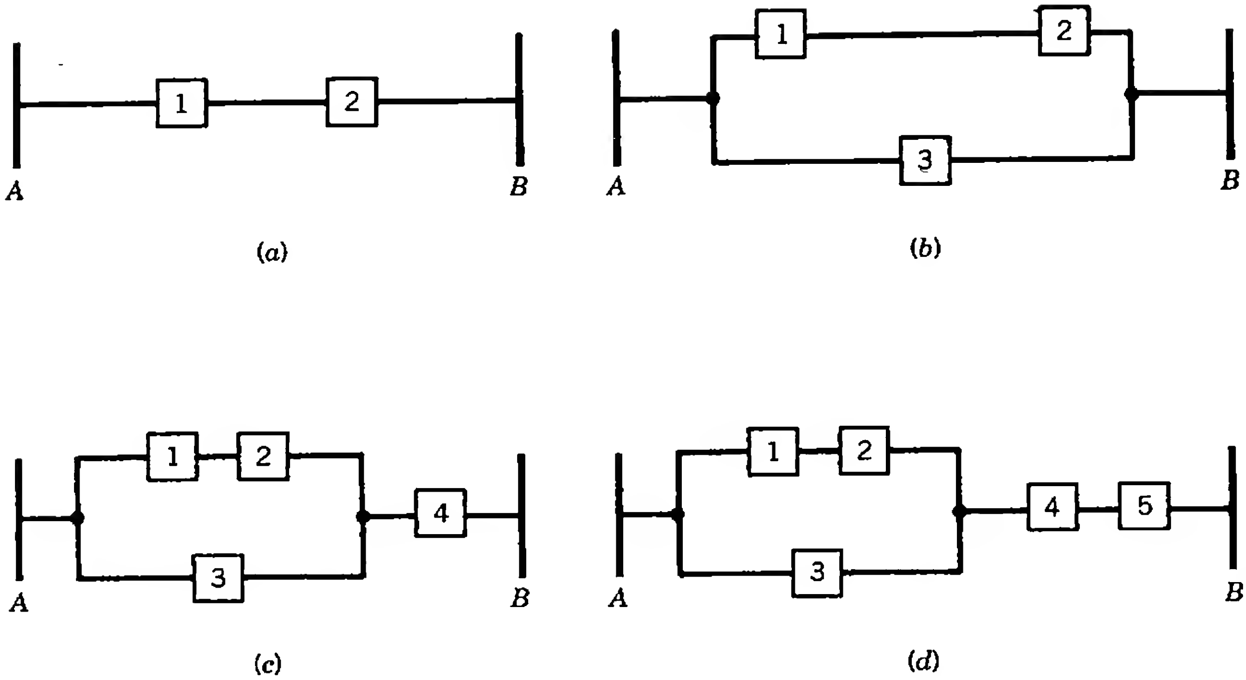


Figure P6.40

the components are given as 2 outages and 8 h, respectively. Expected downtimes for maintenance outages for each of the components are 8 h. The expected durations of normal and stormy weather periods are 3 and 195 h, respectively. Assume that bus A is a 100 percent reliable source bus and that bus B is the load bus. Use the frequency and duration method and determine the following:

(a) Overall annual forced outage rates for each component.

(b) Overall annual forced outage rate for series system.

(c) Annual maintenance outage rate of series system.

(d) Expected outage duration due to forced outage for system.

(e) Expected outage duration due to maintenance outages for system.

(f) Total annual outage rate for system.

(g) Expected restoration time, in hours, for system.

6.41. Assume that the two components given in Problem 6.40 are connected in parallel with respect to each other. Use the frequency and duration method and determine the overall annual failure rate due to normal and stormy weather forced outages for the system.

6.42. Consider Problem 6.40 and determine the average total outage time per year in hours.

6.43. Consider Problem 6.40 and assume that an additional component has been connected in parallel with respect to the previous components, as shown in Figure P6.40(b). Use the results of Problem 6.40 and determine the overall annual failure rate due to normal and stormy weather forced outages for the new system. Assume that the following data has been given for the third component:

$$\lambda_3 = 0.4 \text{ failure/yr of normal weather}$$

$$\lambda_3' = 20 \text{ failures/yr of stormy weather}$$

$$\lambda_3'' = 2 \text{ maintenance outages/yr}$$

$$r_3 = 10 \text{ h}$$

$$r_3' = 12 \text{ h}$$

$$r_3'' = 8 \text{ h}$$

6.44. Consider Problem 6.43 and assume that an additional component has been connected in series with respect to the previous system, as shown in Figure P6.40(*c*). Use the results of Problem 6.43 and determine the overall annual failure rate due to normal and stormy weather forced outages for the new system. Assume that components 3 and 4 are identical.

6.45. Consider Problem 6.44 and assume that an additional component has been connected in series with respect to the previous system, as shown in Figure P6.40(*d*). Use the results of Problem 6.44 and determine the overall annual failure rate due to normal and stormy weather forced outages for the new system. Assume that components 3, 4, and 5 are identical.

6.46. Assume that the following permanent outage data has been given for the subtransmission system shown in Figure P6.39.

System Component	Outage Rate (outages/yr)	Average Repair Time (h)
138 kV breakers	0.0058	66
138 kV line	0.627	10
138/69 kV transformer	0.0119	360
69 kV breakers	0.0045	44
69 kV bus	0.0111	5

Assume that the 138-kV bus is 100 percent reliable and determine the following:

(a) Total failure rate for one parallel line.

(b) Total mean time to repair for one parallel line.

(c) Total failure rate for system.

(d) Total mean time to repair for system.

(e) Total failure time due to overlapping-component permanent failures.

6.47. Consider Problem 6.46 and assume that the following maintenance outage data have been given for the system.

System Component	Outage rate (outages/yr)	Average Repair Time (h)
138-kV breakers	2	10
138-kV line	4	9
138/69-kV transformer	2	9
69-kV bus	1.5	5

Determine the following:

(a) Maintenance outage rate for one parallel line.

(b) Expected maintenance outage duration for one parallel line.

(c) Outage rate due to component permanent outages overlapping-component maintenance outages.

(d) Expected outage duration due to component permanent outages overlapping-component maintenance outages.

(e) Overall annual permanent outage rate.

(f) Overall expected outage duration.

(g) Overall average total outage time per year.

6.48. Consider Problem 6.46 and assume that the following temporary forced outage data have been given for the system.

System Component	Outage Rate (outages/yr)	Average Repair Time (h)
138-kV line	3.069	7
138/69-kV transformer	0.0048	90
69-kV bus	0.0164	7

Determine the following:

(a) Temporary outage rate for one parallel line.

(b) Expected temporary outage duration for one parallel line.

(c) System outage rate due to component temporary outages overlapping-component permanent outages.

(d) Overlapping restoration time due to temporary outage.

6.49. Consider Problem 6.48 and determine the following:

(a) Temporary outage rate due to component temporary outages overlapping-component maintenance outages.

(b) Overlapping restoration time in part (a).

(c) Total temporary outage rate due to component temporary outages overlapping-component permanent and maintenance outages.

(d) Restoration time for temporary outages.

(e) Overall temporary outage rate.

7

TRANSIENT OVERVOLTAGES AND INSULATION COORDINATION

7.1 INTRODUCTION

By definition, a transient phenomenon is an aperiodic function of time and has a short duration. Examples for such transient phenomenon are voltage or current surges. A voltage surge is introduced by a sudden change in voltage at a point in a power system. Its velocity depends on the medium in which the surge is traveling. Such voltage surge always has an associated current surge with which it travels. The current surges are made up of charging or discharging capacitive currents that are introduced by the change in voltages across the shunt capacitances of the transmission system. The surge voltages can be caused by lightning, switching, or faults, etc. High-voltage surges on power systems can be very destructive to system equipment, and thus they must be limited to safe levels.

When lightning strikes a phase conductor or shield wire (overhead ground wire), the current of the lightning stroke tends to divide, half going in each direction. If the overhead ground wire is struck at the tower, current will also flow in the tower, including its footings and counterpoise. The current of the lightning stroke will see the surge impedance of the conductor or conductors so that a voltage will be built up. As stated before, both the voltage and the current will move along the conductor as traveling waves.

Studies of transient disturbances on a transmission system have shown that lightning strokes and switching operations are followed by a traveling wave of a steep wave front. When a voltage wave of this type reaches a power transformer, for example, it causes an unequal stress distribution along its windings and may lead to breakdown of the insulation system.

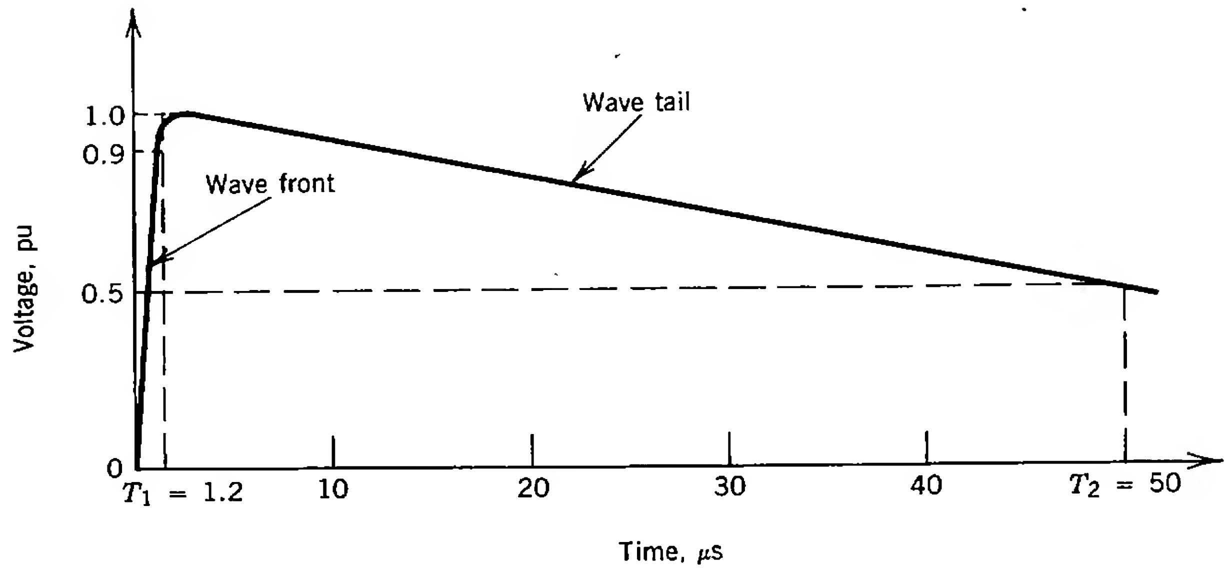


Figure 7.1. Standard impulse voltage waveform.

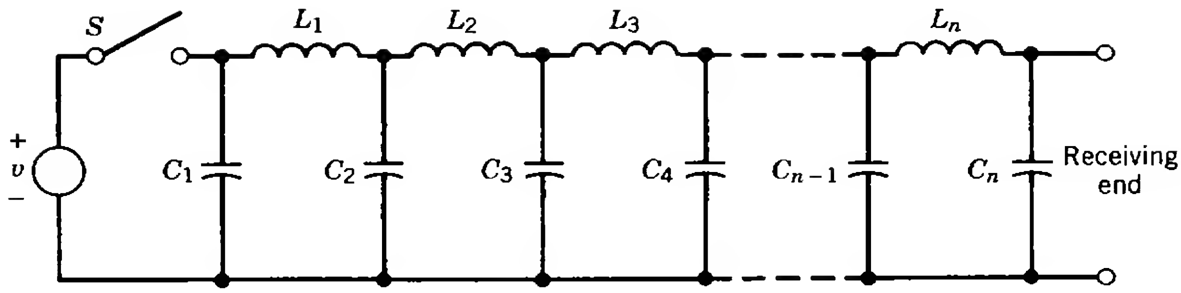
Therefore, it is required that the insulation behavior be studied under such impulse voltages. An impulse voltage is a unidirectional voltage that rises quickly to a maximum value and then decays slowly to zero. The wave shape is referred to as $T_1 \times T_2$, where both values are given in microseconds. For example, for the international standard[†] wave shape (which is also the new U.S. standard waveform) the $T_1 \times T_2$ is 1.2×50, as shown in Figure 7.1. Note that the crest (peak) value of the voltage is reached in 1.2 μs and the 50 percent point on the tail of the wave is reached in 50 μs. Of course, not all of the voltage waves caused by lightning can conform to this specification.

7.2 TRAVELING WAVES

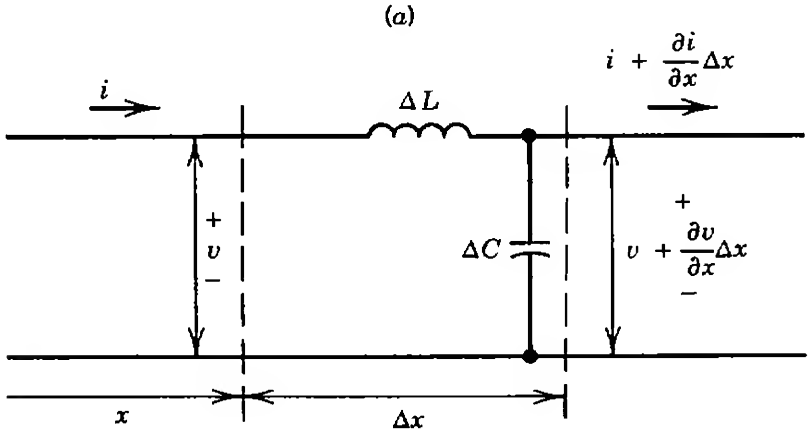
To study transient problems on a transmission line in terms of traveling waves, the line can be represented as incremental sections, as shown in Figure 7.2(*a*). The two-wire line is shown with one phase and neutral return. The parameters L and C are inductance and capacitance of the line (overhead or cable) per-cable length, respectively. To simplify the analysis, the line is assumed to be lossless, that is, its resistance R and conductance G are zero.

Any disturbance on the line can be represented by the closing or opening of the switch S, as shown in Figure 7.2(*a*). For example, when the line is suddenly connected to a voltage source, the whole of the line is not energized instantaneously. In other words, if the voltage v is applied to the sending end of the line by closing switch S, the voltage does not appear

[†] It is a standard set by the International Electrotechnical Commission (IEC).



(a)



(b)

Figure 7.2. Representation of two-wire transmission line for application of traveling waves: (*a*) *lumpy* representation; (*b*) elemental section of line.

instantaneously at the receiving end. When switch S is closed, the first capacitor becomes charged immediately to the instantaneous applied voltage. However, because of the first series inductor (since it acts as an open circuit), the second capacitor does not respond immediately but is delayed. Similarly, the third capacitor is delayed still more by the existence of the second inductor. Therefore, the farther away from the sending end of the line the greater is the delay. This gradual buildup of voltage over the transmission line conductors can be regarded as though a voltage wave is traveling from one end to the other end, and the gradual charging of the capacitances is due to the associated current wave. Of course, if the applied voltage is in the form of a surge, starting from zero and returning again to zero, it can be seen that the voltages on the intermediate capacitors rise to some maximum value and return again to zero. The disturbance of the applied surge is therefore propagated along the line in the form of a wave. Thus, such a propagation of the sending-end voltage and current conditions along the line is called *traveling waves*. Therefore, the voltage and current are functions of both x and t,

$$v = v(x, t) \quad \text{and} \quad i = i_j(x, t)$$

Thus, the series voltage drop along the elemented length of line can be expressed as

$$\Delta v(\Delta x, t) = v_j(x, t) - v_j(x + \Delta x, t)$$
$$= \int_x^{x+\Delta x} L \frac{\partial i}{\partial t} \, dt \tag{7.1}$$

or in the limit as Δx approaches to zero

$$\frac{\partial v_j(x, t)}{\partial x} = -L \frac{\partial i_j(x, t)}{\partial t} \tag{7.2}$$

or

$$\frac{\partial v}{\partial x} = -L \frac{\partial v}{\partial t} \tag{7.3}$$

Similarly, the current to charge the infinitesimal capacitance can be expressed as

$$\frac{\partial i_j(x, t)}{\partial x} = -C \frac{\partial i_j(x, t)}{\partial t} \tag{7.4}$$

or

$$\frac{\partial i}{\partial x} = -C \frac{\partial v}{\partial t} \tag{7.5}$$

Note that the negative signs in equations (7.2) to (7.5) are due to the direction of progress, at distance x, along the line. Figure 7.2 shows that x is increasing to the right. Therefore, based on the given current direction, both voltage v and current i will decrease with increasing x.

The i can be eliminated from equations (7.3) and (7.5) by taking the partial derivative of equation (7.3) with respect to x and equation (7.5) with respect to t so that

$$\frac{\partial^2 v}{\partial x^2} = -L \frac{\partial^2 i}{\partial x \, \partial t} \tag{7.6}$$

$$\frac{\partial^2 i}{\partial x \, \partial t} = -C \frac{\partial^2 v}{\partial t^2} \tag{7.7}$$

Substituting equation (7.7) into equation (7.6),

$$\frac{\partial^2 v}{\partial x^2} = LC \frac{\partial^2 v}{\partial t^2} \tag{7.8}$$

Similarly, it can be shown that

$$\frac{\partial^2 i}{\partial x^2} = LC \frac{\partial^2 i}{\partial t^2} \tag{7.9}$$

Equations (7.8) and (7.9) are known as *transmission line wave equations*. They can be expressed as

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 v}{\partial t^2} \tag{7.10}$$

and

$$\frac{\partial^2 v}{\partial x^2} = \frac{1}{\nu^2} \frac{\partial^2 i}{\partial t^2} \tag{7.11}$$

where

$$\nu \triangleq \frac{1}{\sqrt{LC}} \quad \text{m/s} \tag{7.12}$$

Here ν represents the velocity of the voltage and current (wave) propagation along the line in the positive direction. The dimensions of L and C are in henries per meter and farads per meter.

It can be shown that equation (7.8) can be satisfied by

$$v_f = v_1(x - \nu t) \tag{7.13}$$

and

$$v_b = v_2(x + \nu t) \tag{7.14}$$

where v_f denotes a forward-traveling wave (incident wave) and v_b denotes a backward-traveling wave (i.e., reflected wave). Therefore, the general solution of equation (7.8) can be expressed as

$$v(x, t) = v_f + v_b \tag{7.15}$$

or

$$v(x, t) = v_1(x - \nu t) + v_2(x + \nu t) \tag{7.16}$$

that is, the value of a voltage wave, at a given time t and location x along the line, is the sum of forward- and backward-traveling waves. Of course, the actual shape of each component is defined by the initial and boundary (terminal) conditions of a given problem.

The relationships between the traveling voltage and current waves can be expressed as

$$v_f = Z_c i_f \tag{7.17}$$

$$v_b = -Z_c i_b \tag{7.18}$$

where Z_c is the surge (or characteristic) impedance of the line. Since

$$Z_c = \left(\frac{L}{C}\right)^{1/2} \tag{7.19}$$

Therefore,

$$i_f = \frac{v_f}{Z_c} \tag{7.20}$$

and

$$i_b = -\frac{v_b}{Z_c} \tag{7.21}$$

Hence, the general solution of equation (7.9) can be expressed as

$$i(x, t) = i_f + i_b \tag{7.22}$$

or

$$\begin{aligned} i(x, t) &= \frac{1}{Z_c}(v_f - v_b) \\ &= \frac{1}{Z_c}[v_1(x - \nu t) - v_2(x + \nu t)] \end{aligned} \tag{7.23}$$

or

$$i(x, t) = \left(\frac{C}{L}\right)^{1/2}[v_1(t - \nu x) - v_2(t + \nu x)] \tag{7.24}$$

7.2.1 Velocity of Surge Propagation

The velocity of propagation of any electromagnetic disturbance in air is equal to the speed of light, that is, about 300,000 km/s[†]. As stated before, the velocity of surge propagation along the line can be expressed as

[†] To travel a distance of 2500 km an electric wave requires $\frac{1}{120}$ s, which is equal to a half-period of the 60-Hz ac frequency.

$$\nu \triangleq \frac{1}{\sqrt{LC}} \quad \text{m/s} \tag{7.25}$$

Since inductance of a single-phase overhead line conductor, assuming zero ground resistivity, is

$$L = 2 \times 10^{-7} \ln \frac{2h}{r} \quad \text{H/m} \tag{7.26}$$

and its capacitance is

$$C = \frac{1}{18 \times 10^9 \ln(2h/r)} \quad \text{F/m} \tag{7.27}$$

where h = height of conductor above ground in meters
r = radius of conductor in meters

Therefore, the surge velocity in a single-phase overhead line can be found as

$$\begin{aligned}\nu &= \frac{1}{\sqrt{LC}} \\ &= \left[\left(\frac{2 \times 10^{-7} \ln(2h/r)}{18 \times 10^9 \ln(2h/r)}\right)^{1/2}\right]^{-1} \\ &= 3 \times 10^8 \text{ m/s}\end{aligned}$$

Hence its surge velocity is the same as that of light. If the surge velocity in a three-phase overhead line is calculated, it can be seen that it is the same as for the single-phase overhead line. Furthermore, the surve velocity is independent of the conductor size and spacing between the conductors.

Similarly, the surge velocity in cables can be expressed as

$$\begin{aligned}\nu &= \frac{1}{\sqrt{LC}} \\ &= 3 \times 10^8 \sqrt{K} \quad \text{m/s}\end{aligned}$$

where K is the dielectric constant of the cable insulation, and let say, its value varies from 2.5 to 4.0. Thus, taking it as 4.0, the surge velocity in a cable can be found as 1.5×10^8 m/s. In other words, the surge velocity in a cable is half the one in an overhead line conductor.[†]

[†] Note that in $\frac{1}{120}$ s the surge travels 1250 km in a cable contrary to 2500 km that it can travel in an overhead line.

7.2.2 Surge Power Input and Energy Storage

Consider the two-wire transmission line shown in Figure 7.2. When switch S is closed, a surge voltage and surge current wave of magnitudes v and i, respectively, travel toward the open end of the line at a velocity of ν m/s. Therefore, the surge power input to the line can be expressed as

$$P = vi \quad \text{W} \tag{7.28}$$

Since the receiving end of the line is open-circuited and the line is assumed to be lossless, energy input per second is equal to energy stored per second. The energy stored is, in turn, equal to the sum of the electrostatic and electromagnetic energies stored. The electrostatic component is determined by the voltage and capacitance per-unit length as

$$W_s = \tfrac{1}{2} C v^2 \tag{7.29}$$

Similarly, the electromagnetic component is determined by the current and inductance per-unit length as

$$W_m = \tfrac{1}{2} L i^2 \tag{7.30}$$

Since the two components of energy storage are equal, the total energy content stored per-unit length is

$$W = W_s + W_m \tag{7.31}$$

or

$$W = 2W_s = 2W_m \tag{7.32}$$

that is,

$$W = Cv^2 = Li^2 \tag{7.33}$$

Therefore, the surge power can be expressed in terms of energy content and surge velocity as

$$P = W\nu \tag{7.34}$$

or

$$P = \frac{Li^2}{\sqrt{LC}} = i^2 Z_c \tag{7.35}$$

or

$$P = \frac{v^2}{Z_c} \tag{7.36}$$

It is interesting to note that for a given voltage level the surge power is greater in cables than in overhead line conductors due to the smaller surge impedance of the cables.

Example 7.1

Assume that a surge voltage of 1000 kV is applied to an overhead line with its receiving end open. If the surge impedance of the line is 500 Ω, determine the following:

(a) Total surge power in line.
(b) Surge current in line.

Solution

(a) The total surge power is

$$\begin{aligned} P &= \frac{v^2}{Z_c} \\ &= \frac{1 \times 10^2}{500} = 2000 \text{ MW} \end{aligned} \tag{7.37}$$

(b) Therefore the surge current is

$$\begin{aligned} i &= \frac{v}{Z_c} \\ &= \frac{1 \times 10^6}{500} = 2000 \text{ A} \end{aligned}$$

Example 7.2

Repeat Example 7.1 assuming a cable with surge impedance of 50 Ω.

Solution

(a) The total surge power is

$$\begin{aligned} P &= \frac{v^2}{Z_c} \\ &= \frac{1 \times 10^{12}}{50} = 20{,}000 \text{ MW} \end{aligned}$$

(b) Thus, the surge current is

$$i = \frac{v}{Z_c}$$

$$= \frac{1 \times 10^6}{50} = 20{,}000 \text{ A}$$

7.2.3 Superposition of Forward- and Backward-Traveling Waves

Figure 7.3(a) shows forward-traveling voltage and current waves. Note that x is increasing to the right in the positive direction, as before, according to the sign convention. Figure 7.3(b) shows backward-traveling voltage and current waves. Figure 7.3(c) shows the superposition of forward and backward waves of voltage and current, respectively. It can be shown that on

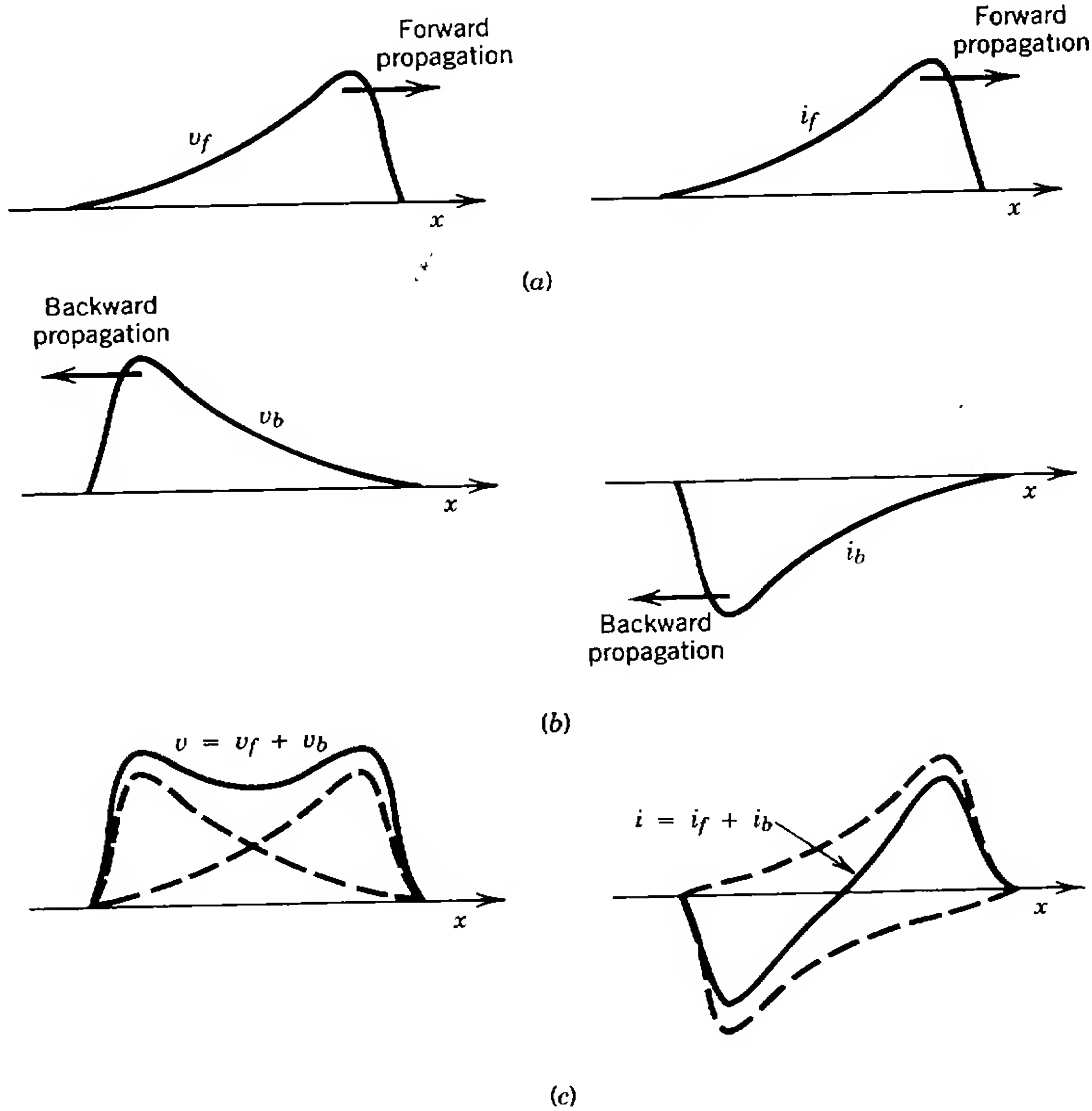


Figure 7.3. Representation of voltage and current waves: (a) in forward direction; (b) in backward direction; (c) superposition of waves.

loss-free transmission lines the voltage and current waves have the same shape, being related to each other by the characteristic impedance of the line, and travel is undistorted. Furthermore, when two waves meet, they do not affect each other but appear to pass through each other without any distortion.

7.3 EFFECTS OF LINE TERMINATIONS

Assume that v_f and i_f and v_b and i_b are the instantaneous voltage and current of the forward and backward waves, respectively, at the point of discontinuity (i.e., at the end of the line). Hence the instantaneous voltage and current at the point of discontinuity can be expressed as

$$v = v_f + v_b \tag{7.38}$$

$$i = i_f + i_b \tag{7.39}$$

Substituting equations (7.20) and (7.21) into equation (7.39),

$$i = \frac{v_f}{Z_c} - \frac{v_b}{Z_c} \tag{7.40}$$

or

$$iZ_c = v_f - v_b \tag{7.41}$$

Adding equations (7.38) and (7.41),

$$v + iZ_c = 2v_f \tag{7.42}$$

or

$$v_f = \tfrac{1}{2}(v + iZ_c) \tag{7.43}$$

Similarly, subtracting equation (7.41) from equation (7.38),

$$v_b = \tfrac{1}{2}(v - iZ_c) \tag{7.44}$$

Alternatively, from equation (7.42),

$$v = 2v_f - iZ_c \tag{7.45}$$

Substituting equation (7.45) into equation (7.44),

$$v_b = v_f - iZ_c \tag{7.46}$$

7.3.1 Line Termination in Resistance

Assume that the receiving end of the line is terminated in a pure resistance so that

$$v = iR \tag{7.47}$$

Substituting this equation into equation (7.42),

$$i = \frac{2}{R + Z_c} v_f \tag{7.48}$$

and from equation (7.47)

$$v = \frac{2R}{R + Z_c} v_f \tag{7.49}$$

or

$$v_f = \frac{R + Z_c}{2R} \tag{7.50}$$

Similarly, substituting equations (7.48) and (7.49) into equation (7.44),

$$v_b = \frac{R - Z_c}{R + Z_c} v_f \tag{7.51}$$

The power transmitted to the termination point by the forward wave is

$$P_f = \frac{v_f^2}{Z_c} \tag{7.52}$$

Whereas, the power transmitted from the termination point by the backward wave is

$$P_b = \frac{v_b^2}{Z_c} \tag{7.53}$$

Therefore, the power absorbed by the resistor R is

$$P_R = \frac{v^2}{R} \tag{7.54}$$

or

$$P_R = \frac{(v_f + v_b)^2}{R} \tag{7.55}$$

so that

$$P_f = P_b + P_R \tag{7.56}$$

7.3.2 Line Termination in Impedance

In the general case of a line of characteristic impedance Z_c terminated in an impedance Z,

$$i = \frac{2}{Z + Z_c} i_f \tag{7.57}$$

$$v = \frac{2Z}{Z + Z_c} v_f \tag{7.58}$$

or

$$v = \tau v_f \tag{7.59}$$

where τ is the *refraction coefficient* or *transmission factor* or simply *coefficient* τ. Thus, for voltge waves,

$$\tau \triangleq \frac{2Z}{Z + Z_c} \tag{7.60}$$

The value of τ varies between zero and two depending on the relative values of Z and Z_c. Alternatively,

$$v_f = \frac{Z + Z_c}{2Z} v \tag{7.61}$$

Similarly,

$$v_b = \frac{Z - Z_c}{Z + Z_c} v_f \tag{7.62}$$

or

$$v_b = \rho v_f \tag{7.63}$$

where ρ is the *reflection coefficient*. Therefore, for voltage waves,

$$\rho \triangleq \frac{Z - Z_c}{Z + Z_c} \tag{7.64}$$

Of course, ρ can be positive or negative depending on the relative values of Z and Z_c. For example, when the line is terminated with its characteristic impedance (i.e., $Z = Z_c$), then $\rho = 0$, that is, no reflection. Thus, $v_b = 0$ and $i_b = 0$. In other words, the line acts as if it is infinitely long. When the line is terminated in an impedance that is larger than its characteristic impedance (i.e., $Z > Z_c$), then v_b is positive and i_b is negative. Therefore, the reflected surges consist of increased voltage and reduced current, as shown in Figure 7.4. On the other hand, when the line is terminated in an impedance that is

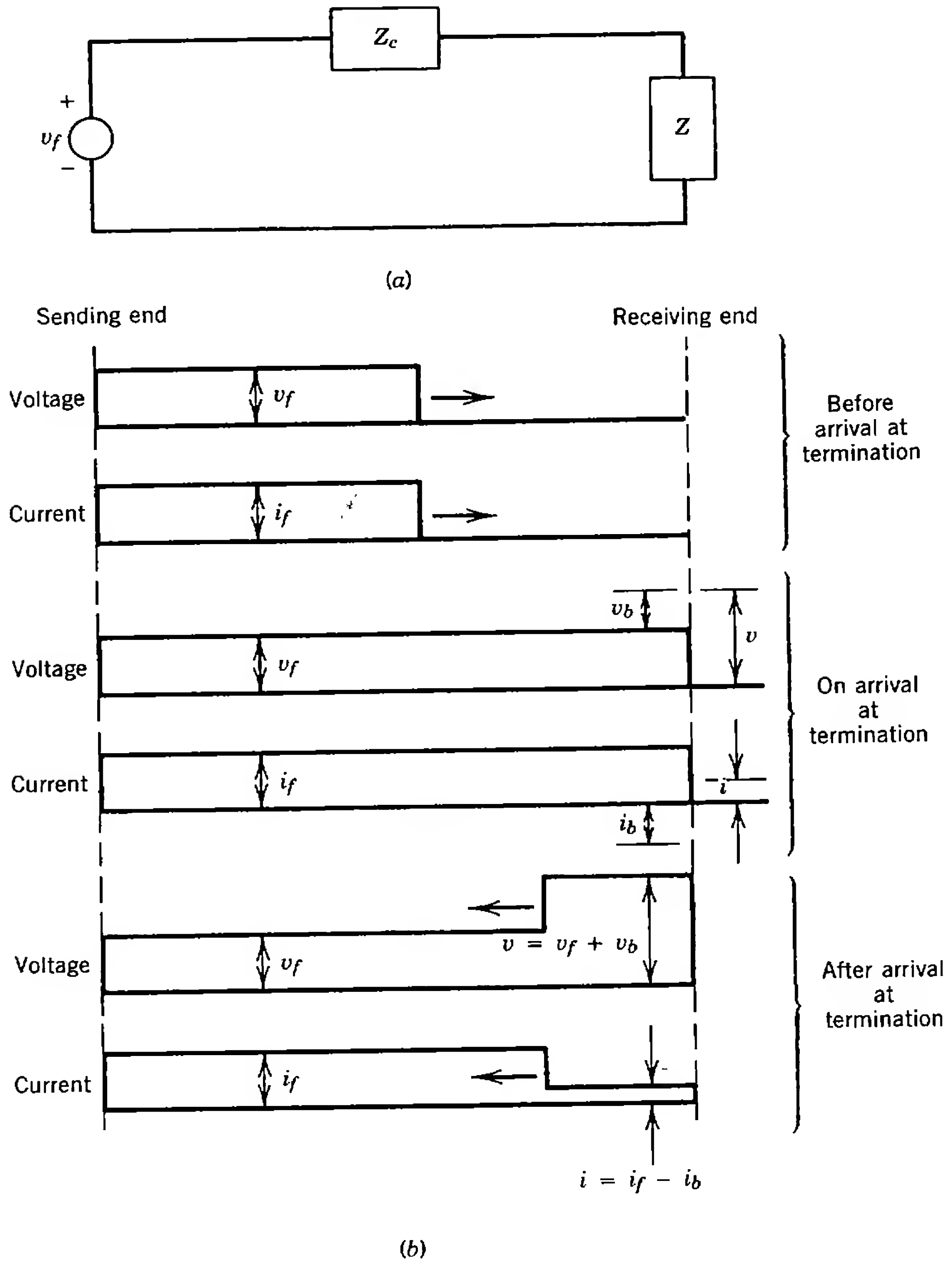


Figure 7.4. Analysis of traveling waves when $Z > Z_c$: (*a*) circuit diagram; (b) voltage and current distributions.

smaller than its characteristic impedance (i.e., $Z < Z_c$), then v_b is negative and i_b is positive. Thus, the reflected surges consist of reduced voltage and increased current, as shown in Figure 7.5.

If Z_s and Z_r are defined as the sending-end and receiving-end Thévenin equivalent impedances, respectively, the sending-end reflection coefficient is

$$\rho_s = \frac{Z_s - Z_c}{Z_s + Z_c} \tag{7.65}$$

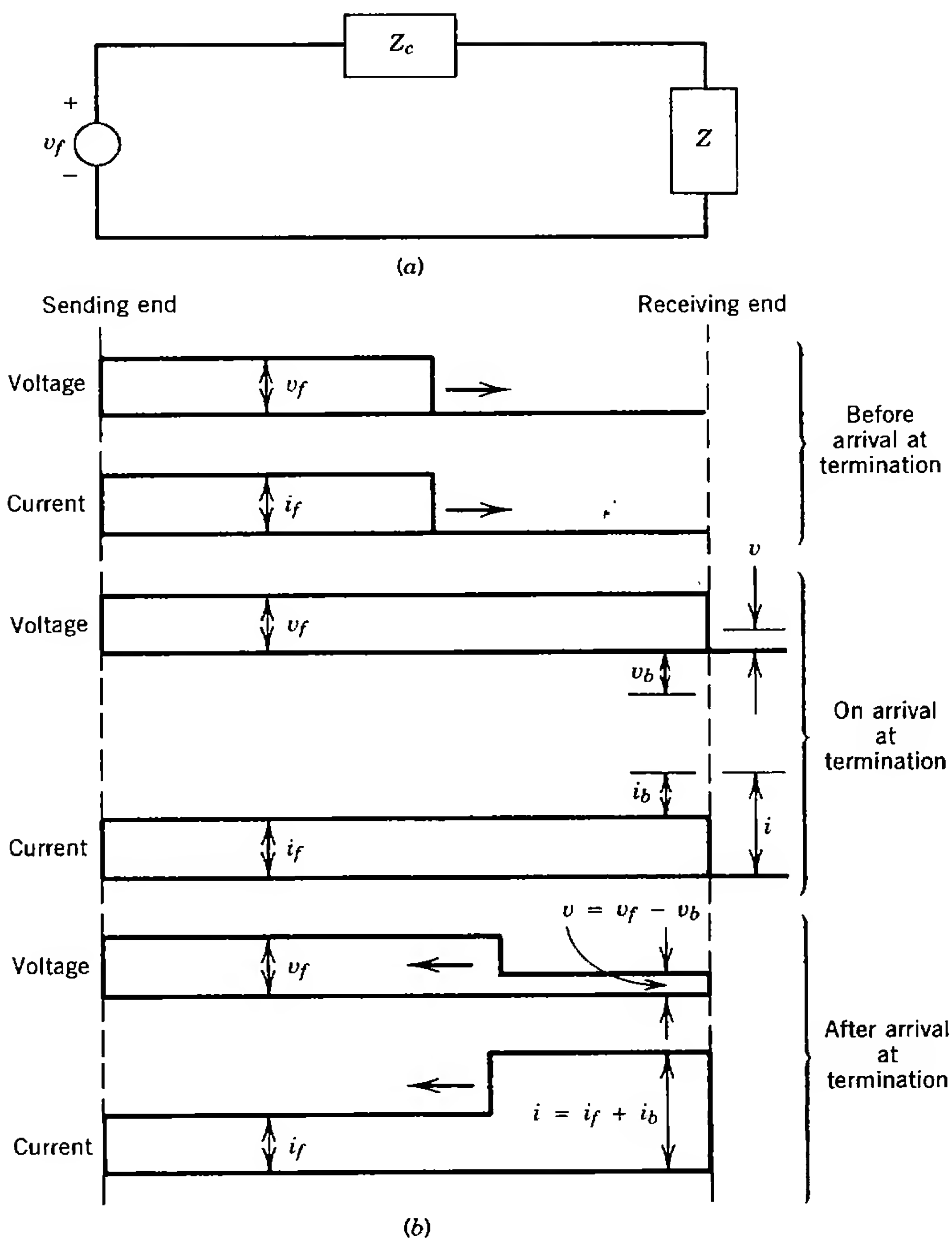


Figure 7.5. Analysis of traveling waves when $Z < Z_c$: (a) circuit diagram; (b) voltage and current distributions.

and the receiving-end reflection coefficient is

$$\rho_r = \frac{Z_r - Z_c}{Z_r + Z_c} \tag{7.66}$$

Note that waves traveling back toward the sending end will result in new reflections as determined by the reflection coefficient at the sending end ρ_s. Furthermore, note that the reflection coefficient for current is always the negative of the reflection coefficient for voltage.

EXAMPLE 7.3

Consider equations (7.60) and (7.62) and verify that

(a) $i_b = \rho i_f$.
(b) $\tau = \rho + 1$.

Solution

(a) Since

$$i_f = \frac{v_f}{Z_c} \quad \text{and} \quad i_b = -\frac{v_b}{Z_c}$$

then

$$v_f = Z_c i_f \quad \text{and} \quad v_b = -Z_c i_b$$

Substituting them into equation (7.62),

$$-Z_c i_b = \rho Z_c i_f$$

Therefore,

$$i_b = -\rho i_f \tag{7.67}$$

(b) Since

$$\rho = \frac{Z - Z_c}{Z + Z_c}$$

then

$$\rho + 1 = \frac{Z - Z_0}{Z + Z_0} + 1$$

$$= \frac{2Z}{Z + Z_0}$$

Therefore,

$$\tau = \rho + 1 \tag{7.68}$$

EXAMPLE 7.4

A line has a characteristic impedance of $400\,\Omega$ and a resistance of $500\,\Omega$. Assume that the magnitudes of forward-traveling voltage and current waves are 5000 V and 12.5 A, respectively. Determine the following:

(a) Reflection coefficient of voltage wave.
(b) Reflection coefficient of current wave.
(c) Backward-traveling voltage wave.
(d) Voltage at end of line.
(e) Refraction coefficient of voltage wave.
(f) Backward-traveling current wave.
(g) Current flowing through resistor.
(h) Refraction coefficient of current wave.

Solution

(a) $\rho = \dfrac{R - Z_c}{R + Z_c} = \dfrac{500 - 400}{500 + 400} = 0.1111$

(b) $\rho = -\dfrac{R - Z_c}{R + Z_c} = -\dfrac{500 - 400}{500 + 400} = -0.1111$

(c) $v_b = \rho v_f = 0.1111 \times 5000 = 555.555\text{ V}$

(d) $v = v_f + v_b = 5000 + 555.555 = 5555.555\text{ V}$

or

$$v = \frac{2R}{R + Z_c}\, v_f = \frac{2 \times 500}{500 + 400} \times 5000 = 5555.555\text{ V}$$

(e) $\tau = \dfrac{2R}{R + Z_c} = \dfrac{2 \times 500}{500 + 400} = 1.1111$

(f) $i_b = -\dfrac{v_b}{Z_c} = -\dfrac{555.555}{400} = -1.3889\text{ A}$

or

$i_b = -\rho i_f = -0.1111 \times 12.5 \cong -1.3889\text{ A}$

(g) $i = \dfrac{v}{R} = \dfrac{5555.555}{500} = 11.1111\text{ A}$

(h) $\tau = \dfrac{2Z_c}{R + Z_c} = \dfrac{2 \times 400}{500 + 400} = 0.8889$

7.3.3 Open-Circuit Line Termination

The boundary condition for current is

$$i = 0 \tag{7.69}$$

Therefore,

$$i_f = -i_b \tag{7.70}$$

Substituting this in equations (7.17) and (7.18),

$$v_b = Z_c i_b = Z i_f = v_f \tag{7.71}$$

Thus the total voltage at the receiving end is

$$v = v_f + v_b = 2v_f \tag{7.72}$$

Therefore, the voltage at the open end of the line is twice the forward voltage wave, as shown in Figure 7.6(*a*).

7.3.4 Short-Circuit Line Termination

The boundary condition for voltage at the short-circuited receiving end is

$$v = 0 \tag{7.73}$$

Therefore,

$$v_f = -v_b \tag{7.74}$$

Substituting this in equations (7.20) and (7.21),

$$i_f = \frac{v_f}{Z_c} = -\frac{v_b}{Z_c} = i_b \tag{7.75}$$

Thus the total current at the receiving end is

$$i = i_f + i_b = 2i_f \tag{7.76}$$

Therefore, the current at the short-circuited end of the line is twice the forward current wave, as shown in Figure 7.6(*b*).

7.3.5 Overhead Line Termination by Transformer

It is a well-known fact that at high frequencies, the voltage distribution across each winding is modified due to the capacitive currents between the transformer windings, between the turns of each winding, and between each winding and the grounded iron core. The impact of high velocity of a surge on a transformer is similar. Therefore, the resulting capacitive voltage distribution can be represented in the same manner as the one for a string of

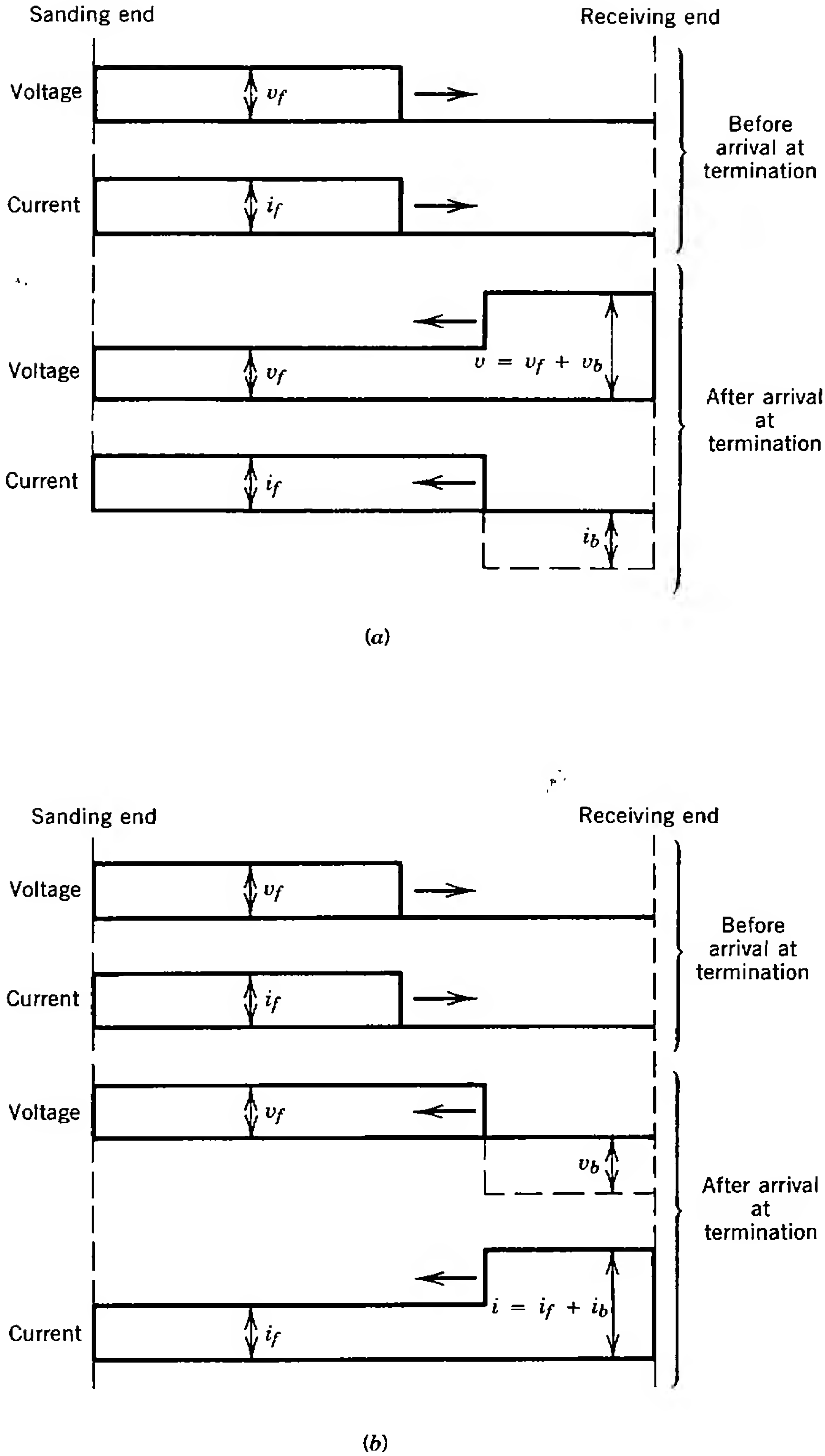


Figure 7.6. Analysis of traveling waves for: (*a*) open-circuit line termination; (*b*) short-circuit line termination.

suspension insulators. Thus, the maximum voltage gradient tàkes place at the winding turns nearest to the line conductor. Hence, when wye-connected transformers are employed in grounded neutral systems, their winding insulations are graded by more heavily insulating the winding turns closer to the line. Furthermore, the magnitude of the voltage surge can be reduced before it arrives at the transformer by putting in a short cable between the overhead line and the transformer. In addition to the reduction in the magnitude of the voltage wave, the steepness is also reduced due to the capacitance of the cable. Therefore, the voltage distribution along the windings of the apparatus is further reduced.

7.4 JUNCTION OF TWO LINES

Figure 7.7 shows a simple junction between two lines. Assume that $Z_{c1} > Z_{c2}$ where Z_{c1} and Z_{c2} are the characteritic impedances of the first and second lines, respectively. For example, Figure 7.7 might represent the

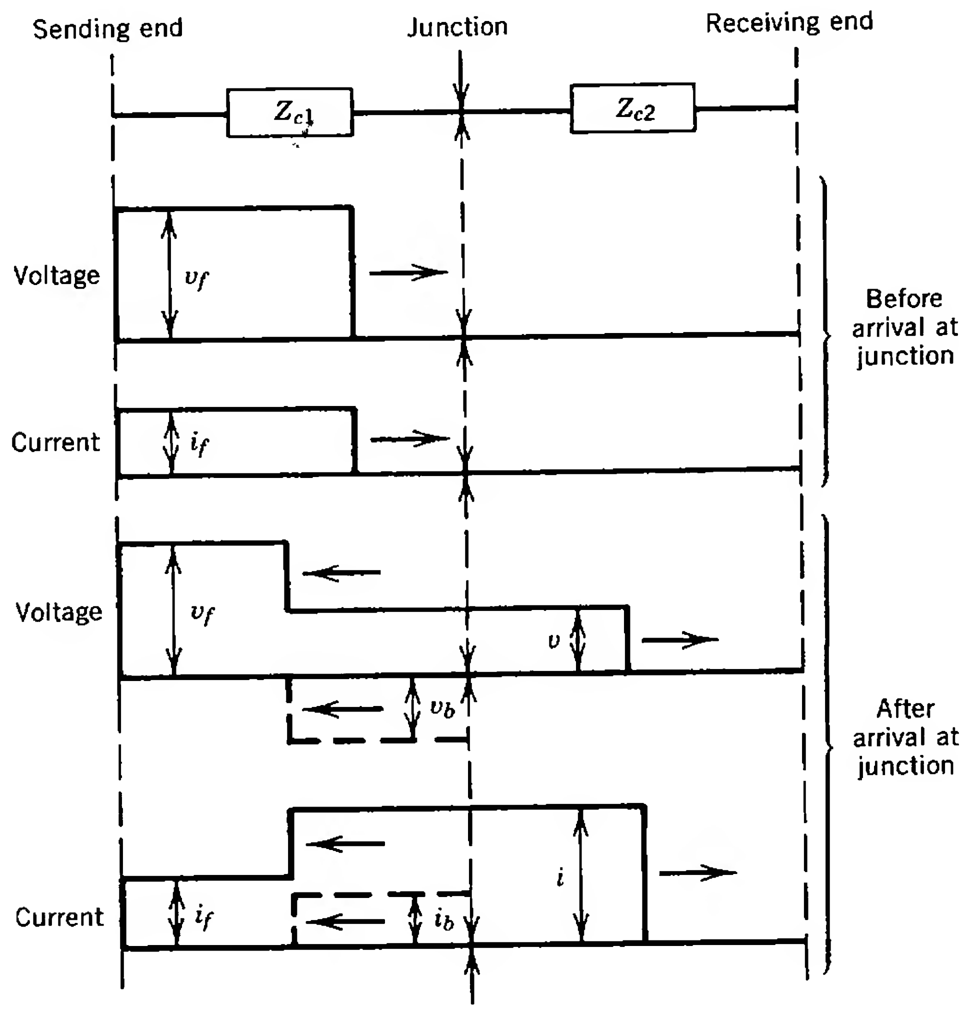


Figure 7.7. Traveling voltage and current waves being reflected and transmitted at junction between two lines.

junction between an overhead line and an underground cable. If a voltage surge of step function form and amplitude v_f approaches the junction along the overhead line, the current wave will have the same shape and an amplitude of

$$i_f = \frac{v_f}{Z_{c1}} \tag{7.77}$$

Therefore, after the arrival at the junction,

$$i_b = \frac{v_b}{Z_{c1}} \tag{7.78}$$

$$i = \frac{v}{Z_{c2}} \tag{7.79}$$

Since

$$\text{Refracted or transmitted wave} = \text{forward wave} + \text{backward wave} \tag{7.80}$$

then

$$v_f + v_b = v \tag{7.81}$$

$$i_f + i_b = i \tag{7.82}$$

Substituting equations (7.77)–(7.79) into equation (7.82),

$$\frac{v_f}{Z_{c1}} - \frac{v_b}{Z_{c1}} = \frac{v}{Z_{c2}} \tag{7.83}$$

by multiplying equation (7.83) by Z_{c1} and adding the resulting equation and equation (7.82) side by side yields

$$2v_f = \left(1 + \frac{Z_{c1}}{Z_{c2}}\right)v \tag{7.84}$$

Thus, the transmitted (i.e., refracted) voltage and current waves can be expressed as

$$v = \frac{2Z_{c2}}{Z_{c1} + Z_{c2}} v_f \tag{7.85}$$

and

$$i = \frac{2Z_{c1}}{Z_{c1} + Z_{c2}} i_f \tag{7.86}$$

The reflected (i.e., backward) voltage and current waves can be expressed as

$$v_b = \frac{Z_{c2} - Z_{c1}}{Z_{c1} + Z_{c2}} v_f \tag{7.87}$$

and

$$i_b = \frac{Z_{c1} - Z_{c2}}{Z_{c1} + Z_{c2}} i_f \tag{7.88}$$

The sign change between equations (7.87) and (7.82) is because of the negative sign in equation (7.20).

The power in the forward wave arriving at the junction is

$$P_f = \frac{v_f^2}{Z_{c1}} \tag{7.89}$$

and the transmitted wave power is

$$P = \frac{v^2}{Z_{c2}} \tag{7.90}$$

Similarly, the power in the backward wave is

$$P_b = \frac{v_b^2}{Z_{c1}} \tag{7.91}$$

EXAMPLE 7.5

Assume that an overhead line is connected in series with an underground cable. The surge (i.e., characteristic) impedances of the overhead line and cable are 400 and 40 Ω, respectively. The forward-traveling surge voltage is 200 kV and is traveling toward the junction from the sending end of the overhead line.

(a) Determine the magnitude of the forward current wave.
(b) Determine the reflection coefficient.
(c) Determine the refraction coefficient.
(d) Determine the surge voltage transmitted forward into the cable.
(e) Determine the surge current transmitted forward into the cable.
(f) Determine the surge voltage reflected back along the overhead line.
(g) Determine the surge current reflected back along the overhead line.
(h) Plot voltage and current surges showing them after arriving at the junction.

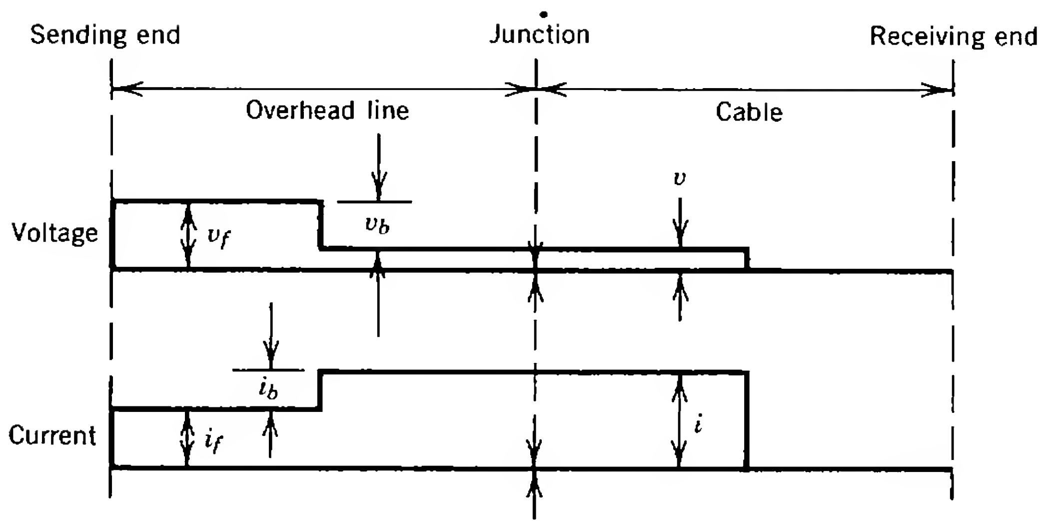


Figure 7.8

Solution

(a) $i_f = \dfrac{v_f}{Z_{c1}} = \dfrac{200{,}000}{400} = 500\text{ A}$

(b) $\rho = \dfrac{Z_{c2} - Z_{c1}}{Z_{c1} + Z_{c2}} = \dfrac{40 - 400}{400 + 40} = -0.8182$

(c) $\tau = \dfrac{2Z_{c2}}{Z_{c1} + Z_{c2}} = \dfrac{2 \times 40}{400 + 40} = 0.1818$

(d) $v = \tau v_f = 0.1818 \times 200 = 36.36\text{ kV}$

(e) $i = \dfrac{v}{Z_{c2}} = \dfrac{36{,}360}{40} = 909\text{ A}$

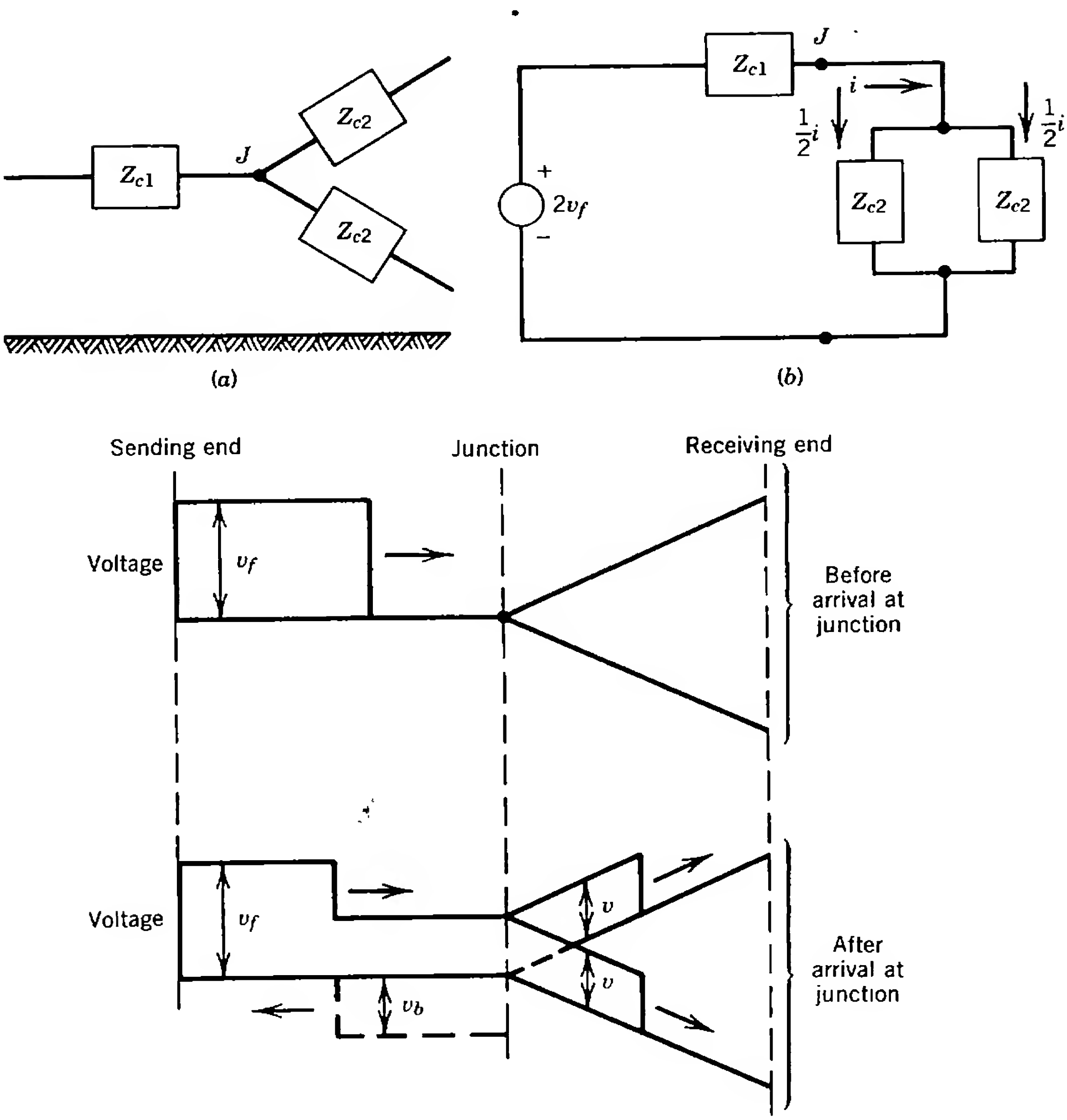
(f) $v_b = \rho v_f = -0.8182 \times 200 = -163.64\text{ kV}$

(g) $i_b = -\rho i_f = 0.8182 \times 500 = 409\text{ A}$

(h) Figure 7.8 shows the plot of the voltage and current surges after arrival at the junction.

7.5 JUNCTION OF SEVERAL LINES

Figure 7.9(a) shows the analysis of a traveling voltage wave encountering a line bifurcation made of two lines having equal surge impedances Z_{c2}. Figure 7.9(b) shows the corresponding equivalent circuit. Figure 7.9(c) shows the traveling voltage wave being reflected and transmitted at the junction J. Note that at the junction, the impedance seen is that due to the two equal surge impedances Z_{c2} in parallel. Therefore, equations developed in the previous section are applicable as long as Z_{c2} is replaced by $\frac{1}{2}Z_{c2}$. For



(c)

Figure 7.9. Traveling voltage wave encountering line bifurcation: (*a*) system; (*b*) equivalent circuit; (*c*) traveling voltage wave reflected and transmitted at junction of three lines.

example, the transmitted (i.e., refracted) voltage and current can be expressed as

$$v = \frac{2v_f}{Z_{c1} + Z_{c2}/2} \frac{Z_{c2}}{2} \tag{7.92}$$

and

$$i = \frac{2Z_{c1}}{Z_{c1} + Z_{c2}/2} i_f \tag{7.93}$$

where

$$i_f = \frac{2v_f}{Z_{c1} + Z_{c2}/2} \tag{7.94}$$

7.6 TERMINATION IN CAPACITANCE AND INDUCTANCE

7.6.1 Termination through Capacitor

Assume that a line is terminated in a capacitor, as shown in Figure 7.10(*a*). Figure 7.10(*b*) shows its equivalent circuit. From equation (7.60), the refraction coefficient can be expressed as

$$\tau = \frac{2Z}{Z + Z_c} \tag{7.95}$$

or in Laplace transform as

$$\tau = \frac{2(1/Cs)}{Z_c + 1/Cs} \tag{7.96}$$

where s is the Laplace transform operator. Therefore, the refracted voltage can be found as

$$v = \tau v_f$$

or

$$\begin{aligned} v(s) &= \frac{2(1/Cs)}{Z_c + 1/Cs}\,\frac{v_f}{s} = \frac{2v_f}{s}\,\frac{1}{Z_cC_s + 1} \\ &= \frac{2v_f}{s}\,\frac{1/Z_cC}{s + 1/Z_cC} \\ &= 2v_f\,\frac{1}{s} - \frac{1}{s + 1/Z_cC} \end{aligned} \tag{7.97}$$

so that

$$v(t) = 2v_f(1 - e^{-t/Z_cC}) \tag{7.98}$$

where $v_f(t)$ is not a traveling wave, but the voltage that will be impressed across the capacitor.

The current flowing through capacitor C is

$$i(t) = \frac{2v_f}{Z_c}\,e^{-t/Z_cC} \tag{7.99}$$

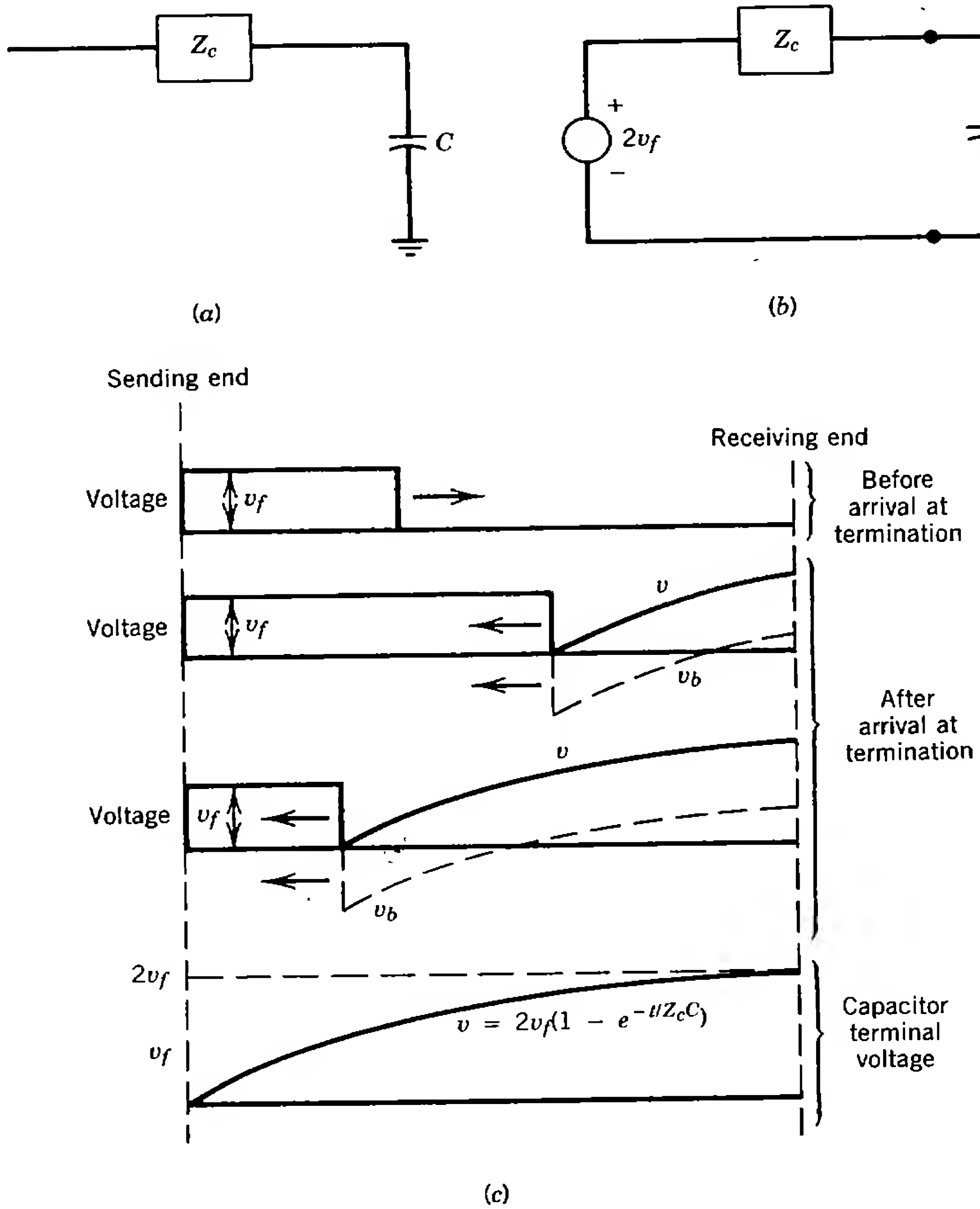


Figure 7.10. Traveling voltage wave on line with capacitive termination: (*a*) system; (*b*) equivalent circuit; (*c*) disposition of wave at various instants.

The reflected voltage wave can be expressed as

$$v_b(t) = v_f(1 - 2e^{-t/Z_cC}) \tag{7.100}$$

Figure 7.10(*c*) shows the disposition of the voltage wave at various instants. Succinctly put, the capacitor acts as a short circuit at the instant of arrival of the forward wave. At this moment, the reflected voltage wave is negative because the terminal voltage is momentarily zero, and the current of the forward wave is momentarily doubled. As the capacitor becomes fully charged, it behaves as an open circuit. Thus, its terminal current becomes zero and its voltage becomes equal to twice of the forward voltage wave.

7.6.2 Termination through Inductor

Assume that the capacitor C shown in Figures 7.10(a) and 7.10(b) has been replaced by an inductor L. The circuit behaves like an open-circuited line initially (since a current cannot flow through the inductor instantaneously) but finally acts like a short-circuited line. Therefore, the inductive termination is the dual of the capacitive termination. From the equivalent circuit, the voltage across the inductor is

$$v(t) = 2v_f e^{-(Z_c/L)t} \tag{7.101}$$

Thus, its voltage starts at a value twice that of the forward wave and eventually becomes zero. At that time, the current flowing through the inductor is

$$i(t) = \frac{2v_f}{Z_c}\left(1 - e^{-(Z_c/L)t}\right) \tag{7.102}$$

The reflected voltage wave is

$$v_b(t) = v(t) - v_f(t) \tag{7.103}$$

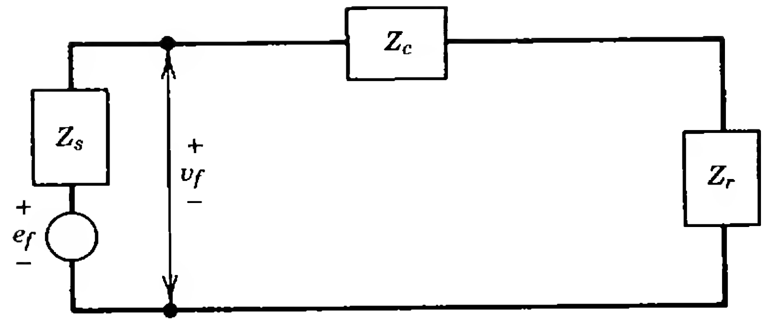
or

$$v_b(t) = v_f(2e^{-(Z_c/L)t} - 1) \tag{7.104}$$

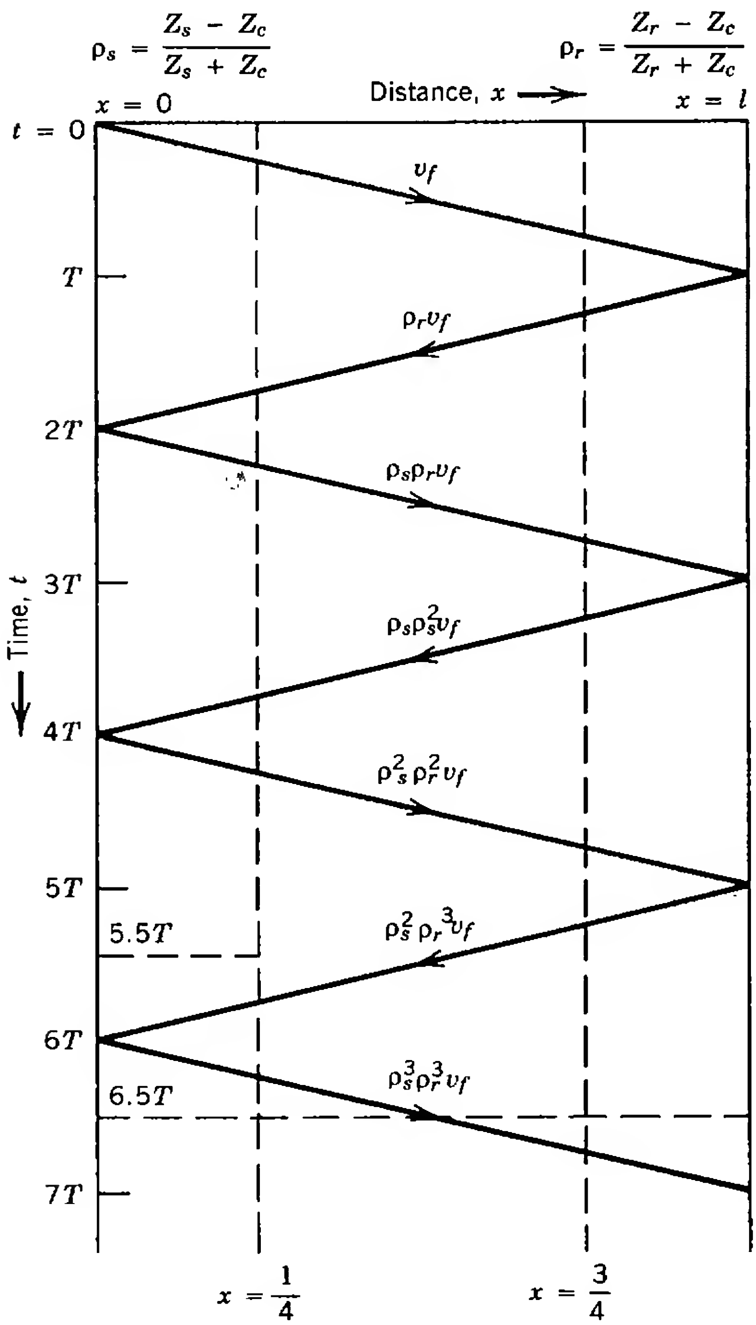
7.7 BEWLEY LATTICE DIAGRAM

The *bounce diagram*, developed by Bewley [1], determines the voltages at a given point and time in a transmission system. It is a useful visual aid to keep track of traveling voltage or current wave as it reflects back and forth from the ends of the line, as shown in Figure 7.11.

Figure 7.11(a) shows the circuit diagram where Z_s and Z_r represent internal source impedance and impedance connected at the end of the line, respectively. In the lattice diagram, the distance between the sending and receiving ends is represented by the horizontal line drawn to scale, and time is represented by the two vertical lines scaled in time. T is the time for a wave to travel the line length. The diagonal zigzag line represents the wave as it travels back and forth between the ends or discontinuities. The slopes of the zigzag lines gives the times corresponding to the distances traveled.



(a)



(b)

Figure 7.11. Bewley lattice diagram: (a) circuit diagram; (b) lattice diagram.

The reflections are determined by multiplying the incident waves by the appropriate reflection coefficient. The voltage at a given point in time and distance is found by adding all terms that are directly above that point. For example, the voltage at $t = 5.5T$ and $x = \frac{1}{4}l$ is

$$v(\tfrac{1}{4}l, 5.5T) = v_f(1 + \rho_r + \rho_s\rho_r + \rho_s\rho_r^2 + \rho_s^2\rho_r^2)$$

whereas the voltage at $t = 6.5T$ and $x = \frac{3}{4}l$ is

$$v(\tfrac{3}{4}l, 6.5T) = v_f(1 + \rho_r + \rho_s\rho_r^2 + \rho_s^2\rho_r^2 + \rho_s^2\rho_r^3)$$

Of course, lattice diagrams for current can also be drawn. However, the fact that the reflection coefficient for current is always the negative of the reflection coefficient for voltage should be taken into account.

Example 7.6

Consider the circuit diagram shown in Figure 7.11(a) and assume that the dc source is a 1000-V ideal voltage source so that its internal impedance Z_s is zero and it is connected to the sending end of an underground cable with characteristic impedance of 40 Ω. Assume the cable is terminated in a 60-Ω resistor.

(a) Determine the reflection coefficient at the sending end.
(b) Determine the reflection coefficient at the receiving end.
(c) Draw the associated lattice diagram showing the value of each reflected voltage.
(d) Determine the value of voltage at $t = 6.5T$ and $x = \frac{3}{4}l$.
(e) Plot the receiving-end voltage versus time.

Solution

(a) $\rho_s = \dfrac{Z_s - Z_c}{Z_s + Z_c} = \dfrac{0 - 40}{0 + 40} = -1$

(b) $\rho_r = \dfrac{Z_r - Z_c}{Z_r + Z_c} = \dfrac{60 - 40}{60 + 40} = 0.2$

(c) The lattice diagram is shown in Figure 7.12(a).
(d) From Figure 7.12(a), the voltage value is 1008 V.
(e) The plot of the receiving-end voltage versus time is shown in Figure 7.12(b):

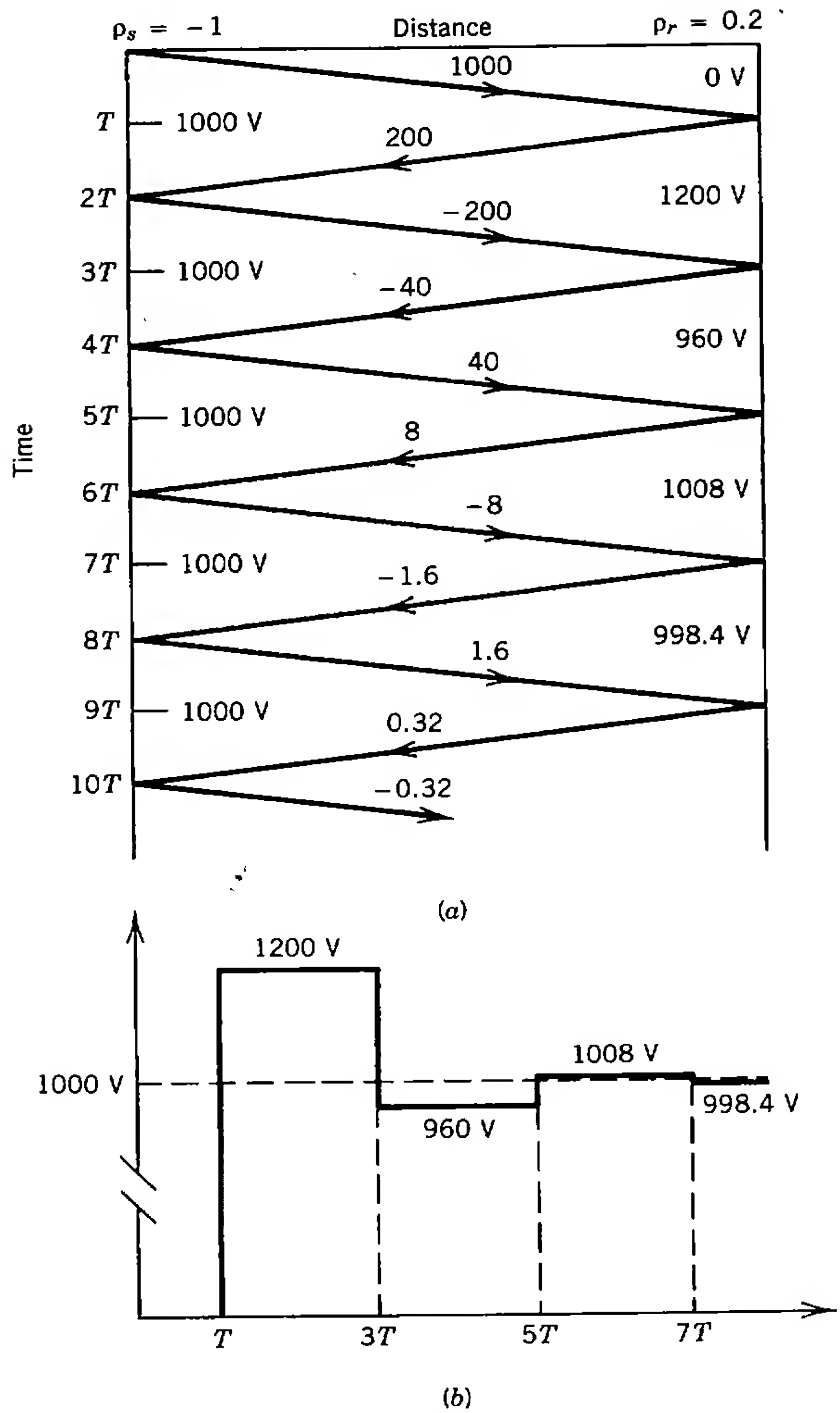


Figure 7.12

7.8 SURGE ATTENUATION AND DISTORTION

In general, in addition to the effects of reflections at transition points, traveling waves are also subject to both attenuation (decrease in magnitude) and distortion (change in shape) as they propagate along the line. They are caused by losses in the energy of the wave due to resistance, leakage, dielectric, and corona. Corona is the main cause of attenution at high voltages. It reduces the magnitude and the steepness of the wavefronts within a few miles to a safe voltage. If the attenuation cannot be neglected, then, in Figure 7.11, the amplitude of the wave after each reflection should be reduced by factor of $e^{-\alpha l}$ due to the fact that the wave is attenuated by that amount of each transit.

The values of voltage and current waves can be found by taking the power and losses into account as they travel over a length dx of a line. The power loss in the differential element dx can be expressed as

$$dp = i^2 R\, dx + v^2 G\, dx \tag{7.105}$$

where R and G are the resistance and conductance of per-unit length of the line, respectively. Since

$$p = vi = i^2 Z_c$$

then the differential power is

$$dp = -2iZ_c\, di \tag{7.106}$$

The negative sign reflects the fact that the dp represents reduction in power. Thus, substituting equation (7.106) into equation (7.105)

$$-2iZ_c\, di = i^2 R\, dx + v^2 G\, dx$$

or

$$\frac{di}{i} = -\frac{R + GZ_c^2\, dx}{2Z_c}$$

At $x = 0$, $i = i_f$, so that

$$i = i_f \exp\left(-\frac{R + GZ_c^2}{2Z_c}\, x\right) \tag{7.107}$$

Similarly,

$$v = v_f \exp\left(-\frac{R + GZ_c^2}{2Z_c}\, x\right) \tag{7.108}$$

7.9 TRAVELING WAVES ON THREE-PHASE LINES

Even though the basic traveling-wave equations remain unchanged when a three-phase system is considered, mutual coupling exists between the phases of the system and must be included in any calculation. In single-phase overhead conductor applications, it can be seen that the presence of line losses attenuates and retards a traveling voltage wave along the line. Whereas, in three-phase overhead conductor applications, the situation is much more complex due to the fact that mutual exists between the phases, which causes second-order changes of voltage on each phase to be functions of the voltages on the other conductors. Therefore, losses cannot be represented by simply attenuating and retarding the voltages in each phase. The solution to this problem can be found using an appropriate transformation matrix to diagonalize the matrix equations. Thus, the line can be represented by a number of modes of propagation so that the associated voltages travel uncoupled, that is, independently of each other. Hence the losses can be introduced to each mode as it is done in the single-phase system.

For completely balanced lines, there are a number of simple transformation matrices that decouple the line equations. One such matrix for three-phase lines is the transformation matrix of the Clark components, that is, α, β, and 0 components. Since its elements are real, it is well suited for transient analyses. Therefore, the phase voltages can be transformed into the modal domain using the modal transformation so that [2]

$$\begin{bmatrix} \mathbf{v}_a \\ \mathbf{v}_b \\ \mathbf{v}_c \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0 \\ \mathbf{v}_\alpha \\ \mathbf{v}_\beta \end{bmatrix} \tag{7.109}$$

or

$$[\mathbf{v}_{abc}] = [T_c][\mathbf{v}_{0\alpha\beta}] \tag{7.110}$$

where the subscripts represent the modal quantities. Since the matrix $[T_c]$ is a unitary matrix, its inverse matrix can be found easily as

$$[T_c]^{-1} = [T_c]^{t} \tag{7.111}$$

Similarly, it can be shown that

$$[\mathbf{i}_{abc}] = [T_c][\mathbf{i}_{0\alpha\beta}] \tag{7.112}$$

Equation (7.109) can be expressed using the Laplace transforms as

$$\begin{bmatrix} \mathbf{v}_a(s) \\ \mathbf{v}_b(s) \\ \mathbf{v}_c(s) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \mathbf{v}_0(s) \\ \mathbf{v}_\alpha(s) \\ \mathbf{v}_\beta(s) \end{bmatrix} \tag{7.113}$$

or

$$[\mathbf{v}_{abc}(s)] = [T_c][\mathbf{v}_{0\alpha\beta}(s)] \tag{7.114}$$

Similarly, equation (7.112) can be expressed as

$$\begin{bmatrix} \mathbf{I}_a(s) \\ \mathbf{I}_b(s) \\ \mathbf{I}_c(s) \end{bmatrix} = \begin{bmatrix} 1/\sqrt{3} & 1/\sqrt{6} & 1/\sqrt{2} \\ 1/\sqrt{3} & -2/\sqrt{6} & 0 \\ 1/\sqrt{3} & 1/\sqrt{6} & -1/\sqrt{2} \end{bmatrix} \begin{bmatrix} \mathbf{I}_0(s) \\ \mathbf{I}_\alpha(s) \\ \mathbf{I}_\beta(s) \end{bmatrix} \tag{7.115}$$

or

$$[\mathbf{I}_{abc}(s)] = [T_c][\mathbf{I}_{0\alpha\beta}(s)] \tag{7.116}$$

Therefore,

$$[\mathbf{v}_{0\alpha\beta}(s)] = [\mathbf{Z}_{0\alpha\beta}(s)][\mathbf{I}_{0\alpha\beta}(s)] \tag{7.117}$$

where the matrix $[\mathbf{Z}_{0\alpha\beta}(s)]$ is found using the similarity transformation. Therefore,

$$[\mathbf{Z}_{0\alpha\beta}(s)] = [T_c]^{-1}[\mathbf{Z}_{abc}][T_c] \tag{7.118}$$

or

$$[\mathbf{Z}_{0\alpha\beta}(s)] = [T_c]^{t}[\mathbf{Z}_{abc}][T_c] \tag{7.119}$$

If the line is completely balanced, the result of equation (7.119) becomes

$$\begin{bmatrix} \mathbf{Z}_0(s) \\ \mathbf{Z}_\alpha(s) \\ \mathbf{Z}_\beta(s) \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_0 & 0 & 0 \\ 0 & \mathbf{Z}_\alpha & 0 \\ 0 & 0 & \mathbf{Z}_\beta \end{bmatrix} \tag{7.120}$$

where

$$\mathbf{Z}_0 = \mathbf{Z}_s + 2\mathbf{Z}_m \tag{7.121}$$

$$\mathbf{Z}_\alpha = \mathbf{Z}_\beta = \mathbf{Z}_s - \mathbf{Z}_m \tag{7.122}$$

$$\mathbf{Z}_s = R_s + sL_s \tag{7.123}$$

$$\mathbf{Z}_m = R_m + sL_m \tag{7.124}$$

$$R_s = 2R \tag{7.125}$$

$$R_m = R \qquad s = jw \tag{7.126}$$

Gross [2, 3] shows that in the lossless case the modal series impedances can be directly determined from sequence impedances as

$$\mathbf{Z}_0 = \frac{s}{jw}\mathbf{Z}_0 \tag{7.127}$$

$$\mathbf{Z}_\alpha = \mathbf{Z}_\beta = \frac{s}{jw}\mathbf{Z}_1 \tag{7.128}$$

After transforming terminal conditions into $0\alpha\beta$ components, the $0\alpha\beta$ decoupled line models can be developed, and therefore the transient response of each mode can be determined separately from each other.

Hedman [4] shows that the multiphase reflection coefficient can be developed from a transmission line terminated in a resistive network. The equations that are applicable for a multiphase network are similar to those for the single-phase case, with the exception that each term is a matrix. Therefore,

$$[v] = [v_f] + [v_b] \tag{7.129}$$

$$[i] = [i_f] + [i_b] \tag{7.130}$$

$$[v_f] = [Z_l][i_f] \tag{7.131}$$

$$[v_b] = -[Z_l][i_b] \tag{7.132}$$

$$[v] = [Z][i] \tag{7.133}$$

where the voltage and current matrices are column vectors and the impedance matrices are square matrices. Eliminating terms in equations (7.129) through (7.133) and expressing the results in terms of $[v_f]$ and $[v_b]$ results in

$$[v_f] + [v_b] = [Z]\{[Z_l]^{-1}[v_f] - [Z_l]^{-1}[v_b]\} \tag{7.134}$$

premultiplying by $[Z]^{-1}$ and getting coefficients for $[v_f]$ and $[v_b]$ results in

$$\{[Z_l]^{-1} - [Z]^{-1}[v_f] = [Z]^{-1} + [Z_l]^{-1}\}[v_b] \tag{7.135}$$

Thus, the reflected voltage matrix in terms of forward voltages can be expressed as

$$[v_b] = \{[Z]^{-1} + [Z_l]^{-1}\}^{-1}\{[Z_l]^{-1} - [Z]^{-1}\}[v_f] \tag{7.136}$$

Therefore, the reflection coefficient for the multiphase case† can be determined from

$$[\rho] = \{[Z]^{-1} + [Z_l]^{-1}\}^{-1}\{[Z_l]^{-1} - [Z]^{-1}\} \tag{7.137}$$

Today, there are various analog or digital computer techniques that have been developed to study the traveling-wave phenomenon on large systems. Existing digital computer programs are based on mathematical models using either differential equations or Fourier or Laplace transforms. It is interest-

† Those who are interested in the application of matrix methods to the solution of traveling waves should also read Wedepohl [5], Uram et al. [6, 7], Hedman [8, 9], Virmani et al. [10], Dommel [11], and Dommel and Meyer [12].

ing to note that the most rapid and least expensive computer solutions now known have been obtained with hybrid computers.

7.10 LIGHTNING AND LIGHTNING SURGES

7.10.1 Lightning

By definition, lightning is an electrical discharge. It is the high-current discharge of an electrostatic electricity accumulation between cloud and earth or between clouds. The mechanism by which a cloud becomes electrically charged is not yet fully understood. However, it is known that the ice crystals in an active cloud are positively charged while the water droplets usually carry negative charges. Therefore, a thundercloud has a positive center in its upper section and a negative charge center in its lower section. Electrically speaking, this constitutes a dipole. An interpretation of particle flow in relation to temperature and height is shown in Figure 7.13. Note that the charge separation is related to the supercooling, and occasionally even the freezing, of droplets. The disposition of charge concentrations is partially due to the vertical circulation in terms of updrafts and downdrafts.

As a negative charge builds up in the cloud base, a corresponding positive charge is induced on earth, as shown in Figure 7.14(*a*). The voltage gradient in the air between charge centers in cloud (or clouds) or between cloud and earth is not uniform, but it is maximum where the charge concentration is greatest. When voltage gradients within the cloud build up to the order of 5 to 10 kV/cm, the air in the region breaks down and an ionized path called *leader* or *leader stroke* starts to form, moving from the cloud up to the earth, as shown in Figure 7.14(*b*). The tip of the leader has a speed between 10^5 and 2×10^5 m/s (i.e., less than one-thousandth of the speed of light of 3×10^8 m/s) and moves in jumps. If photographed by a camera the lens of which is moving from left to right, the leader stroke would appear as shown in Figure 7.15. Therefore, the formation of a lightning stroke is a progressive breakdown of the arc path instead of the complete and instantaneous breakdown of the air path from the cloud to the earth. As the leader strikes the earth, an extremely bright return streamer, called *return stroke*, propagates upward from the earth to the cloud following the same path, as shown in Figures 7.14(*c*) and 7.15. In a sense, the return stroke establishes an electric short circuit between the negative charge disposited along the leader and the electrostatically induced positive charge in the ground. Therefore, the charge energy from the cloud is released into the ground, neutralizing the charge centers. The initial speed of the return stroke is 10^8 m/s. The current involved in the return stroke has a peak value from 1 to 200 kA, lasting about 100 μs. About 40 μs later, a second leader, called *dart leader*, may stroke usually following the same path taken by the first

THE NATURE OF LIGHTNING

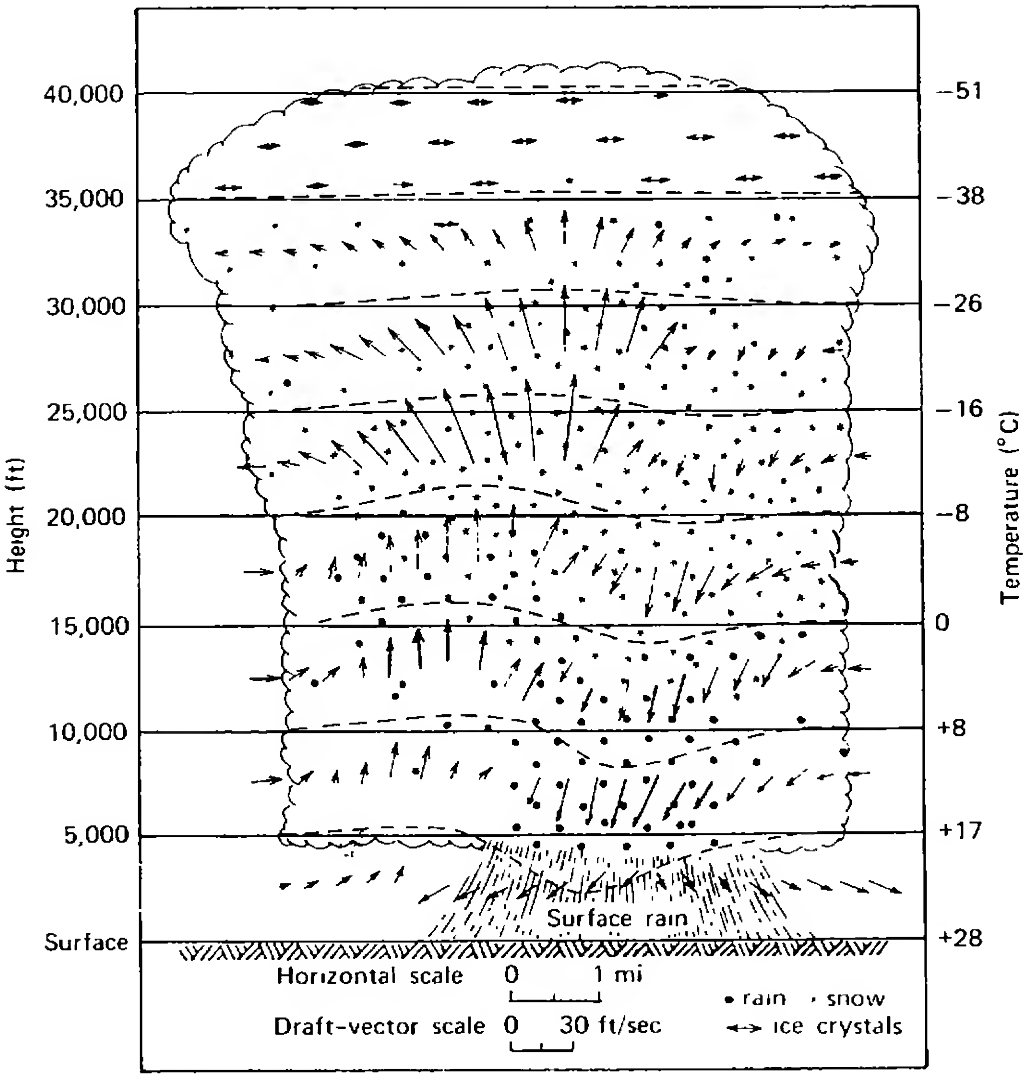


Figure 7.13. Idealized thunderstorm cloud *cell* in its mature state [20].

leader. The dart leader is much faster and has no branches and may be produced by discharge between two charge centers in the cloud, as shown in Figure 7.14(*e*). Note the distribution of the negative charge along the stroke path. The process of dart leader and return stroke [Figure 7.14(*f*)] *can be* repeated several times. The complete process of successive strokes is called *lightning flash*. Therefore, a lightning flash may have a single stroke or a sequence several discrete strokes (as many as 40) separated by about 40 ms, as shown in Figure 7.15.

7.10.2 Lightning Surges

The voltages produced on overhead lines by lightning† may be due to

† On extra-high-voltage lines, lightning is the greatest single cause of outages.

