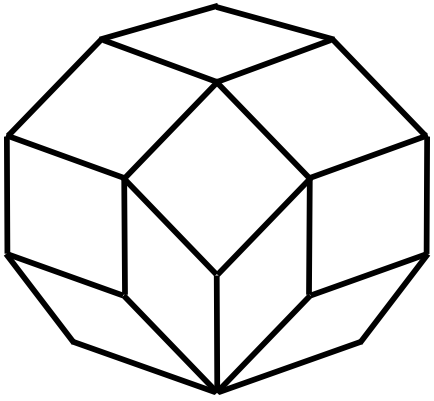
Drawing Hypercubes



A 5 dimensional hypercube

By Stacy David Thurston

Tiger Farm Press

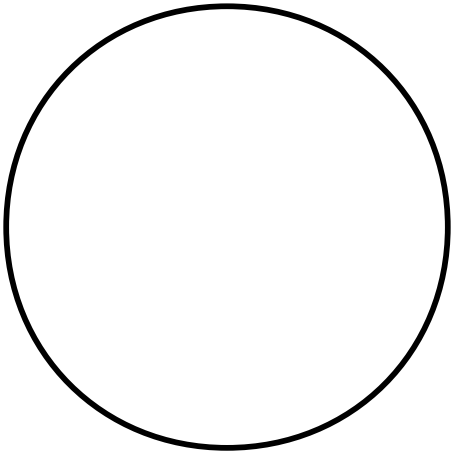
Promotional book created
for Greg Doench,
Executive Editor, Pearson
Technology Group,
Prentice Hall

Dedicated to my
sons
Branden and
Riley.

Cover by Tiger Farm Press, the author's personal publishing company.

TIGER FARM PRESS, CALIFORNIA

What's this?



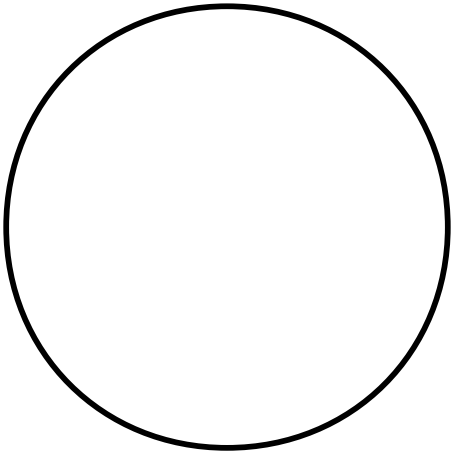
Tiger Farm Press

Did you say, *“A circle?”*

Really?

The title of this book is *Drawing Hypercubes*, not *Drawing Circles*.

It's a hypercube outline and I'll bloody well prove it.



Tiger Farm Press

The Big Bang Theory

To follow this book, rather than having studied the complexities of the Big Bang theory, it's better if you have watched *The Big Bang Theory*.

You know, the TV show, whose guest star, is physic's superhero, Stephen Hawking, author of *A Brief History of Time*.

In the prime time show, Sheldon Cooper studies String Theory, which has 10 dimensions of space and time.

And so, after you manage to draw a 4 dimensional hypercube, you'll move on to draw a 10 dimensional String Theory hypercube.

Tiger Farm