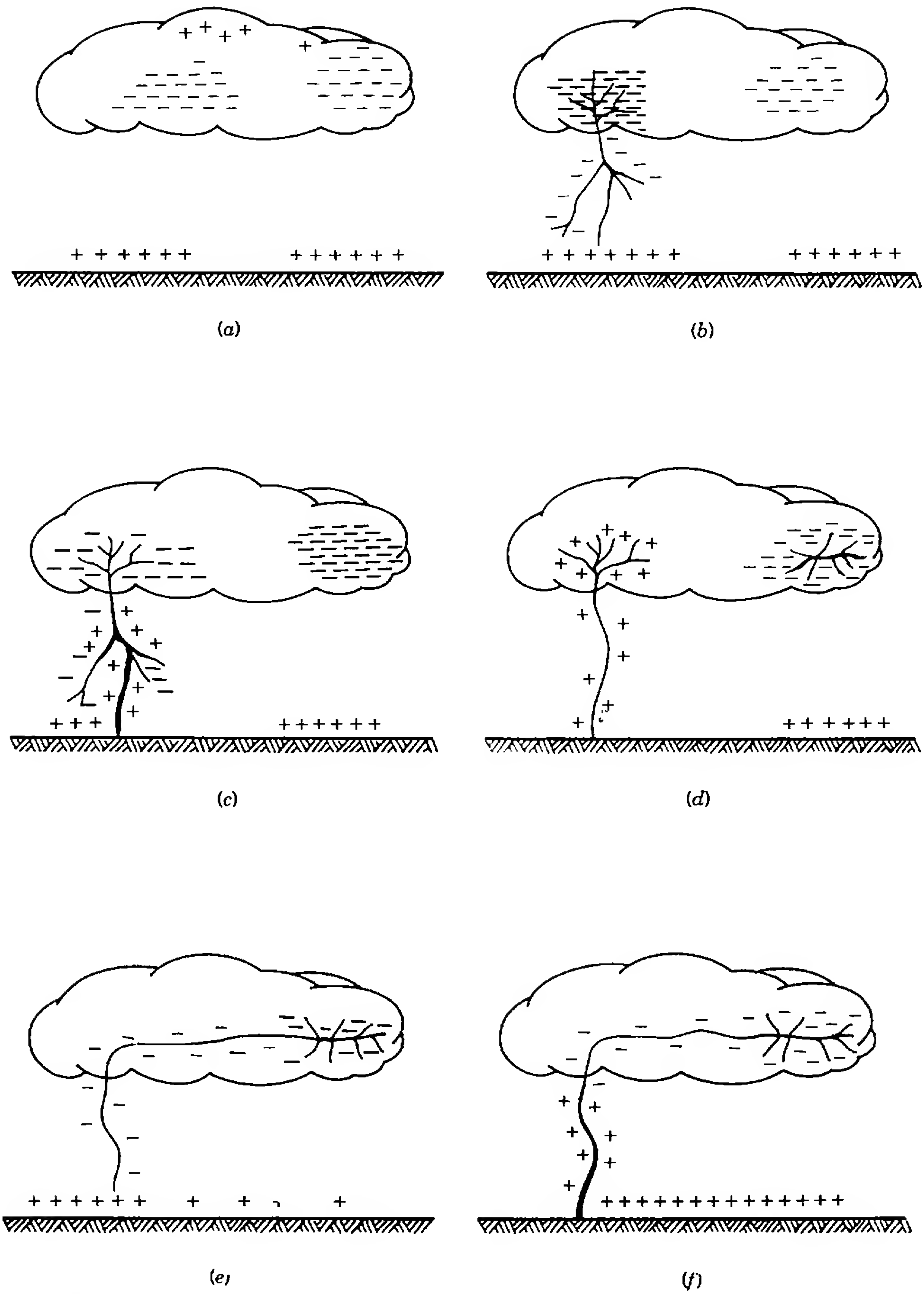


Figure 7.14. Charge distribution at various stages of lightning discharge: (a) charge centers in cloud and induced charge on ground; (b) leader stroke about to strike ground; (c) return stroke; (d) first charge center completely discharged; (e) dart leader; (f) return stroke.

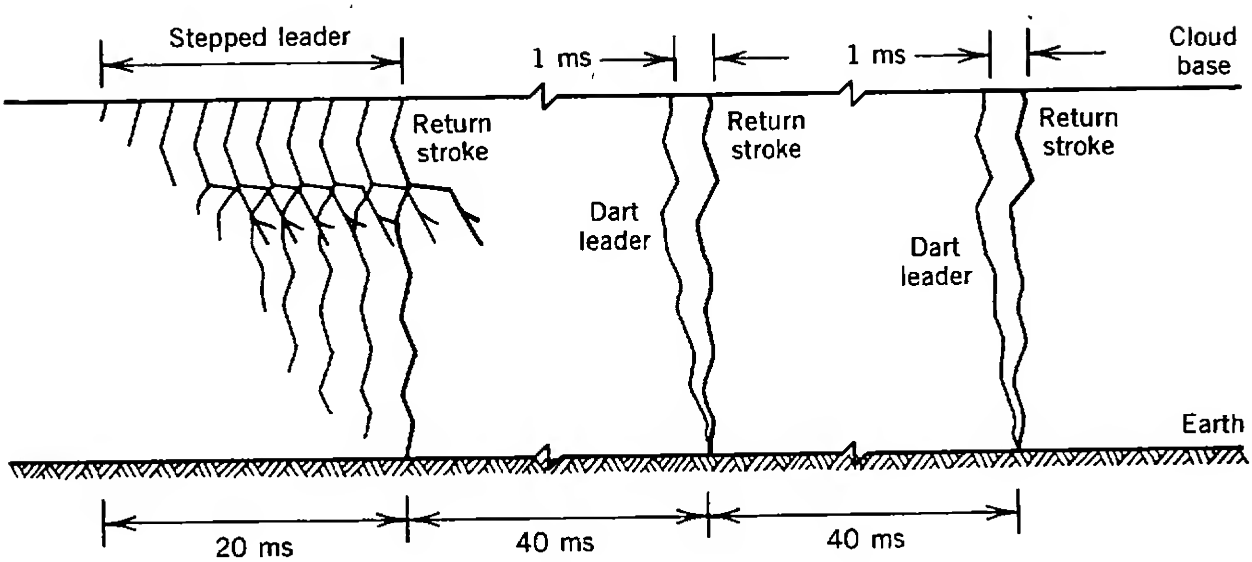


Figure 7.15. Mechanism of lightning flash.

indirect strokes or *direct strokes*. In the indirect stroke, induced charges can take place on the lines as a result of close by lightning strokes to ground. Even though the cloud and earth charges are neutralized through the established cloud-to-earth current path, a charge will be trapped on the line, as shown in Figure 7.16(*a*). Of course, the magnitude of this trapped charge is a function of the initial cloud-to-earth voltage gradient and the closeness of the stroke to the line. Such voltage may also be induced as a result of lightning among clouds, as shown in Figure 7.16(*b*). In any case, the voltage

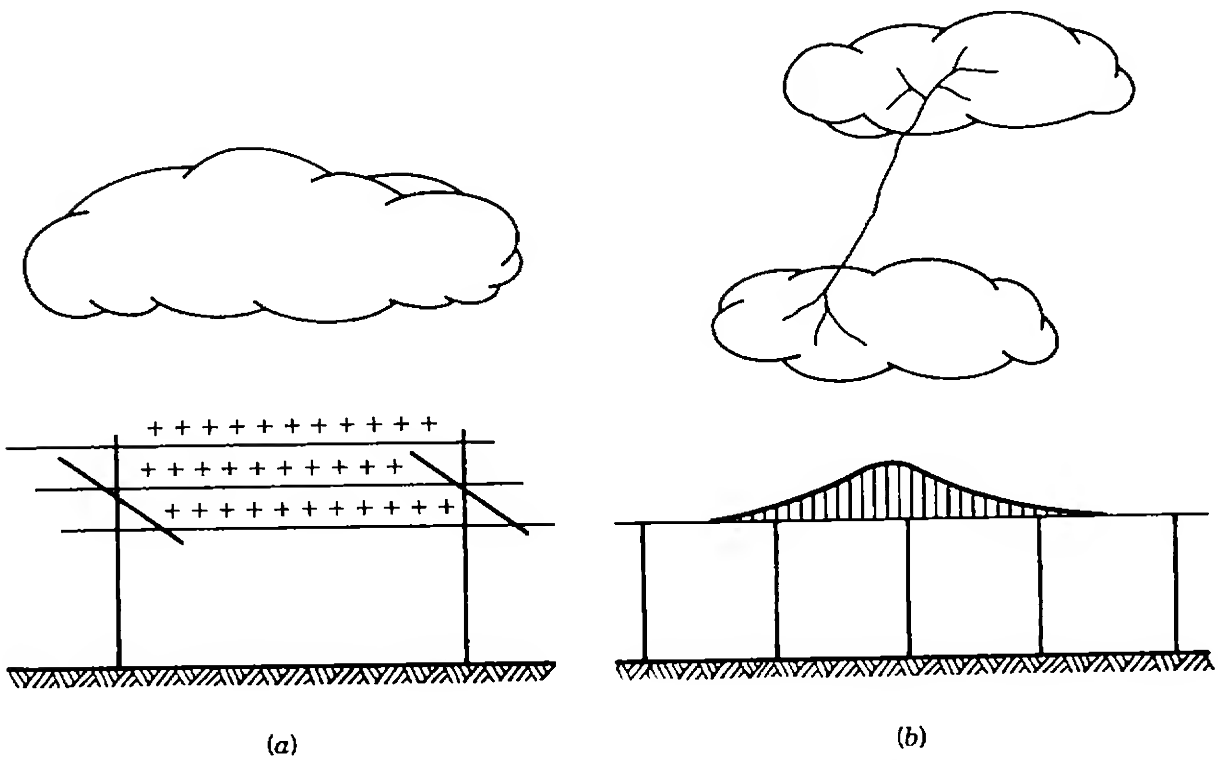


Figure 7.16. Induced line charges due to indirect lightning strokes.

induced on the line propages along the line as a traveling wave until it is dissipated by attenuation, leakage, insulation failure, or arrester operation.

In a direct stroke, the lightning current path is directly from the cloud to the line, causing the voltage to rise rapidly at the contact point. The contact point may be on the top of a tower, on the shield (overhead ground wires) wire, or on a line conductor. If lightning hits a tower top, some of the current may flow through the shield wires, and the remaining current flows through the tower to the earth. If the stroke is average in terms of both current magnitude and rate of rise, the current may flow into the ground without any harm provided that the tower and its footings have low resistance values. Otherwise, the lightning current will raise the tower to a high voltage above the ground, causing a flashover from the tower, over the line insulators, to one or more of the phase conductors. On the other hand, when lightning strikes a line directly, the raised voltage, at the contact point, propagates in the form of a traveling wave in both directions and raises the potential of the line to the voltage of the downward leader. If the line is not properly protected against such overvoltage, such voltage may exceed the line-to-ground withstand voltage of line insulation and cause insulation failure. Therefore, such insulation failure, or preferably arrester operation, establishes a path from the line conductor to ground for the *lightning surge current*.

If the lightning strikes an overhead ground wire somewhere between two adjacent towers, it causes traveling waves along the overhead ground wire. The lightning current flows to the ground at the towers without causing any damage provided that the surge impedance of towers and the resistance tower footings are not too high. Otherwise, the tower-top voltage is impressed across the line insulator strings and can cause a flashover, resulting in a *line outage*. It is possible that the arcing from the ground wire to the phase conductor may be sustained by the 60-Hz line voltage and can only be removed by deenergizing the line. This phenomenon is known as the *backflash*. It is most prevalent when footing resistances are high, but can also occur on tall towers with low footing reistances.

7.10.3 Lightning Performance of Transmission Lines

Figure 7.17 shows an isokeraunic map indicating the frequency of occurrence of thunderstorms throughout the United States. The contours represents the *isokeraunic level*, that is, the average number of thunderstorm days (i.e., days on which thunder could be heard) to be expected each year in different parts of the country. However, the isokeraunic level can vary widely from year to year and does not differentiate between cloud-to-cloud lightning (which cause little harm) and cloud-to-earth lightning (which is very destructive). In a region of average storm intensity of 30 thunderstorm days per year, a 100-mi-long transmission line will be struck on the average of 100 direct strokes per year. One of the most important parameters for the

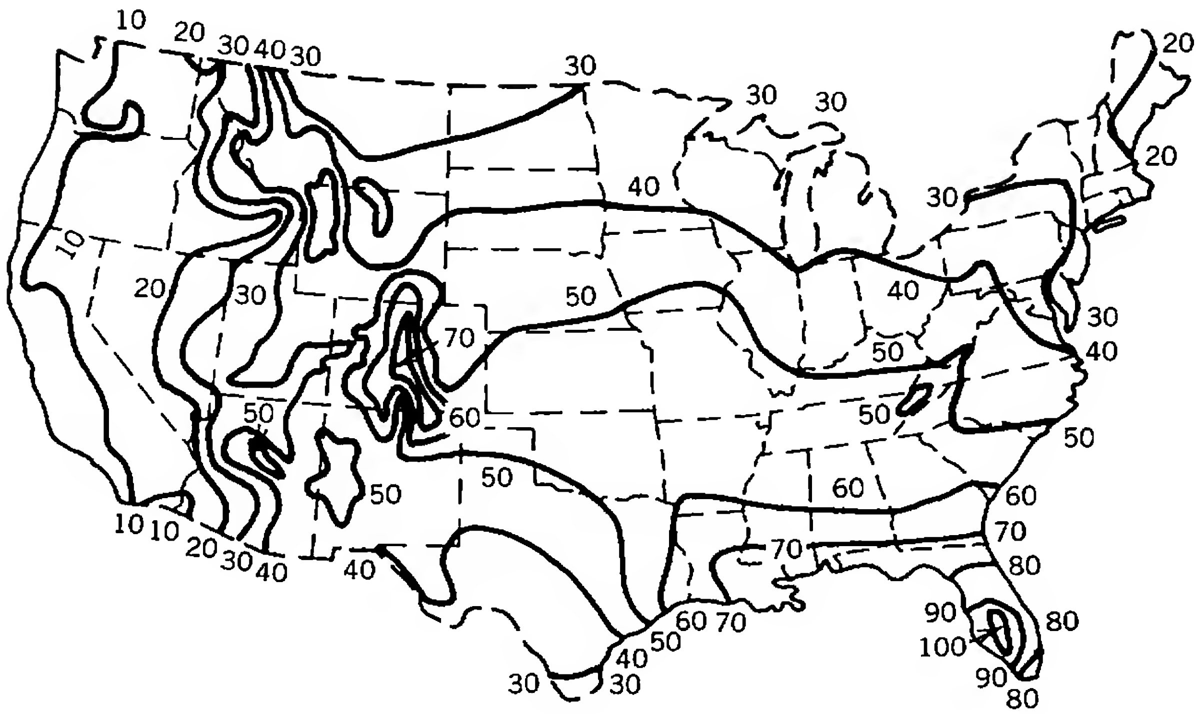


Figure 7.17. Isokeraunic map of United States showing average number of thunderstorm days per year. (U.S. Weather Bureau.)

line design is the probability of the actual flashover rate exceeding some specified value in a given year. The probability of having exactly k no flashovers (successes) for n flashes to the line can be expressed, based on the binomial distribution, as

$$P_k = \frac{n!}{k!(n-k)!} p^k q^{n-k} \tag{7.138}$$

where p = probability of having no flashover (succes) for single flash
q = probability of having flashover (failure) for single flash, $=1-p$
$n-k$ = flashover rate

As stated before, n is the number of flashes to earth per square mile per year in the vicinity of the line. Anderson [13] gives the following equations to estimate the number of flashes to earth per square or per square miles per year, respectively, as

$$n = 0.12T \quad \text{flashes/km}^2\text{-yr} \tag{7.139}$$

and

$$n = 0.31T \quad \text{flashes/mi}^2\text{-yr} \tag{7.140}$$

where T is the keraunic level in thunderstorm days per year in the area.

It is a well-known fact that a transmission line provides a protective shadow area on the earth beneath it so that any stroke normally hitting this area is drawn to the line instead. The line itself is protected by the shield wires. The degree of protection afforded in this way depends on the disposition of such overhead ground wires with respect to the phase conductors. The shield wires must have adequate clearance from the conductors, not only at the towers, but throughout the span. Of course, shield wires inevitably increase the number of strokes likely to terminate somewhere on the line, but this should not increase the number of outages. Figure 7.18 illustrates how such shadow can be determined for a single-circuit line with two ground wires (GW) and horizontal configuration. The width of the shadow is

$$W = D + 4H \tag{7.141}$$

where D is the distance between the two ground wires and H is the *average* height of the ground wire. If there is only one ground wire, the distance D is zero. The average height H of the ground wire can be found from

$$H = H_t - \tfrac{2}{3}(H_t - H_{ms}) \tag{7.142}$$

where H_t is the height of the ground wires at the tower and H_{ms} is the height of the ground wires at midspan with respect to ground.

An improved version of equation (7.141) is suggested by Whitehead [14] as

$$W = D + 4H^{1.09} \quad \text{m} \tag{7.143}$$

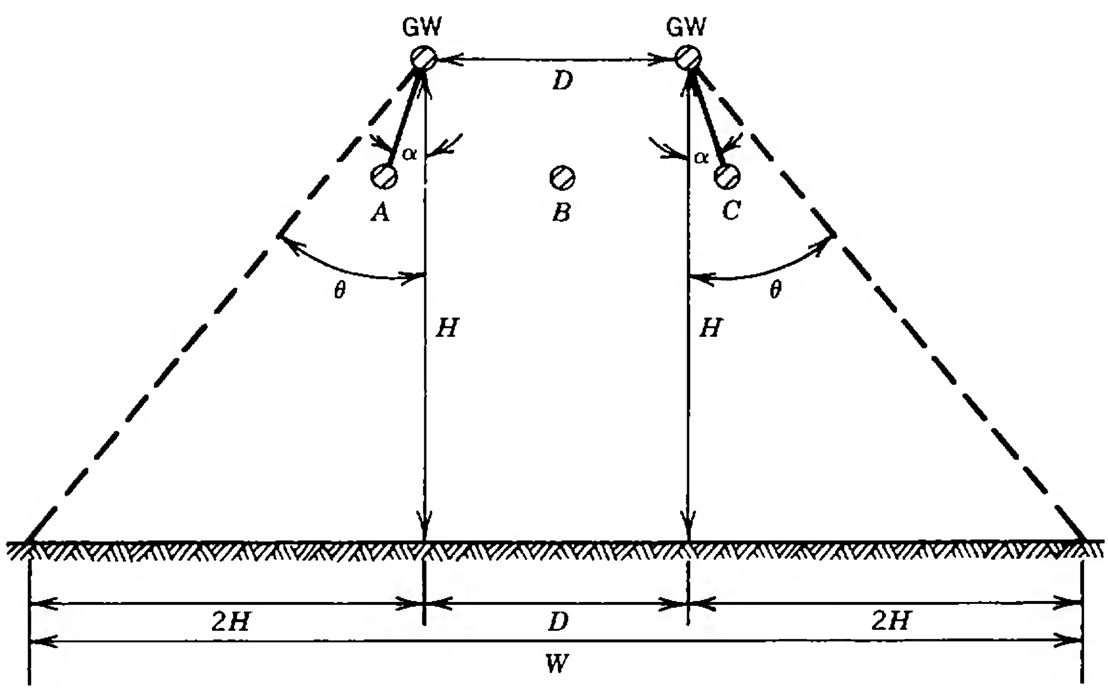


Figure 7.18. Width of right-of-way shielded from lightning strokes.

Therefore, the number of flashes to the line can be expressed as

$$n_{\text{line}} = 0.012T(D + 4H^{1.09}) \tag{7.144}$$

where n_{line} is the number of flashes to the line per 100 km per year and T is the keraunic level in thunderstorm days per year.

Note that, in Figure 7.18, θ is the shadow angle (usually assumed to be 63.5°), α is the shield angle† between shield wire and phase conductor, W is the shadow width on the surface of the earth beneath the line, GW is the ground wire location, and A, B, and C are the phase conductors.

Besides receiving the shock of the direct strokes, the overhead ground wire provides a certain amount of electrostatic screening as this reduces the voltage induced in the phase conductors by the discharge of a nearby cloud. For example, assume that E is the potential difference between the cloud and the earth and that C_1 and C_2 are the capacitances of the cloud to line and the line to ground, respectively. Therefore, the induced voltage between the line to ground can be expressed as

$$v_{\text{L-G}} = E\,\frac{C_1}{C_1 + C_2} \tag{7.145}$$

Thus, the presence of the ground wire above the line causes a considerable increase in C_2 and hence a reduction of the induced voltage of the line.

A properly located shield wire intercepts a great portion (probably above 95 percent) of the strokes that would otherwise hit a phase conductor. Shielding failure is defined as the situation when a stroke passes the shield wire and hits a phase conductor instead. Such shielding failure† might be due to the high wind accompanying the thunderstorm so that the phase conductor is blown out beyond the zone of protection of the shield wire.

Therefore, an evaluation of tower clearances and conductor and overhead ground wire configurations for an acceptable lightning protection design is based on theory and experience. The major factors affecting the lightning performance of a transmission line can be summarized as follows: (1) isokeraunic level, (2) magnitude and waveshape of the stroke current, (3) tower height, (4) resistance of tower and its footings, (5) number and location of overhead ground wires (shield angles to conductors), (6) span length, (7) midspan clearance between conductors and overhead ground wires, and (8) number of insulator units. Figures 7.19 and 7.20 show a 500-kV line under lightning impulse tests. Note also the corona activity shown in Figure 7.20.

Since the basic inputs such as the lightning frequency, lightning current

† Field experience indicates that a shield angle of 30° (from the vertical) reduces the direct stroke chance of a phase conductor by about a factor of 1000.

† An excellent review of this topic is given by Anderson [13], p. 569].

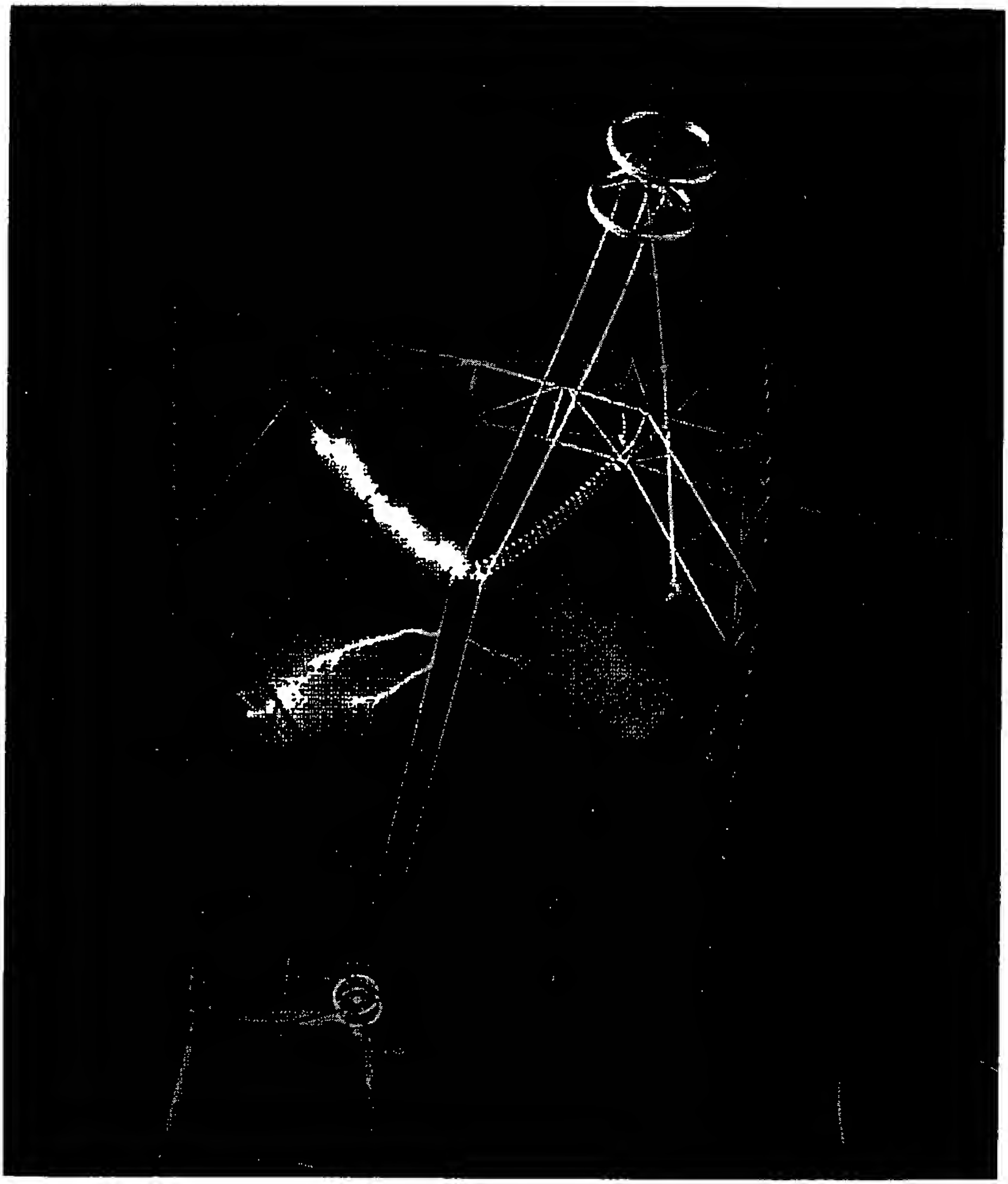


Figure 7.19. View of 500-kV line with two bundled conductors exposed to man-made lightning. (Courtesy of Ohio Brass Company.)

magnitude, wave front time, and incident rate are random variables, the prediction of lightning performance of a line is a probabilistic problem. Therefore, various probabilistic methods of computing the lightning flashover performance of lines have been developed. Among the successful approaches is the Monte Carlo method used by Anderson [15]. In this method, deterministic relationships between surge voltage across the in-

Figure 7.20. View of 500-kV line with three bundled conductors under corona and lightning impulse flashover test. (Courtesy of Ohio Brass Company.)

sulator string or air gap and variables such as lightning current magnitude, wavefront time, stroke location, tower surge impedance, and footing resistance were determined by measurements on miniature physical models representing towers, ground, insulators, conductors, and the lightning path itself. The results of these measurements with appropriate perturbations of the significant variables were then entered as input to a digital program together with statistical distributions of the input variables and estimates of the frequency of lightning strokes terminating on the transmission line. Flashover rates were then computed by Monte Carlo simulations. Today, many digital computer programs exist to determine the response of the line to lightning strokes without requiring any miniature physical models. Based on such computer programs, generalized results in the form of design curves have been developed and published for specific tower types [16, 17].

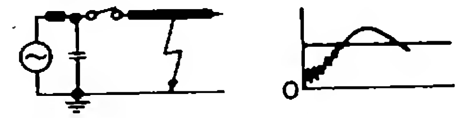
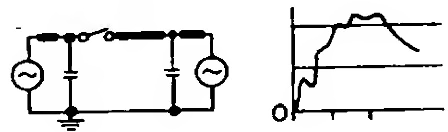
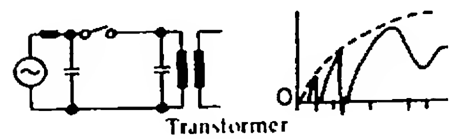
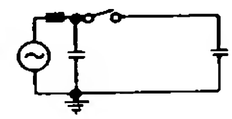
7.11 SWITCHING AND SWITCHING SURGES

7.11.1 Switching

In an ac circuit, energy is continually exchanged cyclically between circuit inductances and capacitances. Depending on resistances present, losses will extract energy that will be supplied by various sources within the system. Each steady-state condition dictates its own unique set of energy storge and exchange rates in and among the various circuit elements. Therefore, as redistribution of energy must take place among the various system elements to change from one steady-state condition to another. Of course, this change cannot take place instantaneously; a finite period of time, called *transient period*, prevails during which transient voltages and currents develop to enforce these changes.

Therefore, whenever a switch in an electric circuit is opened or closed, a switching transient may take place. This is true for transmission as well as distribution circuits. Table 7.1 presents various possible switching operations. Therefore, the interruption by a switching operation of a circuit having inductance and capacitance may result in transient oscillations that can cause overvoltages on the system. Switching surges with high rates of voltage rise may produce repeated restriking of the arc between the opening contacts of a circuit breaker and thus impose such excessive duty on the circuit breaker as to result in its destruction. The interrupting ability of a circuit breaker depends on its capacity to increase the dielectric strength across its contacts at a more rapid rate than the rate at which the voltage is built up. Furthermore, switching surges may result in resonant oscillations in the windings of transformers or machines and therefore such windings may need to be protected using electrostatic shields. As far as transmission line insulation coordination is concerned, two particular types of switching operations are important: (1) energizing a line with no initial voltage and (2) high-speed reclosing following a line tripout. The main difference between these two switching operations is the fact that there may be energy trapped on the line from the previous opening. For example, the switching of a capacitance, such as disconnecting a line or a cable or a capacitor bank, may cause excessive overvoltages across the circuit breaker contacts, especially if restrikes occur in the switching device. Restriking occurs if the recovery voltage across the circuit breaker builds up at a faster rate than the dielectric strength of the interrupting medium causing reestablishment of the arc across the interrupting contacts. Consider the circuit shown in Figure 7.21, where the current drawn by the capacitor leads the voltage by 90°. As the circuit breaker contacts separate from each other, an arc is established

TABLE 7.1 Various Possible Switching Operations

Switching operation	System	Voltage across contacts
1. Terminal short circuit		
2. Short line fault		
3. Two out-of-phase systems—voltage depends on grounding conditions in systems		
4. Small inductive currents, current chopped (unloaded transformer)		
5. Interrupting capacitive currents—capacitor banks, lines and cables on no load		
6. Evolving fault—e.g. flashover across transformer plus arc across contacts when interrupting transformer on no load		
7. Switching—in unloaded e.h.v./u.h.v. line (trapped charge)		

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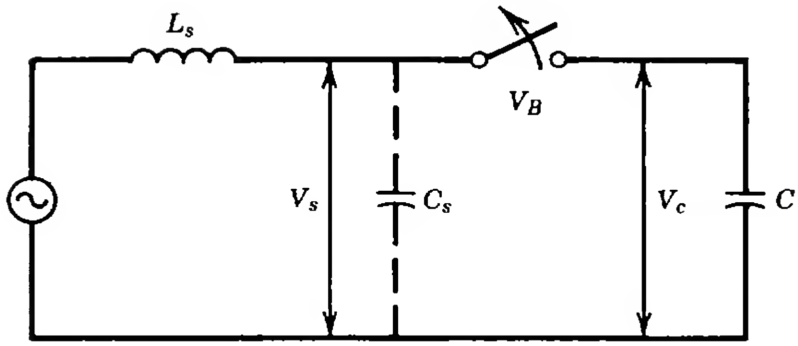
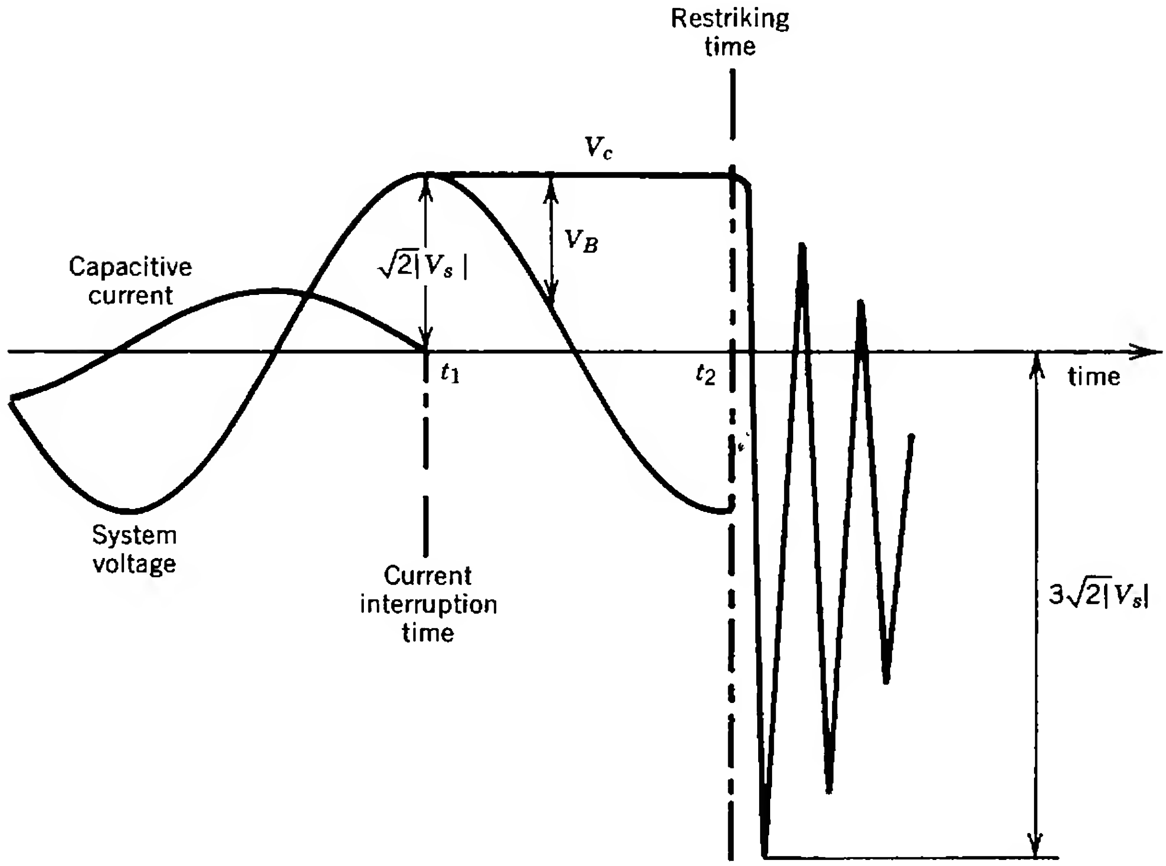
between the contacts, and current continuous to flow. As the current goes through zero, the arc loses conductivity, and the current is interrupted. Note tht C_s represents the stray capacitance. The arc cannot reignite due to the fact that the voltage across the contacts of the circuit breaker V_B'(which is equal to $V_c - V_s$) is too small. Therefore, the capacitor remains charged to a voltage equal to the peak value of the supply voltage, that is, -1.0 pu. In other words, the voltage *trapped* on the capacitor is 1.0 pu. As the supply voltage reverses, the recovery voltage V_B $(=V_c - V_s)$ on the circuit breaker rises. Half a cycle later, when the source voltage V_s has changed to $+1.0$ pu, there will be a voltage of about 2.0 pu across the breaker. In the event that the breaker has regained enough dielectric strength to withstand this voltage, the switching operation will be successful. Otherwise, there will be a restrike, which simply means that the insulation gap collapses and the breaker contacts are basically short-circuited. The restrike causes a fast oscillatory voltage in the LC circuit. Therefore, the voltage overshoot on the capacitor will be as high as $+3.0$ pu (i.e., $3\sqrt{2}|V_s|$). When the voltage on the



(a)



(b)

Figure 7.21. Restriking voltage transient caused by switch opening.

capacitor has reached its maximum, the transient discharge current will pass through zero, and arc may again extinguish, leaving the capacitor charged to +3.0 pu. Since the source voltage is V_s, the voltage across the breaker contacts after another half cycle will be −4.0 pu, which may cause another restrike, leaving the capacitor charged to −5.0 pu. This phenomenon may theoretically continue indefinitely, increasing the voltage by successive increments of 2.0 pu. In practice, however, losses, stray capacitance, and possibly the resulting insulation failure will restrict the overvoltage. Voltages on the order of 2.5 times the V_s are more typical of field measurements. Switching overvoltages may rarely be determined by hand calculations, at least for realistic three-phase transmission-line circuits. Usually, they are determined by employing a transient network analyzer (TNA) or a digital computer program [11].

7.11.2 Causes of Switching Surge Overvoltages

As mentioned in the previous section, the operation of circuit breakers produces a transient overvoltage. However, the concept of switching should not be limited only to the intentional actions of opening and closing circuit breakers and switches but may also include the arcing faults and even lightning. The causes of switching surge overvoltages can be summarized as follows: (1) normal line energizing or deenergizing; (2) high-speed line reclosing; (3) switching cable circuits, capacitor banks, and shunt reactors; (4) load rejection; (5) out-of-phase switching, (6) reinsertion of series capacitors; (7) circuit breaker restriking; and (8) current chopping. Whereas, the 60-Hz voltages are caused by an *abnormal condition* that exists until change in the system alters or removes the condition. Examples of this condition are: (1) voltages on the unfaulted phases during a phase-to-ground fault, (2) load rejection, (3) open end of a long energized line (*Ferranti effect*), and (4) ferroresonance.

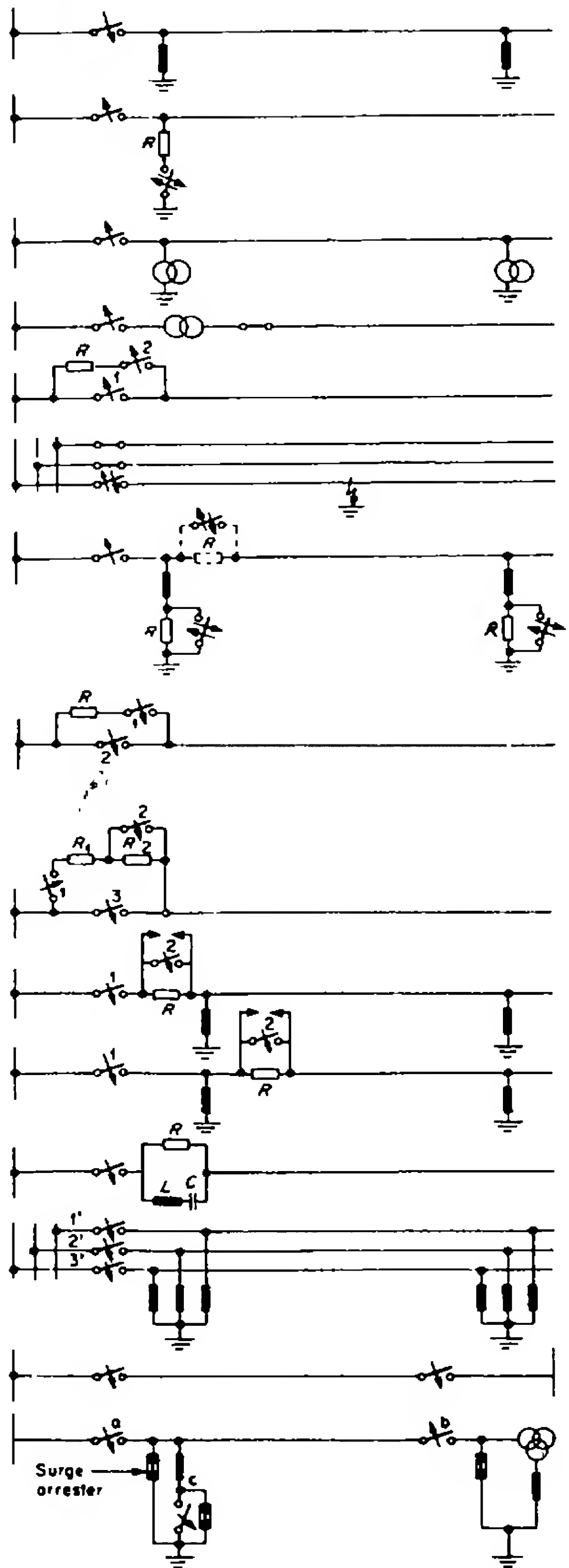
7.11.3 Control of Switching Surges

IEEE Standard 399-1980 [18] recommends the following "philosophy of mitigation and control" of switching surges: (1) minimizing the number and severity of switching events, (2) restriction of the *rate* of exchange of energy that prevails among system elements during the transient periods, (3) extraction of energy, (4) provision of energy reservoirs to contain released or trapped energy within safe limits of current and voltage, (5) provision of preferred paths for elevated-frequency currents attending switching, and (6) shifting particularly offensive frequencies. Furthermore, IEEE Standard 399-1980 [18] recommends the implementation of this control philosophy through the judicious use of the following means: (1) temporary insertion of resistance between circuit elements, for example, insertion resistors in circuit breakers; (2) inrush control reactors; (3) damping resistors in filter circuits and surge-protective circuits; (4) tuning reactors; (5) surge capacitors; (6) filters; (7) surge arrestors; (8) necessary switching only, with properly maintained switching devices; and (9) proper switching sequences. Additional means of reducing switching overvoltages on EHV and UHF lines are given in Table 7.2.

Figure 7.22 shows a digital transient recorder that can be used for the continuous surveillance of a transmission network. The analysis is instanta-

TABLE 7.2 Means of Reducing Switching Overvoltages on Extra-High-Voltage and Ultrahigh-Voltage Lines

Means of reducing switching overvoltages	Basic diagram
1. High-voltage shunt reactors connected to the line to reduce power-frequency overvoltage	
2. Eliminating or reducing trapped charge by:	
2.1 Line shunting after interruption	
2.2 Line discharge by magnetic potential transformers	
2.3 Low-voltage side disconnection of the line	
2.4 Opening resistors	
2.5 Single-phase reclosing	
2.6 Damping of line voltage oscillation after disconnecting a line equipped with h.v. reactors	
3. Damping the transient oscillation of the switching overvoltages	
3.1 Single-stage closing resistor insertion	
3.2 Multi-stage closing resistor insertion	
3.3 Closing resistor in-line between circuit breaker and shunt reactor	
3.4 Closing resistor in-line on the line side of the shunt reactor.	
3.5 Resonance circuit (surge absorber) connected to the line	
4. Switching at favourable switching moments:	
4.1 Synchronized closing	
4.2 Reclosing at voltage minimum of a beat across the breaker	
5. Simultaneous closing at both ends of the line	
6. Limitation by surge arresters when: energizing line at no-load (a) disconnecting reactor loaded transformers (b) disconnecting high-voltage reactors (c)	



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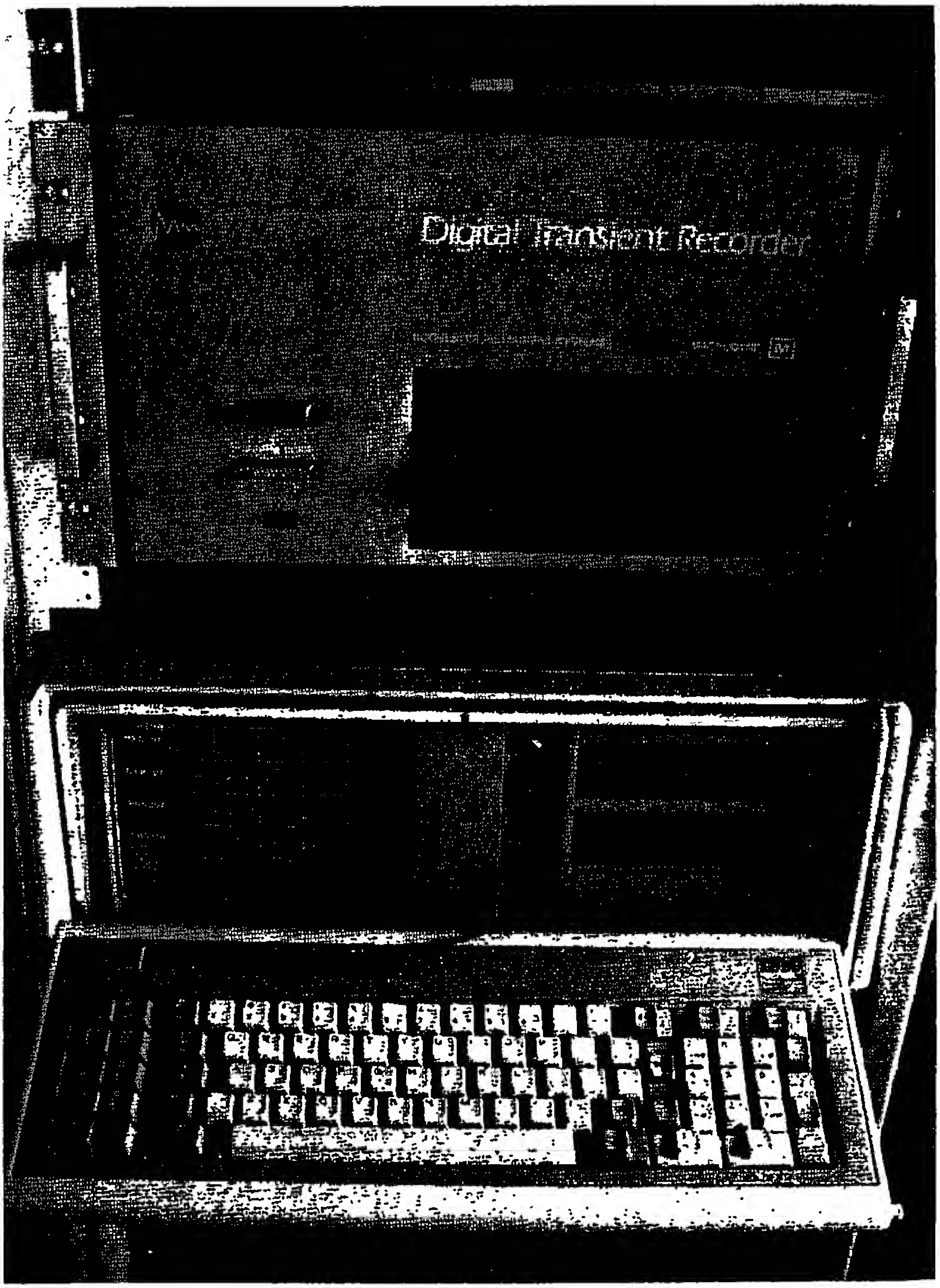
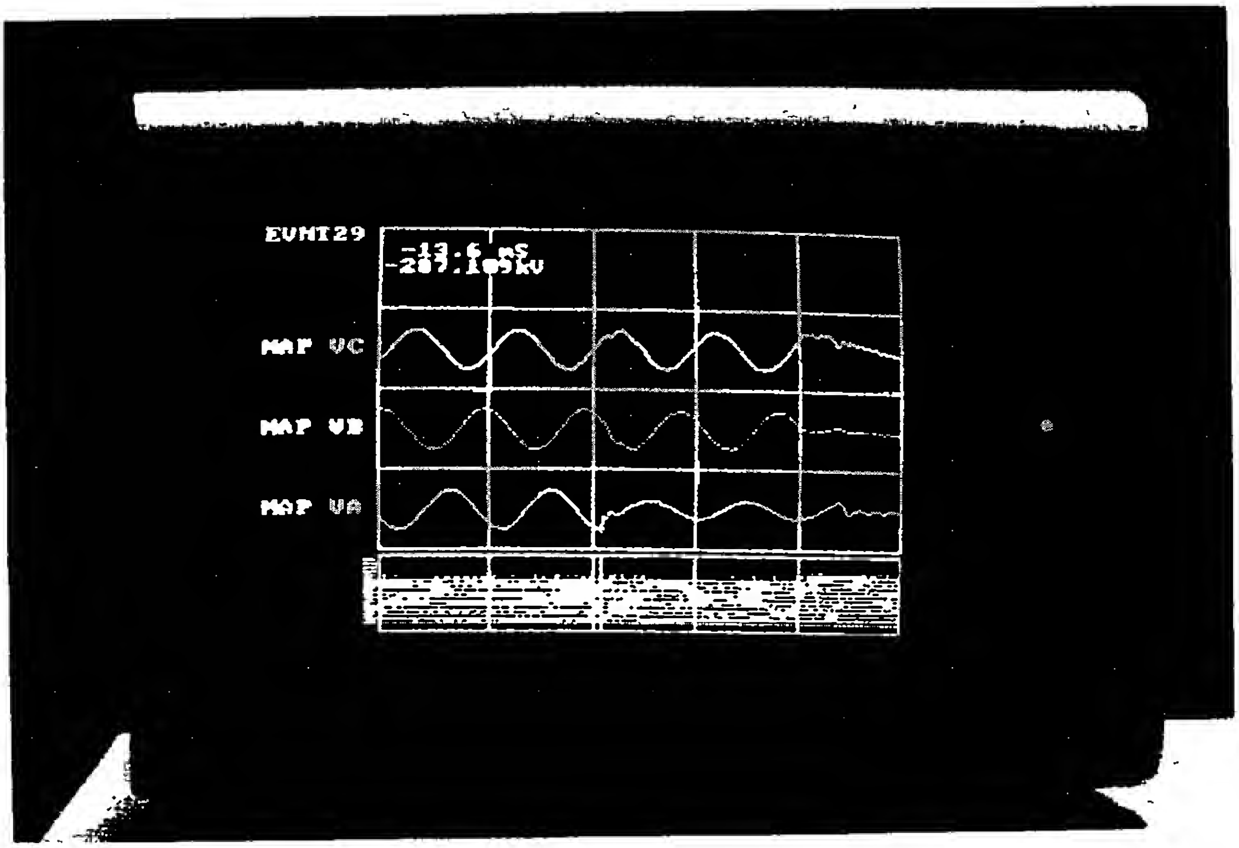


Figure 7.22. Digital transient recorder. (Courtesy of Rochester Instrument Systems Company.)



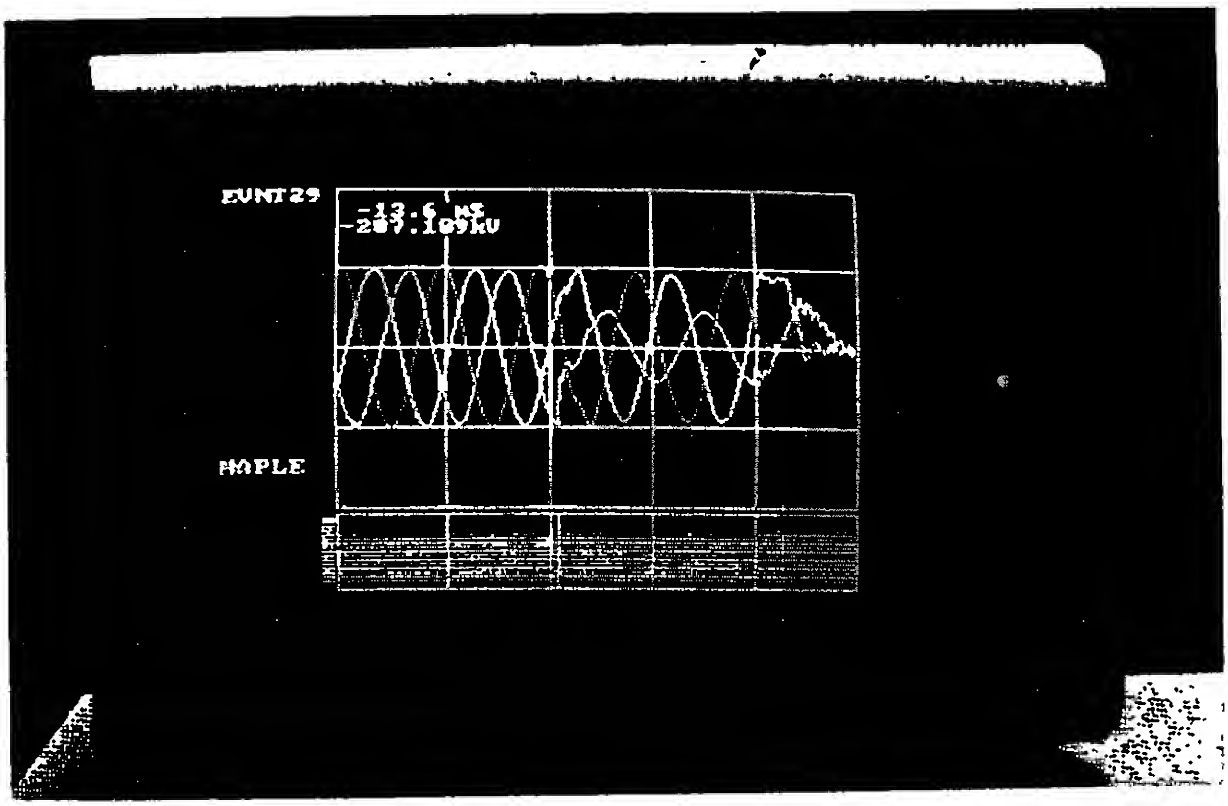


Figure. 7.22 (*Continued*)

neous through remote substation data acquisition units that transmit via telephone line or microwave link to the central station. The transient data is presented on a CRT screen. Waveform amplitude and timing information are derived automatically, without manual measurements and calculations.

7.12 OVERVOLTAGE PROTECTION

In addition to the overhead ground wires or shield wires, air (rod and horn) gaps, surge diverters, and surge arresters are used to protect a power system against severe overvoltages. As stated before, the effectiveness of overhead ground wires for lightning protection depends on a low-impedance path to ground. Therefore, all metal structures in transmission lines having overhead ground wires should be adequately grounded. If there are two overhead ground wires, they should be tied together at the top of each structure to reduce the impedance to ground. Of course, an array of ground wires would be very effective but too expensive. Thus, the maximum number of ground wires used is normally two, even on double-circuit lines.

The rod gap is the simplest form of a diverter. It has a preset air gap designed to flashover first if there is an excessive overvoltage. It is connected between a phase conductor and ground. Unfortunately, the rod gap cannot clear itself and, therefore, it will presist at the 60-Hz voltages. Also its electrodes are damaged in the arcing process. Table 7.3 [19] gives flashover values of various air gaps. For a given air gap, the time for breakdown changes almost inversely with the applied voltage. However, the breakdown time for positive voltages are lower than those for negative. In general, a rod gap is set to a breakdown voltage that is not less than 30 percent below the withstand voltage level of the protected power apparatus. Succinctly put, the surge diverters pass no current at the 60-Hz voltage, prevent the 60-Hz follow-on current after a flashover, and break-down quickly after the arrival of the excessive overvoltage. The surge diverter derives its name from the fact that when a high-voltage surge reaches its gap, sparkover takes place, and therefore the surge energy is diverted to the earth.

Surge arresters are also called *lightning* arresters. They are applied on electric systems to protect equipment such as transformers, rotating machines, etc., against the effects of overvoltages resulting from lightning, switching surges, or other disturbances. Surge arresters are also connected in shunt with equipment to be protected. It has basically a nonlinear resistor so that its resistance decrease rapidly as the voltage across it rises. After the surge ends and the voltage across the arrester returns to the normally 60-Hz line-to-neutral voltage level, the resistance becomes high enough to limit the arc current to a value that can be quenched by the series gap of the arrester. Therefore, the surge arrester provides both gap protection and nonlinear resistance. A disruptive discharge between the electrodes of a surge arrester is called *sparkover*. The highest value of applied voltage t which an arrester

TABLE 7.3 Flashover Values of Air Gaps [19]

Air gap		Flashover		Air gap		Flashover	
mm	in	60-Hz wet, kV	Pos. critical impulse, kV	mm	in	60-Hz wet, kV	Pos. critical impulse, kV
25	1		38	1295	51	438	814
51	2		60	1321	52	447	829
76	3		75	1346	53	455	843
102	4		91-95	1372	54	464	858
127	5		106-114	1397	55	472	872
152	6		128-141	1422	56	481	887
178	7		141-155	1448	57	489	901
203	8		159-166	1473	58	498	916
229	9		175-178	1499	59	506	930
254	10	80	190	1524	60	515	945
279	11	89	207	1549	61	523	960
305	12	98	224	1575	62	532	975
330	13	107	241	1600	63	540	990
356	14	116	258	1626	64	549	1005
381	15	125	275	1651	65	557	1020
406	16	134	290	1676	66	566	1035
432	17	143	305	1702	67	574	1050
457	18	152	320	1727	68	583	1065
483	19	161	335	1753	69	591	1080
508	20	170	350	1778	70	600	1095
533	21	178	365	1803	71	607	1109
559	22	187	381	1829	72	615	1124
584	23	195	396	1854	73	622	1138
610	24	204	412	1880	74	630	1153
635	25	212	427	1905	75	637	1167
660	26	221	443	1930	76	645	1182
686	27	229	458	1956	77	652	1196
711	28	238	474	1981	78	660	1211
737	29	246	489	2007	79	667	1225
762	30	255	505	2032	80	675	1240
787	31	264	519	2057	81	683	1254
813	32	273	534	2083	82	691	1269
838	33	282	548	2108	83	699	1283
864	34	291	563	2134	84	707	1298
889	35	300	577	2159	85	715	1312
914	36	309	592	2184	86	723	1327
940	37	318	606	2210	87	731	1341
965	38	327	621	2235	88	739	1356
991	39	336	635	2261	89	747	1370
1016	40	345	650	2286	90	755	1385
1041	41	353	665	2311	91	763	1399
1067	42	362	680	2337	92	771	1414
1092	43	370	695	2362	93	779	1428
1118	44	379	710	2388	94	787	1443
1143	45	387	725	2413	95	795	1457
1168	46	396	740	2438	96	803	1472
1194	47	404	755	2464	97	811	1486
1219	48	413	770	2489	98	819	1501
1245	49	421	785	2515	99	827	1515
1270	50	430	800	2540	100	835	1530

TABLE 7.3 (*Continued*)

Air gap		Flashover		Air gap		Flashover	
mm	in	60-Hz wet, kV	Pos. critical impulse, kV	mm	in	60-Hz wet, kV	Pos. critical impulse, kV
2565	101	842	1544	3835	151	1176	2269
2591	102	848	1559	3861	152	1182	2284
2616	103	855	1573	3886	153	1188	2298
2642	104	862	1588	3912	154	1194	2313
2667	105	869	1602	3937	155	1200	2327
2692	106	875	1617	3962	156	1206	2342
2718	107	882	1631	3988	157	1212	2356
2743	108	889	1646	4013	158	1218	2371
2769	109	896	1660	4039	159	1224	2385
2794	110	902	1675	4064	160	1230	2400
2819	111	909	1689	4089	161	1236	2414
2845	112	916	1704	4115	162	1242	2429
2870	113	923	1718	4140	163	1248	2443
2896	114	929	1733	4166	164	1254	2458
2921	115	936	1747	4191	165	1260	2472
2946	116	943	1762	4216	166	1266	2487
2972	117	950	1776	4242	167	1272	2501
2997	118	956	1791	4267	168	1278	2516
3023	119	963	1805	4293	169	1284	2530
3048	120	970	1820	4318	170	1290	2545
3073	121	977	1834	4343	171	1296	2559
3099	122	984	1849	4369	172	1302	2574
3124	123	991	1863	4394	173	1308	2588
3150	124	998	1878	4420	174	1314	2603
3175	125	1005	1892	4445	175	1320	2617
3200	126	1012	1907	4470	176	1326	2632
3226	127	1019	1921	4496	177	1332	2646
3251	128	1026	1936	4521	178	1338	2661
3277	129	1033	1950	4547	179	1344	2675
3302	130	1040	1965	4572	180	1350	2690
3327	131	1047	1979	4597	181	1355	2704
3353	132	1054	1994	4623	182	1361	2719
3378	133	1061	2008	4648	183	1366	2733
3404	134	1068	2023	4674	184	1372	2748
3429	135	1075	2037	4699	185	1377	2762
3454	136	1082	2052	4724	186	1383	2777
3480	137	1089	2066	4750	187	1388	2791
3505	138	1096	2081	4775	188	1394	2806
3531	139	1103	2095	4801	189	1399	2820
3556	140	1110	2110	4826	190	1405	2835
3581	141	1116	2124	4851	191	1410	2849
3607	142	1122	2139	4877	192	1416	2864
3632	143	1128	2153	4902	193	1421	2878
3658	144	1134	2168	4928	194	1427	2893
3683	145	1140	2182	4953	195	1432	2907
3708	146	1146	2197	4978	196	1438	2922
3734	147	1152	2211	5004	197	1443	2936
3759	148	1158	2226	5029	198	1449	2951
3785	149	1164	2240	5055	199	1454	2965
3810	150	1170	2255	5080	200	1460	2980

will not flash is the withstand voltage. Current that flows through an arrester, caused by the 60-Hz system voltage across it, during and after the flow of surge current is called *follow current.* Surge arresters, with their controlled breakdown characteristics, sparkover at voltages well below the withstand strength of system insulation.

7.13 INSULATION COORDINATION

7.13.1 Basic Definitions

BASIC IMPULSE INSULATION LEVEL (BIL). Reference insulation levels expressed in impulse crest (peak) voltage with a standard wave not longer than a 1.2×50-μs wave. It is determined by tests made using impulses of a 1.2×50-μs waveshape. BIL is usually defined as a per unit of maximum value of the line-to-neutral voltage. For example, for 345 kV, it is

$$1\ \mathrm{pu} = \sqrt{2}\left(\frac{345}{\sqrt{3}}\right) = 282\ \mathrm{kV}$$

so that a BIL of 2.7 pu = 760 kV.

WITHSTAND VOLTAGE. The BIL that can be repeatedly applied to an equipment without any flashover, disruptive charge, puncture, or other electrical failure, under specified test conditions.

CHOPPED-WAVE INSULATION LEVEL. It is determined by tests using waves of the same shape to determine the BIL, with the exception that the wave is chopped after about 3 μs. If there is no information, the chopped-wave level is assumed to be 1.15 times the BIL for oil-filled equipment, for example, transformers. It is assumed to be equal to the BIL for dry-type insulation. The equipment manufacturer should be consulted for exact values.

CRITICAL FLASHOVER VOLTAGE (CFO). The peak voltage for a 50 percent probability of flashover or disruptive discharge.

IMPULSES RATIO (FOR FLASHOVER OR PUNCTURE OF INSULATION). It is the ratio of impulse peak voltage to the peak value of the 60-Hz voltage to cause flashover or puncture. In other words, it is the ratio of breakdown voltage at surge frequency to breakdown voltage at normal system frequency.

7.13.2 Insulation Coordination

Insulation coordination is the process of determining the proper insulation levels of various components in a power system and their arrangement. In other words, it is the selection of an insulation structure that will withstand the voltage stresses to which the system or equipment will be subjected together with the proper surge arrester. This process is determined from the known characteristics of voltage surges and the characteristics of surge arresters. There are three different voltage stresses to consider when determining insulation and electrical clearance requirements for the design of high-voltage transmission lines: (1) the 60-Hz power voltage, (2) lightning surge voltage, and (3) switching surge voltage. Therefore, insulation for transmission systems must be chosen after a careful study of both the transient and power frequency voltage stresses on each insulation element. In general, lightning impulse voltages have the highest values and the highest rates of voltage rise.

Therefore, a properly done insulation coordination provides the following: (1) the assurance that the insulation provided will withstand all normal operating stresses and a majority of abnormal ones, (2) the efficient discharge of overvoltages due to the lightning and switching surges as well as other internal causes, (3) the breakdown will occur only due to the external flashover, and (4) the positions at which breakdown takes place will be where breakdown may cause no or comparatively minor damage.

Thus, insulation coordination involves the following: (1) determination of line insulation, (2) selection of the BIL and insulation levels of other apparatus, and (3) selection of lightning arresters.

For transmission lines up to 345 kV, the line insulation is determined by the lightning flashover rate. At 345 kV, the line insulation may be dictated by either switching surge considerations or by the lightning flashover rate. Above 345 kV, switching surges become the major factor in flashover considerations and will more likely control the insulation design. The probability of flashover due to a switching surge is a function of the line insulation characteristics and the magnitude of the surges expected. The number of insulators employed may be selected to keep the probability of flashover from switching surges very low. Switching surge impulse insulation strength is based on tests that have been made on simulated towers. At extra-high-voltage levels, an increase in insulation length does not provide a proportional increase in switching surge withstand strength. This is due to the electric field distortion caused by the proximity of the tower surfaces and is called the *proximity effect*. Since the proximity effect is not related to lightning impulses, switching surge considerations dictate the insulation values at the extra-high-voltage levels.

The maximum switching surge level used in the design of a substation is either the maximum surge that can take place on the system or the protective level of the arrester, which is the maximum switching surge the arrester will allow into the station. Therefore, the coordination of insulation in a substation means the selection of the minimum arrester rating applicable to withstand the 60-Hz voltage and the choice of equipment having an insulation level that can be protected by the arrester. Transient network analyzers can also be used to study the switching surge levels that can take place at the substation. The results may be used to determine and coordinate proper impulse insulation and switching surge strength required in substation apparatus. The distance between the arrester location and the protected insulation affects the voltage imposed on insulation due to reflections. The severity of the surge depends on how well the substation is shielded, the insulation level of the substation structure, and the incoming line insulation. For a traveling wave coming into a dead-end station, the discharge current in the arrester is determined by the maximum voltage that the line insulation can allow, by the surge impedance of the line, and by the voltage characteristic of the arrester. Therefore, the discharge current of an arrester can be expressed as

$$I_{ar} = \frac{2V - V_{ar}}{Z_c} \tag{7.146}$$

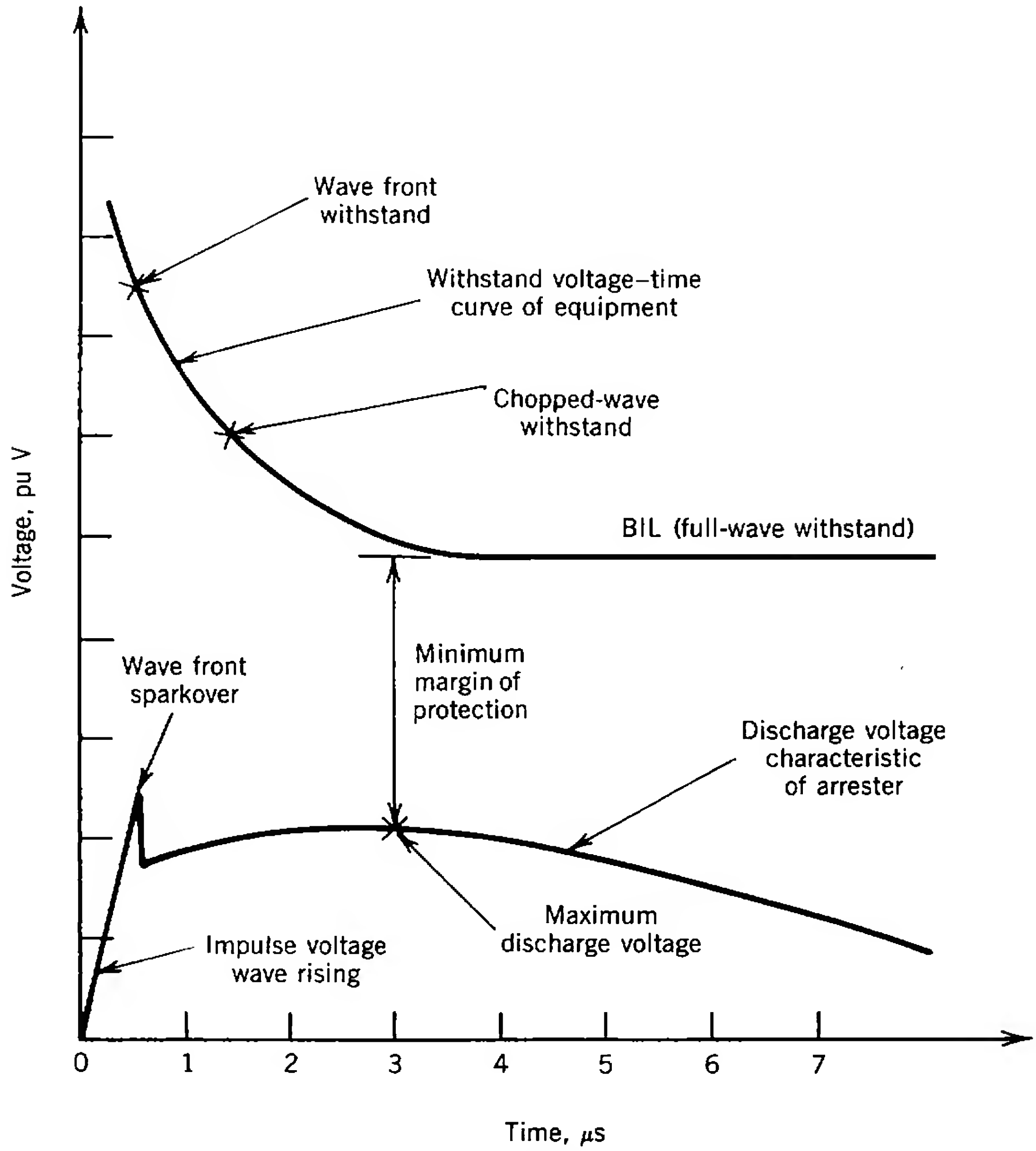


Figure 7.23. Illustration of insulation coordination between oil-filled equipment and surge arrester.

where I_{ar} = arrester current
V = magnitude of incoming surge voltage
V_{ar} = arrester terminal voltage
Z_c = surge impedance of line

Figure 7.23 illustrates the insulation coordination between an oil-filled equipment (e.g., transformer) and a surge arrester. The arrester impulse sparkover voltage is compared to the chopped-wave test level of the transformer. A more meaningful comparison is to compare the arrester sparkover with the wave front test. The difference between arrester dis-

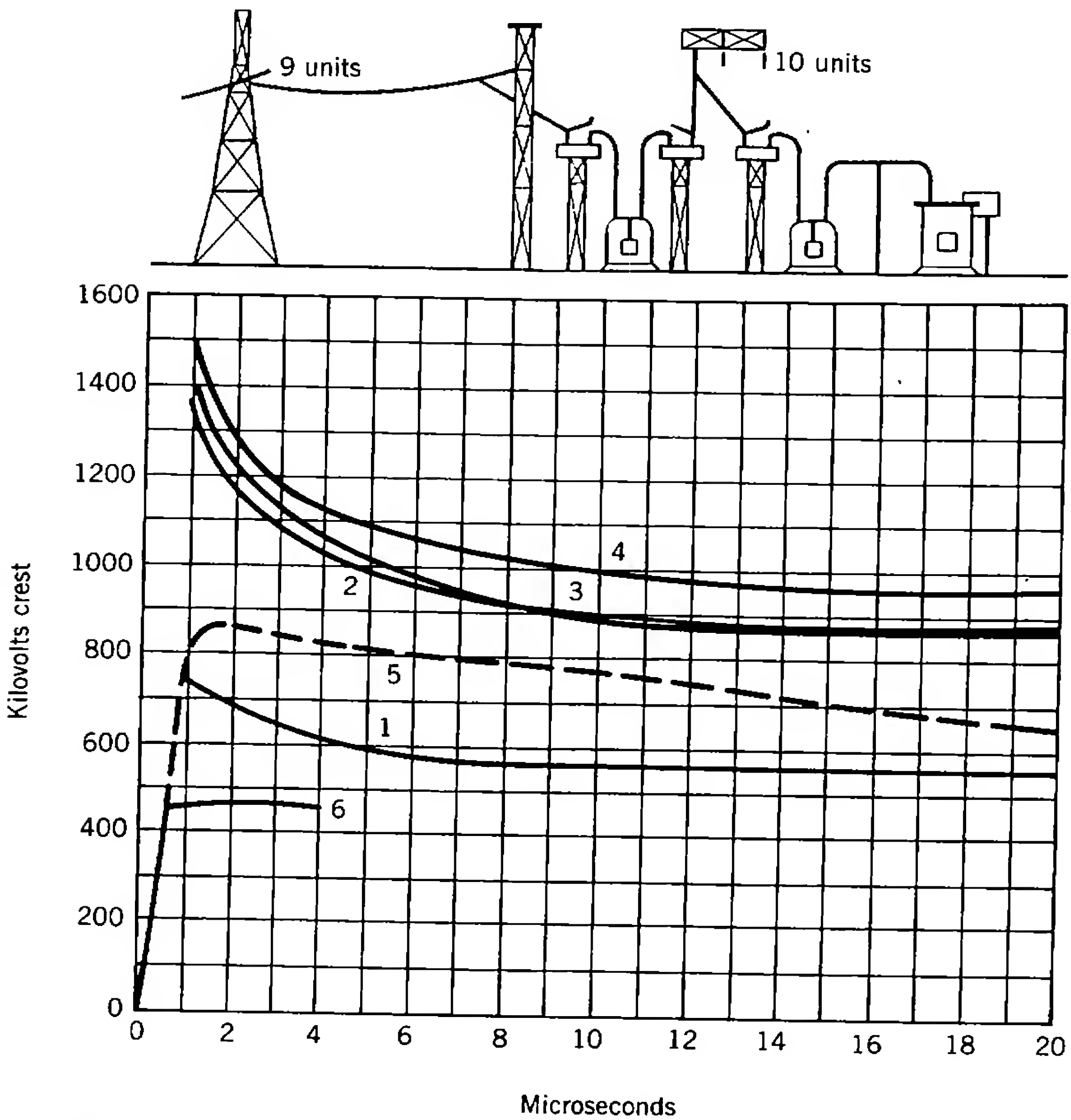


Figure 7.24. Insulation coordination in 138-kV substation for 1.5×40-μs wave: (1) transformer with 550-kV BIL; (2) line insulation of 9 suspension units; (3) disconnect switches on 4 apparatus insulators; (4) bus insulation of 10 suspension units; (5) maximum 1.5×40-μs wave permitted by line insulation; (6) discharge of 121-kV arrester for maximum 1.2×40-μs full wave [16].

charge characteristics and equipment withstand level, at any given instant of time, is called the *margin of protection* (MP) and is expressed as

$$\mathrm{MP} = \frac{\mathrm{BIL}_{\mathrm{equip}} - V_{\mathrm{ar}}}{\mathrm{BIL}_{\mathrm{equip}}} \tag{7.147}$$

where $\mathrm{BIL}_{\mathrm{equip}}$ is the BIL of the equipment and V_{ar} is the discharge of voltage of the arrester. The margin of protection is a safety factor for the equipment protection and should not be less than 0.20. It also takes into account various possible errors and unknown factors. Figure 7.24 illustrates the insulation coordination in a 138-kV substation for $1.5 \times 40\,\mu\mathrm{s}$. The present U.S. standard impulse wave is $1.2 \times 50\,\mu\mathrm{s}$.

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PROBLEMS

7.1. Consider Figure 7.2 and derive equation (7.5).

7.2. Consider equations (7.2) and (7.16) and verify equation (7.23) by using derivatives and integrals.

7.3. Consider Figure 7.2 and assume that switch S is closed and that the surge velocity is ν. Verify that the electrostatic and electromagnetic energy storage (due to the surge phenomenon) are equal.

7.4. Repeat Example 7.1 assuming a surge impedance of 400 Ω.

7.5. Repeat Example 7.2 assuming a surge impedance of 40 Ω.

7.6. Repeat Example 7.2 assuming a surge voltage of 100 kV.

7.7. Consider an open-circuit line termination and verify the following:
(a) $\tau = 2$ for voltage waves.
(b) $\tau = 0$ for current waves.
(c) $\rho = 1$ for voltage waves.
(d) $\rho = -1$ for current waves.

7.8. Assume that a line is terminated in its characteristic impedance and verify the following:

(a) $\tau = 1$ for voltage waves.
(b) $\tau = 1$ for current waves.
(c) $\rho = 0$ for voltage waves.
(d) $\rho = 0$ for current waves.

7.9. Repeat Example 7.4 assuming that the line is terminated in a 200-Ω resistance and that the magnitudes for forward-traveling voltage and current waves are 2000 V and 5 A, respectively.

7.10. Repeat Example 7.4 assuming that the line is terminated in a 400-Ω resistance and that the magnitudes of forward-traveling voltage and current waves are 2000 V and 5 A, respectively.

7.11. A static charge on a line can be represented by the interaction of traveling waves, even though there are no transport phenomena involved. Assume that the line is open at both ends and charged to the voltage v. Determine the following:
(a) $i_b = -i_f$ for every point x along the line.
(b) $v_b = v_f = \frac{1}{2}v$ for every point x along the line.
(c) Plot the voltage and current distributions.

7.12. Assume that a dc source is supplying current to a resistance R over a transmission line having a characteristic impedance of Z_c and that the dc source voltage is v. Verify the following:

(a) $i_b = \dfrac{v}{R} - i_f$

(b) $v = 2Z_c i_f - \dfrac{Z_c}{R}\, v$

(c) $i_f = \left(1 + \frac{R}{Z_c}\right)\frac{i}{2}$

(d) $i_b = \left(1 - \frac{R}{Z_c}\right)\frac{i}{2}$

(e) $v_f = \left(1 + \frac{Z_c}{R}\right)\frac{v}{2}$

(f) $v_b = \left(1 - \frac{Z_c}{R}\right)\frac{v}{2}$.

7.13. Consider Problem 7.11 and Figure 7.16(a) and assume that a static charge along a transmission line is built up below a cloud, which is suddenly released by a lightning discharge at time $t = 0$ into the cloud. The static charge existing in the line below the cloud at time $t = 0$ is essentially released by the lightning discharge and cannot remain static but must propagate along the line to permit transition to a new final steady-state solution. Use the results of Problem 7.11 and verify that

$$i_f = -i_b = \frac{v}{2Z_c}$$

7.14. Verify the following expressions:

(a) $\frac{P}{P_f} = \left(\frac{2}{(Z_{c1}/Z_{c2})^{1/2} + (Z_{c2}/Z_{c1})^{1/2}}\right)^2$

(b) $\frac{P_b}{P_f} = \left(\frac{Z_{c2} - Z_{c1}}{Z_{c1} + Z_{c2}}\right)^2$

7.15. Repeat Example 7.5 assuming that the 200-kV voltage surge is traveling toward the junction from the cable end.

7.16. Vefify equation (7.91) using Laplace transforms.

7.17. Use Laplace transforms and verify the following:

(a) Equation (7.101).

(b) Equation (7.104).

7.18. Consider the junction between two lines with characteristics impedances of Z_{c1} and Z_{c2}, respectively. Assume that a shunt capacitor C is connected at the junction, as shown in Figure P7.18. Verify that the voltage across the capacitor is

$$v(t) = \frac{2v_f Z_{c2}}{Z_{c1} + Z_{c2}}\left[1 - \exp\left(-\frac{Z_{c1} + Z_{c2}}{Z_{c1}Z_{c2}C}\right)t\right]$$

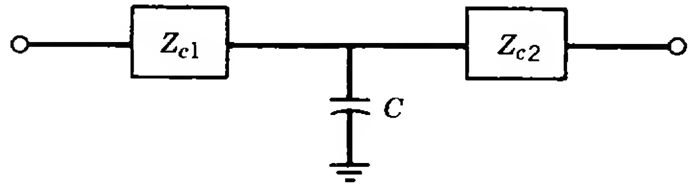


Figure P7.18

7.19. Consider Problem 7.18 and assume that the traveling surge is of finite duration of τ, as shown in Figure P7.19, rather than of infinite length, and that its magnitude is v_f units. The wave can be decomposed into two waves as shown in the figure. Verify that the maximum voltage across the capacitor can be expressed as

$$v(t) = \frac{2v_f Z_{c2}}{Z_{c1} + Z_{c2}} \left[1 - \exp\left(-\frac{Z_{c1} + Z_{c2}}{Z_{c1}Z_{c2}C}\,\tau\right)\right]$$

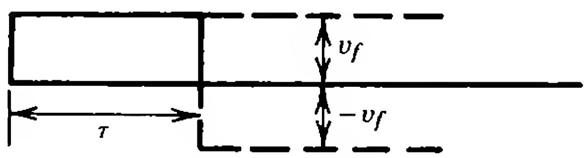


Figure P7.19

7.20. Repeat Example 7.6 assuming that the cable is terminated in a 10-Ω resistor.

7.21. Assume that a new 100-mi-long transmission line is built and that the line is designed to have a flashover rate of 1.0 per 100 mi per year. If the line is located in an area with a keraunic level such that 100 strokes (i.e., flashes) hit the line in an average year, determine the following:

(a) Probability of having zero flashover.
(b) Probability of having one flashover.
(c) Probability of having two flashovers.
(d) Probability of having three flashovers.
(e) Probability of having four flashovers.

7.22. Assume that a 345-kV transmission is built in a location that has a keraunic level of 50 thunderstorm days per year. Each tower has two ground wires separated from each other by 26 ft. The height of the ground wires at the tower and at the midspan are 57 and 48 ft, respectively. Determine the following:

(a) Average height of ground wire.
(b) Width of protective shadow provided by line.
(c) Number of flashes to line that can be expected per 100 km/yr.

7.23. Assume that a thyrite-type nonlinear lightning arrester has a characteristic of $RI^{0.72} = 57{,}000$. Determine the ratio of the voltages appearing at the end of a line having a surge impedance of $400\,\Omega$ due to a 400-kV surge when:

(a) Line is open-circuited.
(b) Line is terminated by arrester.

8

LIMITING FACTORS FOR EXTRA-HIGH AND ULTRAHIGH-VOLTAGE TRANSMISSION: CORONA, RADIO NOISE, AND AUDIBLE NOISE

8.1 INTRODUCTION

A corona is a *partial discharge* and takes place at the surface of a transmission line conductor when the electrical stress, that is, the electric field intensity (or surface potential gradient), of a conductor exceeds the breakdown strength of the surrounding air. In such a nonuniform field, various visual manifestations of locally confined ionization and excitation processes can be viewed. These local breakdowns (i.e., corona or partial discharges) can be either of a transient (nonself-sustaining) or steady-state (self-sustaining) nature. These manifestations are called coronas due to the similarity between them and the glow or corona surrounding the sun (which can only be observed during a total eclipse of the sun). In nature, the corona phenomenon can also be observed between and within electrically charged clouds. According to a theory of cloud electrification, such a corona is not only the effect but also the cause of the appearance of charged clouds and thus of lightning and thunder storms.

Corona† on transmission lines causes power loss, radio and television interference, and audible noise (in terms of buzzing, hissing, or frying sounds) in the vicinity of the line. At extra-high-voltage levels (i.e., at 345 kV and higher), the conductor itself is the major source of audible noise, radio interference, television interference, and corona loss. The audible noise is a relatively new environmental concern and is becoming more important with increasing voltage level. For example, for transmission lines up to 800 kV, audible noise and electric field effects have become major design factors and have received considerable testing and study. It had been observed that the audible noise from the corona process mainly takes place in foul weather. In dry conditions, the conductors normally operate below the corona detection level, and therefore, very few corona sources exist. In wet conditions, however, water drops on the conductors cause large number of corona dicharges and a resulting burst of noise. At ultrahigh-voltage levels (1000 kV and higher), such audible noise is the limiting environmental design factor.

8.2 CORONA

8.2.1 Nature of Corona

Succinctly put, corona is a luminous partial discharge due to ionization of the air surrounding a conductor caused by electrical overstress. Many tests show that dry air at normal atmospheric pressure and temperature (25 °C and 76 cm barometric pressure) breaks down at 29.8 kV/cm (maximum, or peak, value) or 21.1 kV/cm (rms, or effective, value). There are always a few free electrons in the air due to ultraviolet radiation from the sun, cosmic rays from outer space, radioactivity of the earth, etc. As the conductor becomes energized on each half cycle of the ac voltage wave, the electrons in the air near its surface are accelerated toward the conductor on its positive half cycle and away from the conductor on its negative half cycle. The velocity attained by a free electron is dependent on the intentisy of the electric field. If the intensity of the electric field exceeds a certain critical value, any free electron in this field will acquire a sufficient velocity and energy to knock one of the outer orbit electrons clear out of one of the two atoms of the air molecule. This process is called *ionization*, and the molecule with the missing electron is called a *positive ion*. The initial electron, which lost most of its velocity in the collision, and the electron

† According to Nasser [1], "coronas have various industrial applications, such as in high-speed printout devices, in air purification devices by electronic precipitators, in dry-ore separation systems, as chemical catalysts, in radiation detectors and counters, and in discharging undesirable electric charges from airplanes and plastics. Coronas are used as efficient means of discharging other statically electrified surfaces of wool and paper in the manufacturing industry. They are also used successfully in the deemulsification of crude oil–brine mixtures."

knocked out of the air molecule, which also has a low velocity, are both accelerated by the electric field, and therefore, each electron is capable of ionizing an air molecule at the next collision. Of course, after the second collision, there are now four electrons to repeat the process, and so on, the number of electrons doubling after each collision. All this time, the electrons are advancing toward the positive electrode, and after many collisions, their number has grown enormously. Therefore, this process is called the *avalanche process*†. Note that each so-called *electron avalanche* is initiated by a single free electron that finds itself in an intense electrostatic field. Also note that the intensity of the electrostatic field around a conductor is nonuniform. Therefore, it has its maximum strength at the surface of the conductor and its intensity diminishes inversely as the distance increases from the center of the conductor. Thus, as the voltage level in the conductor is increased, the critical field strength is approached, and the initial discharges take place only at or near the conductor surface. For the positive half cycle, the electron avalanches move toward the conductor and continue to grow until they hit the surface. For the negative half cycle, the electron avalanches move way from the conductor surface toward a weaker field and

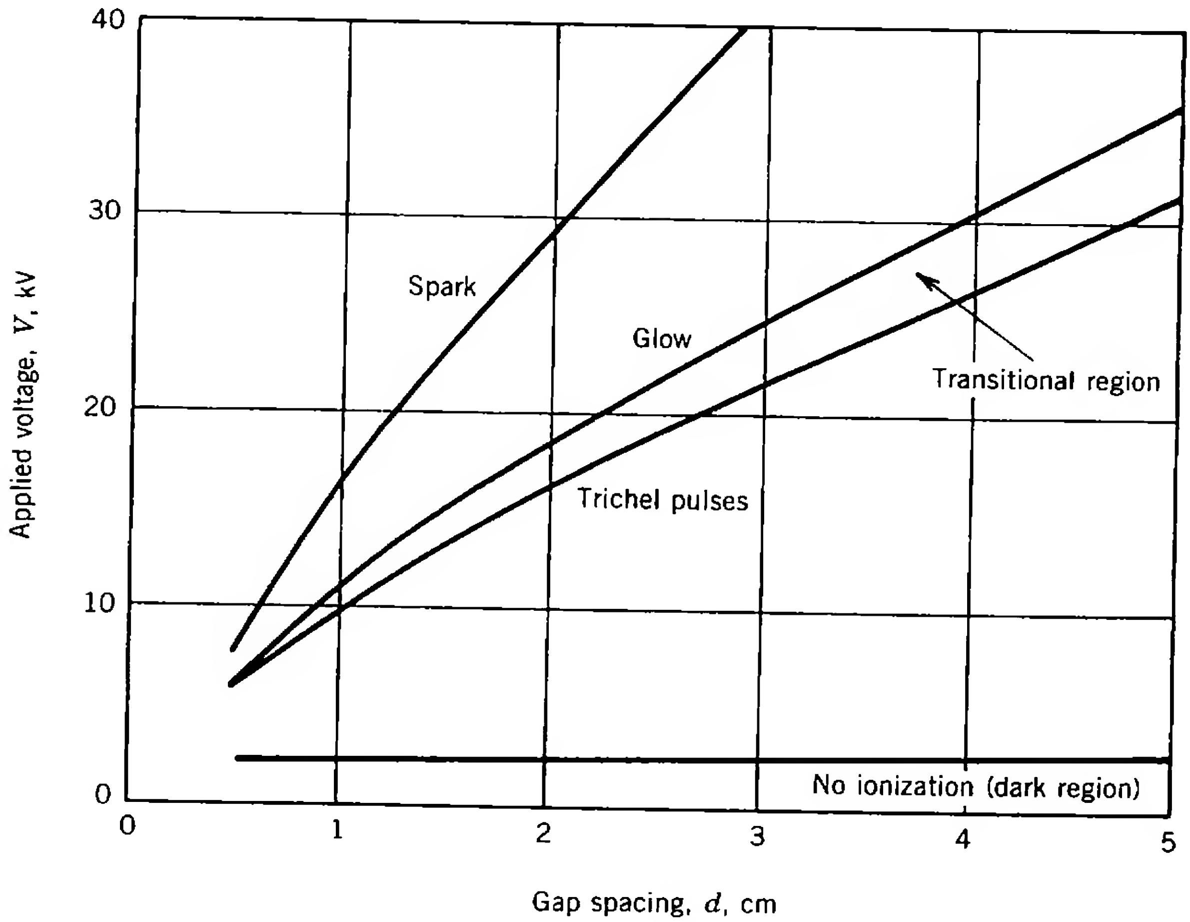


Figure 8.1. Basic negative corona modes and their regions in typical gap with cathode radius of 0.06 mm [1].

† It is also known as *Townsend's avalanche process*, after Townsend [2].

cease to advance when the field becomes too weak to accelerate the electrons to ionizing velocity. Most of the early studies done by Trichel [3], Loeb [4–6], and others [7–9] has been on the negative nonuniforn field-corona processes. For example, Trichel [3, 5] observed a type of negative corona from a point discontinuity. The corona consisted of a sequence of low-amplitude current pulses whose repetition rate depended on the sharpness of the point. Figure 8.1 illustrates the fact that when the voltage across a point-to-plane gap is gradually increased, a current (in the order of 10^{-14} A) is measured. Here there is no ionization that takes place, and this current is known as the *saturation current*. At a certain voltage, an abrupt current increase indicates the development of an ionization form that produces regular current pulses. These pulses were studied in great detail by Trichel in 1938 and are therefore called the *Trichel pulses*. Figure 8.1 shows the onset voltage of different coronas plotted as a function of electrode separation d for a typical example of a cathode of 0.06. Early corona studies included the use of photographs of the coronas known as *Lichtenberg figures* [8, 9]. The ac corona, viewed through a stroboscope, has the same appearance as the dc corona.

8.2.2 Manifestations of Corona

Corona manifests itself by a *visual corona*, which appears as bluish (or violet-colored) tuffs, streamers, and/or glows around the conductor, being more or less concentrated at irregularities on the conductor surface. This light is produced by the recombination of positive nitrogen ions with free electrons. This glow discharge is a very faint (or weak) light that appears to surround the conductor surface. It also may appear on critical regions of insulator surfaces during high humidity conditions. Figures 8.2–8.4 show various corona discharges at extra-high-voltage levels. The streamer-type discharge are also known as the *brush* discharge and are projected radially from the conductor surfaces. The discharge resembling a plume is also known as the *plume* discharge and has a concentrated stem that may be anywhere from a fraction of an inch long to several inches in length depending on the voltage level of the conductor. At its outer end, the stem branches many times and merges into a violet-colored treelike halo that has a length from a few inches at lower voltages to a foot or more at very high voltages. The second manifestation of corona is known as the *audible corona*, which appears as a hissing or frying sound whenever the conductor is energized above its corona threshold voltage. The sound is produced by the disturbances set up in the air in the vicinity of the discharge, possibly by the movement of the positive ions as they are suddenly created in an intense electric field. There is generally no sound associated with glow discharges. The corona phenomenon is also accompanied by the odor of ozone. In the presence of moisture, nitrous acid is produced, and if the corona is heavy enough, corrosion of the conductors will result. Of course, there is always a

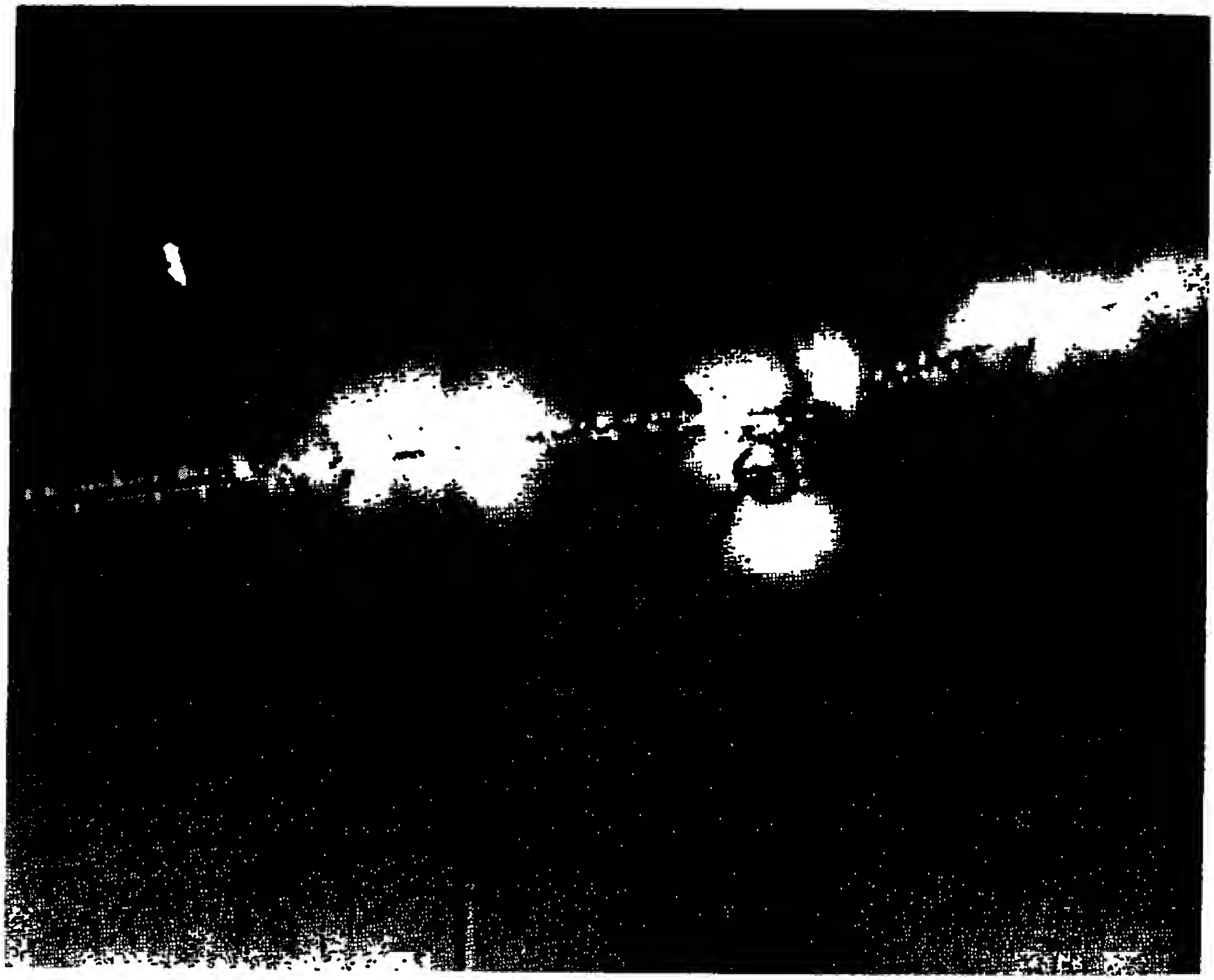


Figure 8.2. Corona testing of conductor in laboratory environment. (Courtesy of Ohio Brass Company.)

power loss associated with corona. Furthermore, the charging current under corona condition increases due to the fact that the corona introduces harmonic currents.

The last and perhaps most serious manifestation of the corona is the electrical effect that causes radio interference (RI) and/or television interference (TVI). The avalanches, being electrons in motion, actually constitute electric currents and therefore produce both magnetic and electrostatic fields in the vicinity. Since they are formed very suddenly and have short duration, these magnetic and electrostatic fields can induce high-frequency voltage pulses in nearby radio (or television) antennas and thus may cause RI (or TVI). These electrical disturbances are usually measured with a radio meter. It is interesting to note that corona will reduce the overvoltage on long open-circuited lines due to lightning or switching surges.

8.2.3 Factors Affecting Corona

As a rule of thumb, if the ratio of spacing between conductors to the radius of the conductor is less than 15, flashover will take place between the

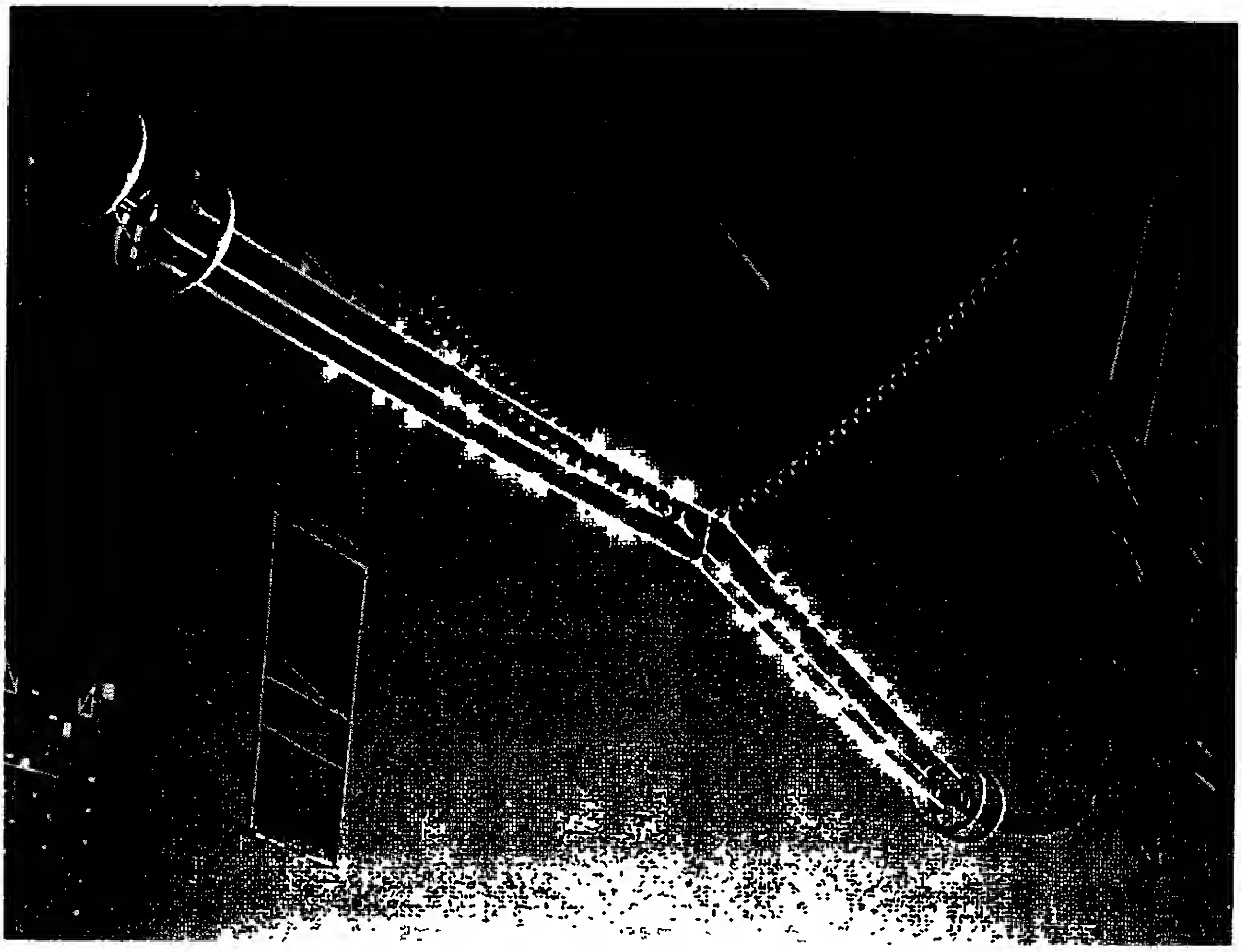


Figure 8.3. Corona testing on 500-kV triple-conductor support at elevated 60-Hz voltage for visual study of corona. (Courtesy of Ohio Brass Company.)

conductors before corona phenomenon occurs. Since for overhead lines this ratio is much greater than 15, the flashover can be considered as impossible under normal circumstances. At a given voltage level, the factors affecting corona include line configuration, conductor type, condition of conductor surface, and weather. In a horizontal configuration, the field near the middle conductor is larger than the field near the outer conductors. Therefore, the disruptive critical voltage is lower for the middle conductor, causing larger corona loss than the ones for the two other conductors. If the conductors are not spaced equilaterally, the surface gradients of the conductors and therefore the corona losses are not equal. Also, the conductor height affects the corona loss, that is, the greater the height, the smaller the corona loss. The corona loss is proportional to the frequency of the voltage. Therefore, the higher the frequency, the higher the corona losses. Thus, the corona loss at 60 Hz is greater than the one at 50 Hz. Of course, the corona loss at zero frequency, that is, direct current, is far less than the one for ac current.

The irregularity of the conductor surface in terms of scratches, raised strands, die burrs, die grease, and the particles of dust and dirt that clog the conductor can significantly increase the corona loss. The smoother the surface of a given cylindrical conductor, the higher the disruptive voltage.

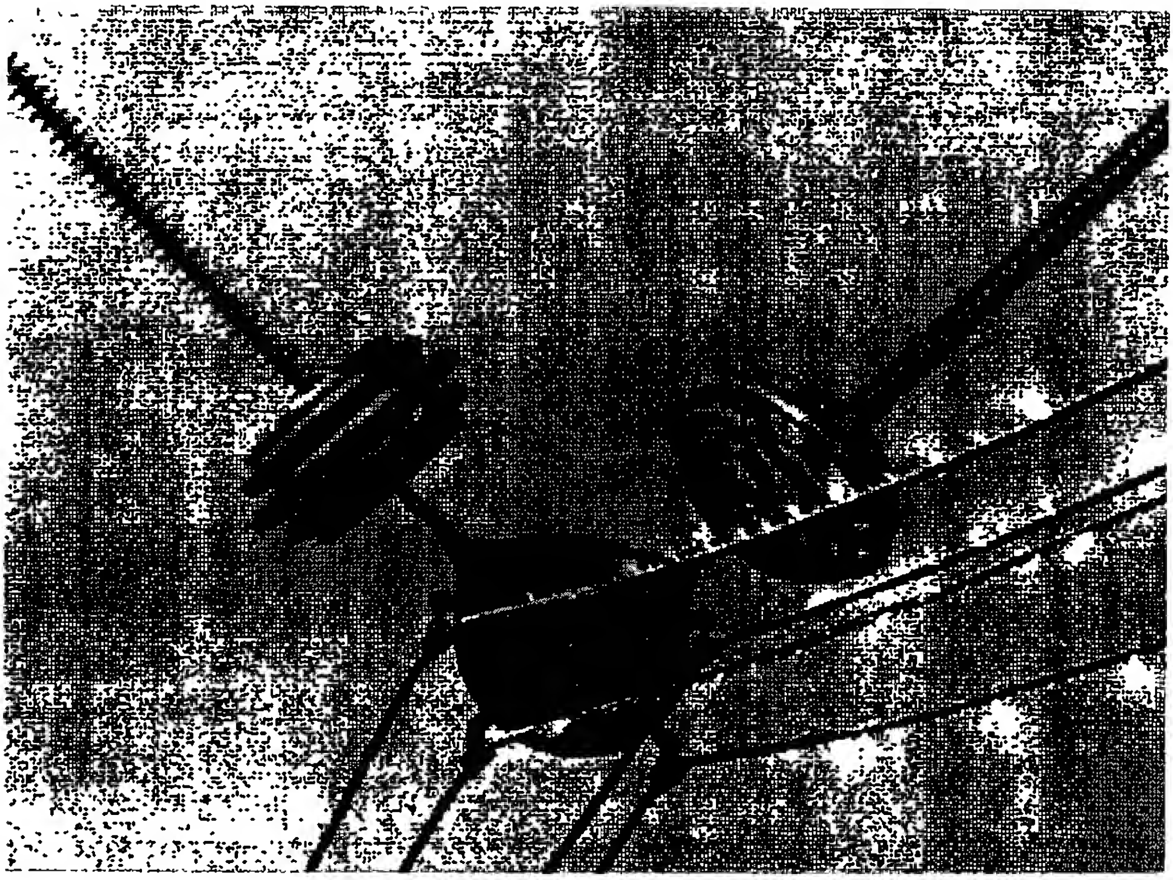


Figure 8.4. Corona testing of line with four bundled conductors. (Courtesy of Ohio Brass Company.)

For the same diameter, a stranded conductor is usually satisfactory for about 80–85 percent of the voltage of a smooth conductor. As said before, the size of the conductors and their spacings also have considerable effect on corona loss. The larger the diameter, the less likelihood of corona. Therefore, the use of conductors with large diameters or the use of hollow conductors or the use of bundled conductors increases the effective diameter by reducing the electric stress at the conductor surfaces.

The breakdown strength of air varies with atmospheric conditions. Therefore, the breakdown strength of air is directly proportional to the density of the air. The air–density factor is defined as

$$\delta = \frac{3.9211p}{273 + t} \tag{8.1}$$

where p = barometric pressure in centimeters of mercury
t = ambient temperature in degrees Celsius

Table 8.1 gives barometric pressures as a function of attitude. Foul weather conditions (e.g., rain, snow, hoarfrost, sleet, and fog) all lower the critical voltage and increase the corona. Rain affects corona loss usually more than

TABLE 8.1 Standard Barometric Pressure as Function of Altitude

Altitude (ft)	Pressure (cm Hg)	Altitude (ft)	Pressure (cm Hg)
−1000	78.79	5,000	63.22
−500	77.40	6,000	60.91
0	76.00	7,000	58.67
1000	73.30	8,000	56.44
2000	70.66	10,000	52.27
3000	68.10	15,000	42.88
4000	65.54	20,000	34.93

any other factor. For example, it may cause the corona loss to be produced on a conductor at voltages as low as 65 percent of the voltage at which the same loss takes place during fair weather. Heavy winds have no effect on the disruptive critical voltage or on the loss, but presence of smoke lowers the critical voltage and increases the loss. Corona in fair weather may be negligible up to a voltage close to the disruptive critical voltage for a particular conductor. Above this voltage, the impacts of corona increase very quickly.

A transmission line should be designed to operate just below the disruptive critical voltage in fair wether so that corona only takes place during adverse atmospheric conditions. Therefore, the calculated disruptive critical voltage is an indicator of corona performance of the line. However, a high value of the disruptive critical voltage is not the only criterion of satisfactory corona performance. The sensitivity of the conductor to foul weather should also be considered (e.g., corona increases more slowly on stranded conductors than on smooth conductors). Due to the numerous factors involved, the precise calculation of the peak value of corona loss is extremely difficult, if not impossible. The minimum voltage at which the ionization occurs in fair weather is called the *disruptive critical voltage* and can be determined from

$$E_0 = \frac{V_0}{r \ln(D/r)} \tag{8.2}$$

as

$$V_0 = E_0 r \ln \frac{D}{r} \tag{8.3}$$

where E_0 = value of electric stress (or critical gradient) at which disruption starts in kilovolts per centimeters
V_0 = disruptive critical voltage to neutral in kilovolts (rms)
r = radius of conductor in centimeters
D = spacing between two conductors in centimeters

Since, in fair weather, the E_0 of air is 21.1 kV/cm rms,

$$V_0 = 21.1 r \ln \frac{D}{r} \quad \text{kV} \tag{8.4}$$

which is correct for normal atmospheric pressure and temperature (76 cm Hg at 25°C). For other atmospheric pressures and temperatures,

$$V_0 = 21.1 \delta r \ln \frac{D}{t} \quad \text{kV} \tag{8.5}$$

where δ is the air density factor given by equation (8.1). Further, according to Peek [10], after making allowance for the surface condition of the conductor by using the irregularity factor, the disruptive critical voltage can be expressed as

$$V_0 = 21.1 \delta m_0 r \ln \frac{D}{r} \quad \text{kV} \tag{8.6}$$

where m = irregularity factor ($0 < m_0 \le 1$)
= 1 for smooth, polished, solids, cylindrical conductors
= 0.93–0.98 for weathered, solid, cylindrical conductors
= 0.87–0.90 for weathered conductors with more than seven strands
= 0.80–0.87 for weathered conductors with up to seven strands

Note that at the disruptive critical voltage V_0, there is no visible corona. In the event that the potential difference (or critical gradient) is further increased, a second point is reached at which a weak luminous glow of violet color can be seen to surround each conductor. The voltage value at this point is called the *visual critical voltage* and is given by Peek [10] as

$$V_v = 21.1 \delta m_v r \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \ln \frac{D}{r} \quad \text{kV} \tag{8.7}$$

where V_v = visual critical voltage in kilovolts (rms)
m_v = irregularity factor for visible corona ($0 < m_v \le 1$)
= 1 for smooth, polished, solid, cylindrical conductors
= 0.93–0.98 for local and general visual corona on weathered, solid, cylindrical conductors
= 0.70–0.75 for local visual corona on weathered stranded conductors
= 0.80–0.85 for general visual corona on weathered stranded conductors

Note that the voltage equations given in this section are for fair weather. For wet weather voltage values, multiply the resulting fair weather voltage values, multiply the resulting fair weather voltage values by 0.80. For a three-phase horizontal conductor configuration, the calculated disruptive critical voltage should be multiplied by 0.96 and 1.06 for the middle conductor and for the two outer conductors, respectively.

EXAMPLE 8.1

Assume that a three-phase overhead transmission line is made up of three equilaterally spaced conductors, each with overall diameter of 3 cm. The equilateral spacing between conductors is 5.5 m. The atmosphere pressure is 74 cm Hg and the temperature is 10 °C. If the irregularity factor of the conductors is 0.90 in each case, determine the following:

(a) Disruptive critical rms line voltage.
(b) Visual critical rms line voltage.

Solution

(a) From equation (8.1),

$$\delta = \frac{3.9211p}{273 + t} = \frac{3{,}9211 \times 74}{273 + 10} = 1.0253$$

$$V_0 = 21.1\delta m_0 r \ln \frac{D}{r}$$

$$= 21.1 \times 1.0253 \times 0.90 \times 1.5 \ln \frac{550}{1.5}$$

$$= 172.4 \text{ kV/phase}$$

Thus, the rms line voltage is

$$V_0 = \sqrt{3} \times 172.4 = 298.7 \text{ kV}$$

(b) The visual critical rms line voltage is

$$V_v = 21.1\delta m_v \left(1 + \frac{0.3}{\sqrt{\delta r}}\right) \ln \frac{D}{r}$$

$$= 21.1 \times 1.0253 \times 0.90 \times 1.5 \times \left(1 + \frac{0.3}{\sqrt{1.0253 \times 1.5}}\right) \ln \frac{550}{1.5}$$

$$= 214.2 \text{ kV/phase}$$

Therefore, the rms line voltage is

$$V_v = \sqrt{3} \times 214.2 = 370.9 \text{ kV}$$

8.2.4 Corona Loss

According to Peek [10], the fair weather corona loss per phase or conductor can be calculated from

$$P_c = \frac{241}{\delta}(f+25)\left(\frac{r}{D}\right)^{1/2}(V-V_0)^2 \times 10^{-5} \text{ kW/km} \tag{8.8}$$

or

$$P_c = \frac{390}{\delta}(f+25)\left(\frac{r}{D}\right)^{1/2}(V-V_0)^2 \times 10^{-5} \text{ kW/mi} \tag{8.9}$$

where f = frequency in hertz
V = line-to-neutral operating voltage in kilovolts
V_0 = disruptive critical voltage in kilovolts

The wet weather corona can be calculated from the above equations by multiplying V_0 by 0.80. Peek's equation gives a correct result if (1) the frequency is between 25 and 120 Hz, (2) the conductor radius is greater than 0.25 cm, and (3) the ratio of V to V_0 is greater than 1.8. From equation (10.8) or (10.9), one can observe that the power loss due to corona is

$$P_c \propto \left(\frac{r}{D}\right)^{1/2}$$

that is, the power loss is proportional to the square root of the size of the conductor. Therefore, the larger the radius of the conductor, the larger the power loss. Also, the larger the spacing between conductors, the smaller the power loss. Similarly,

$$P_c \propto (V-V_0)^2$$

that is, for a given voltage level, the larger the conductor size, the larger the disruptive critical voltage and therefore the smaller the power loss.

According to Peterson [11], the fair weather corona loss per phase or conductor† can be calculated from

$$P_c = \frac{1.11066 \times 10^{-4}}{[\ln(2D/d)]^2} fV^2F \quad \text{kW/km} \tag{8.10}$$

or

$$P_c = \frac{1.78738 \times 10^{-4}}{[\ln(2D/d)]^2} fV^2F \quad \text{kW/mi} \tag{8.11}$$

where d = conductor diameter
D = spacing between conductors
f = frequency in hertz
V = line-to-neutral operating voltage in kilovolts
F = corona factor determined by test and is a function of ratio of V to V_0

† An additional and also popular method to calculate the fair-weather corona loss has been suggested by Carroll and Rockwell [12].

Typically, for fair weather corona†,

(V/V_0)	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0	2.2
F	0.012	0.018	0.05	0.08	0.3	1.0	3.5	6.0	8.0

In general, the corona losses due to fair weather conditions are not significantly large at extra-high-voltage range. Therefore, their effects are not significant from technical and/or economic points of view. Whereas the corona losses due to foul weather conditions are very significant. For lines operating between 400 and 700 kV, the corona loss due to rainy weather is determined from the following expression [12–14]:

$$\mathrm{TP}_{c,\mathrm{RW}} = \mathrm{TP}_{c,\mathrm{FW}} + \left[\frac{V}{\sqrt{3}}\, jr^2 \ln(1 + KR) \right] \sum_{i=1}^{n} E_i^m \tag{8.12}$$

where $\mathrm{TP}_{c,\mathrm{RW}}$ = total three-phase corona losses due to rainy weather in kilowatts per kilometer

$\mathrm{TP}_{c,\mathrm{FW}}$ = total three-phase corona losses due to fair weather in kilowatts per kilometer

V = line-to-line operating voltage in kilovolts

r = conductor radius in centimeters

n = total number of conductors (number of conductors per bundle times 3)

E_i = voltage gradient on underside of conductor i in kilovolts (peak) per centimeter

m = an exponent ($\cong 5$)

J = loss current constant ($\sim 4.37 \times 10^{-10}$ at 400 kV and 3.32×10^{-10} at 500 kV and 700 kV)‡

R = rain rate in millimeters per hour or inches per hour

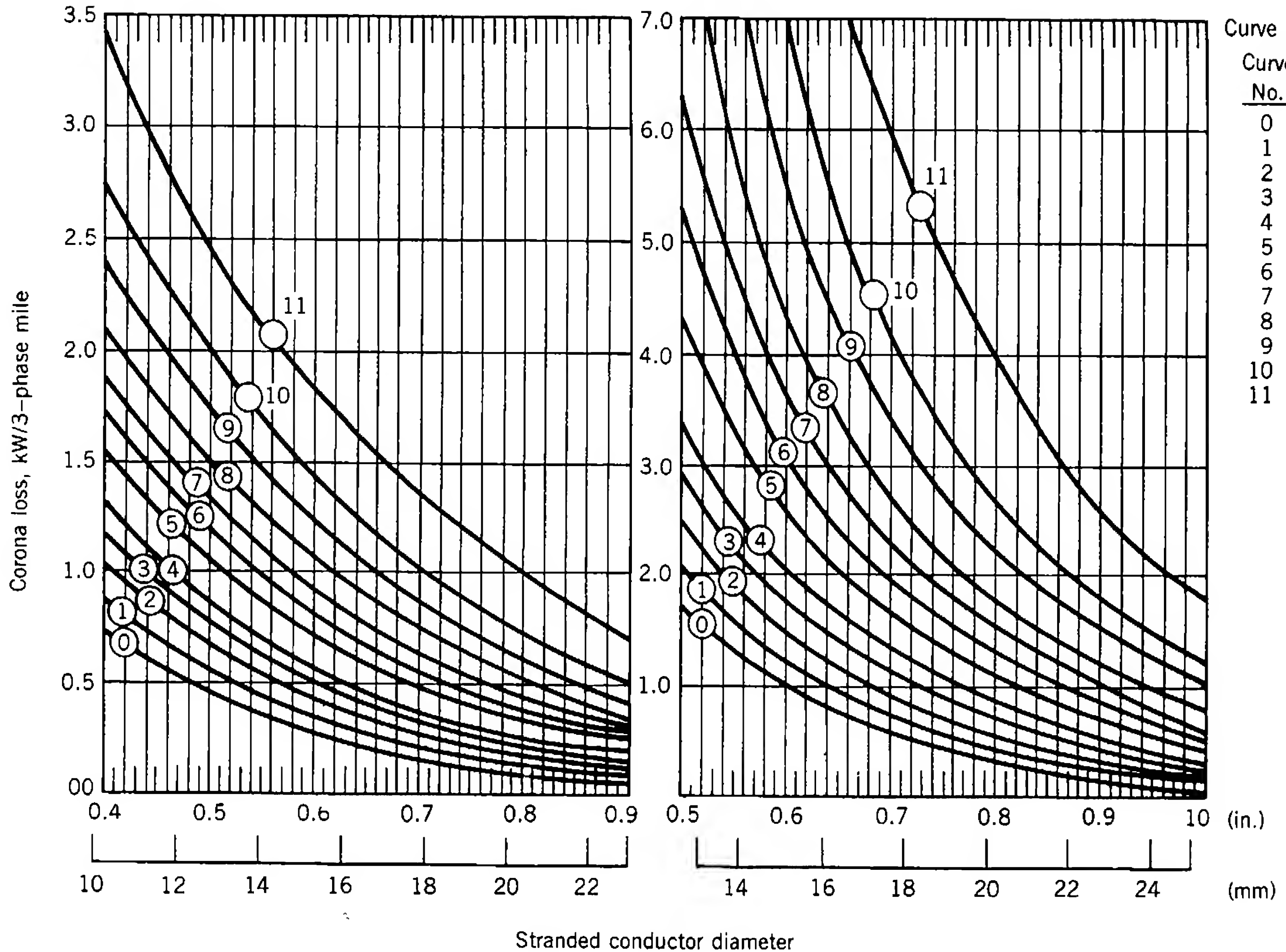
K = wetting coefficient (10 if R is in millimeter per hour or 254 if R is in inches per hour)

Note that the terms given in the square brackets are strictly due to the rain. The *EHV Transmission Line Reference Book* [13] gives a probabilistic method to determine the corona losses on the extra-high-voltage of various standard designs for different climatic regions of the United States.

Figures 8.5 and 8.6 show corona loss curves for 69-, 115-, 161-, and 230-kV lines designed for different elevations [15]. The curves are based on the Carroll–Rockwell method [12] and was developed for fair weather corona at 25 °C (77 °F) using ACSR conductors. Note that for a given

† The loss current constant J is approximately 7.04×10^{-10} at 400 kV and 5.35×10^{-10} at 500 and 700 kV if both the $\mathrm{TP}_{c,\mathrm{FW}}$ and $\mathrm{TP}_{c,\mathrm{RW}}$ are calculated in kilowatts per mile rather than in kilowatts per kilometer.

‡ For wet weather corona, determine the factor F using $V/0.80\,V_0$.



Curve index

Curve No.	Elevation m	Elevation ft
0	Sea level	
1	305	1 000
2	610	2 000
3	914	3 000
4	1218	4 000
5	1524	5 000
6	1829	6 000
7	2134	7 000
8	2438	8 000
9	2743	9 000
10	3048	10 000
11	3658	12 000

Figure 8.5. Corona loss curves for: (*a*) 115-kV line with 12 ft horizontal spacing; (*b*) 161-kV line with 17 ft horizontal spacing [15]. All curves computed by Carroll–Rockwell method for fair weather at 25 °C (77 °F).

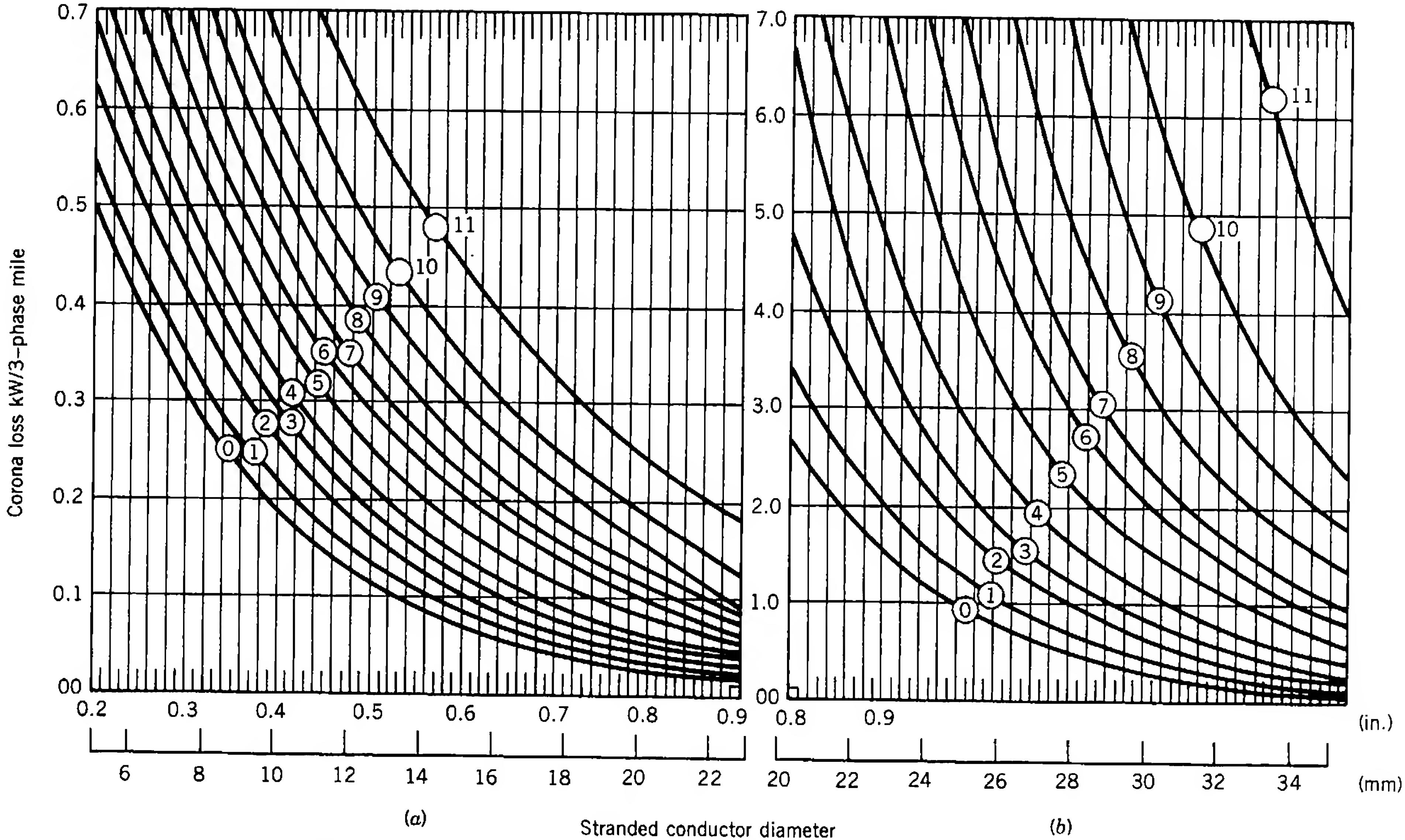


Figure 8.6. Corona loss curves for: (*a*) 69-kV line with 10 ft horizontal spacing; (*b*) 230-kV line with 22 ft horizontal spacing [15].

conductor diameter, the curves give the fair weather corona loss in kilowatts per three-phase mile.[†]

EXAMPLE 8.2

Consider Example 8.1 and assume that the line operates at 345 kV at 60 Hz and the line length is 50 mi. Determine the total fair weather corona loss for the line by using Peek's formula.

Solution

According to Peek, the fair weather corona loss per phase is

$$\begin{aligned} P_c &= \frac{390}{\delta}(f+25)\frac{r^{1/2}}{D}(V-V_0)^2 \times 10^{-15} \\ &= \frac{390}{1.0253}(60+25)\frac{1.5^{1/2}}{550}(199.2-172.4)^2 \times 10^{-5} \\ &= 12.1146 \text{ kW/mi/phase} \end{aligned}$$

or, for the total line length,

$$P_c = 12.1146 \times 50 = 605.7 \text{ kW/phase}$$

Therefore, the total corona loss of the line is

$$P_c = 3 \times 605.7 = 1817.2 \text{ kW}$$

8.3 RADIO NOISE

Radio noise (i.e., electromagnetic interference) from overhead power lines can occur due to partial electrical discharges (i.e., corona) or due to complete electrical discharges across small gaps (i.e., gap discharges, specifically sparking). The gap-type radio noise sources can take place in insulators, at tie wires between hardware parts, at small gaps between neutral or ground wires and hardware, in defective electrical apparatus, and on overhead power lines themselves. Typically, more than 90 percent of the consumer complaints are due to the gap-type radio noise.

Note the fact that *radio noise* (RN) is a general term that can be defined as "any unwanted disturbance within the radio frequency band, such as undesired electric waves in any transmission channel or device" [16]. The corona discharge process produces pulses of current and voltage on the line conductors. The frequency spectrum of such pulses is so large that it can include a significant portion of the radio frequency band, which extends from 3 kHz to 30,000 MHz. Therefore, the term *radio noise* is a general term that includes the terms *radio interference* and *television interference*.

[†]For those readers interested in the design aspects of transmission lines, Farr [15] is highly recommended.

8.3.1 Radio Interference

The radio interference (also called the radio influence) is a noise type that occurs in the AM radio reception, including the standard broadcast band from 0.5 to 1.6 MHz. It does not take place in the FM band.

Figure 8.7 illustrates the manner and paths by which such interference is transmitted to a radio receiver. As succinctly put by Chartier [17], "The interference energy can travel by one, or simultaneously, by two or three of the following means of transmission: (1) It travels by conduction via the transformer or by means of the neutral wire into the receiver power supply or wiring. (2) It travels by induction when the power line conductor or power supply lead carrying the interference energy is near enough to the antenna or some part of the receiver circuit to couple the interference energy into the receiver. (3) It travels by radiation, when the energy is launched into space by the overhead line or lines acting as a broadcasting antenna. In this instance, the energy can be reflected or reradiated from a nearby fence, power line, or metallic structure. Transmission by the first two methods is most important at very low frequencies because conduction current decreases more slowly with distance along the line as the frequency is decreased. At higher frequencies, *radiation* becomes relatively more efficient and is more likely to be the cause of interference than the conduction currents or the induction fields. In any case, however, power line

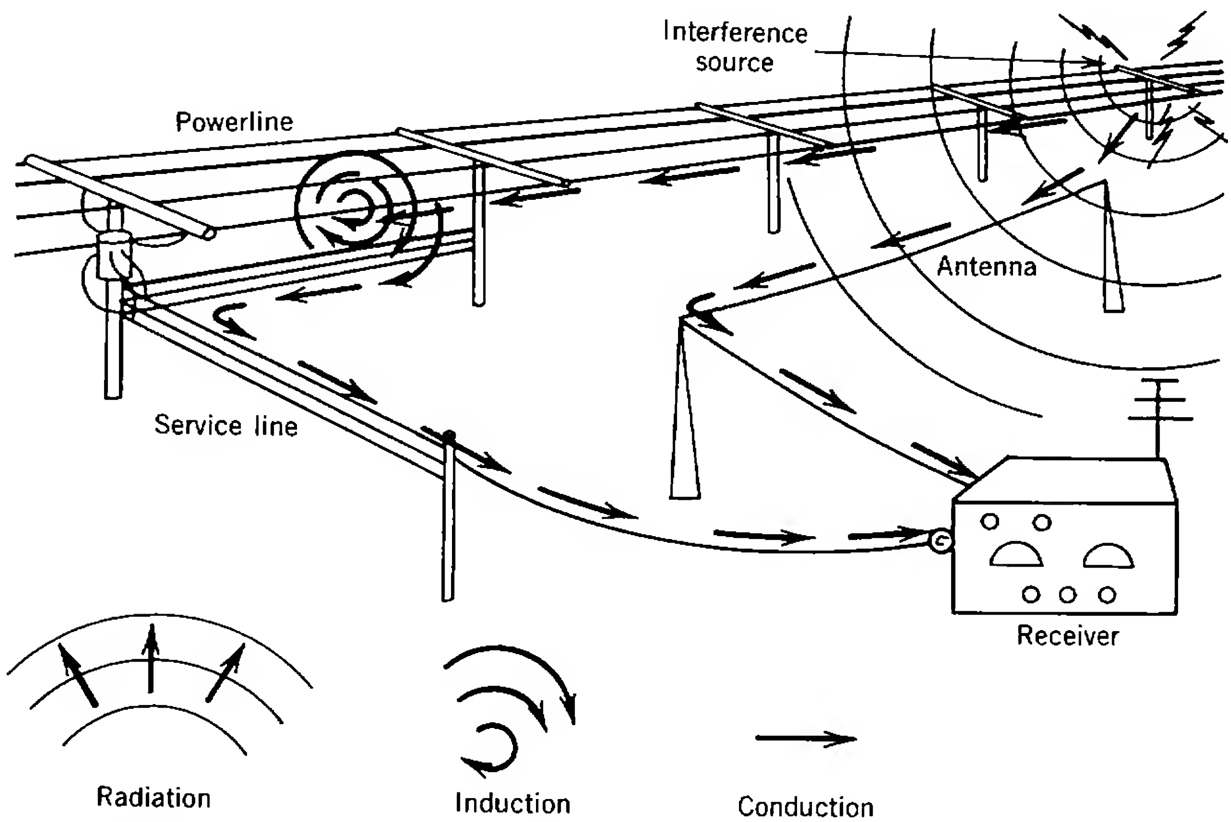


Figure 8.7. Paths by which interference energy travels from source to radio receiver. (From Ref. 17. Used with permission. © 1976 IEEE.)

interference tends to be roughly in inverse proportion to the frequency, that is, the higher the frequency, the lower the absolute interference level. Above a frequency of 100 MHz, conducted power line interference is very likely to have its source within a distance of 6 to 8 pole line spans of the receiver affected. However, in the case of *radiated* power line interference, there have been reports of objectionable interference originating from sources as far as 30 miles away." According to a report published by the Iowa State University [17], 25 percent of all the cases of RI could be traced to household equipment, while 15 percent of the cases were in the receiver itself. The remaining cases are distributed as follows: 30 percent, industrial equipment; 17 percent, generation, transmission, and distribution equipment; and 13 percent, miscellaneous. Figure 8.8 shows the RI and TVI complaints for the years 1959–1975 compared to number of customers of Southern California Edison Company.

The RI properties of a transmission line conductor can be specified by *radio influence voltage* (*RIV*) generated on the conductor surface. This term

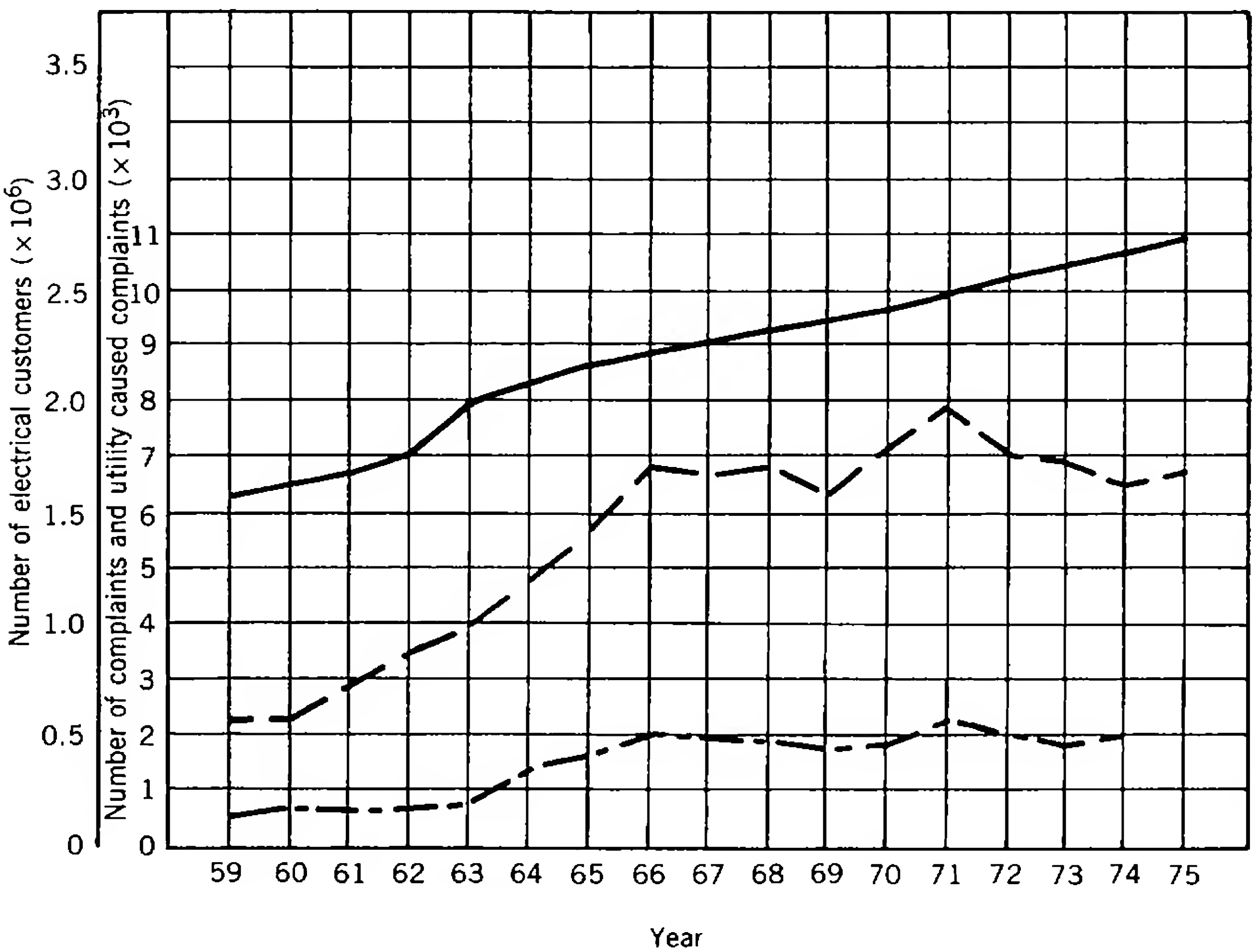


Figure 8.8. Radio and television interference complaints for 1959–1975 compared to number of customers of Southern California Edison Company: ———, number of electrical customers; – – – – –, total RI and TVI complaints; —·—, utility-caused complaints. (From Nelson and Schlinger, 1976. Used with permission. © 1976 IEEE.)

refers to the magnitude of the line-to-ground voltage that exists on a device such as the power line or a station apparatus at any specified frequency below 30 MHz. The threshold of RIV coincides with the appearance of visual corona. At the visual corona voltage, the RIV is negligibly small, but with the initial appearance of corona, RIV level increases quickly, reaching very high values for small increases above the visual corona voltages. The rate of increase in RI is affected by conductor surface and diameter, being higher for smooth conductors and large-diameter conductors. The corona and RI problems can be reduced or avoided by the correct choice of conductor size and the use of *conductor bundling*, often made necessary by other line design requirements. Figure 8.9 shows typical values of conductor diameter that yield acceptable levels of electromagnetic interference. Precipitation increases RI, as does high humidity. Instrumentation to measure the electromagnetic interference field and to determine its frequency spectrum has been developed and standardized. The *quasi-peak* (*QP*) value of the electric field component obtained with a narrow-band amplifier of standard gain is recognized as representing the disturbing effect of typical

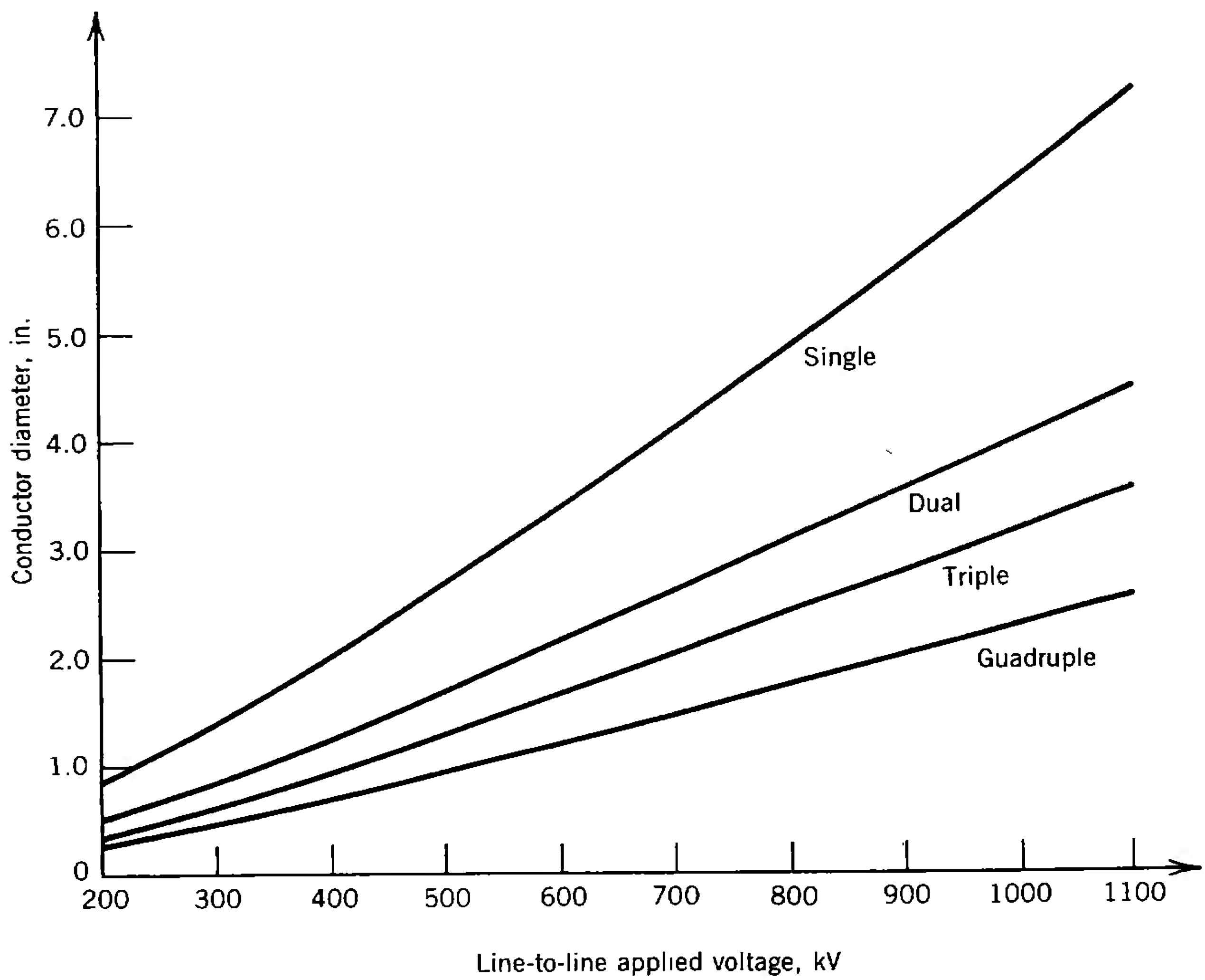


Figure 8.9. Typical values of conductor diameter that yield acceptable levels of interference.

corona noise. Radio noise (RI or TVI) is usually expressed in millivolts per meter or in decibels above 1 μV/m. Figure 8.10 provides a comparison of measured fair weather RN profile and computed heavy rain RN profile for a 735-kV line. As conductors age, RN levels tend to decrease. Since corona is mainly a function of the potential gradients at the conductors and the RN is associated with the corona, the RN as well as corona will increase with higher voltage, other things being equal. The RN depends also on the layout of the line, including the number and location of the phase and ground conductors, and the line length.

The RN is measured adjacent to a transmission line by an antenna equipped with a radio noise meter. The standard noise meter operates at 1 MHz (in the standard AM broadcast band) with a bandwidth of 5 kHz, using a quasi-peak detector having a charging time constant of 1 ms and a discharging time constant of 600 ms. For measurements in the RI range, a *rod antenna* is usually used for determination of the electric field E, and a *loop antenna* is normally used for determintion of the magnetic field component H.

Succinctly put, RI generated by a corona streamer is caused by the movement of space charges in the electric field of the conductor. As

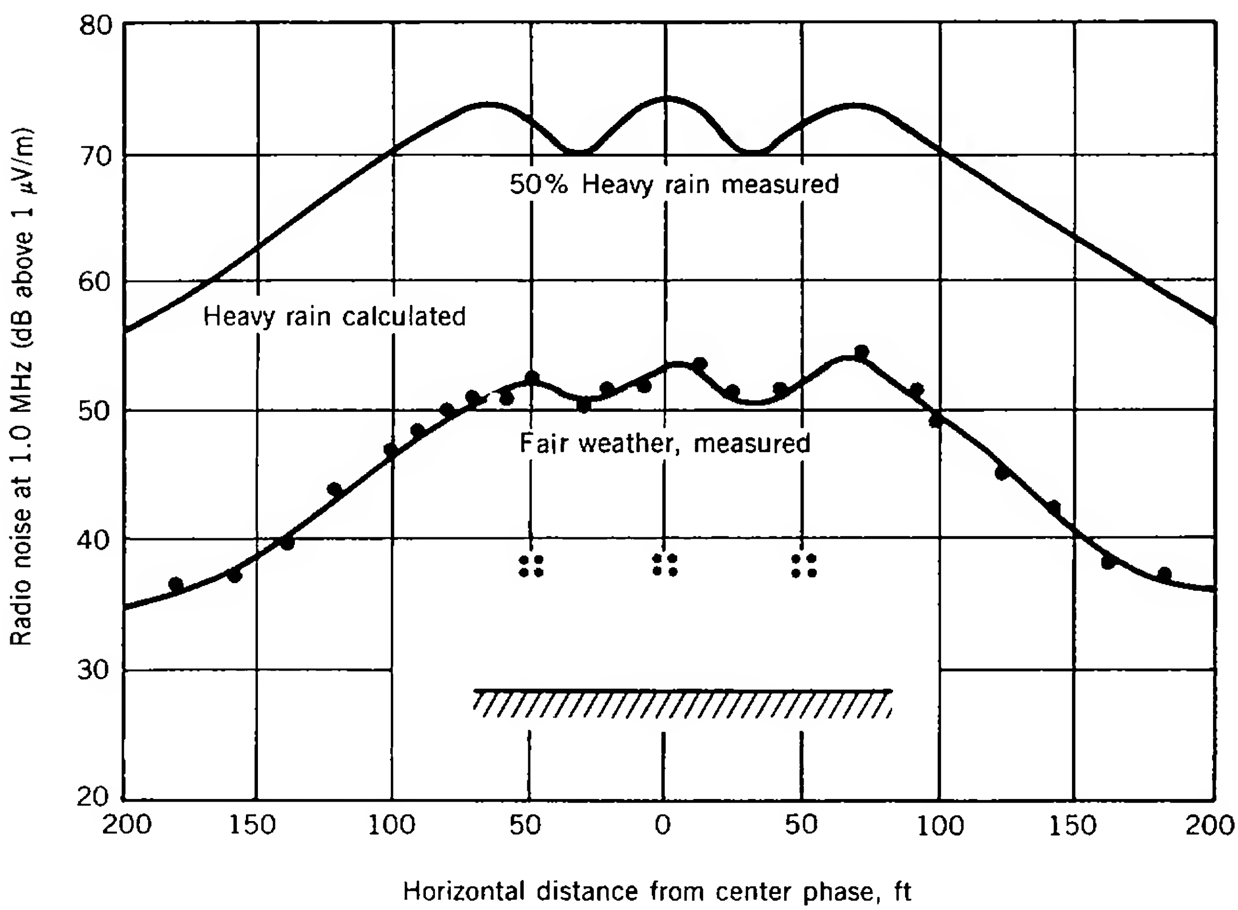


Figure 8.10. Comparison of measured fair weather RN profile and computed heavy rain RN profile for 735-kV line. (From Ref. 27. Used with permission. © 1971 IEEE.)

explained before, these charges are due to the ionization of air in the immediate vicinity of the conductor. As a source of RI, the streamer is usually represented as a *current generator*. Therefore, the current injected from this generator into the conductor depends only on the characteristics of the streamer. Adams has done extensive research on the RI phenomenon [18–23] and has demonstrated [18] that this representation was somewhat imperfect and that, in reality, the corona streamer induced currents in all conductors of a multiwire system and therefore not only in the conductor that produced it. These currents depend on the characteristics of the conductor under corona and the self-capacitance and mutual capacitance of the conductors. Therefore, the RI currents in conductors of different lines are not necessarily equal from one line to the other, even if they are generated by identical corona streamers. According to Adams [18], the term that expresses the characteristics of the corona streamer is *excitation function*. Later, to predict the RI associated with different line designs, measurements were taken in specially built *test cages* for various conductor bundles. From mesurements of the RI produced within these short lengths of enclosed line, the effective noise current (i.e., the excitation function) being injected into each phase of the line was inferred. Figure 8.11 shows this excitation function as a function of the maximum surface gradient for bundled conductors made up of conductors of the diameter shown. Therefore, the noise currents being injected into each phase of the line can be determined for the excitation function.[†] Once the excitation function of a conductor is known, the RI (or RN) of a line having the same conductor can be calculated.

The approximate value of the RI can be determined from the following empirical formula:

$$\mathrm{RI} = 50 + K(E_m - 16.95) + 17.3686 \ln \frac{d}{3.93} + F_n + 13.8949 \ln \frac{20}{D} + F_{\mathrm{FW}} \tag{8.13}$$

where RI = radio noise in decibels above 1 μV/m at 1 MHz

$K = 3$ for 750-kV class

$= 3.5$ for others, gradient limits 15–19 kV/cm

E_m = maximum electric field at conductor (gradient) in kilovolts per centimeter rms

d = (sub)conductor diameter in centimeters

$F_n = -4$ dB for single conductor

$= 4.3422 \ln(n/4)$ for $n > 1$, n = number of conductors in bundle

D = radial distance from conductor to antenna in meters

$= (h^2 + R^2)^{1/2}$

h = line height in meters

[†] For an excellent explanation of the physical meaning of the excitation function, see Gary [24].

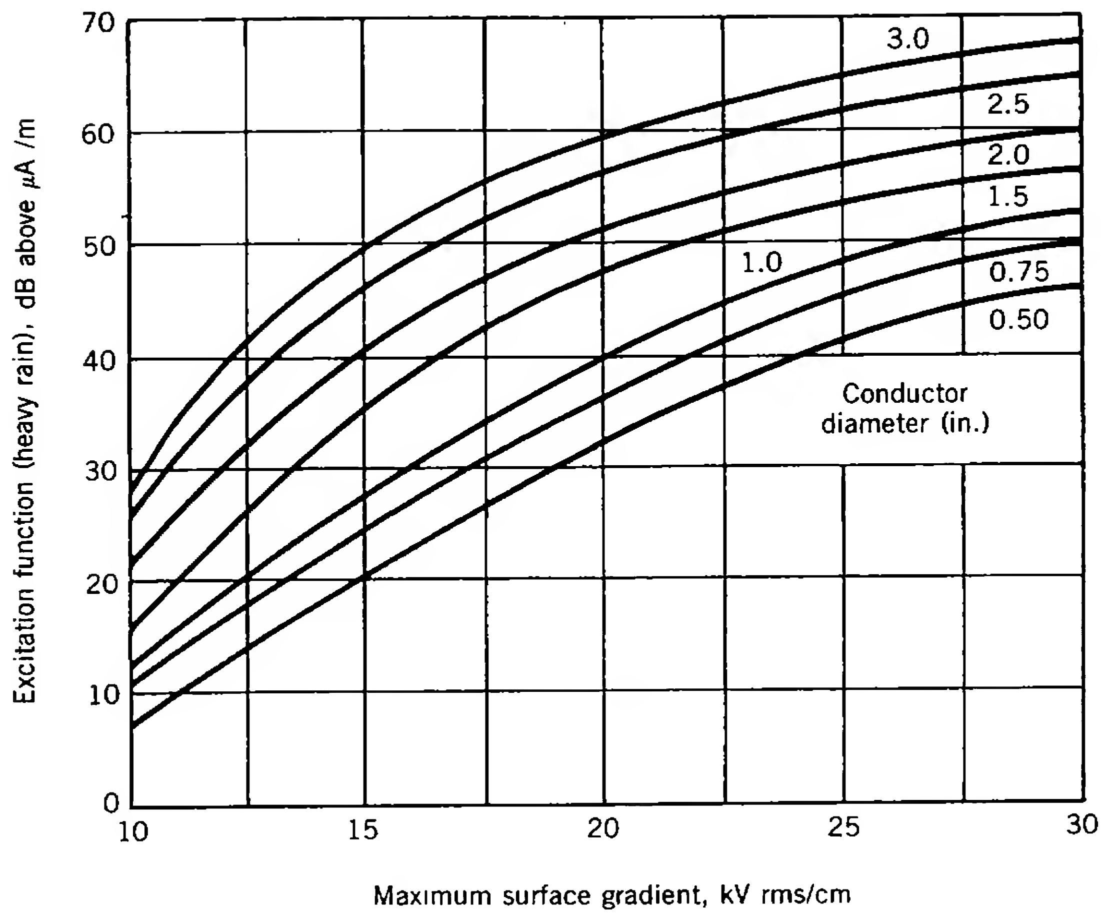


Figure 8.11. Radio interference excitation function in heavy rain of different bundles as function of maximum surface gradient (add 7 dB for $n = 1$, 2 dB for $n = 2$, and 0 dB for $n = 3$, where n is number of subconductors). (From Ref. 27. Used with permission. © 1971 IEEE.)

R = lateral distance from antenna to nearest phase in meters
F_{FW} = 17 for foul weather
= 0 for fair weather

Alternatively, the RI of a transmission line can also be determined from a method adopted by the Booneville Power Administration (BPA) [25]. The method relates the RI of any given line to that of a RI (under the same meteorological conditions) for which the RI is known through measurement. Therefore, the RI of the given line can be determined from

$$\mathrm{RI} = \mathrm{RI}_0 + 120 \log_{10} \frac{g}{g_0} + 40 \log_{10}(d/d_0) + 20 \log_{10} \frac{hD_0^2}{h_0 D^2} \tag{8.14}$$

where RI_0 = radio interference of reference line
g = average maximum (bundle) gradient in kilovolts per centimeter
d = (sub) conductor in millimeters
h = line height in meters
D = direct (radial) distance from conductor to antenna in meters

8.3.2 Television Interference

In general, power line RN sources disturbing television reception are due to noncorona sources. Such power line interference in the VHF (30–300 MHz) and UHF (300–3000 MHz) bands is almost always caused by *sparking*. TVI can be categorized as fair weather TVI and foul weather TVI. Since the sparks are usually shorted out during rain, sparking is considered to be a fair weather problem rather than a foul weather one. The foul weather TVI is basically from water droplet corona on the bottom side of conductors, and therefore, it does not require *source locating*. If the RI of a transmission line is known, its foul weather TVI can be determined from the following [14]

$$\text{TVI} = \text{RI} - 20\log_{10}\left[f\left(\frac{1+(R/h)^2}{1+(15/h)_2}\right)^{1/2}\right] + 3.2 \tag{8.15}$$

where TVI = television interference, in decibels (quasi-peak) above 1 μV/m at a frequency f in megahertz

RI = radio interference in decibels (quasi-peak) above 1 μV/m at 1 MHz and at standard reference location of 15 m laterally from outermost phase

f = frequency in megahertz

R = lateral distance from antenna to nearest phase in meter

h = height of closest phase in meters

Alternatively, the foul weather TVI of a transmission line can also be determined from a method adopted by the BPA [25]. The method relates the TVI of any given line to that of a reference line (under the same meteorological conditions) for which the TVI is known through measurement. Therefore, the TVI of the given line can be determined from

$$\text{TVI} = \text{TVI}_0 + 120\log_{10}\frac{g}{g_0} + 40\log_{10}\frac{d}{d_0} + 20\log_{10}\frac{D}{D_0} \tag{8.16}$$

where TVI_0 = television interference of reference line

g = average maximum (bundle) gradient in kilovolts per centimeter rms

d = (sub)conductor diameter in millimeters

D = direct (radial) distance from conductor to antenna in meters

8.4 AUDIBLE NOISE

With increasing transmission system voltages, audible noise produced by corona on the transmission line conductors has become a significant design factor. Audible noise (AN) from transmission lines occurs primarily in foul weather. In fair weather, the conductors usually operate below the corona inception level, and very few corona sources exist. Therefore, the emission from well-designed ultrahigh-voltage bundle conductor in fair weather is quite low. In wet weather, however, water drops impinging or collecting on the conductors produce a large number of corona discharges, each of them creating a burst of noise. Therefore, the AN increases to such an extent that it represents one of the most serious limitations to the use of ultrahigh voltage. It has been shown that the broadband component of random noise generated by corona may extend to frequencies well beyond the sonic range [26]. The noise manifests itself as a sizzle, crackle, or hiss. Additionally, corona creates low-frequency pure tones (hum), basically 120 and 240 Hz, which are caused by the movement of the space charge surrounding the conductor. Figure 8.12 shows a typical random-noise portion of the AN frequency spectrum measured near an ultrahigh-voltage test line having 4×2-in. conductors per phase. The plotted test results can be expressed by the following empirical equation [27]:

$$\text{AN} = knd^{2.2}E^{3.6} \tag{8.17}$$

where k = coefficient of proportionality
n = number of conductors
d = diameter of conductors
E = field strength at conductor surface (potential gradient)

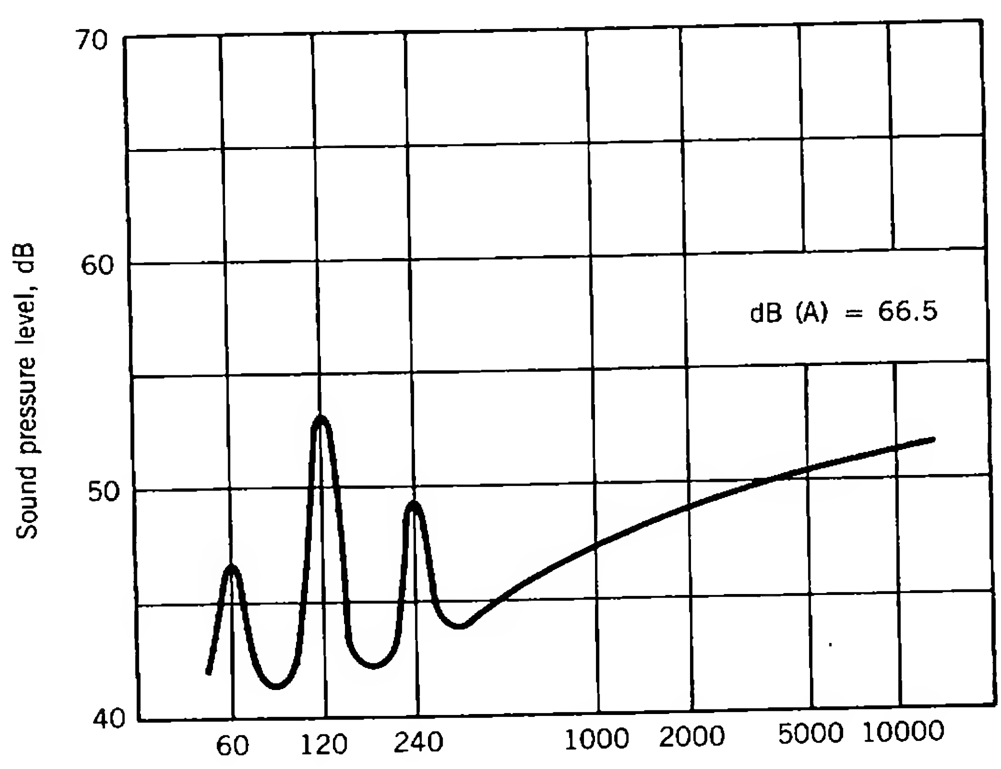


Figure 8.12. Audible noise frequency spectrum during rain (1/10 octave bandwidth dB above 0.0002 μbar General Radio meter) of a UHV line 4×2-in. bundle. (From Ref. 27. Used with permission. © 1971 IEEE.)

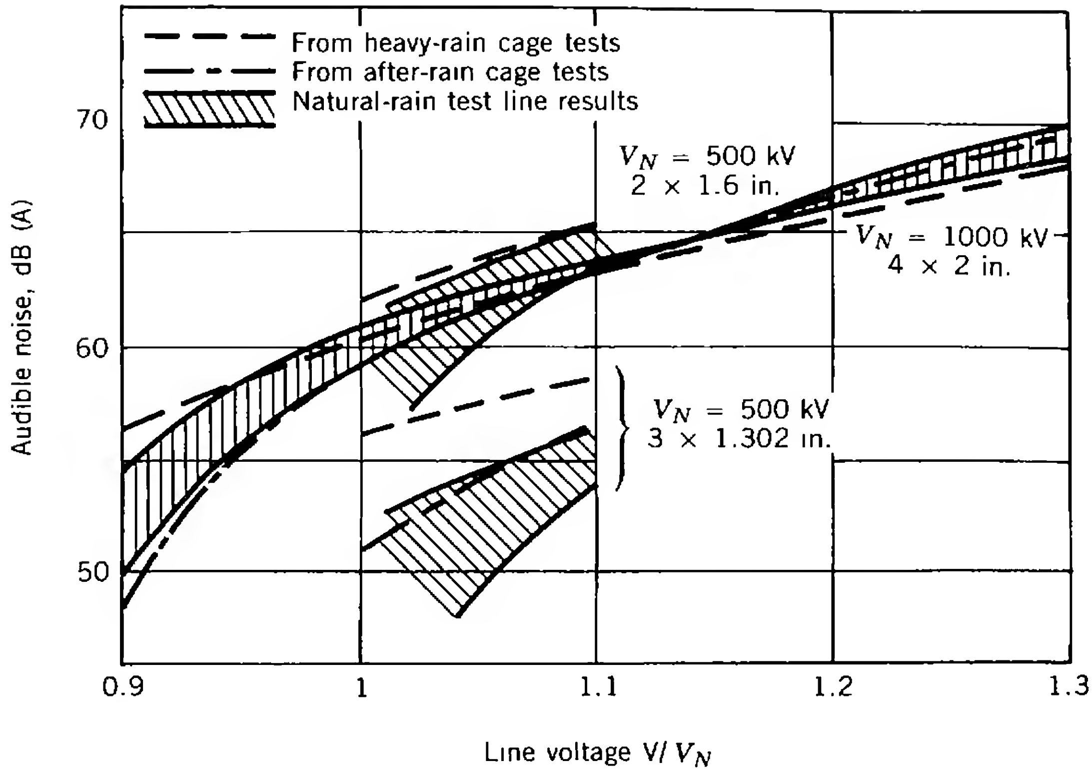


Figure 8.13. Comparison line between audible noise results with test lines under natural rain and computed values from cage tests with artificial rain. Line results are from two 500-kV lines of BPA (measuring point under outside phase) and from single-phase test line of Project UHV (measuring point at 100 ft from line). (From Ref. 27. Used by permission. © 1971 IEEE.)

It has been determined that the bundle diameter has relatively little effect on the noise produced. Figure 8.13 shows a direct comparison of noise predicted from cage tests and actual noise found during overhead line tests. Instrumentation and AN measurements have been described in American National Standard Institute (ANSI) standards and procedures [26].

8.5 CONDUCTOR SIZE SELECTION

In the past, RI mitigation, rather than economic requirements, dictated the conductor size. This was due to the fact that (1) energy was inexpensive and (2) smaller conductor sizes would have facilitated an optimum balance between initial investment cost and operation cost. Today, due to increasing energy costs, the lower future energy and demand losses on a line more than offset the greater initial investment cost. As conductor size increases, the investment cost increases, whereas the costs of energy and demand losses will decrease. Therefore, the total annual equivalent cost of a line with given conductor size for year n can be expressed as

$$\text{TAC}_n = \text{AIC}_n + \text{AEC}_n + \text{ADC}_n \quad \text{dollars/mi} \tag{8.18}$$

where TAC_n = total annual equivalent cost of line in dollars per mile
AIC_n = annual equivalent investment cost of line in dollars per mile
AEC_n = annual equivalent energy cost due to I^2R losses in line conductors in dollars per mile
ADC_n = annual equivalent demand cost incurred to maintain adequate system capacity to supply I^2R losses in line conductors in dollars per mile

The annual equivalent investment of a given line for year n can be expressed as

$$\text{AIC}_n = \text{IC}_L \frac{i_L}{100} \quad \text{dollars/m} \tag{8.19}$$

where IC_L = total investment cost of line in dollars per mile
i_L = annual fixed charge rate applicable to line in percent

The annual equivalent energy cost due to I^2R losses in line conductors for year n can be expressed as

$$\text{AEC}_n = \frac{C_{\text{MWh}}\text{inf}_n}{10^6} I_L^2 \frac{R}{N_c} N_{\text{ckt}} N_p \frac{F_{\text{LS}}}{100} 8760 \quad \text{dollars/mi} \tag{8.20}$$

where C_{MWh} = cost of generating energy in dollars per megawatt hours.
inf_n = inflation cost factor for year n
I_L = phase current in amperes per circuit
R = single conductor resistance in ohms per mile
N_c = number of conductors per phase
N_{ckt} = number of circuits
N_p = number of phases
F_{LS} = loss factor in percent

The annual equivalent demand cost incurred to maintain adequate system capacity to supply the I^2R losses in the line conductors for year n can be expressed as

$$\text{ADC}_n = \frac{C_{\text{kW}}\text{inf}_n}{1000}\left[1 + F_{\text{res}} I_L^2 \frac{R}{N_c} N_{\text{ckt}} N\right] \frac{i_G}{100} \quad \text{dollars/mi} \tag{8.21}$$

where C_{kW} = installed generation cost in dollars per kilowatts
F_{res} = required generation reserve (factor) in percent
i_G = generation fixed charge rate in percent

The *inflation cost factor* (also called *escalation cost factor*) for year n can be determined from

$$\text{inf}_n = \left(1 + \frac{\text{inf}}{100}\right)^{n-1} \tag{8.22}$$

where inf is inflation rate in percent. Therefore, the present equivalent (or worth) cost of the line can be expressed as

$$\mathrm{PEC} = \sum_{i=1}^{N} \left(1 + \frac{i}{100}\right)^{-n} (\mathrm{AIC}_n + \mathrm{AEC}_n + \mathrm{ADC}_n) \quad \text{dollars/mi} \tag{8.23}$$

where PEC = present equivalent cost of line in dollars per mile
N = study period in years
i = annual discount rate in percent

Thus, the present equivalent of revenue required is the sum of the present equivalent of levelized annual fixed charges on the total line capital investment plus annual expenses for line losses [28].

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PROBLEMS

8.1. Repeat Example 8.2 using Peterson's formula.

8.2. Assume that a three-phase overhead transmission line is made up of three equilaterally spaced conductors each with an overall diameter of 2.5 cm. The equilateral spacing between conductors is 2.5 m. The 40-mi-long line is located at an altitude of 10,000 ft with an average air temperature of 20 °C. If the irregularity factor of the conductors is 0.85 in each case, determine the following:

(a) Disruptive critical rms line voltage.

(b) Visual critical rms line voltage.

8.3. Consider the results of Problem 8.2 and assume that the line operates at 161 kV at 60 Hz. Determine the total fair weather corona loss for the line using:

(a) Peek's formula.

(b) Peterson's formula.

8.4. Solve Problem 8.2 assuming a conductor diameter of 1 cm.

8.5. Consider the results of Problem 8.4 and assume that the line operates at 69 kV at 60 Hz. Determine the total fair weather corona loss for the line using Peek's formula.

8.6. Consider the results of Problem 8.4 and assume that the line operate at 69 kV at 60 Hz. Determine the total fair weather corona loss for the line using Peterson's formula.

8.7. Determine the approximate greatest operating voltage that can be applied to a three-phase line having smooth, solid, and cylindrical conductors each 14 mm in dimeter and spaced 3 m apart in equilateral configuration. Neglect conductor irregularity and assume that the electrical strength of air is 30 kV/cm.

8.8. Assume that a three-phase overhead transmission line has weathered solid cylindrical conductors of 14 mm in diameter and spaced 9 ft apart in an equilateral configuration. The regularity factor for the conductors is 0.95. Determine the maximum operating voltage at which the electrical stress at the surface of a conductor will not exceed the electrical strength of air at 30 °C and 27 in. Hg.

8.9. Consider a three-phase 150-km-long line with smooth and clean copper conductors having a diameter of 15 mm and spaced 4 m apart in equilateral configuration. The line-to-line operating voltage is 230 kV at 60 Hz. The barometric pressure is 73 cm Hg at −5 °C. Use Peek's formula and determine the total corona losses of the line.

8.10. Consider a three-phase transmission line having stranded copper conductors 12 mm in dimeter and spaced 4 m apart in equilateral configuration. The barometric pressure is 78 cm Hg at an air temperature of 29 °C. Assume that the irregularity factors are 0.90, 0.72, and 0.82 for the disruptive critical voltage, local visual corona, and general visual corona, respectively. Determine the following:

(a) Disruptive critical rms line voltage.
(b) The rms line voltage for local visual corona.
(c) The rms line voltage for general visual corona.

8.11. Solve Problem 8.9 assuming wet weather corona.

8.12. Solve Problem 8.9 using Peterson's formula under the assumption of fair weather.

8.13. Solve Problem 8.9 using Peterson's formula under the assumption of wet weather.

8.14. Assume that a prediction of the foul weather TVI is required for an antenna location 90 m from a three-phase 1100-kV transmission line and for a TV channel 6 signal (carrier frequency 83.25 MHz). The average foul weather RI at the standard reference location of 15 m laterally from the outmost phase is 58 dB above 1 μV/m. If the height of the closest phase is 25 m, determine the foul weather TVI of the line.

PART II

MECHANICAL DESIGN AND ANALYSIS

9

CONSTRUCTION OF OVERHEAD LINES

9.1 INTRODUCTION

Overhead construction[†] is only 15 to 60 percent as costly as underground and is therefore more economical. The first consideration in the design of an overhead line, of course, is its electrical characteristics. As explained in the previous chapters, the electrical design of the line must be sufficient for the required power to be transmitted without excessive voltage drop and/or energy losses, and the line insulation must be adequate to cope with the system voltage. The mechanical factors influencing the design must then be considered. For example, the poles supporting the conductors must have sufficient mechanical strength to withstand all expected loads. Another example is that the material chosen for the conductors must be strong enough to withstand the forces to which it is subjected.

The conductors and poles must have sufficient strength with a predetermined safety factor to withstand the loads due to the line itself and stresses imposed by ice and wind loads. Thus, the overhead line should provide satisfactory service over a long period of time without the necessity for too much maintanence. Ultimate economy is provided by a good construction since excessive maintenance or especially short life can easily more than overbalance a saving in the first cost.

The overhead line must have a proper strength to withstand the stresses imposed on its component parts by the line itself. These include stresses set up by the tension in conductors at dead-end points, compression stresses

[†] In this chapter, the emphasis has been placed on the lower voltage high-voltage overhead lines, including overhead distribution lines.

due to guy tension, transverse loads due to angles in the line, vertical stresses due to the weight of conductors, and the vertical component of conductor tension. The tension in the conductors should be adjusted so that it is well within the permissible load of the material. This will mean in practice that one must allow for an appreciable amount of sag.

The poles must have sufficient height and be so located, taking into account the topography of the land, as to provide adequte ground clearances at both maximum loading and maximum temperature condition. The conductor ground clearance for railroad tracks and wire line crossings, as well as from buildings and other objects, must meet the requirements of the National Electric Safety Code (NESC).†

A proper mechanical design is one of the essentials in providing good service to customers. A large majority of service interruptions can be traced to physical failures on the distribution system, broken wires, broken poles, damaged insulation, damaged equipment, etc. Of course, many of these service interruptions are more or less unavoidable. But their numbers can be reduced if the design and construction of the various physical parts can withstand, with reasonable safety factors, not only normal conditions but also some probable abnormal conditions.

Therefore, the overhead line must be designed from the mechanical point of view to withstand the worst probable, but not the worst possible, conditions. For example, the cost of an overhead line that would withstand a severe hurricane would be tremendous, and thus from the economical point of view, it may be justifiable to run the risk of failure under such conditions.

Example 9.1

The No Power & No Light (NP&NL) utility company provide service by an overhead line to a small number of farms at a remote area at the outskirts of Ghost City. The past experiences with the line indicate that frequent repairs of the line are needed as a result of lightning, windstorms, and snow. Each repair, on the average, costs the company \$1500. The probability of damage to the line in a given year is

Number of times repair required	0	1	2	3
Probability of exactly that number of repairs	0.4	0.3	0.2	0.1

† It is important to note that the material in this book, especially in this and following chapters, illustrates only some selected requirements of the NESC. It is not advocated that the material in this book be used as the sole basis for line design in practice. Any line design must satisfy all applicable laws, ordinances, commission rules and orders, etc. While every precaution has been taken in the preparation of this book, the author and the publisher assume no responsibility for errors or omissions. Neither is any liability assumed for damages resulting from the use of information contained herein.

The distribution engineer of the NP&NL Company estimates that relocating and rebuilding of the line could reduce the probabilities to

Number of times repair required	0	1
Probability of exactly that number of repairs	0.9	0.1

Assuming a useful life of 25 years, zero salvage value, a carrying charge rate of 20 percent, and that operating costs other than those for repairs are unaffected by the proposed change, find how much the NP&NL Company could afford to pay for relocating and rebuilding the line.

Solution

Let B be the affordable cost of relocating the line. At the break-even point, the present equivalent of savings should be equal to the present equivalent of the added costs as a result of relocating the line. Therefore,†

$$\begin{aligned} B &= \$1500[3(0.1) + 2(0.2) + 1(0.3) + 0(0.4) - 1(0.1) \\ &\quad -0(0.9)](P/A)_{25}^{20\%} \\ &= \$1500[0.9](4.948) \\ &= \$6679.8 \end{aligned}$$

Since the actual relocating and rebuilding of the line would cost much more than the amount found, the distribution engineer decides to keep the status quo.

9.2 FACTORS AFFECTING MECHANICAL DESIGN OF OVERHEAD LINES

In general, the factors affecting a mechanical design of the overhead lines are:

1. Character of line route.
2. Right-of-way.
3. Mechanical loading.
4. Required clearances.
5. Type of supporting structures.
6. Grade of construction.
7. Conductors.
8. Type of insulators.
9. Joint use by other utilities.

† $(P/A)_n^{i\%}$ is the present worth of a uniform series, and it is the reciprocal of the capital recovery factor. It is also called the discount factor. For a given interest rate and number of years, its value can be found from interest tables or it can be calculated from $(P/A)i\%n = [(1+i)^n - 1]/i(1+i)^n$.

9.3 CHARACTER OF LINE ROUTE

The routes of overhead transmission lines are usually selected across the country on private right-of-way in order to obtain the most direct route and proper space for towers as well as to avoid buildings, roads, highways, and low-voltage lines. Lower voltage overhead distribution lines are run along streets and highways, as much as possible, in order to reach customers more easily and to make the lines accessible for maintenance. In urban and suburban areas, poles are spaced 100 to about 150 ft apart to provide convenient points for service attachments or service drops and to keep the service lengths to a minimum. Usually, poles are set from half to one foot inside the curb when along streets. Transmission lines may have spans of several hundred feet. Of course the general character of the country in which the overhead line is to be located affects the design primarily in terms of selecting the conductor and type of supporting structures. The line location task requires judgment, experience, and skill in minimizing the costs of right-of-way and construction but also in providing convenience in maintenance and eliminating some possible operational bottlenecks that might occur in the future. In general, the factors affecting the length of a span are:

1. Character of route.
2. Proper clearance between conductors.
3. Excessive tensions under maximum load.
4. Structures adequate to carry additional loads.

It is usually not recommended, especially in mountainous country or in heavily populated areas, to choose a direct route or try to locate the line on long tangents.

9.4 RIGHT-OF-WAY

It is important to have all right-of-way and easements for a given line secured before final plans, designs, and specifications for construction. Higher voltage transmission lines on private right-of-way are usually built with long spans, and the type of terrain covered by the line has impact on the selection of the construction type. Of course, existing right-of-way should be utilized whenever possible, for especially augmented transmission systems, and in many cases this is done with less environmental disruption than would occur with the acquisition of a new right-of-way. Advance planning and scheduling of road, pipeline, telephone, and electrical transmission is imperative for the future. In general, rather than purchasing the right-of-way in fee, a permanent easement is obtained in which the owner or owners permits the necesary rights to construct and operate the line but keeps ownership and use of the land. The easement secured must stipulate the following:

1. Permission to build all supporting structures.
2. Permission for a means of access to each supporting structure.
3. Permission to clear all trees and brush over a width of at least 10 ft larger than the spread of the conductors in order to allow sufficient working space for construction.
4. Permission to remove all trees, which might violate the minimum required clearance to the conductors if they were to fall.
5. Permission to remove all trees, which might violate the minimum required clearance to conductors if the conductor were to swing out under maximum wind.
6. Permission to remove all obstacles, for example, buildings, lumber piles, haystacks, etc., which might cause a fire.

As a rule, trees that may interfere with conductors should be trimmed or removed. Normal tree growth, the combined movement of trees and conductors under adverse conditions, voltage, and sagging of a conductor at elevated temperatures are among the factors to be considered in determining the extent of trimming required. Where trimming or removal is not practical, especially in distribution lines, the conductor should be separated from the tree with suitable insulating materials or devices to prevent conductor damage by abrasion and grounding of the circuit through the tree.

9.5 MECHANICAL LOADING

9.5.1 Definitions of Stresses

The term *mechanical loading* refers to the external conditions that produce mechanical stresses in the line conductors and supports, that is, poles or towers. Mechanical loading also includes the weight of the conductors and structures themselves. Structures are subject to vertical and horizontal loads. Vertical loading includes the dead weight of equipment such as crossarms, insulators, conductors, transformers, etc. It also includes ice and snow clinging on the structures and the conductors.

Poles supporting overhead conductors and other equipment are subjected to strains from the tension with which they are strung. When a force is applied against an object, it produces *stress* within the object. There are five kinds of stress:

1. *Tensile Stress*. Caused by the forces acting in opposite directions, away from the body and along the same straight line to elongate, or stretch, the body beyond its normal length, as shown in Figure 9.1.

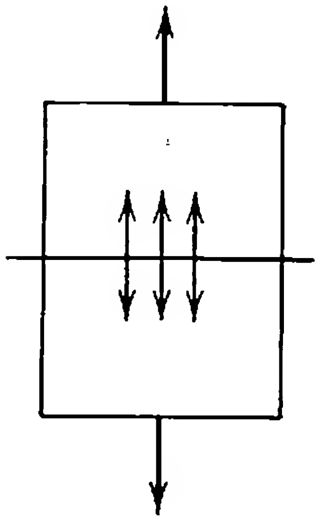


Figure 9.1. Tension.

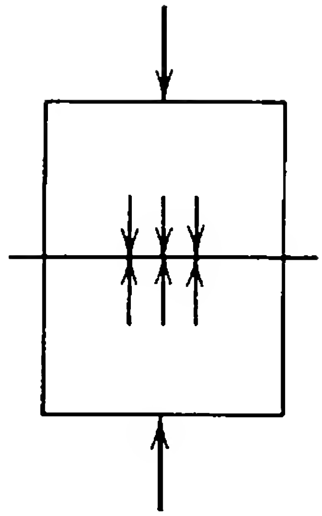


Figure 9.2. Compression.

For example, a conductor strung between two poles or a guy wire, under tension, is subjected to tensile stress.

2. *Compressive Stress*. The opposite of the tensile stress. Caused by the forces acting toward the body to shorten the body, as shown in Figure 9.2. For example, a distribution transformer hung on a pole subjects the pole to a compressive stress.
3. *Shearing Stress*. Caused by the forces, not in the same straight line, that tend to cut the body in two, as shown in Figure 9.3. For example, bolts attaching a cross-arm to a pole are subjected to a shearing stress between the cross-arm and the pole.
4. *Bending Stress*. Caused by the forces acting along a body. For example, a pole supporting a corner in the line, and not guyed, is subject to a bending stress.
5. *Twisting Stress* or *Torque*. Caused by line tensions that are not equal on the two sides of a pole. For example, a pole may be subjected to a twisting stress when a conductor breaks between supports.

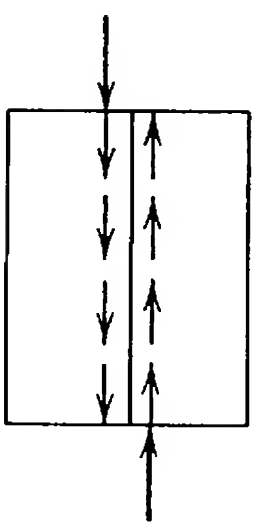


Figure 9.3. Shear.

9.5.2 Elasticity and Ultimate Strength

Elasticity is the property of a material that enables it to recover its original shape and size after being stressed. The ratio of normal stress (in pounds per square inch) to strain (in inches per inch) is called *Young's modulus* or the *modulus of elasticity*. It is constant for a given material up to the proportional limit, as shown in Figure 9.4. Up to a certain limit, stress applied to a material causes deformation, which disappears when the stress is removed.

Every material has a stress limit, and stress beyond this point causes a certain amount of permanent deformation. This limit is the *elastic limit* of the material. When the stress is less than the elastic limit of the material, the deformation is directly proportional to the unit stress. When the stress exceeds the elastic limit, the material still resists the stress but has lost certain of its original characteristics. The deformation increases until failure occurs. The stress that causes failure or rupture is the ultimate stress of the material. For some materials, for example, glass, the elastic limit and the ultimate strength are very nearly the same. However, most materials show the deformation or yield point of elastic limit at a lower value than the breaking or ultimate strength.

In the design of mechanical structures, there are a number of variables and possibilities that make the exact determination of stresses and strengths difficult. The maximum stress at which a structure is designed to operate normally is the allowable or working stress. The ratio of working stress to the ultimate strength of the material is the *design safety factor*. It is usual practice to design for the assumed loading conditions, and to use this safety factor, or constant, to make reasonable provision for unusual and unforseen

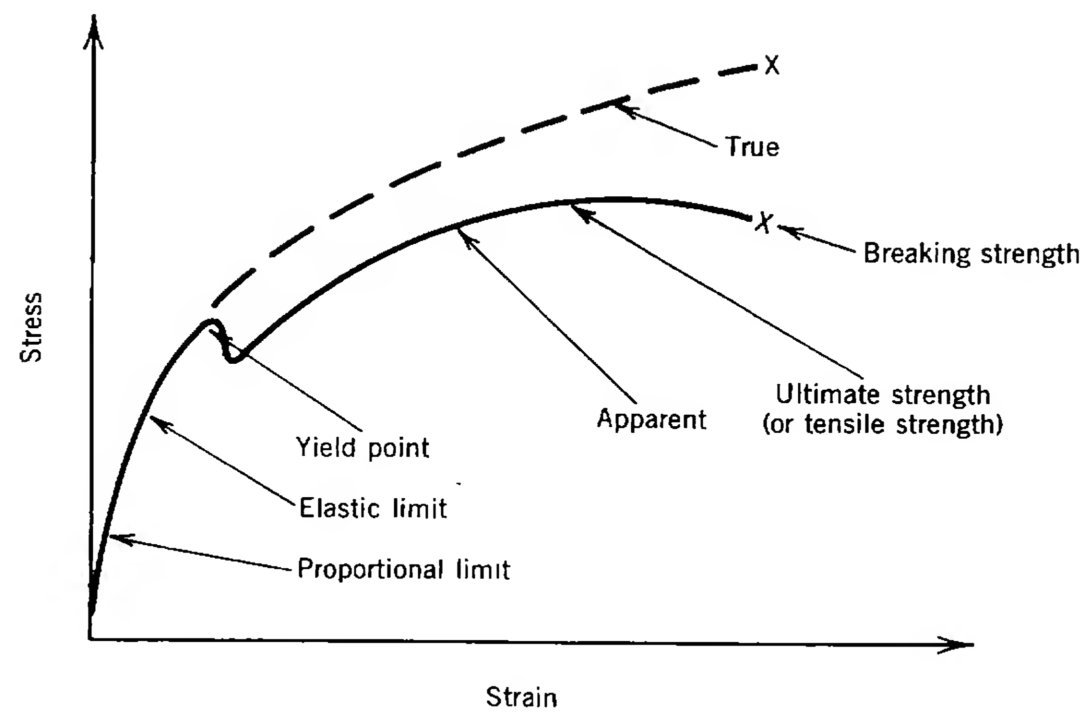


Figure 9.4. Stress–strain diagram.

conditions and hazards to which the structure may be subjected. Furthermore, these safety factors make allowance for the difference between elastic limit and ultimate strength and make allowance for variations from average quality. The NESC and/or the local rules and regulations provide the minimum required safety factors. Where the NESC or the local regulations are not in effect, to specify safety factors that must be applied under various conditions, the design engineer must use his own engineering judgment in choosing such safety factors as will best meet the existing conditions.

9.5.3 NESC Loadings

In general, the map shown in Figure 9.5 is taken as a basis for determining the thickness of ice, wind velocity, and temperature for a given overhead line in any region of the country. In the design of an important overhead line, the past records of the local weather bureau should be studied. When the local conditions are found to be different from the general conditions in the surrounding area indicated on this map, the line should be designed and constructed to meet such conditions. For example, certain districts are more subject to sleet storms than others even in the same general locality. In general, sleet storms are most frequent in the moderate climates, since precipitation takes place more often at freezing temperatures. In large cities, sleet formation is much less likely to occur than in rural areas. Also, the exposure of lines to wind is extremely different since hills, buildings, trees,

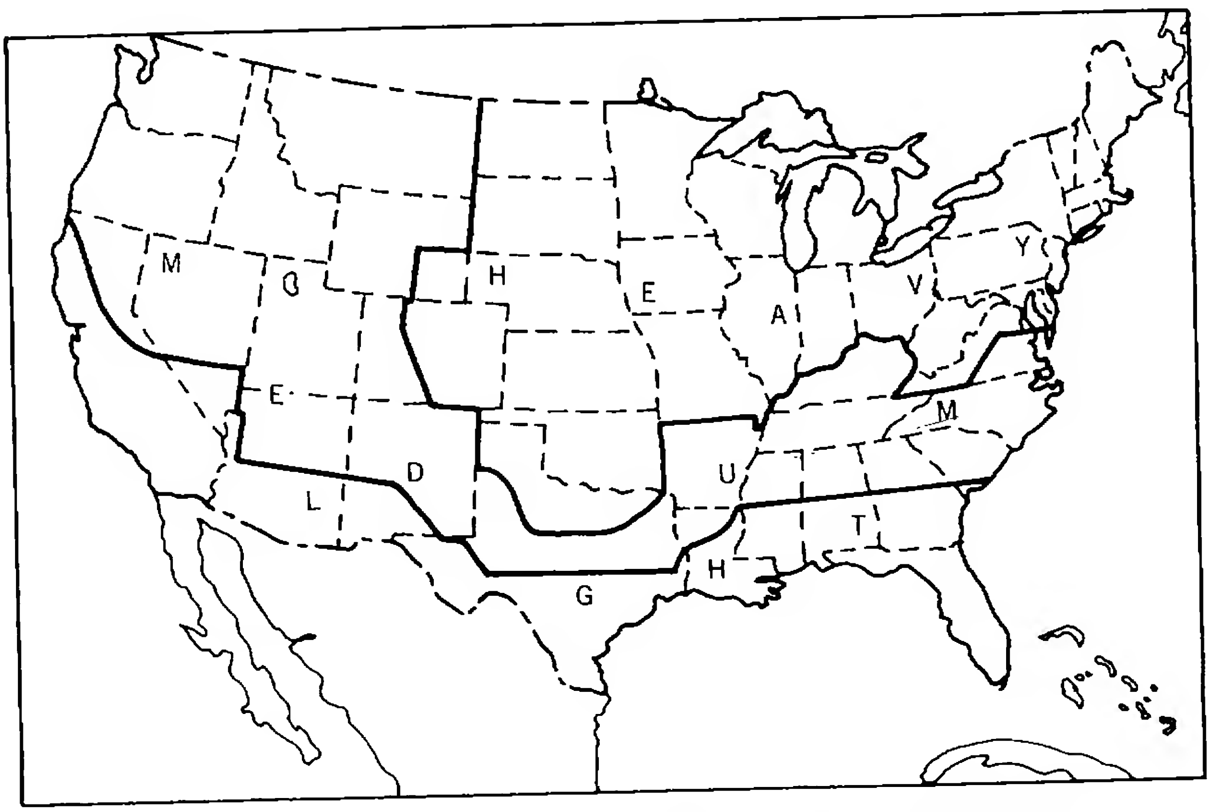


Figure 9.5. NESC's mechanical loading map for overhead lines.

etc., make a fair amount of shields against the full pressure that a wind might cause in open country. It is possible to have very heavy ice formation without much wind or very strong winds in warm weather with no ice formation. The NESC defines three conditions of loading, namely, heavy, medium, and light and divides the country into three areas in which these loadings are possibly taking place.

9.5.4 Wind Pressure

The method of calculating wind pressure on cyclindrical surfaces, for example, conductors and wood poles, was developed by H. W. Buck and is given by Buck's formula [1]

$$p = 0.00256V^2 \quad \text{lb/ft}^2 \tag{9.1}$$

where V is actual wind velocity in miles per hour. It is, in general, accepted in span calculations.

The pressure on flat surfaces, for example, cross-arms and towers, normal to the direction of the wind can be calculated by using the following formula developed by C. F. Marvin

$$p = 0.004 \frac{B}{30} V^2 \quad \text{lb/ft}^2 \tag{9.2}$$

where B = barometric pressure in millimeters of mercury
V = actual wind velocity in miles per hour

This equation can be written as

$$p = 0.004V^2 \quad \text{lb/ft}^2 \tag{9.3}$$

since $B/30$ is, in general, equal to unity.

When using the data for V's given by the U.S. Weather Bureau, the necessary correction factors should also be obtained since the observed values are taken at elevations considerably different than those on which conductors are usually hung. For the minimum required wind pressures by the NESC see Chapter 10.

In still air the conductor is subject to its own weight only, and, if the temperature is high at the same time, the resulting sag gives a low tension. Therefore, the combination of still air with high temperature gives the easiest conditions. The worst conditions are a combination of low temperature, which lessens the sag of the line, and accumulations of ice or snow, which increases the weight per-unit length and consequently the effect of the wind blowing against the conductor. This topic is discussed in greater detail in Chapter 10.

9.6 REQUIRED CLEARANCES

In general, the following clearances need to be considered: ground, tracks, buildings, trees, conductors and structures of another line, other conductors on the same structure, the structure itself, guy wires and other equipment on the structure, and the edge of the right-of-way. The NESC gives the minimum required clearances. Space does not permit the tabulation here of clearances for all these conditions for all voltages used for overhead lines. However, some information is summarized in the following sections.

9.6.1 Horizontal Clearances

Briefly, the location of poles must be chosen to provide sufficient clearance from driveways, fire hydrants, street traffic, railroad tracks, buildings, fire escapes, etc. Table 9.1 gives the clearance of conductors passing by but not attached to building and other installations except bridges. The given clearances are taken from the NESC, 1984 edition.

Conductors of one line should be not less than 4 ft from those of another and conflicting line. If conductors pass near the pole of another overhead line, providing that they are not attached, they should not interfere with the climbing space.

9.6.2 Vertical Clearances

Table 9.2, taken from the NESC, 1984 edition, presents vertical clearances. The given values are applicable to crossings where span lengths do not exceed 175 ft in heavy-loading districts, 250 ft in medium-loading districts, or 350 ft in light-loading districts. The given clearances are based on a temperature of 60°F with no wind and voltages not over 50 kV to ground. For longer spans and higher voltages, larger clearances are required, depending upon sag and tension in the span.

9.6.3 Clearances at Wire Crossings

Crossings should be made on a common crossing structure where practical, If not practical, the clearance between any two wires, conductors, or cables crossing each other and carried on different supports should be not less than the values given in Table 9.3 in order to prevent the possibility of accidental contact under varying wing, temperature, and ice loading. The given clearances apply at 60 °F with no wind and for spans not exceeding 175, 250, or 350 ft, in heavy-, medium, and light-loading districts, respectively. For longer spans and higher voltages, greater clearances are required, depending on sag and tension in the span.

TABLE 9.1 Clearance of Conductors Passing By But Not Attached to Buildings (ft)

		Open-Supply Conductors with Phase-to-Ground Voltages			
Clearance of	Communication Conductors, etc.	0–750 V	750 V–8.7 kV	8.7–15 kV	15–50 kV
Buildings					
Horizontal					
To walls and projections	3	5	5	8	10
To unguarded windows	3	5	5	8	10
To balconies, etc.	3	5	5	8	10
Vertical					
Above or below roofs, etc. (accessible to pedestrians)	3	10	10	10	12
Above or below balconies and roofs (accessible to pedestrians)	8	15	15	15	17
Above roofs (accessible to vehicular traffic)	18	18	20	20	22
Chimneys, antennas, etc.					
Horizontal	3	5	5	8	10
Vertical, above or below	3	5	8	8	10

TABLE 9.2 Minimum Vertical Clearances of Conductors above Ground or Rails (ft)

Location Type	Communication Conductors, Guys, Messengers, etc.	Open-Supply Conductors with Phase-to-Ground Voltages			Trolley Conductors with Phase-to-Ground Voltages	
		0–750 V	750 V–15 kV	15–50 kV	0–750 V	Over 750 V
When crossing above						
Railroads	27	27	28	30	22	22
Streets, alleys, and roadways	18	18	20	22	18	20
Private driveways	10	15	20	22	18	20
Walks for pedestrians only	15	15	15	17	16	18
When conductors are along						
Streets or alleys	18	18	20	22	18	20
Roads in rural districts	14	15	18	20	18	20

TABLE 9.3 Crossing Clearances of Wires Carried on Different Supports (ft)

Nature of Wires Crossed Over	Communication Conductors	Services, Supply Cables 0–750 V	Open-Supply Conductors with Phase-to-Ground Voltages		
			0–750 V	750 V–8.7 kV	8.7–50 kV
Communication wires	2	2	4	4	6
Aerial-supply cables	4	2	2	2	4
Open-supply wires, 0–750 V	4	2	2	2	4
Open-supply wires, 750 V–8.7 kV	4	4	2	2	4
Open-supply wires, 8.7–50 kV	6	6	4	4	4
Trolley conductors	4	4	4	6	6
Guys, lightning protection wires	2	2	2	4	4

9.6.4 Horizontal Separation of Conductors from Each Other

The NESC requires that for supply conductors of the same circuit, at voltages up to 8.7 kV, the minimum horizontal clearances between the conductors should be 12 in., and for higher voltages should be 12 in. plus 0.4 in. per kilovolt over 8.7 kV.

It is required that for supply conductors of different circuits, at voltages up to 8.7 kV, the minimum horizontal clearances between the conductors should be 12 in.; for voltages between 8.7 and 50 kV, the clearances should be 12 in. plus 0.4 in. per kilovolt over 8.7 kV; and for voltages between 50 and 814 kV, the clearances should be 28.5 in. plus 0.4 in. per kilovolt over 50 kV.

The minimum required horizontal clearances by the NESC for line conductors smaller than No. 2 AWG can be calculated by using the following formula:

$$\text{Minimum clearance} = 0.3\ \text{in./kV} + 7\left(\tfrac{1}{3}S - 8\right)^{1/2} \tag{9.4}$$

where S is apparent sag of the conductor in inches.

Table 9.4 gives the minimum horizontal clearances between the conductors up to 46 kV.

The minimum required horizontal clearances by the NESC for line conductors of No. 2 AWG or larger can be calculated by using the following formula:

TABLE 9.4 Horizontal Clearances at Supports between Line Conductors Smaller Than No. 2 AWG Based on Sags

Voltage between Conductors (kV)	Sag (in.) 36	48	72	96	120	180	240	But Not Less Than
	Horizontal Clearance (in.)							
2.4	14.7	20.5	28.7	35.0	40.3	51.2	60.1	12.0
4.16	15.3	21.1	29.3	35.6	40.9	51.8	60.7	12.0
12.47	17.7	23.5	31.7	38.0	43.3	54.2	63.1	13.5
13.2	18.0	23.8	32.0	38.3	43.6	54.5	63.4	13.8
13.8	18.1	23.9	32.1	38.4	43.7	54.6	63.5	14.0
14.4	18.3	24.1	32.3	38.6	43.9	54.8	63.7	14.3
24.94	21.5	27.3	35.5	41.8	47.1	58.0	66.9	18.5
34.5	24.4	30.2	38.4	44.7	50.0	60.9	69.8	22.4
46	27.8	33.6	41.8	48.1	53.4	64.3	73.2	26.9

Source: The National Electric Safety Code [2].

TABLE 9.5 Horizontal Clearance at Supports between Line Conductors No. 2 AWG or Larger Based on Sags

Voltage between Conductors (kV)	Sag. (in.) 36	48	72	96	120	180	240	But Not Less Than
	Horizontal Clearance (in.)							
2.4	14.6	16.7	20.2	23.3	26.0	31.7	36.5	12.0
4.16	15.1	17.3	20.8	23.8	26.5	32.2	37.0	12.0
12.47	17.6	19.7	23.6	26.3	29.0	34.7	39.5	13.5
13.2	17.8	20.0	23.5	26.5	29.2	34.9	39.7	13.8
13.8	18.0	23.7	26.7	29.4	35.1	39.9	14.0	
14.4	18.2	20.3	23.8	26.9	29.6	35.3	40.1	14.3
24.94	21.3	23.5	27.0	30.0	32.8	38.4	43.2	18.5
34.5	24.2	26.2	29.9	32.9	35.6	41.3	46.1	22.4
46	27.7	29.8	33.3	36.4	39.1	44.8	49.6	26.9

Source: The National Electrical Safety Code [2].

$$\text{Minimum clearance} = 0.3\ \text{in./kV} + 8(\tfrac{1}{12}S)^{1/2} \tag{9.5}$$

where S is the apparent sag of the conductor in inches.

Table 9.5 gives the minimum horizontal clearances between the conductors up to 46 kV.

In addition to those clearance requirements included here, the NESC provides other minimum requirements such as for climbing space through lower wires on a pole to gain access to wires on upper arms or for vertical separtion of cross-arms. For further information, see the current edition of the National Electrical Safety Code and local rules and regulations.

9.7 TYPE OF SUPPORTING STRUCTURES

9.7.1 Pole Types

There are basically four different pole types: (1) wood poles, (2) concrete poles, (3) steel poles, and (4) aluminum poles. In general, wood poles are preferred over others for overhead distribution lines because of the abundance of the material, ease of handling, and cost. Concrete poles reinforced with steel have been used for street lighting because of their neat appearances. Steel poles have been used to support trolley overheads and street and parkway lighting. Both concrete and steel poles have been used to a limited extent for distribution. Aluminum poles are used basically for parkway lighting.

The life of wood poles is materially extended by impregnation with wood preservatives. Wood that has been properly treated for the environment in which it will be used will resist decay and maintain its mechanical strength for many years. A minimum life expectancy of 35 years has been accepted by the wood industry [3]. Cedar, pine, and fir are best suited by their proportions and properties for use as distribution poles. Besides their usage in distribution systems, wood structures have been utilized for many years as a means of supporting single- and double-circuit transmission lines at voltages of 115 through 230 kV and single circuit of 345 kV. As a result of developing technology, wood structures have recently been designed for applications up to 765 kV and tested for 500 kV [3]. Wood structures design is based on an assigned or calculated ultimate stress for the species used. The inherent flexibility of wood adds a certain degree of cushion when severe loadings are imposed. This property provides wood construction the ability to absorb shock loads and longitudinal load capability not found in rigid structures.

Figure 9.6 shows some typical single-column wood structure designs used in distribution systems. Figure 9.7 shows typical single-column structure designs. Single wood column designs have been used for double-circuit lines through 230 kV and appear feasible for 345 kV. Structures using two columns, as shown in Figure 9.8, provide the basis for conventional H-frame designs with variations. Wood cross-arms are normally used, although metal arms are sometimes specified. Double-circuit structures have been built using two columns for voltages through 230 kV and appear feasible for double circuits of 345 kV.

In distribution systems, single poles are widely used to support three-transformer banks and their fused disconnects and surge arresters. A-frame poles are used where greater strength is required, and H-frame poles are used where it is necessary to support switching equipment and/or a transformer as well as the line.

The poles must have sufficient height and be so located as to provide adequate ground clearance at either maximum-loading or maximum-temperature condition. The conductor ground clearance for railroad tracks and wire line crossings, as well as from buildings and other subjects, must meet the requirements of the NESC and other local rules and regulations. In essence, the height of a pole required for a particular location is determined by the following factors:

1. Length of vertical pole space required for wires and equipment.
2. Clearance required above ground or obstructions for wires and equipment.
3. Sag of conductors.
4. Depth of pole to be set in ground.

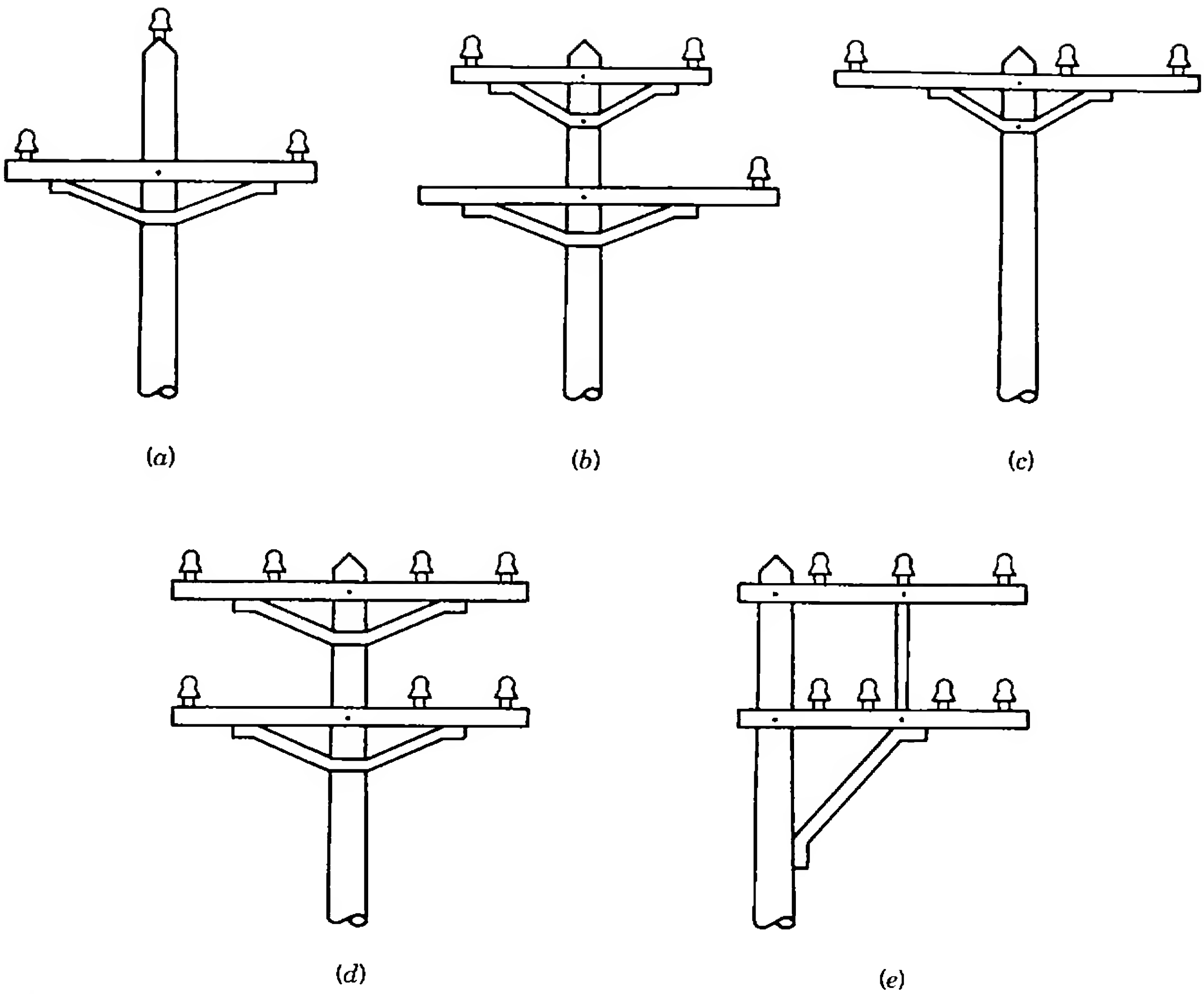


Figure 9.6. Typical single-pole designs used in distribution systems: (*a*) pole top; (*b*) two arms; (*c*) single arm; (d) line arms; (*e*) side arms.

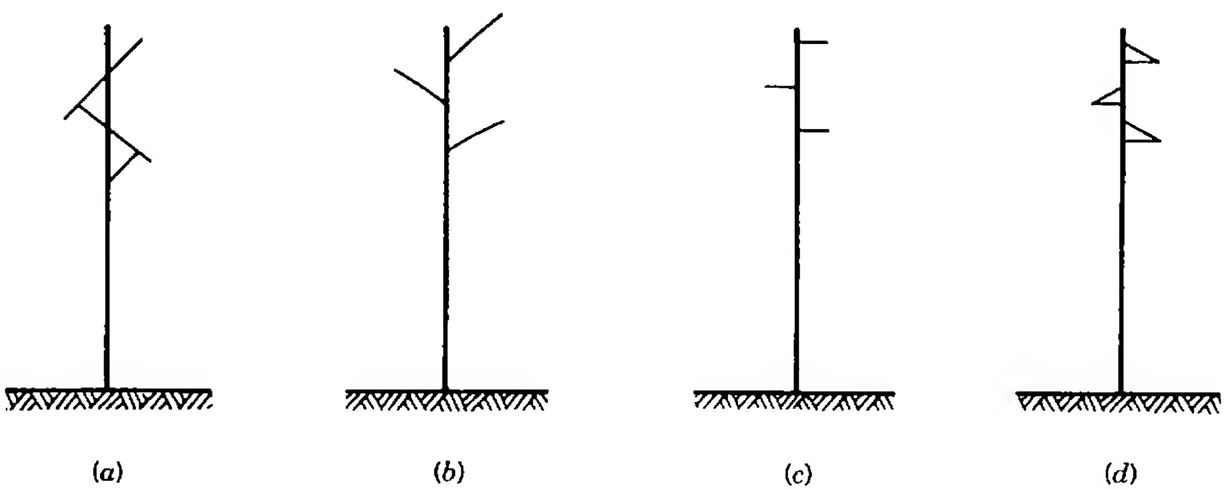


Figure 9.7. Typical single-column designs: (*a*) wishbone design; (*b*) unbraced up-swept arms; (*c*) horizontal line post; (*d*) braced horizontal arms.

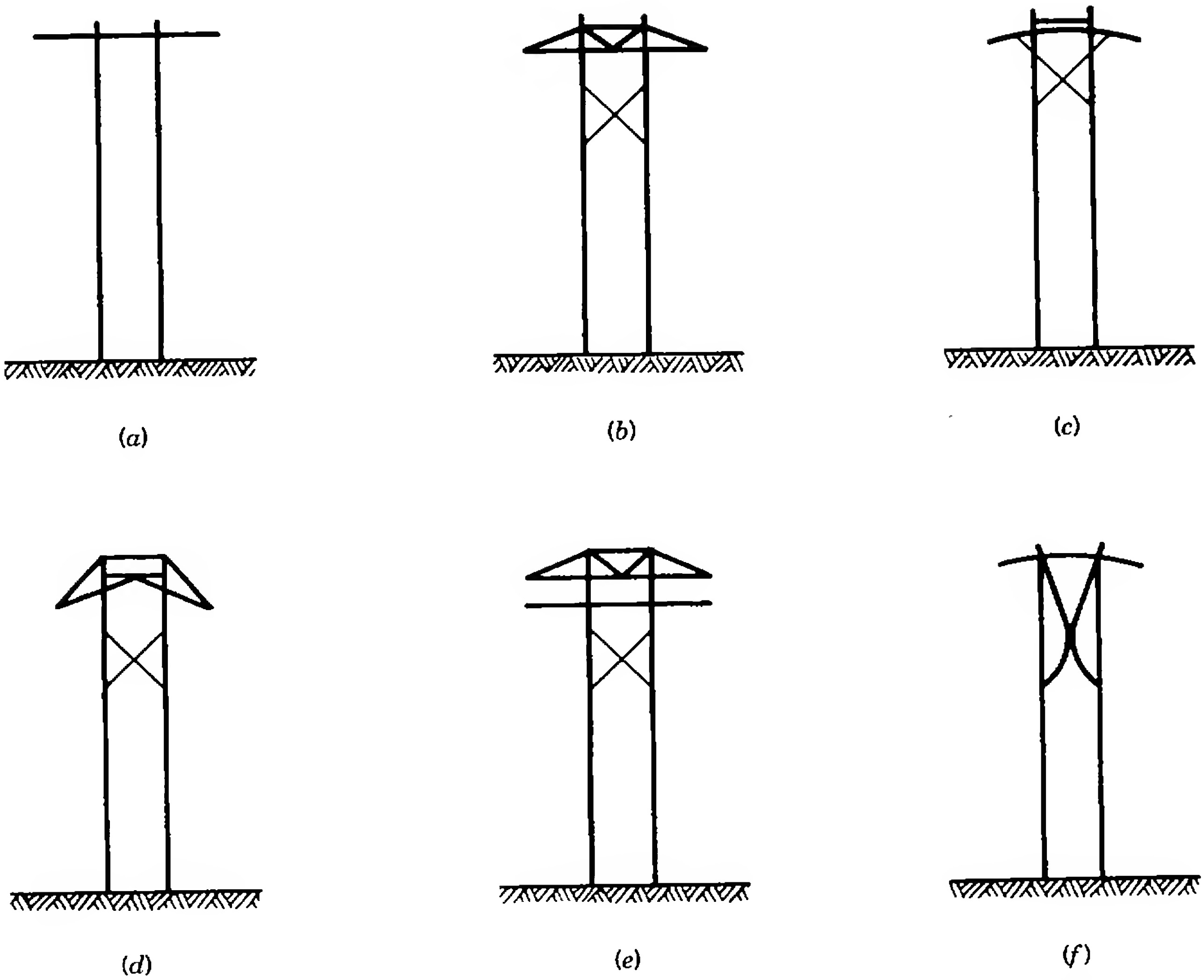


Figure 9.8. Typical two-column designs: (*a*) unbraced H-frame; (*b*) H-frame with wood (solid or laminated) cross-arm; (*c*) H-frame with curved laminated cross-arm; (*d*) K-frame; (*e*) double-circuit H-frame; (*f*) Dreyfus design.

In distribution systems, the most commonly used pole in the 35-ft pole, and poles shorter than 30 ft are generally not used. The 30ft pole may be used in alleys and on rear-lot lines. The larger sizes are, of course, used for providing clearance over obstructions, for heavier leads, etc.

The size (i.e., class), or diameter, of the pole is determined by the strength required to endure the mechanical loading imposed upon it. The critical point of strength for an unguyed pole at or near the ground line, therefore, the circumference of the pole at this point determines the resisting moment of the pole when bending as a cantilever. However, if a pole is guyed, the diameter of the pole at the point of attachment of the guy is the measure of its strength. The resisting moment at the point of guy attachment must be sufficient to endure the bending stresses caused at that point. Also, the top of the pole must be of adequate circumference to permit the attachment of cross-arms without excessively weakening the pole near the top. The wood poles are divided into several classes according to top circumference and the circumference 6 ft from the butt end for each

TABLE 9.6 Standard Wood Pole Dimensions

Class		1	2	3	4	5	6	7
Minimum Top Circumference (in.)		27	25	23	21	19	17	15
Minimum Top Diameter (in.)		8.6	8.0	7.3	6.7	6.1	5.4	4.8
Pole Length (ft)	Wood Type[a]	Minimum Circumference at Ground Level (in.)						
25	P	34.5	32.5	30.0	28.0	26.0	24.0	22.0
	C	37.0	34.5	32.5	30.0	28.0	25.5	24.0
	W	38.0	35.5	33.0	30.5	28.5	26.0	24.5
30	P	37.5	35.0	32.5	30.0	28.0	26.0	24.0
	C	40.0	37.5	35.0	32.5	30.0	28.0	26.0
	W	41.0	38.5	35.5	33.0	30.5	28.5	26.5
35	P	40.0	37.5	35.0	32.0	30.0	27.5	25.5
	C	42.5	40.0	37.5	34.5	32.0	30.0	27.5
	W	43.5	41.0	38.0	35.5	32.5	30.5	28.0
40	P	42.0	39.5	37.0	34.0	31.5	29.0	27.0
	C	45.0	42.5	39.5	36.5	34.0	31.5	29.5
	W	46.0	43.5	40.5	37.5	34.5	32.0	30.0
45	P	44.0	41.5	38.5	36.0	33.0	30.5	28.5
	C	47.5	44.5	41.5	38.5	36.0	33.0	31.0
	W	48.5	45.5	42.5	39.5	36.5	33.5	31.5
50	P	46.0	43.0	40.0	37.5	34.5	32.0	29.5
	C	49.5	46.5	43.5	40.0	37.5	34.5	32.0
	W	50.5	47.5	44.5	41.0	38.0	35.0	32.5
55	P	47.5	44.5	41.5	39.0	36.0	33.5	
	C	51.5	48.5	45.0	42.0	39.0	36.0	
	W	52.5	49.5	46.0	42.5	39.5	36.5	
60	P	49.5	46.0	43.0	40.0	37.0	34.5	
	C	53.5	50.0	46.5	43.0	40.0	37.5	
	W	54.5	51.0	47.5	44.0	41.0	38.5	
65	P	51.0	47.5	44.5	41.5	38.5		
	C	55.0	51.5	48.0	45.0	42.0		
	W	56.0	52.5	49.0	45.5	42.5		
70	P	52.5	49.0	46.0	42.5	39.5		
	C	56.5	53.0	48.5	45.5	43.5		
	W	57.5	54.0	50.5	47.0	45.0		
75	P	54.0	50.5	47.0	44.0			
	C	59.0	54.0	50.0	47.0			
	W	59.5	55.5	52.0	48.5			

[a] Yellow pine, chestnut, and western cedar are denoted as P, C, and W, respectively.

TABLE 9.7 Minimum Required Setting Depths

Pole Size (ft)	30	35	40	45	50	55	60	70
Setting Depth (ft)	5	5.5	6	6.5	6.5	7	7	7.5

nominal length. The word *class* refers to the dimensional classifications set up by the American Standards Association. The classes are numbered from 1 to 10. Class 1 provides the largest ground circumference, and class 7 the smallest. Classes 8 to 10 inclusive specify minimum top circumferences only. All poles in a given class, regardless of length, have approximately the same strength against load applied horizontally at the top. Table 9.6 gives the standard pole dimensions for yellow pine, chestnut, and western cedar. In order to identify any particular wood pole, its class, pole length, and wood type should be given.

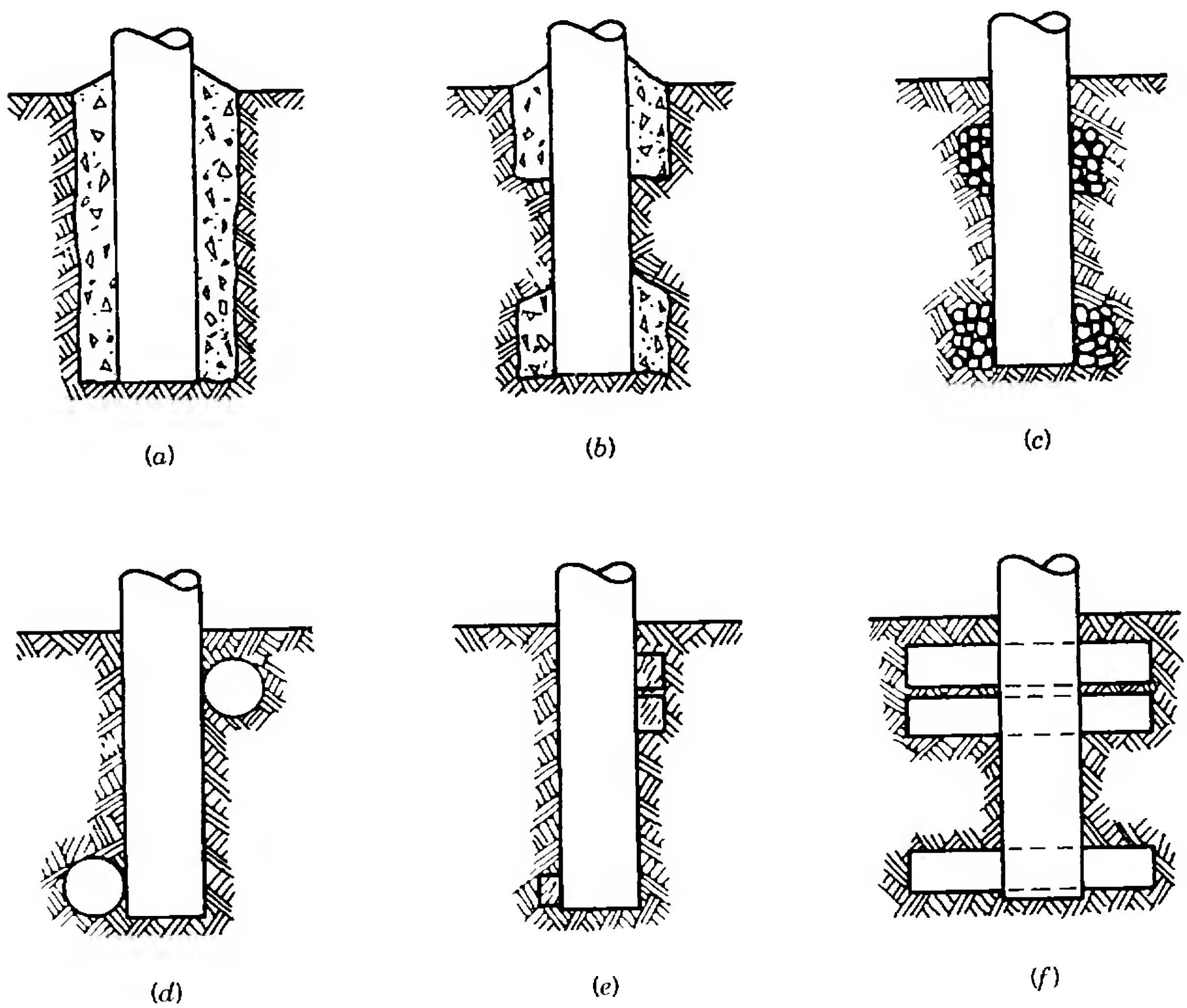


Figure 9.9. Setting techniques: (*a*) full-concrete setting; (*b*) concrete setting; (*c*) crushed stone setting; (*d*) plain earth setting; (*e*) heel-and-breast concrete blocks setting; (*f*) bolted-timber setting.

TABLE 9.8 Various Earth Resistance to Displacement [4]

Class	Earth Type	Percentage of Pole Resisting Moment, S_e
1	Hard rock	50
2	Shale, sandstone, or soft rock	50
3	Hard, dry hardpan	50
4	Crumbly, damp	40
5	Firm, moist	35
6	Plastic, wet	30
7	Loose, dry of loose, wet	25
8	Swamps, marshes	20

9.7.2 Soil Types and Pole Setting

A stable pole must have sufficient setting depth. Table 9.7 provides minimum depth of pole settings. However, the distribution engineer chooses the depth of settings as the situation dictates. For example, corner poles should have about 6 in. deeper settings. Of course, the stability or rigidity of the pole depends not only on the depth of setting but also on the type of earth, moisture content of soil, size of pole butt, and setting technique used. Figure 9.9 shows some of the pole butt, and setting technique used. Figure 9.9 shows some of the setting techniques.

Earth can be classified into eight different groups, as given in Table 9.8, for the purpose of settings. Table 9.8 [4] also gives the resistance S_e, as percentage of pole ultimate resisting moments, that the earth around the pole base shows to displacement for various earth types. The values given in the table are somewhat arbitrary and based on the assumptions that the pole setting is standard, the hole diameter is minimum, and the backfilling is properly tamped.

9.8 MECHANICAL CALCULATIONS

9.8.1 Introduction

In general, the forces acting on a given supporting structure, for example, the pole, are:

1. Vertical forces due to weight of pole, conductors, ice clinging to conductors.
2. Vertical forces due to downward pull of guys.

3. Lateral horizontal forces due to wind across line pole, conductors, ice, etc.
4. Longitudinal horizontal forces due to unbalanced pull of conductors.
5. Torsional forces due to unbalanced pull of conductors.

Any given pole is strong for vertical forces but weak for horizontal forces, and any given cross-arm is weak for the torsional forces. Thus, in order to achieve a good line design, the horizontal and torsional forces should be reduced to a minimum by balancing the stresses, and the remains of unbalanced horizontal stresses should be transformed into vertical stresses on the pole by the use of guys. Therefore, the strength of a wood pole must be sufficient to withstand transverse forces, such as wind pressure, on the pole and conductors, unbalanced pull on conductors when they are broken, and side pull on curves and corners where guys cannot be used. These forces place the fiber of the wood under tension, and the load a pole can carry is determined by the inherent strength of its wood fiber under tension and the moments of forces.

However, the calculations for the strength of a given pole, at best, give only approximate results since there will usually be a slight movement of the pole at the ground level. Therefore, the calculated fiber stress can be different than the actual value. In order to find the length of an unguyed span for a given height, kind, and class of pole, the bending moment of the pole at the ground level, which is usually the point of failure, is calculated. It is assumed that the pole is set in firm soil. The minimum radial thicknesses of ice and the wind pressures to be used in calculating loadings for the specified wind loading under light-, medium-, and heavy-loading conditions are given by the NESC.

There are two bending moments of wind affecting the pole:

1. The bending moment due to wind on the conductors.
2. The bending moment due to wind on the pole itself.

9.8.2 Bending Moment Due to Wind on Conductors

The bending moment is equal to the force applied times its distance in inches (at right angles to its direction) from the point, that is, its moment arm, whose strength is being considered (see Figure 9.10). Therefore, the total bending moment due to wind on the conductors is

$$M_{\mathrm{tc}} = \sum_{i=1}^{m} \sum_{j=1}^{n} m_i n_{ij} P L_{\mathrm{avg}} h_{ij} \quad \text{lb-ft} \tag{9.6}$$

where M_{tc} = total bending moment due to wind on conductors in pound feet

m = number of cross-arms on pole

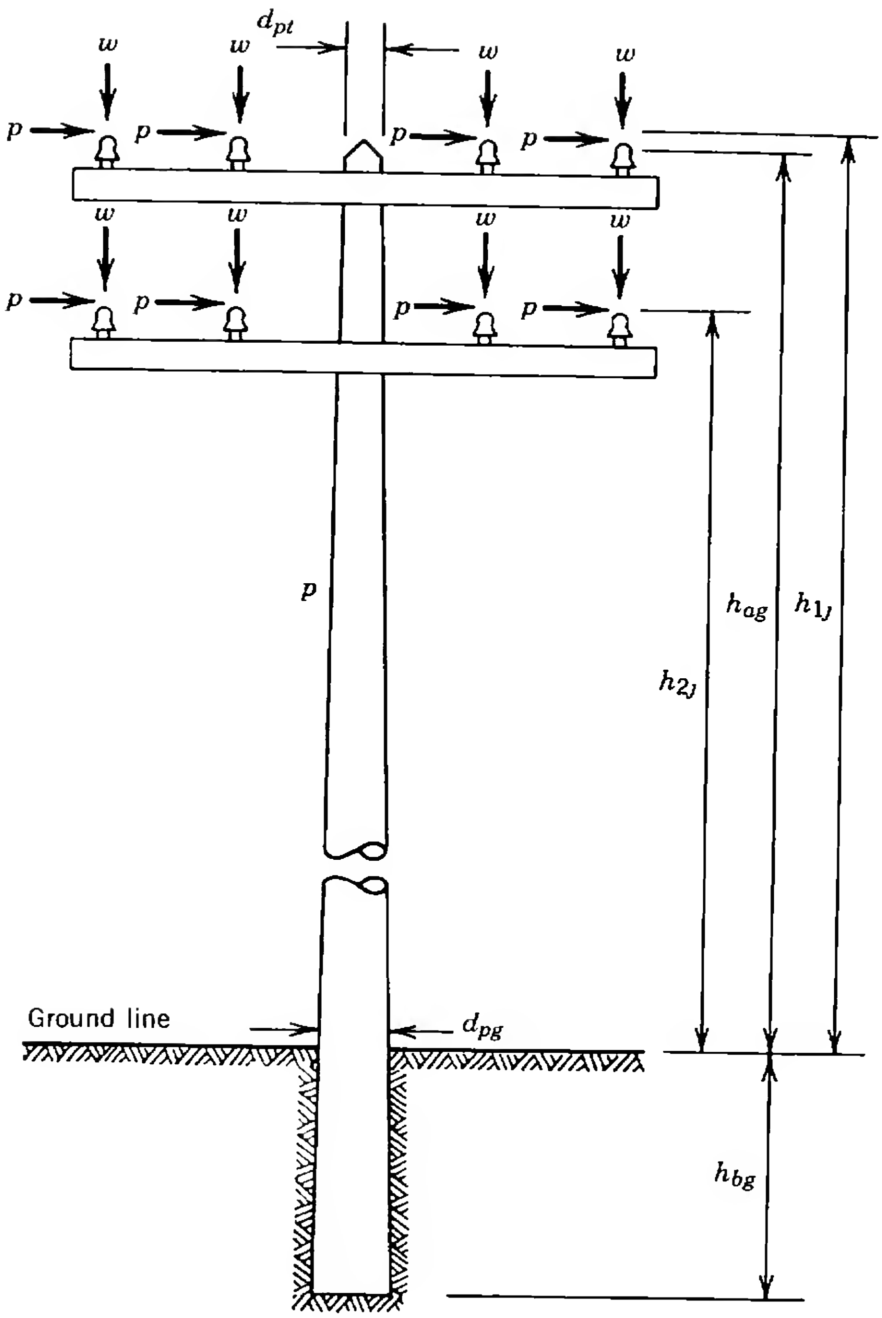


Figure 9.10. Schematic of pole with two cross-arms (not to scale).

n = number of conductors on each cross-arm
P = transversal and horizontal wind force (i.e., load) exerted on line in pounds per feet
L_{avg} = average span in feet
h_{ij} = height of conductor j on cross-arm i in feet

The amount of transversal and horizontal wind load exerted on the conductors depends on whether or not the conductors are covered with ice. This topic is discussed in greater detail in Section 10.5.

In Figure 9.11, L_{avg} represents the average horizontal span, and it is

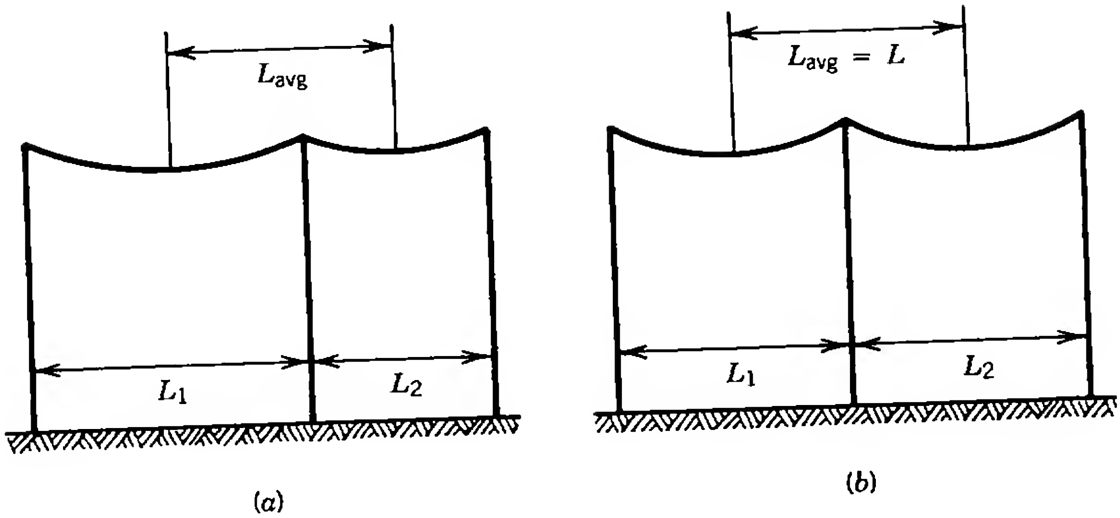


Figure 9.11. Pole-loading diagrams: (*a*) when two adjacent spans are not equal to each other; (*b*) when two adjacent spans are equal to each other.

equal to one-half the length of the two adjacent spans L_1 and L_2. Of course, the L_{avg} can be calculated as

$$L_{avg} = \tfrac{1}{2}(L_1 + L_2) \quad \text{when } L_1 \neq L_2 \tag{9.7}$$

or

$$L_{avg} = L_1 = L_2 = L \quad \text{when } L_1 = L_2$$

9.8.3 Bending Moment Due to Wind on Poles

The bending moment due to wind on the pole (see Figure 9.10), which is usually a maximum at the ground level of an unguyed pole, is

$$M_{gp} = \frac{ph_{ag}^2}{72}(d_{pg} + 2d_{pt}) \quad \text{lb-ft} \tag{9.8}$$

or

$$M_{gp} = \frac{ph_{ag}^2}{72\pi}(c_{pg} + 2c_{pt}) \quad \text{lb-ft} \tag{9.9}$$

where M_{gp} = bending moment due to wind on pole in pound feet
h_{ag} = height of pole above ground in feet
d_{pg} = diameter of pole at ground line in inches
d_{pt} = diameter of pole at pole top in inches
p = wind pressure in pounds per square feet
c_{pg} = circumference of pole at ground line in inches
c_{pt} = circumference of pole at pole top in inches

The internal resisting moment of the wood pole, when the maximum stress is at the ground line

$$M = \frac{1}{3790} Sc_{pg}^3 \quad \text{lb-ft} \tag{9.10}$$

or

$$M = 2.6385 \times 10^{-4} Sc_{pg}^3 \quad \text{lb-ft} \tag{9.11}$$

Where the maximum stress is above ground line,

$$M = 2.6385 \times 10^{-4} Sc_1^2(c_{pg} - c_1) \quad \text{lb-ft} \tag{9.12}$$

where M = bending moment at ground line in pound feet
c_{pg} = circumference of pole at ground line in inches
c_1 = circumference of pole at point of maximum stress
S = allowable maximum fiber stress in pounds per square inch

where

$$S = \frac{\text{ultimate fiber strength of pole}}{\text{safety factor}} \tag{9.13}$$

The minimum safety factors required, according to the grades of construction, are given by the NESC. Table 9.9 [4] provides the resisting moments of wood poles given circumference of pole at the ground line and ultimate fiber stress rating of the pole.

The condition that a given pole will not break is

$$M > M_{tc} + M_{gp}$$

Fiber stress should not be more than 15 percent that at the breaking point for normal unbalanced forces, for example, forces affecting an unguyed corner pole. For wind pressures and an unbalanced force of broken conductors that are abnormal and do not persist, the stress is usually used at about 50 percent of that at breaking point.

Equation (9.11) is based on the assumption that the ground line is the weakest point of the pole. This is not so correct, especially for northern cedar or other wood poles tapering 1 in. in 5 to 6 ft of length and being approximately a truncated cone in shape. For a bending loading, that is, force applied at one end, such a cone is weakest at the point where the diameter is $\frac{3}{2}$, or 1.5 the diameter at the point (near the small end) where the result and load or force is applied. For example, a pole with a 10-in. diameter at the cross-arm is weakest where it is 15 in. in diameter. If a northern cedar pole, having a taper of 1 in. in 6 ft of length, the weakest

TABLE 9.9 Resisting Moments of Wood Poles

Pole Circumference at Ground (in.)	Pound Feet at One-half Ultimate Fiber Stress Ratings of (lb/in.^{2})						
	8400	8000	7400	6600	6000	5600	4000
28	24,300	23,150	21,400	19,150	17,350	16,250	11,550
30	29,900	28,500	26,350	23,500	21,350	19,950	14,250
32	36,300	34,600	32,000	28,500	25,950	24,200	17,300
34	43,600	41,500	38,400	34,200	31,150	29,000	20,750
36	51,750	49,300	45,600	40,700	36,950	34,450	24,650
38	60,750	57,850	53,500	47,800	43,400	40,550	28,900
40	70,950	67,550	62,500	55,700	50,650	47,250	33,800
42	82,200	78,250	72,400	64,400	58,700	54,700	39,150
44	94,450	89,950	83,200	74,100	67,450	62,900	44,950
46	107,950	102,800	95,100	84,600	77,100	71,900	51,400
48	122,600	116,750	108,000	96,200	87,550	81,700	58,350
50	138,600	132,000	122,100	108,700	99,000	92,300	66,000
52	155,850	148,400	137,300	122,400	111,300	103,900	74,200

Source: From Fink and Carroll [5].

point would be $6 \times (15 - 10) = 30$ ft below the cross-arms. However, in practice, the weakest section of the pole is taken at ground line since the pole at that point tends to become weaker than any point above ground as a result of its greater moisture content and its greater tendency to decay as the pole ages.

Since pole-top transformers impose not only vertical but also transversal loading on poles, the wood poles used to carry transformers greater than 25 kVA are usually selected having a pole-top diameter of 1 in. or larger than would be required elsewhere. Usually, a transformer of 300 kVA or greater is installed on a platform supported by two wood poles placed 10 to 15 ft apart.

Example 9.2

Assume a 35-ft pole set 6 ft in ground, with a 28-in. circumference at the pole top and a 40-in. circumference at ground level. Also assume a wind velocity of 40 mi/h and an average span of 120 ft. The conductor used is 4/0 copper of 0.81 in. diameter. There are eight conductors on the line, as shown in Figure 9.10. Calculate the following:

(a) Total pressure of wind on pole.

(b) Total pressure of wind on conductors

Solution

(a) Using equation (9.1),

$$\begin{aligned} p &= 0.00256V^2 \\ &= 0.00256 \times 40^2 \\ &= 4.096 \text{ lb/ft}^2 \end{aligned}$$

The projected area of the pole is

$$S_{\text{pni}} = \frac{d_{\text{pg}} + d_{\text{pt}}}{2} h_{\text{ag}} \times 12 \text{ in.}^2 \tag{9.14}$$

where d_{pg} = diameter of pole at ground line in inches
d_{pt} = diameter of pole at pole top in inches
h_{ag} = height of pole above ground in feet

Therefore,

$$\begin{aligned} S_{\text{pni}} &= \tfrac{1}{2}(12.7 + 8.9) \times 29 \times 12 \\ &= 3758.4 \text{ in.}^2 \end{aligned}$$

or

$$= 3758.4 \text{ in.}^2 \times 0.0069444 \text{ ft}^2/\text{in.}^2 \cong 26.1 \text{ ft}^2$$

Hence, the total pressure of the wind on the pole is

$$\begin{aligned} P &= S_{\text{pn}i}p \\ &= 26.1 \times 4.096 \\ &\cong 106.9 \text{ lb} \end{aligned} \tag{9.15}$$

(b) The diameter of the conductor is 0.810 in. Therefore, the projected area of the conductor, a 120-ft span, is

$$\begin{aligned} S_{\text{n}i} &= 0.810 \text{ in} \times 120 \text{ ft} \times 12 \text{ in./ft} \\ &= 1166.4 \text{ in.}^2 \end{aligned}$$

or

$$= 1166 \text{ in.}^2 \times 0.006944 \text{ ft}^2/\text{in.}^2 \cong 8.1 \text{ ft}^2$$

Thus, the total pressure of the wind on the conductors is

$$\begin{aligned} P &= S_{\text{n}i}p \\ &= 8 \times 8.1 \text{ ft}^2 \times 4.096 \text{ lb/ft}^2 \\ &\cong 265.4 \text{ lb} \end{aligned} \tag{9.16}$$

Example 9.3

A section of an overhead line needs to be built on 45-ft wood poles set 6.5 ft deep in the ground. Each pole will carry two cross-arms, and each cross-arm has four conductors, as shown in Figure 9.10. The top cross-arm will be 1 ft below the pole top, and the lower cross-arm will be 3 ft below the pole top. The conductors located on the top cross-arm and on the lower cross-arm have transverse wind loads of 0.6861 and 0.4769 lb/ft of conductor, respectively. The rated ultimate strength of the wood pole is 8000 lb/in.^{2} and a safety factor of 2. Assume that the transverse wind load on the wood poles will not exceed 9 lb/ft^2. Calculate the minimum required pole circumference at the ground line if the average span is 250 ft.

Solution

First, let us find the moment arms:

$$\text{For top arm:} \quad h_{1j} = 45 - 6.5 - 1 = 37.5 \text{ ft}$$

$$\text{For lower arm:} \quad h_{2j} = 45 - 6.5 - 3 = 35.5 \text{ ft}$$

By using equation (9.6), the total bending moment due to wind on the conductors is

$$M_{\text{tc}} = \sum_{i=1}^{2} \sum_{j=1}^{4} m_i n_{ij} P L_{\text{avg}} h_{ij} \quad \text{lb-ft}$$

For the top arm,

$$M_{tc} = 1 \times 4 \times 0.6861 \times 250 \times 37.5 \cong 25{,}729 \text{ lb-ft}$$

For the lower arm

$$M_{tc} = 1 \times 4 \times 0.4769 \times 250 \times 35.5 \cong 16{,}930 \text{ lb-ft}$$

Then, for both cross-arms together,

$$M_{tc} = 25{,}729 + 16{,}930 = 42{,}659 \text{ lb-ft}$$

Using equation (9.11), the internal resisting moment of the pole is

$$M = 2.6385 \times 10^{-4} S c^3$$

where

$$M = 42{,}659 \text{ lb-ft}$$

$$S = \frac{8000}{2} = 400 \text{ psi}$$

Therefore,

$$\begin{aligned} c_{pg}^3 &= \frac{M}{2.6385 \times 10^{-4} s} \\ &= \frac{42{,}659}{2.6385 \times 10^{-4} \times 4000} \\ &\cong 40{,}419.8 \end{aligned}$$

Hence,

$$c_{pg} = 34.3 \text{ in.}$$

Assume that, from the proper tables, the minimum pole-top circumference for the kind of pole corresponding to an ultimate fiber stress of 8000 psi is found to be 22 in. Therefore, the bending moment due to wind on the pole, using equation (9.9), is

$$\begin{aligned} M_{gp} &= \frac{p h_{ag}^2}{72\pi} (c_{pg} + 2c_{pt}) \\ &= \frac{9 \times (38.5)^2}{72\pi} (34.3 + 2 \times 22) \\ &\cong 4618 \text{ lb-ft} \end{aligned}$$

Therefore, the total bending moment due to wind on conductors and pole is

$$\begin{aligned} M_T &= M_{tc} + M_{gp} \\ &= 42{,}659 + 4{,}618 \\ &= 47{,}277 \text{ lb-ft} \end{aligned}$$

Using equation (9.11),

$$c_{pg}^3 = \frac{42{,}277}{2.6485 \times 10^{-4} \times 400}$$
$$\cong 44{,}795$$

or, the required minimum circumference of the pole at the ground line is

$$c \cong 35.5 \text{ in.}$$

Therefore, the nearest standard size pole, which has a ground-line circumference larger that 35.5 in., has to be used. Instead of using equation (9.11) to find the required minimum circumference of the pole at ground level, one can find it directly from Table 9.9 as 36 in., which is a standard size.

9.8.4 Stress Due to Angle in Line

If there is an angle in the line, an additional stress is imposed upon the supporting structure at the angle point because of the tensions in the conductors. Figure 9.12 shows a plan view of a diagram of the forces acting on an angle pole.

If the conductors in the adjacent spans have equal tensions of T and the angle of departure of the line is α, the resultant side pull force on the pole is T_r. This force can be calculated by using the following formula:

$$T_r = 2nT_1 \sin\frac{\alpha}{2} \quad \text{lb} \tag{9.17}$$

where T_r = resultant side pull force due to angle in line in pounds
n = number of conductors on pole
T_1 = maximum tension in conductors in pounds
α = angle departure of line in degrees

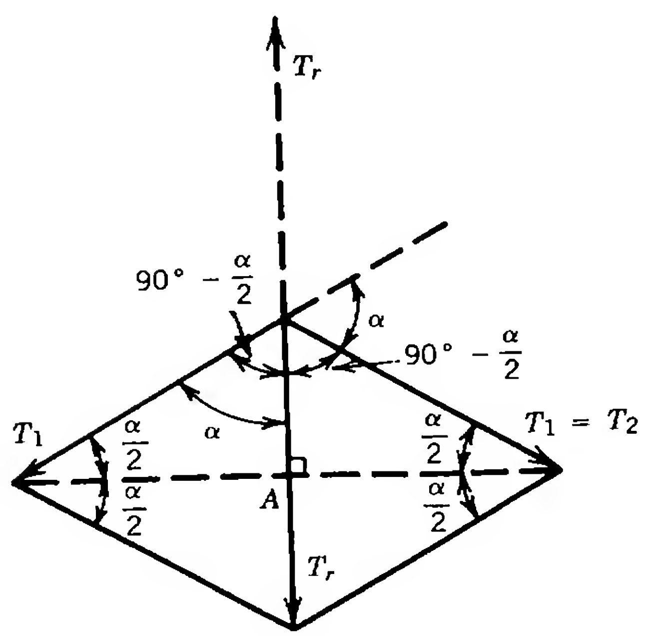


Figure 9.12. Plan view of angle pole and diagram of forces.

When this force is large enough, the bending stress may become greater than the allowable working stress or even the ultimate fiber strength of the pole. Because of this, it must be balanced by the guy wire.

If the conductor tensions in the adjacent spans are not equal, then the resultant side pull force is

$$T_r = \sqrt{T_1^2 + T_2^2 - 2T_1T_2\cos\alpha} \quad lb \tag{9.18}$$

and the angle γ between the resultant and the span in which the tension T_1 is obtained from can be determined from

$$\cos\gamma = \frac{T_r^2 + T_1^2 - T_2^2}{2T_rT_1} \tag{9.19}$$

where γ = angle between direction of resultant and direction of either span, that is, $90° - \frac{1}{2}\alpha$.

If the angle departure of the line is less than 60°, the resultant side pull force is less than the maximum tension of the conductors in the adjacent spans. Therefore, one single guy installed in the opposite direction of the resultant side pull force, as shown in Figure 9.13, will be sufficient. However, if the angle departure of the line is larger than 60°, the resultant side pull force is larger than the maximum tension of the conductors in the adjacent spans. In order to stop a tendency to displace the pole if the angle does not exactly bisect the line angle and not use a guy of extreme strength, install two guys each located in the opposite direction of the line, as shown in Figure 9.13.

9.8.5 Strength Determintion of Angle Pole

In order to determine whether a given pole, which will be used as an angle pole in the line, has the required strength to meet the NESC requirements, the following equation is used [5]:

$$M = \frac{M_{gp} + M_{tc}}{S_1} \times 100 + \frac{M_r}{S_2} \times 100 \text{ lb-ft} \tag{9.20}$$

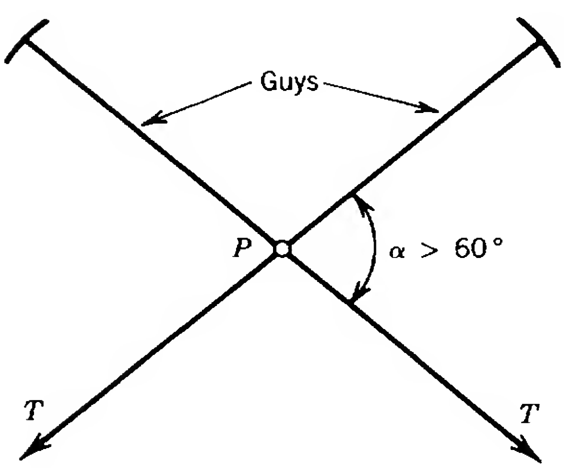


Figure 9.13. Installment of two guys when angle of departure of line is larger than 60°.

where M = required internal resisting moment of pole in pound feet
M_{gp} = total bending moment due to wind on pole in pound feet
M_{tc} = total bending moment due to wind on conductors in pound feet
M_r = bending moment due to tensions in conductors in pound feet
S_1 = permissible stress in pole for transverse loading, in percentage of ultimate fiber strength
S_2 = permissible stress in pole, for longitudinal loading at dead ends, in percentage of ultimate fiber strength

Here

$$M_r = T_r h \quad \text{lb-ft} \tag{9.21}$$

or

$$M_r = 2nhT \sin\frac{\alpha}{2} \quad \text{lb-ft} \tag{9.22}$$

which has to be calculated for every conductor and added together. Theoretically, if M is larger than the ultimate resisting moment of the pole, use guy, otherwise, do not. However, in practice, if the pole does not have a proper rigid setting, there is still a need for the guy.

9.8.6 Permissible Maximum Angle without Guys

It is almost impossible to build an overhead line of any considerable length, especially transmission lines, without several angles, which may vary in magnitude from only a few degrees to 90° or more. The earth settings, depending on earth type, may allow a certain amount of pole stresses, which are caused by angles in the line, to be in excess of the permissible fiber stress of a given wood pole. If the conductor tensions in the adjacent spans are equal, the allowable maximum angle, without any side guying, in a given line can be found from the following equation:

$$M_{tc} + M_{gp} + 2nhTs\text{ in}\,\frac{\alpha}{2} = \frac{S_e}{100}\,M \tag{9.23}$$

where M_{tc} = total bending moment due to wind on conductors in pound feet
M_{gp} = bending moment due to wind on pole in pound feet
T = maximum tension of conductors in adjacent spans in pounds
h_{ag} = height of pole above ground in feet
α = angle of departure of line
S_e = earth resistance to displacement, in percentage of pole internal resisting moment
M = required internal resisting moment of pole in pound feet

If a given angle in the line is larger than the allowable maximum angle obtained from equation (9.23) or if the earth resistance to displacement is not large enough, then a guy or guys need to be used.

9.8.7 Guying

Whenever a pole is not strong enough to endure the bending stresses imposed on it by unbalanced forces, it should be guyed. For example, at the pole where the direction of a line changes, tension of the conductor should be supported by guying to other poles, to a ground anchor, or to a stub. Another common usage of guys is at dead-ends.

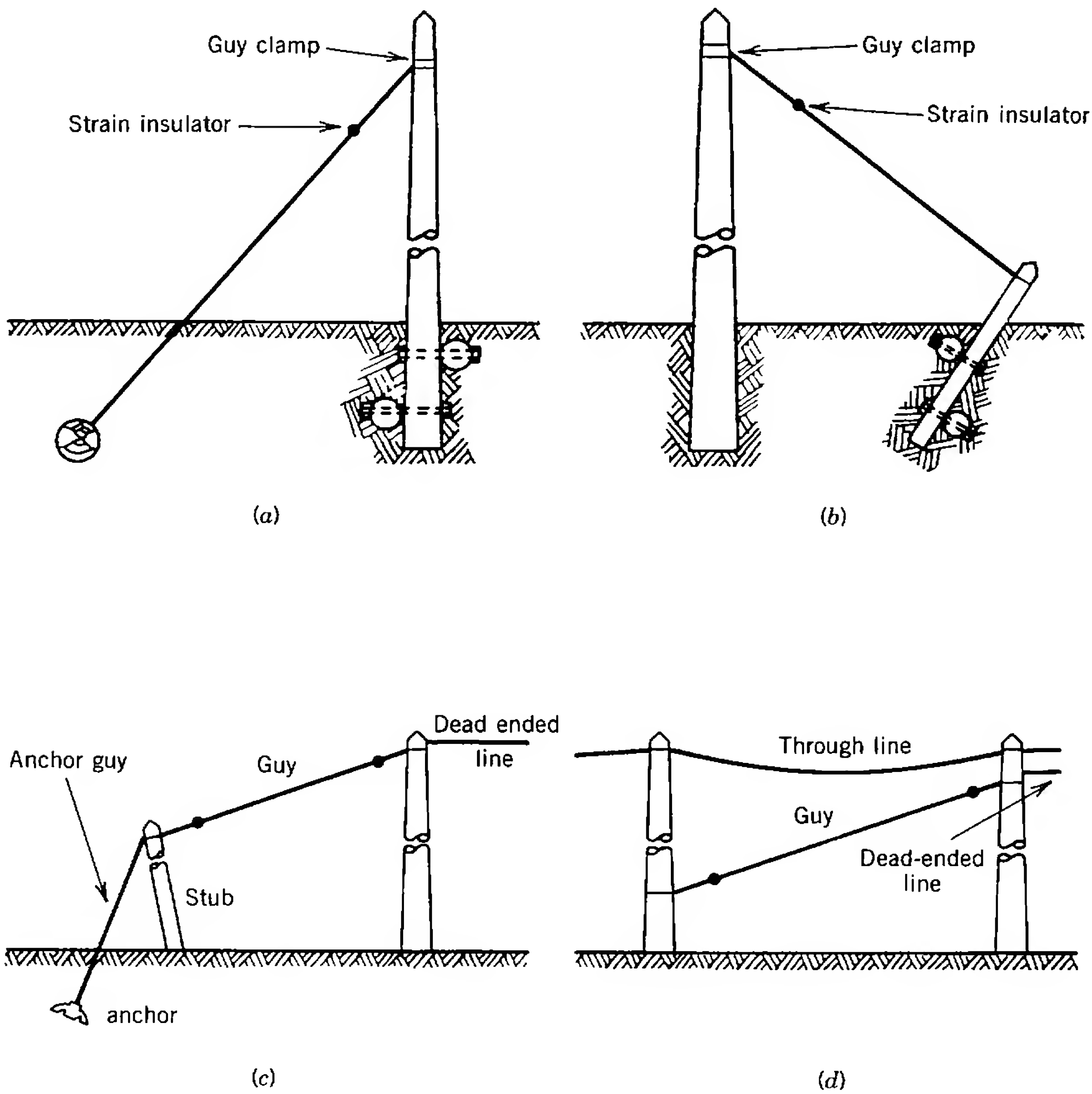


Figure 9.14. Various guying techniques: (*a*) anchor guy; (*b*) stub guy; (*c*) pole-to-stub-to-anchor guy; (*d*) pole-to-pole guy.

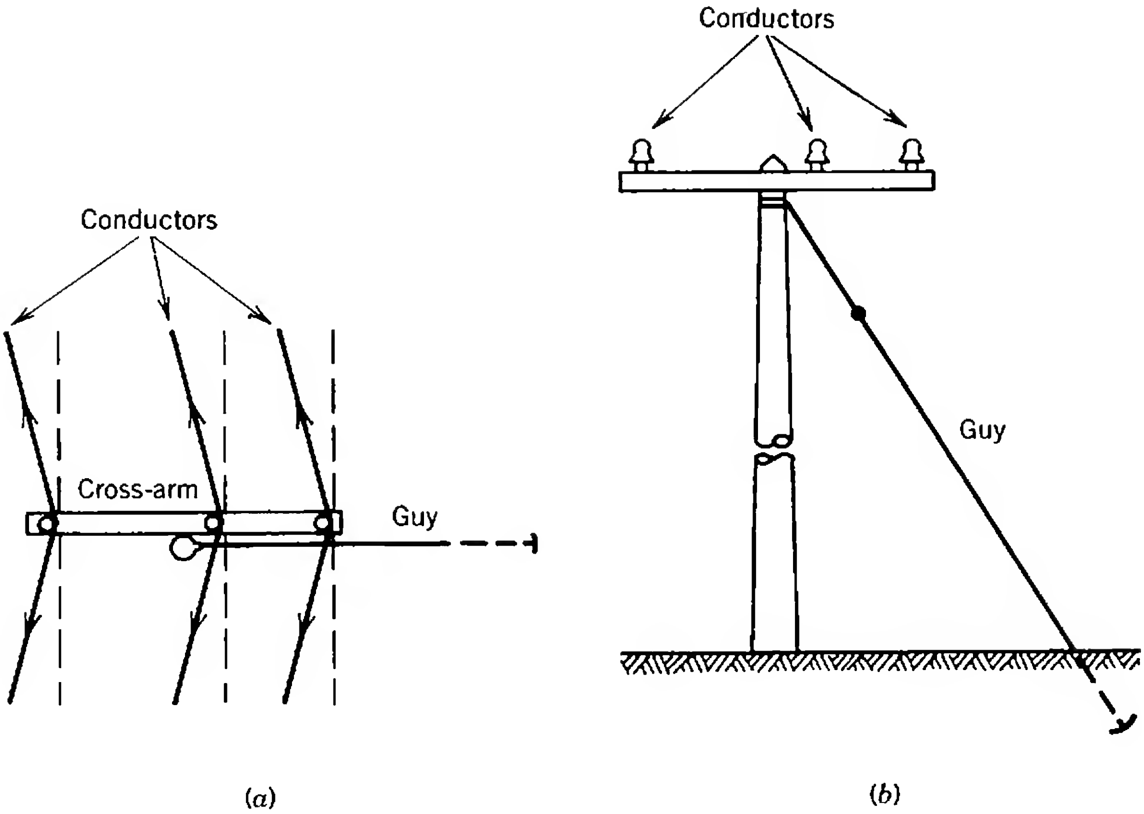


Figure 9.15. Guy installation at an angle: (*a*) plan; (*b*) elevation.

The strength of a guy should be large enough to take the entire horizontal stress in the direction in which it acts, the pole acting only as a strut taking only the vertical component of guy tension. Special structures such as A-frames, push braces, etc., are sometimes used instead of guys in some applications; but the most common technique is to install guys or steel wire or other high-strength material to take the stress. Figure 9.14 illustrates various guying techniques. Figure 9.15 shows a plan and elevation views of a guy installation at an angle. Figure 9.16 shows a dead-end guy installation.

Guys are firmly attached to poles by wrapping the end of the guy wire twice or more around the pole and clamping the free end to the main section of the guy, usually, by means of one or more guy clamps. However, nowadays, the guy is usually attached to the pole by a thimble-eye or by a guy eye bolt and a stubbing washer, as shown in Figure 9.17. The attachment point of a guy should be as close as possible to the point where the resultant side pull force is imposed upon the supporting structure. If there are several cross-arms mounted on the pole at different elevations, then the load at those elevations has to be converted to an equivalent load applied at the level where the guy is attached.

Usually one or two strain insulators† are installed in guys to prohibit the

† However, there is also the widespread use of uninsulated, grounded guys on multigrounded neutral distribution systems.

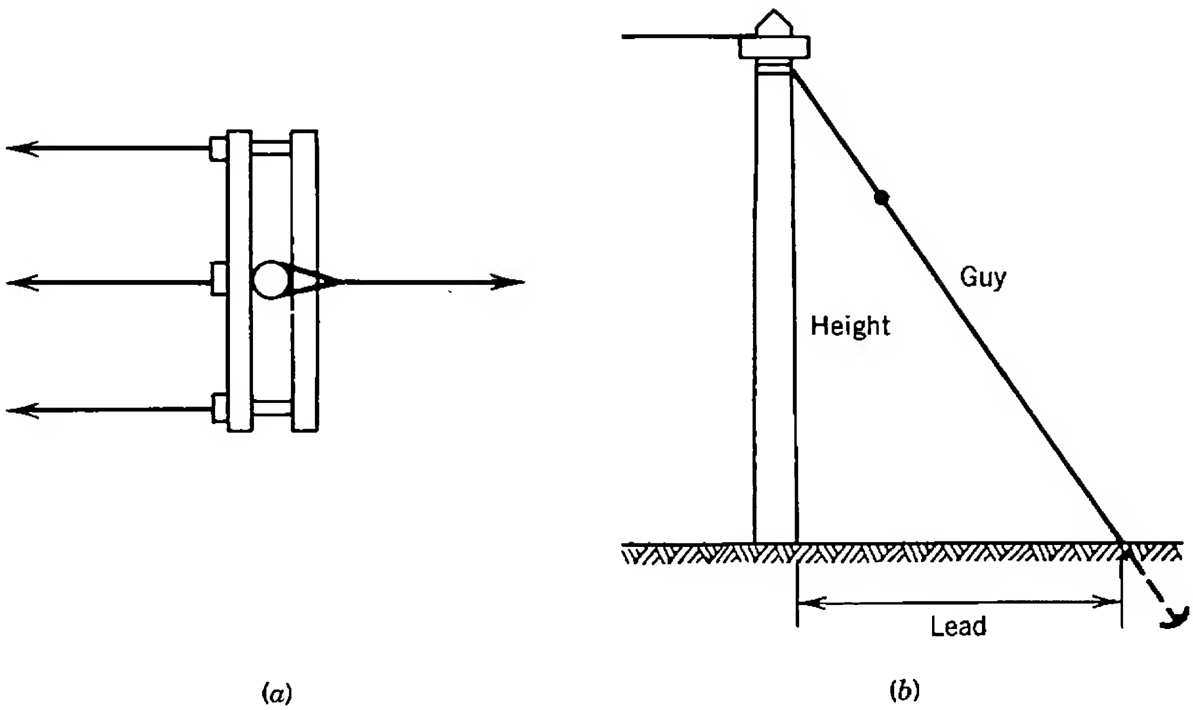


Figure 9.16. Dead-end guy installation: (*a*) plan; (*b*) elevation.

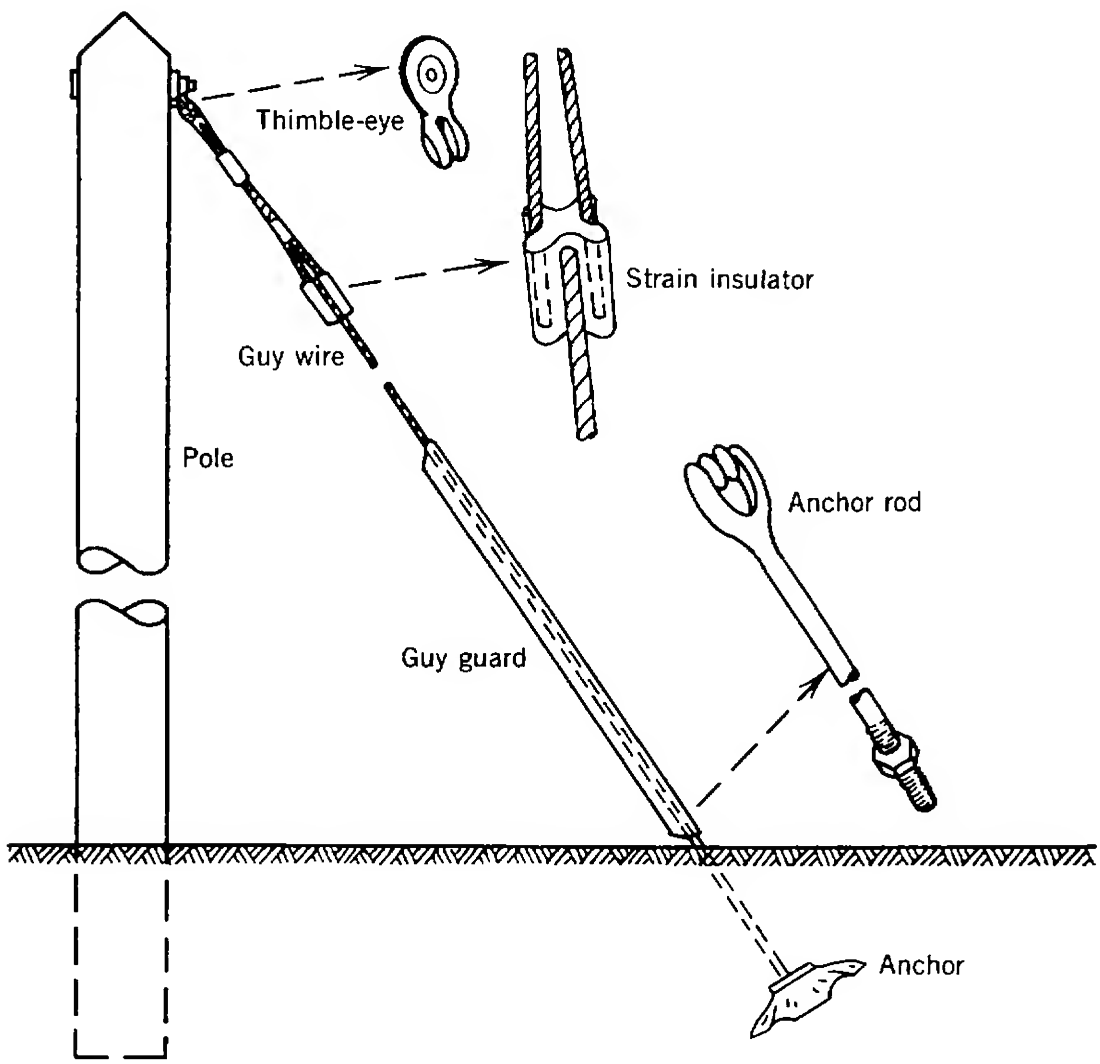


Figure 9.17. Components of anchor guy installation.

lower part from becoming electrically energized by contact of the upper part with conductors or by leakage. Figure 9.17 shows the basic components of an anchor wire guy, which include the guy wire, clamps, the anchor, and a strain insulator. The guy wire is usually copperweld, galvanized, or bethanized steel.

Burring logs, which were called dead-men, in the ground to anchor the guy wire, as shown in Figure 9.14(c), has been abandoned since the soil conditions often deteriorated the wood. Instead, in the present practice, metal anchors are used in any type of ground from swamp to solid rock.

9.8.8 Calculation of Guy Tension

Consider the dead-end pole supported by a guy wire, as shown in Figure 9.18. Assume that the line conductors are carried by the pole at two different heights. The resultant side pull is counterbalanced by the tension in the guy wire. This tension T_g can be resolved into two components, T_h and T_v. The summation of the bending moments created by T_1 and T_2 loads at heights h_1 and h_2, respectively, must be balanced by the bending moment created by T_h:

$$T_h h_g = T_r h_g \tag{9.24}$$

or

$$T_h h_g = T_1 h_1 + T_2 h_2 \tag{9.25}$$

Therefore, the horizontal component of the tension in the guy wire is

$$T_h = \frac{1}{h_g}(T_1 h_1 + T_2 h_2) \tag{9.26}$$

where T_h = horizontal component of guy wire tension in pounds
T_1 = horizontal load at height h_1 in pounds
T_2 = horizontal load at height h_2 in pounds
h_g = height of attachment point of guy in feet
h_1 = height of horizontal load T_1 in feet
h_2 = height of horizontal load T_2 in feet

From Figure 9.18

$$\tan \beta = \frac{h_g}{L} \tag{9.27}$$

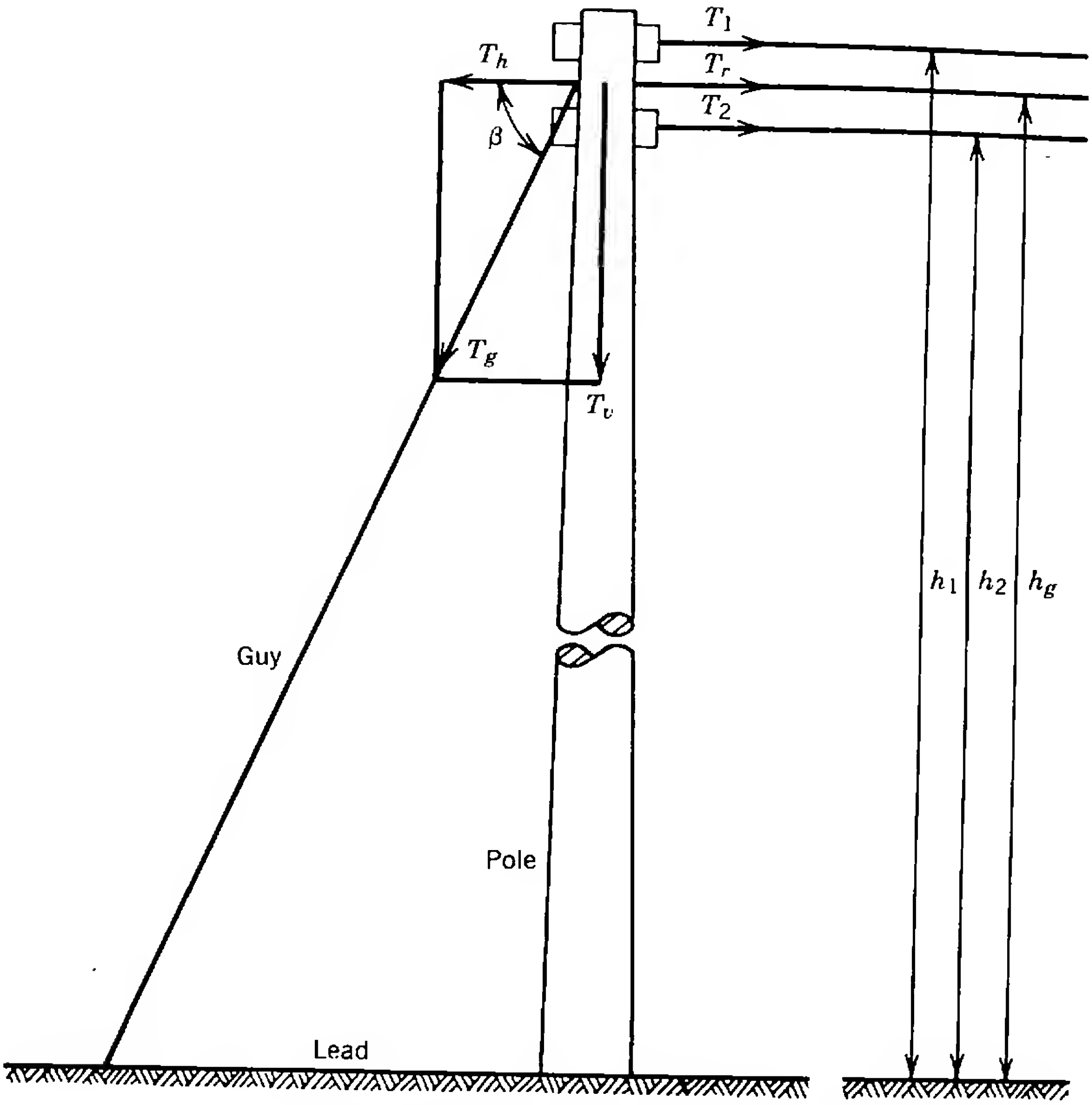


Figure 9.18. Guy-loading diagram.

or

$$\beta = \arctan \frac{h_g}{L} \tag{9.28}$$

where L is the lead of the guy in feet. Then, the tension in the guy wire is

$$T_g = \frac{T_h}{\cos \beta} \tag{9.29}$$

or

$$T_g = T_h \sec \beta \tag{9.30}$$

Also,

$$T_g = T_r \sqrt{1 + \left(\frac{h_g}{L}\right)^2} \tag{9.31}$$

since

$$T_r = T_h \quad \text{and} \quad \sec\beta = \sqrt{1 + \left(\frac{h_g}{L}\right)^2}$$

Further,

$$\tan\beta = \frac{h_g}{L} \tag{9.32}$$

or

$$\tan\beta = \frac{T_v}{T_h} \tag{9.33}$$

and the vertical component of the tension in the guy wire is

$$T_v = T_h \tan\beta \tag{9.34}$$

or

$$T_v = T_h \frac{h_g}{L} \tag{9.35}$$

Therefore, the total vertical load on the pole is

$$W_v = \frac{T_h h_g}{L} + W_e + W_p \quad \text{lb} \tag{9.36}$$

where W_v = total vertical load on pole in pounds
W_p = weight of pole in pounds
W_e = weight of equipment, hardware, and conductors on pole in pounds

As the angle β decreases, the tension T_g in the guy wire and its vertical component T_v also decreases, despite the fact that the horizontal component of the guy wire tension T_h stays the same. Therefore, in practice, the tangent of the angle β should be held minimum.

If the attachment point of a given guy is too far from the center of the horizontal loads T_1 and T_2, the stress in the pole at that point may become important. Thus, the bending moment of the pole at the point of attachment becomes

$$M = T_1(h_1 - h_g) + T_2(h_2 - h_g) \quad \text{lb-ft} \tag{9.37}$$

It should be kept less than the required minimum pole resisting moment.

EXAMPLE 9.4

A dead-end pole is supported by a guy wire, as shown in Figure 9.18. The bending moments are created by 3000 and 2500 lb at heights of 37.5 and 35.5 ft, respectively. The guy is attached to a pole at the height of 36.5 ft. The lead of the guy is 15 ft. Calculate the following:

(a) Horizontal component of tension in guy wire.
(b) Angle β.
(c) Vertical component of tension in guy wire.
(d) Tension in guy wire.

Solution

(a) Using equation (9.26),

$$
\begin{aligned}
T_h &= \frac{1}{h_g}(T_1 h_1 + T_2 h_2) \\
&= \frac{1}{36.5}(3000 \times 37.5 + 2500 \times 35.5) \\
&\cong 5513.7 \text{ lb}
\end{aligned}
$$

(b) Using equation (9.28),

$$
\begin{aligned}
\beta &= \arctan \frac{h_g}{L} \\
&= \arctan \frac{36.5}{15} \\
&\cong 67.6^\circ
\end{aligned}
$$

(c) Using equation (9.34),

$$
\begin{aligned}
T_v &= T_h \tan \beta \\
&= 5513.7 \tan 67.6^\circ \\
&\cong 13{,}416.67 \text{ lb}
\end{aligned}
$$

(d) Using equation (9.29),

$$
\begin{aligned}
T_g &= \frac{T_h}{\cos \beta} \\
&= \frac{5513.7}{\cos 67.6^\circ} \\
&\cong 14{,}505.4 \text{ lb}
\end{aligned}
$$

or from

$$
\begin{aligned}
T_g &= \sqrt{T_h^2 + T_v^2} \\
&= \sqrt{5513.7^2 + 13{,}416.67^2} \\
&\cong 14{,}505.4 \text{ lb}
\end{aligned}
$$

9.9 GRADE OF CONSTRUCTION

The criterion used for the strength of requirements of a line is called the *grade of construction*. The grades of construction are specified on the basis of the required strengths for safety. The NESC designates the grades for supply and communication lines by the letters B, C, D, E, and N. Grade B is the highest and requires the greatest strength. Grade D is specified only for communication lines, and it is higher than grade N. The grade used depends on the type of circuit, the voltage, and the surroundings of the line. For the example, a power line of any voltage crossing over a main track of a railroad requires grade B construction, but under certain other conditions may be as low as grade N. In addition to the NESC requirements, there are also local rules and regulations for the grades of construction.

9.10 LINE CONDUCTORS

Copper and aluminum are the metals most frequently used as conductors in distribution systems. The selection criteria include conductivity, cost, mechanical strength, and weight. According to these selection criteria, copper conductor is the best and aluminum conductor is the second best conductor in terms of conductivity and availability. Aluminum has the advantage of about 70 percent less weight for a given size, but its conductivity is only about 61 percent that of annealed copper. Its breaking strength is about 43 percent that of hard-drawn copper. In general, aluminum conductor is rated as equivalent to a copper conductor two AWG sizes smaller, which has almost identical resistance.

The factors affecting voltage drop, power loss, and mechanical strength to prevent excessive sag are important in selecting the type of conductor for overhead lines. In order to obtain proper ground clearance without excessively increasing the height of poles, for rural overhead distribution lines with lower load densities and longer spans, conductors of high tensile strength are usually preferrable. However, for urban underground distribution, serving high-load density areas, current-carrying capacity and voltages drop are more important in selecting the conductor type.

The relatively small diameter of copper conductors, solid or stranded, in comparison with their current-carrying capacity, allows a minimum of projected area to wind and ice loads. This provides a greater safety factor for the poles and demands only a minimum of guying against transverse loading. However, because of its comparatively low ratio of strength to weight, copper conductors necessarily required greater sag for a given span length when compared with copperweld or ASCR conductors. Because of this greater sag, higher poles or shorter spans have been used to provide adequate ground clearance at maximum temperature conditions.

Copper wires or cables are made in three standard degrees of hardness:

(1) hard drawn, (2) medium hard drawn, and (3) soft drawn. Hard-drawn copper has greatest tensile strength and is used for overhead lines with span lengths of 200 ft or more. Medium hard-drawn copper has less tensile strength and is used for common types of local distribution overhead lines with shorter span lengths. Soft-drawn copper has the least tensile strength and is used almost only for underground cables because of its greater flexibility. Maximum transmission capacity for a given power loss and voltage drop is the largest for hard-drawn copper conductors. Hard-drawn wire is cold drawn to size from a stock copper bar. This cold-drawing process increases the tensile strength of the copper, it hardens it, and slightly decreases its conductivity. If a hard-drawn copper wire is heated at the proper temperature for a specific time period, its small tensile strength decreases and the wire becomes softer and more ductile, and is said to be annealed. Medium-hard drawn copper is annealed after being cold drawn to size and then cold drawn to size and strength. Whereas soft-drawn copper wire is cold drawn to size and then annealed.

Aluminum stranded around a steel core sized to give the required strength is especially used in rural overhead lines. It is called *aluminum cable steel-reinforced* and is commonly designated as ACSR. Development of high-strength aluminum alloys has led to such alternative cables as *aluminum conductor alloy-reinforced* (ACAR) and *all-aluminum-alloy conductor* (AAAC), which also combine conductivity with tensile strength.

Because of their high resistance steel conductors are rarely used in distribution lines. But a high-strength steel strand covered with a thin sheet of copper welded on, known as *copperweld*, or with a thin sheet of aluminum welded on, known as *alumoweld*, has conductivity of about 40 percent that of copper and are used. When a conductor of high conductivity and high tensile strength is required, copper strands combined with the copperweld strands form a conductor called *copperweld-copper*. Another composite conductor is made out of hard-draw aluminum strands combined with the alumoweld strands. Some guy cables are made out of copperweld or alumoweld since they are more durable than galvanized-steel cables.

In general, the size of conductors used for an overhead line is determined by the electric power to be transmitted and permissible voltage drop. The requirements for mechanical strength, however, place a minimum on the conductor size that is practical to use. The NESC specifies the minimum conductor sizes that are permissible to use.

9.11 INSULATOR TYPES

The overhead line insulators are classified as (1) pin-type insulators, (2) suspension insulators, and (3) strain insulators. Pin-type insultors are used for low- and medium-voltage distribution lines. Suspension insulators are used for all voltage lines. Strain insulators are used in guys and for dead-ending low-voltage lines. Usually, pin-type or post-type insulators are used on the overhead lines with not more than 70 kV. Above 70 kV, suspension insulators are used for dead-ending lines of any voltage, although small conductors of low-voltage distribution lines are often dead-ended on double-arm construction using pin-type insulators. Suspension insulators are also used for tangent and angle construction for practically all voltage lines. Suspension insulators are manufactured either with clevis-and-pin connec-

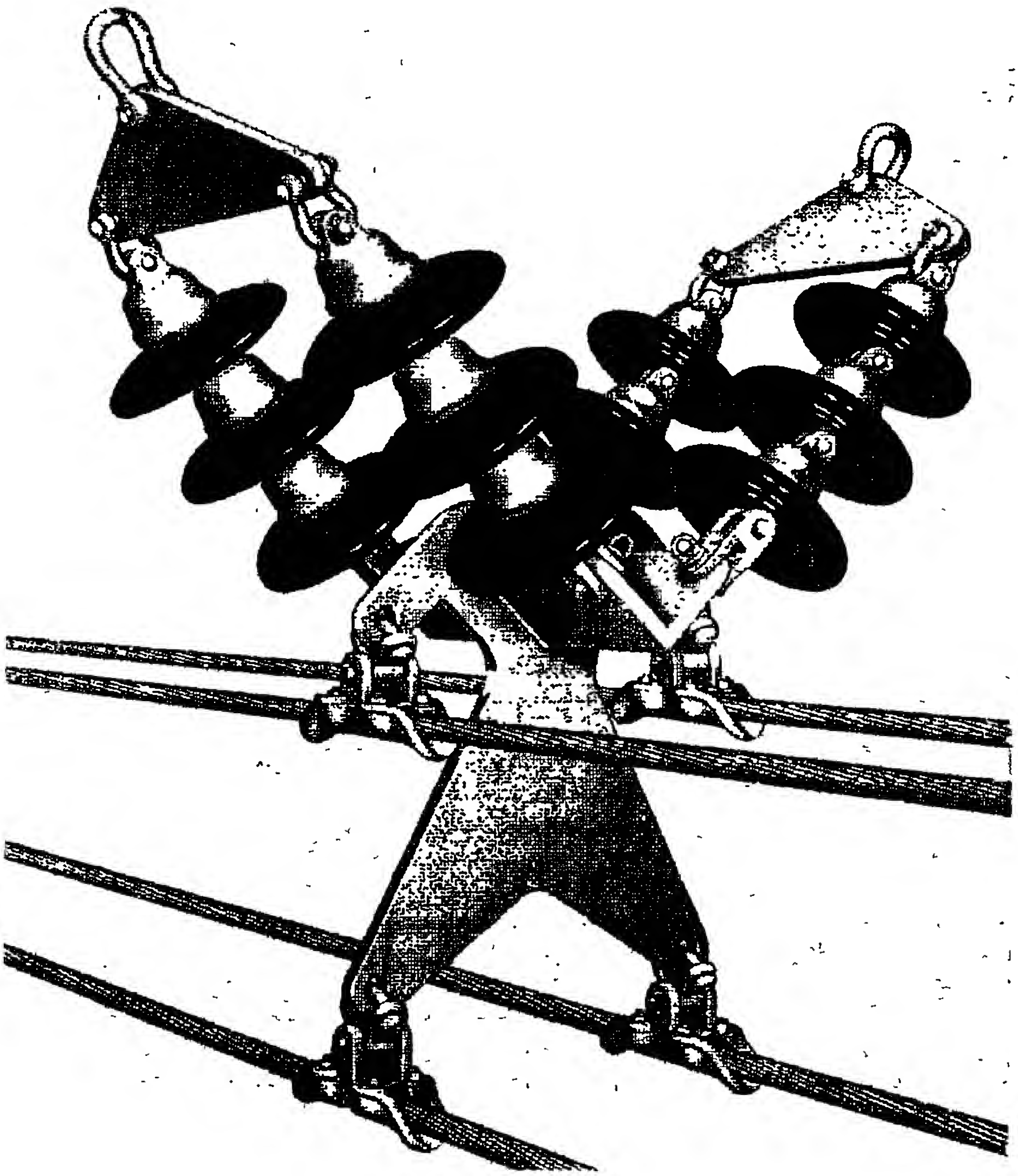


Figure 9.19. Vee arrangement of suspension insulator strings carrying four bundled conductors per phase. (Courtesy of Ohio Brass Company.)

Figure 9.20. Installing vee arrangement of suspension insulator strings carrying four bundled conductors per phase at 500 kV. (Courtesy of Ohio Brass Company.)

tions or with ball-and-socket connections. Both connection types are commonly used.

Pin insulators are generally mounted on pins bolted directly to the cross-arms of the pole or tower. Pin insulators mounted on metal cross-arms should be provided with metal pins that have sufficient length above the cross-arm to ensure that flashover will take place to the pin rather than the cross-arm. The method of attaching suspension insulator strings varies. In one of the attachment methods, a U-bolt is fastened on the underside of the cross-arm to which the insulator hardware is attached. This would provide flexibility both longitudinally and transversely. In another method, an attachment, in the form of a bent plate or angle, is fastened to the underside of the cross-arm with a sufficient hole to receive a hook or shackle at the top of the insulator string.

Furthermore, the minimum weight that should be allowed on a supporting structure may be obtained by calculating the transverse angle to which a suspension insulator string may swing without lessening the clearance from the conductor to the structure too much and by requiring that the ratio of vertical weight to horizontal wind load be maintained such as not to permit the insulator to swing beyond this angle. The maximum wind is assumed at a temperature of 60 °F. However, the wind pressure in pounds per square feet to be applied in sag calculations is somewhat arbitrary, and it depends on local conditions. The required minimum angle of conductor swing to be used in calculations, where nearness to other circuits is involved, is 30° according to the NESC. Usually, a clearance corresponding to about 75 percent of the flashover value of the insulator is sufficient. A suspension insulator swings in the direction of the resultant of the vertical and horizontal forces affecting the insulator swing. For further information see Chapter 10.

Figure 9.19 shows a vee arrangement (V-strings) of suspension insulator strings carrying four bundled conductors per phase. Figure 9.20 shows a vee arrangement of suspension insulator strings carrying four bundled conductors per phase at 500 kV being installed. From a contamination point of view, the V-strings are more effective than vertical strings because of a "self-cleaning" possibility. Both sides of each insulator V-strings are exposed to the rain, allowing contaminants to be shed more effectively.

9.12 JOINT USE BY OTHER UTILITIES

There are advantages in the use of joint poles. However, when supporting structures of the overhead lines are used jointly by other utilities, such as telephone or other communication systems, additional factors are introduced into the problem of line design beside those needing consideration in the case of power lines alone. For example, often a higher grade of construction is necessary, and consideration must be given the required separations between the conductors and equipment of the two utilities.

The cost of providing the pole is borne jointly by the companies that share in its ownership. In general, the allocation of the expense is made in proportion to the space assigned to owners. The cost of the clearance between higher voltage and lower voltage power circuits is usually charged to the higher voltage circuits. However, the required clearance space, between power and communication circuits or between the lowest attachment and ground, is disregarded in determining percentage ownership. it is also possible that poles are used jointly under a lease agreement in which case the leasee has only the right to occupy a designated space.

In general, conductors are placed in such an order so that the higher voltage conductors are at the higher levels. As a result of this, the highest

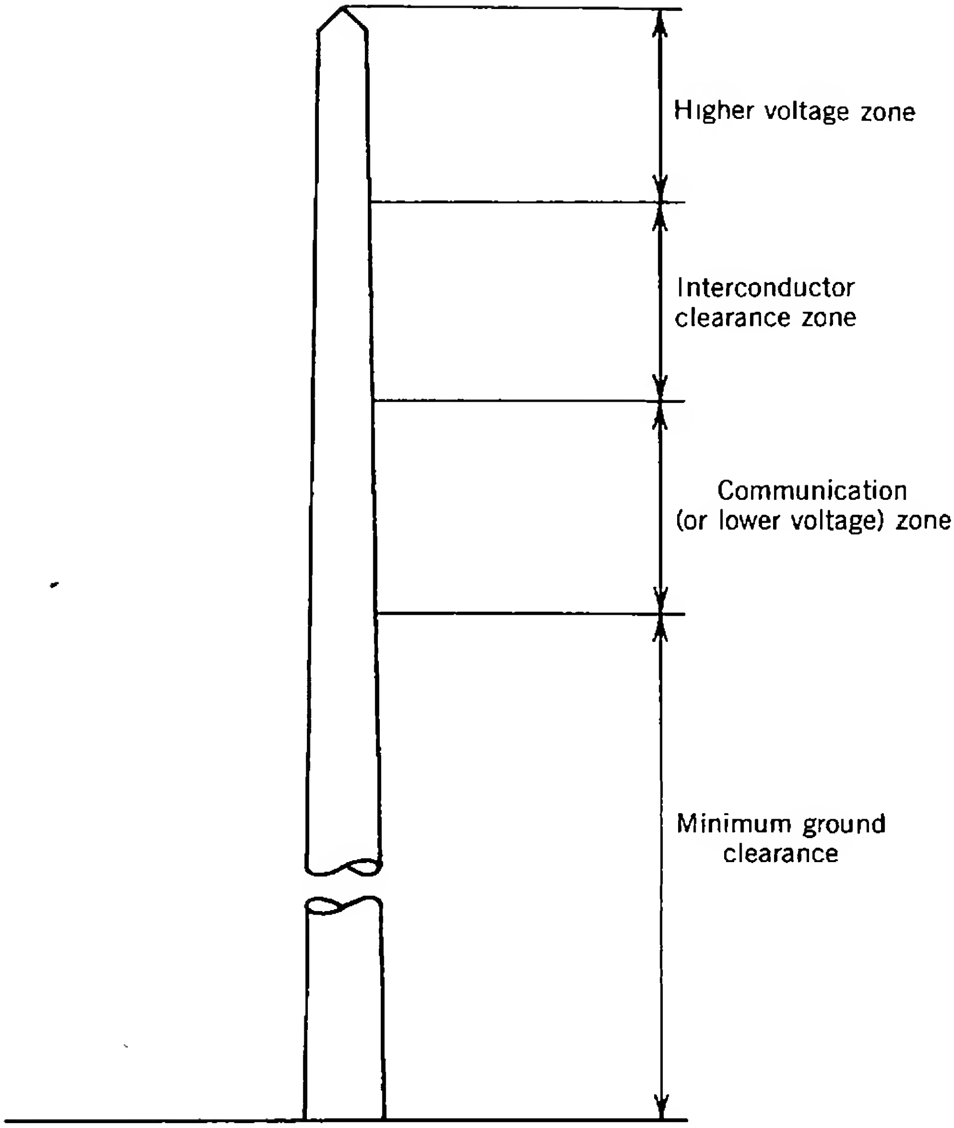


Figure 9.21. Allotment of pole space.

voltage circuits are near the pole top, and communication circuits are at a lower level, as shown in Figure 9.21.

9.13 CONDUCTOR VIBRATION

The failure of conductors under tensions that are much below maximum design stresses has been caused by fatigue due to very fast vertical vibrations of the conductor (from 15 to possibly 100 Hz) caused by steady nonturbulent winds blowing across the line. In general, the mechanical vibrations in overhead conductors and ground wires are of six types.

1. *Aeolian Vibration*. It is a resonant oscillation caused by vortex shedding from the leeward side of a conductor in a steady wind. Its amplitudes are of approximately one conductor diameter, with frequencies of oscillation on the order of 2–150 Hz. If uncorrected, these vibrations can cause chafing

and fatigue failures of conductor strands, typically at oscillation nodes such as splices and suspension points. It causes wires to hum in the wind and the *whistle of wind* in the rigging of ships. Aeolian vibrations can be controlled by adding energy-dissipating vibrations dampers (usually Stockbridge), which are attached to the conductor. Such dampers are a compound-pendulum type of arrangement that detunes the vibrating conductor and absorbs enough of the the energy to stop or greatly lessen vibration. Aeolian vibration can also be prevented by the use of armor rod and/or reduced conductor tensions, and by the use of self-damped conductors that have recently been developed as another control alternative.

2. *Swinging of Conductors Caused by Changes in Wind Pressure*. This type vibration is not harmful provided there is enough clearance left between conductors to prevent flashover.

3. *Galloping (or Dancing)*. It is usually created by a nonuniform airfoil surface formed around the conductor by ice. It can be very severe, is of a very low frequency, and is extremely difficult to control since the shapes of the ice and the wind velocity combine to result in a critical stability condition. Therefore, winds from 15 mph upwards can create violent conductor motion with amplitudes measuring up to two times the amount of conductor sag. Cases have occurred where this galloping has displaced the phase conductor all the way up to the ground wires, causing multiple trip-outs. It has also resulted in damage to conductors, spacers, and towers. Although galloping produces severe motions, it is almost always limited to areas of icing and is dependent on terrain and wind exposure. It is more common in regions (e.g., Nebraska, Iowa, etc.) where steady, moderate-velocity winds (19–35 mph) occur. Luckily, it occurs very infrequently.

4. *Conductor Ice Loading and Shedding*. Conductor icing and the subsequent shedding of ice loads can cause large vertical conductor motions (i.e., jumping). The worst jumping takes place when ice melts from the center span of a section after it has fallen from the other spans. Serious jumping also occurs when ice slips down the conductor toward midspan. The conductor jumping can be controlled by fitting special insulator assemblies at the suspension points and by increasing the mass per-unit length of line at midspan. Of course, vertical motion of the conductor due to ice shedding is dependent on span length, tension, conductor size, ice thickness, and the amount of ice shed at any one time. EPRI [6] suggests the following criteria for ice shedding on 138 kV lines:

1. Assume a maximum sag error of 6 in.
2. Assume the upper conductor has an ice load equal to 50 percent of the criterion for unequal static ice load (usually 0.5×1 in. or 0.5 in.). Ice is assumed to weight 57 lb/ft^3.

3. Assume that the lower conductor, previously with the same ice load as cited above, has already shed 25 percent.
4. Assume that the remaining 75 percent of the ice on lower conductor is shed at one time.
5. Provide sufficient initial separation to ensure that the minimum clearance during the subsequent jump is 16 in., adequate for 60 Hz withstand.

5. *Subconductor Vibration*. It is only possible on bundled conductor arrangements. On a bundled conductor, the windward conductor has a wake that spreads out its leeward side. One or more of the leeward conductors is riding in a wake that has shear flow, and different velocities of wind cross over the top and bottom of the conductor. Depending on the conductor position, this can result in either negative or positive lift. There is also a decreased drag on the leeward conductor compared with the windward conductor, tending to displace the conductors horizontally with respect to each other. Therefore, for example, in a horizontally twin-bundled conductor system, the windward conductor is exposed to the wind velocity, while the leeward conductor is exposed to the wake of the wind. The interaction of resonant forces due to the wind and mechanical coupling by spacers may cause an elliptical motion of the system. Amplitude may be in the range of 2 to 5 ft for winds 20 mph or greater. Subconductor vibration occurs at lower frequency (2–4 Hz) than aeolian vibration and is more difficult to control. It may lead to the ultimate breaking of spacers and, in some cases, destruction of suspension points at the insulators. In general, it can be controlled by the use of vibration dampers, the spacing of subconductors as far apart as practical, orienting the subconductors so that they are at advantageous points in the wind wake, and more frequent use of spacers within each span.

6. *Corona Vibration*. It is usually takes place in wet weather when water drops clinging to the underside of the conductor are forced off by an expulsion action due to the electrostatic field forces at the bottom of the conductor. Therefore, vibrational displacements of a few inches can appear between vibration nodes on any span. Corona vibration has relatively low amplitude and does not occur often; however, it may be important on the UHV lines.

With the exception of the corona vibration, the aforementioned vibration problems can also be controlled by the use of self-damped conductors that have recently been developed. For example, since two factors can cause conductor motion, namely, conductor shape and weather, Kaiser Aluminum engineers deduced that wind-induced motion can be controlled by changing the shape of the conductor. It is obvious that a round conductor presents the same profile to the wind along its entire length between structures. There-

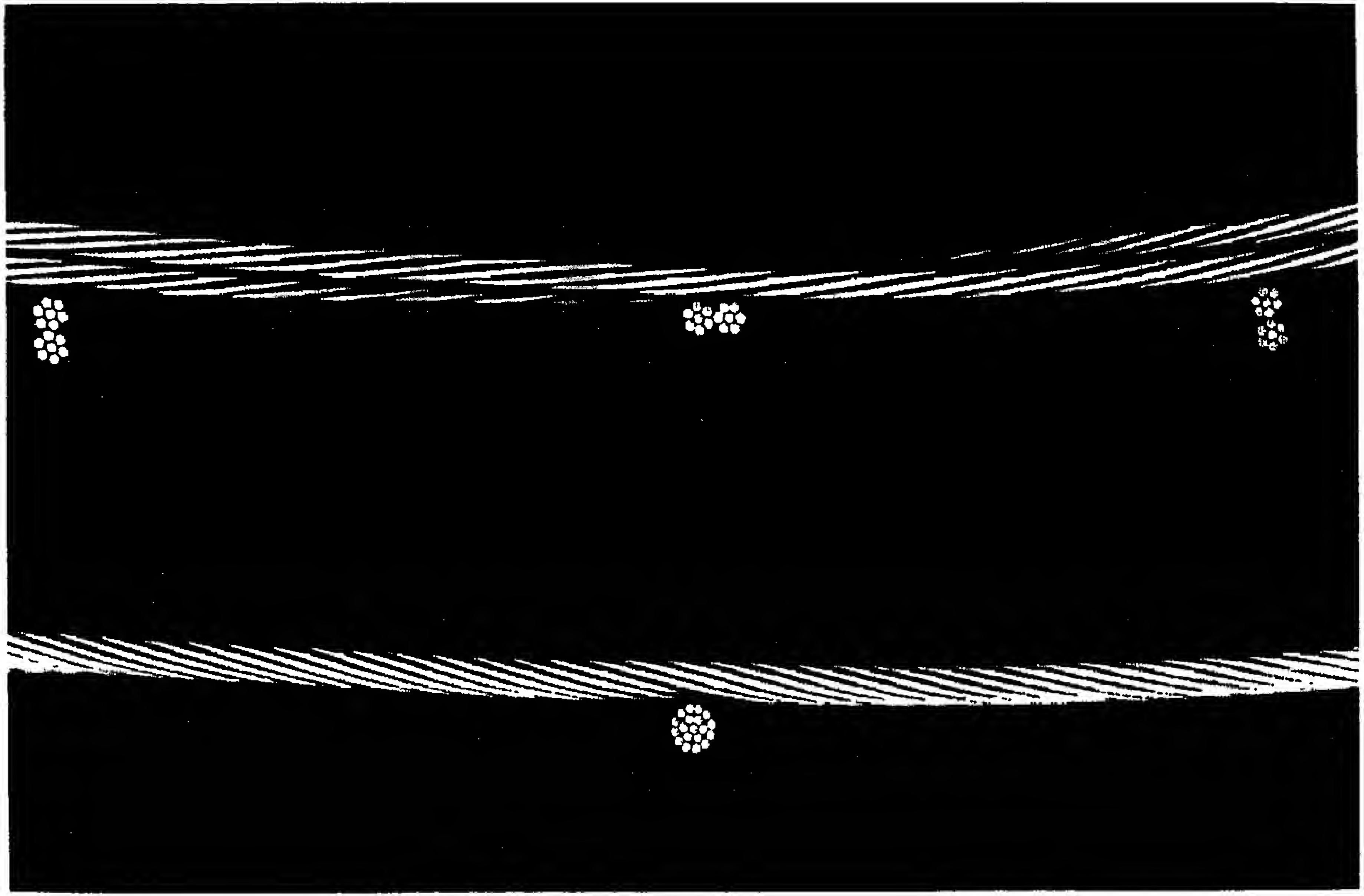


Figure 9.22. Cross-sectional comparison of round conductor and equivalent T_2 conductor, showing continually changing profile of T_2. (Courtesy of Kaiser Aluminum.)

fore, it was reasoned that a twisted double conductor would present a continually changing profile to the wind, thus preventing the buildup of resonant vibrations. Hence, under ice buildup conditions, the ice-coated figure 8 shape and constantly changing profile of the T_2 *conductor*, as shown in Figure 9.22, would act as a spoiler rather than as the airfoil shape associated with violent conductor galloping. A T_2 conductor is made up of two round aluminum conductors twisted at the factory to make one complete 360° revolution gradually over approximately every 9 ft of length. Its profile is like a figure 8. This configuration results in a continually changing orientation of major and minor axes. This ever-changing profile to the wind interferes with the wind forces that create conductor motion. Because of its profile, the T_2 conductor has a lower operating temperature than the standard round conductor of equal aluminum circular mil area. Of course, lower operating temperature mean lower operating resistance, less sag, less loss in strength, and less creep. Figure 9.23 illustrates the installation of T_2 conductors. Figure 9.24 shows a corner tower carrying T_2 conductors.

9.14 CONDUCTOR MOTION CAUSED BY FAULT CURRENTS

Two parallel and current-carrying conductors are under a force of attraction or repulsion, depending on current direction. The magnitude of the force on each conductor can be expressed as

$$F \propto \frac{I^2}{d} \tag{9.38}$$

where I = current in each conductor
d = distance (spacing) between conductors

In the event the current flow in each conductor is in the same direction, the resulting force will cause attraction. Otherwise, the force will cause repulsion. During short circuits, these forces may be great enough to cause significant conductor movement, especially where conductors are closely spaced (e.g., in EHV or UHV conductor bundles). Such conductor movement depends on the fault current magnitude and the fault duration and therefore the interruption time of the circuit breaker involved. A line-to-line fault will cause current in the two affected phases to flow in opposite directions. The two conductors will then be repelled and, on interruption of the fault current, will swing together. If the fault is on an adjacent line section, the motion may be serious since it might cause interruption on the unfaulted section. Therefore, such conductor movement should be taken into account in determining the phase-to-phase spacing or in establishing the need for insulating spacers.

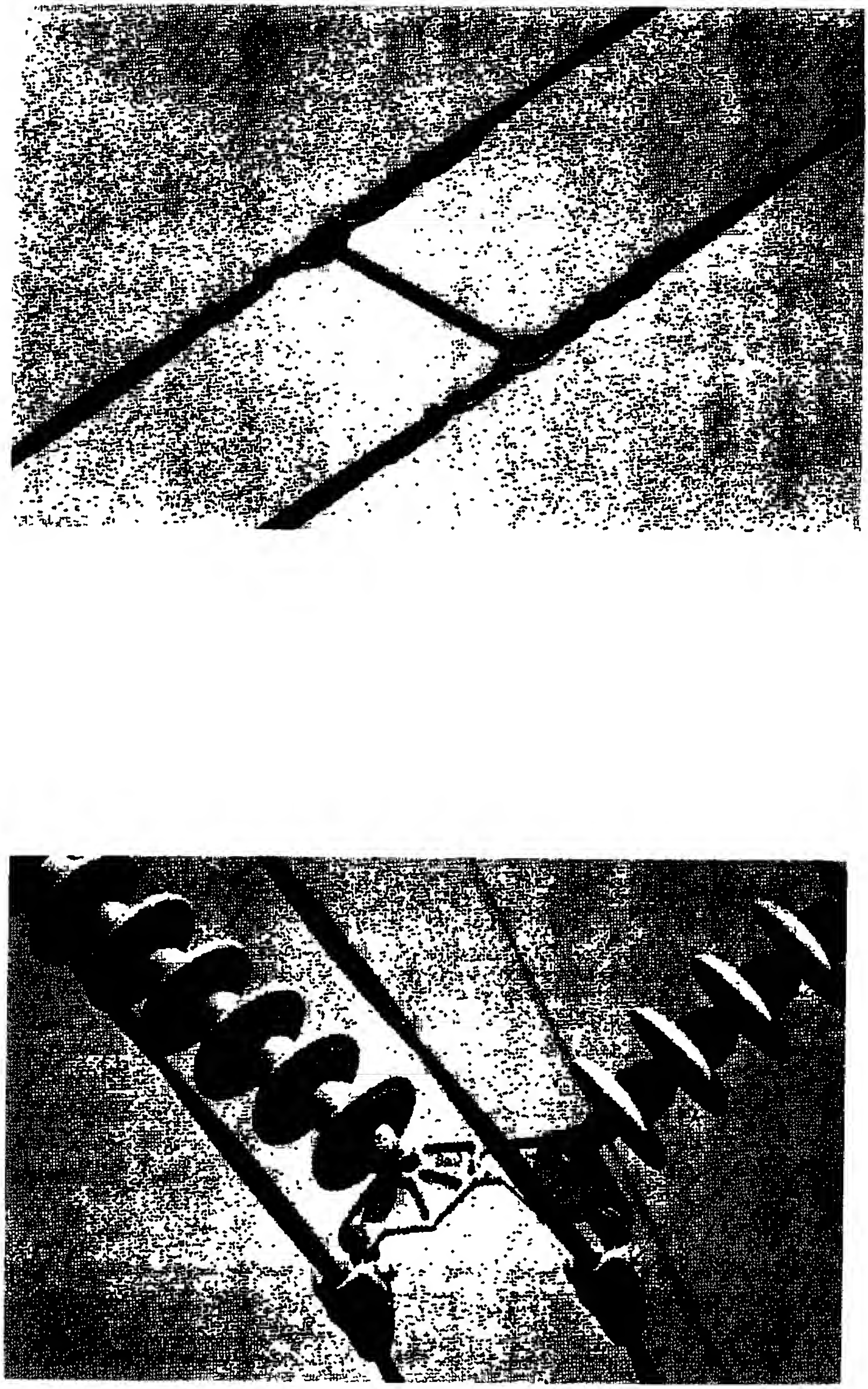
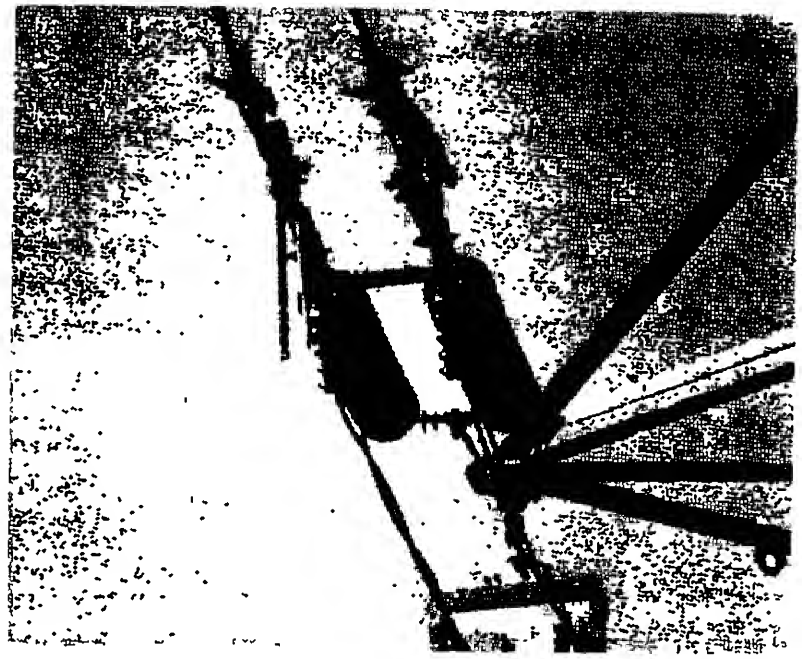


Figure 9.23. Installation of T_2 conductors. (*a*) clipping crew installing suspension clamp; (*b*) typical 345-kV two-conductor suspension clamp and yoke plate arrangement; (*c*) rigged used to dead end T_2 conductor; (*d*) strain clamp dead ends. (Courtesy of Kaiser Aluminum.)



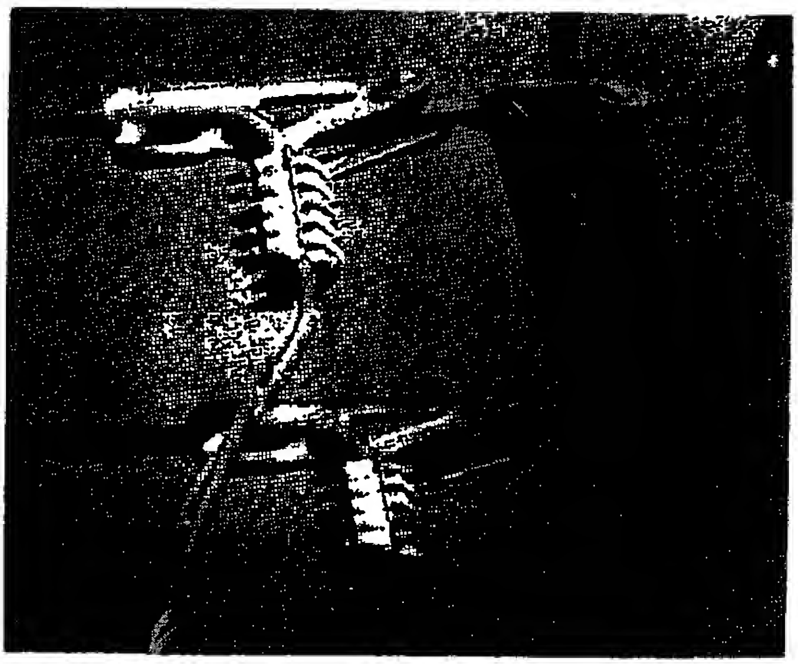


Figure 9.23 (*Continued*)

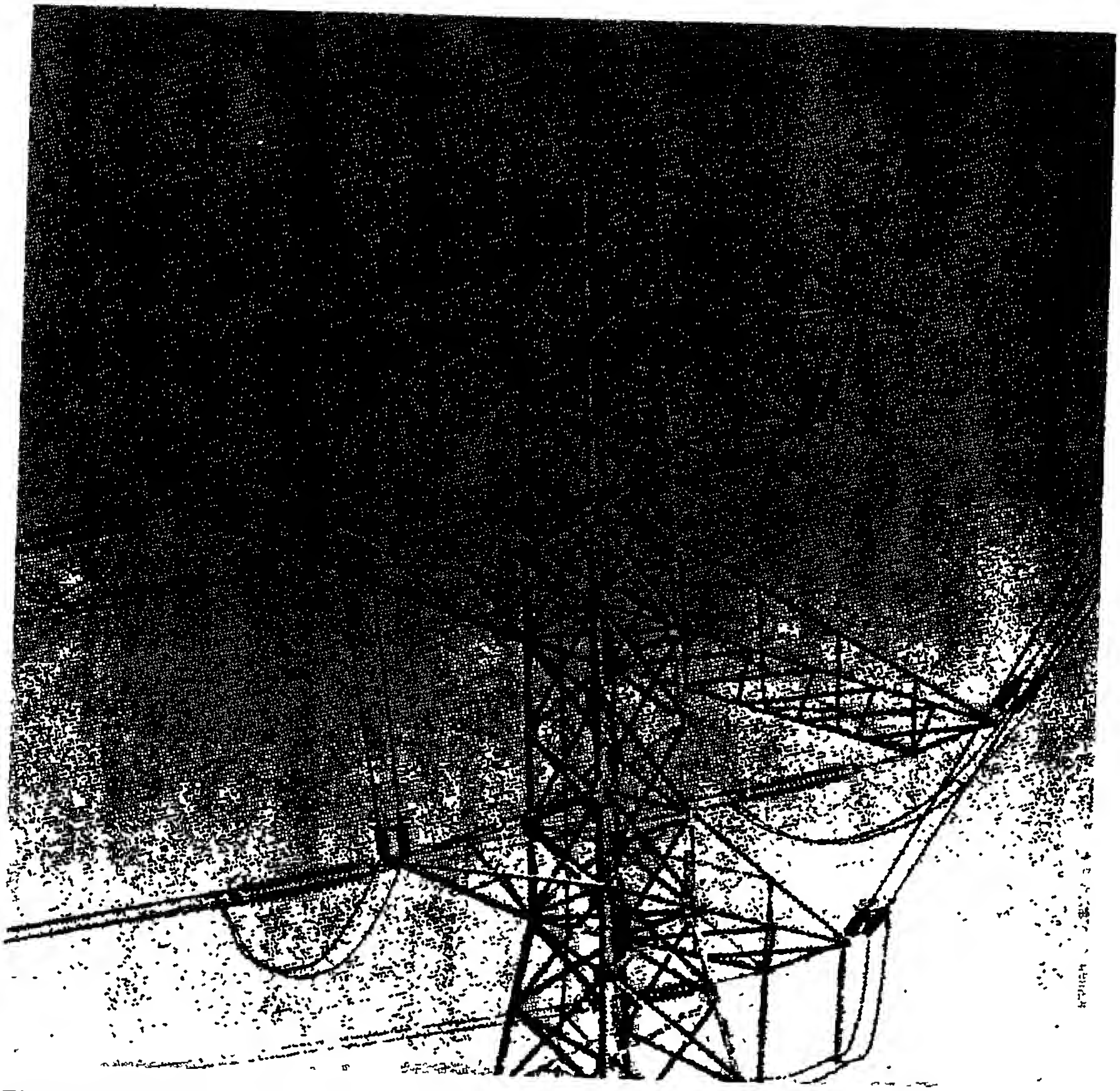


Figure 9.24. View of corner tower carrying T_2 conductors. (Courtesy of Kaiser Aluminum.)

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PROBLEMS

9.1. A 3/0 overhead conductor is subjected to a tension of 2000 lb. Find the unit tensile stress of the following conductors.

(a) Copper with seven strands.

(b) ACSR.

(c) Copperweld-copper, 3/0 E.

9.2. A heavy transformer hung on a wood cross-arm attached to a wood pole by a steel bolt sets up a shearing stress in the bolt between the cross-arm and the pole. Calculate the shearing stress in the bolt. Assume that the transformer weighs 400 lb and a $\frac{3}{4}$ in. bolt, having a cross-sectional area of 0.302 in.^{2}, fastens the cross-arm to the pole.

9.3. Assume that a western cedar pole carries two cross-arms, one of which is subjected to a normal transverse force of 450 lb at a height of 34 ft and the other 225 lb at 32 ft. Assume that the pole tapers 3 in. in circumference for each 8 ft of length and has an ultimate fiber stress rating of 5600 psi. Calculate the following:

(a) Minimum required pole diameter at ground line.

(b) Relevant pole-top diameter of pole.

(c) Permissible bending moment during storms.

9.4. Repeat Problem 9.3 if the pole used is a northern white cedar with an ultimate fiber stress rating of 3600 psi.

9.5. Assume a 46-ft pole with 24-in. circumference at the pole top and 50-in. circumference at the ground line. Use a wind velocity of 60 mph and an average span of 250 ft. The pole carries two cross-arms, and each cross-arm carries three conductors. The top cross-arm is 1 ft below the pole top, and the lower cross-arm is 2 ft below the top cross-arm. Use hard-drawn copper conductors of 250 kcmil with 12 strands and 4/0 with 7 strands for the top and lower conductors, respectively. Assume that 6.5 ft of the pole is below the ground and find the following:

(a) Total pressure due to wind on pole.

(b) Total pressure due to wind on conductors.

(c) Total pressure due to wind on pole and conductors.

9.6. Assume that the conductors in the adjacent spans have equal tensions of 400 lb and the angle of departure of the line is 46°. The pole, which carries the conductors of the adjacent spans, has a cross-arm with four conductors. Calculate the additional stress imposed upon the supporting pole due to the angle in the line.

9.7. If the conductors in the adjacent spans of Problem 9.6 have different tensions of 400 and 300 lb for T_1 and T_2 and the angle of departure of the line is 46° as before, calculate the following:

(a) Resultant side pull force.

(b) Angle γ.

9.8. Repeat Problem 9.6 assuming the angle of departure of the line is 80°.

9.9. Consider Problem 9.5. Assume that the conductors in the adjacent spans have different tensions of 300 and 350 lb and the angle of departure of the line is 40°. Also assume that the permissible stress in pole is 50 and 60 percent of ultimate fiber strength for transverse loading and longitudinal loading, respectively. Find the required internal resisting moment of the pole in pound feet.

9.10. Find the allowable maximum angle of departure of the line, without any side guying, for the angle pole of Problem 9.9 if the pole is to be set in firm soil.

9.11. Repeat Example 9.4 if the bending moments are created by 4000 and 3000 lb at heights of 45 and 40 ft, respectively. The guy is attached to the pole at the heights of 45 ft. The lead of the guy is 20 ft.

9.12. Consider Figure 9.12 and assume that $T_2 \neq T_1$ and verify equation (9.19).

10

SAG AND TENSION ANALYSIS

10.1 INTRODUCTION

Conductor sag and tension analysis is an important consideration in overhead distribution line design as well as in overhead transmission line design. The quality and continuity of electric service supplied over a line (regardless of whether it is a distribution, a subtransmission, or a transmission line) depend largely on whether the conductors have been properly installed. Thus, the designing engineer must determine in advance the amount of sag and tension to be given the wires or cables of a particular line at a given temperature. In order to specify the tension to be used in stringing the line conductors, the values of sag and tension for winter and summer conditions must be known. Tension in the conductors contributes to the mechanical load on structures at angles in the line and at dead ends. Excessive tension may cause mechanical failure of the conductor itself.

The factors affecting the sag of a conductor strung between supports are:

1. Conductor load per unit length.
2. Span, that is, distance between supports.
3. Temperature.
4. Conductor tension.

In order to determine the conductor load properly, the factors that need to be taken into account are:

1. Weight of conductor itself.
2. Weight of ice or snow clinging to wire.
3. Wind blowing against wire.

The maximum effective weight of the conductor is the vector sum of the vertical weight and the horizontal wind pressure. It is very important to include the most adverse condition. The wind is considered to be blowing at right angles to the line and to act against the projected area of the conductor, including the projected area of ice or snow that may be clinging to it.

Economic design dictates that conductor sag should be minimum to refrain from extra pole height, to provide sufficient clearance above ground level, and to avoid providing excessive horizontal spacing between conductors to prevent them swinging together in midspan.

Conductor tension pulls the conductor up and decreases its sag. At the same time, tension elongates the conductor, from elastic stretching, which tends to relieve tension and increase sag. The elastic property of metallic wire is measured by its modulus of elasticity. The modulus of elasticity of a material equals the stress per unit of area divided by the deformation per unit of length. That is, since

$$\sigma = \frac{T}{A} \quad \text{psi} \tag{10.1}$$

where σ = stress per unit area in pounds per square inches
T = conductor tension in pounds
A = actual metal cross section of conductor in square inches, in.^{2} = cmil/1,273,000

The resultant elongation e of the conductor due to the tension is

$$e = \frac{\text{stress}}{\text{modulus of elasticity}}$$

Of course, if the modulus of elasticity is low, the elongation is high, and vice versa. Thus, a small change in conductor length has a comparatively large effect on conductor sag and tension.

Sags and stresses in conductors are dependent on the initial tension put on them when they are clamped in place and are due to the weight of the conductors themselves, to ice or sleet clinging to them, and to wind pressure.

The stress in the conductor is the parameter on which everything else is based. But the stress itself is determined by the sag in the conductor as it hangs between adjacent poles or towers. Since the stress depends on sag, any span can be used provided the poles or towers are high enough and strong enough. The matter is merely one of extending the catenary in both directions. But the cost of poles or towers sharply increases with height and loading. Thus, the problem becomes the balancing of a larger number of lighter and shorter poles or towers against a smaller number of heavier and taller ones.

10.2 EFFECT OF CHANGE IN TEMPERATURE

Sags and stresses vary with temperature on account of the thermal expansion and contraction of the conductors. A temperature rise increases conductor length, with resulting increase in sag and decrease in tension. A temperature drop causes reverse effects. The change in length per unit of conductor length per degree fahrenheit of temperature change is the temperature coefficient of linear expansion. The maximum stress occurs at the lowest temperature, when the line has contracted and is also possibly covered with ice and sleet.

1. If the conductor *unstressed* or the conductor stress is *constant* while the temperature changes, the change in length of the conductor is

$$\Delta l = l_0 \alpha \, \Delta t \tag{10.2}$$

where

$$\Delta t = t_1 - t_0 \qquad \Delta l = l_1 - l_0$$

where t_0 = initial temperature
l_0 = conductor length at initial temperature t_0
l_1 = conductor length at t_1
α = coefficient of linear expansion of conductor per degree fahrenheit
Δt = change in temperature
Δl = change in conductor length in feet

2. If the temperature is *constant* while the conductor stress changes (i.e. loading), the change in length of the conductor is

$$\Delta l = l_0 \frac{\Delta T}{MA} \tag{10.3}$$

where

$$\Delta T = T_1 - T_0$$

where T_0 = conductor initial tension in pounds
ΔT = change in conductor tension in pounds
M = modulus of elasticity of conductor in pound inches
A = actual metal cross section of conductor in square inches

10.3 LINE SAG AND TENSION CALCULATIONS

A conductor suspended freely from two supports, which are at the same level and spaced L unit length apart, as shown in Figure 10.1, takes the form of a catenary curve providing the conductor is perfectly flexible and its weight is uniformly distributed along its length. If the conductor is tightly stretched (i.e., when sag d is very small in comparison to span L), the resultant curve can be considered a parabola. If the conductor's sag is less than 6 percent of its span length, the error in sag computed by the parabolic equations is less than 0.5 percent. If the conductor's sag is less than 10 percent of the span, the error is about 2 percent.

In distribution systems, determining accurate values of sag is not so important as it is in transmission systems. Nevertheless. even in the distribution lines, if the conductor is strung with too low tension, the resultant sag will be excessive, with the likelihood of wires swinging together and short-circuited. The usual tendency, however, is to pull the conductor too tight, which causes the conductor to be overstressed and stretched when the heaviest loading takes place and the normal sag after this loading becomes excessive. Then the excessive sag needs to be pulled out of the conductor, a process that also causes the conductor to be overstressed on heaviest loading. This process of overstressing and pulling up may cause the conductors, especially the smaller ones, to be broken. This can be eliminated by measuring the line tension more accurately.

10.3.1 Supports at Same Level

Catenary Method

Figure 10.1 shows a span of conductor with two supports at the same level and separated by a horizontal distance L. Let 0 be the lowest point on the catenary curve and 1 be the length of the conductor between two supports. Let w be the weight of the conductor per unit length, T be the tension of the conductor at any point P in the direction of the curve, and H be the tension at origin 0. Further, let s be the length of the curve between points 0 and P, so that the weight of the portion s is ws.

Tension T can be resolved into two components, T_x the horizontal component and T_y the vertical component. Then, for equilibrium,

$$T_x = H \quad \text{and} \quad T_y = ws$$

Thus, the portion OP of the conductor is in equilibrium under the tension T at P, the weight ws acting vertically downward, and the horizontal tension H.

In the triangle shown in Figure 10.2, ds represents a very short portion of the conductor, in the region of point P. When s is increased by ds, the corresponding x and y are increased by dx and dy, respectively. Hence,

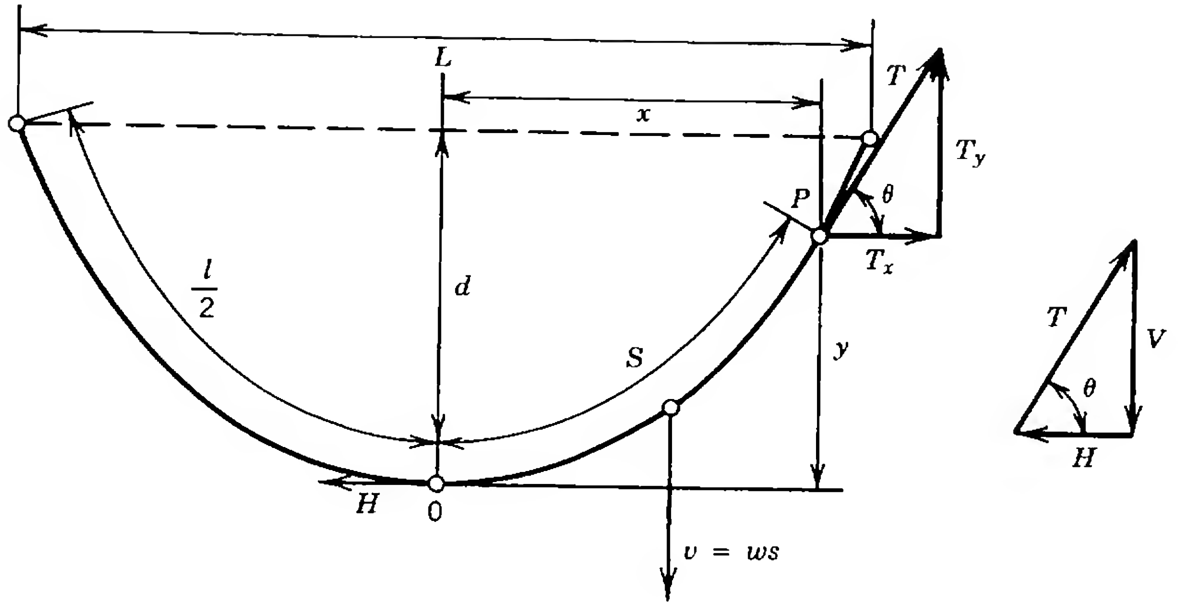

Figure 10.1. Conductor suspended between supports at same elevation.

$$\tan\theta = \frac{dy}{dx} = \frac{ws}{H}$$

since

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{dy}{dx}\right)^2$$

then

$$\left(\frac{ds}{dx}\right)^2 = 1 + \left(\frac{ws}{H}\right)^2$$

Therefore,

$$dx = \frac{ds}{\sqrt{1 + (ws/H)^2}}$$

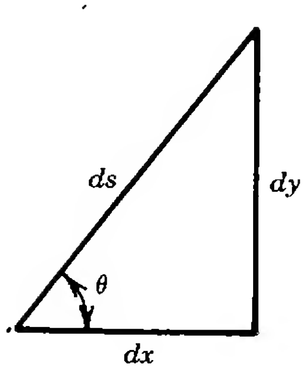

Figure 10.2

Integrating both sides gives

$$x = \int \frac{1}{\sqrt{1 + (ws/H)^2}}\, ds$$

Therefore,

$$x = \frac{H}{w} \sinh^{-1} \frac{ws}{H} + K$$

where K is the constant of integration. When $x = 0$, $s = 0$, and $K = 0$,

$$s = \frac{H}{w} \sinh \frac{wx}{H} \tag{10.4}$$

When $x = \frac{1}{2}L$,

$$s = \frac{l}{2} = \frac{H}{w} \sinh \frac{wL}{2H} \tag{10.5}$$

Therefore,

$$l = \frac{2H}{w} \sinh \frac{wL}{2H} \tag{10.6}$$

or

$$l = \frac{2H}{w} \left[\frac{1}{1!} \frac{wL}{2H} + \frac{1}{3!} \left(\frac{wL}{2H} \right)^3 + \cdots \right] \tag{10.7}$$

or approximately

$$l \cong L\left(1 + \frac{w^2L^2}{24H^2}\right) \tag{10.8}$$

From equations (10.3) and (10.4),

$$\frac{dy}{dx} = \frac{ws}{H} = \sinh \frac{wx}{H}$$

or

$$dy = \sinh \frac{wx}{H}\, dx$$

Integrating both sides,

$$y = \int \sinh \frac{wx}{H}\, dx$$

or

$$y = \frac{H}{w} \cos \frac{wx}{H} + K_1 \tag{10.9}$$

If the lowest point of the curve is taken as the origin, when $x = 0$, $y = 0$, then $K_1 = -H/w$, since, by the series, $\cosh 0 = 1$. Therefore,

$$y = \frac{H}{w} \left(\cosh \frac{wx}{H} - 1 \right) \tag{10.10}$$

is the equation of the curve that is called a catenary. Equation (10.10) can also be written as

$$y = \frac{H}{w} \left[1 + \frac{1}{2!} \left(\frac{wx}{H} \right)^2 + \cdots - 1 \right] \tag{10.11}$$

or in approximate form,

$$y \cong \frac{wx^2}{2H} \tag{10.12}$$

The total tension in the conductor at any point x is

$$T = H \sqrt{1 + \left(\frac{dy}{dx} \right)^2}$$

or

$$T = H \cos \frac{wx}{H} \tag{10.13}$$

whereas the total tension in the conductor at the support is

$$T = H \cos \frac{wL}{2H} \tag{10.14}$$

or

$$T = H \left[1 + \frac{1}{2!} \left(\frac{wL}{2H} \right)^2 + \frac{1}{4!} \left(\frac{wL}{2H} \right)^4 + \cdots \right] \tag{10.15}$$

The sag, or deflection, of the conductor for a span of length L between supports on the same level is

$$d = \frac{H}{w} \left(\cosh \frac{wL}{2H} - 1 \right) \tag{10.16}$$

or

$$d = \frac{L}{2}\left[\frac{1}{2}\,\frac{wL}{2H} + \frac{1}{4!}\left(\frac{wL}{2H}\right)^3 + \frac{1}{6!}\left(\frac{wL}{2H}\right)^5 + \cdots\right] \tag{10.17}$$

The National Electric Safety Code (NESC) gives the minimum (required) clearance height for the line above ground, and if to this is added the sag, the minimum height of the insulator support points can be found.

In Figure 10.3, it can be observed that c is the ordinate of the lowest point of the curve with respect to the directrix and y is the ordinate of the point of tangency with respect to the directrix. The form of the curve depends on the slope of the conductor at a support. In turn, the slope itself is the factor to determine the conductor tension. As previously mentioned, the tension T can be resolved into two components, T_x and T_y, where

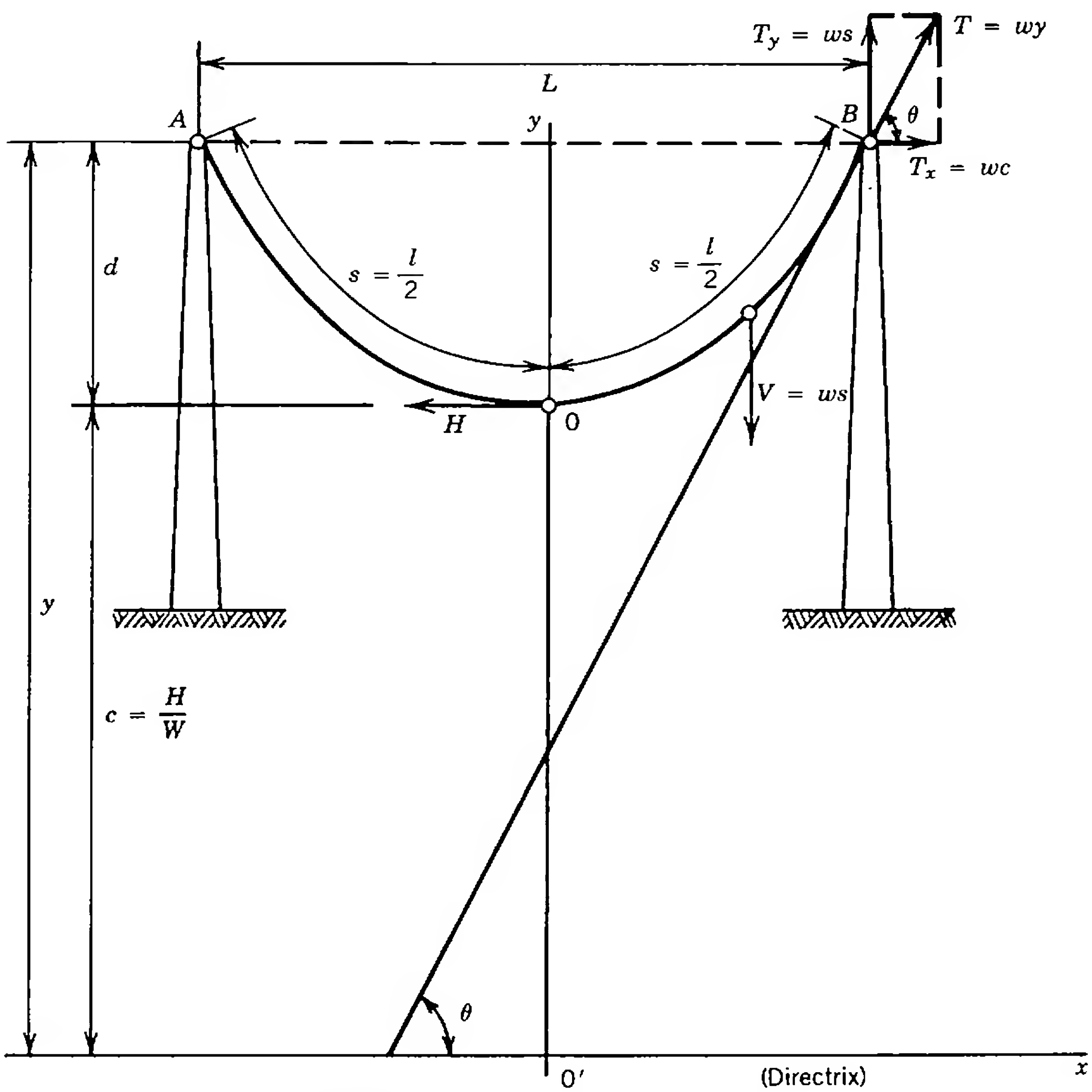

Figure 10.3. Parameters of catenary.

$$T_y = \frac{1}{2} w$$

or if $s = \frac{1}{2}l$,

$$T_y = ws \tag{10.18}$$

$$T_x = wc \tag{10.19}$$

where T_x can be defined as the mass of some unknown length c of the conductor and T_y and T can also be defined similarly. Then for equilibrium,

$$T_x = H \quad \text{and} \quad T_y = V$$

where H = horizontal tension in conductor at origin 0
V = weight of wire per foot of span times distance from point of maximum sag to support

Thus, from the triangle of forces,

$$T^2 = H^2 + V^2$$

so that

$$T = \sqrt{H^2 + V^2} \tag{10.20}$$

or

$$T = \sqrt{(wc)^2 + (ws)^2}$$

from which

$$T = w\sqrt{c^2 + s^2} \tag{10.21}$$

Equation (10.4) can be written as

$$s = c\left(\sinh \frac{x}{c}\right) \tag{10.22}$$

since $c = H/w$. Also, equation (10.9) can be written as

$$y = c\left(\cosh \frac{x}{c}\right) + K_1 \tag{10.23}$$

where x is half of the span length $(L/2)$. From Figure 10.3, when $x = 0$, equation (10.23) becomes

$$c = c(\cosh 0) + K_1$$

Thus, $K_1 = 0$, and therefore,

$$y = c\left(\cosh \frac{x}{c}\right) \tag{10.24}$$

If both sides of equations (10.22) and (10.24) are squared,

$$s^2 = c^2\left(\sinh^2 \frac{x}{c}\right) \tag{10.25}$$

and

$$y^2 = c^2\left(\cosh^2 \frac{x}{c}\right) \tag{10.26}$$

Subtracting equation (10.26) from equation (10.25),

$$y^2 - s^2 = c^2\left[\cosh^2 \frac{x}{c} - \sinh^2 \frac{x}{c}\right]$$

or

$$y^2 - s^2 = c^2 \tag{10.27}$$

since

$$\cosh^2 \frac{x}{c} - \sinh^2 \frac{x}{c} = 0$$

From equation (10.27),

$$y = \sqrt{c^2 + s^2} \tag{10.28}$$

By substituting equation (10.28) into equation (10.21),

$$T_{\max} = wy \tag{10.29}$$

Also

$$T_{\max} = w\sqrt{c^2 + s^2} \tag{10.30}$$

Thus, according to equation (10.29), the maximum tension T occurs at the supports where the conductor is at an angle to the horizontal whose tangent is V/H, or s/c, since

$$V = ws \quad \text{and} \quad H = wc$$

At supports,

$$y = c + d \tag{10.31}$$

thus, equation (10.28) can be written as

$$c + d = \sqrt{c^2 + s^2}$$

from which

$$c = \frac{s^2 - d^2}{2d} \tag{10.32}$$

Also, equation (10.29) can be written as

$$T_{\max} = w(c + d) \tag{10.33}$$

Substituting equation (10.32) into equation (10.33),

$$T_{\max} = \frac{w}{2d}(s^2 + d^2) \tag{10.34}$$

which gives the maximum value of the conductor tension.

A line tangent to the conductor is horizontal at the points of maximum sag and has the greatest angle from the horizontal at the supports. Since the supports are the same level, the weight of the conductor in one half span on each side is supported at each pole. At midspan, or point of maximum sag, the vertical component of tension equals zero. Thus, the minimum tension occurs at the point of maximum sag. The tension at this point (the point at which $y = c$) acts in a horizontal direction and equals the horizontal component of total tension. Therefore,

$$T_{\min} = H$$

or since $H = wc$,

$$T_{\min} = wc \tag{10.35}$$

or

$$T_{\min} = \frac{w}{2d}(s^2 - d^2) \tag{10.36}$$

Also from Figure 10.3,

$$y = \tfrac{1}{2}c(e^{x/c} + e^{-x/c}) \tag{10.37}$$

or

$$y = c\left(\cosh \frac{x}{c}\right) \tag{10.38}$$

where

$$C = \frac{H}{w}$$

Also,

$$d = y - c \quad \text{or} \quad c = y - d \tag{10.39}$$

The conductor length is

$$l = 2s$$

or

$$l = 2[\tfrac{1}{2}c(e^{x/c} - e^{-x/c}] \tag{10.40}$$

or

$$l = 2c\left(\sinh \frac{x}{c}\right) \tag{10.41}$$

Another useful equation can be developed by substituting $c = H/w$ into equation (10.33),

$$T_{\text{max}} = H + wd \tag{10.42}$$

or

$$T_{\text{max}} = T_{\text{min}} + wd \tag{10.43}$$

since

$$T_{\text{min}} = H$$

Parabolic Method

In the case of short spans with small sags, the curve can be considered as a parabola. When the horizontal tension is the same, the radius of curvature at the lowest point of the conductor is the same for both the parabola and the catenary, but the outlines of the two curves are different at all points between the lowest point of the curve and the point of support. As the span length and sag are increased, the difference in outline of the two curve becomes more significant. Changes in the loading will produce different changes in the length of the two curves to make the sags different.

Since the sag by the parabola solution is smaller than the sag by the catenary solution for the same horizontal tension, the angle θ will be smaller. Thus, the vertical component of tension is smaller for the parabola solution than for the catenary. This difference increases along the curve toward the support, becoming maximum at the supports. However, for the sake of simplicity, the following assumptions are made:

1. The tension is considered uniform throughout the span, the slight excess of tension at the supports over that in the middle being neglected.
2. The change in length of the conductor due to elastic stretch or temperature expansion is taken as equal to the change of length of a conductor equal in length to the horizontal distance between the points of supports.

In Figure 10.4, let P be any point on the parabolic curve such that arc $0P$ is equal to x. The portion $0P$ of the conductor is in equilibrium under the action of T, H, and wx. As previously done, the tension T can be resolved into two components, T_x and T_y. Then, for equilibrium,

$$T_x = H \quad \text{and} \quad T_y = wx$$

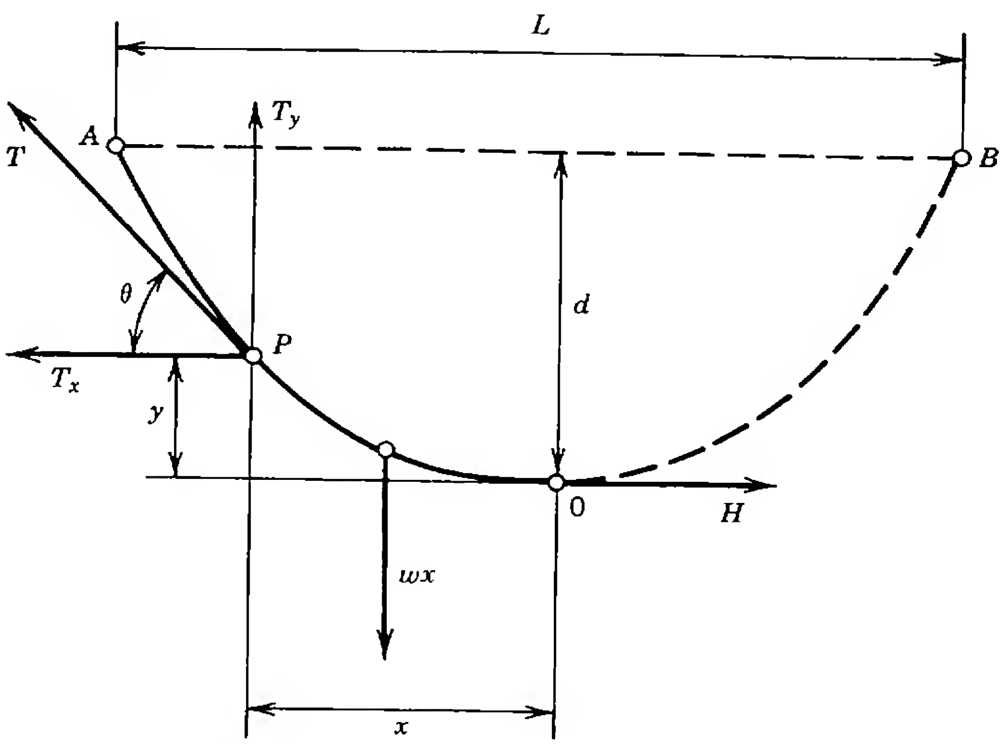

Figure 10.4. Parameters of parabola.

Taking moments about P,

$$\text{moments clockwise} = \text{moments anticlockwise}$$

or

$$H_y = wx\,\frac{x}{2}$$

from which

$$y = \frac{wx^2}{2H} \tag{10.44}$$

Distribution lines usually have comparatively short spans with small sag. The difference between the maximum tension T and the horizontal tension H is relatively small because of short spans and small sags. Under such conditions, a slight error will result if T is substituted for H in the equation for sag. Therefore,

$$y = \frac{wx^2}{2T} \tag{10.45}$$

When $x = \frac{1}{2}L$, y is equal to sag or deflection d; therefore, the sag is

$$d = \frac{wL^2}{8T} \tag{10.46}$$

It is interesting to note that if the terms $(wL/2H)^3$ and $(wL/2H)^4$ are omitted in equation (10.17), as is appropriate for spans of only a few hundred feet, the very same equation (10.46) can be obtained.

After replacing T with H in equation (10.46), if it is combined with equation (10.8), the following equation can be derived for the conductor length (i.e., the perimeter of the conductor in the span):

$$l = L\left(1 + \frac{8d^2}{3L^2}\right) \tag{10.47}$$

EXAMPLE 10.1

A subtransmission line conductor has been suspended freely from two towers and has taken the form of a catenary that has $c = 1600$ ft. The span between the two towers is 500 ft, and the weight of the conductor is 4122 lb/mi.

Calculate the following:

(a) Length of conductor by using equations (10.6) and (10.8).

(b) Sag.

(c) Maximum and minimum values of conductor tension using catenary method.

(d) Approximate value of tension by using parabolic method.

Solution

(a) Using equation (10.6),

$$l = \frac{2H}{w} \sinh \frac{wL}{2H}$$

or

$$l = 2c\left(\sinh \frac{L}{2c}\right)$$

since

$$c = \frac{H}{w}$$

Therefore,

$$\begin{aligned} l &= 2 \times 1600 \sinh \frac{500}{2 \times 1600} \\ &= 3200 \sinh 0.15625 \\ &= 502.032 \text{ ft} \end{aligned}$$

Using equation (10.8),

$$l = L\left(1 + \frac{w^2L^2}{24H^2}\right)$$

or

$$\begin{aligned} l &= L\left(1 + \frac{L^2}{24c^2}\right) \\ &= 500\left(1 + \frac{500^2}{24 \times 1600^2}\right) \\ &= 502.0345 \text{ ft} \end{aligned}$$

(b) Using equation (10.16),

$$d = \frac{H}{w}\left(\cosh \frac{wL}{2H} - 1\right)$$

or

$$d = c\left(\cosh \frac{L}{2c} - 1\right)$$

since

$$c = \frac{H}{w}$$

Therefore,

$$\begin{aligned} d &= 1600\left(\cosh \frac{500}{2 \times 1600} - 1\right) \\ &= 1600(\cosh 0.15625 - 1) \\ &\cong 19.6 \text{ ft} \end{aligned}$$

(c) Using equation (10.33),

$$\begin{aligned} T_{\max} &= w(c + d) \\ &= \frac{4122}{5280}(1600 + 19.6) \\ &\cong 1264.4 \; lb \end{aligned}$$

Using equation (10.35),

$$\begin{aligned} T_{\min} &= wc \\ &= \frac{4122}{5280} \times 1600 \\ &\cong 1249.1 \text{ lb} \end{aligned}$$

(d) From equation (10.46),

$$\begin{aligned} T &\cong \frac{wL^2}{8d} \\ &\cong \frac{(4122/5280) \times 500^2}{8 \times 19.6} \\ &\cong 1244.7 \text{ lb} \end{aligned}$$

10.3.2 Supports at Different Levels: Unsymmetrical Spans

Consider a span L between two supports, as shown in Figure 10.5, whose elevations differ by a distance h. Let the horizontal distance from the lowest point of the curve to the lower and the higher supports be x_1 and x_2, respectively.

By using equation (10.46), that is,

$$y = \frac{wx^2}{2T}$$

d_1 and d_2 sags can be found as

$$d_1 = \frac{wx^2}{2T} \tag{10.48}$$

and

$$d_2 = \frac{wx_2^2}{2T} \tag{10.49}$$

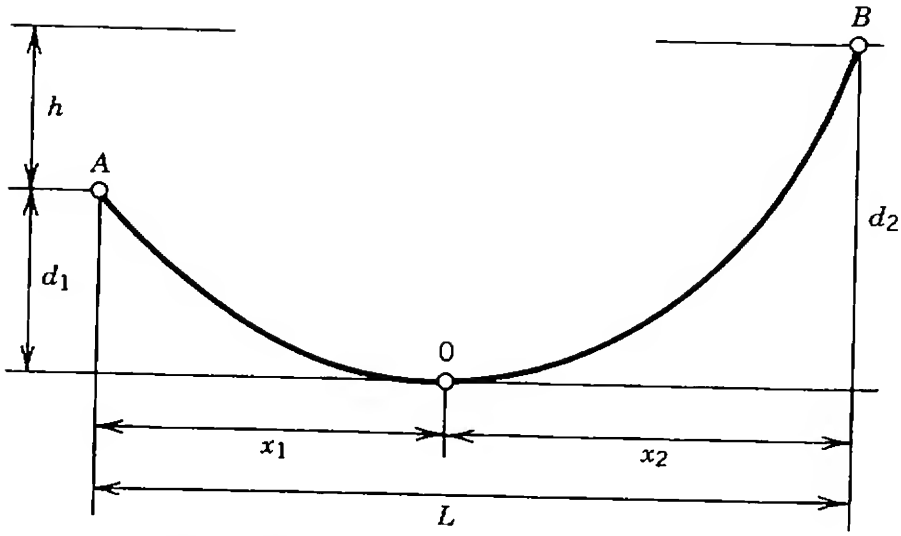

Figure 10.5. Supports at different levels.

Therefore,

$$h = d_2 - d_1 \tag{10.50}$$

or

$$h = \frac{w}{2T}(x_2^2 - x_1^2) \tag{10.51}$$

or

$$h = \frac{wL}{2T}(x_2 - x_1) \tag{10.52}$$

since

$$L = x_1 + x_2 \tag{10.53}$$

Therefore,

$$\frac{2Th}{wL} = x_2 - x_1 \tag{10.54}$$

By adding equations (10.53) and (10.54),

$$2x_2 = L + \frac{2Th}{wL}$$

or

$$x_2 = \frac{L}{2} + \frac{Th}{wL} \tag{10.55}$$

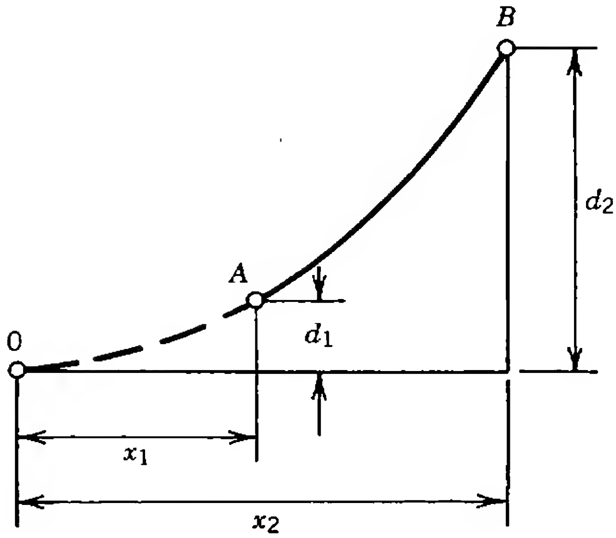

Figure 10.6. Case of negative x_1.

Also by subtracting equation (10.54) from equation (10.53),

$$2x_1 = L - \frac{2Th}{wL}$$

or

$$x_1 = \frac{L}{2} - \frac{Th}{wL} \tag{10.56}$$

In equation (10.56),

$$\text{if } \frac{L}{2} > \frac{Th}{wL}, \quad \text{then } x_1 \text{ is positive}$$
$$\text{if } \frac{L}{2} = \frac{Th}{wL}, \quad \text{then } x_1 \text{ is zero}$$
$$\text{if } \frac{L}{2} < \frac{Th}{wL}, \quad \text{then } x_1 \text{ is negative}$$

If x_1 is negative, the lowest point (the point 0) of the imaginary curve lies outside the actual span, as shown in Figure 10.6.

10.4 SPANS OF UNEQUAL LENGTH: RULING SPAN

When a line consists of spans of unequal length, each span should theoretically be tensioned according to its own length. However, this is not possible with suspension insultors since the insulator strings would swing so as to equalize the tension in each span. It is impractical to dead-end and erect each span separately. However, it is possible to assume a uniform tension between dead-end supports by defining an equivalent span, which is called a *ruling span*,[†] and basing all the calculations on this equivalent span.

If the actual spans are known, the ruling span can be calculated from the equation

$$L_e = \sqrt{\frac{L_1^3 + L_2^3 + L_2^3 + \cdots + L_n^3}{L_1 + L_2 + L_3 + \cdots + L_n}} \tag{10.57}$$

where L_e = ruling span or equivalent span
L_i = each individual span in line

Generally, an exact value of the ruling span is not necessary. An approximate ruling span can be calculated as

$$L_e = L_{\text{avg}} + \frac{2}{3}(L_{\max} - L_{\text{avg}}) \tag{10.58}$$

where L_{avg} = average span in line
$L_{\max}$ = maximum span in line

The line tension T can be estimated using this equivalent span length, and then the sag for each actual span can be calculated from

$$d = \frac{wL^2}{8T} \tag{10.59}$$

10.5 EFFECTS OF ICE AND WIND LOADING

The span design consists in determining the sag at which the line is constructed so that heavy winds, accumulations of ice or snow, and excessive temperature changes will not stress the conductor beyond its elastic limit, cause a serious permanent stretch, or result in fatigue failures from continued vibrations. In other words, the lines will be erected under warmer and nearly still-air conditions and yet must comply with the worst conditions.

10.5.1 Effect of Ice

In mountainous geographic areas, the thickness of ice formed on the conductor becomes very significant. Depending on the circumstances, it might be as much as several times the diameter of the conductor. Ice accumulations on the conductor affect the design of the line (1) by increasing the dead weight per foot of the line and (2) by increasing the projected surface of the line subject to wind pressure.

[†] Mostly used for distribution lines.

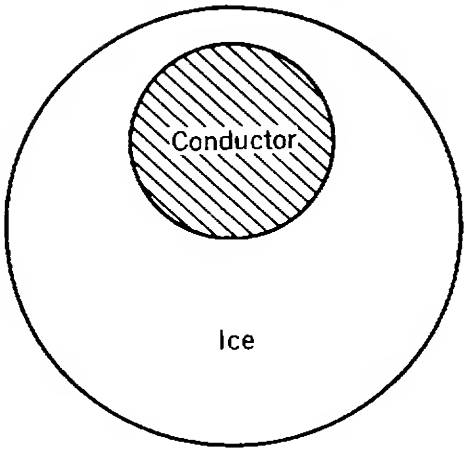

Figure 10.7. Probable configuration of ice covered conductor cross-sectional area.

Even though the more likely configuration of a conductor with a coating of ice is as shown in Figure 10.7, for the sake of simplicity, it can be assumed that the ice coating, of thickness t_i inches, is uniform over the surface of a conductor, as shown in Figure 10.8.

Then the cross-sectional area of the ice is

$$A_i = \tfrac{1}{4}\pi[(d_c + 2t_i)^2 - d_c^2]$$

or

$$A_i = \pi t_i(d_c + t_i) \quad \text{in.}^2 \tag{10.60}$$

or

$$A_i = \tfrac{1}{144}\pi t_i(d_c + t_i) \quad \text{ft}^2 \tag{10.61}$$

where d_c = diameter of conductor in inches
t_i = radial thickness of ice coating in inches

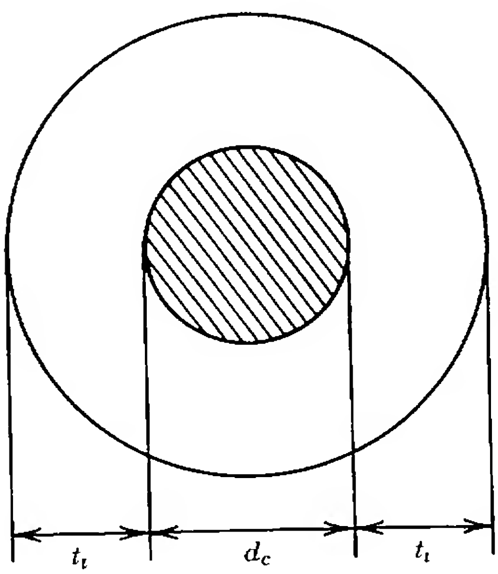

Figure 10.8. Assumed configuration of ice-covered conductor cross-sectional area.

If the ice load is assumed to be uniform throughout the length of the conductor, the volume of ice per foot is

$$V_t = \frac{1}{144}\pi \times \frac{1}{1}t_t(d_c + t_t) \quad \text{ft}^3/\text{ft} \tag{10.62}$$

The weight of the ice is 57 lb/ft^3, so that the weight of ice per foot is

$$w_t = \frac{57}{144}\pi t_t(d_c + t_t)$$

or approximately

$$w_t = 1.25t_t(d_c + t_i) \quad \text{lb/ft} \tag{10.63}$$

Therefore, the total vertical load on the conductor per unit length is

$$w_T = w + w_t \tag{10.64}$$

where w_T = total vertical load on conductor per unit length
w = weight of conductor per unit length
w_i = weight of ice per unit length

10.5.2 Effect of Wind

It is customary to assume that the wind blows uniformly and horizontally across the projected area of the conductor covered with no ice and ice, respectively.

The projected area per unit length of the conductor with no ice is

$$S_{ni} = A_{ni}l \tag{10.65}$$

where S_{ni} = projected area of conductor covered with no ice in square feet per unit length
A_{ni} = cross-sectional area of conductor covered with no ice in square feet
l = length of conductor in unit length

for a 1-ft length of conductor with no ice,

$$S_{ni} = \frac{1}{12}d_c l \tag{10.66}$$

whereas with ice, it is

$$S_{wi} = A_{wi}l \tag{10.67}$$

where S_{wi} = projected area of conductor covered with ice in square feet per unit length

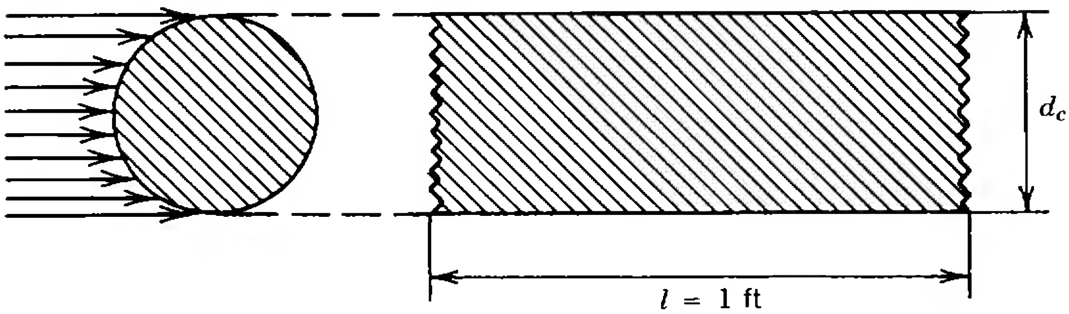

Figure 10.9. Force of wind on conductor covered with no ice.

A_{wi} = cross-sectional area of conductor covered with ice in square feet
l = length of conductor in unit length

for a 1-ft length of conductor,

$$S_{\text{wi}} = \frac{d_c + 2t_t}{12}\, l \quad \text{ft}^2/\text{ft} \tag{10.68}$$

Therefore, the horizontal force exerted on the line as a result of the wind pressure with no ice (Figure 10.9) is

$$P = S_{\text{ni}}\, p \quad \text{lb/unit length} \tag{10.69}$$

for a 1-ft length of conductor,

$$P = \tfrac{1}{12} d_c p \quad \text{lb/ft} \tag{10.70}$$

where P = horizontal wind force (i.e., load) exerted on line in pounds per feet
p = wind pressure in pounds per squre feet

whereas with ice (Figure 10.10), it is

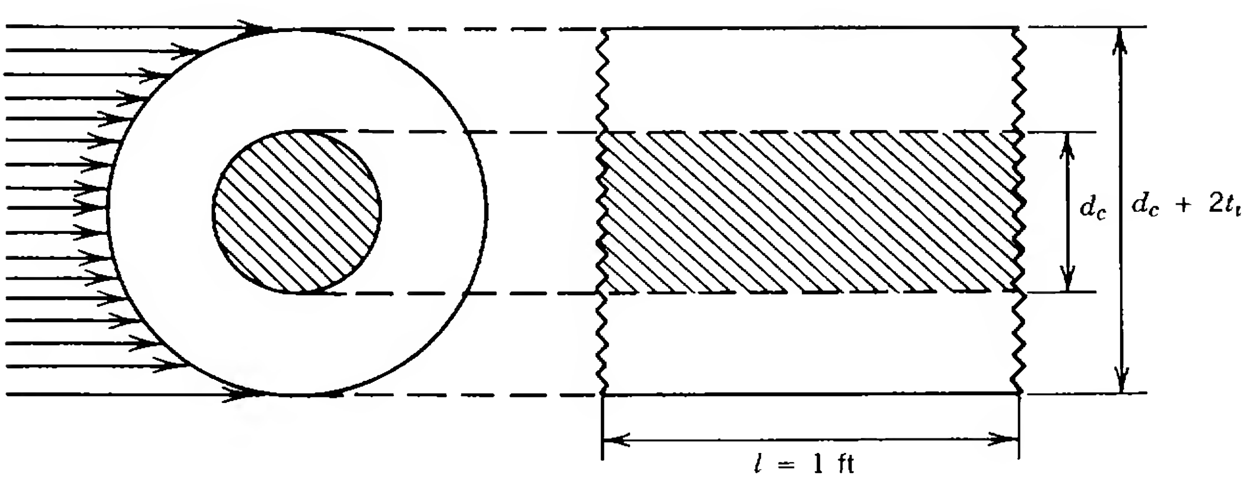

Figure 10.10. Force of wind on conductor covered with ice.

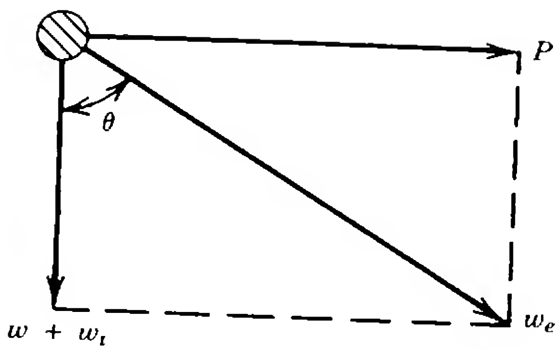

Figure 10.11

$$P = S_{\text{wi}} p \quad \text{lb/unit length} \tag{10.71}$$

for a 1-ft length of conductor,

$$P = \frac{d_c + 2t_i}{12} p \quad \text{lb/ft} \tag{10.72}$$

Therefore, the effective load acting on the conductor is

$$w_e = \sqrt{P^2 + (w + w_i)^2} \quad \text{lb/ft} \tag{10.73}$$

acting at an angle θ to the vertical, as shown in Figure 10.11.

By replacing w by w_e in the previously derived equations for tension and sag of the line in still air, these equations can be applied to a wind- and ice-loaded line. For example. the sag equation (10.46) becomes

$$d = \frac{w_e L^2}{8T} \quad \text{ft} \tag{10.74}$$

Example 10.2

A stress-crossing overhead subtransmission line has a span of 500 ft over the stream. The line is located in a heavy-loading district in which the horizontal wind pressure is 4 lb/ft^2 and the radial thickness of the ice is 0.50 in. Use an ACSR conductor of 795 kcmil having an outside diameter of 1.093 in., a weight of 5399 lb/mi, and an ultimate strength of 28,500 lb. Also use a safety factor of 2 and 57 lb/ft^3 for the weight of ice. Using the parabolic method, calculate the following:

(a) Weight of ice in pounds per feet.
(b) Total vertical load on conductor in pounds per feet.
(c) Horizontal wind force exerted on line in pounds per feet.
(d) Effective load acting on conductor in pounds per feet.
(e) Sag in feet
(f) Vertical sag in feet.

Solution

(a) Using equation (10.63),

$$w_t = 1.25t_t(d_c + t_t)$$
$$= 1.25 \times 0.50(1.093 + 0.50)$$
$$\cong 0.9956 \text{ lb}$$

(b) Using equation (10.64)

$$w_T = w + w_i$$

The weight of the conductor is

$$w = 5399 \text{ lb/mi}$$

or

$$w = \frac{1}{5280} 5399 \cong 1.0225 \text{ lb/ft}$$

Therefore,

$$W_T = 1.0225 + 0.9956$$
$$= 2.0181 \text{ lb/ft}$$

(c) From equation (10.72),

$$P = \frac{d_c + 2t_t}{12} p$$
$$= 1.093 + 2 \times \frac{0.50}{12} \times 4$$
$$= 0.6977 \text{ lb/ft}$$

(d) Using equation (10.73) and Figure 10.11,

$$w_e = \sqrt{P^2 + (w + w_t)^2}$$
$$= \sqrt{0.6977^2 + 2.0181^2}$$
$$= 2.1353 \text{ lb/ft}$$

as shown in Figure 10.12

(e) $T = \dfrac{28{,}500}{2} = 14{,}250 \text{ lb}$

From equation (10.74),

$$d = \frac{w_e L^2}{8T}$$
$$= \frac{2.1353 \times 500^2}{8 \times 14{,}250}$$
$$= 4.68 \text{ ft}$$

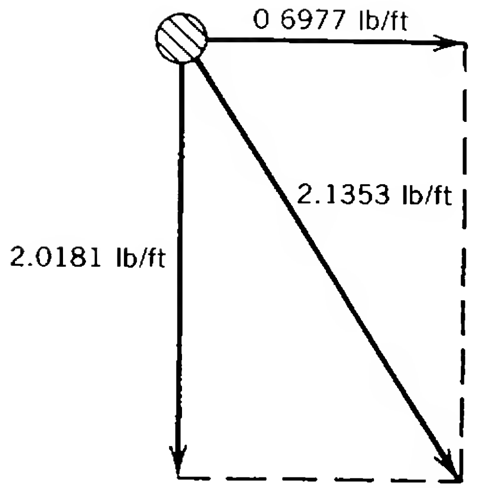

Figure 10.12

(f) Vertical sag $= d \cos \theta$

$$= 4.68 \times \frac{2.0181}{2.1353}$$
$$= 4.42 \text{ ft}$$

10.6 NATIONAL ELECTRIC SAFETY CODE

More than 60 percent of the states in the United States use the National Electrical Safety Code (NESC) as either an official state code or a guide [1]. Therefore, the design of the overhead lines should comply with NESC rules and regulations.

The following information has been based on the 1984 edition of the National Electrical Safety Code [1]. Figure 10.13 shows the general loading map of the U.S.A. with respect to loading of overhead lines. Table 10.1 gives the corresponding normal ice and wind loads for each of the loading districts. However, the NESC recognizes the possible variations in the ice and wind loads due to regional differences.

Table 10.1 shows the minimum radial thickness of ice and the wind pressures to be used in determining loadings. Here, ice is assumed to weigh $57\ \text{lb/ft}^3$.

Figure 10.14 shows the NESC's wind map of the United States. It shows the minimum horizontal wind pressures to be used for determining loads on tall structures. If any portion of a structure of supported facilities is located in excess of 60 ft above ground, these wind pressures are applied to the entire structure and supported facilities without ice covering. Note the fact that wind velocity usually increases with height. Therefore, the given wind pressures are required to be increased accordingly.

The total load on a conductor or messenger is the sum of vertical loading and horizontal loading components calculated at the temperature specified in Table 10.2, to which resultant has been added the constant given in Table

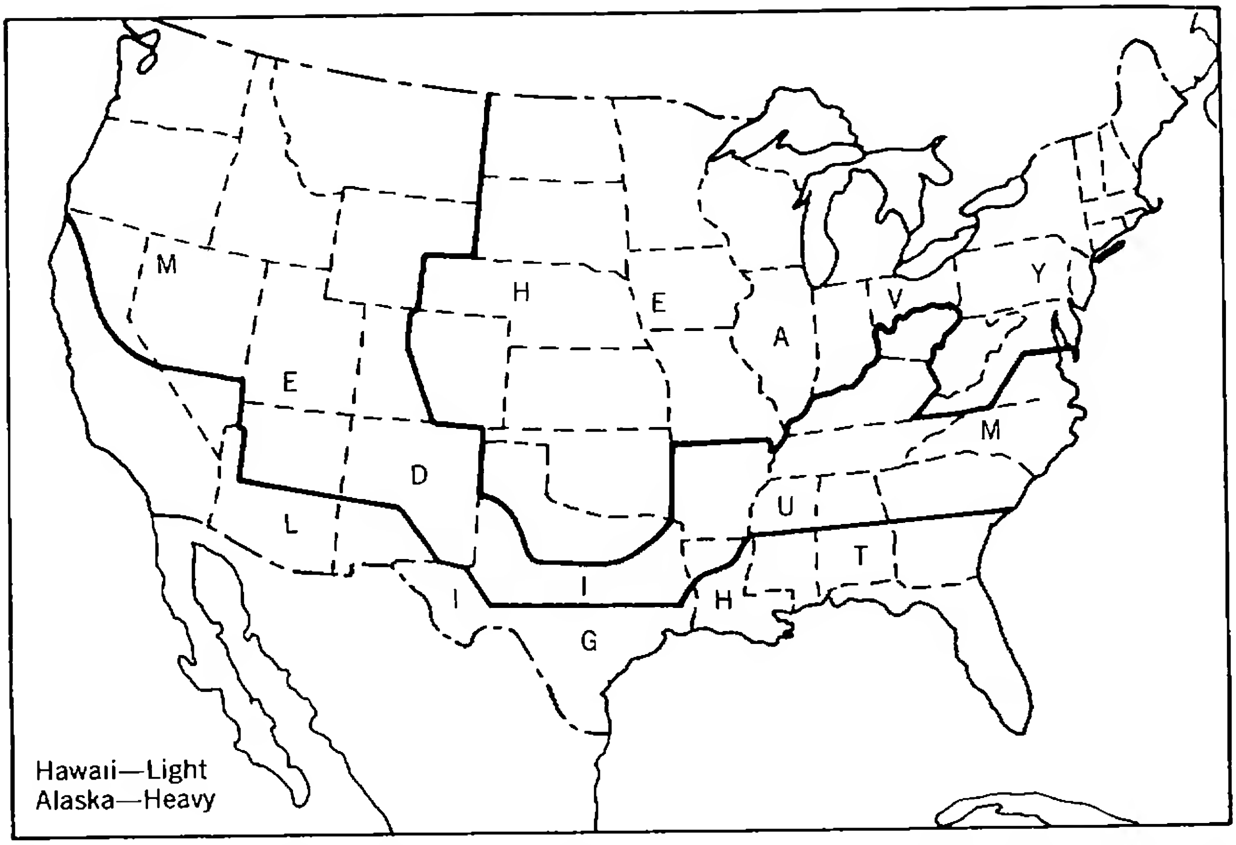

Figure 10.13. NESC's general loading map of United States with respect to loading of overhead lines.

TABLE 10.1 Ice and Wind Loads for Specified Districts According to NESC

	Loading District		
	Heavy	Medium	Light
Radial thickness of ice, in.	0.50	0.25	0
Horizontal wind pressure, lb/ft	4	4	9

TABLE 10.2 Temperatures and Constants as Specified by NESC

	Loading Districts			
	Heavy	Medium	Light	Extreme Wind Loading
Temperature, °F	0	+15	+30	+60
Constant to be added to resultant in pounds per foot for all conductors	0.30	0.20	0.05	0.0

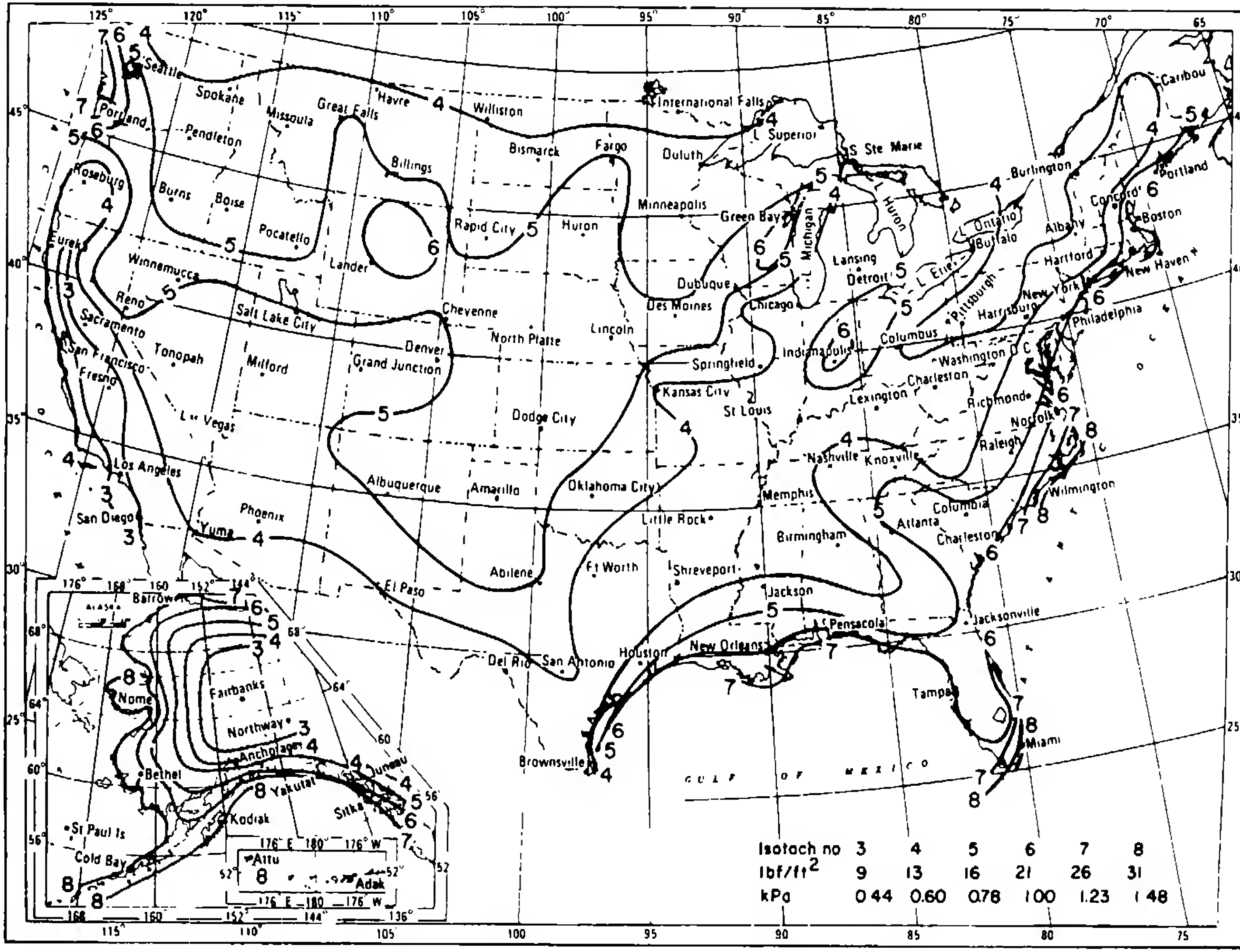

Figure 10.14. Extreme wind pressure in lb/ft^2 at 30 ft above ground, 50-yr mean recurrence interval (based on fastest mile of wind).

10.1. In all cases, therefore, the conductor or messenger tension is determined from this total loading.

10.7 LINE LOCATION

The routing of a transmission line or a rural distribution line requires thorough investigation and study with several different routings explored to assure that the most desirable and practical route is selected, taking into consideration not only cost of construction, cost of easements, and cost of clearing but also environmental and maintenance requirements. Therefore, the line location is a matter of judgment and requires sound evalution of divergent requirements. In practice, the ground is rarely level, and the poles are actually located to conform to the irregularities of the ground. The irregularities of the ground are ordinarily advantageous and allow slightly longer spans on the average than could be obtained with the same height of structures on level ground.

10.7.1 Profile and Plan of Right-of-Way

The basic purpose of the profile and the final plan of the right-of-way of a given line is to ensure correct design and economical construction. They are permanent drawings, as they are the key construction drawings, right-of-way record, and permanent property record. Accurate collection of field data and translation of such data to plan and profile drawings are of utmost importance since errors in this work will defeat the purposes of the drawings and cancel out the accuracy of structure spotting with a template. In general, profiles are plotted on standard ruled profile section paper to a vertical scale of 40 ft to the inch and a horizontal scale of 400 ft to the inch. This scaling arrangement provides a most compact drawing with sufficient accuracy for most conditions. For some lines, three profiles may be desirable, one along the center of the line and one on each side, that is, at each edge of right-of-way. The two side profiles indicate the amount and direction of the slope of the ground across the line that must be allowed for in determining ground clearance.

Of course, a plan of the secured right-of-way is also required for determining the construction at angles in the line and the clearances from the conductor to the edge of the right-of-way when the conductor is deflected by the wind.

The planning engineer selects a tentative line route, making use of aerial photographs and the following maps: geological survey, county soil, county plat, road, post office routes, the U.S. Forest Service, Bureau of Indian Affairs, aerial strip, and any other maps that may be available. These maps contain detailed topography from which a tentative paper location can be made. Also, aerial reconnaissance may be desirable over hilly and mountainous country.

Based on this tentative line route, a more detailed survey needs to be prepared to show the prosed high-voltage line and the angle positions, together with the obstructions mentioned. The angles in the line must be more accurately positioned by the surveyor according to site conditions. The surveyor measures the angles of deviation of the line at each angle or control point (marked by hub stakes, with tack points of alignment) and measures the straight-line distance between them. He has to take levels, entering them in a level book, along the measuring chain at intervals depending on the gradient of the land at the point concerned. Other methods of survey are:

1. *Slight Grading*. It is cheaper and much faster if it is done by an experienced surveyor. However, of course, it is not economic if each pole ends up being much longer than it needs to be. Thus, it may be used only for level ground.
2. *Tacheometric Survey*. It is done by using a theodolite if the ground is hilly and the measuring chain is not used.
3. *Aerial Survey*. It may be useful for very long high-voltage transmission lines, but it is hardly applicable to ordinary rural distribution

10.7.2 Templates for Locating Structures

The location of structures on the profile with a template is essential for both correct design and economy. The sag template is a convenient device used in the design of a transmission line to determine graphically on plan and profile drawings the location and height of structures. It is cut from a transparent plastic material approximately 0.72 mm in thickness. It has the same horizontal and vertical scales as used for the profile and plan of the right-of-way.

With reasonable competence and some experience, this method can be relied upon to provide the following:

1. Maintenance of proper clearance from conductor to ground and to crossing conductors.
2. Economic layout.
3. Minimum possibility of errors in design and layout.
4. Proper grading of structures.
5. Prevention of excessive insulator swing or uplift at structures.
6. Exactly the correct quantity of material purchased and delivered to the proper site.

A sag template is cut as a parabola on the maximum sag, usually at 120 °F, of the ruling span, and is extended by calculating the sag as proportional to square of the span for spans both shorter and longer than the ruling span. Since the curvature of the catenary or parabola in which the conductor hangs depends only on the tension and loading and not on the length of the span or on the difference in elevation of the points of support, all spans having the same tension and loading can be drawn from a single template, irrespective of their lengths or of the differences in elevation of the support points. However, when the elevations of the support points are not the same, the lowest point of the curve is shifted from the middle of the span toward the lower support, but the axis of the curve remains vertical.

The sag template has three curves on it. One for ground clearance with maximum sag, one for uplift at times of minimum sag, and one for maximum side swing. Therefore, the required curves are:

1. *Hot Curve.* It is drawn for 120 °F, no ice, no wind, final sag curve. It is used to locate position of structure and to check clearance, insulator swing, and structure height on the plotted profile.
2. *Cold Curve.* It is drawn for minimum temperature (0 °F), no ice, no wind, minimum initial sag curve. It is used to check for uplift and insulator swing.
3. *Normal Curve.* It is drawn for 60 °F, no ice, no wind, final sag curve. It is used to check normal clearances and insultor swing.

Uplift conditions for the overhead conductors must be avoided. It can be checked by the cold curve. Conductors of underbuild lines may be of different sizes. The hot curve of the lowest conductor should be used for checking ground clearance. Cold curves are required for each size of conductor to check for uplift or insulator swing.

A sag template drawing, as shown in Figure 10.15, is prepared for each line as a guide in cutting the template. A new template has to be prepared for each line where there is any variation in conductor size, conductor configuration, assumed loading conditions, design tension, ruling span, or voltage because of a change in any one of these factors will change the characteristics of the template. The sags in Table 10.3 are determined as follows [3]:

1. Read applicable ruling span sag from sag and tension chart furnished by conductor manufacturer.
2. Calculate the other sag by using the formula

$$d = \frac{L^2 d_e}{L_e^2}$$

where d = sag of other span in feet
d_e = sag of ruling span in feet
L = length of other span in feet
L_e = length of ruling span in feet

3. Apply catenary sag correction to long spans having large sags.

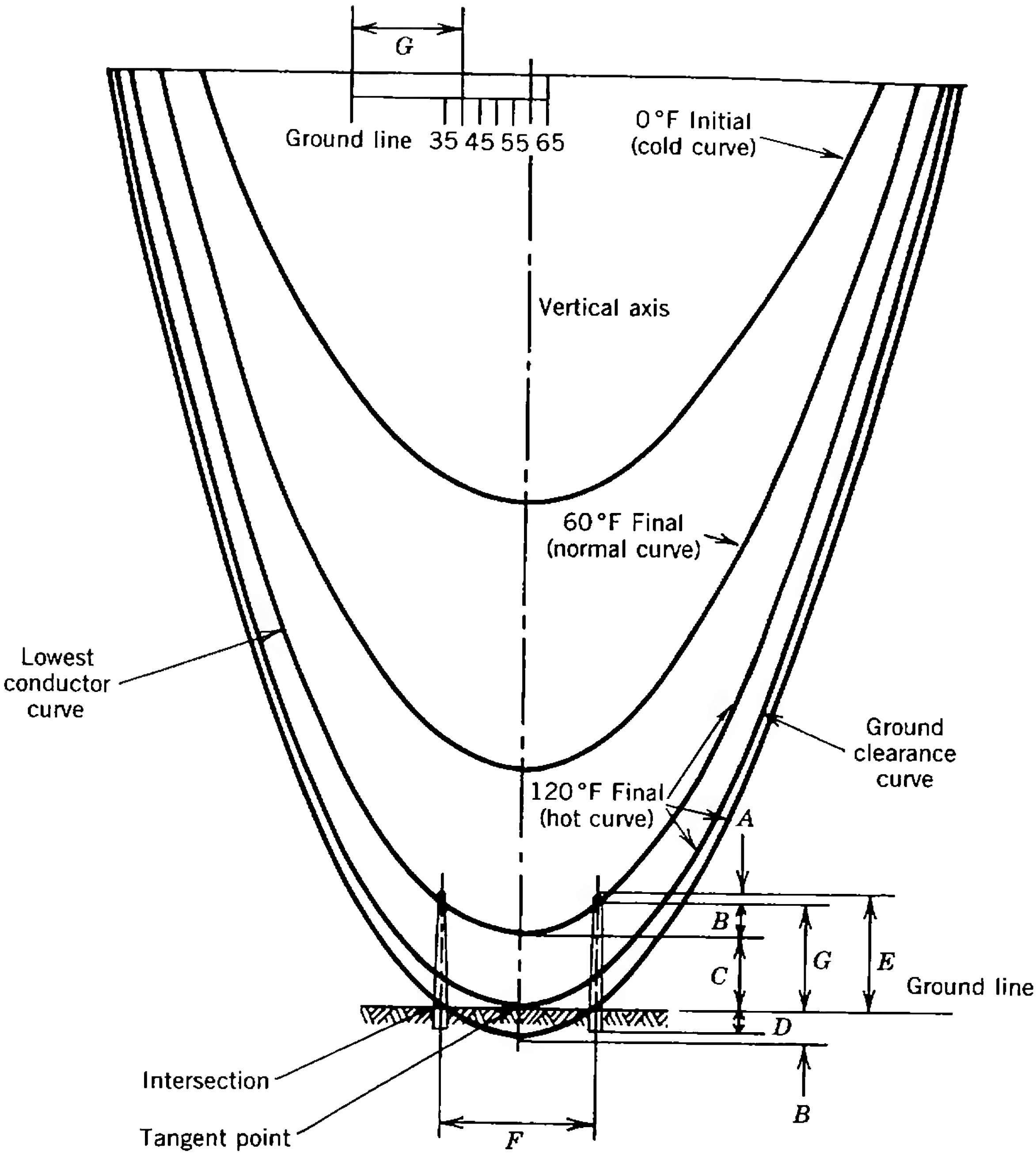

Figure 10.15. Sag template based on values given in Table 10.3: A, dimension from top of pole to point of attachment of lowest conductor; B, sag in level ground; C, ground clearance; D, setting depth of pole; E, length of pole; F, level ground span; G, dimension from ground to point of attachment of lowest conductor.

Table 10.3 gives sags for preparing a template for 4/0 ACSR conductor with 4210 lb design tension and 600 ft ruling span and for heavy-loading conditions. The template allows 1 ft greater clearances than the ones required by the NESC to allow minor shifts in structure location.

As shown in Figure 10.15, the intersection of the template with ground line determines the location of the structure. The vertical distance between this intersection and the lowest conductor curve gives the basic height of the pole. In order to achieve this, the ground clearance curve should be tangent to the ground profile. The vertical axis aids in holding the template vertical when spotting structures.

TABLE 10.3 Sags for Preparing Spotting Template

Condition	0 °F Initial	60 °F Final	120 °F Final
Tension, lb	2150	1174	981
Span (ft)	Sag (ft)	Sag (ft)	Sag (ft)
200	0.68	1.24	1.49
400	2.71	4.98	5.96
600	6.11	11.2	13.4
800	10.9	19.9	23.8
1000	16.9	31.1	37.2
1200	24.4	44.8	53.6
1400	33.2	60.9	73.2[a]
1600	43.4	79.8[a]	95.7[a]
1800	55.0	101.2[a]	121.3[a]
2000	67.9	125.0[a]	150.0[a]
2200	82.1	151.5[a]	181.7[a]
2400	98.0	180.4[a]	216.6[a]
2600	115.0	212.0[a]	254.7[a]
2800	133.5	246.3[a]	296.0[a]
3000	153.3	283.2[a]	340.6[a]

[a] Corrected by catenary method; a maximum of 1.64%.

Ice and snow on the conductors may cause weights several times that of the 0.5-in. ice loading, and conductors have been known to sag to within reach of the ground. Such occurrences are not considered in line design, and when they happen, the line is taken out of service until the ice or snow falls. Checks made afterward have nearly always shown no permanent deformation.

The template must be used subject to a "creep" correction when aluminum conductors are involved. The creep is a constant nonelastic conductor stretch that continues for the life of the line. It causes a continuous slow increase in the sag of the line that must be estimated and allowed for. The manufacturers of aluminum conductors provide creep-estimating curves. The remedy is to check all close clearance points on the profile with a template made with no creep allowance and to choose higher structure at these points if the addition of extra creep sag encroach on the required clearances.

The lowest point of the sag, on step and inclined spans, may fall beyond the lower support. This indicates that the conductor in the uphill span exerts a negative or upward pull on the lower structure, that is, the pole or tower. The amount of this upward pull is equal to the weight of the conductor from the structure to the low point in the sag. If the upward pull of the uphill span is greater than the downward load of the next adjacent span, actual uplift is caused, and the conductor tends to swing clear of the structure.

Therefore, the designer should not allow sudden changes in elevation of the structures.

In the design of lines with suspension insulators, each structure must carry a considerable weight of the conductor, and the uplift condition should not even be approached. The minimum weight that each structure must carry can be determined by finding the transverse angle to which the insulator string may swing without reducing the clearance from the conductor to the structure too greatly and by maintaining the ratio of vertical weight to horizontal wind load such that the insulator is not permitted to swing beyond this angle. Usually, the maximum wind is assumed at 60 °F. The insulator swings in the direction of the resultant of the vertical and horizontal forces acting on the insulator string. The minimum angle of conductor swing to be used in computations, where proximity to other circuits is involved, is 30 ° according to sixth edition of the NESC. In general, a clearance corresponding to approximately 75 percent of the flashover value of the insulator is appropriate.

10.7.3 Supporting Structures

The structure heights, locations, and types can be determined by use of plan and profile sheets. It requires both engineering and economics. The following design factors should be considered:

1. Conductor sag and tension limitations.
2. Clearance involved.
3. Separation distances between conductors.
4. Uplift or insulator swing.
5. Loading of conductors.
6. Height and strength of structures.
7. Soil conditions.
8. Control, or angle, points.
9. Operation and maintenance problems.

In order to spot or locate structures, the template is held tangent to the profile, and the ground clearance curve is held tangent to the profile. The edge of the template intersects the ground profile at points where structures of the basic height should be set. Here the ground clearance curve represents the actual position of the lowest conductor. The procedure that is illustrated in Figure 10.15 for a level span is the same for any given type of terrain. Once the height and the location of the structure is determined, the template should be shifted so that the clearance curve barely touches tangent to the profile, and the point where the edge of the template intersects the profile determines the location of the next structure of basic

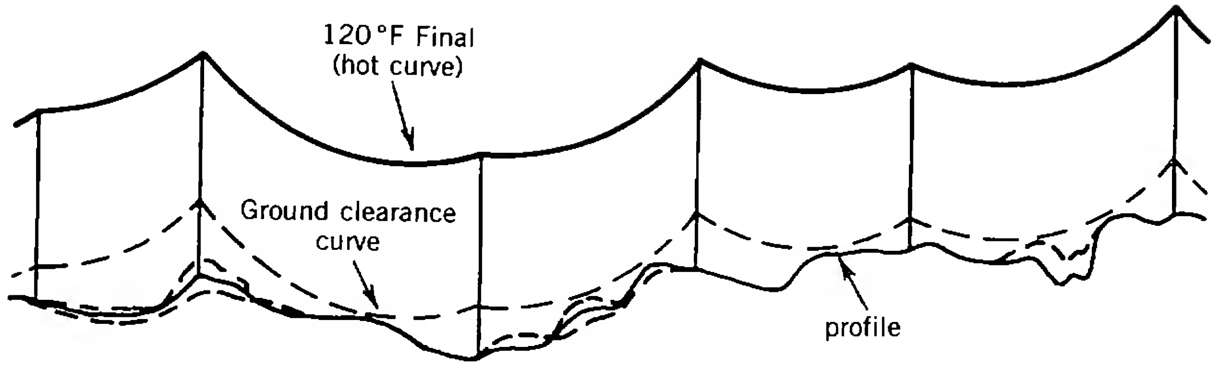

Figure 10.16. Locating structures by templates.

height. The point may be marked by drawing an arc along the edge of the template where it intersects the profile. The template should then be shifted and adjusted so that with the opposite edge of the template held on the point previously located, the clearance curve will again barely touch the profile. As is done before, an arc should be drawn to mark the location of the next structure of basic height, and the process is repeated to locate each structure of the line, as shown in Figure 10.16.

After all structures are located, the structures and lowest conductor's arc should be drawn in. When line angles, broken terrain, and crossings are encountered, it may be necessary to cut and try several different arrangements of structure heights, increased clearances, and locations to determine the best arrangement. In addition to maintaining clearances, uplift must be avoided on lines with pin-type insulators, and excessive insulator swings must be avoided on lines with suspension-type insulators. On the lines with pin-type insulators, there is no danger of uplift if the low-temperature initial curve does not pass above the point of conductor support on the two adjacent structures.

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PROBLEMS

10.1. A single span of an overhead line is 250 ft in length, having the supporting poles at the same level. The maximum tension in the conductor is 855 lb. The weight of the conductor is 0.125 lb/ft. Calculate the sag.

(a) Using the catenary method.

(b) Using the parabolic method.

10.2. An overhead line conductor has taken the form of a catenary of $y = c\cosh(x/c)$, where c is 925 ft. The span between the poles is 235 ft, and the weight of the conductor is 0.139 lb/ft. Calculate the following:

(a) Length of conductor by using equations (10.24) and (10.27).

(b) Sag.

(c) Maximum tension of conductor.

(d) Minimum tension of conductor.

10.3. An overhead line over a hillside is supported from two towers, as shown in Figure P10.3, at levels of 28.1 and 49.6 ft above a horizontal datum line. The span is 350 ft. The copper conductor of the line has a breaking strength of 15,140 lb and a weight of 5706 lb/mi. Use the parabolic method.

(a) Calculate the location of the lowest point of the curve.

(b) Calculate sags d_1 and d_2.

(c) Which of the sags should be considered as the principal sag.

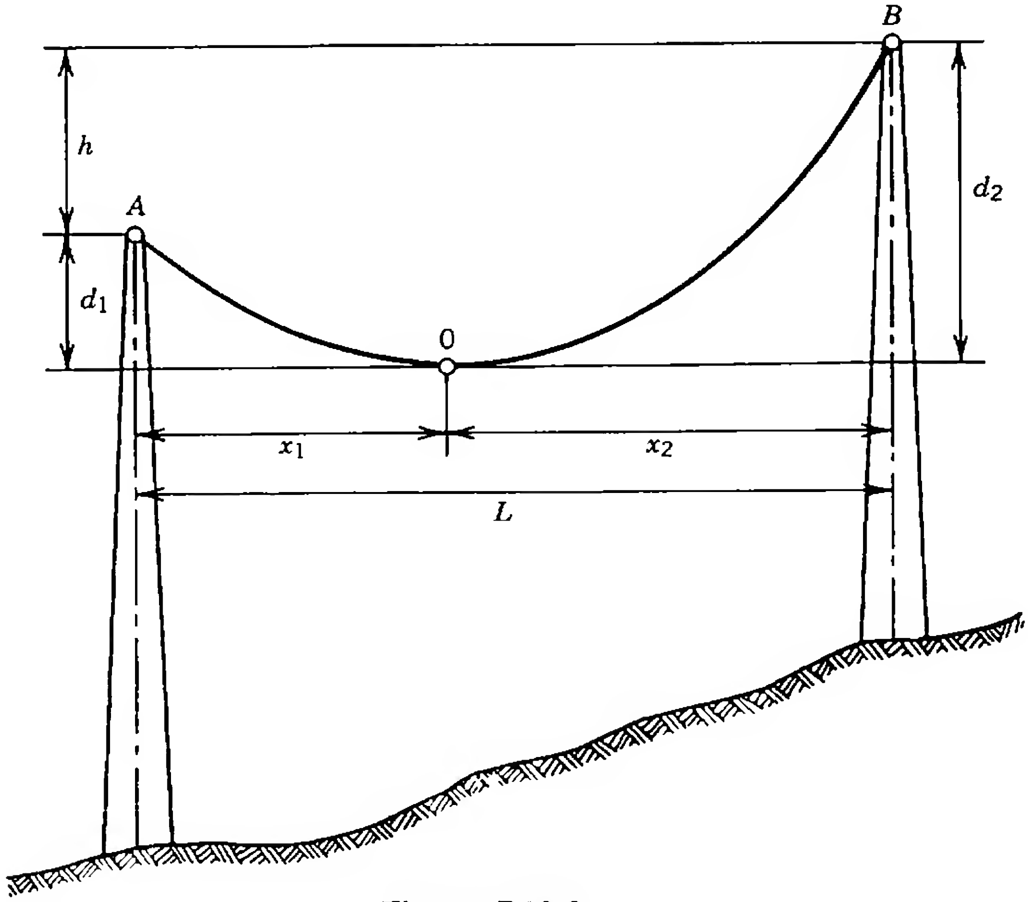

Figure P10.3

10.4. An overhead line conductor is supported at a water crossing from two towers, the heights of the supports being 30 and 34.2 m, respectively, above water level, with a horizontal span of 325 m. The conductor weighs 8.42 N/m, and its tension is not to exceed 4.44×10^4 N. Calculate the following

(a) Clearance between lowest point of conductor and water,

(b) Horizontal distance of this point from lower support.

10.5. A river-crossing overhead line is supported from two poles at heights of 30 and 40 ft above the water level, as shown in Figure P10.5. The conductor of the line is a 1/0 copper conductor. Use a safety factor of 2. Calculate the clearance between the water level and the conductor at a point midway between the poles.

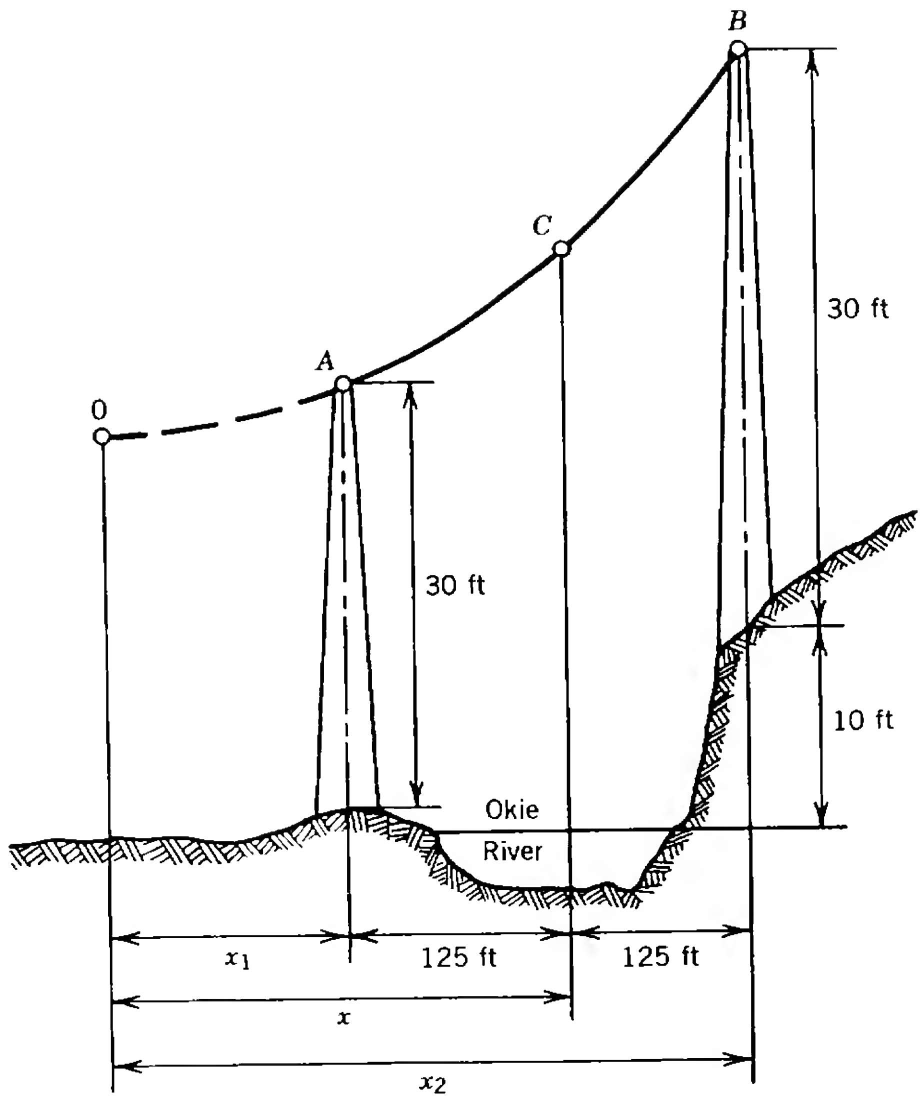

Figure P10.5

10.6. The ACSR conductors of an overhead line have 1.63 cm overall diameter and are supported by suspension insulator strings of 1.02 m long. The permissible tension is 2.50×10^4 N, the conductor weighs 5.35 N/m, and the ice weighs 8920 N/m^3. Find the height of the lowest cross-arm above ground level if the minimum clearance between conductor and ground is 6.9 m when there is an ice load 1.27 cm thick and a horizontal wind pressure of 461 N/m^2 on span lengths of 195 m.

10.7. Find the minimum permissible sag for a span of 225 m using a copper conductor with 1.2 cm diameter and allowing a maximum conductor tension of 2200 kg/cm^2. The horizontal wind pressure is 6 g/cm^2, and the specific gravity of copper is 8.9.

10.8. Calculate the sag in an 90-yard span of a 0.035-in.^{2} overhead line when covered with ice to a radial thickness of $\frac{5}{8}$ in, in a wind that exerts a pressure of 9 lb/ft^2. Assume the breaking load as 1620 lb and the weight of the conductor as 0.1 lb/ft. Allow a safety factor of 2.

APPENDIX A

IMPEDANCE TABLES FOR OVERHEAD LINES, TRANSFORMERS, AND UNDERGROUND CABLES

TABLE A.1 Characteristics of Copper Conductors, Hard Drawn, 97.3% Conductivity [1, 2]

Size of Conductor: Circular Mils	Size of Conductor: A.W.G. or B. & S.	Number of Strands	Diameter of Individual Strands Inches	Outside Diameter Inches	Breaking Strength Pounds	Weight Pounds per Mile	Approx. Current Carrying Capacity* Amps	Geometric Mean Radius at 60 Cycles Feet	r_a Resistance Ohms per Conductor per Mile, 25°C. (77°F.): d-c	25°C: 25 cycles	25°C: 50 cycles	25°C: 60 cycles	50°C (122°F.): d-c	50°C: 25 cycles	50°C: 50 cycles	50°C: 60 cycles	x_a Inductive Reactance Ohms per Conductor Per Mile At 1 Ft. Spacing: 25 cycles	x_a: 50 cycles	x_a: 60 cycles	x_a' Shunt Capacitive Reactance Megohms per Conductor Per Mile At 1 Ft Spacing: 25 cycles	x_a': 50 cycles	x_a': 60 cycles
1 000 000		37	0.1644	1.151	43 830	16 300	1 300	0.0368	0.0585	0.0594	0.0620	0.0634	0.0640	0.0648	0.0672	0.0685	0.1666	0.333	0.400	0.216	0.1081	0.0901
900 000		37	0.1560	1.092	39 510	14 670	1 220	0.0349	0.0650	0.0658	0.0682	0.0695	0.0711	0.0718	0.0740	0.0752	0.1693	0.339	0.406	0.220	0.1100	0.0916
800 000		37	0.1470	1.029	35 120	13 040	1 130	0.0329	0.0731	0.0739	0.0760	0.0772	0.0800	0.0806	0.0826	0.0837	0.1722	0.344	0.413	0.224	0.1121	0.0934
750 000	.	37	0.1424	0.997	33 400	12 230	1 090	0.0319	0.0780	0.0787	0.0807	0.0818	0.0853	0.0859	0.0878	0.0888	0.1739	0.348	0.417	0.226	0.1132	0.0943
700 000	..	37	0.1375	0.963	31 170	11 410	1 040	0.0308	0.0836	0.0842	0.0861	0.0871	0.0914	0.0920	0.0937	0.0947	0.1759	0.352	0.422	0.229	0.1145	0.0954
600 000	.	37	0.1273	0.891	27 020	9 781	940	0.0285	0.0975	0.0981	0.0997	0.1006	0.1066	0.1071	0.1086	0.1095	0.1799	[illegible]	0.432	0.235	0.1173	[illegible]
500 000		37	0.1162	0.814	22 510	8 151	840	0.0260	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1845	0.369	0.443	0.241	0.1206	0.1004
500 000	. .	19	0.1622	0.811	21 590	8 151	840	0.0256	0.1170	0.1175	0.1188	0.1196	0.1280	0.1283	0.1296	0.1303	0.1853	0.371	0.445	0.241	0.1206	0.1005
450 000	...	19	0.1539	0.770	19 750	7 336	780	0.0243	0.1300	0.1304	0.1316	0.1323	0.1422	0.1426	0.1437	0.1443	0.1879	0.376	0.451	0.245	0.1224	0.1020
400 000	.	19	0.1451	0.726	17 560	6 521	730	0.0229	0.1462	0.1466	0.1477	0.1484	[illegible]	0.1603	0.1613	0.1619	[illegible]	[illegible]	0.458	0.249	0.1245	0.1038
350 000		19	0.1357	0.679	15 590	5 706	670	0.0214	0.1671	0.1675	0.1684	0.1690	[illegible]	0.1831	0.1840	0.1845	[illegible]	0.363	0.466	0.254	0.1269	0.1058
350 000	.	12	0.1708	0.710	15 140	5 706	670	0.0225	0.1671	0.1675	0.1684	0.1690	0.1828	0.1831	0.1840	0.1845	0.1918	0.384	0.460	0.251	0.1253	0.1044
300 000	..	19	0.1257	0.629	13 510	4 891	610	0.01987	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1982	0.396	0.476	0.259	0.1296	0.1080
300 000	.	12	0.1581	0.657	13 170	4 891	610	0.0208	0.1950	0.1953	0.1961	0.1966	0.213	0.214	0.214	0.215	0.1957	0.392	0.470	0.256	0.1281	0.1068
250 000		19	0.1147	0.574	11 360	4 076	540	0.01813	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.203	0.406	0.487	0.266	0.1329	0.1108
250 000		12	0.1443	0.600	11 130	4 076	540	0.01902	0.234	0.234	0.235	0.235	0.256	0.256	0.257	0.257	0.200	0.401	0.481	0.263	0.1313	0.1094
211 600	4/0	19	0.1055	0.528	9 617	3 450	480	0.01668	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.207	0.414	0.497	0.272	0.1359	0.1132
211 600	4/0	12	0.1328	0.552	9 483	3 450	490	0.01750	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.205	0.409	0.491	0.269	0.1343	0.1119
211 600	4/0	7	0.1739	0.522	9 154	3 450	480	0.01579	0.276	0.277	0.277	0.278	0.302	0.303	0.303	0.303	0.210	0.420	0.503	0.273	0.1363	0.1136
167 800	3/0	12	0.1183	0.492	7 556	2 736	420	0.01559	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.210	0.421	0.505	0.277	0.1384	0.1153
167 800	3/0	7	0.1548	0.464	7 366	2 736	420	0.01404	0.349	0.349	0.349	0.350	0.381	0.381	0.382	0.382	0.216	0.431	0.518	0.281	0.1405	0.1171
133 100	2/0	7	0.1379	0.414	5 926	2 170	360	0.01252	0.440	0.440	0.440	0.440	0.481	0.481	0.481	0.481	0.222	0.443	0.532	0.289	0.1445	0.1205
105 500	1/0	7	0.1228	0.368	4 752	1 720	310	0.01113	0.555	0.555	0.555	0.555	0.606	0.607	0.607	0.607	0.227	0.455	0.546	0.298	0.1488	0.1240
83 690	1	7	0.1093	0.328	3 804	1 364	270	0.00992	0.699	0.699	0.699	0.699	0.765				0.233	0.467	0.560	0.306	0.1528	0.1274
83 690	1	3	0.1670	0.360	3 620	1 351	270	0.01016	0.692	0.692	0.692	0.692	0.757				0.232	0.464	0.557	0.299	0.1495	0.1246
66 370	2	7	0.0974	0.292	3 045	1 082	230	0.00883	0.881	0.882	0.882	0.882	0.964				0.239	0.478	0.574	0.314	0.1570	0.1308
66 370	2	3	0.1487	0.320	2 913	1 071	240	0.00903	0.873				0.955				0.238	0.476	0.571	0.307	0.1537	0.1281
66 370	2	1		0.258	3 003	1 061	220	0.00836	0.864				0.945				0.242	0.484	0.581	0.323	0.1614	0.1345
52 630	3	7	0.0867	0.260	2 433	858	200	0.00787	1.112				1.216				0.245	0.490	0.588	0.322	0.1611	0.1343
52 630	3	3	0.1325	0.285	2 359	850	200	0.00805	1.101				1.204				0.244	0.488	0.585	0.318	0.1578	0.1315
52 630	3	1		0.229	2 439	841	190	0.00745	1.090		Same as d-c		1.192		Same as d-c		0.248	0.496	0.595	0.331	0.1656	0.1380
41 740	4	3	0.1180	0.254	1 879	674	180	0.00717	1.388				1.518				0.250	0.499	0.599	0.324	0.1619	0.1349
41 740	4	1		0.204	1 970	667	170	0.00663	1.374				1.503				0.254	0.507	0.609	0.339	0.1697	0.1415
33 100	5	3	0.1050	0.226	1 505	534	150	0.00638	1.750				1.914				0.256	0.511	0.613	0.332	0.1661	0.1384
33 100	5	1		0.1819	1 591	529	140	0.00590	1.733				1.895				0.260	0.519	0.623	0.348	0.1738	0.1449
26 250	6	3	0.0934	0.201	1 205	424	130	0.00568	2.21				2.41				0.262	0.523	0.628	0.341	0.1703	0.1419
26 250	6	1		0.1620	1 280	420	120	0.00526	2.18				2.39				0.265	0.531	0.637	0.356	0.1779	0.1483
20 820	7	1		0.1443	1 030	333	110	0.00468	2.75				3.01				0.271	0.542	0.651	0.364	0.1821	0.1517
16 510	8	1		0.1285	826	264	90	0.00417	3.47				3.80				0.277	0.554	0.665	0.372	0.1862	0.1552

*For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency = 60 cycles.

TABLE A.2 Characteristics of Aluminum Conductors, Hard Drawn, 61% Conductivity [1, 2] (Aluminum Company of America)

Size of Conductor: Circular Mils or A W G	No of Strands	Diameter of Individual Strands Inches	Outside Diameter Inches	Ultimate Strength Pounds	Weight Pounds Per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx Current Carrying Capacity* Amps
6	7	0.0612	0.184	528	130	0.00556	100
4	7	0.0772	0.232	826	207	0.00700	134
3	7	0.0867	0.260	1022	261	0.00787	155
2	7	0.0974	0.292	1266	329	0.00883	180
1	7	0.1094	0.328	1537	414	0.00992	209
1/0	7	0.1228	0.368	1865	523	0.01113	242
1/0	19	0.0745	0.373	2090	523	0.01177	244
2/0	7	0.1379	0.414	2350	659	0.01251	282
2/0	19	0.0837	0.419	2594	659	0.01321	283
3/0	7	0.1548	0.464	2845	831	0.01404	327
3/0	19	0.0940	0.470	3200	832	0.01483	328
4/0	7	0.1739	0.522	3590	1049	0.01577	380
4/0	19	0.1055	0.528	3890	1049	0.01666	381
250 000	37	0.0822	0.575	4860	1239	0.01841	425
266 800	7	0.1953	0.586	4525	1322	0.01771	441
266 800	37	0.0849	0.594	5180	1322	0.01902	443
300 000	19	0.1257	0.629	5100	1487	0.01983	478
300 000	37	0.0900	0.630	5830	1487	0.02017	478
336 400	19	0.1331	0.666	5940	1667	0.02100	514
336 400	37	0.0954	0.668	6400	1687	0.02135	514
350 000	37	0.0973	0.681	6680	1735	0.02178	528
397 500	19	0.1447	0.724	6880	1967	0.02283	575
477 000	19	0.1585	0.793	8090	2364	0.02501	646
500 000	19	0.1623	0.812	8476	2478	0.02560	664
500 000	37	0.1162	0.813	9010	2478	0.02603	664
556 500	19	0.1711	0.856	9440	2758	0.02701	710
636 000	37	0.1311	0.918	11240	3152	0.02936	776
715 500	37	0.1391	0.974	12640	3546	0.03114	817
750 000	37	0.1424	0.997	12980	3717	0.03188	864
750 000	61	0.1109	0.998	13510	3717	0.03211	864
795 000	37	0.1466	1.026	13770	3940	0.03283	897
874 500	37	0.1539	1.077	14830	4334	0.03443	949
954 000	37	0.1606	1.024	16180	4728	0.03594	1000
1 000 000	61	0.1280	1.152	17670	4956	0.03707	1030
1 000 000	91	0.1048	1.153	18350	4956	0.03720	1030
1 033 500	37	0.1672	1.170	18260	5122	0.03743	1050
1 113 000	61	0.1351	1.216	19660	5517	0.03910	1110
1 192 500	61	0.1398	1.258	21000	5908	0.04048	1160
1 192 500	91	0.1145	1.259	21400	5908	0.04082	1160
1 272 000	61	0.1444	1.300	22000	6299	0.04180	1210
1 351 500	61	0.1489	1.340	23400	6700	0.04309	1250
1 431 000	61	0.1532	1.379	24100	7091	0.04434	1300
1 510 500	61	0.1574	1.417	25600	7487	0.04558	1320
1 590 000	61	0.1615	1.454	27000	7883	0.04674	1380
1 590 000	91	0.1322	1.454	28100	7883	0.04691	1380

Size of Conductor	r_a Resistance Ohms Per Conductor Per Mile, 25°C (77°F) d-c	25°C 25 cycles	25°C 50 cycles	25°C 60 cycles	50°C (122°F) d-c	50°C 25 cycles	50°C 50 cycles	50°C 60 cycles	x_a Inductive Reactance Ohms per Conductor Per Mile At 1 Ft Spacing, 25 cycles	x_a 50 cycles	x_a 60 cycles	x'_a Shunt Capacitive Reactance Megohms Per Conductor Per Mile At 1 Ft Spacing, 25 cycles	x'_a 50 cycles	x'_a 60 cycles
6	3.56	3.56	3.56	3.56	3.91	3.91	3.91	3.91	0.2626	0.5251	0.6301	0.3468	0.1734	0.1445
4	2.24	2.24	2.24	2.24	2.46	2.46	2.46	2.46	0.2509	0.5017	0.6201	0.3302	0.1651	0.1376
3	1.77	1.77	1.77	1.77	1.95	1.95	1.95	1.95	0.2450	0.4899	0.5879	0.3221	0.1610	0.1342
2	1.41	1.41	1.41	1.41	1.55	1.55	1.55	1.55	0.2391	0.4782	0.5739	0.3139	0.1570	0.1308
1	1.12	1.12	1.12	1.12	1.23	1.23	1.23	1.23	0.2333	0.4665	0.5598	0.3055	0.1528	0.1273
1/0	0.885	0.8851	0.8853	0.885	0.973	0.9731	0.9732	0.973	0.2264	0.4528	0.5434	0.2976	0.1488	0.1240
1/0	0.885	0.8851	0.8853	0.885	0.973	0.9731	0.9732	0.973	0.2246	0.4492	0.5391	0.2964	0.1482	0.1235
2/0	0.702	0.7021	0.7024	0.702	0.771	0.7711	0.7713	0.771	0.2216	0.4431	0.5317	0.2890	0.1445	0.1204
2/0	0.702	0.7021	0.7024	0.702	0.771	0.7711	0.7713	0.771	0.2188	0.4376	0.5251	0.2882	0.1441	0.1201
3/0	0.557	0.5571	0.5574	0.558	0.612	0.6121	0.6124	0.613	0.2157	0.4314	0.5177	0.2810	0.1405	0.1171
3/0	0.557	0.5571	0.5574	0.558	0.612	0.6121	0.6124	0.613	0.2129	0.4258	0.5110	0.2801	0.1400	0.1167
4/0	0.441	0.4411	0.4415	0.442	0.485	0.4851	0.4855	0.486	0.2099	0.4196	0.5036	0.2726	0.1363	0.1136
4/0	0.441	0.4411	0.4415	0.442	0.485	0.4851	0.4855	0.486	0.2071	0.4141	0.4969	0.2717	0.1359	0.1132
250 000	0.374	0.3741	0.3746	0.375	0.411	0.4111	0.4115	0.412	0.2020	0.4040	0.4848	0.2657	0.1329	0.1107
266 800	0.350	0.3502	0.3506	0.351	0.385	0.3852	0.3855	0.386	0.2040	0.4079	0.4895	0.2642	0.1321	0.1101
266 800	0.350	0.3502	0.3506	0.351	0.385	0.3852	0.3855	0.386	0.2004	0.4007	0.4809	0.2633	0.1316	0.1097
300 000	0.311	0.3112	0.3117	0.312	0.342	0.3422	0.3426	0.343	0.1993	0.3985	0.4758	0.2592	0.1296	0.1080
300 000	0.311	0.3112	0.3117	0.312	0.342	0.3422	0.3426	0.343	0.1974	0.3947	0.4737	0.2592	0.1296	0.1080
336 400	0.278	0.2782	0.2788	0.279	0.306	0.3062	0.3067	0.307	0.1953	0.3907	0.4688	0.2551	0.1276	0.1063
336 400	0.278	0.2782	0.2788	0.279	0.306	0.3062	0.3067	0.307	0.1945	0.3890	0.4668	0.2549	0.1274	0.1062
350 000	0.267	0.2672	0.2678	0.268	0.294	0.2942	0.2947	0.295	0.1935	0.3870	0.4644	0.2537	0.1268	0.1057
397 500	0.235	0.2352	0.2359	0.236	0.258	0.2582	0.2589	0.259	0.1911	0.3822	0.4587	0.2491	0.1246	0.1038
477 000	0.196	0.1963	0.1971	0.198	0.215	0.2153	0.2160	0.216	0.1865	0.3730	0.4476	0.2429	0.1214	0.1012
500 000	0.187	0.1873	0.1882	0.189	0.206	0.2062	0.2070	0.208	0.1853	0.3707	0.4446	0.2412	0.1206	0.1005
500 000	0.187	0.1873	0.1882	0.189	0.206	0.2062	0.2070	0.208	0.1845	0.3689	0.4427	0.2410	0.1205	0.1004
556 500	0.168	0.1683	0.1693	0.170	0.185	0.1853	0.1862	0.187	0.1826	0.3652	0.4383	0.2374	0.1187	0.0989
636 000	0.147	0.1474	0.1484	0.149	0.162	0.1623	0.1633	0.164	0.1785	0.3569	0.4283	0.2323	0.1162	0.0968
715 500	0.131	0.1314	0.1326	0.133	0.144	0.1444	0.1455	0.146	0.1754	0.3508	0.4210	0.2285	0.1141	0.0951
750 000	0.125	0.1254	0.1267	0.127	0.137	0.1374	0.1385	0.139	0.1743	0.3485	0.4182	0.2266	0.1133	0.0944
750 000	0.125	0.1254	0.1267	0.127	0.137	0.1374	0.1385	0.139	0.1739	0.3477	0.4173	0.2283	0.1132	0.0943
795 000	0.117	0.1175	0.1188	0.120	0.129	0.1294	0.1306	0.131	0.1728	0.3455	0.4146	0.2244	0.1122	0.0935
874 500	0.107	0.1075	0.1089	0.110	0.118	0.1185	0.1198	0.121	0.1703	0.3407	0.4088	0.2210	0.1105	0.0921
954 000	0.0979	0.0985	0.1002	0.100	0.108	0.1085	0.1100	0.111	0.1682	0.3363	0.4038	0.2179	0.1090	0.0908
1 000 000	0.0934	0.0940	0.0956	0.0966	0.103	0.1035	0.1050	0.106	0.1666	0.3332	0.3999	0.2162	0.1081	0.0901
1 000 000	0.0934	0.0940	0.0956	0.0966	0.103	0.1035	0.1050	0.106	0.1664	0.3328	0.3994	0.2160	0.1080	0.0900
1 033 500	0.0904	0.0910	0.0927	0.0936	0.0994	[illegible]	0.1015	0.102	0.1661	0.3322	0.3987	0.2150	0.1075	0.0896
1 113 000	0.0839	0.0845	0.0864	0.0874	0.0922	0.0928	0.0945	0.0954	0.1639	0.3278	0.3934	0.2124	0.1062	0.0885
1 192 500	0.0783	0.0790	0.0810	0.0821	0.0860	0.0866	0.0884	0.0895	0.1622	0.3243	0.3892	0.2100	0.1050	0.0875
1 192 500	0.0783	0.0790	0.0810	0.0821	0.0860	0.0866	0.0884	0.0895	0.1620	0.3240	0.3888	0.2098	0.1049	0.0874
1 272 000	0.0734	0.0741	0.0762	0.0774	0.0806	0.0813	0.0832	0.0843	0.1606	0.3211	0.3853	0.2076	0.1038	0.0865
1 351 500	0.0691	0.0699	0.0721	0.0733	0.0760	0.0767	0.0787	0.0798	0.1590	0.3180	0.3816	0.2054	0.1027	0.0856
1 431 000	0.0653	0.0661	0.0685	0.0697	0.0719	0.0725	0.0747	0.0759	0.1576	0.3152	0.3782	0.2033	0.1016	0.0847
1 510 500	0.0618	0.0627	0.0651	0.0665	0.0679	0.0687	0.0710	0.0722	0.1562	0.3123	0.3748	0.2014	0.1007	0.0839
1 590 000	0.0597	0.0598	0.0622	0.0636	0.0645	0.0653	0.0677	0.0690	0.1549	0.3098	0.3718	0.1997	0.0998	0.0832
1 590 000	0.0587	0.0596	0.0622	0.0638	0.0645	0.0653	0.0677	0.0690	0.1547	0.3094	0.3713	0.1997	0.0998	0.0832

*For conductor at 75°C, wind 1.4 miles per hour (2 ft/sec), frequency = 60 cycles.

TABLE A.3 Characteristics of Aluminum 25le, Steel Reinforced [1] (Aluminum Company of America)

Circular Mils or A.W.G. Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent* Circular Mils or A.W.G.	Ultimate Strength Pounds	Wt Lbs per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity† Amps	r_a 25°C (77°F) Small Currents d-c	r_a 25°C 25 cycles	r_a 25°C 50 cycles	r_a 25°C 60 cycles	r_a 50°C (122°F) Current Approx. 75% Capacity‡ d-c	r_a 50°C 25 cycles	r_a 50°C 50 cycles	r_a 50°C 60 cycles	x_a 25 cycles	x_a 50 cycles	x_a 60 cycles	x_a' 25 cycles	x_a' 50 cycles	x_a' 60 cycles
1 590 000	54	3	0.1716	19	0.1030	1.545	1 000 000	56 000	10 777	0.0520	1 380	0.0587	0.0588	0.0590	0.0591	0.0646	0.0656	0.0675	0.0684	0.1495	0.299	0.359	0.1953	0.0977	0.0814
1 510 500	54	3	0.1673	19	0.1004	1.506	950 000	53 200	10 237	0.0507	1 340	0.0618	0.0619	0.0621	0.0622	0.0680	0.0690	0.0710	0.0720	0.1508	0.302	0.362	0.1971	0.0986	0.0821
1 431 000	54	3	0.1628	19	0.0977	1.465	900 000	50 400	9 699	0.0493	1 300	0.0652	0.0653	0.0655	0.0656	0.0718	0.0729	0.0749	0.0760	0.1522	0.304	0.365	0.1991	0.0996	0.0830
1 351 000	54	3	0.1582	19	0.0949	1.424	850 000	47 600	9 160	0.0479	1 250	0.0691	0.0692	0.0694	0.0695	0.0761	0.0771	0.0792	0.0803	0.1536	0.307	0.369	0.201	0.1006	0.0838
1 272 000	54	3	0.1535	19	0.0921	1.382	800 000	44 800	8 621	0.0465	1 200	0.0734	0.0735	0.0737	0.0738	0.0808	0.0819	0.0840	0.0851	0.1551	0.310	0.372	0.203	0.1015	0.0847
1 192 500	54	3	0.1486	19	0.0892	1.338	750 000	43 100	8 082	0.0450	1 160	0.0783	0.0784	0.0786	0.0788	0.0862	0.0872	0.0894	0.0906	0.1568	0.314	0.376	0.206	0.1028	0.0857
1 113 000	54	3	0.1436	19	0.0862	1.293	700 000	40 200	7 544	0.0435	1 110	0.0839	0.0840	0.0842	0.0844	0.0924	0.0935	0.0957	0.0969	0.1585	0.317	0.380	0.208	0.1040	0.0867
1 033 500	54	3	0.1384	7	0.1384	1.246	650 000	37 100	7 019	0.0420	1 060	0.0903	0.0905	0.0907	0.0909	0.0994	0.1005	0.1025	0.1035	0.1603	0.321	0.385	0.211	0.1053	0.0878
954 000	54	3	0.1329	7	0.1329	1.196	600 000	34 200	6 479	0.0403	1 010	0.0979	0.0980	0.0981	0.0982	0.1078	0.1088	0.1118	0.1128	0.1624	0.325	0.390	0.214	0.1068	0.0890
900 000	54	3	0.1291	7	0.1291	1.162	566 000	32 300	6 112	0.0391	970	0.104	0.104	0.104	0.104	0.1145	0.1155	0.1175	0.1185	0.1639	0.328	0.393	0.216	0.1078	0.0898
874 500	54	3	0.1273	7	0.1273	1.146	550 000	31 400	5 940	0.0386	950	0.107	0.107	0.107	0.108	0.1178	0.1188	0.1218	0.1228	0.1646	0.329	0.395	0.217	0.1083	0.0903
795 000	54	3	0.1214	7	0.1214	1.093	500 000	28 500	5 399	0.0368	900	0.117	0.118	0.118	0.119	0.1288	0.1308	0.1358	0.1378	0.1670	0.334	0.401	0.220	0.1100	0.0917
795 000	26	2	0.1749	7	0.1360	1.108	500 000	31 200	5 770	0.0375	900	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1660	0.332	0.399	0.219	0.1095	0.0912
795 000	30	2	0.1628	19	0.0977	1.140	500 000	38 400	6 517	0.0393	910	0.117	0.117	0.117	0.117	0.1288	0.1288	0.1288	0.1288	0.1637	0.327	0.393	0.217	0.1085	0.0904
715 500	54	3	0.1151	7	0.1151	1.036	450 000	26 300	4 859	0.0349	830	0.131	0.131	0.131	0.132	0.1442	0.1452	0.1472	0.1482	0.1697	0.339	0.407	0.224	0.1119	0.0932
715 500	26	2	0.1659	7	0.1290	1.051	450 000	28 100	5 193	0.0355	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1687	0.337	0.405	0.223	0.1114	0.0928
715 500	30	2	0.1544	19	0.0926	1.081	450 000	34 600	5 865	0.0372	840	0.131	0.131	0.131	0.131	0.1442	0.1442	0.1442	0.1442	0.1664	0.333	0.399	0.221	0.1104	0.0920
666 600	54	3	0.1111	7	0.1111	1.000	419 000	24 500	4 527	0.0337	800	0.140	0.140	0.141	0.141	0.1541	0.1571	0.1591	0.1601	0.1715	0.343	0.412	0.226	0.1132	0.0943
636 000	54	3	0.1085	7	0.1085	0.977	400 000	23 600	4 319	0.0329	770	0.147	0.147	0.148	0.148	0.1618	0.1638	0.1678	0.1688	0.1726	0.345	0.414	0.228	0.1140	0.0950
636 000	26	2	0.1564	7	0.1216	0.990	400 000	25 000	4 616	0.0335	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1718	0.344	0.412	0.227	0.1135	0.0946
636 000	30	2	0.1456	19	0.0874	1.019	400 000	31 500	5 213	0.0351	780	0.147	0.147	0.147	0.147	0.1618	0.1618	0.1618	0.1618	0.1693	0.339	0.406	0.225	0.1125	0.0937
605 000	54	3	0.1059	7	0.1059	0.953	380 500	22 500	4 109	0.0321	750	0.154	0.155	0.155	0.155	0.1695	0.1715	0.1755	0.1775	0.1739	0.348	0.417	0.230	0.1149	0.0957
605 000	26	2	0.1525	7	0.1186	0.966	380 500	24 100	4 391	0.0327	760	0.154	0.154	0.154	0.154	0.1700	0.1720	0.1720	0.1720	0.1730	0.346	0.415	0.229	0.1144	0.0953
556 500	26	2	0.1463	7	0.1138	0.927	350 000	22 400	4 039	0.0313	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1751	0.350	0.420	0.232	0.1159	0.0965
556 500	30	2	0.1362	7	0.1362	0.953	350 000	27 200	4 588	0.0328	730	0.168	0.168	0.168	0.168	0.1849	0.1859	0.1859	0.1859	0.1728	0.346	0.415	0.230	0.1149	0.0957
500 000	30	2	0.1291	7	0.1291	0.904	314 500	24 400	4 122	0.0311	690	0.187	0.187	0.187	0.187	0.206				0.1754	0.351	0.421	0.234	0.1167	0.0973
477 000	26	2	0.1355	7	0.1054	0.858	300 000	19 430	3 462	0.0290	670	0.196	0.196	0.196	0.196	0.216				0.1790	0.358	0.430	0.237	0.1186	0.0988
477 000	30	2	0.1261	7	0.1261	0.883	300 000	23 300	3 933	0.0304	670	0.196	0.196	0.196	0.196	0.216				0.1766	0.353	0.424	0.238	0.1176	0.0980
397 500	26	2	0.1236	7	0.0961	0.783	250 000	16 190	2 885	0.0265	590	0.235				0.259				0.1836	0.367	0.441	0.244	0.1219	0.1015
397 500	30	2	0.1151	7	0.1151	0.806	250 000	19 980	3 277	0.0278	600	0.235	Same as d-c			0.259	Same as d-c			0.1812	0.362	0.435	0.242	0.1208	0.1006
336 400	26	2	0.1138	7	0.0885	0.721	4/0	14 050	2 442	0.0244	530	0.278				0.306				0.1872	0.376	0.451	0.250	0.1248	0.1039
336 400	30	2	0.1059	7	0.1059	0.741	4/0	17 040	2 774	0.0255	530	0.278				0.306				0.1855	0.371	0.445	0.248	0.1238	0.1032
300 000	26	2	0.1074	7	0.0835	0.680	188 700	12 650	2 178	0.0230	490	0.311				0.342				0.1908	0.382	0.458	0.254	0.1269	0.1057
300 000	30	2	0.1000	7	0.1000	0.700	188 700	15 430	2 473	0.0241	500	0.311				0.342				0.1883	0.377	0.452	0.252	0.1258	0.1049
266 800	26	2	0.1013	7	0.0788	0.642	3/0	11 250	1 936	0.0217	460	0.350				0.385				0.1936	0.387	0.465	0.258	0.1289	0.1074

Single Layer Conductors

Circular Mils or A.W.G. Aluminum	Aluminum Strands	Aluminum Layers	Aluminum Strand Dia. Inches	Steel Strands	Steel Strand Dia. Inches	Outside Diameter Inches	Copper Equivalent*	Ultimate Strength Pounds	Wt Lbs per Mile	Geometric Mean Radius at 60 Cycles Feet, For Current Approx. 75% Capacity‡	Approx. Current Carrying Capacity† Amps	r_a 25°C d-c	r_a 25°C 25 cycles	r_a 25°C 50 cycles	r_a 25°C 60 cycles	r_a 50°C d-c	r_a 50°C 25 cycles	r_a 50°C 50 cycles	r_a 50°C 60 cycles	x_a Small Currents 25 cycles	x_a Small Currents 50 cycles	x_a Small Currents 60 cycles	x_a Current Approx. 75% Capacity‡ 25 cycles	x_a Current Approx. 75% Capacity‡ 50 cycles	x_a Current Approx. 75% Capacity‡ 60 cycles	x_a' 25 cycles	x_a' 50 cycles	x_a' 60 cycles
266 800	6	1	0.2109	7	0.0703	0.633	3/0	9 645	1 802	0.00684	460	0.351	0.351	0.351	0.352	0.386	0.430	0.510	0.552	0.194	0.388	0.466	0.252	0.504	0.605	0.259	0.1294	0.1079
4/0	6	1	0.1878	1	0.1878	0.563	2/0	8 420	1 542	0.00814	340	0.441	0.442	0.444	0.445	0.485	0.514	0.567	0.592	0.218	0.437	0.524	0.242	0.484	0.581	0.267	0.1336	0.1113
3/0	6	1	0.1672	1	0.1672	0.502	1/0	6 675	1 223	0.00600	300	0.556	0.557	0.559	0.560	0.612	0.642	0.697	0.723	0.225	0.450	0.540	0.259	0.517	0.621	0.275	0.1377	0.1147
2/0	6	1	0.1490	1	0.1490	0.447	1	5 345	970	0.00510	270	0.702	0.702	0.704	0.706	0.773	0.806	0.866	0.895	0.231	0.462	0.554	0.267	0.534	0.641	0.284	0.1418	0.1182
1/0	6	1	0.1327	1	0.1327	0.398	2	4 280	769	0.00446	230	0.885	0.885	0.887	0.888	0.974	1.01	1.08	1.12	0.237	0.473	0.568	0.273	0.547	0.656	0.292	0.1460	0.1216
1	6	1	0.1182	1	0.1182	0.355	3	3 480	610	0.00418	200	1.12	1.12	1.12	1.12	1.23	1.27	1.34	1.38	0.242	0.483	0.580	0.277	0.554	0.665	0.300	0.1500	0.1250
2	6	1	0.1052	1	0.1052	0.316	4	2 790	484	0.00418	180	1.41	1.41	1.41	1.41	1.55	1.59	1.56	1.69	0.247	0.493	0.592	0.277	0.554	0.665	0.306	0.1542	0.1285
2	7	1	0.0974	1	0.1299	0.325	4	3 525	566	0.00504	180	1.41	1.41	1.41	1.41	1.55	1.59	1.62	1.65	0.247	0.493	0.592	0.267	0.535	0.642	0.306	0.1532	0.1276
3	6	1	0.0937	1	0.0937	0.281	5	2 250	384	0.00430	160	1.78	1.78	1.78	1.78	1.95	1.98	2.04	2.07	0.252	0.503	0.604	0.275	0.551	0.661	0.317	0.1583	0.1320
4	6	1	0.0834	1	0.0834	0.250	6	1 830	304	0.00437	140	2.24	2.24	2.24	2.24	2.47	2.50	2.54	2.57	0.257	0.514	0.611	0.274	0.549	0.659	0.325	0.1627	0.1356
4	7	1	0.0772	1	0.1029	0.257	6	2 288	356	0.00452	140	2.24	2.24	2.24	2.24	2.47	2.50	2.53	2.55	0.257	0.515	0.618	0.273	0.545	0.655	0.323	0.1615	0.1346
5	6	1	0.0743	1	0.0743	0.223	7	1 460	241	0.00416	120	2.82	2.82	2.82	2.82	3.10	3.12	3.16	3.18	0.262	0.525	0.630	0.279	0.557	0.665	0.333	0.1666	0.1388
6	6	1	0.0661	1	0.0661	0.198	8	1 170	191	0.00394	100	3.56	3.56	3.56	3.56	3.92	3.94	3.97	3.98	0.268	0.536	0.643	0.281	0.561	0.673	0.342	0.1708	0.1423

Column headings: r_a = Resistance, Ohms per Conductor per Mile; x_a = Inductive Reactance, Ohms per Conductor per Mile at 1 Ft. Spacing, All Currents; x_a' = Shunt Capacitive Reactance, Megohms per Conductor per Mile at 1 Ft. Spacing.

*Based on copper 97 per cent, aluminum 61 per cent conductivity.
†For conductor at 75°C., air at 25°C., wind 1.4 miles per hour (2 ft/sec), frequency = 60 cycles.
‡"Current Approx. 75% Capacity" is 75% of the "Approx. Current Carrying Capacity in Amps." and is approximately the current which will produce 50°C. conductor temp. (25°C. rise) with 25°C. air temp., wind 1.4 miles per hour.

TABLE A.4 Characteristics of "Expanded" Aluminum Cable, Steel Reinforced [1] (Aluminum Company of America)

Circular Mils or A.W.G. Aluminum	Aluminum			Steel		Filler Section				Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Ultimate Strength Pounds	Weight Pounds per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx. Current Carrying Capacity Amps	r_a Resistance Ohms per Conductor per Mile								x_a Inductive Reactance Ohms per Conductor per Mile at 1 Ft. Spacing All Currents			x_a' Shunt Capacitive Reactance Megohms per Conductor per Mile at 1 Ft. Spacing		
						Aluminum		Paper								25°C. (77°F.) Small Currents				50°C. (122°F.) Current Approx. 75% Capacity									
	Strands	Layers	Strand Dia. Inches	Strands	Strand Dia. Inches	Strands	Strand Dia. Inches	Strands	Layers							d-c	25 cycles	50 cycles	60 cycles	d-c	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles	25 cycles	50 cycles	60 cycles
850 000	54	2	0.1255	19	0.0834	4	0 1182	23	2	1.38	534 000	35 371	7 200																
1 150 000	54	2	0 1409	19	0 0921	4	0.1353	24	2	1 55	724 000	41 900	9 070	(1)	(1)	(1)				(1)				(1)	(1)	(1)	(1)	(1)	(1)
1 338 000	66	2	0 1350	19	0 100	4	0.184	18	2	1 75	840 000	49 278	11 340																

(1) Electrical Characteristics not available until laboratory measurements are completed.

TABLE A.5 Characteristics of Copperweld-Copper Conductors [1, 2] (Copperweld Steel Company)

Size of Conductor: Nominal Designation	Number and Diameter of Wires: Copperweld	Number and Diameter of Wires: Copper	Outside Diameter Inches	Copper Equivalent Circular Mils or A.W.G.	Rated Breaking Load Lbs	Weight Lbs per Mile	Geometric Mean Radius at 60 Cycles Feet	Approx Current Carrying Capacity at 60 Cycles Amps*	r_a Resistance Ohms per Conductor per Mile at 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a Resistance Ohms per Conductor per Mile at 50°C (122°F) Current Approx. 75% of Capacity**: d-c	25 cycles	50 cycles	60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile One ft. Spacing Average Currents: 25 cycles	50 cycles	60 cycles	x_a' Capacitive Reactance Megohms per Conductor per Mile One ft. Spacing: 25 cycles	50 cycles	60 cycles
350 E	7x.1576"	12x.1576"	0.788	350 000	32 420	7 409	0.0220	660	0.1658	0.1728	0.1789	0.1812	0.1812	0.1915	0.201	0.204	0.1929	0.386	0.463	0.243	0.1216	0.1014
350 EK	4x.1470"	15x.1470"	0.735	350 000	23 850	6 536	0.0245	680	0.1658	0.1682	0.1700	0.1705	0.1812	0.1845	0.1873	0.1882	0.1875	0.375	0.450	0.248	0.1241	0.1034
350 V	3x.1751"	9x.1893"	0.754	350 000	23 480	6 578	0.0226	650	0.1655	0.1725	0.1800	0.1828	0.1809	0.1910	0.202	0.206	0.1915	0.383	0.460	0.246	0.1232	0.1027
300 E	7x.1459"	12x.1459"	0.729	300 000	27 770	6 351	0.0204	600	0.1934	0.200	0.207	0.209	0.211	0.222	0.232	0.235	0.1969	0.394	0.473	0.249	0.1244	0.1037
300 EK	4x.1361"	15x.1361"	0.680	300 000	20 960	5 602	0.0227	610	0.1934	0.1958	0.1976	0.198	0.211	0.215	0.218	0.219	0.1914	0.383	0.460	0.254	0.1269	0.1057
300 V	3x.1621"	9x.1752"	0.698	300 000	20 730	5 639	0.0209	590	0.1930	0.200	0.208	0.210	0.211	0.222	0.233	0.237	0.1954	0.391	0.469	0.252	0.1259	0.1050
250 E	7x.1332"	12x.1332"	0.666	250 000	23 920	5 292	0.01859	540	0.232	0.239	0.245	0.248	0.254	0.265	0.275	0.279	0.202	0.403	0.484	0.255	0.1276	0.1064
250 EK	4x.1242"	15x.1242"	0.621	250 000	17 840	4 669	0.0207	540	0.232	0.235	0.238	0.237	0.254	0.258	0.261	0.261	0.1960	0.392	0.471	0.260	0.1301	0.1084
250 V	3x.1480"	9x.1600"	0.637	250 000	17 420	4 699	0.01911	530	0.232	0.239	0.246	0.249	0.253	0.264	0.276	0.281	0.200	0.400	0.480	0.258	0.1292	0.1077
4/0 E	7x.1225"	12x.1225"	0.613	4/0	20 730	4 479	0.01711	480	0.274	0.281	0.287	0.290	0.300	0.312	0.323	0.326	0.208	0.411	0.493	0.261	0.1306	0.1088
4/0 G	2x.1944"	5x.1944"	0.583	4/0	15 640	4 168	0.01409	460	0.273	0.284	0.294	0.298	0.299	0.318	0.336	0.342	0.215	0.431	0.517	0.265	0.1324	0.1103
4/0 EK	4x.1143"	15x.1143"	0.571	4/0	15 370	3 851	0.01903	490	0.274	0.277	0.278	0.279	0.300	0.304	0.307	0.308	0.200	0.401	0.481	0.266	0.1331	0.1109
4/0 V	3x.1361"	9x.1472"	0.586	4/0	15 000	3 877	0.01758	470	0.274	0.281	0.288	0.291	0.299	0.311	0.323	0.328	0.204	0.409	0.490	0.264	0.1322	0.1101
4/0 F	1x.1833"	6x.1833"	0.550	4/0	12 290	3 750	0.01558	470	0.273	0.280	0.285	0.287	0.299	0.309	0.318	0.322	0.210	0.421	0.505	0.269	0.1344	0.1220
3/0 E	7x.1091"	12x.1091"	0.545	3/0	16 800	3 552	0.01521	420	0.346	0.353	0.359	0.361	0.378	0.391	0.402	0.407	0.212	0.423	0.508	0.270	0.1348	0.1123
3/0 J	3x.1851"	4x.1851"	0.555	3/0	16 170	3 732	0.01150	410	0.344	0.356	0.367	0.372	0.377	0.398	0.419	0.428	0.225	0.451	0.541	0.268	0.1341	0.1118
3/0 G	2x.1731"	2x.1731"	0.519	3/0	12 860	3 305	0.01254	400	0.344	0.355	0.365	0.369	0.377	0.397	0.416	0.423	0.221	0.443	0.531	0.273	0.1365	0.1137
3/0 EK	4x.1018"	4x.1018"	0.509	3/0	12 370	3 134	0.01607	420	0.346	0.348	0.350	0.351	0.378	0.382	0.386	0.388	0.206	0.412	0.495	0.274	0.1372	0.1143
3/0 V	3x.1311"	9x.1311"	0.522	3/0	12 220	3 154	0.01588	410	0.345	0.352	0.360	0.362	0.377	0.390	0.403	0.408	0.210	0.420	0.504	0.273	0.1363	0.1138
3/0 F	1x.1632"	6x.1632"	0.490	3/0	9 980	2 974	0.01388	410	0.344	0.351	0.356	0.358	0.377	0.388	0.397	0.401	0.216	0.432	0.519	0.277	0.1385	0.1155
2/0 K	4x.1780"	3x.1780"	0.534	2/0	17 600	3 411	0.00912	360	0.434	0.447	0.459	0.466	0.475	0.499	0.524	0.535	0.237	0.475	0.570	0.271	0.1355	0.1129
2/0 J	3x.1648"	4x.1648"	0.494	2/0	13 430	2 960	0.01029	350	0.434	0.446	0.457	0.462	0.475	0.498	0.520	0.530	0.231	0.463	0.555	0.277	0.1383	0.1162
2/0 G	2x.1542"	5x.1542"	0.463	2/0	10 510	2 622	0.01119	350	0.434	0.445	0.456	0.459	0.475	0.497	0.518	0.525	0.227	0.454	0.545	0.281	0.1406	0.1171
2/0 V	3x.1080"	9x.1167"	0.465	2/0	9 846	2 502	0.01395	360	0.433	0.442	0.450	0.452	0.476	0.489	0.504	0.509	0.216	0.432	0.518	0.281	0.1404	0.1170
2/0 F	1x.1454"	6x.1454"	0.436	2/0	8 094	2 359	0.01235	350	0.434	0.441	0.446	0.448	0.475	0.487	0.497	0.501	0.222	0.444	0.533	0.285	0.1427	0.1189
1/0 K	4x.1585"	3x.1585"	0.475	1/0	14 490	2 703	0.00812	310	0.549	0.560	0.573	0.579	0.599	0.625	0.652	0.664	0.243	0.487	0.584	0.279	0.1397	0.1164
1/0 J	3x.1467"	4x.1467"	0.440	1/0	10 970	2 348	0.00917	310	0.548	0.559	0.570	0.576	0.599	0.624	0.648	0.659	0.237	0.474	0.569	0.285	0.1423	0.1186
1/0 G	2x.1373"	5x.1373"	0.412	1/0	8 563	2 078	0.00998	310	0.548	0.559	0.568	0.573	0.599	0.623	0.645	0.654	0.233	0.466	0.559	0.289	0.1447	0.1206
1/0 F	1x.1294"	6x.1294"	0.388	1/0	6 536	1 870	0.01099	310	0.548	0.554	0.559	0.562	0.599	0.612	0.622	0.627	0.228	0.456	0.547	0.294	0.1469	0.1224
1 N	5x.1546"	2x.1546"	0.464	1	15 410	2 541	0.00638	280	0.691	0.705	0.719	0.726	0.755	0.787	0.818	0.832	0.256	0.512	0.614	0.281	0.1405	0.1171
1 K	4x.1412"	3x.1412"	0.423	1	11 900	2 144	0.00721	270	0.691	0.704	0.716	0.722	0.755	0.784	0.813	0.825	0.249	0.498	0.598	0.288	0.1438	0.1198
1 J	3x.1307"	4x.1307"	0.392	1	9 000	1 881	0.00817	270	0.691	0.703	0.714	0.719	0.755	0.783	0.808	0.820	0.243	0.486	0.583	0.293	0.1465	0.1221
1 G	2x.1222"	5x.1222"	0.367	1	6 956	1 649	0.00867	260	0.691	0.702	0.712	0.716	0.755	0.781	0.805	0.815	0.219	0.478	0.573	0.298	0.1488	0.1240
1 F	1x.1153"	6x.1153"	0.346	1	5 266	1 483	0.00980	270	0.691	0.698	0.704	0.705	0.755	0.769	0.781	0.786	0.234	0.468	0.561	0.302	0.1509	0.1258
2 P	6x.1540"	1x.1540"	0.462	2	16 870	2 487	0.00501	250	0.871	0.886	0.901	0.909	0.952	0.988	1.024	1.040	0.268	0.536	0.643	0.281	0.1406	0.1172
2 N	5x.1377"	2x.1377"	0.413	2	12 680	2 015	0.00568	240	0.871	0.885	0.899	0.906	0.952	0.986	1.020	1.035	0.261	0.523	0.627	0.289	0.1446	0.1205
2 K	4x.1257"	3x.1257"	0.377	2	9 730	1 701	0.00644	240	0.871	0.884	0.896	0.902	0.952	0.983	1.014	1.028	0.255	0.510	0.612	0.296	0.1479	0.1232
2 J	3x.1164"	4x.1164"	0.349	2	7 322	1 476	0.00727	230	0.871	0.883	0.894	0.899	0.952	0.982	1.010	1.022	0.249	0.498	0.598	0.301	0.1506	0.1255
2 A	1x.1699"	2x.1699"	0.366	2	6 876	1 356	0.00763	240	0.869	0.875	0.880	0.882	0.950	0.962	0.973	0.979	0.247	0.493	0.592	0.298	0.1489	0.1241
2 G	2x.1089"	5x.1089"	0.327	2	5 626	1 307	0.00790	230	0.871	0.882	0.892	0.896	0.952	0.980	1.006	1.016	0.245	0.489	0.587	0.306	0.1529	0.1275
2 F	1x.1026"	6x.1026"	0.308	2	4 233	1 176	0.00873	230	0.871	0.878	0.884	0.886	0.952	0.967	0.979	0.985	0.230	0.479	0.575	0.310	0.1551	0.1292
3 P	6x.1371"	1x.1371"	0.411	3	13 910	1 973	0.00445	220	1.098	1.113	1.127	1.136	1.200	1.239	1.273	1.296	0.274	0.547	0.657	0.290	0.1448	0.1207
3 N	5x.1226"	2x.1226"	0.368	3	10 390	1 598	0.00506	210	1.098	1.112	1.126	1.133	1.200	1.237	1.273	1.289	0.267	0.534	0.641	0.298	0.1487	0.1239
3 K	4x.1120"	3x.1120"	0.336	3	7 910	1 349	0.00574	210	1.098	1.111	1.123	1.129	1.200	1.233	1.267	1.281	0.261	0.522	0.626	0.304	0.1520	0.1266
3 J	3x.1036"	4x.1036"	0.311	3	5 955	1 171	0.00648	200	1.098	1.110	1.121	1.126	1.200	1.232	1.262	1.275	0.255	0.509	0.611	0.309	0.1547	0.1289
3 A	1x.1513"	2x.1513"	0.326	3	4 810	1 076	0.00679	210	1.096	1.102	1.107	1.109	1.198	1.211	1.225	1.229	0.252	0.505	0.606	0.306	0.1531	0.1275
4 P	6x.1221"	1x.1221"	0.366	4	11 420	1 664	0.00397	190	1.385	1.400	1.411	1.423	1.514	1.555	1.598	1.616	0.280	0.559	0.671	0.298	0.1489	0.1241
4 N	5x.1092"	2x.1092"	0.328	4	8 460	1 267	0.00451	180	1.385	1.390	1.413	1.420	1.514	1.554	1.593	1.610	0.273	0.546	0.656	0.306	0.1528	0.1274
4 D	2x.1615"	1x.1615"	0.348	4	7 340	1 191	0.00566	190	1.382	1.389	1.396	1.399	1.511	1.529	1.544	1.542	0.262	0.523	0.628	0.301	0.1507	0.1256
4 A	1x.1347"	2x.1347"	0.290	4	3 938	853	0.00604	180	1.382	1.388	1.393	1.395	1.511	1.525	1.540	1.545	0.258	0.517	0.620	0.314	0.1572	0.1310
5 P	6x.1087"	1x.1087"	0.326	5	9 311	1 240	0.00353	160	1.747	1.762	1.776	1.785	1.909	1.954	2.00	2.02	0.286	0.571	0.686	0.306	0.1531	0.1276
5 D	2x.1438"	1x.1438"	0.310	5	6 035	944	0.00504	160	1.742	1.749	1.756	1.759	1.905	1.924	1.941	1.939	0.268	0.535	0.642	0.310	0.1548	0.1290
5 A	1x.1200"	2x.1200"	0.258	5	3 193	676	0.00538	160	1.742	1.748	1.753	1.755	1.905	1.920	1.936	1.941	0.264	0.528	0.634	0.323	0.1614	0.1345
6 D	2x.1281"	1x.1281"	0.276	6	4 942	749	0.00449	140	2.20	2.21	2.21	2.22	2.40	2.42	2.44	2.44	0.273	0.547	0.656	0.318	0.1590	0.1326
6 A	1x.1068"	2x.1068"	0.230	6	2 585	536	0.00479	140	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.270	0.540	0.648	0.331	0.1655	0.1379
6 C	1x.1046"	2x.1046"	0.225	6	2 143	514	0.00469	130	2.20	2.20	2.21	2.21	2.40	2.42	2.44	2.44	0.271	0.542	0.651	0.333	0.1663	0.1386
7 D	2x.1141"	1x.1141"	0.246	7	4 022	594	0.00400	120	2.77	2.78	2.79	2.79	3.03	3.05	3.07	3.07	0.279	0.558	0.670	0.326	0.1631	0.1359
7 A	1x.1266"	2x.0895"	0.223	7	2 754	495	0.00441	120	2.77	2.78	2.78	2.78	3.03	3.06	3.07	3.07	0.274	0.548	0.658	0.333	0.1666	0.1388
8 D	2x.1016"	1x.1016"	0.219	8	3 284	471	0.00356	110	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.285	0.570	0.684	0.334	0.1672	0.1393
8 A	1x.1127"	2x.0797"	0.199	8	2 233	392	0.00394	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.87	0.280	0.560	0.672	0.341	0.1706	0.1422
8 C	1x.0808"	2x.0834"	0.179	8	1 362	320	0.00373	100	3.49	3.50	3.51	3.51	3.82	3.84	3.86	3.86	0.283	0.565	0.679	0.349	0.1744	0.1453
9½ D	2x.0808"	1x.0808"	0.174	9½	1 743	298	0.00283	85	4.91	4.92	4.92	4.93	5.37	5.39	5.42	5.42	0.297	0.593	0.712	0.351	0.1754	0.1462

*Based on a conductor temperature of 75°C. and an ambient of 25°C., wind 1.4 miles per hour (2 ft/sec.), frequency = 60 cycles, average tarnished surface.

**Resistances at 50°C. total temperature, based on an ambient of 25°C. plus 25°C. rise due to heating effect of current. The approximate magnitude of current necessary to produce the 25°C. rise is 75% of the "Approximate Current Carrying Capacity at 60 cycles."

TABLE A.6 Characteristics of Copperweld Conductors [1, 2] (Copperweld Steel Company)

Nominal Conductor Size	Number and Size of Wires	Outside Diameter Inches	Area of Conductor Circular Mils	Rated Breaking Load Pounds, Strength: High	Rated Breaking Load Pounds, Strength: Extra High	Weight Pounds per Mile	Geometric Mean Radius at 60 cycles and Average Currents Feet	Approx Current Carrying Capacity* Amps at 60 Cycles	r_a Resistance Ohms per Conductor per Mile at 25°C (77°F) Small Currents: d-c	25 cycles	50 cycles	60 cycles	r_a Resistance Ohms per Conductor per Mile at 75°C (167°F) Current Approx. 75% of Capacity**: d-c	25 cycles	50 cycles	60 cycles	x_a Inductive Reactance Ohms per Conductor per Mile One Ft. Spacing Average Currents: 25 cycles	50 cycles	60 cycles	x_a' Capacitive Reactance Megohms per Conductor per Mile One Ft. Spacing: 25 cycles	50 cycles	60 cycles
30% Conductivity																						
7/8"	19 No. 5	0.910	628 900	55 570	66 910	9 344	0.00758	620	0.306	0.316	0.326	0.331	0.363	0.419	0.476	0.499	0.261	0.493	0.592	0.233	0.1165	0.0971
13/16"	19 No. 6	0.810	498 800	45 830	55 530	7 410	0.00675	540	0.386	0.396	0.406	0.411	0.458	0.518	0.580	0.605	0.267	0.505	0.606	0.241	0.1206	0.1005
23/32"	19 No. 7	0.721	395 500	37 740	45 850	5 877	0.00601	470	0.486	0.496	0.506	0.511	0.577	0.643	0.710	0.737	0.273	0.517	0.621	0.250	0.1248	0.1040
21/32"	19 No. 8	0.642	313 700	31 040	37 690	4 660	0.00535	410	0.613	0.623	0.633	0.638	0.728	0.799	0.872	0.902	0.279	0.529	0.635	0.258	0.1289	0.1074
9/16"	19 No. 9	0.572	248 800	25 500	30 610	3 696	0.00477	360	0.773	0.783	0.793	0.798	0.917	0.995	1.075	1.106	0.285	0.541	0.649	0.266	0.1330	0.1109
5/8"	7 No. 4	0.613	292 200	24 780	29 430	4 324	0.00511	410	0.656	0.664	0.672	0.676	0.778	0.824	0.870	0.887	0.281	0.533	0.640	0.261	0.1306	0.1088
9/16"	7 No. 5	0.546	231 700	20 470	24 650	3 429	0.00455	360	0.827	0.835	0.843	0.847	0.981	1.030	1.080	1.099	0.287	0.545	0.654	0.269	0.1347	0.1122
1/2"	7 No. 6	0.486	183 800	16 890	20 460	2 719	0.00405	310	1.042	1.050	1.058	1.062	1.237	1.290	1.343	1.364	0.293	0.557	0.668	0.278	0.1388	0.1157
7/16"	7 No. 7	0.433	145 700	13 910	16 890	2 157	0.00361	270	1.315	1.323	1.331	1.335	1.560	1.617	1.675	1.697	0.299	0.569	0.683	0.286	0.1429	0.1191
3/8"	7 No. 8	0.385	115 600	11 440	13 890	1 710	0.00321	230	1.658	1.666	1.674	1.678	1.967	2.03	2.09	2.12	0.305	0.581	0.697	0.294	0.1471	0.1226
11/32"	7 No. 9	0.343	91 650	9 393	11 280	1 356	0.00286	200	2.09	2.10	2.11	2.11	2.48	2.55	2.61	2.64	0.311	0.592	0.711	0.303	0.1512	0.1260
5/16"	7 No. 10	0.306	72 680	7 758	9 196	1 076	0.00255	170	2.64	2.64	2.65	2.66	3.13	3.20	3.27	3.30	0.316	0.604	0.725	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	9 262	11 860	1 467	0.00457	220	1.926	1.931	1.936	1.938	2.29	2.31	2.34	2.35	0.289	0.545	0.654	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	7 639	9 754	1 163	0.00407	190	2.43	2.43	2.44	2.44	2.88	2.91	2.94	2.95	0.295	0.556	0.668	0.301	0.1506	0.1255
3 No. 7	3 No. 7	0.311	62 450	6 291	7 922	922.4	0.00363	160	3.06	3.07	3.07	3.07	3.63	3.66	3.70	3.71	0.301	0.568	0.682	0.310	0.1547	0.1289
3 No. 8	3 No. 8	0.277	49 530	5 174	6 282	731.5	0.00323	140	3.86	3.87	3.87	3.87	4.58	4.61	4.65	4.66	0.307	0.580	0.696	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	4 250	5 129	580.1	0.00288	120	4.87	4.87	4.88	4.88	5.78	5.81	5.85	5.86	0.313	0.591	0.710	0.326	0.1629	0.1358
3 No. 10	3 No. 10	0.220	31 150	3 509	4 160	460.0	0.00257	110	6.14	6.14	6.15	6.15	7.28	7.32	7.36	7.38	0.319	0.603	0.724	0.334	0.1671	0.1392
40% Conductivity																						
7/8"	19 No. 5	0.910	628 900	50 240		9 344	0.01175	690	0.229	0.239	0.249	0.254	0.272	0.321	0.371	0.391	0.236	0.449	0.539	0.233	0.1165	0.0971
13/16"	19 No. 6	0.810	498 800	41 600		7 410	0.01046	610	0.289	0.299	0.309	0.314	0.343	0.396	0.450	0.472	0.241	0.461	0.553	0.241	0.1206	0.1005
23/32"	19 No. 7	0.721	395 500	34 390		5 877	0.00931	530	0.365	0.375	0.385	0.390	0.433	0.490	0.549	0.573	0.247	0.473	0.567	0.250	0.1248	0.1040
21/32"	19 No. 8	0.642	313 700	28 380		4 660	0.00829	470	0.460	0.470	0.480	0.485	0.546	0.608	0.672	0.698	0.253	0.485	0.582	0.258	0.1289	0.1074
9/16"	19 No. 9	0.572	248 800	23 390		3 696	0.00739	410	0.580	0.590	0.600	0.605	0.688	0.756	0.826	0.753	0.259	0.496	0.595	0.266	0.1330	0.1109
5/8"	7 No. 4	0.613	292 200	22 310		4 324	0.00792	470	0.492	0.500	0.508	0.512	0.584	0.624	0.664	0.680	0.255	0.489	0.587	0.261	0.1306	0.1088
9/16"	7 No. 5	0.546	231 700	18 510		3 429	0.00705	410	0.620	0.628	0.636	0.640	0.736	0.780	0.843	0.840	0.261	0.501	0.601	0.269	0.1347	0.1122
1/2"	7 No. 6	0.486	183 800	15 330		2 719	0.00628	350	0.782	0.790	0.798	0.802	0.928	0.975	1.021	1.040	0.267	0.513	0.615	0.278	0.1388	0.1157
7/16"	7 No. 7	0.433	145 700	12 570		2 157	0.00559	310	0.986	0.994	1.002	1.006	1.170	1.220	1.271	1.291	0.273	0.524	0.629	0.286	0.1429	0.1191
3/8"	7 No. 8	0.385	115 600	10 460		1 710	0.00497	270	1.244	1.252	1.260	1.264	1.476	1.530	1.584	1.606	0.279	0.536	0.644	0.294	0.1471	0.1226
11/32"	7 No. 9	0.343	91 650	8 616		1 356	0.00443	230	1.568	1.576	1.584	1.588	1.861	1.919	1.978	2.00	0.285	0.548	0.658	0.303	0.1512	0.1260
5/16"	7 No. 10	0.306	72 680	7 121		1 076	0.00395	200	1.978	1.986	1.994	1.998	2.35	2.41	2.47	2.50	0.291	0.559	0.671	0.311	0.1553	0.1294
3 No. 5	3 No. 5	0.392	99 310	8 373		1 467	0.00621	250	1.445	1.450	1.455	1.457	1.714	1.738	1.762	1.772	0.269	0.514	0.617	0.293	0.1465	0.1221
3 No. 6	3 No. 6	0.349	78 750	6 934		1 163	0.00553	220	1.821	1.826	1.831	1.833	2.16	2.19	[illegible]	2.22	0.275	0.526	0.631	0.301	0.1506	[illegible]
3 No. 7	3 No. 7	0.311	62 450	5 732		922.4	0.00492	190	2.30	2.30	2.31	2.31	2.73	2.75	2.78	2.79	0.281	0.537	0.645	0.310	0.1547	[illegible]
3 No. 8	3 No. 8	0.277	49 530	4 730		731.5	0.00439	160	2.90	2.90	2.91	2.91	3.44	3.47	3.50	3.51	0.286	0.549	0.659	0.318	0.1589	0.1324
3 No. 9	3 No. 9	0.247	39 280	3 898		580.1	0.00391	140	3.65	3.66	3.66	3.66	4.33	4.37	4.40	4.41	0.292	0.561	0.673	0.326	0.1629	0.1358
3 No. 10	3 No. 10	1.220	31 150	3 221		460.0	0.00348	120	4.61	4.61	4.62	4.62	5.46	5.50	5.53	5.55	0.297	0.572	0.687	0.334	0.1671	0.1392
3 No. 12	3 No. 12	0.174	19 590	2 236		289.3	0.00276	90	7.32	7.33	7.33	7.34	8.69	8.73	8.77	8.78	0.310	0.596	0.715	0.331	0.1754	0.1462

*Based on conductor temperature of 125°C. and an ambient of 25°C

**Resistance at 75°C total temperature, based on an ambient of 25°C, plus 50°C rise due to heating effect of current. The approximate magnitude of current necessary to produce the 50°C. rise is 75% of the "Approximate Current Carrying Capacity at 60 Cycles."

TABLE A.7 Electrical Characteristics of Overhead Ground Wires [3]

Part A: Alumoweld strand

Strand (AWG)	Resistance, Ω/mi: Small currents, 25°C oc	Resistance, Ω/mi: Small currents, 25°C 60 Hz	Resistance, Ω/mi: 75% of cap., 75°C oc	Resistance, Ω/mi: 75% of cap., 75°C 60 Hz	60-Hz reactance for 1-ft radius: Inductive, Ω/mi	60-Hz reactance for 1-ft radius: Capacitive, MΩ·mi	60-Hz geometric mean radius, ft
7 NO. 5	1.217	1.240	1.432	1.669	0.707	0.1122	0.002958
7 NO. 6	1.507	1.536	1.773	2.010	0.721	0.1157	0.002633
7 NO. 7	1.900	1.937	2.240	2.470	0.735	0.1191	0.002345
7 NO. 8	2.400	2.440	2.820	3.060	0.749	0.1226	0.002085
7 NO. 9	3.020	3.080	3.560	3.800	0.763	0.1260	0.001858
7 NO. 10	3.810	3.880	4.480	4.730	0.777	0.1294	0.001658
3 NO. 5	2.780	2.780	3.270	3.560	0.707	0.1221	0.002940
3 NO. 6	3.510	3.510	4.130	4.410	0.721	0.1255	0.002618
3 NO. 7	4.420	4.420	5.210	5.470	0.735	0.1289	0.002333
3 NO. 8	5.580	5.580	6.570	6.820	0.749	0.1324	0.002078
3 NO. 9	7.040	7.040	8.280	8.520	0.763	0.1358	0.001853
3 NO. 10	8.870	8.870	10.440	10.670	0.777	0.1392	0.001650

Part B: Single-layer ACSR

Code	Resistance, Ω/mi: 25°C dc	Resistance, Ω/mi: 60 Hz, 75°C, $I = 0$ A	Resistance, Ω/mi: 60 Hz, 75°C, $I = 100$ A	Resistance, Ω/mi: 60 Hz, 75°C, $I = 200$ A	60-Hz reactance for 1-ft radius: Inductive, Ω/mi at 75°C, $I = 0$ A	Inductive, $I = 100$ A	Inductive, $I = 200$ A	Capacitive, MΩ·mi
Brahma	0.394	0.470	0.510	0.565	0.500	0.520	0.545	0.1043
Cochin	0.400	0.480	0.520	0.590	0.505	0.515	0.550	0.1065
Dorking	0.443	0.535	0.575	0.650	0.515	0.530	0.565	0.1079
Dotterel	0.479	0.565	0.620	0.705	0.515	0.530	0.575	0.1091
Guinea	0.531	0.630	0.685	0.780	0.520	0.545	0.590	0.1106
Leghorn	0.630	0.760	0.810	0.930	0.530	0.550	0.605	0.1131
Minorca	0.765	0.915	0.980	1.130	0.540	0.570	0.640	0.1160
Petrel	0.830	1.000	1.065	1.220	0.550	0.580	0.655	0.1172
Grouse	1.080	1.295	1.420	1.520	0.570	0.640	0.675	0.1240

TABLE A.7 (Continued)

Part C: Steel conductors

Grade (7-strand)	Dia., in	Resistance, Ω/mi, at 60 Hz			60-Hz reactance for 1-ft radius: Inductive, Ω/mi			Capacitive, MΩ · mi
		$I = 0$ A	$I = 30$ A	$I = 60$ A	$I = 0$ A	$I = 30$ A	$I = 60$ A	
Ordinary	1/4	9.5	11.4	11.3	1.3970	3.7431	3.4379	0.1354
Ordinary	9/32	7.1	9.2	9.0	1.2027	3.0734	2.5146	0.1319
Ordinary	5/16	5.4	7.5	7.8	0.8382	2.5146	2.0409	0.1288
Ordinary	3/8	4.3	6.5	6.6	0.8382	2.2352	1.9687	0.1234
Ordinary	1/2	2.3	4.3	5.0	0.7049	1.6893	1.4236	0.1148
E.B.	1/4	8.0	12.0	10.1	1.2027	4.4704	3.1565	0.1354
E.B.	9/32	6.0	10.0	8.7	1.1305	3.7783	2.6255	0.1319
E.B.	5/16	4.9	8.0	7.0	0.9843	2.9401	2.5146	0.1288
E.B.	3/8	3.7	7.0	6.3	0.8382	2.5997	2.4303	0.1234
E.B.	1/2	2.1	4.9	5.0	0.7049	1.8715	1.7616	0.1148
E.B.B.	1/4	7.0	12.8	10.9	1.6764	5.1401	3.9482	0.1354
E.B.B.	9/32	5.4	10.9	8.7	1.1305	4.4833	3.7783	0.1319
E.B.B.	5/16	4.0	9.0	6.8	0.9843	3.6322	3.0734	0.1288
E.B.B.	3/8	3.5	7.9	6.0	0.8382	3.1168	2.7940	0.1234
E.B.B.	1/2	2.0	5.7	4.7	0.7049	2.3461	2.2352	0.1148

TABLE A.8 Inductive Reactance Spacing Factor X_d (Ω/mi/conductor) at 60 Hz [1]

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		−0.2794	−0.1953	−0.1461	−0.1112	−0.0841	−0.0620	−0.0433	−0.0271	−0.0128
1	0.0	0.0116	0.0221	0.0318	0.0408	0.0492	0.0570	0.0644	0.0713	0.0779
2	0.0841	0.0900	0.0957	0.1011	0.1062	0.1112	0.1159	0.1205	0.1249	0.1292
3	0.1333	0.1373	0.1411	0.1449	0.1485	0.1520	0.1554	0.1588	0.1620	0.1651
4	0.1682	0.1712	0.1741	0.1770	0.1798	0.1825	0.1852	0.1878	0.1903	0.1928
5	0.1953	0.1977	0.2001	0.2024	0.2046	0.2069	0.2090	0.2112	0.2133	0.2154
6	0.2174	0.2194	0.2214	0.2233	0.2252	0.2271	0.2290	0.2308	0.2326	0.2344
7	0.2361	0.2378	0.2395	0.2412	0.2429	0.2445	0.2461	0.2477	0.2493	0.2508
8	0.2523	0.2538	0.2553	0.2568	0.2582	0.2597	0.2611	0.2625	0.2639	0.2653
9	0.2666	0.2680	0.2693	0.2706	0.2719	0.2732	0.2744	0.2757	0.2769	0.2782
10	0.2794	0.2806	0.2818	0.2830	0.2842	0.2853	0.2865	0.2876	0.2887	0.2899
11	0.2910	0.2921	0.2932	0.2942	0.2953	0.2964	0.2974	0.2985	0.2995	0.3005
12	0.3015	0.3025	0.3035	0.3045	0.3055	0.3065	0.3074	0.3084	0.3094	0.3103
13	0.3112	0.3122	0.3131	0.3140	0.3149	0.3158	0.3167	0.3176	0.3185	0.3194
14	0.3202	0.3211	0.3219	0.3228	0.3236	0.3245	0.3253	0.3261	0.3270	0.3278
15	0.3286	0.3294	0.3302	0.3310	0.3318	0.3326	0.3334	0.3341	0.3349	0.3357
16	0.3364	0.3372	0.3379	0.3387	0.3394	0.3402	0.3409	0.3416	0.3424	0.3431
17	0.3438	0.3445	0.3452	0.3459	0.3466	0.3473	0.3480	0.3487	0.3494	0.3500
18	0.3507	0.3514	0.3521	0.3527	0.3534	0.3540	0.3547	0.3554	0.3560	0.3566
19	0.3573	0.3579	0.3586	0.3592	0.3598	0.3604	0.3611	0.3617	0.3623	0.3629
20	0.3635	0.3641	0.3647	0.3653	0.3659	0.3665	0.3671	0.3677	0.3683	0.3688
21	0.3694	0.3700	0.3706	0.3711	0.3717	0.3723	0.3728	0.3734	0.3740	0.3745
22	0.3751	0.3756	0.3762	0.3767	0.3773	0.3778	0.3783	0.3789	0.3794	0.3799
23	0.3805	0.3810	0.3815	0.3820	0.3826	0.3831	0.3836	0.3841	0.3846	0.3851
24	0.3856	0.3861	0.3866	0.3871	0.3876	0.3881	0.3886	0.3891	0.3896	0.3901
25	0.3906	0.3911	0.3916	0.3920	0.3925	0.3930	0.3935	0.3939	0.3944	0.3949
26	0.3953	0.3958	0.3963	0.3967	0.3972	0.3977	0.3981	0.3986	0.3990	0.3995
27	0.3999	0.4004	0.4008	0.4013	0.4017	0.4021	0.4026	0.4030	0.4035	0.4039
28	0.4043	0.4048	0.4052	0.4056	0.4061	0.4065	0.4069	0.4073	0.4078	0.4082
29	0.4086	0.4090	0.4094	0.4098	0.4103	0.4107	0.4111	0.4115	0.4119	0.4123
30	0.4127	0.4131	0.4135	0.4139	0.4143	0.4147	0.4151	0.4155	0.4159	0.4163
31	0.4167	0.4171	0.4175	0.4179	0.4182	0.4186	0.4190	0.4194	0.4198	0.4202
32	0.4205	0.4209	0.4213	0.4217	0.4220	0.4224	0.4228	0.4232	0.4235	0.4239
33	0.4243	0.4246	0.4250	0.4254	0.4257	0.4261	0.4265	0.4268	0.4272	0.4275
34	0.4279	0.4283	0.4286	0.4290	0.4293	0.4297	0.4300	0.4304	0.4307	0.4311
35	0.4314	0.4318	0.4321	0.4324	0.4328	0.4331	0.4335	0.4338	0.4342	0.4345
36	0.4348	0.4352	0.4355	0.4358	0.4362	0.4365	0.4368	0.4372	0.4375	0.4378
37	0.4382	0.4385	0.4388	0.4391	0.4395	0.4398	0.4401	0.4404	0.4408	0.4411
38	0.4414	0.4417	0.4420	0.4423	0.4427	0.4430	0.4433	0.4436	0.4439	0.4442
39	0.4445	0.4449	0.4452	0.4455	0.4458	0.4461	0.4464	0.4467	0.4470	0.4473
40	0.4476	0.4479	0.4492	0.4485	0.4488	0.4491	0.4494	0.4497	0.4500	0.4503
41	0.4506	0.4509	0.4512	0.4515	0.4518	0.4521	0.4524	0.4527	0.4530	0.4532
42	0.4535	0.4538	0.4541	0.4544	0.4547	0.4550	0.4553	0.4555	0.4558	0.4561
43	0.4564	0.4567	0.4570	0.4572	0.4575	0.4578	0.4581	0.4584	0.4586	0.4589
44	0.4592	0.4595	0.4597	0.4600	0.4603	0.4606	0.4608	0.4611	0.4614	0.4616
45	0.4619	0.4622	0.4624	0.4627	0.4630	0.4632	0.4635	0.4638	0.4640	0.4643
46	0.4646	0.4648	0.4651	0.4654	0.4656	0.4659	0.4661	0.4664	0.4667	0.4669
47	0.4672	0.4674	0.4677	0.4680	0.4682	0.4685	0.4687	0.4690	0.4692	0.4695
48	0.4697	0.4700	0.4702	0.4705	0.4707	0.4710	0.4712	0.4715	0.4717	0.4720
49	0.4722	0.4725	0.4727	0.4730	0.4732	0.4735	0.4737	0.4740	0.4742	0.4744
50	0.4747	0.4749	0.4752	0.4754	0.4757	0.4759	0.4761	0.4764	0.4766	0.4769
51	0.4771	0.4773	0.4776	0.4778	0.4780	0.4783	0.4785	0.4787	0.4790	0.4792
52	0.4795	0.4797	0.4799	0.4801	0.4804	0.4806	0.4808	0.4811	0.4813	0.4815
53	0.4818	0.4820	0.4822	0.4824	0.4827	0.4829	0.4831	0.4834	0.4836	0.4838
54	0.4840	0.4843	0.4845	0.4847	0.4849	0.4851	0.4854	0.4856	0.4858	0.4860
55	0.4863	0.4865	0.4867	0.4869	0.4871	0.4874	0.4876	0.4878	0.4880	0.4882
56	0.4884	0.4887	0.4889	0.4891	0.4893	0.4895	0.4897	0.4900	0.4902	0.4904
57	0.4906	0.4908	0.4910	0.4912	0.4914	0.4917	0.4919	0.4921	0.4923	0.4925
58	0.4927	0.4929	0.4931	0.4933	0.4935	0.4937	0.4940	0.4942	0.4944	0.4946
59	0.4948	0.4950	0.4952	0.4954	0.4956	0.4958	0.4960	0.4962	0.4964	0.4966
60	0.4968	0.4970	0.4972	0.4974	0.4976	0.4978	0.4980	0.4982	0.4984	0.4986

TABLE A.8 (Continued)

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
61	0.4988	0.4990	0.4992	0.4994	0.4996	0.4998	0.5000	0.5002	0.5004	0.5006
62	0.5008	0.5010	0.5012	0.5014	0.5016	0.5018	0.5020	0.5022	0.5023	0.5025
63	0.5027	0.5029	0.5031	0.5033	0.5035	0.5037	0.5039	0.5041	0.5043	0.5045
64	0.5046	0.5048	0.5050	0.5052	0.5054	0.5056	0.5058	0.5060	0.5062	0.5063
65	0.5065	0.5067	0.5069	0.5071	0.5073	0.5075	0.5076	0.5078	0.5080	0.5082
66	0.5084	0.5086	0.5087	0.5089	0.5091	0.5093	0.5095	0.5097	0.5098	0.5100
67	0.5102	0.5104	0.5106	0.5107	0.5109	0.5111	0.5113	0.5115	0.5116	0.5118
68	0.5120	0.5122	0.5124	0.5125	0.5127	0.5129	0.5131	0.5132	0.5134	0.5136
69	0.5138	0.5139	0.5141	0.5143	0.5145	0.5147	0.5148	0.5150	0.5152	0.5153
70	0.5155	0.5157	0.5159	0.5160	0.5162	0.5164	0.5166	0.5167	0.5169	0.5171
71	0.5172	0.5174	0.5176	0.5178	0.5179	0.5181	0.5183	0.5184	0.5186	0.5188
72	0.5189	0.5191	0.5193	0.5194	0.5196	0.5198	0.5199	0.5201	0.5203	0.5204
73	0.5206	0.5208	0.5209	0.5211	0.5213	0.5214	0.5216	0.5218	0.5219	0.5221
74	0.5223	0.5224	0.5226	0.5228	0.5229	0.5231	0.5232	0.5234	0.5236	0.5237
75	0.5239	0.5241	0.5242	0.5244	0.5245	0.5247	0.5249	0.5250	0.5252	0.5253
76	0.5255	0.5257	0.5258	0.5260	0.5261	0.5263	0.5265	0.5266	0.5268	0.5269
77	0.5271	0.5272	0.5274	0.5276	0.5277	0.5279	0.5280	0.5282	0.5283	0.5285
78	0.5287	0.5288	0.5290	0.5291	0.5293	0.5294	0.5296	0.5297	0.5299	0.5300
79	0.5302	0.5304	0.5305	0.5307	0.5308	0.5310	0.5311	0.5313	0.5314	0.5316
80	0.5317	0.5319	0.5320	0.5322	0.5323	0.5325	0.5326	0.5328	0.5329	0.5331
81	0.5332	0.5334	0.5335	0.5337	0.5338	0.5340	0.5341	0.5343	0.5344	0.5346
82	0.5347	0.5349	0.5350	0.5352	0.5353	0.5355	0.5356	0.5358	0.5359	0.5360
83	0.5362	0.5363	0.5365	0.5366	0.5368	0.5369	0.5371	0.5372	0.5374	0.5375
84	0.5376	0.5378	0.5379	0.5381	0.5382	0.5384	0.5385	0.5387	0.5388	0.5389
85	0.5391	0.5392	0.5394	0.5395	0.5396	0.5398	0.5399	0.5401	0.5402	0.5404
86	0.5405	0.5406	0.5408	0.5409	0.5411	0.5412	0.5413	0.5415	0.5416	0.5418
87	0.5419	0.5420	0.5422	0.5423	0.5425	0.5426	0.5427	0.5429	0.5430	0.5432
88	0.5433	0.5434	0.5436	0.5437	0.5438	0.5440	0.5441	0.5442	0.5444	0.5445
89	0.5447	0.5448	0.5449	0.5451	0.5452	0.5453	0.5455	0.5456	0.5457	0.5459
90	0.5460	0.5461	0.5463	0.5464	0.5466	0.5467	0.5468	0.5470	0.5471	0.5472
91	0.5474	0.5475	0.5476	0.5478	0.5479	0.5480	0.5482	0.5483	0.5484	0.5486
92	0.5487	0.5488	0.5489	0.5491	0.5492	0.5493	0.5495	0.5496	0.5497	0.5499
93	0.5500	0.5501	0.5503	0.5504	0.5505	0.5506	0.5508	0.5509	0.5510	0.5512
94	0.5513	0.5514	0.5515	0.5517	0.5518	0.5519	0.5521	0.5522	0.5523	0.5524
95	0.5526	0.5527	0.5528	0.5530	0.5531	0.5532	0.5533	0.5535	0.5536	0.5537
96	0.5538	0.5540	0.5541	0.5542	0.5544	0.5545	0.5546	0.5547	0.5549	0.5550
97	0.5551	0.5552	0.5554	0.5555	0.5556	0.5557	0.5559	0.5560	0.5561	0.5562
98	0.5563	0.5565	0.5566	0.5567	0.5568	0.5570	0.5571	0.5572	0.5573	0.5575
99	0.5576	0.5577	0.5578	0.5579	0.5581	0.5582	0.5583	0.5584	0.5586	0.5587
100	0.5588	0.5589	0.5590	0.5592	0.5593	0.5594	0.5595	0.5596	0.5598	0.5599

	ρ, Ω m	r_e, x_e (f = 60 Hz)
r_e	All	0.2860
	1	2.050
	5	2.343
	10	2.469
x_e	50	2.762
	100†	2.888†
	500	3.181
	1000	3.307
	5000	3.600
	10,000	3.726

* From formulas

$$r_e = 0.004764f$$

$$x_e = 0.006985 f \log_{10} 4{,}665{,}600 \frac{\rho}{f}$$

where f = frequency

ρ = resistivity, Ω m

† This is an average value which may be used in the absence of definite information

Fundamental equations

$$z_1 = z_2 = r_a + j(x_a + x_d)$$

$$z_0 = r_a + r_e + j(x_a + x_e - 2x_d)$$

where $x_d = \omega k \ln d$

d = separation, ft

TABLE A.9 Shunt Capacitive Reactance Spacing Factor X'_d (MΩ/mi/conductor) at 60 Hz [1]

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0		−0.0683	−0.0477	−0.0357	−0.0272	−0.0206	−0.0152	−0.0106	−0.0066	−0.0031
1	0.0000	0.0028	0.0054	0.0078	0.0100	0.0120	0.0139	0.0157	0.0174	0.0190
2	0.0206	0.0220	0.0234	0.0247	0.0260	0.0272	0.0283	0.0295	0.0305	0.0316
3	0.0326	0.0336	0.0345	0.0354	0.0363	0.0372	0.0380	0.0388	0.0396	0.0404
4	0.0411	0.0419	0.0426	0.0433	0.0440	0.0446	0.0453	0.0459	0.0465	0.0471
5	0.0477	0.0483	0.0489	0.0495	0.0500	0.0506	0.0511	0.0516	0.0521	0.0527
6	0.0532	0.0536	0.0541	0.0546	0.0551	0.0555	0.0560	0.0564	0.0569	0.0573
7	0.0577	0.0581	0.0586	0.0590	0.0594	0.0598	0.0602	0.0606	0.0609	0.0613
8	0.0617	0.0621	0.0624	0.0628	0.0631	0.0635	0.0638	0.0642	0.0645	0.0649
9	0.0652	0.0655	0.0658	0.0662	0.0665	0.0668	0.0671	0.0674	0.0677	0.0680
10	0.0683	0.0686	0.0689	0.0692	0.0695	0.0698	0.0700	0.0703	0.0706	0.0709
11	0.0711	0.0714	0.0717	0.0719	0.0722	0.0725	0.0727	0.0730	0.0732	0.0735
12	0.0737	0.0740	0.0742	0.0745	0.0747	0.0749	0.0752	0.0754	0.0756	0.0759
13	0.0761	0.0763	0.0765	0.0768	0.0770	0.0772	0.0774	0.0776	0.0779	0.0781
14	0.0783	0.0785	0.0787	0.0789	0.0791	0.0793	0.0795	0.0797	0.0799	0.0801
15	0.0803	0.0805	0.0807	0.0809	0.0811	0.0813	0.0815	0.0817	0.0819	0.0821
16	0.0823	0.0824	0.0826	0.0828	0.0830	0.0832	0.0833	0.0835	0.0837	0.0839
17	0.0841	0.0842	0.0844	0.0846	0.0847	0.0849	0.0851	0.0852	0.0854	0.0856
18	0.0857	0.0859	0.0861	0.0862	0.0864	0.0866	0.0867	0.0869	0.0870	0.0872
19	0.0874	0.0875	0.0877	0.0878	0.0880	0.0881	0.0883	0.0884	0.0886	0.0887
20	0.0889	0.0890	0.0892	0.0893	0.0895	0.0896	0.0898	0.0899	0.0900	0.0902
21	0.0903	0.0905	0.0906	0.0907	0.0909	0.0910	0.0912	0.0913	0.0914	0.0916
22	0.0917	0.0918	0.0920	0.0921	0.0922	0.0924	0.0925	0.0926	0.0928	0.0929
23	0.0930	0.0931	0.0933	0.0934	0.0935	0.0937	0.0938	0.0939	0.0940	0.0942
24	0.0943	0.0944	0.0945	0.0947	0.0948	0.0949	0.0950	0.0951	0.0953	0.0954
25	0.0955	0.0956	0.0957	0.0958	0.0960	0.0961	0.0962	0.0963	0.0964	0.0965
26	0.0967	0.0968	0.0969	0.0970	0.0971	0.0972	0.0973	0.0974	0.0976	0.0977
27	0.0978	0.0979	0.0980	0.0981	0.0982	0.0983	0.0984	0.0985	0.0986	0.0987
28	0.0989	0.0990	0.0991	0.0992	0.0993	0.0994	0.0995	0.0996	0.0997	0.0998
29	0.0999	0.1000	0.1001	0.1002	0.1003	0.1004	0.1005	0.1006	0.1007	0.1008
30	0.1009	0.1010	0.1011	0.1012	0.1013	0.1014	0.1015	0.1016	0.1017	0.1018
31	0.1019	0.1020	0.1021	0.1022	0.1023	0.1023	0.1024	0.1025	0.1026	0.1027
32	0.1028	0.1029	0.1030	0.1031	0.1032	0.1033	0.1034	0.1035	0.1035	0.1036
33	0.1037	0.1038	0.1039	0.1040	0.1041	0.1042	0.1043	0.1044	0.1044	0.1045
34	0.1046	0.1047	0.1048	0.1049	0.1050	0.1050	0.1051	0.1052	0.1053	0.1054
35	0.1055	0.1056	0.1056	0.1057	0.1058	0.1059	0.1060	0.1061	0.1061	0.1062
36	0.1063	0.1064	0.1065	0.1066	0.1066	0.1067	0.1068	0.1069	0.1070	0.1070
37	0.1071	0.1072	0.1073	0.1074	0.1074	0.1075	0.1076	0.1077	0.1078	0.1078
38	0.1079	0.1080	0.1081	0.1081	0.1082	0.1083	0.1084	0.1085	0.1085	0.1086
39	0.1087	0.1088	0.1088	0.1089	0.1090	0.1091	0.1091	0.1092	0.1093	0.1094
40	0.1094	0.1095	0.1096	0.1097	0.1097	0.1098	0.1099	0.1100	0.1100	0.1101
41	0.1102	0.1102	0.1103	0.1104	0.1105	0.1105	0.1106	0.1107	0.1107	0.1108
42	0.1109	0.1110	0.1110	0.1111	0.1112	0.1112	0.1113	0.1114	0.1114	0.1115
43	0.1116	0.1117	0.1117	0.1118	0.1119	0.1119	0.1120	0.1121	0.1121	0.1122
44	0.1123	0.1123	0.1124	0.1125	0.1125	0.1126	0.1127	0.1127	0.1128	0.1129
45	0.1129	0.1130	0.1131	0.1131	0.1132	0.1133	0.1133	0.1134	0.1135	0.1135
46	0.1136	0.1136	0.1137	0.1138	0.1138	0.1139	0.1140	0.1140	0.1141	0.1142
47	0.1142	0.1143	0.1143	0.1144	0.1145	0.1145	0.1146	0.1147	0.1147	0.1148
48	0.1148	0.1149	0.1150	0.1150	0.1151	0.1152	0.1152	0.1153	0.1153	0.1154
49	0.1155	0.1155	0.1156	0.1156	0.1157	0.1158	0.1158	0.1159	0.1159	0.1160
50	0.1161	0.1161	0.1162	0.1162	0.1163	0.1164	0.1164	0.1165	0.1165	0.1166
51	0.1166	0.1167	0.1168	0.1168	0.1169	0.1169	0.1170	0.1170	0.1171	0.1172
52	0.1172	0.1173	0.1173	0.1174	0.1174	0.1175	0.1176	0.1176	0.1177	0.1177
53	0.1178	0.1178	0.1179	0.1180	0.1180	0.1181	0.1181	0.1182	0.1182	0.1183
54	0.1183	0.1184	0.1184	0.1185	0.1186	0.1186	0.1187	0.1187	0.1188	0.1188
55	0.1189	0.1189	0.1190	0.1190	0.1191	0.1192	0.1192	0.1193	0.1193	0.1194
56	0.1194	0.1195	0.1195	0.1196	0.1196	0.1197	0.1197	0.1198	0.1198	0.1199
57	0.1199	0.1200	0.1200	0.1201	0.1202	0.1202	0.1203	0.1203	0.1204	0.1204
58	0.1205	0.1205	0.1206	0.1206	0.1207	0.1207	0.1208	0.1208	0.1209	0.1209
59	0.1210	0.1210	0.1211	0.1211	0.1212	0.1212	0.1213	0.1213	0.1214	0.1214
60	0.1215	0.1215	0.1216	0.1216	0.1217	0.1217	0.1218	0.1218	0.1219	0.1219

TABLE A.9 (Continued)

Ft	0.0	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
61	0.1220	0.1220	0.1221	0.1221	0.1221	0.1222	0.1222	0.1223	0.1223	0.1224
62	0.1224	0.1225	0.1225	0.1226	0.1226	0.1227	0.1227	0.1228	0.1228	0.1229
63	0.1229	0.1230	0.1230	0.1231	0.1231	0.1231	0.1232	0.1232	0.1233	0.1233
64	0.1234	0.1234	0.1235	0.1235	0.1236	0.1236	0.1237	0.1237	0.1237	0.1238
65	0.1238	0.1239	0.1239	0.1240	0.1240	0.1241	0.1241	0.1242	0.1242	0.1242
66	0.1243	0.1243	0.1244	0.1244	0.1245	0.1245	0.1246	0.1246	0.1247	0.1247
67	0.1247	0.1248	0.1248	0.1249	0.1249	0.1250	0.1250	0.1250	0.1251	0.1251
68	0.1252	0.1252	0.1253	0.1253	0.1254	0.1254	0.1254	0.1255	0.1255	0.1256
69	0.1256	0.1257	0.1257	0.1257	0.1258	0.1258	0.1259	0.1259	0.1260	0.1260
70	0.1260	0.1261	0.1261	0.1262	0.1262	0.1262	0.1263	0.1263	0.1264	0.1264
71	0.1265	0.1265	0.1265	0.1266	0.1266	0.1267	0.1267	0.1268	0.1268	0.1268
72	0.1269	0.1269	0.1270	0.1270	0.1270	0.1271	0.1271	0.1272	0.1272	0.1272
73	0.1273	0.1273	0.1274	0.1274	0.1274	0.1275	0.1275	0.1276	0.1276	0.1276
74	0.1277	0.1277	0.1278	0.1278	0.1278	0.1279	0.1279	0.1280	0.1280	0.1280
75	0.1281	0.1281	0.1282	0.1282	0.1282	0.1283	0.1283	0.1284	0.1284	0.1284
76	0.1285	0.1285	0.1286	0.1286	0.1286	0.1287	0.1287	0.1288	0.1288	0.1288
77	0.1289	0.1289	0.1289	0.1290	0.1290	0.1291	0.1291	0.1291	0.1292	0.1292
78	0.1292	0.1293	0.1293	0.1294	0.1294	0.1294	0.1295	0.1295	0.1296	0.1296
79	0.1296	0.1297	0.1297	0.1297	0.1298	0.1298	0.1299	0.1299	0.1299	0.1300
80	0.1300	0.1300	0.1301	0.1301	0.1301	0.1302	0.1302	0.1303	0.1303	0.1303
81	0.1304	0.1304	0.1304	0.1305	0.1305	0.1306	0.1306	0.1306	0.1307	0.1307
82	0.1307	0.1308	0.1308	0.1308	0.1309	0.1309	0.1309	0.1310	0.1310	0.1311
83	0.1311	0.1311	0.1312	0.1312	0.1312	0.1313	0.1313	0.1313	0.1314	0.1314
84	0.1314	0.1315	0.1315	0.1316	0.1316	0.1316	0.1317	0.1317	0.1317	0.1318
85	0.1318	0.1318	0.1319	0.1319	0.1319	0.1320	0.1320	0.1320	0.1321	0.1321
86	0.1321	0.1322	0.1322	0.1322	0.1323	0.1323	0.1324	0.1324	0.1324	0.1325
87	0.1325	0.1325	0.1326	0.1326	0.1326	0.1327	0.1327	0.1327	0.1328	0.1328
88	0.1328	0.1329	0.1329	0.1329	0.1330	0.1330	0.1330	0.1331	0.1331	0.1331
89	0.1332	0.1332	0.1332	0.1333	0.1333	0.1333	0.1334	0.1334	0.1334	0.1335
90	0.1335	0.1335	0.1336	0.1336	0.1336	0.1337	0.1337	0.1337	0.1338	0.1338
91	0.1338	0.1339	0.1339	0.1339	0.1340	0.1340	0.1340	0.1340	0.1341	0.1341
92	0.1341	0.1342	0.1342	0.1342	0.1343	0.1343	0.1343	0.1344	0.1344	0.1344
93	0.1345	0.1345	0.1345	0.1346	0.1346	0.1346	0.1347	0.1347	0.1347	0.1348
94	0.1348	0.1348	0.1348	0.1349	0.1349	0.1349	0.1350	0.1350	0.1350	0.1351
95	0.1351	0.1351	0.1352	0.1352	0.1352	0.1353	0.1353	0.1353	0.1353	0.1354
96	0.1354	0.1354	0.1355	0.1355	0.1355	0.1356	0.1356	0.1356	0.1357	0.1357
97	0.1357	0.1357	0.1358	0.1358	0.1358	0.1359	0.1359	0.1359	0.1360	0.1360
98	0.1360	0.1361	0.1361	0.1361	0.1361	0.1362	0.1362	0.1362	0.1363	0.1363
99	0.1363	0.1364	0.1364	0.1364	0.1364	0.1365	0.1365	0.1365	0.1366	0.1366
100	0.1366	0.1366	0.1367	0.1367	0.1367	0.1368	0.1368	0.1368	0.1369	0.1369

Conductor height above ground, ft	x'_0 (f = 60 Hz)
10	0.267
15	0.303
20	0.328
25	0.318
30	0.364
40	0.390
50	0.410
60	0.426
70	0.440
80	0.452
90	0.462
100	0.472

$$x'_0 = \frac{12.30}{f} \log_{10} 2h$$

where h = height above ground
f = frequency

Fundamental equations

$$x'_1 = x'_2 = x'_a = x'_d$$

$$x'_0 = x'_a + x'_c - 2x'_d$$

where $x'_d = (1/\omega k') \ln d$
d = separation, ft

TABLE A.10 Standard Impedances for Power Transformers 10,000 kVA and Below [4]

Highest-voltage winding, BIL kV	Low-voltage winding, BIL kV (For intermediate BIL, use value for next higher BIL listed)	At kVA base equal to 55°C rating of largest capacity winding Self-cooled (OA), self-cooled rating of self-cooled/ forced-air cooled (OA/FA) Standard impedance, percent	
		Ungrounded neutral operation	Grounded neutral operation
110 and below	45 60, 75, 95, 110	5.75 5.5	
150	45 60, 75, 95, 110	5.75 5.5	
200	45 60, 75, 95, 110 150	6.25 6.0 6.5	
250	45 60, 150 200	6.75 6.5 7.0	
350	200 250	7.0 7.5	
450	200 250 350	7.5 8.0 8.5	7.00 7.50 8.00
550	200 350 450	8.0 9.0 10.0	7.50 8.25 9.25
650	200 350 550	8.5 9.5 10.5	8.00 8.50 9.50
750	250 450 650	9.0 10.0 11.0	8.50 9.50 10.25

TABLE A.11 Standard Impedances Limits for Power Transformers above 10,000 kVA [4]

Highest-voltage winding, BIL kV	Low-voltage winding, BIL kV (For intermediate BIL, use value for next higher BIL listed)	At kVA base equal to 55°C rating of largest capacity winding							
		Self-cooled (OA), self-cooled rating of self-cooled/forced-air cooled (OA/FA), self-cooled rating of self-cooled, forced-air, forced-oil cooled (OA/FOA) Standard impedance, percent				Forced-oil cooled (FOA and FOW) Standard impedance, percent			
		Ungrounded neutral operation		Grounded neutral operation		Ungrounded neutral operation		Grounded neutral operation	
		Min	Max	Min	Max	Min	Max	Min	Max
110 and below	110 and below	5.0	6.25			8.25	10.5		
150	110	5.0	6.25			8.25	10.5		
200	110	5.5	7.0			9.0	12.0		
	150	5.75	7.5			9.75	12.75		
250	150	5.75	7.5			9.5	12.75		
	200	6.25	8.5			10.5	14.25		
350	200	6.25	8.5			10.25	14.25		
	250	6.75	9.5			11.25	15.75		
450	200	6.75	9.5	6.0	8.75	11.25	15.75	10.5	14.5
	250	7.25	10.75	6.75	9.5	12.0	17.25	11.25	16.0
	350	7.75	11.75	7.0	10.25	12.75	18.0	12.0	17.25
550	200	7.25	10.75	6.5	9.75	12.0	18.0	10.75	16.5
	350	8.25	13.0	7.25	10.75	13.25	21.0	12.0	18.0
	450	8.5	13.5	7.75	11.75	14.0	22.5	12.75	19.5
650	200	7.75	11.75	7.0	10.75	12.75	19.5	11.75	18.0
	350	8.5	13.5	7.75	12.0	14.0	22.5	12.75	19.5
	450	9.25	14.0	8.5	13.5	15.25	24.5	14.0	22.5
750	250	8.0	12.75	7.5	11.5	13.5	21.25	12.5	19.25
	450	9.0	13.75	8.25	13.0	15.0	24.0	13.75	21.5
	650	10.25	15.0	9.25	14.0	16.5	25.0	15.0	24.0
825	250	8.5	13.5	7.75	12.0	14.25	22.5	13.0	20.0
	450	9.5	14.25	8.75	13.5	15.75	24.0	14.5	22.25
	650	10.75	15.75	9.75	15.0	17.25	26.25	15.75	24.0
900	250			8.25	12.5			13.75	21.0
	450			9.25	14.0			15.25	23.5
	750			10.25	15.0			16.5	25.5
1050	250			8.75	13.5			14.75	22.0
	550			10.0	15.0			16.75	25.0
	825			11.0	16.5			18.25	27.5
1175	250			9.25	14.0			15.5	23.0
	550			10.5	15.75			17.5	25.5
	900			12.0	17.5			19.5	29.0
1300	250			9.75	14.5			16.25	24.0
	550			11.25	17.0			18.75	27.0
	1050			12.5	18.25			20.75	30.5

TABLE A.12 60-Hz Characteristics of Three-Conductor Belted Paper-Insulated Cables [1]

Voltage Class	Insulation Thickness Mils: Conductor	Insulation Thickness Mils: Belt	Circular Mils or AWG (B. & S.)	Type of Conductor [6]	Weight per 1000 Feet	Diameter or Sector Depth [5] inches	Resistance, Ohms Per Mile [1]	GMR of One Conductor—inches [2]	Positive & Negative—Seq.: Series Reactance Ohms per Mile	Positive & Negative—Seq.: Shunt Capacitive Reactance Ohms per Mile [3]	GMR—Three Conductors	Zero—Sequence: Series Resistance Ohms per Mile [4]	Zero—Sequence: Series Reactance Ohms per Mile [4]	Zero—Sequence: Shunt Capacitive Reactance Ohms per Mile [3]	Sheath: Thickness Mils	Sheath: Resistance, Ohms per Mile at 50°C
1 Kv	60	35	6	SR	1 500	0.184	2.50	0.067	0.185	6300	0.184	10.66	0.315	11 600	85	2.69
	60	35	4	SR	1 910	0.232	1.58	0.084	0.175	5400	0.218	8.39	0.293	10 200	90	2.27
	60	35	2	SR	2 390	0.292	0.987	0.106	0.165	4700	0.262	6.99	0.273	9 000	90	2.00
	60	35	1	SR	2 820	0.332	0.786	0.126	0.155	4300	0.295	6.07	0.256	8 400	95	1.76
	60	35	0	SR	3 210	0.373	0.622	0.142	0.152	4000	0.326	5.54	0.246	7 900	95	1.64
	60	35	00	CS	3 160	0.323	0.495	0.151	0.138	2800	0.290	5.96	0.250	5 400	95	1.82
	60	35	000	CS	3 650	0.364	0.392	0.171	0.134	2300	0.320	5.46	0.241	4 500	95	1.69
	60	35	0000	CS	4 390	0.417	0.310	0.191	0.131	2000	0.355	4.72	0.237	4 000	100	1.47
	60	35	250 000	CS	4 900	0.455	0.263	0.210	0.129	1800	0.387	4.46	0.224	3 600	100	1.40
	60	35	300 000	CS	5 660	0.497	0.220	0.230	0.128	1700	0.415	3.97	0.221	3 400	105	1.25
	60	35	350 000	CS	6 310	0.539	0.190	0.249	0.126	1500	0.446	3.73	0.216	3 100	105	1.18
	60	35	400 000	CS	7 080	0.572	0.166	0.265	0.124	1500	0.467	3.41	0.214	2 900	110	1.08
	60	35	500 000	CS	8 310	0.642	0.134	0.297	0.123	1300	0.517	3.11	0.208	2 600	110	0.993
	65	40	600 000	CS	9 800	0.700	0.113	0.327	0.122	1200	0.567	2.74	0.197	2 400	115	0.877
	65	40	750 000	CS	11 800	0.780	0.091	0.366	0.121	1100	0.623	2.40	0.194	2 100	120	0.771
3 Kv	70	40	6	SR	1 680	0.184	2.50	0.067	0.192	6700	0.192	9.67	0.322	12 500	90	2.39
	70	40	4	SR	2 030	0.232	1.58	0.084	0.181	5800	0.227	8.06	0.298	11 200	90	2.16
	70	40	2	SR	2 600	0.292	0.987	0.106	0.171	5100	0.271	6.39	0.278	9 800	95	1.80
	70	40	1	SR	2 930	0.332	0.786	0.126	0.161	4700	0.304	5.83	0.263	9 200	95	1.68
	70	40	0	SR	3 440	0.373	0.622	0.142	0.156	4400	0.335	5.06	0.256	8 600	100	1.48
	70	40	00	CS	3 300	0.323	0.495	0.151	0.142	3500	0.297	5.69	0.259	6 700	95	1.73
	70	40	000	CS	3 890	0.364	0.392	0.171	0.138	2700	0.329	5.28	0.246	5 100	95	1.63
	70	40	0000	CS	4 530	0.417	0.310	0.191	0.135	2400	0.367	4.57	0.237	4 600	100	1.42
	70	40	250 000	CS	5 160	0.455	0.263	0.210	0.132	2100	0.396	4.07	0.231	4 200	105	1.27
	70	40	300 000	CS	5 810	0.497	0.220	0.230	0.130	[illegible]	0.424	3.82	0.228	3 800	105	1.20
	70	40	350 000	CS	6 470	0.539	0.190	0.249	0.129	1800	0.455	3.61	0.219	3 700	105	1.14
	70	40	400 000	CS	7 240	0.572	0.166	0.265	0.128	1700	0.478	3.32	0.218	3 400	110	1.05
	70	40	500 000	CS	8 660	0.642	0.134	0.297	0.126	1500	0.527	2.89	0.214	3 000	115	0.918
	75	40	600 000	CS	9 910	0.700	0.113	0.327	0.125	1400	0.577	2.68	0.210	2 800	115	0.855
	75	40	750 000	CS	11 920	0.780	0.091	0.366	0.123	1300	0.633	2.37	0.204	2 500	120	0.758
5 Kv	105	55	6	SR	2 150	0.184	2.50	0.067	0.215	8500	0.218	8.14	0.342	15 000	95	1.88
	100	55	4	SR	2 470	0.232	1.58	0.084	0.199	7600	0.250	6.86	0.317	13 600	95	1.76
	95	50	2	SR	2 900	0.292	0.987	0.106	0.184	6100	0.291	5.88	0.290	11 300	95	1.63
	90	45	1	SR	3 280	0.332	0.786	0.126	0.171	5400	0.321	5.23	0.270	10 200	100	1.48
	90	45	0	SR	3 660	0.373	0.622	0.142	0.165	5000	0.352	4.79	0.259	9 600	100	1.39
	85	45	00	CS	3 480	0.323	0.495	0.151	0.148	3600	0.312	5.42	0.263	9 300	95	1.64
	85	45	000	CS	4 080	0.364	0.392	0.171	0.143	3200	0.343	4.74	0.254	8 700	100	1.45
	85	45	0000	CS	4 720	0.417	0.310	0.191	0.141	2800	0.380	4.33	0.245	8 300	100	1.34
	85	45	250 000	CS	5 370	0.455	0.263	0.210	0.138	2600	0.410	3.89	0.237	7 800	105	1.21
	85	45	300 000	CS	6 050	0.497	0.220	0.230	0.135	2400	0.438	3.67	0.231	7 400	105	1.15
	85	45	350 000	CS	6 830	0.539	0.190	0.249	0.133	2200	0.470	3.31	0.225	7 000	110	1.04
	85	45	400 000	CS	7 480	0.572	0.166	0.265	0.131	2000	0.493	3.17	0.221	6 700	110	1.00
	85	45	500 000	CS	8 890	0.642	0.134	0.297	0.129	[illegible]	0.542	2.79	0.216	6 200	115	0.885
	85	45	600 000	CS	10 300	0.700	0.113	0.327	0.128	1600	0.587	2.51	0.210	5 800	120	0.798
	85	45	750 000	CS	12 340	0.780	0.091	0.366	0.125	1500	0.643	2.21	0.206	5 400	125	0.707
8 Kv	130	65	6	SR	2 450	0.184	2.50	0.067	0.230	9600	0.236	7.57	0.353	16 300	95	1.69
	125	65	4	SR	2 900	0.232	1.58	0.084	0.212	8300	0.269	6.08	0.329	14 500	100	1.50
	115	60	2	SR	3 280	0.292	0.987	0.106	0.193	6800	0.307	5.25	0.302	12 500	100	1.42
	110	55	1	SR	3 560	0.332	0.786	0.126	0.179	6100	0.338	4.90	0.280	11 400	100	1.37
	110	55	0	SR	4 090	0.373	0.622	0.142	0.174	[illegible]	0.368	4.31	0.272	10 700	105	1.23
	105	55	00	CS	3 870	0.323	0.495	0.151	0.158	[illegible]	0.330	4.79	0.273	8 300	100	1.43
	105	55	000	CS	4 390	0.364	0.392	0.171	0.151	3800	0.362	4.41	0.263	7 400	100	1.34
	105	55	0000	CS	5 150	0.417	0.310	0.191	0.147	3500	0.399	3.88	0.254	6 600	105	1.19
	105	55	250 000	CS	5 830	0.455	0.263	0.210	0.144	3200	0.428	3.50	0.246	6 200	110	1.08
	105	55	300 000	CS	6 500	0.497	0.220	0.230	0.141	2900	0.458	3.31	0.239	5 600	110	1.03
	105	55	350 000	CS	7 160	0.539	0.190	0.249	0.139	2700	0.489	3.12	0.233	5 200	110	0.978
	105	55	400 000	CS	7 980	0.572	0.166	0.265	0.137	2500	0.513	2.86	0.230	4 900	115	0.899
	105	55	500 000	CS	9 430	0.642	0.134	0.297	0.135	2200	0.563	2.53	0.224	4 300	120	0.800
	105	55	600 000	CS	10 680	0.700	0.113	0.327	0.132	2000	0.606	2.39	0.218	3 900	120	0.758
	105	55	750 000	CS	12 740	0.780	0.091	0.366	0.129	1800	0.663	2.11	0.211	3 500	125	0.673
15 Kv	170	85	2	SR	4 350	0.292	0.987	0.106	0.217	8600	0.349	4.20	0.323	15 000	110	1.07
	165	80	1	SR	4 640	0.332	0.786	0.126	0.202	7800	0.381	3.88	0.305	13 800	110	1.03
	160	75	0	SR	4 990	0.373	0.622	0.142	0.193	7100	0.409	3.62	0.288	12 800	110	1.00
	155	75	00	SR	5 600	0.419	0.495	0.159	0.185	6500	0.439	3.25	0.280	12 000	115	0.918
	155	75	000	SR	6 230	0.470	0.392	0.178	0.180	6000	0.476	2.99	0.272	11 300	115	0.867
	155	75	0000	SR	7 180	0.528	0.310	0.200	0.174	5600	0.520	2.64	0.263	10 600	120	0.778
	155	75	250 000	SR	7 840	0.575	0.263	0.218	0.168	5300	0.555	2.50	0.256	10 200	120	0.744
	155	75	300 000	CS	7 480	0.497	0.220	0.230	0.155	5400	0.507	2.79	0.254	7 900	115	0.855
	155	75	350 000	CS	8 340	0.539	0.190	0.249	0.152	5100	0.536	2.54	0.250	7 200	120	0.784
	155	75	400 000	CS	9 030	0.572	0.166	0.265	0.149	4900	0.561	2.44	0.245	6 900	120	0.758
	155	75	500 000	CS	10 550	0.642	0.134	0.297	0.145	4600	0.611	2.26	0.239	6 200	125	0.680
	155	75	600 000	CS	12 030	0.700	0.113	0.327	0.142	[illegible]	0.656	1.97	0.231	5 700	130	0.620
	155	75	750 000	CS	14 190	0.780	0.091	0.366	0.139	4000	0.712	1.77	0.226	5 100	135	0.558

[1] A-c resistance based upon 100% conductivity at 65°C, including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant = 3.7.
[4] Based upon all return current in the sheath; none in ground
[5] See Fig. 7.
[6] The following symbols are used to designate the cable types; SR—Stranded Round; CS—Compact Sector.

TABLE A.13 60-Hz Characteristics of Three-Conductor Shielded Paper-Insulated Cables [1]

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Type of Conductor [6]	Weight per 1000 Feet	Diameter or Sector Depth [5] inches	Resistance—Ohms per Mile [1]	GMR of one Conductor [2] inches	Positive & Negative Sequence: Series Reactance Ohms per Mile	Positive & Negative Sequence: Shunt-Capacitive Reactance—Ohms per Mile [3]	GMR—Three Conductors	Zero—Sequence: Series Resistance Ohms per Mile [4]	Zero—Sequence: Series Reactance Ohms per Mile [4]	Zero—Sequence: Shunt-Capacitive Reactance—Ohms per Mile [3]	Sheath: Thickness Mils	Sheath: Resistance Ohms per Mile at 50°C
15 Kv	205	4	SR	[illegible]	0.232	1.58	0.084	0.248	8200	0.328	5.15	0.325	8200	105	1.19
	190	2	SR	[illegible]	0.292	0.987	0.106	0.226	6700	0.365	4.44	0.298	6700	105	1.15
	185	1	SR	4 740	0.332	0.786	0.126	0.210	6000	0.398	3.91	0.285	6000	110	1.04
	180	0	SR	5 090	0.373	0.622	0.141	0.201	5400	0.425	3.65	0.275	5400	110	1.01
	175	00	CS	4 790	0.323	0.495	0.151	0.178	5200	0.397	3.95	0.268	5200	105	1.15
	175	000	CS	5 510	0.364	0.392	0.171	0.170	4800	0.432	3.48	0.256	4800	110	1.03
	175	0000	CS	6 180	0.417	0.310	0.191	0.166	4400	0.468	3.24	0.249	4400	110	0.975
	175	250 000	CS	6 910	0.455	0.267	0.210	0.158	4100	0.498	2.95	0.243	4100	115	0.807
	175	300 000	CS	7 610	0.497	0.220	[illegible]	[illegible]	3800	0.530	2.80	0.237	3800	115	[illegible]
	175	350 000	CS	8 480	0.539	[illegible]	0.249	[illegible]	3600	0.561	2.53	0.233	3600	120	0.783
	175	400 000	CS	9 170	[illegible]	[illegible]	0.265	0.151	3400	0.585	2.45	0.228	3400	120	0.761
	175	500 000	CS	10 710	0.642	0.134	[illegible]	[illegible]	3100	0.636	2.19	0.222	3100	125	0.684
	175	600 000	CS	12 230	0.700	0.113	[illegible]	0.143	[illegible]	0.681	1.98	0.215	2900	130	0.623
	175	750 000	CS	14 380	0.780	0.091	[illegible]	0.139	[illegible]	0.737	1.78	0.211	2600	135	0.562
23 Kv	265	2	SR	5 590	0.292	0.987	0.106	0.250	8300	0.418	3.60	0.317	8300	115	0.870
	250	1	SR	5 800	0.332	0.786	0.126	0.232	7500	0.450	3.26	0.298	7500	115	0.851
	250	0	SR	[illegible]	0.373	0.622	0.141	0.222	[illegible]	0.477	2.99	[illegible]	[illegible]	120	0.788
	240	00	CS	[illegible]	0.323	0.495	0.151	0.196	[illegible]	0.446	3.16	[illegible]	[illegible]	115	0.890
	240	000	CS	[illegible]	0.364	0.392	0.171	0.188	[illegible]	0.480	2.95	[illegible]	6000	115	0.851
	240	0000	CS	[illegible]	0.410	0.310	0.191	0.181	5600	0.515	2.64	[illegible]	5600	120	0.775
	240	250 000	CS	8 070	0.447	0.263	0.210	0.177	5200	0.545	2.50	0.261	5200	120	0.747
	240	300 000	CS	8 990	0.490	[illegible]	0.230	0.171	4900	0.579	2.29	[illegible]	4900	125	0.690
	240	350 000	CS	9 720	0.532	[illegible]	0.249	0.167	[illegible]	[illegible]	2.10	[illegible]	4600	125	0.665
	240	400 000	CS	[illegible]	0.566	0.166	0.265	0.165	4400	[illegible]	2.03	[illegible]	4400	130	0.620
	240	500 000	CS	[illegible]	0.635	0.134	0.297	0.159	3900	[illegible]	1.82	0.237	3900	135	0.562
	240	600 000	CS	[illegible]	0.690	0.113	0.327	0.154	3700	[illegible]	1.73	0.230	3700	135	0.540
	240	750 000	CS	[illegible]	0.767	0.091	0.366	0.151	3400	0.787	1.56	0.225	3400	140	0.488
35 Kv	[illegible]	0	SR	[illegible]	0.288	0.622	0.141	0.239	9900	[illegible]	2.40	0.330	9900	130	0.594
	[illegible]	00	SR	[illegible]	0.323	0.495	0.159	0.226	[illegible]	0.548	2.17	0.322	9100	135	0.559
	[illegible]	000	[illegible]	[illegible]	0.364	0.392	0.178	0.217	[illegible]	[illegible]	2.01	0.312	8500	135	0.538
	[illegible]	0000	CS	9 830	0.410	0.310	0.191	0.204	7200	[illegible]	2.00	0.290	7200	135	0.563
	345	250 000	CS	10 470	0.447	0.263	0.210	0.197	6800	0.628	1.90	0.280	6800	135	0.545
	345	300 000	CS	[illegible]	0.490	0.220	0.230	0.191	6400	0.663	1.80	0.273	6400	135	0.527
	345	350 000	CS	[illegible]	0.532	0.190	0.249	0.187	6000	0.693	1.66	0.270	6000	140	0.491
	345	400 000	CS	[illegible]	0.566	0.166	0.265	0.183	5700	0.721	1.61	0.265	5700	140	0.480
	345	500 000	CS	14 760	0.635	0.134	0.297	0.177	5200	0.773	1.46	0.257	5200	145	0.441
	345	600 000	CS	16 420	0.690	0.113	0.327	0.171	4900	0.819	1.35	0.248	4900	150	0.412
	345	750 000	CS	18 860	0.767	0.091	0.366	0.165	4500	0.879	1.22	0.243	4500	155	0.377

[1] A-c resistance based on 100% conductivity at 65°C including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant = 3.7
[4] Based on all return current in the sheath, none in ground.
[5] See Fig. 7
[6] The following symbols are used to designate conductor types: SR—Stranded Round, CS—Compact Sector.

TABLE A.14 60-Hz Characteristics of Three-Conductor Oil-Filled Paper-Insulated Cables [1]

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B. & S.)	Type of Conductor (6)	Weight per 1000 Feet	Diameter or Sector Depth (5)—inches	Resistance-Ohms Per Mile (1)	GMR of One Conductor (2)—inches	POSITIVE & NEGATIVE SEQ. Series Reactance Ohms Per Mile	POSITIVE & NEGATIVE SEQ. Shunt Capacitive Reactance-Ohms Per Mile (3)	GMR—Three Conductors	ZERO—SEQUENCE Series Resistance Ohms Per Mile (4)	ZERO—SEQUENCE Series Reactance Ohms Per Mile (4)	ZERO—SEQUENCE Shunt Capacitive Reactance-Ohms Per Mile (3)	SHEATH Thickness Mils	SHEATH Resistance-Ohms Per Mile at 50°C
35 Kv	190	00	CS	5 590	0.323	0.495	0.151	0.185	6030	0.406	3.56	0.265	6030	115	1.02
		000	CS	6 150	0.364	0.392	0.171	0.178	5480	0.439	3.30	0.256	5480	115	0.970
		0000	CS	6 860	0.417	0.310	0.191	0.172	4840	0.478	3.06	0.243	4840	115	0.918
		250 000	CS	7 680	0.455	0.263	0.210	0.168	4570	0.508	2.72	0.238	4570	125	0.820
		300 000	CS	9 090	0.497	0.220	0.230	0.164	4200	0.539	2.58	0.232	4200	125	0.788
		350 000	CS	9 180	0.539	0.190	0.249	0.160	3900	0.570	2.44	0.227	3900	125	0.752
		400 000	CS	9 900	0.572	0.166	0.266	0.157	3690	0.595	2.35	0.223	3690	125	0.729
		500 000	CS	11 550	0.642	0.134	0.297	0.153	3400	0.646	2.04	0.217	3400	135	0.636
		600 000	CS	12 900	0.700	0.113	0.327	0.150	3200	0.691	1.94	0.210	3200	135	0.608
		750 000	CS	15 660	0.780	0.091	0.366	0.148	3070	0.763	1.73	0.202	3070	140	0.548
46 Kv	225	00	CS	6 360	0.323	0.495	0.151	0.195	6700	0.436	3.28	0.272	6700	115	0.928
		000	CS	6 940	0.364	0.392	0.171	0.188	6100	0.468	2.87	0.265	6100	125	0.826
		0000	CS	7 660	0.410	0.310	0.191	0.180	5620	0.503	2.67	0.256	6520	125	0.788
		250 000	CS	8 280	0.447	0.263	0.210	0.177	5180	0.533	2.55	0.247	5180	125	0.761
		300 000	CS	9 690	0.490	0.220	0.230	0.172	4820	0.566	2.41	0.241	4820	125	0.729
		350 000	CS	10 100	0.532	0.190	0.249	0.168	4490	0.596	2.16	0.237	4490	135	0.658
		400 000	CS	10 820	0.566	0.166	0.265	0.165	4220	0.623	2.08	0.232	4220	135	0.639
		500 000	CS	12 220	0.635	0.134	0.297	0.160	3870	0.672	1.04	0.226	3870	135	0.603
		600 000	CS	13 930	0.690	0.113	0.327	0.156	3670	0.718	1.74	0.219	3670	140	0.542
		750 000	CS	16 040	0.767	0.091	0.366	0.151	3350	0.773	1.62	0.213	3350	140	0.510
		1 000 000	CS												
69 Kv	315	00	CR	8 240	0.376	0.495	0.147	0.234	8330	0.532	2.41	0.290	8330	135	0.639
		000	CS	8 830	0.364	0.392	0.171	0.208	7560	0.538	2.32	0.284	7560	135	0.642
		0000	CS	9 660	0.410	0.310	0.191	0.200	6840	0.575	2.16	0.274	6840	135	0.618
		250 000	CS	10 330	0.447	0.263	0.210	0.195	6500	0.607	2.06	0.266	6500	135	0.597
		300 000	CS	11 540	0.490	0.220	0.230	0.190	6030	0.640	1.85	0.260	6030	140	0.543
		350 000	CS	12 230	0.532	0.190	0.249	0.185	5700	0.672	1.77	0.254	5700	140	0.527
		400 000	CS	13 040	0.566	0.166	0.265	0.181	5410	0.700	1.55	0.248	5430	140	0.513
		500 000	CS	14 880	0.635	0.134	0.297	0.176	5050	0.750	1.51	0.242	5050	150	0.460
		600 000	CS	16 320	0.690	0.113	0.327	0.171	4740	0.797	1.44	0.235	4740	150	0.442
		750 000	CS	18 980	0.767	0.091	0.366	0.165	4360	0.854	1.29	0.230	4360	155	0.399
		1 000 000													

[1] A-c resistance based on 100% conductivity at 65°C, including 2% allowance for stranding.
[2] GMR of sector-shaped conductors is an approximate figure close enough for most practical applications.
[3] For dielectric constant = 3.5
[4] Based on all return current in sheath; none in ground.
[5] See Fig. 7
[6] The following symbols are used to designate the cable types: CR—Compact Round; CS—Compact Sector.

TABLE A.15 60-Hz Characteristics of Single-Conductor Concentric-Strand Paper-Insulated Cables [1]

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight Per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[1]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_a Resistance of One Conductor—Ohms Per Phase Per Mile[2]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[3]	Lead Sheath Thickness—Mils
1 Kv	60	6	560	0.184	0.067	0.628	0.489	2.60	6.20	4040	75
	60	4	670	0.232	0.084	0.602	0.475	1.58	5.56	3360	75
	60	2	880	0.292	0.106	0.573	0.458	0.987	4.65	2760	80
	60	1	990	0.332	0.128	0.552	0.450	0.786	4.25	2490	80
	60	0	1 110	0.373	0.141	0.539	0.442	0.622	3.61	2250	80
	60	00	1 270	0.418	0.159	0.524	0.434	0.495	3.34	2040	80
	60	000	1 510	0.470	0.178	0.512	0.425	0.392	3.23	1840	85
	60	0000	1 740	0.528	0.200	0.498	0.414	0.310	2.98	1650	85
	60	250 000	1 930	0.575	0.221	0.484	0.408	0.263	2.81	1530	85
	60	350 000	2 490	0.681	0.262	0.464	0.392	0.190	2.31	1300	90
	60	500 000	3 140	0.814	0.313	0.442	0.378	0.134	2.08	1090	90
	60	750 000	4 380	0.998	0.385	0.417	0.358	0.091	1.65	885	95
	60	1 000 000	5 560	1.152	0.445	0.400	0.344	0.070	1.40	800	100
	60	1 500 000	8 000	1.412	0.543	0.374	0.319	0.050	1.05	645	110
	60	2 000 000	10 190	1.632	0.633	0.356	0.305	0.041	0.894	555	115
3 Kv	75	6	600	0.184	0.067	0.628	0.481	2.50	5.80	4810	75
	75	4	720	0.232	0.084	0.602	0.467	1.58	5.23	4020	75
	75	2	930	0.292	0.106	0.573	0.453	0.987	4.31	3300	80
	75	1	1 040	0.332	0.126	0.552	0.445	0.786	4.03	2990	80
	75	0	1 170	0.373	0.141	0.539	0.436	0.622	3.79	2670	80
	75	00	1 320	0.418	0.159	0.524	0.428	0.495	3.52	2450	80
	75	000	1 570	0.470	0.178	0.512	0.420	0.392	3.10	2210	85
	75	0000	1 800	0.528	0.200	0.498	0.412	0.310	2.87	2010	85
	75	250 000	1 990	0.575	0.221	0.484	0.403	0.263	2.70	1860	85
	75	350 000	2 550	0.681	0.262	0.464	0.389	0.190	2.27	1610	90
	75	500 000	3 340	0.814	0.313	0.442	0.375	0.134	1.89	1340	95
	75	750 000	4 570	0.998	0.385	0.417	0.352	0.091	1.53	1060	100
	75	1 000 000	5 640	1.152	0.445	0.400	0.341	0.070	1.37	980	100
	75	1 500 000	8 090	1.412	0.543	0.374	0.316	0.050	1.02	805	110
	75	2 000 000	10 300	1.632	0.633	0.358	0.302	0.041	0.877	685	115
5 Kv	120	6	740	0.184	0.067	0.628	0.456	2.50	4.47	6700	80
	115	4	890	0.232	0.084	0.573	0.447	1.58	4.17	5540	80
	110	2	1 040	0.292	0.106	0.573	0.439	0.987	3.85	4520	80
	110	1	1 160	0.332	0.126	0.552	0.431	0.786	3.62	4100	80
	105	0	1 270	0.373	0.141	0.539	0.425	0.622	3.47	3600	80
	100	00	1 520	0.418	0.159	0.524	0.420	0.495	3.09	3140	85
	100	000	1 710	0.470	0.178	0.512	0.412	0.392	2.91	2860	85
	95	0000	1 870	0.525	0.200	0.496	0.406	0.310	2.74	2460	85
	90	250 000	2 080	0.575	0.221	0.484	0.400	0.263	2.62	2180	85
	90	350 000	2 620	0.681	0.262	0.464	0.386	0.190	2.20	1890	90
	90	500 000	3 410	0.814	0.313	0.442	0.369	0.134	1.85	1610	95
	90	750 000	4 650	0.998	0.385	0.417	0.350	0.091	1.49	1350	100
	90	1 000 000	5 850	1.152	0.445	0.400	0.339	0.070	1.27	1140	105
	90	1 500 000	8 180	1.412	0.543	0.374	0.316	0.050	1.02	950	110
	90	2 000 000	10 370	1.632	0.663	0.356	0.302	0.041	0.870	820	115
8 Kv	150	6	890	0.184	0.067	0.628	0.431	2.50	3.62	7780	80
	150	4	1 010	0.232	0.084	0.602	0.425	1.58	3.[illegible]2	6660	85
	140	2	1 150	0.292	0.106	0.573	0.417	0.987	3.06	5400	85
	140	1	1 330	0.332	0.126	0.552	0.411	0.786	2.91	4920	85
	135	0	1 450	0.373	0.141	0.539	0.408	0.622	2.83	4390	85
	130	00	1 590	0.418	0.159	0.524	0.403	0.495	2.70	3890	85
	125	000	1 760	0.470	0.178	0.512	0.397	0.392	2.59	3440	85
	120	0000	1 980	0.528	0.200	0.496	0.389	0.310	2.29	3020	90
	120	250 000	2 250	0.575	0.221	0.484	0.383	0.263	2.18	2790	90
	115	350 000	2 730	0.681	0.262	0.464	0.376	0.190	1.90	2350	95
	115	500 000	3 530	0.814	0.313	0.442	0.361	0.134	1.69	2010	95
	115	750 000	4 790	0.998	0.385	0.417	0.341	0.091	1.39	1670	100
	115	1 000 000	6 000	1.152	0.445	0.400	0.330	0.070	1.25	1470	105
	115	1 500 000	8 250	1.412	0.543	0.374	0.310	0.050	0.975	1210	110
	115	2 000 000	10 480	1.632	0.663	0.356	0.297	0.041	0.797	1055	120

Voltage Class	Insulation thickness Mils	Circular Mils or AWG (B & S)	Weight Per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[1] inches	x_a Reactance at 12 inches—Ohms per Phase per Mile	x_s Reactance of Sheath Ohms per Phase Per Mile	r_a Resistance of One Conductor—Ohms per Phase per Mile[2]	r_s Resistance of Sheath Ohms per phase per Mile at 50°C	Shunt-Capacitive Reactance Ohms per phase per Mile[3]	Lead Sheath Thickness—Mils
15 Kv	220	4	1 340	0.232	0.084	0.602	0.412	1.58	2.91	8560	85
	215	2	1 500	0.292	0.106	0.573	0.406	0.987	2.74	7270	85
	210	1	1 610	0.332	0.126	0.552	0.400	0.786	2.64	6580	85
	200	0	1 710	0.373	0.141	0.539	0.397	0.622	2.59	5880	85
	195	00	1 940	0.418	0.159	0.524	0.391	0.495	2.32	5290	90
	185	000	2 1[illegible]	0.470	0.178	0.512	0.386	0.392	2.24	4680	90
	180	0000	2 3[illegible]	0.528	0.200	0.498	0.380	0.310	2.14	4200	90
	175	250 000	2 600	0.575	0.221	0.484	0.377	0.263	2.06	3820	90
	175	350 000	3 110	0.681	0.262	0.464	0.366	0.190	1.98	3340	95
	175	500 000	3 940	0.814	0.313	0.442	0.352	0.134	1.61	2870	100
	175	750 000	5 240	0.998	0.385	0.417	0.336	0.091	1.26	2420	105
	175	1 000 000	6 350	1.152	0.445	0.400	0.325	0.070	1.15	2130	105
	175	1 500 000	8 810	1.412	0.546	0.374	0.305	0.050	0.90	1790	115
	175	2 000 000	11 080	1.632	0.633	0.358	0.294	0.041	0.772	1570	120
23 Kv	295	2	1 920	0.292	0.106	0.573	0.383	0.987	2.18	8890	90
	285	1	2 010	0.332	0.126	0.552	0.380	0.786	2.12	8050	90
	275	0	2 120	0.373	0.141	0.539	0.377	0.622	2.08	7300	90
	265	00	2 250	0.418	0.159	0.524	0.375	0.495	2.02	6580	90
	260	000	2 530	0.470	0.178	0.512	0.370	0.392	1.65	6000	95
	250	0000	2 740	0.528	0.200	0.496	0.366	0.310	1.78	5350	95
	245	250 000	2 930	0.575	0.221	0.484	0.361	0.263	1.72	4950	95
	240	350 000	3 550	0.681	0.262	0.464	0.352	0.190	1.51	4310	100
	240	500 000	4 300	0.814	0.313	0.442	0.341	0.134	1.38	3720	100
	240	750 000	5 630	0.998	0.385	0.417	0.325	0.091	1.16	3170	105
	240	1 000 000	6 910	1.152	0.445	0.400	0.313	0.070	1.01	2800	110
	240	1 500 000	9 460	1.412	0.546	0.374	0.296	0.050	0.806	2350	120
	240	2 000 000	11 790	1.632	0.633	0.358	0.285	0.041	0.697	2070	125
35 Kv	395	0	2 900	0.373	0.141	0.539	0.352	0.622	1.61	9150	100
	385	00	3 040	0.418	0.159	0.524	0.350	0.495	1.45	8420	100
	370	000	3 190	0.470	0.178	0.512	0.347	0.392	1.46	7620	100
	355	0000	3 380	0.528	0.200	0.498	0.344	0.310	1.43	6870	100
	350	250 000	3 590	0.575	0.221	0.484	0.342	0.263	1.39	6410	100
	345	350 000	4 230	0.681	0.262	0.464	0.366	0.190	1.24	5640	105
	345	500 000	5 040	0.814	0.313	0.442	0.325	0.134	1.15	4940	105
	345	750 000	6 430	0.998	0.385	0.417	0.311	0.091	0.975	4250	110
	345	1 000 000	7 780	1.152	0.445	0.400	0.302	0.070	0.866	3780	115
	345	1 500 000	10 420	1.412	0.546	0.374	0.285	0.050	0.700	3210	125
	345	2 000 000	12 830	1.632	0.633	0.356	0.274	0.041	0.611	2830	130
46 Kv	475	000	3 910	0.470	0.178	0.512	0.331	0.392	1.20	8890	105
	460	0000	4 080	0.528	0.200	0.496	0.329	0.310	1.19	8100	105
	450	250 000	4 290	0.575	0.221	0.484	0.326	0.263	1.16	7570	105
	445	350 000	4 990	0.681	0.262	0.464	0.319	0.190	1.06	6720	110
	445	500 000	5 620	0.814	0.313	0.442	0.310	0.134	0.930	5950	115
	445	750 000	7 450	0.998	0.385	0.417	0.298	0.091	0.807	5130	120
	445	1 000 000	8 680	1.152	0.445	0.400	0.290	0.070	0.752	4610	120
	445	1 500 000	11 420	1.412	0.546	0.374	0.275	0.050	0.615	3930	130
	445	2 000 000	13 910	1.632	0.633	0.356	0.264	0.041	0.543	3520	135
69 Kv	650	350 000	6 720	0.681	0.262	0.464	0.292	0.190	0.773	8590	120
	650	500 000	7 810	0.814	0.313	0.442	0.284	0.134	0.695	7680	125
	650	750 000	9 420	0.998	0.385	0.417	0.275	0.091	0.615	6[illegible]00	130
	650	1 000 000	10 940	1.152	0.445	0.400	0.267	0.070	0.537	6060	135
	650	1 500 000	13 680	1.412	0.546	0.374	0.256	0.050	0.488	5250	140
	650	2 000 000	16 320	1.632	0.633	0.356	0.246	0.041	0.437	4710	145

TABLE A.16 60-Hz Characteristics of Single-Conductor Oil-Filled (Hollow-Core) Paper-Insulated Cables [1]

INSIDE DIAMETER OF SPRING CORE = 0.5 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[3]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[1]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	00	3 980	0.736	0.345	0.431	0.333	0.495	1.18[illegible]	5240	110
		000	4 090	0.768	0.356	0.427	0.331	0.392	1.157	5070	110
		0000	4 720	0.807	0.373	0.421	0.328	0.310	1.130	4900	110
		250 000	4 650	0.837	0.381	0.418	0.325	0.263	1.057	4790	115
		350 000	5 180	0.918	0.408	0.410	0.320	0.188	1.009	4470	115
		500 000	6 [illegible]00	1.028	0.448	0.399	0.312	0.133	0.905	4070	120
		750 000	7 310	1.180	0.505	0.384	0.302	0.089	0.838	3820	120
		1 000 000	8 630	1.310	0.550	0.374	0.294	0.068	0.752	3380	125
		1 500 000	11 090	1.547	0.639	0.356	0.281	0.048	0.649	2920	130
		2 000 000	13 750	1.760	0.716	0.342	0.270	0.039	0.550	2570	140
115 Kv	480	0000	5 720	0.807	0.373	0.421	0.305	0.310	0.805	6650	120
		250 000	5 930	0.837	0.381	0.418	0.303	0.263	0.793	5540	120
		350 000	6 390	0.918	0.408	0.410	0.298	0.188	0.730	[illegible]	125
		500 000	7 480	1.028	0.448	0.399	0.291	0.133	0.692	5600	125
		750 000	8 950	1.180	0.505	0.384	0.283	0.089	0.625	5040	130
		1 000 000	10 350	1.310	0.550	0.374	0.276	0.068	0.568	4700	135
		1 500 000	12 460	1.547	0.639	0.356	0.265	0.048	0.500	4110	140
		2 000 000	15 530	1.760	0.716	0.342	0.255	0.039	0.447	3710	145
138 Kv	560	0000	6 480	0.807	0.373	0.421	0.295	0.310	0.758	7410	125
		250 000	6 700	0.837	0.381	0.418	0.293	0.263	0.718	7240	125
		350 000	7 460	0.918	0.408	0.410	0.288	0.188	0.[illegible]	6820	130
		500 000	8 310	1.028	0.448	0.399	0.282	0.133	0.[illegible]	6260	130
		750 000	9 800	1.180	0.505	0.384	0.274	0.089	0.592	5680	135
		1 000 000	11 270	1.310	0.550	0.374	0.268	0.068	0.541	5240	140
		1 500 000	13 720	1.547	0.639	0.356	0.257	0.048	0.477	4670	145
		2 000 000	16 080	1.760	0.716	0.342	0.248	0.039	0.427	4170	150
161 Kv	650	250 000	7 600	0.837	0.381	0.418	0.283	0.263	0.660	7980	130
		350 000	8 390	0.918	0.408	0.410	0.279	0.188	0.611	7520	135
		500 000	9 270	1.028	0.448	0.399	0.273	0.133	0.5[illegible]	6980	135
		750 000	10 840	1.180	0.505	0.384	0.266	0.089	0.532	6320	140
		1 000 000	12 340	1.310	0.550	0.374	0.259	0.068	0.463	5880	145
		1 500 000	15 090	1.547	0.639	0.356	0.246	0.048	0.433	5190	150
		2 000 000	18 000	1.760	0.716	0.342	0.241	0.039	0.391	4710	155

INSIDE DIAMETER OF SPRING CORE = 0.69 INCHES

Voltage Class	Insulation Thickness Mils	Circular Mils or AWG (B & S)	Weight per 1000 Feet	Diameter of Conductor—inches	GMR of One Conductor[3]—inches	x_a Reactance at 12 Inches—Ohms Per Phase Per Mile	x_s Reactance of Sheath—Ohms Per Phase Per Mile	r_c Resistance of One Conductor—Ohms Per Phase Per Mile[1]	r_s Resistance of Sheath Ohms Per Phase Per Mile at 50°C.	Shunt Capacitive Reactance—Ohms Per Phase Per Mile[2]	Lead Sheath Thickness—Mils
69 Kv	315	000	4 860	0.924	0.439	0.399	0.320	0.392	1.007	4450	115
		0000	5 090	0.956	0.450	0.398	0.317	0.310	0.983	4350	115
		250 000	5 290	0.983	0.460	0.396	0.315	0.263	0.975	4230	115
		350 000	5 950	1.050	0.483	0.390	0.310	0.189	0.897	4000	120
		500 000	6 700	1.145	0.516	0.182	0.304	0.132	0.850	3700	120
		750 000	8 080	1.286	0.550	0.174	0.295	0.089	0.759	3410	125
		1 000 000	9 440	1.416	0.612	0.160	0.288	0.067	0.688	3140	130
		1 500 000	11 970	1.635	0.692	0.146	0.276	0.047	0.601	2750	135
		2 000 000	14 450	1.835	0.763	0.134	0.266	0.038	0.533	2310	140
115 Kv	480	0000	6 590	0.956	0.450	0.398	0.295	0.310	0.760	5950	125
		250 000	6 800	0.983	0.460	0.396	0.294	0.263	0.732	5790	125
		350 000	7 340	1.050	0.483	0.390	0.290	0.188	0.729	6540	125
		500 000	8 320	1.145	0.516	0.382	0.284	0.132	0.669	6150	130
		750 000	9 790	1.286	0.550	0.374	0.277	0.089	0.606	4770	135
		1 000 000	11 060	1.416	0.612	0.360	0.270	0.087	0.573	4430	135
		1 500 000	13 900	1.635	0.692	0.346	0.260	0.047	0.490	3620	145
		2 000 000	16 610	1.835	0.763	0.334	0.251	0.038	0.440	3360	150
138 Kv	560	0000	7 390	0.956	0.450	0.398	0.286	0.310	0.678	6590	130
		250 000	7 610	0.983	0.460	0.396	0.285	0.263	0.669	6480	130
		350 000	8 170	1.050	0.483	0.390	0.281	0.188	0.649	6180	130
		500 000	9 180	1.145	0.516	0.382	0.276	0.132	0.601	5790	136
		750 000	10 660	1.286	0.550	0.374	0.269	0.089	0.545	5320	140
		1 000 000	12 010	1.416	0.612	0.360	0.263	0.067	0.519	4940	140
		1 500 000	14 450	1.635	0.692	0.346	0.253	0.047	0.462	4460	145
		2 000 000	16 820	1.835	0.763	0.334	0.245	0.038	0.404	4060	155
161 Kv	650	250 000	8 560	0.983	0.460	0.396	0.275	0.263	0.696	7210	135
		350 000	9 140	1.050	0.483	0.390	0.272	0.188	0.580	6860	135
		500 000	10 280	1.145	0.516	0.382	0.267	0.132	0.537	6430	140
		750 000	11 370	1.286	0.[illegible]	0.374	0.261	0.089	0.492	5980	145
		1 000 000	13 110	1.416	0.612	0.360	0.255	0.067	0.469	5540	145
		1 500 000	15 840	1.635	0.692	0.346	0.246	0.047	0.421	[illegible]	150
		2 000 000	18 840	1.835	0.763	0.334	0.238	0.038	0.369	4600	160
230 Kv	925	750 000	15 360	1.286	0.550	0.374	0.233	0.089	0.369	7610	160
		1 000 000	15 790	1.416	0.612	0.360	0.233	0.067	0.355	7140	160
		2 000 000	22 990	1.835	0.763	0.334	0.219	0.038	0.315	5960	170

[1] A-c Resistance based on 100% conductivity at 65°C, including 2% allowance for stranding. Above values calculated from "A Set of Curves for Skin Effect in Isolated Tubular Conductors" by A. W. Ewan, *G. E. Review*, Vol. 33, April 1930.
[2] For dielectric constant = 3.5
[3] Calculated for circular tube as given in *Symmetrical Components* by Wagner & Evans, Ch. VII, page 138.

TABLE A.17 Current-Carrying Capacity of Three-Conductor Belted Paper-Insulated Cables [1]

Conductor Size AWG or 1000 CM	Conductor type[1]	Number of Equally Loaded Cables in Duct Bank: ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor: 30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
4500 Volts		Copper Temperature 85°C																			
6	S	82	80	78	75	81	78	73	68	79	74	68	63	78	72	65	58	76	69	61	54
4	SR	109	106	103	98	108	102	96	89	104	97	89	81	102	94	84	74	100	90	79	69
2	SR	143	139	134	128	139	133	124	115	136	127	115	104	133	121	108	95	130	117	101	89
1	SR	164	161	153	146	159	152	141	130	156	145	130	118	152	138	122	108	148	133	115	100
0	CS	189	184	177	168	184	175	162	149	180	166	149	134	175	159	140	122	170	152	130	114
00	CS	218	211	203	192	211	201	185	170	208	190	170	152	201	181	158	138	195	173	148	128
000	CS	250	242	232	219	242	229	211	193	237	217	193	172	223	206	179	156	223	197	167	145
0000	CS	286	276	264	249	276	260	240	218	270	246	218	194	261	234	202	176	254	223	189	163
250	CS	316	305	291	273	305	288	263	239	297	271	239	212	288	258	221	192	279	244	206	177
300	CS	354	340	324	304	340	321	292	264	332	301	264	234	321	285	245	211	310	271	227	195
350	CS	392	376	357	334	375	353	320	288	366	330	288	255	351	311	266	229	341	296	248	211
400	CS	424	408	385	359	406	380	344	309	395	355	309	272	380	334	285	244	367	317	264	224
500	CS	487	465	439	408	465	433	390	348	451	403	348	305	433	378	320	273	417	357	296	251
600	CS	544	517	487	450	517	480	430	383	501	444	383	334	480	416	350	298	462	393	323	273
750	CS	618	581	550	505	585	541	482	427	566	500	427	371	541	466	390	331	519	439	359	302
		(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[3]			
7500 Volts		Copper Temperature 83°C																			
6	S	81	80	77	74	79	76	72	67	78	74	67	62	77	71	64	57	75	69	60	53
4	SR	107	105	101	97	104	100	94	87	103	96	87	79	100	92	82	73	98	89	77	68
2	SR	140	137	132	126	136	131	122	113	134	125	113	102	130	119	105	93	127	114	99	87
1	SR	161	156	150	143	156	149	138	128	153	142	128	115	149	136	120	105	145	130	112	98
0	CS	186	180	174	165	180	172	156	148	177	163	148	131	172	155	136	120	167	149	128	111
00	CS	214	206	198	188	206	196	181	166	202	186	166	148	196	177	155	135	191	169	145	125
000	CS	243	236	226	214	236	224	206	188	230	211	188	168	223	200	174	152	217	192	163	141
0000	CS	280	270	258	243	270	255	235	214	264	241	213	190	255	229	198	172	247	218	184	159
250	CS	311	300	287	269	300	283	259	235	293	266	235	208	282	252	217	188	273	240	202	174
300	CS	349	316	320	300	335	316	288	260	320	296	259	230	315	279	240	207	304	[illegible]	223	190
350	CS	385	309	351	328	369	346	315	283	359	323	282	219	345	305	261	224	333	[illegible]	242	206
400	CS	417	399	378	353	398	373	338	303	388	348	303	267	371	317	279	239	360	309	257	220
500	CS	476	454	429	399	454	423	381	341	440	392	340	298	422	369	312	267	406	348	288	245
600	CS	534	508	479	443	507	471	422	376	491	438	375	327	469	408	343	291	451	384	315	267
750	CS	607	576	540	[illegible]	575	532	473	419	555	489	418	363	529	455	381	323	507	428	350	295
		(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]				(1.08 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.72 at 50°C)[3]			
15 000 Volts		Copper Temperature 75°C																			
6	S	78	77	74	71	76	74	69	64	75	70	64	59	73	68	61	54	72	65	57	50
4	SR	102	99	96	92	98	95	89	83	97	91	83	75	95	87	78	69	93	85	73	64
2	SR	132	129	125	119	129	123	115	106	128	117	106	96	123	112	99	88	120	108	93	82
1	SR	151	147	142	135	146	140	131	120	144	133	120	109	140	128	112	99	136	122	107	92
0	CS	175	170	163	155	169	161	150	138	166	153	137	123	161	146	128	112	156	139	120	104
00	CS	200	194	187	177	194	184	170	156	189	175	156	139	183	166	145	127	178	158	135	117
000	CS	230	223	214	202	222	211	195	178	217	199	177	158	210	189	165	143	203	180	153	132
0000	CS	266	257	245	232	253	242	222	202	249	228	201	179	240	215	187	158	233	205	173	149
250	CS	295	284	271	255	281	268	245	221	276	251	220	196	266	239	204	177	257	225	189	163
300	CS	330	317	301	283	316	297	271	245	307	278	244	215	295	264	225	194	285	248	208	178
350	CS	365	349	332	310	318	327	297	267	339	305	266	235	324	289	245	211	313	271	227	193
400	CS	394	377	357	333	375	352	319	286	365	327	285	251	349	307	262	224	336	290	241	206
500	CS	449	429	406	377	428	399	359	321	414	396	319	280	396	346	293	250	379	326	269	229
600	CS	502	479	450	417	476	443	396	352	459	409	351	306	438	380	319	273	420	358	294	249
750	CS	572	543	510	468	540	499	444	393	520	458	391	341	494	425	356	302	471	399	326	275
		(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types:
S—Solid copper, SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions
a. Ambient earth temperature = 20°C
b. 60 cycle alternating current
c. Ratings include dielectric loss and all induced a-c losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

TABLE A.18 Current-Carrying Capacity of Three-Conductor Shielded Paper-Insulated Cables [1]

Conductor Size AWG or 1000 CM	Conductor Type[1]	Number of Equally Loaded Cables in Duct Bank																			
		ONE				THREE				SIX				NINE				TWELVE			
		Per Cent Load Factor																			
		30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
		AMPERES PER CONDUCTOR[2]																			
15 000 Volts		Copper Temperature 81°C																			
6	S	94	91	88	83	91	87	81	75	89	83	74	66	87	78	69	60	84	75	64	56
4	SR	123	120	115	107	119	114	104	95	116	108	95	85	113	102	89	77	109	96	83	72
2	SR	159	154	146	137	153	144	139	121	149	136	120	107	144	129	112	97	139	123	104	90
1	SR	179	174	166	156	172	163	149	136	168	153	136	121	162	145	125	109	158	138	117	100
0	CS	203	195	182	176	196	185	169	154	190	173	154	137	183	164	141	122	178	156	131	112
00	CS	234	224	215	202	225	212	193	175	218	198	174	156	211	187	162	139	203	177	148	127
000	CS	270	258	245	230	258	242	220	198	249	225	198	174	241	212	182	157	232	202	168	144
0000	CS	308	295	281	261	295	276	250	223	285	257	224	196	275	241	205	176	265	227	189	162
250	CS	341	327	310	290	325	305	276	246	315	283	245	215	303	265	224	193	291	250	207	177
300	CS	383	365	344	320	364	339	305	272	351	313	271	236	337	293	246	211	322	276	227	194
350	CS	417	397	375	346	397	369	330	293	383	340	293	255	366	318	267	227	350	301	245	208
400	CS	453	428	403	373	429	396	354	314	413	366	313	273	394	340	285	242	376	320	262	222
500	CS	513	487	450	418	483	446	399	350	467	410	350	303	444	381	318	269	419	358	292	247
600	CS	567	537	501	460	534	491	437	385	513	450	384	330	488	416	346	293	465	390	317	269
750	CS	643	606	562	514	602	551	485	426	576	502	423	365	545	464	383	323	519	432	348	293
		(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[3]				(1.08 at 10°C, 0.91 at 30°C, 0.81 at 40°C, 0.70 at 50°C)[3]			
23 000 Volts		Copper Temperature 77°C																			
2	SR	156	150	143	134	149	141	130	117	145	132	117	105	140	125	107	84	134	119	100	86
1	SR	177	170	162	152	170	160	145	133	164	149	132	117	159	140	121	105	154	133	112	97
0	CS	200	192	183	172	192	182	168	149	186	169	147	132	178	158	136	118	173	149	126	109
00	CS	227	220	210	197	221	208	189	170	212	193	168	149	202	181	156	134	196	172	144	123
000	CS	262	251	238	223	254	238	216	191	242	220	191	169	230	206	175	150	222	195	162	139
0000	CS	301	289	271	251	291	273	246	219	278	250	215	190	264	233	197	169	255	221	182	157
250	CS	334	315	298	277	321	299	270	239	308	275	236	207	290	258	216	184	279	242	199	170
300	CS	373	349	328	306	354	329	297	263	341	302	259	227	320	283	232	202	309	266	217	186
350	CS	405	379	358	331	384	356	318	283	369	327	[illegible]	243	347	305	255	217	335	285	233	199
400	CS	434	409	386	356	412	379	340	302	396	348	298	260	374	325	273	232	359	303	247	211
500	CS	492	465	436	401	461	427	379	335	443	391	333	288	424	363	302	257	400	336	275	230
600	CS	543	516	484	440	512	470	414	366	489	428	365	313	464	398	329	279	441	367	299	248
750	CS	616	583	541	495	577	528	465	407	550	479	402	347	520	439	364	306	490	408	329	276
		(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.67 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.66 at 50°C)[3]				(1.09 at 10°C, 0.90 at 30°C, 0.79 at 40°C, 0.65 at 50°C)[3]			
34 500 Volts		Copper Temperature 70°C																			
0	CS	193	185	176	165	184	174	158	141	178	161	140	124	171	149	129	111	164	142	119	103
00	CS	219	209	199	187	208	197	178	160	202	182	158	140	194	170	145	126	185	161	134	115
000	CS	250	238	225	211	238	222	202	182	229	206	179	158	220	193	165	141	209	182	152	128
0000	CS	288	275	260	241	273	256	229	205	263	234	203	179	251	219	186	160	236	205	170	144
250	CS	316	302	285	266	301	280	253	224	289	258	222	196	276	240	202	174	262	222	187	157
300	CS	352	335	315	293	334	310	278	246	320	284	244	213	304	264	221	190	288	244	203	171
350	CS	384	364	342	318	363	336	301	267	316	308	264	229	329	285	238	204	311	263	217	184
400	CS	413	392	367	341	384	360	321	284	372	329	281	244	352	303	254	216	334	282	232	195
500	CS	468	442	414	381	436	402	358	317	418	367	312	271	393	337	281	238	372	313	256	215
600	CS	514	487	455	416	481	440	391	344	459	401	340	294	430	367	304	259	406	340	277	232
750	CS	584	548	510	466	541	496	435	383	515	447	378	324	481	409	337	284	452	377	304	255
		(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.61 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.89 at 30°C, 0.76 at 40°C, 0.60 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.75 at 40°C, 0.58 at 50°C)[3]				(1.10 at 10°C, 0.88 at 30°C, 0.74 at 40°C, 0.56 at 50°C)[3]			

[1] The following symbols are used here to designate conductor types
S—solid copper, SR—standard round concentric-stranded, CS—compact-sector stranded.

[2] Current ratings are based on the following conditions:
a. Ambient earth temperature = 20°C.
b. 60 cycle alternating current.
c. Ratings include dielectric loss, and all induced a-c losses.
d. One cable per duct, all cables equally loaded and in outside ducts only.

[3] Multiply tabulated currents by these factors when earth temperature is other than 20°C.

TABLE A.19 Current-Carrying Capacity of Single-Conductor Solid Paper-Insulated Cables

Conductor Size AWG or MCM	Number of Equally Loaded Cables in Duct Bank: THREE				SIX				NINE				TWELVE			
	Per Cent Load Factor															
	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
	AMPERES PER CONDUCTOR[1]															
7500 Volts									Copper Temperature, 85°C							
6	116	113	109	103	115	110	103	96	113	107	98	90	111	104	94	85
4	154	149	142	135	152	144	134	125	149	140	128	116	147	136	122	110
2	202	196	186	175	199	189	175	162	196	183	167	151	192	178	159	142
1	234	226	214	201	230	218	201	185	226	210	190	172	222	204	181	162
0	270	262	245	232	266	251	231	212	261	242	219	196	256	234	208	184
00	311	300	283	[illegible]	309	290	270	241	303	278	250	224	295	268	236	208
000	356	344	324	300	356	333	303	275	348	310	265	255	340	308	270	238
0000	412	395	371	345	408	380	347	314	398	364	[illegible]	290	390	352	307	269
250	456	438	409	379	449	418	379	344	437	400	356	316	427	386	336	294
300	512	491	459	423	499	464	420	380	486	442	394	349	474	428	371	325
350	561	537	500	460	546	507	457	403	532	483	429	379	518	466	403	352
400	607	580	540	496	593	548	493	445	576	522	461	407	560	502	434	378
500	692	660	611	561	679	626	560	504	659	597	524	459	641	571	490	427
600	772	735	679	621	757	696	621	557	733	663	579	506	714	632	542	470
700	846	804	741	677	827	758	674	604	802	721	629	548	779	688	587	508
750	881	837	771	702	860	789	700	627	835	750	651	568	810	714	609	526
800	914	866	797	725	[illegible]	817	725	648	865	776	674	588	840	740	[illegible]	544
1000	1037	980	898	816	1012	922	815	725	980	874	756	657	950	832	705	606
1250	1176	1108	1012	914	1145	1039	914	809	1104	981	845	730	1068	941	784	673
1500	1300	1224	1110	1000	1268	1146	1000	884	1220	1078	922	794	1178	1032	855	731
1750	1420	1332	1204	1080	1382	1240	1078	949	1342	1166	992	851	1280	1103	919	783
2000	1546	1442	1300	1102	1500	1343	1162	1019	1442	1260	1068	914	1385	1190	986	839
	(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]				(1.07 at 10°C, 0.92 at 30°C, 0.83 at 40°C, 0.73 at 50°C)[2]			
15 000 Volts									Copper Temperature, 81°C							
6	113	110	105	100	112	107	100	93	110	104	96	87	108	101	92	83
4	149	145	138	131	147	140	131	117	144	136	125	114	142	132	119	107
2	195	190	180	170	193	183	170	157	189	177	161	146	186	172	154	137
1	226	218	208	195	222	211	195	179	218	204	185	167	214	197	175	157
0	256	248	234	220	252	239	220	203	247	230	209	188	242	223	198	177
00	297	287	271	254	295	278	253	232	287	265	239	214	283	257	226	202
000	344	330	312	290	341	320	293	267	333	306	274	245	327	296	260	230
0000	399	384	361	335	392	367	[illegible]	305	383	352	315	280	374	340	298	263
250	440	423	396	367	432	404	367	334	422	387	345	306	412	372	325	286
300	490	470	439	408	481	449	408	369	470	429	382	338	457	413	359	316
350	539	516	481	444	527	491	443	401	514	468	416	367	501	450	391	342
400	586	561	522	480	572	530	478	432	558	506	447	395	542	485	419	366
500	669	639	592	543	655	605	542	488	[illegible]	577	507	445	618	551	474	412
600	746	710	656	601	727	668	598	537	705	637	557	488	685	608	521	452
700	810	772	712	652	790	726	647	581	760	691	604	528	744	659	564	488
750	840	797	736	674	821	753	672	602	795	716	625	547	772	684	584	505
800	869	825	762	698	850	780	695	[illegible]	823	741	640	565	800	707	604	522
1000	991	939	864	785	968	882	782	697	933	832	724	631	903	794	675	581
1250	1130	1067	975	864	1102	1000	883	784	1063	941	816	706	1026	898	759	650
1500	1250	1176	1072	966	1220	1105	972	[illegible]	1175	1037	892	772	1133	987	828	707
1750	1368	1282	1162	1044	1330	1198	1042	919	1278	1124	958	824	1230	1063	866	755
2000	1464	1368	1233	1106	1422	1274	1105	970	1360	1192	1013	869	1308	1125	935	795
	(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]				(1.08 at 10°C, 0.92 at 30°C, 0.82 at 40°C, 0.71 at 50°C)[2]			
23 000 Volts									Copper Temperature, 77°C							
2	186	181	172	162	184	175	162	150	180	169	154	140	178	164	147	132
1	214	207	197	188	211	200	185	171	206	193	176	159	203	187	167	150
0	247	239	227	213	244	230	213	196	239	222	197	182	234	216	192	171
00	283	273	258	242	278	263	243	221	275	253	225	205	267	245	217	193
000	326	314	296	277	320	302	276	252	315	290	259	233	307	280	247	220
0000	376	362	340	317	367	345	315	288	360	332	297	265	351	320	281	250
250	412	396	373	346	405	380	346	316	396	365	326	290	386	351	307	272
300	463	444	416	386	450	422	382	349	438	404	360	319	428	389	340	301
350	508	488	466	422	493	461	418	380	481	442	393	347	468	424	369	326
400	548	525	491	454	536	498	451	409	521	478	423	373	507	458	398	349
500	627	600	559	514	615	570	514	464	597	546	480	423	580	521	450	392
600	695	663	616	566	684	632	568	511	663	603	529	466	645	577	496	431
700	765	729	675	620	744	689	617	554	725	656	574	503	703	627	538	467
750	797	759	702	643	779	717	641	574	754	681	596	527	732	650	558	483
800	826	786	726	665	808	743	663	595	782	706	617	540	759	674	576	500
1000	946	898	827	752	921	842	747	667	889	797	692	603	860	759	646	560
1250	1080	1020	935	848	1052	957	845	751	1014	904	781	676	980	858	725	630
1500	1192	1122	1025	925	1162	1053	926	818	1118	993	855	738	1081	940	791	682
1750	1296	1215	1108	994	1256	1130	991	875	1206	1067	911	785	1162	1007	843	720
2000	1390	1302	1180	1058	1352	1213	1053	928	1293	1137	967	831	1240	1073	893	760
	(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.68 at 50°C)[2]				(1.09 at 10°C, 0.90 at 30°C, 0.80 at 40°C, 0.62 at 50°C)[2]			

Continued

TABLE A.19 (Continued)

Conductor Size AWG or MCM	THREE				SIX				NINE				TWELVE			
	Number of Equally Loaded Cables in Duct Bank — Per Cent Load Factor															
	30	50	75	100	30	50	75	100	30	50	75	100	30	50	75	100
	AMPERES PER CONDUCTOR[1]															

34 500 Volts — Copper Temperature, 70°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
0	227	221	209	197	225	213	197	182	220	205	187	169	215	199	177	158
00	200	251	239	224	255	242	224	205	249	234	211	190	245	226	200	179
000	299	290	273	256	295	278	256	235	288	268	242	217	282	259	230	204
0000	341	330	312	291	336	317	291	267	328	304	274	246	321	293	259	230
250	380	367	345	322	374	352	321	294	364	337	303	270	356	324	286	253
300	422	408	382	355	416	390	356	324	405	374	334	298	395	359	315	278
350	464	446	419	389	455	426	388	353	443	408	364	324	432	392	343	302
400	502	484	451	419	491	460	417	379	478	440	390	347	466	421	368	323
500	575	551	514	476	562	524	474	429	547	500	442	392	532	479	418	364
600	644	616	573	528	629	584	520	475	610	556	491	433	593	532	459	401
700	710	675	626	577	690	639	574	517	[illegible]	608	535	470	649	580	500	435
750	736	702	651	598	718	664	595	535	[illegible]	631	554	486	675	602	518	450
800	765	730	676	620	747	690	617	555	723	654	574	503	700	624	535	465
1000	875	832	766	701	852	783	698	624	823	741	646	564	796	706	601	520
1250	994	941	864	786	967	882	782	696	930	833	722	628	898	790	670	577
1500	1098	1036	949	859	1068	972	856	760	1025	914	788	682	[illegible]	865	730	626
1750	1192	1123	1023	925	1156	1048	919	814	1109	984	845	730	[illegible]	929	780	668
2000	1275	1197	1088	981	1234	1115	975	860	1182	1045	893	770	1135	985	824	704
2500	1418	1324	1196	1072	1367	1225	1064	936	1305	1144	973	834	1248	1075	893	760
	(1 10 at 10°C, 0 89 at 30°C, 0 76 at 40°C, 0.81 at 50°C)[2]				(1 10 at 10°C, 0 89 at 30°C, 0 76 at 40°C, 0 61 at 50°C)[2]				(1 10 at 10°C, 0 89 at 30°C, 0 76 at 40°C, 0 60 at 50°C)[2]				(1.10 at 10°C, 0 89 at 30°C, 0 76 at 40°C, 0 60 at 50°C)[2]			

46 000 Volts — Copper Temperature, 65°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
000	279	270	256	240	274	259	230	221	268	249	226	204	262	241	214	191
0000	322	312	294	276	317	299	274	251	309	287	259	232	302	276	244	217
250	352	340	321	300	346	326	290	274	336	313	282	252	329	301	266	236
300	394	380	358	334	385	364	332	304	377	349	313	280	367	335	295	260
350	433	417	392	365	425	398	364	331	413	382	341	304	403	366	321	283
400	469	451	423	393	459	430	391	356	446	411	367	326	433	394	344	307
500	534	512	482	444	522	487	441	400	506	464	412	365	492	444	386	339
600	602	577	538	496	589	546	404	447	570	520	[illegible]	406	553	497	430	377
700	663	633	589	542	645	598	518	486	626	569	[illegible]	441	605	542	468	408
750	689	658	611	561	672	622	559	504	650	590	520	457	629	562	485	422
800	717	683	638	583	698	645	578	522	674	612	[illegible]	472	652	582	501	438
1000	816	776	718	657	794	731	653	585	766	691	[illegible]	528	740	657	562	487
1250	927	879	810	738	900	825	732	654	865	777	675	589	834	736	626	541
1500	1020	968	887	805	992	904	799	703	951	850	735	638	914	802	679	585
1750	1110	1047	959	867	1074	976	859	762	1028	915	788	682	987	862	726	623
2000	1184	1115	1016	918	1144	1035	909	805	1094	970	833	718	1048	913	766	656
2500	1314	1232	1115	1002	1265	1138	994	875	1205	1062	905	778	1151	996	830	708
	(1.11 at 10°C, 0 87 at 30°C, 0 73 at 40°C, 0.54 at 50°C)[2]				(1 11 at 10°C, 0 87 at 30°C, 0.72 at 40°C, 0 53 at 50°C)[2]				(1 11 at 10°C, 0 87 at 30°C, 0 72 at 40°C, 0 52 at 50°C)[2]				(1 12 at 10°C, 0 87 at 30°C, 0 70 at 40°C, 0.51 at 50°C)[2]			

69 000 Volts — Copper Temperature, 60°C

Conductor Size AWG or MCM	THREE 30	THREE 50	THREE 75	THREE 100	SIX 30	SIX 50	SIX 75	SIX 100	NINE 30	NINE 50	NINE 75	NINE 100	TWELVE 30	TWELVE 50	TWELVE 75	TWELVE 100
350	395	382	360	336	387	364	333	305	375	348	312	279	365	332	293	259
400	428	413	389	362	418	393	358	328	405	375	335	300	394	358	315	278
500	489	470	441	409	477	446	406	370	461	425	379	337	447	405	354	312
600	545	524	490	454	532	496	450	409	513	471	419	371	497	448	391	343
700	599	573	536	495	582	543	490	444	[illegible]	514	455	403	542	489	425	372
750	623	597	556	514	605	542	508	460	583	533	472	417	563	506	439	384
800	644	617	575	531	626	582	525	475	603	554	487	430	[illegible]	523	453	396
1000	736	702	652	599	713	660	592	533	685	622	547	481	[illegible]	589	508	442
1250	832	792	734	672	806	742	664	595	772	698	610	535	741	659	564	489
1500	918	872	804	733	886	814	724	647	848	763	664	580	812	718	612	529
1750	994	942	865	788	957	876	776	692	913	818	711	618	873	770	653	563
2000	1066	1008	924	840	1020	931	822	732	972	868	750	651	927	814	688	592
2500	1163	1096	1001	903	1115	1013	892	791	1060	942	811	700	1007	880	741	635
	(1 13 at 10°C 0 85 at 30°C, 0 67 at 40°C 0 42 at 50°C)[2]				(1 13 at 10°C, 0 85 at 30°C 0 66 at 40°C, 0 40 at 50°C)[2]				(1 13 at 10°C, 0 84 at 30°C, 0.65 at 40°C, 0 36 at 50°C)[2]				(1 14 at 10°C, 0 84 at 30°C, 0 64 at 40°C, 0 32 at 50°C)[2]			

[1]Current ratings are based on the following conditions:
- a Ambient earth temperature = 20°C.
- b 60 cycle alternating current.
- c. Sheaths bonded and grounded at one point only (open circuited sheaths).
- d Standard concentric stranded conductors.
- e. Ratings include dielectric loss and skin effect.
- f. One cable per duct, all cables equally loaded and in outside ducts only.

[2]Multiply tabulated values by these factors when earth temperature is other than 20°C.

TABLE A.20 60-Hz Characteristics of Self-Supporting Rubber-Insulated, Neoprene-Jacketed Aerial Cable [1]

Voltage Class	Conductor Size	Stranding	Insulation Thickness	Shielding	Jacket Thickness	Diameter	Messenger Used with Copper Conductors	Wt. Per 1000 Ft, Messenger and Copper	Messenger Used with Aluminum Conductors	Wt. Per 1000 Ft, Messenger and Aluminum	Positive Sequence 60~ AC Ohms/Mi: Resistance (1) Copper	Positive Sequence Resistance (1) Aluminum	Positive Sequence Reactance Series Inductive	Positive Sequence Reactance Shunt Capacitive(2)	Zero Sequence(3) 60~ AC Ohms/Mi: Resistance (1) Copper	Zero Sequence Resistance (1) Aluminum	Zero Sequence Reactance Series Inductive Copper	Zero Sequence Reactance Series Inductive Aluminum	Zero Sequence Reactance Shunt Capacitive(2)
2-kv Ungrounded Neutral 5-kv Grounded Neutral	6	7	10/64	No	4/64	0.59	1/4" 30% CCS	1020	1/4" 30% CCS	854	2.52	4.13	0.258	...	3.592	5.082	3.712	3.712	...
	4	7	10/64	No	4/64	0.67	1/4" 30% CCS	1230	1/4" 30% CCS	956	1.58	2.58	0.246	...	2.632	3.572	3.662	3.662	...
	2	7	10/64	No	4/64	0.73	1/4" 30% CCS	1530	1/4" 30% CCS	1100	1.00	1.64	0.229	...	2.025	2.605	3.615	3.615	...
	1	19	10/64	No	4/64	0.77	1/4" 30% CCS	1780	1/4" 30% CCS	1250	0.791	1.29	0.211	...	1.815	2.275	3.582	3.582	...
	1/0	19	10/64	No	4/64	0.81	1/4" 30% CCS	2070	1/4" 30% CCS	1390	0.635	1.03	0.207	...	1.844	2.015	3.555	3.555	...
	2/0	19	10/64	No	4/64	0.85	1/4" 30% CCS	2510	1/4" 30% CCS	1530	0.501	0.816	0.200	...	1.622	1.903	3.162	3.526	...
	3/0	19	10/64	No	4/64	0.91	1/4" 30% CCS	2890	1/4" 30% CCS	1690	0.402	0.644	0.194	...	1.517	1.637	3.135	1.499	...
	4/0	19	10/64	No	4/64	0.99	3/8" 30% CCS	3570	1/4" 30% CCS	1900	0.318	0.518	0.191	...	1.401	1.508	2.665	3.459	...
	250	37	11/64	No	4/64	1.08	3/8" 30% CCS	4080	1/4" 30% CCS	2160	0.269	0.437	0.189	...	1.351	1.430	2.635	3.429	...
	300	37	11/64	No	4/64	1.13	3/8" 30% CCS	4620	1/4" 30% CCS	2500	0.228	0.366	0.184	...	1.308	1.485	2.612	3.042	...
	350	37	11/64	No	4/64	1.18	3/8" 30% CCS	5290	1/4" 30% CCS	2760	0.197	0.316	0.180	...	1.277	1.415	2.591	3.021	...
	400	37	11/64	No	4/64	1.23	3/8" 30% CCS	5800	1/4" 30% CCS	3040	0.172	0.276	0.176	...	1.252	1.377	2.576	3.006	...
	500	37	11/64	No	4/64	1.32	3/8" 30% CCS	6860	3/8" 30% CCS	3650	0.141	0.223	0.172	...	1.219	1.290	2.543	2.543	...
5-kv Ungrounded Neutral	6	7	10/64	Yes	4/64	0.74	1/4" 30% CCS	1310	1/4" 30% CCS	1140	2.52	4.13	0.292	4970	...	...	...	...	...
	4	7	10/64	Yes	4/64	0.79	1/4" 30% CCS	1540	1/4" 30% CCS	1270	1.58	2.58	0.272	4320	...	...	...	...	...
	2	7	10/64	Yes	4/64	0.88	1/4" 30% CCS	1950	1/4" 30% CCS	1520	1.00	1.64	0.257	3830	...	...	...	...	...
	1	19	10/64	Yes	4/64	0.92	1/4" 30% CCS	2180	1/4" 30% CCS	1640	0.791	1.29	0.241	3330	...	...	...	...	...
	1/0	19	10/64	Yes	4/64	0.96	1/4" 30% CCS	2450	1/4" 30% CCS	1770	0.655	1.03	0.233	3080	...	...	...	...	...
	2/0	19	10/64	Yes	4/64	1.00	1/4" 30% CCS	2910	1/4" 30% CCS	1930	0.501	0.816	0.223	2830	...	...	...	...	...
	3/0	19	10/64	Yes	4/64	1.06	1/4" 30% CCS	3320	1/4" 30% CCS	2120	0.402	0.644	0.215	2590	...	...	...	...	...
	4/0	19	10/64	Yes	4/64	1.11	1/4" 30% CCS	4030	1/4" 30% CCS	2350	0.318	0.518	0.207	2380	...	...	...	...	...
	250	37	11/64	Yes	4/64	1.20	3/8" 30% CCS	4570	3/8" 30% CCS	2770	0.269	0.437	0.206	2380	...	...	...	...	...
	300	37	11/64	Yes	4/64	1.29	3/8" 30% CCS	5260	3/8" 30% CCS	3140	0.228	0.366	0.203	2280	...	...	...	...	...
	350	37	11/64	Yes	4/64	1.34	3/8" 30% CCS	5840	3/8" 30% CCS	3380	0.197	0.316	0.199	2000	...	...	...	...	...
	400	37	11/64	Yes	4/64	1.39	3/8" 30% CCS	6380	3/8" 30% CCS	3610	0.172	0.276	0.194	1890	...	...	...	...	...
	500	37	11/64	Yes	4/64	1.47	3/8" 30% CCS	7470	3/8" 30% CCS	4240	0.141	0.223	0.187	1740	...	...	...	...	...
15-kv Grounded Neutral	6	19	19/64	Yes	4/64	1.05	1/4" 30% CCS	2090	1/4" 30% CCS	1920	2.52	4.13	0.326	7150	3.348	5.348	3.396	3.396	7150
	4	19	19/64	Yes	4/64	1.10	1/4" 30% CCS	2350	1/4" 30% CCS	2080	1.58	2.58	0.302	6260	2.901	3.831	3.364	3.364	6260
	2	19	19/64	Yes	4/64	1.16	1/4" 30% CCS	2560	3/8" 30% CCS	2430	1.00	1.64	0.279	5460	2.419	3.039	2.851	2.851	5460
	1	19	19/64	Yes	4/64	1.20	1/4" 30% CCS	3120	3/8" 30% CCS	2580	0.791	1.29	0.268	5110	2.238	2.701	2.837	2.837	5110
	1/0	19	19/64	Yes	4/64	1.27	1/4" 30% CCS	3560	3/8" 30% CCS	2880	0.655	1.03	0.260	4720	2.052	2.426	2.825	2.825	4720
	2/0	19	19/64	Yes	4/64	1.32	3/8" 30% CCS	4120	3/8" 30% CCS	3070	0.501	0.816	0.249	4370	1.808	2.214	2.251	2.801	4370
	3/0	19	19/64	Yes	4/64	1.37	3/8" 30% CCS	4580	3/8" 30% CCS	3510	0.402	0.644	0.241	4120	1.762	2.008	2.240	2.240	4120
	4/0	19	19/64	Yes	4/64	1.43	3/8" 30% CCS	5150	3/8" 30% CCS	3790	0.318	0.518	0.231	3770	1.681	1.864	2.235	2.235	3770
	250	37	19/64	Yes	4/64	1.47	3/8" 30% CCS	5590	3/8" 30% CCS	3980	0.269	0.437	0.223	3570	1.630	1.782	2.227	2.227	3570
	300	37	19/64	Yes	4/64	1.53	3/8" 30% CCS	6260	3/8" 30% CCS	4330	0.228	0.366	0.217	3330	1.577	1.701	2.226	2.226	3330
	350	37	19/64	Yes	4/64	1.59	3/8" 30% CCS	6870	3/8" 30% CCS	4600	0.197	0.316	0.212	3130	1.536	1.640	2.226	2.226	3130
	400	37	19/64	Yes	4/64	1.63	3/8" 30% CCS	7450	3/8" 30% CCS	4860	0.172	0.276	0.208	2980	1.500	1.592	2.216	2.216	2980
	500	37	19/64	Yes	1/4	1.75	3/8" 30% CCS	8970	3/8" 30% CCS	5560	0.141	0.223	0.204	2830	1.454	1.524	2.198	2.198	2830

(1) AC resistance based on 65°C with allowance for stranding, skin effect and proximity effect.

(2) Dielectric constant assumed 6.0.

(3) Zero-sequence impedance based on return current both in the messenger and in 100 meter-ohm earth.

TABLE A.21 Inductive Reactance of ACSR Bundled Conductors at 60 Hz [3]

						60 Hz INDUCTIVE REACTANCE (1) X_A IN OHMS/MILE FOR 1 FOOT RADIUS										
							2 - CONDUCTOR SPACING (IN.)					3 - CONDUCTOR SPACING (IN.)				
CODE	AREA CMIL	STRANDS AL	STRANDS ST	DIA. IN.	GMR FT.	SINGLE COND.	6	9	12	15	18	6	9	12	15	18
EXPANDED	3108000	62/8	19	2.500	0.0900	0.2922	0.1881	0.1635	0.1461	0.1326	0.1215	0.1535	0.1207	0.0974	0.0793	0.0646
EXPANDED	2294000	66/6	19	2.320	0.0858	0.2980	0.1910	0.1664	0.1490	0.1355	0.1244	0.1554	0.1226	0.0993	0.0813	0.0665
EXPANDED	1414000	58/4	19	1.750	0.0640	0.3336	0.2088	0.1842	0.1668	0.1532	0.1422	0.1673	0.1345	0.1112	0.0931	0.0784
EXPANDED	1275000	50/4	19	1.600	0.0578	0.3459	0.2150	0.1904	0.1730	0.1594	0.1484	0.1714	0.1386	0.1153	0.0973	0.0825
KIWI	2167000	72	7	1.737	0.0571	0.3474	0.2158	0.1912	0.1737	0.1602	0.1491	0.1719	0.1391	0.1158	0.0977	0.0830
BLUEBIRD	2156000	84	19	1.762	0.0588	0.3438	0.2140	0.1894	0.1719	0.1584	0.1473	0.1707	0.1379	0.1146	0.0966	0.0818
CHUKAR	1780000	84	19	1.602	0.0536	0.3551	0.2196	0.1950	0.1775	0.1640	0.1529	0.1744	0.1416	0.1184	0.1003	0.0856
FALCON	1590000	54	19	1.545	0.0523	0.3580	0.2211	0.1965	0.1790	0.1655	0.1544	0.1754	0.1426	0.1193	0.1013	0.0866
LAPWING	1590000	45	7	1.502	0.0498	0.3640	0.2241	0.1995	0.1820	0.1685	0.1574	0.1774	0.1446	0.1213	0.1033	0.0885
PARROT	1510500	54	19	1.506	0.0506	0.3621	0.2231	0.1985	0.1810	0.1675	0.1564	0.1768	0.1440	0.1207	0.1026	0.0879
NUTHATCH	1510500	45	7	1.466	0.0486	0.3670	0.2255	0.2009	0.1835	0.1699	0.1589	0.1784	0.1456	0.1223	0.1043	0.0895
PLOVER	1431000	54	19	1.465	0.0494	0.3650	0.2245	0.1999	0.1825	0.1689	0.1579	0.1777	0.1449	0.1217	0.1036	0.0889
BOBOLINK	1431000	45	7	1.427	0.0470	0.3710	0.2276	0.2030	0.1855	0.1720	0.1609	0.1797	0.1469	0.1237	0.1056	0.0909
MARTIN	1351500	54	19	1.424	0.0482	0.3680	0.2260	0.2014	0.1840	0.1704	0.1594	0.1787	0.1459	0.1227	0.1046	0.0899
DIPPER	1351500	45	7	1.385	0.0459	0.3739	0.2290	0.2044	0.1869	0.1734	0.1623	0.1807	0.1479	0.1246	0.1066	0.0918
PHEASANT	1272000	54	19	1.382	0.0466	0.3721	0.2281	0.2035	0.1860	0.1725	0.1614	0.1801	0.1473	0.1240	0.1060	0.0912
BITTERN	1272000	45	7	1.345	0.0444	0.3779	0.2310	0.2064	0.1890	0.1754	0.1644	0.1820	0.1492	0.1260	0.1079	0.0932
GRACKLE	1192500	54	19	1.333	0.0451	0.3760	0.2301	0.2055	0.1880	0.1745	0.1634	0.1814	0.1486	0.1253	0.1073	0.0925
BUNTING	1192500	45	7	1.302	0.0429	0.3821	0.2331	0.2085	0.1910	0.1775	0.1664	0.1834	0.1506	0.1274	0.1093	0.0946
FINCH	1113000	54	19	1.293	0.0436	0.3801	0.2321	0.2075	0.1901	0.1765	0.1655	0.1828	0.1500	0.1267	0.1087	0.0939
BLUEJAY	1113000	45	7	1.259	0.0415	0.3861	0.2351	0.2105	0.1931	0.1795	0.1685	0.1848	0.1520	0.1287	0.1107	0.0959
CURLEW	1033500	54	7	1.246	0.0420	0.3847	0.2344	0.2098	0.1923	0.1788	0.1677	0.1843	0.1515	0.1282	0.1102	0.0954
ORTOLAN	1033500	45	7	1.213	0.0402	0.3900	0.2370	0.2124	0.1950	0.1815	0.1704	0.1861	0.1533	0.1300	0.1119	0.0972
TANAGER	1033500	36	1	1.186	0.0384	0.3955	0.2398	0.2152	0.1978	0.1842	0.1732	0.1879	0.1551	0.1318	0.1138	0.0990
CARDINAL	954000	54	7	1.196	0.0402	0.3900	0.2370	0.2124	0.1950	0.1815	0.1704	0.1861	0.1533	0.1300	0.1119	0.0972
RAIL	954000	45	7	1.165	0.0386	0.3949	0.2395	0.2149	0.1975	0.1839	0.1729	0.1877	0.1549	0.1316	0.1136	0.0988
CATBIRD	954000	36	1	1.140	0.0370	0.4000	0.2421	0.2175	0.2000	0.1865	0.1754	0.1894	0.1566	0.1333	0.1153	0.1005
CANARY	900000	54	7	1.162	0.0392	0.3930	0.2386	0.2140	0.1965	0.1830	0.1719	0.1871	0.1543	0.1310	0.1130	0.0982
RUDDY	900000	45	7	1.131	0.0374	0.3987	0.2414	0.2168	0.1994	0.1858	0.1748	0.1890	0.1562	0.1329	0.1149	0.1001
MALLARD	795000	30	19	1.140	0.0392	0.3930	0.2386	0.2140	0.1965	0.1830	0.1719	0.1871	0.1543	0.1310	0.1130	0.0982
DRAKE	795000	26	7	1.108	0.0373	0.3991	0.2416	0.2170	0.1995	0.1860	0.1749	0.1891	0.1563	0.1330	0.1150	0.1002
CONDOR	795000	54	7	1.093	0.0370	0.4000	0.2421	0.2175	0.2000	0.1865	0.1754	0.1894	0.1566	0.1333	0.1153	0.1005
CUCKOO	795000	24	7	1.092	0.0366	0.4014	0.2427	0.2181	0.2007	0.1871	0.1761	0.1899	0.1571	0.1338	0.1157	0.1010
TERN	795000	45	7	1.063	0.0352	0.4061	0.2451	0.2205	0.2030	0.1895	0.1784	0.1914	0.1586	0.1354	0.1173	0.1026
COOT	795000	36	1	1.040	0.0377	0.3978	0.2409	0.2163	0.1989	0.1853	0.1743	0.1887	0.1559	0.1326	0.1145	0.0998
REDWING	715500	30	19	1.081	0.0373	0.3991	0.2416	0.2170	0.1995	0.1860	0.1749	0.1891	0.1563	0.1330	0.1150	0.1002
STARLING	715500	26	7	1.051	0.0355	0.4051	0.2446	0.2200	0.2025	0.1890	0.1779	0.1911	0.1583	0.1350	0.1170	0.1022
STILT	715500	24	7	1.036	0.0347	0.4078	0.2460	0.2214	0.2039	0.1904	0.1793	0.1920	0.1592	0.1359	0.1179	0.1031
GANNET	666600	26	7	1.014	0.0343	0.4092	0.2467	0.2221	0.2046	0.1911	0.1800	0.1925	0.1597	0.1364	0.1184	0.1036
FLAMINGO	666600	24	7	1.000	0.0355	0.4121	0.2481	0.2235	0.2061	0.1925	0.1815	0.1934	0.1606	0.1374	0.1193	0.1046
--------	653900	18	3	0.953	0.0308	0.4223	0.2532	0.2286	0.2111	0.1976	0.1865	0.1968	0.1640	0.1408	0.1227	0.1080

EGRET	636000	30	19	1.019	0.0352	0.4061	0.2451	0.2205	0.2030	0.1895	0.1784	0.1919	0.1586	0.1354	0.1173	0.1026
GROSBEAK	636000	26	7	0.990	0.0335	0.4121	0.2481	0.2235	0.2061	0.1925	0.1815	0.1934	0.1606	0.1374	0.1193	0.1046
ROOK	636000	24	7	0.977	0.0327	0.4150	0.2496	0.2250	0.2075	0.1940	0.1829	0.1944	0.1616	0.1383	0.1203	0.1055
KINGBIRD	636000	18	1	0.940	0.0304	0.4239	0.2540	0.2294	0.2119	0.1984	0.1873	0.1974	0.1646	0.1413	0.1232	0.1085
SWIFT	636000	36	1	0.930	0.0301	0.4251	0.2546	0.2300	0.2125	0.1990	0.1879	0.1978	0.1650	0.1417	0.1236	0.1089
TEAL	605000	30	19	0.994	0.0341	0.4099	0.2470	0.2224	0.2050	0.1914	0.1804	0.1927	0.1599	0.1366	0.1186	0.1038
SQUAB	605000	26	7	0.966	0.0327	0.4150	0.2496	0.2250	0.2075	0.1940	0.1829	0.1944	0.1616	0.1383	0.1203	0.1055
PEACOCK	605000	24	7	0.953	0.0319	0.4180	0.2511	0.2265	0.2090	0.1955	0.1844	0.1954	0.1626	0.1393	0.1213	0.1065
EAGLE	556500	30	7	0.953	0.0327	0.4150	0.2496	0.2250	0.2075	0.1940	0.1829	0.1944	0.1616	0.1383	0.1203	0.1055
DOVE	556500	26	7	0.927	0.0314	0.4200	0.2520	0.2274	0.2100	0.1964	0.1854	0.1961	0.1633	0.1400	0.1219	0.1072
PARAKEET	556500	24	7	0.914	0.0306	0.4231	0.2536	0.2290	0.2115	0.1980	0.1869	0.1971	0.1643	0.1410	0.1230	0.1082
OSPREY	556500	18	1	0.879	0.0284	0.4321	0.2581	0.2335	0.2161	0.2025	0.1915	0.2001	0.1673	0.1440	0.1260	0.1112
HEN	477000	30	7	0.883	0.0304	0.4239	0.2540	0.2294	0.2119	0.1984	0.1873	0.1974	0.1646	0.1413	0.1232	0.1085
HAWK	477000	26	7	0.858	0.0289	0.4300	0.2571	0.2325	0.2150	0.2015	0.1904	0.1994	0.1666	0.1433	0.1253	0.1105
FLICKER	477000	24	7	0.846	0.0284	0.4321	0.2581	0.2335	0.2161	0.2025	0.1915	0.2001	0.1673	0.1440	0.1260	0.1112
PELICAN	477000	18	1	0.814	0.0264	0.4410	0.2626	0.2380	0.2205	0.2070	0.1959	0.2031	0.1703	0.1470	0.1289	0.1142
LARK	397500	30	7	0.806	0.0277	0.4352	0.2596	0.2350	0.2176	0.2040	0.1930	0.2011	0.1683	0.1451	0.1270	0.1123
IBIS	397500	26	7	0.783	0.0264	0.4410	0.2626	0.2380	0.2205	0.2070	0.1959	0.2031	0.1703	0.1470	0.1289	0.1142
BRANT	397500	24	7	0.772	0.0258	0.4438	0.2639	0.2394	0.2219	0.2084	0.1973	0.2040	0.1712	0.1479	0.1299	0.1151
CHICKADEE	397500	18	1	0.743	0.0241	0.4521	0.2681	0.2435	0.2260	0.2125	0.2014	0.2068	0.1740	0.1507	0.1326	0.1179
ORIOLE	336400	30	7	0.741	0.0255	0.4452	0.2647	0.2401	0.2226	0.2091	0.1980	0.2045	0.1717	0.1484	0.1304	0.1156
LINNET	336400	26	7	0.721	0.0243	0.4511	0.2676	0.2430	0.2255	0.2120	0.2009	0.2064	0.1736	0.1504	0.1323	0.1176
MERLIN	336400	18	1	0.684	0.0222	0.4620	0.2731	0.2485	0.2310	0.2175	0.2064	0.2101	0.1773	0.1540	0.1360	0.1212
OSTRICH	300000	26	7	0.680	0.0229	0.4583	0.2712	0.2466	0.2291	0.2156	0.2045	0.2088	0.1760	0.1528	0.1347	0.1200

(1) X_A IS THE COMPONENT OF INDUCTIVE REACTANCE DUE TO THE MAGNETIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF INDUCTIVE REACTANCE, X_D IS THAT DUE TO OTHER PHASES. THE TOTAL INDUCTIVE REACTANCE PER PHASE IS THE SUM OF X_A AND X_D. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A. X_D IS OBTAINED FROM THE FORMULA BELOW.

$$X_A = 0.2794 \log_{10}\left[\frac{1}{\left[N\,(GMR)\,(A)^{N-1}\right]^{\frac{1}{N}}}\right] \text{ OHMS PER MILE}$$

WHERE: GMR = GEOMETRIC MEAN RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \sin(\pi/N))$; $N > 1$
$A = 0$; $0^0 \equiv 1$: $N = 1$
S = BUNDLE SPACING IN FEET

$$X_D = 0.2794 \log_{10}(GMD) \text{ OHMS PER MILE}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

TABLE A.22 Inductive Reactance of ACSR Bundled Conductors at 60 Hz [3]

CODE	AREA CMIL	STRANDS AL	STRANDS ST	DIA. IN.	GMR FT.	60 Hz INDUCTIVE REACTANCE (1) X_A IN OHMS/MILE FOR 1 FOOT RADIUS — 4 - CONDUCTOR SPACING (IN.) 6	9	12	15	18	6 - CONDUCTOR SPACING (IN.) 6	9	12	15	18
EXPANDED	3108000	62/8	19	2.500	0.0900	0.1256	0.0887	0.0625	0.0422	0.0256	0.0826	0.0416	0.0125	-0.0101	-0.0285
EXPANDED	2294000	66/6	19	2.320	0.0858	0.1271	0.0902	0.0640	0.0437	0.0271	0.0835	0.0425	0.0134	-0.0091	-0.0276
EXPANDED	1414000	58/4	19	1.750	0.0640	0.1360	0.0991	0.0729	0.0526	0.0360	0.0894	0.0484	0.0194	-0.0032	-0.0216
EXPANDED	1275000	50/4	19	1.600	0.0578	0.1390	0.1021	0.0760	0.0557	0.0391	0.0915	0.0505	0.0214	-0.0011	-0.0196
KIWI	2167000	72	7	1.737	0.0571	0.1394	0.1025	0.0763	0.0560	0.0394	0.0918	0.0508	0.0217	-0.0009	-0.0193
BLUEBIRD	2156000	84	19	1.762	0.0588	0.1385	0.1016	0.0754	0.0551	0.0385	0.0912	0.0502	0.0211	-0.0015	-0.0199
CHUKAR	1780000	84	19	1.602	0.0536	0.1413	0.1044	0.0783	0.0579	0.0414	0.0930	0.0520	0.0229	0.0004	-0.0181
FALCON	1590000	54	19	1.545	0.0523	0.1421	0.1052	0.0790	0.0587	0.0421	0.0935	0.0525	0.0234	0.0009	-0.0176
LAPWING	1590000	45	7	1.502	0.0498	0.1436	0.1067	0.0805	0.0602	0.0436	0.0945	0.0535	0.0244	0.0019	-0.0166
PARROT	1510500	54	19	1.506	0.0506	0.1431	0.1062	0.0800	0.0597	0.0431	0.0942	0.0532	0.0241	0.0015	-0.0169
NUTHATCH	1510500	45	7	1.466	0.0486	0.1443	0.1074	0.0812	0.0609	0.0443	0.0950	0.0540	0.0249	0.0024	-0.0161
PLOVER	1431000	54	19	1.465	0.0494	0.1438	0.1069	0.0807	0.0604	0.0438	0.0947	0.0537	0.0246	0.0020	-0.0164
BOBOLINK	1431000	45	7	1.427	0.0470	0.1453	0.1084	0.0822	0.0619	0.0453	0.0957	0.0547	0.0256	0.0030	-0.0154
MARTIN	1351500	54	19	1.424	0.0482	0.1446	0.1077	0.0815	0.0612	0.0446	0.0952	0.0542	0.0251	0.0025	-0.0159
DIPPER	1351500	45	7	1.385	0.0459	0.1460	0.1091	0.0830	0.0627	0.0461	0.0962	0.0552	0.0261	0.0035	-0.0149
PHEASANT	1272000	54	19	1.382	0.0466	0.1456	0.1087	0.0825	0.0622	0.0456	0.0959	0.0549	0.0258	0.0032	-0.0152
BITTERN	1272000	45	7	1.345	0.0444	0.1470	0.1101	0.0840	0.0637	0.0471	0.0968	0.0558	0.0268	0.0042	-0.0142
GRACKLE	1192500	54	19	1.333	0.0451	0.1466	0.1097	0.0835	0.0632	0.0466	0.0965	0.0555	0.0264	0.0039	-0.0146
BUNTING	1192500	45	7	1.302	0.0429	0.1481	0.1112	0.0850	0.0647	0.0481	0.0975	0.0565	0.0274	0.0049	-0.0136
FINCH	1113000	54	19	1.293	0.0436	0.1476	0.1107	0.0845	0.0642	0.0476	0.0972	0.0562	0.0271	0.0046	-0.0139
BLUEJAY	1113000	45	7	1.259	0.0415	0.1491	0.1122	0.0860	0.0657	0.0491	0.0982	0.0572	0.0281	0.0056	-0.0129
CURLEW	1033500	54	7	1.246	0.0420	0.1487	0.1118	0.0857	0.0653	0.0488	0.0980	0.0570	0.0279	0.0053	-0.0131
ORTOLAN	1033500	45	7	1.213	0.0402	0.1501	0.1132	0.0870	0.0667	0.0501	0.0989	0.0579	0.0288	0.0062	-0.0122
TANAGER	1033500	36	1	1.186	0.0384	0.1515	0.1146	0.0884	0.0681	0.0515	0.0998	0.0588	0.0297	0.0071	-0.0113
CARDINAL	954000	54	7	1.196	0.0402	0.1501	0.1132	0.0870	0.0667	0.0501	0.0989	0.0579	0.0288	0.0062	-0.0122
RAIL	954000	45	7	1.165	0.0386	0.1513	0.1144	0.0882	0.0679	0.0513	0.0997	0.0587	0.0296	0.0070	-0.0114
CATBIRD	954000	36	1	1.140	0.0370	0.1526	0.1157	0.0895	0.0692	0.0526	0.1005	0.0595	0.0304	0.0079	-0.0106
CANARY	900000	54	7	1.162	0.0392	0.1508	0.1139	0.0877	0.0674	0.0508	0.0994	0.0584	0.0293	0.0067	-0.0117
RUDDY	900000	45	7	1.131	0.0374	0.1523	0.1154	0.0892	0.0689	0.0523	0.1003	0.0593	0.0302	0.0077	-0.0108
MALLARD	795000	30	19	1.140	0.0392	0.1508	0.1139	0.0877	0.0674	0.0508	0.0994	0.0584	0.0293	0.0067	-0.0117
DRAKE	795000	26	7	1.108	0.0373	0.1523	0.1154	0.0893	0.0689	0.0524	0.1004	0.0594	0.0303	0.0077	-0.0107
CONDOR	795000	54	7	1.093	0.0370	0.1526	0.1157	0.0895	0.0692	0.0526	0.1005	0.0595	0.0304	0.0079	-0.0106
CUCKOO	795000	24	7	1.092	0.0366	0.1529	0.1160	0.0898	0.0695	0.0529	0.1007	0.0597	0.0307	0.0081	-0.0103
TERN	795000	45	7	1.063	0.0352	0.1541	0.1172	0.0910	0.0707	0.0541	0.1015	0.0605	0.0314	0.0089	-0.0096
COOT	795000	36	1	1.040	0.0377	0.1520	0.1151	0.0889	0.0686	0.0520	0.1001	0.0591	0.0301	0.0075	-0.0109
REDWING	715500	30	19	1.081	0.0373	0.1523	0.1154	0.0893	0.0689	0.0524	0.1004	0.0594	0.0303	0.0077	-0.0107
STARLING	715500	26	7	1.051	0.0355	0.1538	0.1169	0.0908	0.0704	0.0539	0.1014	0.0604	0.0313	0.0087	-0.0097
STILT	715500	24	7	1.036	0.0347	0.1545	0.1176	0.0914	0.0711	0.0545	0.1018	0.0608	0.0317	0.0092	-0.0093
GANNET	666600	26	7	1.014	0.0343	0.1549	0.1180	0.0918	0.0715	0.0549	0.1021	0.0611	0.0320	0.0094	-0.0090
FLAMINGO	666600	24	7	1.000	0.0355	0.1556	0.1187	0.0925	0.0722	0.0556	0.1025	0.0615	0.0324	0.0099	-0.0086
--------	653900	18	3	0.953	0.0308	0.1581	0.1212	0.0951	0.0748	0.0582	0.1042	0.0632	0.0341	0.0116	-0.0069

EGRET	636000	30	19	1.019	0.0352	0.1541	0.1172	0.0910	0.0707	0.0541	0.1015	0.0605	0.0314	0.0089	-0.0096
GROSBEAK	636000	26	7	0.990	0.0335	0.1556	0.1187	0.0925	0.0722	0.0556	0.1025	0.0615	0.0324	0.0099	-0.0086
ROOK	636000	24	7	0.977	0.0327	0.1563	0.1194	0.0932	0.0729	0.0563	0.1030	0.0620	0.0329	0.0104	-0.0081
KINGBIRD	636000	18	1	0.940	0.0304	0.1585	0.1216	0.0955	0.0752	0.0586	0.1045	0.0635	0.0344	0.0118	-0.0066
SWIFT	636000	36	1	0.930	0.0301	0.1588	0.1219	0.0958	0.0755	0.0589	0.1047	0.0637	0.0346	0.0120	-0.0064
TEAL	605000	30	19	0.994	0.0341	0.1551	0.1182	0.0920	0.0717	0.0551	0.1022	0.0612	0.0321	0.0095	-0.0089
SQUAB	605000	26	7	0.966	0.0327	0.1563	0.1194	0.0932	0.0729	0.0563	0.1030	0.0620	0.0329	0.0104	-0.0081
PEACOCK	605000	24	7	0.953	0.0319	0.1571	0.1202	0.0940	0.0737	0.0571	0.1035	0.0625	0.0334	0.0109	-0.0076
EAGLE	556500	30	7	0.953	0.0327	0.1563	0.1194	0.0932	0.0729	0.0563	0.1030	0.0620	0.0329	0.0104	-0.0081
DOVE	556500	26	7	0.927	0.0314	0.1576	0.1207	0.0945	0.0742	0.0576	0.1038	0.0628	0.0338	0.0112	-0.0072
PARAKEET	556500	24	7	0.914	0.0306	0.1583	0.1214	0.0953	0.0750	0.0584	0.1044	0.0634	0.0343	0.0117	-0.0067
OSPREY	556500	18	1	0.879	0.0284	0.1606	0.1237	0.0975	0.0772	0.0606	0.1059	0.0649	0.0358	0.0132	-0.0052
HEN	477000	30	7	0.883	0.0304	0.1585	0.1216	0.0955	0.0752	0.0586	0.1045	0.0635	0.0344	0.0118	-0.0066
HAWK	477000	26	7	0.858	0.0289	0.1601	0.1232	0.0970	0.0767	0.0601	0.1055	0.0645	0.0354	0.0129	-0.0056
FLICKER	477000	24	7	0.846	0.0284	0.1606	0.1237	0.0975	0.0772	0.0606	0.1059	0.0649	0.0358	0.0132	-0.0052
PELICAN	477000	18	1	0.814	0.0264	0.1628	0.1259	0.0997	0.0794	0.0628	0.1074	0.0664	0.0373	0.0147	-0.0037
LARK	397500	30	7	0.806	0.0277	0.1614	0.1245	0.0983	0.0780	0.0614	0.1064	0.0654	0.0363	0.0137	-0.0047
IBIS	397500	26	7	0.783	0.0264	0.1628	0.1259	0.0997	0.0794	0.0628	0.1074	0.0664	0.0373	0.0147	-0.0037
BRANT	397500	24	7	0.772	0.0258	0.1635	0.1266	0.1004	0.0801	0.0635	0.1078	0.0668	0.0377	0.0152	-0.0033
CHICKADEE	397500	18	1	0.743	0.0241	0.1656	0.1287	0.1025	0.0822	0.0656	0.1092	0.0682	0.0391	0.0165	-0.0019
ORIOLE	336400	30	7	0.741	0.0255	0.1639	0.1270	0.1008	0.0805	0.0639	0.1081	0.0671	0.0380	0.0154	-0.0030
LINNET	336400	26	7	0.721	0.0243	0.1653	0.1284	0.1023	0.0819	0.0654	0.1090	0.0680	0.0389	0.0164	-0.0021
MERLIN	336400	18	1	0.684	0.0222	0.1681	0.1312	0.1050	0.0847	0.0681	0.1109	0.0699	0.0408	0.0182	-0.0002
OSTRICH	300000	26	7	0.680	0.0229	0.1671	0.1302	0.1041	0.0837	0.0672	0.1102	0.0692	0.0401	0.0176	-0.0009

(1) X_A IS THE COMPONENT OF INDUCTIVE REACTANCE DUE TO THE MAGNETIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF INDUCTIVE REACTANCE, X_D IS THAT DUE TO OTHER PHASES. THE TOTAL INDUCTIVE REACTANCE PER PHASE IS THE SUM OF X_A AND X_D. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A. X_D IS OBTAINED FROM THE FORMULA BELOW.

$$X_A = 0.2794 \ \mathrm{LOG}_{10} \left[\frac{1}{\left[N\ (GMR)\ (A)^{N-1} \right]^{\frac{1}{N}}} \right] \text{ OHMS PER MILE}$$

$$X_D = 0.2794 \ \mathrm{LOG}_{10}(GMD) \text{ OHMS PER MILE}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

WHERE: GMR = GEOMETRIC MEAN RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \sin(\pi/N))$; $N > 1$
$A = 0$; $0^0 \equiv 1$: $N=1$
S = BUNDLE SPACING IN FEET

TABLE A.23 Inductive Reactance of ACAR Bundled Conductors at 60 Hz [3]

AREA 62% EQ. EC - AL CMIL	STRANDS EC/6201		DIA. IN.	GMR FT.	60 Hz INDUCTIVE REACTANCE (1) X_A IN OHMS/MILE FOR 1 FOOT RADIUS										
					SINGLE COND.	2 - CONDUCTOR SPACING (IN.)					3 - CONDUCTOR SPACING (IN.)				
						6	9	12	15	18	6	9	12	15	18
2413000	72	19	1.821	0.0596	0.3422	0.2132	0.1886	0.1711	0.1576	0.1465	0.1701	0.1373	0.1141	0.0960	0.0813
2375000	63	28	1.821	0.0596	0.3422	0.2132	0.1886	0.1711	0.1576	0.1465	0.1701	0.1373	0.1141	0.0960	0.0813
2338000	54	37	1.821	0.0599	0.3416	0.2128	0.1882	0.1708	0.1573	0.1462	0.1699	0.1371	0.1139	0.0958	0.0811
2297000	54	7	1.762	0.0571	0.3474	0.2158	0.1912	0.1737	0.1602	0.1491	0.1719	0.1391	0.1158	0.0977	0.0830
2262000	48	13	1.762	0.0578	0.3459	0.2150	0.1904	0.1730	0.1594	0.1484	0.1714	0.1386	0.1153	0.0973	0.0825
2226000	42	19	1.762	0.0578	0.3459	0.2160	0.1904	0.1730	0.1594	0.1484	0.1714	0.1386	0.1153	0.0973	0.0825
2227000	54	7	1.735	0.0561	0.3495	0.2168	0.1922	0.1748	0.1612	0.1502	0.1726	0.1398	0.1165	0.0985	0.0837
2193000	48	13	1.735	0.0530	0.3564	0.2203	0.1957	0.1782	0.1647	0.1536	0.1749	0.1421	0.1188	0.1008	0.0860
2159000	42	19	1.735	0.0568	0.3480	0.2161	0.1915	0.1740	0.1605	0.1494	0.1721	0.1393	0.1160	0.0980	0.0832
1899000	54	7	1.602	0.0519	0.3590	0.2215	0.1969	0.1795	0.1660	0.1549	0.1757	0.1429	0.1197	0.1016	0.0869
1870000	48	13	1.602	0.0490	0.3660	0.2250	0.2004	0.1830	0.1694	0.1584	0.1781	0.1453	0.1220	0.1039	0.0892
1841000	42	19	1.602	0.0526	0.3574	0.2207	0.1961	0.1787	0.1651	0.1541	0.1752	0.1424	0.1191	0.1011	0.0863
1673000	54	7	1.504	0.0486	0.3670	0.2255	0.2009	0.1835	0.1699	0.1589	0.1784	0.1456	0.1223	0.1043	0.0895
1647000	48	13	1.504	0.0461	0.3734	0.2287	0.2041	0.1867	0.1731	0.1621	0.1805	0.1477	0.1245	0.1064	0.0917
1622000	42	19	1.504	0.0495	0.3647	0.2244	0.1998	0.1824	0.1688	0.1578	0.1776	0.1448	0.1216	0.1035	0.0888
1337000	54	7	1.345	0.0436	0.3801	0.2321	0.2075	0.1901	0.1765	0.1655	0.1828	0.1500	0.1267	0.1087	0.0939
1296000	42	19	1.345	0.0440	0.3790	0.2316	0.2070	0.1895	0.1760	0.1649	0.1824	0.1496	0.1263	0.1083	0.0935
1243000	30	7	1.302	0.0421	0.3844	0.2342	0.2096	0.1922	0.1786	0.1676	0.1842	0.1514	0.1281	0.1101	0.0953
1211000	24	13	1.302	0.0417	0.3855	0.2348	0.2102	0.1928	0.1792	0.1682	0.1846	0.1518	0.1285	0.1105	0.0957
1179000	18	19	1.302	0.0426	0.3829	0.2335	0.2089	0.1915	0.1779	0.1669	0.1837	0.1509	0.1276	0.1096	0.0948
1163000	30	7	1.259	0.0407	0.3885	0.2363	0.2117	0.1942	0.1807	0.1696	0.1856	0.1528	0.1295	0.1114	0.0967
1133000	24	13	1.259	0.0408	0.3882	0.2361	0.2115	0.1941	0.1806	0.1695	0.1855	0.1527	0.1294	0.1113	0.0966
1104000	18	19	1.259	0.0412	0.3870	0.2356	0.2110	0.1935	0.1800	0.1689	0.1851	0.1523	0.1290	0.1109	0.0962
1153000	33	4	1.246	0.0401	0.3903	0.2372	0.2126	0.1951	0.1816	0.1705	0.1862	0.1534	0.1301	0.1120	0.0973
1138000	30	7	1.246	0.0403	0.3897	0.2369	0.2123	0.1948	0.1813	0.1702	0.1860	0.1532	0.1299	0.1118	0.0971
1109000	24	13	1.246	0.0405	0.3891	0.2366	0.2120	0.1945	0.1810	0.1699	0.1858	0.1530	0.1297	0.1116	0.0969
1080000	18	19	1.246	0.0407	0.3885	0.2363	0.2117	0.1942	0.1807	0.1696	0.1856	0.1528	0.1295	0.1114	0.0967
1077000	30	7	1.212	0.0393	0.3927	0.2384	0.2138	0.1964	0.1828	0.1718	0.1870	0.1542	0.1309	0.1129	0.0981
1049000	24	13	1.212	0.0389	0.3940	0.2390	0.2144	0.1970	0.1834	0.1724	0.1874	0.1546	0.1313	0.1133	0.0985
1022000	18	19	1.212	0.0396	0.3918	0.2380	0.2134	0.1959	0.1824	0.1713	0.1867	0.1539	0.1306	0.1125	0.0978
1050000	30	7	1.196	0.0388	0.3943	0.2392	0.2146	0.1971	0.1836	0.1725	0.1875	0.1547	0.1314	0.1134	0.0986
1023000	24	13	1.196	0.0384	0.3955	0.2398	0.2152	0.1978	0.1842	0.1732	0.1879	0.1551	0.1318	0.1138	0.0990
996000	18	19	1.196	0.0391	0.3933	0.2387	0.2141	0.1967	0.1831	0.1721	0.1872	0.1544	0.1311	0.1131	0.0983

994800	30	7	1.165	0.0376	0.3981	0.2411	0.2165	0.1990	0.1855	0.1744	0.1888	0.1560	0.1327	0.1146	0.0999
954600	30	7	1.141	0.0369	0.4004	0.2422	0.2176	0.2002	0.1866	0.1756	0.1895	0.1567	0.1335	0.1154	0.1007
969300	24	13	1.165	0.0374	0.3987	0.2414	0.2168	0.1994	0.1858	0.1748	0.1890	0.1562	0.1329	0.1149	0.1001
958000	24	13	1.158	0.0371	0.3997	0.2419	0.2173	0.1999	0.1863	0.1753	0.1893	0.1565	0.1332	0.1152	0.1004
943900	18	19	1.165	0.0381	0.3965	0.2403	0.2157	0.1982	0.1847	0.1736	0.1882	0.1554	0.1322	0.1141	0.0994
900300	30	7	1.108	0.0358	0.4040	0.2441	0.2195	0.2020	0.1885	0.1774	0.1908	0.1580	0.1347	0.1166	0.1019
795000	30	7	1.042	0.0334	0.4125	0.2483	0.2237	0.2062	0.1927	0.1816	0.1936	0.1608	0.1375	0.1194	0.1047
877300	24	13	1.108	0.0355	0.4051	0.2446	0.2200	0.2025	0.1890	0.1779	0.1911	0.1583	0.1350	0.1170	0.1022
795000	24	13	1.055	0.0339	0.4107	0.2474	0.2228	0.2053	0.1918	0.1807	0.1930	0.1602	0.1369	0.1188	0.1041
854200	18	19	1.108	0.0361	0.4030	0.2436	0.2190	0.2015	0.1880	0.1769	0.1904	0.1576	0.1343	0.1163	0.1015
795000	18	19	1.069	0.0349	0.4071	0.2456	0.2210	0.2036	0.1900	0.1790	0.1918	0.1590	0.1357	0.1177	0.1029
829000	30	7	1.063	0.0343	0.4092	0.2467	0.2221	0.2046	0.1911	0.1800	0.1925	0.1597	0.1364	0.1184	0.1036
807700	24	13	1.063	0.0342	0.4096	0.2468	0.2222	0.2048	0.1913	0.1802	0.1926	0.1598	0.1365	0.1185	0.1037
786500	18	19	1.063	0.0348	0.4075	0.2458	0.2212	0.2037	0.1902	0.1791	0.1919	0.1591	0.1358	0.1178	0.1030
727500	33	4	0.990	0.0319	0.4180	0.2511	0.2265	0.2090	0.1955	0.1844	0.1954	0.1626	0.1393	0.1213	0.1065
718300	30	7	0.990	0.0320	0.4177	0.2509	0.2263	0.2088	0.1953	0.1842	0.1953	0.1625	0.1392	0.1212	0.1064
700000	24	13	0.990	0.0317	0.4188	0.2515	0.2269	0.2094	0.1959	0.1848	0.1957	0.1629	0.1396	0.1215	0.1068
681600	18	19	0.990	0.0324	0.4162	0.2501	0.2255	0.2081	0.1945	0.1835	0.1948	0.1620	0.1387	0.1207	0.1059
632000	15	4	0.927	0.0296	0.4271	0.2556	0.2310	0.2136	0.2000	0.1890	0.1984	0.1656	0.1424	0.1243	0.1096
616200	12	7	0.927	0.0291	0.4292	0.2566	0.2320	0.2146	0.2011	0.1900	0.1991	0.1663	0.1431	0.1250	0.1103
487400	15	4	0.814	0.0260	0.4429	0.2635	0.2389	0.2214	0.2079	0.1968	0.2037	0.1709	0.1476	0.1296	0.1148
475200	12	7	0.814	0.0261	0.4424	0.2632	0.2386	0.2212	0.2077	0.1966	0.2035	0.1707	0.1475	0.1294	0.1147
343600	15	4	0.684	0.0221	0.4626	0.2733	0.2487	0.2313	0.2177	0.2067	0.2103	0.1775	0.1542	0.1361	0.1214
335000	12	7	0.684	0.0219	0.4637	0.2739	0.2493	0.2318	0.2183	0.2072	0.2106	0.1778	0.1546	0.1365	0.1218

(1) X_A IS THE COMPONENT OF INDUCTIVE REACTANCE DUE TO THE MAGNETIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF INDUCTIVE REACTANCE, X_D IS THAT DUE TO OTHER PHASES. THE TOTAL INDUCTIVE REACTANCE PER PHASE IS THE SUM OF X_A AND X_D. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A. X_D IS OBTAINED FROM THE FORMULA BELOW.

$$X_A = 0.2794 \ \mathrm{LOG}_{10} \left[\frac{1}{\left[N\ (GMR)\ (A)^{N-1} \right]^{\frac{1}{N}}} \right] \text{ OHMS PER MILE}$$

WHERE: GMR = GEOMETRIC MEAN RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
A = $S/(2 \sin(\pi/N))$; $N > 1$
A = 0; $0^0 \equiv 1$: $N=1$
S = BUNDLE SPACING IN FEET

$$X_D = 0.2794 \ \mathrm{LOG}_{10}(GMD) \text{ OHMS PER MILE}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

TABLE A.24 Inductive Reactance of ACAR Bundled Conductors at 60 Hz [3]

AREA 62% EQ. EC - AL CMIL	STRANDS EC/6201		DIA. IN.	GMR FT.	60 Hz INDUCTIVE REACTANCE (1) X_A IN OHMS/MILE FOR 1 FOOT RADIUS: 4 - CONDUCTOR SPACING (IN.) 6	9	12	15	18	6 - CONDUCTOR SPACING (IN.) 6	9	12	15	18
2413000	72	19	1.821	0.0596	0.1381	0.1012	0.0750	0.0547	0.0381	0.0909	0.0499	0.0208	-0.0018	-0.0202
2375000	63	28	1.821	0.0596	0.1381	0.1012	0.0750	0.0547	0.0381	0.0909	0.0499	0.0208	-0.0018	-0.0202
2338000	54	37	1.821	0.0599	0.1380	0.1011	0.0749	0.0546	0.0380	0.0908	0.0498	0.0207	-0.0019	-0.0203
2297000	54	7	1.762	0.0571	0.1394	0.1025	0.0763	0.0560	0.0394	0.0918	0.0508	0.0217	-0.0009	-0.0193
2262000	48	13	1.762	0.0578	0.1390	0.1021	0.0760	0.0557	0.0391	0.0915	0.0505	0.0214	-0.0011	-0.0196
2226000	42	19	1.762	0.0578	0.1390	0.1021	0.0760	0.0557	0.0391	0.0915	0.0505	0.0214	-0.0011	-0.0196
2227000	54	7	1.735	0.0561	0.1400	0.1031	0.0769	0.0566	0.0400	0.0921	0.0511	0.0220	-0.0005	-0.0190
2193000	48	13	1.735	0.0530	0.1417	0.1048	0.0786	0.0583	0.0417	0.0933	0.0523	0.0232	0.0006	-0.0178
2159000	42	19	1.735	0.0568	0.1396	0.1027	0.0765	0.0562	0.0396	0.0919	0.0509	0.0218	-0.0008	-0.0192
1899000	54	7	1.602	0.0519	0.1423	0.1054	0.0792	0.0589	0.0423	0.0937	0.0527	0.0236	0.0010	-0.0174
1870000	48	13	1.602	0.0490	0.1441	0.1072	0.0810	0.0607	0.0441	0.0948	0.0538	0.0248	0.0022	-0.0162
1841000	42	19	1.602	0.0526	0.1419	0.1050	0.0788	0.0585	0.0419	0.0934	0.0524	0.0233	0.0008	-0.0177
1673000	54	7	1.504	0.0486	0.1443	0.1074	0.0812	0.0609	0.0443	0.0950	0.0540	0.0249	0.0024	-0.0161
1647000	48	13	1.504	0.0461	0.1459	0.1090	0.0828	0.0625	0.0459	0.0961	0.0551	0.0260	0.0034	-0.0150
1622000	42	19	1.504	0.0495	0.1437	0.1068	0.0807	0.0604	0.0438	0.0946	0.0536	0.0246	0.0020	-0.0164
1337000	54	7	1.345	0.0436	0.1476	0.1107	0.0845	0.0642	0.0476	0.0972	0.0562	0.0271	0.0046	-0.0139
1296000	42	19	1.345	0.0440	0.1473	0.1104	0.0842	0.0639	0.0473	0.0970	0.0560	0.0269	0.0044	-0.0141
1243000	30	7	1.302	0.0421	0.1487	0.1118	0.0856	0.0653	0.0487	0.0979	0.0569	0.0278	0.0053	-0.0132
1211000	24	13	1.302	0.0417	0.1490	0.1121	0.0859	0.0656	0.0490	0.0981	0.0571	0.0280	0.0055	-0.0130
1179000	18	19	1.302	0.0426	0.1483	0.1114	0.0852	0.0649	0.0483	0.0977	0.0567	0.0276	0.0050	-0.0134
1163000	30	7	1.259	0.0407	0.1497	0.1128	0.0866	0.0663	0.0497	0.0986	0.0576	0.0285	0.0059	-0.0125
1133000	24	13	1.259	0.0408	0.1496	0.1127	0.0865	0.0662	0.0496	0.0986	0.0576	0.0285	0.0059	-0.0125
1104000	18	19	1.259	0.0412	0.1493	0.1124	0.0862	0.0659	0.0493	0.0984	0.0574	0.0283	0.0057	-0.0127
1153000	33	4	1.246	0.0401	0.1501	0.1132	0.0871	0.0667	0.0502	0.0989	0.0579	0.0288	0.0062	-0.0122
1138000	30	7	1.246	0.0403	0.1500	0.1131	0.0869	0.0666	0.0500	0.0988	0.0578	0.0287	0.0061	-0.0123
1109000	24	13	1.246	0.0405	0.1498	0.1129	0.0868	0.0664	0.0499	0.0987	0.0577	0.0286	0.0060	-0.0124
1080000	18	19	1.246	0.0407	0.1497	0.1128	0.0866	0.0663	0.0497	0.0986	0.0576	0.0285	0.0059	-0.0125
1077000	30	7	1.212	0.0393	0.1507	0.1138	0.0877	0.0674	0.0508	0.0993	0.0583	0.0292	0.0067	-0.0118
1049000	24	13	1.212	0.0389	0.1511	0.1142	0.0880	0.0677	0.0511	0.0995	0.0585	0.0294	0.0069	-0.0116
1022000	18	19	1.212	0.0396	0.1505	0.1136	0.0874	0.0671	0.0505	0.0992	0.0582	0.0291	0.0065	-0.0119
1050000	30	7	1.196	0.0388	0.1511	0.1142	0.0881	0.0677	0.0512	0.0996	0.0586	0.0295	0.0069	-0.0115
1023000	24	13	1.196	0.0384	0.1515	0.1146	0.0884	0.0681	0.0515	0.0998	0.0588	0.0297	0.0071	-0.0113
996000	18	19	1.196	0.0391	0.1509	0.1140	0.0878	0.0675	0.0509	0.0994	0.0584	0.0293	0.0068	-0.0117

994800	30	7	1.165	0.0376	0.1521	0.1152	0.0890	0.0687	0.0521	0.1002	0.0592	0.0301	0.0075	-0.0109
954600	30	7	1.141	0.0369	0.1527	0.1158	0.0896	0.0693	0.0527	0.1006	0.0596	0.0305	0.0079	-0.0105
969300	24	13	1.165	0.0374	0.1523	0.1154	0.0892	0.0689	0.0523	0.1003	0.0593	0.0302	0.0077	-0.0108
958000	24	13	1.158	0.0371	0.1525	0.1156	0.0894	0.0691	0.0525	0.1005	0.0595	0.0304	0.0078	-0.0106
943900	18	19	1.165	0.0381	0.1517	0.1148	0.0886	0.0683	0.0517	0.0999	0.0589	0.0298	0.0073	-0.0112
900300	30	7	1.108	0.0358	0.1536	0.1167	0.0905	0.0702	0.0536	0.1012	0.0602	0.0311	0.0085	-0.0099
795000	30	7	1.042	0.0334	0.1557	0.1188	0.0926	0.0723	0.0557	0.1026	0.0616	0.0325	0.0099	-0.0085
877300	24	13	1.108	0.0355	0.1538	0.1169	0.0908	0.0704	0.0539	0.1014	0.0604	0.0313	0.0087	-0.0097
795000	24	13	1.055	0.0339	0.1552	0.1183	0.0922	0.0718	0.0553	0.1023	0.0613	0.0322	0.0096	-0.0088
854200	18	19	1.108	0.0361	0.1533	0.1164	0.0902	0.0699	0.0533	0.1010	0.0600	0.0309	0.0084	-0.0101
795000	18	19	1.069	0.0349	0.1544	0.1175	0.0913	0.0710	0.0544	0.1017	0.0607	0.0316	0.0091	-0.0094
829000	30	7	1.063	0.0343	0.1549	0.1180	0.0918	0.0715	0.0549	0.1021	0.0611	0.0320	0.0094	-0.0090
807700	24	13	1.063	0.0342	0.1550	0.1181	0.0919	0.0716	0.0550	0.1021	0.0611	0.0320	0.0095	-0.0090
786500	18	19	1.063	0.0348	0.1544	0.1175	0.0914	0.0710	0.0545	0.1018	0.0608	0.0317	0.0091	-0.0093
727500	33	4	0.990	0.0319	0.1571	0.1202	0.0940	0.0737	0.0571	0.1035	0.0625	0.0334	0.0109	-0.0076
718300	30	7	0.990	0.0320	0.1570	0.1201	0.0939	0.0736	0.0570	0.1035	0.0625	0.0334	0.0108	-0.0076
700000	24	13	0.990	0.0317	0.1573	0.1204	0.0942	0.0739	0.0573	0.1037	0.0627	0.0336	0.0110	-0.0074
681600	18	19	0.990	0.0324	0.1566	0.1197	0.0935	0.0732	0.0566	0.1032	0.0622	0.0331	0.0106	-0.0079
632000	15	4	0.927	0.0296	0.1593	0.1224	0.0963	0.0760	0.0594	0.1050	0.0640	0.0350	0.0124	-0.0060
616200	12	7	0.927	0.0291	0.1599	0.1230	0.0968	0.0765	0.0599	0.1054	0.0644	0.0353	0.0127	-0.0057
487400	15	4	0.814	0.0260	0.1633	0.1264	0.1002	0.0799	0.0633	0.1077	0.0667	0.0376	0.0150	-0.0034
475200	12	7	0.814	0.0261	0.1632	0.1263	0.1001	0.0798	0.0632	0.1076	0.0666	0.0375	0.0149	-0.0035
343600	15	4	0.684	0.0221	0.1682	0.1313	0.1051	0.0848	0.0682	0.1109	0.0699	0.0409	0.0183	-0.0001
335000	12	7	0.684	0.0219	0.1685	0.1316	0.1054	0.0851	0.0685	0.1111	0.0701	0.0410	0.0185	0.0000

(1) X_A IS THE COMPONENT OF INDUCTIVE REACTANCE DUE TO THE MAGNETIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF INDUCTIVE REACTANCE, X_D IS THAT DUE TO OTHER PHASES. THE TOTAL INDUCTIVE REACTANCE PER PHASE IS THE SUM OF X_A AND X_D. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A. X_D IS OBTAINED FROM THE FORMULA BELOW

$$X_A = 0.2794 \ \mathrm{LOG}_{10} \left[\frac{1}{\left[N \ (GMR) \ (A)^{N-1} \right]^{\frac{1}{N}}} \right] \text{ OHMS PER MILE}$$

WHERE: GMR = GEOMETRIC MEAN RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \sin(\pi/N))$: $N > 1$
$A = 0$: $0^0 \equiv 1$: $N = 1$
S = BUNDLE SPACING IN FEET

$$X_D = 0.2794 \ \mathrm{LOG}_{10}(GMD) \text{ OHMS PER MILE}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

TABLE A.25 Inductive Reactance of ACAR Bundled Conductors at 60 Hz [3]

CODE	AREA CMIL	STRANDS AL	STRANDS ST	DIA. IN.	60 Hz CAPACITIVE REACTANCE [1] X_A' IN MEGOHM-MILES FOR 1 FOOT RADIUS: SINGLE COND.	2 - CONDUCTOR SPACING (IN.) 6	9	12	15	18	3 - CONDUCTOR SPACING (IN.) 6	9	12	15	18
EXPANDED	3108000	62/8	19	2.500	0.0671	0.0438	0.0378	0.0336	0.0302	0.0275	0.0361	0.0281	0.0224	0.0180	0.0143
EXPANDED	2294000	66/6	19	2.320	0.0693	0.0449	0.0389	0.0347	0.0313	0.0286	0.0368	0.0288	0.0231	0.0187	0.0151
EXPANDED	1414000	58/4	19	1.750	0.0777	0.0491	0.0431	0.0388	0.0355	0.0328	0.0396	0.0316	0.0259	0.0215	0.0179
EXPANDED	1275000	50/4	19	1.600	0.0803	0.0505	0.0444	0.0402	0.0369	0.0342	0.0405	0.0325	0.0268	0.0224	0.0188
KIWI	2167000	72	7	1.737	0.0779	0.0492	0.0432	0.0390	0.0356	0.0329	0.0397	0.0317	0.0260	0.0216	0.0179
BLUEBIRD	2156000	84	19	1.762	0.0775	0.0490	0.0430	0.0387	0.0354	0.0327	0.0395	0.0315	0.0258	0.0214	0.0178
CHUKAR	1780000	84	19	1.602	0.0803	0.0504	0.0444	0.0402	0.0368	0.0341	0.0405	0.0325	0.0268	0.0224	0.0187
FALCON	1590000	54	19	1.545	0.0814	0.0510	0.0450	0.0407	0.0374	0.0347	0.0408	0.0328	0.0271	0.0227	0.0191
LAPWING	1590000	45	7	1.502	0.0822	0.0514	0.0454	0.0411	0.0378	0.0351	0.0411	0.0331	0.0274	0.0230	0.0194
PARROT	1510500	54	19	1.506	0.0821	0.0514	0.0453	0.0411	0.0378	0.0351	0.0411	0.0331	0.0274	0.0230	0.0194
NUTHATCH	1510500	45	7	1.466	0.0829	0.0518	0.0457	0.0415	0.0382	0.0355	0.0414	0.0333	0.0276	0.0232	0.0196
PLOVER	1431000	54	19	1.465	0.0830	0.0518	0.0457	0.0415	0.0382	0.0355	0.0414	0.0333	0.0277	0.0232	0.0196
BOBOLINK	1431000	45	7	1.427	0.0837	0.0522	0.0461	0.0419	0.0386	0.0359	0.0416	0.0336	0.0279	0.0235	0.0199
MARTIN	1351500	54	19	1.424	0.0838	0.0522	0.0462	0.0419	0.0386	0.0359	0.0416	0.0336	0.0279	0.0235	0.0199
DIPPER	1351500	45	7	1.385	0.0846	0.0526	0.0466	0.0423	0.0390	0.0363	0.0419	0.0339	0.0282	0.0238	0.0202
PHEASANT	1272000	54	19	1.382	0.0847	0.0526	0.0466	0.0423	0.0390	0.0363	0.0419	0.0339	0.0282	0.0238	0.0202
BITTERN	1272000	45	7	1.345	0.0855	0.0530	0.0470	0.0427	0.0394	0.0367	0.0422	0.0342	0.0285	0.0241	0.0205
GRACKLE	1192500	54	19	1.333	0.0858	0.0532	0.0471	0.0429	0.0396	0.0369	0.0423	0.0343	0.0286	0.0242	0.0206
BUNTING	1192500	45	7	1.302	0.0865	0.0535	0.0475	0.0432	0.0399	0.0372	0.0425	0.0345	0.0288	0.0244	0.0208
FINCH	1113000	54	19	1.293	0.0867	0.0536	0.0476	0.0433	0.0400	0.0373	0.0426	0.0346	0.0289	0.0245	0.0209
BLUEJAY	1113000	45	7	1.259	0.0875	0.0540	0.0480	0.0437	0.0404	0.0377	0.0429	0.0348	0.0292	0.0247	0.0211
CURLEW	1033500	54	7	1.246	0.0878	0.0542	0.0481	0.0439	0.0406	0.0379	0.0430	0.0349	0.0293	0.0248	0.0212
ORTOLAN	1033500	45	7	1.213	0.0886	0.0546	0.0485	0.0443	0.0410	0.0383	0.0432	0.0352	0.0295	0.0251	0.0215
TANAGER	1033500	36	1	1.186	0.0892	0.0549	0.0489	0.0446	0.0413	0.0386	0.0435	0.0354	0.0297	0.0253	0.0217
CARDINAL	954000	54	7	1.196	0.0890	0.0548	0.0488	0.0445	0.0412	0.0385	0.0434	0.0353	0.0297	0.0252	0.0216
RAIL	954000	45	7	1.165	0.0898	0.0552	0.0491	0.0449	0.0416	0.0389	0.0436	0.0356	0.0299	0.0255	0.0219
CATBIRD	954000	36	1	1.140	0.0904	0.0555	0.0495	0.0452	0.0419	0.0392	0.0438	0.0358	0.0301	0.0257	0.0221
CANARY	900000	54	7	1.162	0.0898	0.0552	0.0492	0.0449	0.0416	0.0389	0.0437	0.0356	0.0299	0.0255	0.0219
RUDDY	900000	45	7	1.131	0.0906	0.0556	0.0496	0.0453	0.0420	0.0393	0.0439	0.0359	0.0302	0.0258	0.0222
MALLARD	795000	30	19	1.140	0.0904	0.0555	0.0495	0.0452	0.0419	0.0392	0.0438	0.0358	0.0301	0.0257	0.0221
DRAKE	795000	26	7	1.108	0.0912	0.0559	0.0499	0.0456	0.0423	0.0396	0.0441	0.0361	0.0304	0.0260	0.0224
CONDOR	795000	54	7	1.093	0.0916	0.0561	0.0501	0.0458	0.0425	0.0398	0.0443	0.0362	0.0305	0.0261	0.0225
CUCKOO	795000	24	7	1.092	0.0917	0.0561	0.0501	0.0458	0.0425	0.0398	0.0443	0.0362	0.0306	0.0261	0.0225
TERN	795000	45	7	1.063	0.0925	0.0565	0.0505	0.0462	0.0429	0.0402	0.0445	0.0365	0.0308	0.0264	0.0228
COOT	795000	36	1	1.040	0.0931	0.0568	0.0508	0.0466	0.0433	0.0405	0.0448	0.0367	0.0310	0.0266	0.0230

REDWING	715500	30	19	1.081	0.0920	0.0563	0.0503	0.0460	0.0427	0.0400	0.0444	0.0363	0.0307	0.0262	0.0226
STARLING	715500	26	7	1.051	0.0928	0.0567	0.0507	0.0464	0.0431	0.0404	0.0446	0.0366	0.0309	0.0265	0.0229
STILT	715500	24	7	1.036	0.0932	0.0569	0.0509	0.0466	0.0433	0.0406	0.0448	0.0368	0.0311	0.0267	0.0231
GANNET	666600	26	7	1.014	0.0939	0.0572	0.0512	0.0469	0.0436	0.0409	0.0450	0.0370	0.0313	0.0269	0.0233
FLAMINGO	666600	24	7	1.000	0.0943	0.0574	0.0514	0.0471	0.0438	0.0411	0.0451	0.0371	0.0314	0.0270	0.0234
--------	653900	18	3	0.953	0.0957	0.0581	0.0521	0.0479	0.0445	0.0418	0.0456	0.0376	0.0319	0.0275	0.0239
EGRET	636000	30	19	1.019	0.0937	0.0571	0.0511	0.0469	0.0436	0.0408	0.0450	0.0369	0.0312	0.0268	0.0232
GROSBEAK	636000	26	7	0.990	0.0946	0.0576	0.0516	0.0473	0.0440	0.0413	0.0452	0.0372	0.0315	0.0271	0.0235
ROOK	636000	24	7	0.977	0.0950	0.0578	0.0518	0.0475	0.0442	0.0415	0.0454	0.0373	0.0317	0.0272	0.0236
KINGBIRD	636000	18	1	0.940	0.0961	0.0583	0.0523	0.0481	0.0448	0.0420	0.0458	0.0377	0.0320	0.0276	0.0240
SWIFT	636000	36	1	0.930	0.0964	0.0585	0.0525	0.0482	0.0449	0.0422	0.0459	0.0378	0.0321	0.0277	0.0241
TEAL	605000	30	19	0.994	0.0945	0.0575	0.0515	0.0472	0.0439	0.0412	0.0452	0.0372	0.0315	0.0271	0.0235
SQUAB	605000	26	7	0.966	0.0953	0.0579	0.0519	0.0477	0.0443	0.0416	0.0455	0.0375	0.0318	0.0274	0.0238
PEACOCK	605000	24	7	0.953	0.0957	0.0581	0.0521	0.0479	0.0445	0.0418	0.0456	0.0376	0.0319	0.0275	0.0239
EAGLE	556500	30	7	0.953	0.0957	0.0581	0.0521	0.0479	0.0445	0.0418	0.0456	0.0376	0.0319	0.0275	0.0239
DOVE	556500	26	7	0.927	0.0965	0.0586	0.0525	0.0483	0.0450	0.0423	0.0459	0.0379	0.0322	0.0278	0.0242
PARAKEET	556500	24	7	0.914	0.0970	0.0588	0.0527	0.0485	0.0452	0.0425	0.0460	0.0380	0.0323	0.0279	0.0243
OSPREY	556500	18	1	0.879	0.0981	0.0593	0.0533	0.0491	0.0457	0.0430	0.0464	0.0384	0.0327	0.0283	0.0247
HEN	477000	30	7	0.883	0.0980	0.0593	0.0533	0.0490	0.0457	0.0430	0.0464	0.0383	0.0327	0.0282	0.0246
HAWK	477000	26	7	0.858	0.0988	0.0597	0.0537	0.0494	0.0461	0.0434	0.0467	0.0386	0.0329	0.0285	0.0249
FLICKER	477000	24	7	0.846	0.0992	0.0599	0.0539	0.0496	0.0463	0.0436	0.0468	0.0388	0.0331	0.0287	0.0251
PELICAN	477000	18	1	0.814	0.1004	0.0605	0.0545	0.0502	0.0469	0.0442	0.0472	0.0392	0.0335	0.0291	0.0254
LARK	397500	30	7	0.806	0.1007	0.0606	0.0546	0.0503	0.0470	0.0443	0.0473	0.0393	0.0336	0.0291	0.0255
IBIS	397500	26	7	0.783	0.1015	0.0611	0.0550	0.0508	0.0475	0.0448	0.0476	0.0395	0.0338	0.0294	0.0258
BRANT	397500	24	7	0.772	0.1020	0.0613	0.0552	0.0510	0.0477	0.0450	0.0477	0.0397	0.0340	0.0296	0.0260
CHICKADEE	397500	18	1	0.743	0.1031	0.0618	0.0558	0.0516	0.0482	0.0455	0.0481	0.0401	0.0344	0.0300	0.0263
ORIOLE	336400	30	7	0.741	0.1032	0.0619	0.0559	0.0516	0.0483	0.0456	0.0481	0.0401	0.0344	0.0300	0.0264
LINNET	336400	26	7	0.721	0.1040	0.0623	0.0563	0.0520	0.0487	0.0460	0.0484	0.0404	0.0347	0.0303	0.0266
MERLIN	336400	18	1	0.684	0.1056	0.0631	0.0570	0.0528	0.0495	0.0468	0.0489	0.0409	0.0352	0.0308	0.0272
OSTRICH	300000	26	7	0.680	0.1057	0.0631	0.0571	0.0529	0.0496	0.0468	0.0490	0.0409	0.0352	0.0308	0.0272

(1) X_A' IS THE COMPONENT OF CAPACITIVE REACTANCE DUE TO THE ELECTROSTATIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF CAPACITIVE REACTANCE, X_D' ACCOUNTS FOR THE FLUX BETWEEN THE 1 FOOT RADIUS AND THE OTHER PHASES. THE TOTAL CAPACITIVE REACTANCE PER PHASE IS THE SUM OF X_A' AND X_D'. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A'. X_D' IS OBTAINED FROM THE FORMULA BELOW.

$$X_A' = 0.0683 \ \mathrm{LOG}_{10} \left[\frac{1}{\left[N \ (r) \ (A)^{N-1} \right]^{\frac{1}{N}}} \right] \ \text{MEGOHM-MILES}$$

WHERE: r = CONDUCTOR RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \ \mathrm{SIN} \ (\pi/N))$: $N>1$
$A = 0$: $0^0 \equiv 1$: $N=1$
S = BUNDLE SPACING IN FEET

$$X_D' = 0.0683 \ \mathrm{LOG}_{10}(\mathrm{GMD}) \ \text{MEGOHM-MILES}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

TABLE A.26 Capacitive Reactance of ACSR Bundled Conductors at 60 Hz [3]

					60 Hz CAPACITIVE REACTANCE (1) X_A' IN MEGOHM-MILES FOR 1 FOOT RADIUS									
					4 - CONDUCTOR SPACING (IN.)					6 - CONDUCTOR SPACING (IN.)				
CODE	AREA CMIL	STRANDS AL	STRANDS ST	DIA. IN.	6	9	12	15	18	6	9	12	15	18
EXPANDED	3108000	62/8	19	2.500	0.0296	0.0206	0.0142	0.0092	0.0052	0.0195	0.0094	0.0023	-0.0032	-0.0077
EXPANDED	2294000	66/6	19	2.320	0.0302	0.0212	0.0148	0.0098	0.0057	0.0198	0.0098	0.0027	-0.0028	-0.0073
EXPANDED	1414000	58/4	19	1.750	0.0323	0.0233	0.0169	0.0119	0.0078	0.0212	0.0112	0.0041	-0.0014	-0.0059
EXPANDED	1275000	50/4	19	1.600	0.0329	0.0239	0.0175	0.0126	0.0085	0.0217	0.0116	0.0045	-0.0010	-0.0055
KIWI	2167000	72	7	1.737	0.0323	0.0233	0.0169	0.0119	0.0079	0.0213	0.0112	0.0041	-0.0014	-0.0059
BLUEBIRD	2156000	84	19	1.762	0.0322	0.0232	0.0168	0.0118	0.0078	0.0212	0.0112	0.0041	-0.0015	-0.0060
CHUKAR	1780000	84	19	1.602	0.0329	0.0239	0.0175	0.0125	0.0085	0.0217	0.0116	0.0045	-0.0010	-0.0055
FALCON	1590000	54	19	1.545	0.0332	0.0242	0.0178	0.0128	0.0088	0.0218	0.0118	0.0047	-0.0008	-0.0053
LAPWING	1590000	45	7	1.502	0.0334	0.0244	0.0180	0.0130	0.0090	0.0220	0.0120	0.0048	-0.0007	-0.0052
PARROT	1510500	54	19	1.506	0.0334	0.0244	0.0180	0.0130	0.0089	0.0220	0.0119	0.0048	-0.0007	-0.0052
NUTHATCH	1510500	45	7	1.466	0.0336	0.0246	0.0182	0.0132	0.0091	0.0221	0.0121	0.0050	-0.0006	-0.0051
PLOVER	1431000	54	19	1.465	0.0336	0.0246	0.0182	0.0132	0.0091	0.0221	0.0121	0.0050	-0.0006	-0.0051
BOBOLINK	1431000	45	7	1.427	0.0338	0.0248	0.0184	0.0134	0.0093	0.0222	0.0122	0.0051	-0.0004	-0.0049
MARTIN	1351500	54	19	1.424	0.0338	0.0248	0.0184	0.0134	0.0094	0.0222	0.0122	0.0051	-0.0004	-0.0049
DIPPER	1351500	45	7	1.385	0.0340	0.0250	0.0186	0.0136	0.0096	0.0224	0.0124	0.0052	-0.0003	-0.0048
PHEASANT	1272000	54	19	1.382	0.0340	0.0250	0.0186	0.0136	0.0096	0.0224	0.0124	0.0053	-0 0003	-0.0048
BITTERN	1272000	45	7	1.345	0.0342	0.0252	0.0188	0.0138	0.0098	0.0225	0.0125	0.0054	-0.0001	-0.0046
GRACKLE	1192500	54	19	1.333	0.0343	0.0253	0.0189	0.0139	0.0098	0.0226	0.0125	0.0054	-0.0001	-0.0046
BUNTING	1192500	45	7	1.302	0.0345	0.0254	0.0190	0.0141	0.0100	0.0227	0.0127	0.0055	0.0000	-0.0045
FINCH	1113000	54	19	1.293	0.0345	0.0255	0.0191	0.0141	0.0101	0.0227	0.0127	0.0056	0.0001	-0.0044
BLUEJAY	1113000	45	7	1.259	0.0347	0.0257	0.0193	0.0143	0.0103	0.0229	0.0128	0.0057	0.0002	-0.0043
CURLEW	1033500	54	7	1.246	0.0348	0.0258	0.0194	0.0144	0.0103	0.0229	0.0129	0.0058	0.0003	-0.0043
ORTOLAN	1033500	45	7	1.213	0.0350	0.0260	0.0196	0.0146	0.0105	0.0230	0.0130	0.0059	0.0004	-0.0041
TANAGER	1033500	36	1	1.186	0.0352	0.0261	0.0197	0.0148	0.0107	0.0231	0.0131	0.0060	0.0005	-0.0040
CARDINAL	954000	54	7	1.196	0.0351	0.0261	0.0197	0.0147	0.0107	0.0231	0.0131	0.0060	0.0005	-0.0041
RAIL	954000	45	7	1.165	0.0353	0.0263	0.0199	0.0149	0.0108	0.0232	0.0132	0.0061	0.0006	-0.0039
CATBIRD	954000	36	1	1.140	0.0355	0.0264	0.0200	0.0151	0.0110	0.0233	0.0133	0.0062	0.0007	-0.0038
CANARY	900000	54	7	1.162	0.0353	0.0263	0.0199	0.0149	0.0109	0.0232	0.0132	0.0061	0.0006	-0.0039
RUDDY	900000	45	7	1.131	0.0355	0.0265	0.0201	0.0151	0.0111	0.0234	0.0134	0.0062	0.0007	-0.0038
MALLARD	795000	30	19	1.140	0.0355	0.0264	0.0200	0.0151	0.0110	0.0233	0.0133	0.0062	0.0007	-0.0038
DRAKE	795000	26	7	1.108	0.0357	0.0266	0.0202	0.0153	0.0112	0.0235	0.0135	0.0063	0.0008	-0.0037
CONDOR	795000	54	7	1.093	0.0358	0.0267	0.0203	0.0154	0.0113	0.0236	0.0135	0.0064	0.0009	-0.0036
CUCKOO	795000	24	7	1.092	0.0358	0.0267	0.0203	0.0154	0.0113	0.0236	0.0135	0.0064	0.0009	-0.0036
TERN	795000	45	7	1.063	0.0360	0.0269	0.0205	0.0156	0.0115	0.0237	0.0137	0.0066	0.0010	-0.0035
COOT	795000	36	1	1.040	0.0361	0.0271	0.0207	0.0157	0.0117	0.0238	0.0138	0.0067	0.0011	-0.0034
REDWING	715500	30	19	1.081	0.0358	0.0268	0.0204	0.0155	0.0114	0.0236	0.0136	0.0066	0.0010	-0.0036
STARLING	715500	26	7	1.051	0.0361	0.0270	0.0206	0.0157	0.0116	0.0237	0.0137	0.0066	0.0011	-0.0034
STILT	715500	24	7	1.036	0.0362	0.0271	0.0207	0.0158	0.0117	0.0238	0.0138	0.0067	0.0012	-0.0033
GANNET	666600	26	7	1.014	0.0363	0.0273	0.0209	0.0159	0.0119	0.0239	0.0139	0.0068	0.0013	-0.0032
FLAMINGO	666600	24	7	1.000	0.0364	0.0274	0.0210	0.0160	0.0120	0.0240	0.0140	0.0069	0.0013	-0.0032
-------	653900	18	3	0.953	0.0368	0.0278	0.0214	0.0164	0.0123	0.0242	0.0142	0.0071	0.0016	-0.0029

EGRET	636000	30	19	1.019	0.0363	0.0273	0.0209	0.0159	0.0118	0.0239	0.0139	0.0068	0.0012	-0.0033
GROSBEAK	636000	26	7	0.990	0.0365	0.0275	0.0211	0.0161	0.0121	0.0240	0.0140	0.0069	0.0014	-0.0031
ROOK	636000	24	7	0.977	0.0366	0.0276	0.0212	0.0162	0.0122	0.0241	0.0141	0.0070	0.0015	-0.0031
KINGBIRD	636000	18	1	0.940	0.0369	0.0279	0.0215	0.0165	0.0124	0.0243	0.0143	0.0072	0.0016	-0.0029
SWIFT	636000	36	1	0.930	0.0370	0.0279	0.0215	0.0166	0.0125	0.0244	0.0143	0.0072	0.0017	-0.0028
TEAL	605000	30	19	0.994	0.0365	0.0274	0.0210	0.0161	0.0120	0.0240	0.0140	0.0069	0.0014	-0.0031
SQUAB	605000	26	7	0.966	0.0367	0.0277	0.0213	0.0163	0.0122	0.0242	0.0141	0.0070	0.0015	-0.0030
PEACOCK	605000	24	7	0.953	0.0368	0.0278	0.0214	0.0164	0.0123	0.0242	0.0142	0.0071	0.0016	-0.0029
EAGLE	556500	30	7	0.953	0.0368	0.0278	0.0214	0.0164	0.0123	0.0242	0.0142	0.0071	0.0016	-0.0029
DOVE	556500	26	7	0.927	0.0370	0.0280	0.0216	0.0166	0.0125	0.0244	0.0143	0.0072	0.0017	-0.0028
PARAKEET	556500	24	7	0.914	0.0371	0.0281	0.0217	0.0167	0.0126	0.0244	0.0144	0.0073	0.0018	-0.0027
OSPREY	556500	18	1	0.879	0.0374	0.0284	0.0220	0.0170	0.0129	0.0246	0.0146	0.0075	0.0020	-0.0025
HEN	477000	30	7	0.883	0.0373	0.0283	0.0219	0.0170	0.0129	0.0246	0.0146	0.0075	0.0020	-0.0026
HAWK	477000	26	7	0.858	0.0376	0.0285	0.0221	0.0172	0.0131	0.0247	0.0147	0.0076	0.0021	-0.0024
FLICKER	477000	24	7	0.846	0.0377	0.0286	0.0222	0.0173	0.0132	0.0248	0.0148	0.0077	0.0022	-0.0023
PELICAN	477000	18	1	0.814	0.0380	0.0289	0.0225	0.0176	0.0135	0.0250	0.0150	0.0079	0.0024	-0.0022
LARK	397500	30	7	0.806	0.0380	0.0290	0.0226	0.0176	0.0136	0.0251	0.0150	0.0079	0.0024	-0.0021
IBIS	397500	26	7	0.783	0.0382	0.0292	0.0228	0.0179	0.0138	0.0252	0.0152	0.0081	0.0025	-0.0020
BRANT	397500	24	7	0.772	0.0383	0.0293	0.0229	0.0180	0.0139	0.0253	0.0152	0.0081	0.0026	-0.0019
CHICKADEE	397500	18	1	0.743	0.0386	0.0296	0.0232	0.0182	0.0142	0.0255	0.0154	0.0083	0.0028	-0.0017
ORIOLE	336400	30	7	0.741	0.0386	0.0296	0.0232	0.0183	0.0142	0.0255	0.0154	0.0083	0.0028	-0.0017
LINNET	336400	26	7	0.721	0.0389	0.0298	0.0234	0.0185	0.0144	0.0256	0.0156	0.0085	0.0030	-0.0016
MERLIN	336400	18	1	0.684	0.0392	0.0302	0.0238	0.0189	0.0148	0.0259	0.0158	0.0087	8.0032	-0.0013
OSTRICH	300000	26	7	0.680	0.0393	0.0303	0.0239	0.0189	0.0148	0.0259	0.0159	0.0088	0.0032	-0.0013

(1) X_A' IS THE COMPONENT OF CAPACITIVE REACTANCE DUE TO THE ELECTROSTATIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF CAPACITIVE REACTANCE. X_D' ACCOUNTS FOR THE FLUX BETWEEN THE 1 FOOT RADIUS AND THE OTHER PHASES. THE TOTAL CAPACITIVE REACTANCE PER PHASE IS THE SUM OF X_A' AND X_D'. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A'. X_D' IS OBTAINED FROM THE FORMULA BELOW.

$$X_A' = 0.0683 \text{ LOG}_{10} \left[\frac{1}{\left[N\,(r)\,(A)^{N-1}\right]^{\frac{1}{N}}} \right] \text{ MEGOHM MILES}$$

$$X_D' = 0.0683 \text{ LOG}_{10}(\text{GMD}) \text{ MEGOHM-MILES}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

WHERE: r = CONDUCTOR RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \text{ SIN } (\pi/N))$: $N>1$
$A = 0$: $0^0 \equiv 1$: $N=1$
S = BUNDLE SPACING IN FEET

TABLE A.27 Capacitive Reactance of ACAR Bundled Conductors at 60 Hz [3]

AREA 62% EQ. EC - AL CMIL	STRANDS EC/6201		DIA. IN.	60 Hz CAPACITIVE REACTANCE (1) X_A' IN MEGOHM-MILES FOR 1 FOOT RADIUS										
				SINGLE COND.	2 - CONDUCTOR SPACING (IN.)					3 - CONDUCTOR SPACING (IN.)				
					6	9	12	15	18	6	9	12	15	18
2413000	72	19	1.821	0.0765	0.0485	0.0425	0.0383	0.0349	0.0322	0.0392	0.0312	0.0255	0.0211	0.0175
2375000	63	28	1.821											
2338000	54	37	1.821											
2297000	54	7	1.762	0.0775	0.0490	0.0430	0.0387	0.0354	0.0327	0.0395	0.0315	0.0258	0.0214	0.0178
2262000	48	13	1.762											
2226000	42	19	1.762											
2227000	54	7	1.735	0.0779	0.0493	0.0432	0.0390	0.0357	0.0330	0.0397	0.0317	0.0260	0.0216	0.0180
2193000	48	13	1.735											
2159000	42	19	1.735											
1899000	54	7	1.602	0.0803	0.0504	0.0444	0.0402	0.0368	0.0341	0.0405	0.0325	0.0268	0.0224	0.0187
1870000	48	13	1.602											
1841000	42	19	1.602											
1673000	54	7	1.504	0.0822	0.0514	0.0454	0.0411	0.0378	0.0351	0.0411	0.0331	0.0274	0.0230	0.0194
1647000	48	13	1.504											
1622000	42	19	1.504											
1337000	54	7	1.345	0.0855	0.0530	0.0470	0.0427	0.0394	0.0367	0.0422	0.0342	0.0285	0.0241	0.0205
1296000	42	19	1.345											
1243000	30	7	1.302	0.0865	0.0535	0.0475	0.0432	0.0399	0.0372	0.0425	0.0345	0.0288	0.0244	0.0208
1211000	24	13	1.302											
1179000	18	19	1.302											
1163000	30	7	1.259	0.0875	0.0540	0.0480	0.0437	0.0404	0.0377	0.0429	0.0348	0.0292	0.0247	0.0211
1133000	24	13	1.259											
1104000	18	19	1.259											
1153000	33	4	1.246	0.0878	0.0542	0.0481	0.0439	0.0406	0.0379	0.0430	0.0349	0.0293	0.0248	0.0212
1138000	30	7	1.246											
1109000	24	13	1.246											
1080000	18	19	1.246											
1077000	30	7	1.212	0.0886	0.0546	0.0486	0.0443	0.0410	0.0383	0.0432	0.0352	0.0295	0.0251	0.0215
1049000	24	13	1.212											
1022000	18	19	1.212											
1050000	30	7	1.196	0.0890	0.0548	0.0488	0.0445	0.0412	0.0385	0.0434	0.0353	0.0297	0.0252	0.0216
1023000	24	13	1.196											
996000	18	19	1.196											

994800	30	7	1.165	0.0898	0.0552	0.0491	0.0449	0.0416	0.0389	0.0436	0.0356	0.0299	0.0255	0.0219
954600	30	7	1.141	0.0904	0.0555	0.0495	0.0452	0.0419	0.0392	0.0438	0.0358	0.0301	0.0257	0.0221
969300	24	13	1.165	0.0898	0.0552	0.0491	0.0449	0.0416	0.0389	0.0436	0.0356	0.0299	0.0255	0.0219
958000	24	13	1.158	0.0899	0.0552	0.0492	0.0450	0.0417	0.0390	0.0437	0.0357	0.0300	0.0256	0.0220
943900	18	19	1.165	0.0898	0.0552	0.0491	0.0449	0.0416	0.0389	0.0436	0.0356	0.0299	0.0255	0.0219
900300	30	7	1.108	0.0912	0.0559	0.0499	0.0456	0.0423	0.0396	0.0441	0.0361	0.0304	0.0260	0.0224
795000	30	7	1.042	0.0931	0.0568	0.0508	0.0465	0.0432	0.0405	0.0447	0.0367	0.0310	0.0266	0.0230
877300	24	13	1.108	0.0912	0.0559	0.0499	0.0456	0.0423	0.0396	0.0441	0.0361	0.0304	0.0260	0.0224
795000	24	13	1.055	0.0927	0.0566	0.0506	0.0463	0.0430	0.0403	0.0446	0.0366	0.0309	0.0265	0.0229
854200	18	19	1.108	0.0912	0.0559	0.0499	0.0456	0.0423	0.0396	0.0441	0.0361	0.0304	0.0260	0.0224
795000	18	19	1.069	0.0923	0.0564	0.0504	0.0462	0.0428	0.0401	0.0445	0.0365	0.0308	0.0264	0.0227
829000	30	7	1.063	0.0925	0.0565	0.0505	0.0462	0.0429	0.0402	0.0445	0.0365	0.0308	0.0264	0.0228
807700	24	13	1.063											
786500	18	19	1.063											
727500	33	4	0.990	0.0946	0.0576	0.0516	0.0473	0.0440	0.0413	0.0452	0.0372	0.0315	0.0271	0.0235
718300	30	7	0.990											
700000	24	13	0.990											
681600	18	19	0.990											
632000	15	4	0.927	0.0965	0.0586	0.0525	0.0483	0.0450	0.0423	0.0459	0.0379	0.0322	0.0278	0.0242
616200	12	7	0.927											
487400	15	4	0.814	0.1004	0.0605	0.0545	0.0502	0.0469	0.0442	0.0472	0.0392	0.0335	0.0291	0.0254
475200	12	7	0.814											
343600	15	4	0.684	0.1056	0.0631	0.0570	0.0528	0.0495	0.0468	0.0489	0.0409	0.0352	0.0308	0.0272
335000	12	7	0.684											

(1) X_A' IS THE COMPONENT OF CAPACITIVE REACTANCE DUE TO THE ELECTROSTATIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF CAPACITIVE REACTANCE, X_D' ACCOUNTS FOR THE FLUX BETWEEN THE 1 FOOT RADIUS AND THE OTHER PHASES. THE TOTAL CAPACITIVE REACTANCE PER PHASE IS THE SUM OF X_A' AND X_D'. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A'. X_D' IS OBTAINED FROM THE FORMULA BELOW

$$X_A' = 0.0683 \ \mathrm{LOG}_{10}\left[\frac{1}{\left[N\ (r)\ (A)^{N-1}\right]^{\frac{1}{N}}}\right] \text{ MEGOHM-MILES}$$

WHERE: r = CONDUCTOR RADIUS IN FEET
N = NUMBER OF CONDUCTORS PER PHASE
$A = S/(2 \sin(\pi/N))$: $N>1$
$A = 0$: $0^0 \equiv 1$: $N=1$
S = BUNDLE SPACING IN FEET

$$X_D' = 0.0683 \ \mathrm{LOG}_{10}(\mathrm{GMD}) \text{ MEGOHM-MILES}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

TABLE A.28 Capacitive Reactance of ACAR Bundled Conductors at 60 Hz [3]

AREA 62% EQ. EC - AL CMIL	STRANDS EC	STRANDS 6201	DIA. IN.	60 Hz CAPACITIVE REACTANCE (1) X_A' IN MEGOHM-MILES FOR 1 FOOT RADIUS — 4 - CONDUCTOR SPACING (IN.) 6	9	12	15	18	6 - CONDUCTOR SPACING (IN.) 6	9	12	15	18
2413000	72	19	1.821	0.0320	0.0230	0.0166	0.0116	0.0075	0.0210	0.0110	0.0039	-0.0016	-0.0061
2375000	63	28	1.821										
2338000	54	37	1.821										
2297000	54	7	1.762	0.0322	0.0232	0.0168	0.0118	0.0078	0.0212	0.0112	0.0041	-0.0015	-0.0060
2262000	48	13	1.762										
2226000	42	19	1.762										
2227000	54	7	1.735	0.0323	0.0233	0.0169	0.0119	0.0079	0.0213	0.0112	0.0041	-0.0014	-0.0059
2193000	48	13	1.735										
2159000	42	19	1.735										
1899000	54	7	1.602	0.0329	0.0239	0.0175	0.0125	0.0085	0.0217	0.0116	0.0045	-0.0010	-0.0055
1870000	48	13	1.602										
1841000	42	19	1.602										
1673000	54	7	1.504	0.0334	0.0244	0.0180	0.0130	0.0090	0.0220	0.0119	0.0048	-0.0007	-0.0052
1647000	48	13	1.504										
1622000	42	19	1.504										
1337000	54	7	1.345	0.0342	0.0252	0.0188	0.0138	0.0098	0.0225	0.0125	0.0054	-0.0001	-0.0046
1296000	42	19	1.345										
1243000	30	7	1.302	0.0345	0.0254	0.0190	0.0141	0.0100	0.0227	0.0127	0.0055	0.0000	-0.0045
1211000	24	13	1.302										
1179000	18	19	1.302										
1163000	30	7	1.259	0.0347	0.0257	0.0193	0.0143	0.0103	0.0229	0.0128	0.0057	0.0002	-0.0043
1133000	24	13	1.259										
1104000	18	19	1.259										
1153000	33	4	1.246	0.0348	0.0258	0.0194	0.0144	0.0103	0.0229	0.0129	0.0058	0.0003	-0.0043
1138000	30	7	1.246										
1109000	24	13	1.246										
1080000	18	19	1.246										
1077000	30	7	1.212	0.0350	0.0260	0.0196	0.0146	0.0106	0.0230	0.0130	0.0059	0.0004	-0.0041
1049000	24	13	1.212										
1022000	18	19	1.212										
1050000	30	7	1.196	0.0351	0.0261	0.0197	0.0147	0.0107	0.0231	0.0131	0.0060	0.0005	-0.0041
1023000	24	13	1.196										
996000	18	19	1.196										

994800	30	7	1.165	0.0353	0.0263	0.0199	0.0149	0.0108	0.0232	0.0132	0.0061	0.0006	-0.0039
954600	30	7	1.141	0.0354	0.0264	0.0200	0.0151	0.0110	0.0233	0.0133	0.0062	0.0007	-0.0038
969300	24	13	1.165	0.0353	0.0263	0.0199	0.0149	0.0108	0.0232	0.0132	0.0061	0.0006	-0.0039
958000	24	13	1.158	0.0353	0.0263	0.0199	0.0149	0.0109	0.0233	0.0132	0.0061	0.0006	-0.0039
943900	18	19	1.165	0.0353	0.0263	0.0199	0.0149	0.0108	0.0232	0.0132	0.0061	0.0006	-0.0039
900300	30	7	1.108	0.0357	0.0266	0.0202	0.0153	0.0112	0.0235	0.0135	0.0063	0.0008	-0.0037
795000	30	7	1.042	0.0361	0.0271	0.0207	0.0157	0.0117	0.0238	0.0138	0.0067	0.0011	-0.0034
877300	24	13	1.108	0.0357	0.0266	0.0202	0.0153	0.0112	0.0235	0.0135	0.0063	0.0008	-0.0037
795000	24	13	1.055	0.0360	0.0270	0.0206	0.0156	0.0116	0.0237	0.0137	0.0066	0.0011	-0.0034
854200	18	19	1.108	0.0357	0.0266	0.0202	0.0153	0.0112	0.0235	0.0135	0.0063	0.0008	-0.0037
795000	18	19	1.069	0.0359	0.0269	0.0205	0.0155	0.0115	0.0237	0.0136	0.0065	0.0010	-0.0035
829000	30	7	1.063	0.0360	0.0269	0.0205	0.0156	0.0115	0.0237	0.0137	0.0066	0.0010	-0.0035
807700	24	13	1.063										
786500	18	19	1.063										
727500	33	4	0.990	0.0365	0.0275	0.0211	0.0161	0.0121	0.0240	0.0140	0.0069	0.0014	-0.0031
718300	30	7	0.990										
700000	24	13	0.990										
681600	18	19	0.990										
632000	15	4	0.927	0.0370	0.0280	0.0216	0.0166	0.0125	0.0244	0.0143	0.0072	0.0017	-0.0028
616200	12	7	0.927										
487400	15	4	0.814	0.0380	0.0289	0.0225	0.0176	0.0135	0.0250	0.0150	0.0079	0.0024	-0.0022
475200	12	7	0.814										
343600	15	4	0.684	0.0392	0.0302	0.0238	0.0189	0.0148	0.0259	0.0158	0.0087	0.0032	-0.0013
335000	12	7	0.684										

(1) X_A' IS THE COMPONENT OF CAPACITIVE REACTANCE DUE TO THE ELECTROSTATIC FLUX WITHIN A 1 FOOT RADIUS. THE REMAINING COMPONENT OF CAPACITIVE REACTANCE, X_D' ACCOUNTS FOR THE FLUX BETWEEN THE 1 FOOT RADIUS AND THE OTHER PHASES. THE TOTAL CAPACITIVE REACTANCE PER PHASE IS THE SUM OF X_A' AND X_D'. THE FOLLOWING FORMULA CAN BE USED TO CALCULATE ADDITIONAL VALUES OF X_A'. X_D' IS OBTAINED FROM THE FORMULA BELOW.

$$= 0.0683 \ \mathrm{LOG}_{10} \left[\frac{1}{\left[N\ (r)\ (A)^{N-1}\right]^{\frac{1}{N}}} \right] \text{ MEGOHM MILES}$$

$$X_D' = 0.0683 \ \mathrm{LOG}_{10}(\mathrm{GMD}) \text{ MEGOHM-MILES}$$

WHERE: GMD = GEOMETRIC MEAN DISTANCE BETWEEN PHASES IN FEET

HERE:
- r = CONDUCTOR RADIUS IN FEET
- N = NUMBER OF CONDUCTORS PER PHASE
- $A = S/(2 \sin(\pi/N))$: $N>1$
- $A = 0$: $0^0 \equiv 1$: $N=1$
- S = BUNDLE SPACING IN FEET

TABLE A.29 Resistance of ACSR Conductors (Ω/mi)

EXPANDED	3108000	62/8	19	2.500	0.0294	0.0333	0.0362	0.0389	0.0418
EXPANDED	2294000	66/6	19	2.320	0.0399	0.0412	0.0453	0.0493	0.0533
EXPANDED	1414000	58/4	19	1.750	0.0644	0.0663	0.0728	0.0793	0.0859
EXPANDED	1275000	50/4	19	1.600	0.0716	0.0736	0.0808	0.0881	0.0953
KIWI	2167000	72	7	1.737	0.0421	0.0473	0.0515	0.0552	0.0593
BLUEBIRD	2156000	84	19	1.762	0.0420	0.0464	0.0507	0.0545	0.0586
CHUKAR	1780000	84	19	1.602	0.0510	0.0548	0.0599	0.0647	0.0696
FALCON	1590000	54	19	1.545	0.0567	0.0594	0.0653	0.0707	0.0763
LAPWING	1590000	45	7	1.502	0.0571	0.0608	0.0664	0.0719	0.0774
PARROT	1510500	54	19	1.506	0.0597	0.0625	0.0686	0.0744	0.0802
NUTHATCH	1510500	45	7	1.466	0.0602	0.0636	0.0697	0.0755	0.0813
PLOVER	1431000	54	19	1.465	0.0630	0.0657	0.0721	0.0782	0.0843
BOBOLINK	1431000	45	7	1.427	0.0636	0.0668	0.0733	0.0794	0.0856
MARTIN	1351500	54	19	1.424	0.0667	0.0692	0.0760	0.0825	0.0890
DIPPER	1351500	45	7	1.385	0.0672	0.0705	0.0771	0.0836	0.0901
PHEASANT	1272000	54	19	1.382	0.0709	0.0732	0.0805	0.0874	0.0944
BITTERN	1272000	45	7	1.345	0.0715	0.0746	0.0817	0.0886	0.0956
GRACKLE	1192500	54	19	1.333	0.0756	0.0778	0.0855	0.0929	0.1000
BUNTING	1192500	45	7	1.302	0.0762	0.0792	0.0867	0.0942	0.1002
FINCH	1113000	54	19	1.293	0.0810	0.0832	0.0914	0.0993	0.1080
BLUEJAY	1113000	45	7	1.259	0.0818	0.0844	0.0926	0.1010	0.1090
CURLEW	1033500	54	7	1.246	0.0871	0.0893	0.0979	0.1070	0.1150
ORTOLAN	1033500	45	7	1.213	0.0881	0.0905	0.0994	0.1080	0.1170
TANAGER	1033500	36	1	1.186	0.0885	0.0915	0.1010	0.1090	0.1180
CARDINAL	954000	54	7	1.196	0.0944	0.0963	0.1060	0.1150	0.1250
RAIL	954000	45	7	1.165	0.0954	0.0978	0.1080	0.1170	0.1260
CATBIRD	954000	36	1	1.140	0.0959	0.0987	0.1090	0.1180	0.1270
CANARY	900000	54	7	1.162	0.1000	0.1020	0.1120	0.1220	0.1320
RUDDY	900000	45	7	1.131	0.1010	0.1030	0.1130	0.1230	0.1340

MALLARD	795000	30	19	1.140	0.111	0.114	0.125	0.137	0.147
DRAKE	795000	26	7	1.108	0.112	0.114	0.125	0.137	0.147
CONDOR	795000	54	7	1.093	0.113	0.115	0.127	0.138	0.149
CUCKOO	795000	24	7	1.092	0.113	0.114	0.127	0.137	0.148
TERN	795000	45	7	1.063	0.114	0.116	0.128	0.139	0.150
COOT	795000	36	1	1.040	0.115	0.117	0.129	0.141	0.152
REDWING	715500	30	19	1.081	0.124	0.126	0.139	0.151	0.164
STARLING	715500	26	7	1.051	0.125	0.126	0.139	0.151	0.164
STILT	715500	24	7	1.036	0.126	0.127	0.141	0.153	0.165
GANNET	666600	26	7	1.014	0.134	0.135	0.149	0.162	0.176
FLAMINGO	666600	24	7	1.000	0.135	0.137	0.151	0.164	0.177
--------	653900	18	3	0.953	0.140	0.142	0.156	0.171	0.184
EGRET	636000	30	19	1.019	0.139	0.143	0.157	0.172	0.186
GROSBEAK	636000	26	7	0.990	0.140	0.142	0.156	0.170	0.184
ROOK	636000	24	7	0.977	0.142	0.143	0.157	0.172	0.186
KINGBIRD	636000	18	1	0.940	0.143	0.145	0.160	0.174	0.188
SWIFT	636000	36	1	0.930	0.144	0.146	0.161	0.175	0.189
TEAL	605000	30	19	0.994	0.146	0.150	0.165	0.180	0.195
SQUAB	605000	26	7	0.966	0.147	0.149	0.164	0.179	0.193
PEACOCK	605000	24	7	0.953	0.149	0.150	0.165	0.180	0.195
EAGLE	556500	30	7	0.953	0.158	0.163	0.179	0.196	0.212
DOVE	556500	26	7	0.927	0.160	0.162	0.178	0.194	0.211
PARAKEET	556500	24	7	0.914	0.162	0.163	0.179	0.196	0.212
OSPREY	556500	18	1	0.879	0.163	0.166	0.183	0.199	0.215
HEN	477000	30	7	0.883	0.185	0.190	0.209	0.228	0.247
HAWK	477000	26	7	0.858	0.187	0.188	0.207	0.226	0.245
FLICKER	477000	24	7	0.846	0.189	0.190	0.209	0.228	0.247
PELICAN	477000	18	1	0.814	0.191	0.193	0.212	0.232	0.250
LARK	397500	30	7	0.806	0.222	0.227	0.250	0.273	0.295
IBIS	397500	26	7	0.783	0.224	0.226	0.249	0.271	0.294
BRANT	397500	24	7	0.772	0.226	0.227	0.250	0.273	0.295
CHICKADEE	397500	18	1	0.743	0.229	0.231	0.254	0.277	0.300
ORIOLE	336400	30	7	0.741	0.262	0.268	0.295	0.322	0.349
LINNET	336400	26	7	0.721	0.265	0.267	0.294	0.321	0.347
MERLIN	336400	18	1	0.684	0.270	0.273	0.300	0.328	0.355
OSTRICH	300000	26	7	0.680	0.297	0.299	0.329	0.359	0.389

TABLE A.30 Resistance of ACAR Conductors (Ω/mi)

AREA 62% EQ. EC - AL CMIL	STRANDS EC/6201		DIA. IN.	DC 20°C	AC - 60 Hz 30°C	50°C	75°C	100°C
2413000	72	19	1.821	0.0373	0.0456	0.0483	0.0516	0.0565
2375000	63	28	1.821	0.0379	0.0462	0.0488	0.0521	0.0555
2338000	54	37	1.821	0.0385	0.0467	0.0493	0.0527	0.0561
2297000	54	7	1.762	0.0392	0.0474	0.0502	0.0538	0.0573
2262000	48	13	1.762	0.0399	0.0479	0.0507	0.0543	0.0580
2226000	42	19	1.762	0.0405	0.0485	0.0513	0.0549	0.0585
2227000	54	7	1.735	0.0405	0.0485	0.0514	0.0551	0.0589
2193000	48	13	1.735	0.0411	0.0491	0.0520	0.0557	0.0595
2159000	42	19	1.735	0.0417	0.0497	0.0526	0.0562	0.0601
1899000	54	7	1.602	0.0474	0.0550	0.0585	0.0629	0.0674
1870000	48	13	1.602	0.0482	0.0557	0.0592	0.0636	0.0682
1841000	42	19	1.602	0.0489	0.0564	0.0599	0.0644	0.0689
1673000	54	7	1.504	0.0539	0.0611	0.0651	0.0702	0.0754
1647000	48	13	1.504	0.0546	0.0619	0.0659	0.0711	0.0762
1622000	42	19	1.504	0.0555	0.0627	0.0667	0.0719	0.0771
1337000	54	7	1.345	0.0674	0.0742	0.0794	0.0860	0.0925
1296000	42	19	1.345	0.0695	0.0763	0.0815	0.0881	0.0947
1243000	30	7	1.302	0.0725	0.0793	0.0849	0.0919	0.0989
1211000	24	13	1.302	0.0744	0.0812	0.0868	0.0937	0.1008
1179000	18	19	1.302	0.0764	0.0831	0.0887	0.0957	0.1028
1163000	30	7	1.259	0.0775	0.0842	0.0902	0.0977	0.1052
1133000	24	13	1.259	0.0795	0.0862	0.0922	0.0997	0.1073
1104000	18	19	1.259	0.0816	0.0882	0.0942	0.1018	0.1095
1153000	33	4	1.246	0.0781	0.0850	0.0910	0.0987	0.1064
1138000	30	7	1.246	0.0791	0.0859	0.0920	0.0997	0.1074
1109000	24	13	1.246	0.0812	0.0880	0.0941	0.1017	0.1095
1080000	18	19	1.246	0.0834	0.0900	0.0962	0.1039	0.1117

1077000	30	7	1.212	0.0836	0.0904	0.0969	0.1050	0.1132
1049000	24	13	1.212	0.0859	0.0926	0.0991	0.1072	0.1154
1022000	18	19	1.212	0.0882	0.0948	0.1013	0.1096	0.1177
1050000	30	7	1.196	0.0859	0.0926	0.0993	0.1076	0.1160
1023000	24	13	1.196	0.0881	0.0948	0.1015	0.1099	0.1183
996000	18	19	1.196	0.0904	0.0971	0.1038	0.1123	0.1207
994800	30	7	1.165	0.0906	0.0974	0.1044	0.1133	0.1221
954600	30	7	1.141	0.0944	0.1012	0.1086	0.1178	0.1271
969300	24	13	1.165	0.0929	0.0997	0.1068	0.1156	0.1246
958000	24	13	1.158	0.0941	0.1008	0.1080	0.1170	0.1260
943900	18	19	1.165	0.0955	0.1021	0.1092	0.1182	0.1271
900300	30	7	1.108	0.1001	0.1070	0.1148	0.1246	0.1345
795000	30	7	1.042	0.1133	0.1204	0.1293	0.1404	0.1516
877300	24	13	1.108	0.1027	0.1096	0.1174	0.1272	0.1372
795000	24	13	1.055	0.1133	0.1204	0.1290	0.1400	0.1509
854200	18	19	1.108	0.1054	0.1123	0.1201	0.1300	0.1400
795000	18	19	1.069	0.1134	0.1203	0.1288	0.1394	0.1501
829000	30	7	1.063	0.1087	0.1157	0.1243	0.1350	0.1456
807700	24	13	1.063	0.1115	0.1185	0.1271	0.1378	0.1486
786500	18	19	1.063	0.1145	0.1216	0.1302	0.1410	0.1518
727500	33	4	0.990	0.1238	0.1312	0.1411	0.1534	0.1658
718300	30	7	0.990	0.1254	0.1327	0.1427	0.1550	0.1675
700000	24	13	0.990	0.1287	0.1360	0.1459	0.1584	0.1709
681600	18	19	0.990	0.1322	0.1394	0.1494	0.1619	0.1743
632000	15	4	0.927	0.1425	0.1503	0.1615	0.1756	0.1897
616200	12	7	0.927	0.1462	0.1539	0.1652	0.1794	0.1935
487400	15	4	0.814	0.1849	0.1938	0.2085	0.2268	0.2453
475200	12	7	0.814	0.1896	0.1986	0.2133	0.2317	0.2501
343600	15	4	0.684	0.2623	0.2739	0.2948	0.3209	0.3470
335000	12	7	0.684	0.2690	0.2806	0.3015	0.3278	0.3540

APPENDIX B

METHODS FOR ALLOCATING TRANSMISSION LINE FIXED CHARGES AMONG JOINT USERS

In general, interconnections can help to achieve the two fundamental objectives of power systems operations, the economy of power production and the continuity of service. Therefore, interchanges between adjacent utilities are scheduled to take advantage of load diversity or available low-cost generating capacity, allowing lower overall operating costs and possible deferment of capital investment for new plants. Thus, interconnections provide the ability to use larger plants and relative flexibility in locating them and the ability to share spinning reserve capacity during emergencies for continuities of service. While sharing in the benefits of interconnected operation, each participating utility is expected to share its responsibilities.

B.1 METHODS FOR ALLOCATING DEMAND COSTS

In general, the fixed charges of a transmission line includes the return of investment, taxes, depreciation, insurance costs, and operating and maintenance costs. The methods† used in the past to allocate fixed charges (i.e., demand costs) are:

1. Energy method.
2. Peak responsibility method.

† The methods presented are useful in rate design work as well as the design of interconnection agreements.

3. Maximum demand method.
4. Green method.
5. Eisenmenger method.
6. Phantom method.
7. Weighted peak method.

B.1.1 Energy Method

The energy method is the simplest and one of the most commonly used methods. It allocates the demand costs in proportion to the energy used by each class of consumer during a period of months or years [1–2]. It is a simple method due to the fact that the values of energy consumed by various consumer classes during the past periods are readily available from the records. However, it is not fair to all users involved since it does not take into account the cost of providing service that is largely dependent on short-time power demands rather than energy. If all customers had 100-percent load factor, the method would be perfectly fair to all consumers. Therefore, the method is usually not an appropriate one because demand costs are not basically proportional to the energy used but rather to the maximum demand of the class of consumers. With this method, the class with the large energy consumption would be overburdened.

B.1.2 Peak Responsibility Method

The peak responsibility method [3] allocates the demand costs in proportion to the demand made by each class of consumer on the system at the time of system maximum demand. It attempts to place the burden on those classes of consumers responsible for the large amount of investment required to serve the peak-load period. If a company serves classes of consumers whose peaks are coincident in forming the annual peak on the company's system, the peak responsibility method is fair and just. In the early days, when the principal load was lighting, this condition existed.

It is obviously unfair to charge one class of customers who happen to use energy at the time of the annual system peak with all of the demand costs and let the other customers use the equipment for nothing. This is not only unfair but it is impracticable. For example, a peak due to one class of customers may coincide with the system annual peak, and in the following year, the system peak may be caused by different classes.

B.1.3 Maximum-Demand Method

The criticism of the peak responsibility method suggested that the demand costs may be more equitably allocated by the ratio of the maximum demand of the class under consideration to the summation of the maximum demands of all classes. However, the maximum-demand method [3] gives correct results only in certain isolated cases. If the customer's peaks coincide, it agrees with the peak responsibility. In cases where the customer maximum demands are not coincident, there is no overlapping of curves, and the load factors (averge load/maximum load) of all customers are the same, this method is applicable, and the results are just and fair.

However, there are two important aspects that are neglected in the method. First, it ignores the important item of time that the peaks occur. Second, it entirely neglects the energy required by those classes. Therefore, it encourages long hour use of the individual demand because all consumers who have a load factor higher than the average are charged too little, and all who have a load factor lower than the average are charged too much.

B.1.4 Greene's Method

The Greene method [4] uses a combination of the maximum-demand and the energy methods. Part of the demand costs are a direct function of the maximum demands, and the remainder is a direct function of energy. The proper values can be obtained by solving the following equations:

$$Kx + Dy = C \tag{B.1}$$

$$8760x + y = \frac{C}{P} \tag{B.2}$$

where x = cost per kilowatt-hour of that portion of demand costs that functions with kilowatt-hours supplied consumers

y = demand cost per kilowatt of portion of demand costs that function with maximum demand of customers

D = sum of consumers' maximum demands

P = maximum coincident demand or peak responsibility of all consumers on sources of supply

K = kilowatt-hours used by all consumers in a year

C = total annual demand costs of all consumers

8760 = kilowatt-hours in a year for 1-kW load operated at 100 percent power factor and 100 percent load factor (number of hours in year)

Without any doubt, this is a fairer method than any one of the aforementioned methods. It is a simple method. However, it neglects a very important parameter, the time at which the individual maximum demands occur, even though it does recognize the duration of such load.

B.1.5 Eisenmenger's Method

Eisenmenger [5] made a most elaborate study of central station (i.e., power plant) load curves and their relative contribution to the demand costs of the system. He advocated the following simplified method of allocation. Eisenmenger's method is usually a more appropriate one than the three previous methods. This is due to the fact that it takes into consideration not only the on-peak but also the off-peak load of the various consumer classes and their duration. If the proportionality factors of the classes of consumers sharing the annual demand costs is represented by F_{class} and the total demand costs are divided by their sum, the demand costs to be allocated to each class can be expressed as

$$\text{Demand costs allocated to class i} = \frac{F_{\text{class } i}}{\sum_{i=1}^{n} F_{\text{class } i}} \times (\text{total demand costs}) \tag{B.3}$$

From an elaborate graphical analysis of many load curves, the following empirical formula has been developed for determining F_{class} factors:

$$\begin{aligned} F_{\text{class}} &= \text{MD}_{\text{class}} \times \frac{\%\text{SP}_{\text{class}}}{100} \\ &+ \text{MD}_{\text{class}} \times \left(1.0 - \frac{\%\text{SP}_{\text{class}}}{100}\right) \times \frac{\text{peak hours}}{24} \\ &+ (\text{MD}_{\text{class}}\text{OP}) \times \frac{\text{OP}_{\text{hours}}}{24} \end{aligned} \tag{B.4}$$

This equation states that the proportionality factor of a class is equal to the sum of the following terms:

1. Maximum demand of class (MD_{class}) times percentage of station peak of class ($\%\text{SP}_{\text{class}}/100$).
2. Maximum demand of class times remainder percentage of station peak of class times ratio of hours per day to 24 h during which the class peak and station peak coincide.
3. Maximum demand off-peak (MD_{class} OP) of class times ratio of hours per day to 24 h during which the class peak and the station peak do not overlap.

For off-peak consumers this method gives correct results. However, it does not divide the demand costs correctly among those consumers who are on at the time of the station peak. In this method, every customer who is on at the time of station peak contributes to that peak. However, the favorable 100 percent load factor consumer has no peak in his individual demand curve. Therefore, the method burdens rather heavily this favorable class of consumer who has a steady load.

B.1.6 Phantom Method

If a public utility could operate steadily at its maximum demand for 24 h a day every day (i.e., at 100 percent load factor), its investment in equipment would be used most economically. The loss of any customer will affect the load factor, or efficiency of plant use, regardless of the fact that one might have twice the demand of the other. This was the conclusion of Hills [6] when he inserted that a fair and just division of cost will be on a kilowatt-hour basis, for every block of energy used is just as important and every other block of the same size as far as costs to the central station is concerned. Therefore, with a plant operating at 100 percent load factor, the demand costs divided by the number of kilowatt-hours generated and multiplied by the consumption of each customer at the generating plant will give the true demand costs that should be allocated to each customer. Therefore, under these conditions, the demand costs per kilowatt-hour can be expressed as

$$\text{Demand costs} = \frac{\text{total annual demand costs}}{8760\ (\text{max demand station})} \tag{B.5}$$

In actual practice, the load factor is usually not 100 percent. Here, the demand costs are divided among the groups of customers according to their kilowatt-hour consumption, charging this phantom customer in the same way as the real customers. Therefre, now the problem is to divide the bill of this phantom customer, which would be required to operate the existing plant at 100 percent load factor, among the existing customers in an equitable manner. Certainly, the customers who already have a 100 percent load factors are not responsible for the bill and neither are those customers who are off-peak, for they are doing their share toward reducing the size of this phantom. Those customers that cuse the peak are responsible since they use more than their average demand during the period of that peak load. Furthermore, their degree of responsibility is limited to the excess demand during the period of the station peak load over the average demand.

In many cases, it may be that there is not only one station peak during the year, due to one set of conditions, but perhaps two or more peaks at other times due to different groups of customers or under different conditions. It often happens that the annual station peak is just as likely to occur because of one group of customers as because of another. This is a case where the phantom method can be applied with accuracy and ease.

B.1.7 Weighted Peak Method

In 1927, Reed [7], in an effort to correct some of the defects of Greene's [4], Eisenmenger's [5], and Hills' [6] methods in overcharging the off-peak customers, presented a new method called the *weighted peak method*. This method allocates the demand costs to the various classes of consumer according to the share of each class in the total weighted peak. The weighted peak of any class of consumer is taken as equal to the demand of that class at the time of the plant peak plus a fraction of the difference between the maximum demand of that class of consumers and its demand at the time of the plant peak. This fraction that is added is the ratio of the plant demand at the time of the class maximum demand to the total peak demand.

B.2 METHODS FOR PLANNING FUTURE INVESTMENTS

In 1950, Watchorn [8] developed a method for determining capacity benefits resulting from an interconnection of generating systems and used this to justify the installation of transmission facilities as a substitute for generating capacity. Also, he described several possible bases for allocating such benefits. He pointed out that when only two systems are involved in an interconnection, the resulting capacity benefit should be divided equally between them. However, in the event that more than two systems are involved, the benefit allocated to any of the participating systems should not be reduced by the addition of any new participants into the interconnection. He suggested what may be defined as the mutual benefits method of allocation, which recognizes that the benefit should be divided among the participating systems in proportion to the benefits for all combinations of two's among them. This method meets the two basic requirements so long as the installed capacity requirements are determined on the basis of consistent application of probability methods.

In 1957, Phillips [9] developed a method that he asserted to be more equitable to allocate saving from energy interchange in power pools where more than three companies are involved. He pointed out that it is a generally accepted principle throughout the United States that on interchange where only two parties are involved, the savings are divided equally between buyer and seller. The accounting involved in applying this theory is given by a simple equation for the billing rate, which is a function of energy interchange, replacement cost of purchasing company, and supplying cost of selling company. When the magnitude of the interconnection grows to include three companies, the accounting is slightly more complicated, since for any specified period, either one company is buying and two are selling or two companies are buying and one is selling. Again, the total interchange can be broken down into seprate two-party transactions, and no arbitrary method is involved for determining the distribution of energy. A similar equation for billing rate can be applied in the three-company interconnection. When the magnitude of the power pool grows to four companies, it is no longer possible to say which company receives a given block of power except in those hours when only one company is buying or only one company is selling. He suggested that if more than one company is buying during a particular period, each buying company's replacement cost is compared with the weighted average of all the selling companies in order to determine the billing rate. Conversely, in any period, the selling cost of any selling company is compared with the weighted average of the replacement costs of all the buying companies for that specific period in order to determine the billing rate for that company.

It is very difficult to determine an equitable method of allocating the fixed charges of the interconnection facilities for power interchange among the various participants. Suggestions have been made that such fixed charges be divided annually among participating parties of the interconnection arrangement on the basis of the actual dollar benefits derived by the individual members from power interchange transactions. Watchorn [8] recommended that such allocation may well be on approximately the same basis as the allocation of the capacity benefits. However, Bary (in his discussion in Phillips [9]) suggested that the disposition of fixed charges on interconnection facilities should be made at the time they enter into an interconnection agreement. Furthermore, he suggested that benefits should be allocated on an equitable basis with the amounts applicable to each participating system to remain fixed for a prolonged period and be subjected to modification only as a result of future changes in the scope or extent of the facilities involved in the interconnection or due to major changes in the components of fixed charges (i.e., return on investment, taxes, depreciation, insurance, and maintenance). He argued that the disposition of fixed charges should not be made automatically dependent on the actual day-to-day or year-to-year operational benefits of power interchanges.

Anthony [10] described the exchange of seasonal diversity capacity between Tennessee Valley Authority (TVA) and the South Central Electric Companies (SCEC). Basically, each SCEC company was to own, operate, and maintain those extra-high-voltage (EHV) facilities required in its "service area." Financing was to be handled on a group basis. The annual cost of ownership, operation, and maintenance of individual company facilities was to be prorated to each company by an arbitrary formula based on the portion of such facilities installed by that company compared with the total EHV facilities installed by all SCEC companies and the percentage of participation by that company in diversity capacity exchange of TVA power. Since the company in whose service area EHV facilities are installed is in a position to use the facilities for purposes other than the interchange of power with TVA, each company owning EHV facilities was to begin to absorb 5 percent of the annual charges of those EHV facilities in its service area. Each year thereafter, for a total of 19 yr, the amount to be absorbed was to be increased by 5 percent. Consequently, at the end of the 19-yr period, annual charges to be shared by the companies was projected to be 50 percent of the initial annual charges. Incremental losses occasioned by the receipt or delivery of power under the agreements were to be distributed in proportion to each company's participation in each power transfer.

Firestone et al. [11] extended the use of probability techniques for analyzing a system's generation reserve position and applied this method to the Central Area Power Coordination Group (CAPCO) system. A probabilistic capacity model is merged with a load model to develop the expected frequency distribution of daily capacity margins. The daily capacity margin is considered to be the difference between the load that exists during a daily peak period and the operable capacity at that time. Operable capacity for this purpose is the normal rating of installed generating capacity, adjusted for various limitations, plus purchases of firm power from other utilities less outages both planned and forced. Each of these capacity margins is, of course, associated with the probability of the corresponding capacity level.

The CAPCO group, like other power pools, required a mechanism for ensuring the equitable sharing of benefits and responsibilities arising from such an association. The fundamental basis of equity adopted by the CAPCO group was that each party should contribute to the group reserve in the same proportion as he expected to utilize it. Negative margins were quite useful as the measure of a system's need for help from outside the pool, whereas the positive margins were used as the measure of a system's ability to provide help to outside systems. An energy quantity called *megawatt-days* was developed as a useful measurement here. *Positive megawatt-days* are equal to the sum of the products of each positive margin and its respective frequency. *Negative megawatt-days* are calculated in a similar manner, from the negative margin data. By proper distribution of capacity responsibility, it is possible to make the relationship of each party's contribution to the group reserve (positive-megawatt-day value) to his potential use of the group reserve (negative-megawatt-day value) equal to that for each of the other parties. The capacity responsibility assigned represents the power in megawatts for which the individual party bears financial responsibility.

In 1967, Rincliffe [12] described the Pennsylvania–New Jersey–Maryland (PJM) policy for allocating the annual costs of the 500-kV transmission system to all pool members. The 500-kV transmission system, owned by six companies, was being constructed to bring power from the mine-mouth stations to the load centers and to provide high-capacity interpool tie lines. The total cost of the transmission system was divided into an interarea tie function and a generation delivery function. The interarea function was allocated to all PJM members and associated systems in proportion to their sizes as measured by peak loads. The generation delivery function was allocated to the owners of the stations in proportion to ownership of the combined capacity of these stations.

The methods that have been described so far take into account the benefits of interconnection facilities from the savings to the participating systems due to power interchange transactions. However, Brandt [13] defined the benefits to be gained from transmission facilities in a more comprehenive way in an Edison Electric Institute's (EEI) committee report.

According to the report, the benefits that a company receives from an interconnection includes distribution benefits, wheeling benefits, and pool benefits (i.e., the value associated with increased reliability of the pool).

Therefore, the possible bases for ownership of transmission lines, based on the aforementioned methods, can be summarized as follows:

1. Ownership may be divided equally among the members.
2. Ownership may be proportional to the installed capacity requirements of each company when operating separately.
3. Ownership may be proportional to the peak load of each for separate operation.
4. Ownership may be proportional to a combination of energy consumed, average load, and the difference between the maximum load and the average load of each company (excess demand).
5. Ownership may be proportional to the distribution and the reliability benefits each member company derives from using the transmission system.

The first ownership base is equitable in a very uncommon situation, that is, when the companies involved have nearly the same size and similar fundamental characteristics. However, the second and third ownership bases are not equitable as far as a transmission line is concerned because the line is not planned for construction on these bases. On the other hand, if the problem was one of adding generating units, these bases may be more equitable. The fourth ownership base appears to be more equitable if one can find an appropriate way to formulate the concept. The phantom customer method of allocating fixed charges is suggested. It allocates the costs from both the energy and the power point of views. It may be an appropriate method for operating purpose on a day-to-day or month-to-month basis. However, the fifth ownership base appears to be more equitable and logical for planning purposes.

Therefore, in the event that there are n member companies in a given power pool, based on energy and excess demand considerations, the fixed charges of path (i.e., tie-line) j allocated to service area i can be expressed as [14]

$$F_{ij} = \frac{L_i}{\sum_{i=1}^{n} L_i} P_{aj} + \frac{\Delta P_{cj}}{\sum_{i=1}^{n} \Delta P_{cj}} (P_{pj} - P_{aj}) \tag{B.6}$$

where F_{ij} = fixed charge of path j allocated to service area i in megawatts
L_i = total load of area i at pool average load level in megawatts
P_{aj} = magnitude of real power flow in path j at pool average load level in megawatts

ΔP_{cj} = incremental real power flow in path j due to any load condition designated as C (positive value only) in megawatts
P_{pj} = magnitude of real power flow in path j at pool peak load level in megawatts

Equation (B.6) is based on the phantom method. Note that the first term of the equation represents the energy charge in megawatts, which is equivalent to the power flow in that particular line when the pool is operating at the average load level. The second term of the equation represents the fixed charges in megawatts due to excess demand and is called the *phantom demand charge* of the line. The demand charge in megawatts is equivalent to the difference between the power flows in the line at the pool peak load level and at the pool average load level. The difference should have only a positive value in order to be significant. A negative value means that there is no excess demand at the time of pool peak.

Of course, the reliance on the transmission lines (tie lines) in a modern pool is more crucial than just providing help in an emergency. It applies whenever generating capacity is insufficient for any reason (e.g., during the refueling of a nuclear unit), and physical back-up has now become as necessary to reliable operation as emergency assistance. This is the reason the reliability benefit should be included in the allocation. Therefore, the fifth ownership base appears to be more equitable than the other four bases for planning purposes. Thus, it can be used by the planning engineer to allocate fixed charges of jointly used transmission lines.

Therefore, in the event that there are n member companies in a given pool, based on distribution and reliability benefits, the fixed charges of path j allocated to service area i can be expressed as [14]

$$F_{ij} = \frac{1}{2}\left(\frac{R_{ij}}{\sum_{i=1}^{n} R_{ij}} + \frac{D_{ij}}{\sum_{i=1}^{n} D_{ij}}\right)F_j \tag{B.7}$$

where F_{ij} = fixed charges of path j allocated to area i in dollars
R_{ij} = reliability benefit to area i from path j
D_{ij} = distribution benefit to area i from path j in megawatts
F_j = fixed charges of path j in dollars

Note that a 50–50 split of the fixed charges is assumed in equation (B.7). The base value to determine the per-unit quantity of the distribution benefit is the sum of the distribution benefits of that particular line to each separate area in the pool. Similarly, the base value to determine the per-unit quantity of the reliability benefit is the sum of the reliability benefits for the entire system,

$$\text{Base reliability benefit} = \sum_{i=1}^{n} R_{ij} \tag{B.8}$$

Therefore, the fixed charge allocation of company i for the transmission line j can be expressed as [14]

$$F_{ij} = \tfrac{1}{2}(R_{ij} + D_{ij})F_j \tag{B.9}$$

where R_{ij} = reliability benefit in per units
D_{ij} = distribution benefit in per units

The distribution benefit of a transmission line to a power system is defined as the increment of real power flowing over that line when the total load of the power system is changed from one arbitrary load level to a higher arbitrary load level under economic production schedules from the dispatch control center under normal conditions, measured either in physical units (i.e., megawatts or kilowatts) or in per units on some appropriate base. The per-unit distribution benefit of a line can be expressed in several ways, depending on how the base value is chosen. For example, if the base value may be chosen to be some arbitrary number (e.g., 100 MW), the per-unit distribution benefit of a line (i.e., per unit ΔP_{line}) can be expressed as

$$D_{\text{line}} = \frac{\Delta P_{\text{line}} \ \text{MW}}{100\ \text{MW base}} \quad \text{pu} \tag{B.10}$$

In the event that the base value is chosen to be the total change in area load in megawatts, it can be calculated as

$$D_{\text{line}} = \frac{\Delta P_{\text{line}} \ \text{MW}}{\Delta P_{\text{area}} \ \text{MW}} \quad \text{pu} \tag{B.11}$$

Finally, if the base value is chosen to be the sum of the changes in every line flow (megawatts) in the area, the per-unit distribution benefit of a line can be expressed as

$$D_{\text{line}} = \frac{\Delta P_{\text{line}} \ \text{MW}}{\sum \Delta \text{P in all lines}} \quad \text{pu} \tag{B.12}$$

Note that the base value may be chosen for each individual line separately to be the total sum of the distribution benefits of that particular line to each separate area in the pool. The increments of real power flowing over the line can be found from load-flow studies.

The reliability benefit of a transmission line to a power system is defined as an increment of the total probability of system failure, calculated at the load buses in the power system, with the line in service and with the line out of service. The total probability of system failure in the system is the sum of the probabilities of system failure to serve the load at each load bus in the system. The probabilities are calculated at each of the individual load buses in the power system. The conditional probability approach can be used to calculate the probability of system failure.

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APPENDIX C

CONVERSION FACTORS, PREFIXES, AND THE GREEK ALPHABET

C.1 CONVERSION FACTORS

Table C.1 gives some factors for conversion into units of the International System (SI).

TABLE C.1 Factors for Conversion into Units of International System

1 in. = 2.54 cm
1 ft = 0.3048 m
1 mi = 1609.3 m
1 in.^{2} = 6.4516 cm^2
1 ft^2 = 0.092903 m^2
1 cmil = 5.0671 × 10^{-4} mm^2 (cmil is circular mil)
1 cmil = 0.7854 × 10^{-6} in.^{2}
1 in.^{3} = 16.387 cm^3
1 ft^3 = 0.028317 m^3
1 ft/m = 5.08 mm/s
1 mi/h = 0.44704 m/s
1 km/h = 0.27778 m/s
1 lb = 0.45359 kg
1 lb/ft^3 = 16.018 kg/m^3
1 lb/in^3. = 27680 kg/m^3
1 pdl = 0.13825 N (pdl is poundal)
1 lbf = 4.4482 N (lbf is pound force)
1 kgf = 9.80665 N
1 pdl/ft^2 = 1.4882 N/m^2
1 ft · lbf = 1.3558 J
1 ft · lbf/s = 1.3558 W
1 hp = 746 W

C.2 PREFIXES

The prefixes indicating decimal multiples or submultiples of units and their symbols are given in Table C.2.

TABLE C.2 Recommend Prefixes

Multiple	Prefix	Symbol
10^{12}	tera	T
10^{9}	giga	G
10^{6}	mega	M
10^{3}	kilo	k
10^{2}	hecto	h
10	deca	da
10^{-1}	deci	d
10^{-2}	centi	c
10^{-3}	mili	m
10^{-6}	micro	μ
10^{-9}	nano	n
10^{-12}	pico	p
10^{-15}	femto	f
10^{-18}	atto	a

C.3 GREEK ALPHABET USED FOR SYMBOLS

Table C.3 presents capital and lower case Greek alphabet symbols.

TABLE C.3 Greek Alphabet Symbols

Greek Letter	Greek Name	English Equivalent	Greek Letter	Greek Name	English Equivalent
$\mathrm{A}\alpha$	Alpha	a	$\mathrm{N}\nu$	Nu	n
$\mathrm{B}\beta$	Beta	b	$\Xi\xi$	Xi	x
$\Gamma\gamma$	Gamma	g	$\mathrm{O}o$	Omicron	ŏ
$\Delta\delta$	Delta	d	$\Pi\pi$	Pi	p
$\mathrm{E}\epsilon$	Epsilon	ĕ	$\mathrm{P}\rho$	Rho	r
$\mathrm{Z}\zeta$	Zeta	z	$\Sigma\sigma\varsigma$	Sigma	s
$\mathrm{H}\eta$	Eta	ē	$\mathrm{T}\tau$	Tau	t
$\Theta\theta\vartheta$	Theta	th	$\Upsilon\upsilon$	Upsilon	u
$\mathrm{I}\iota$	Iota	i	$\Phi\phi\varphi$	Phi	ph
$\mathrm{K}\kappa$	Kappa	k	$\mathrm{X}\chi$	Chi	ch
$\Omega\lambda$	Lambda	l	$\Psi\psi$	Psi	ps
$\mathrm{M}\mu$	Mu	m	$\Omega\omega$	Omega	ō

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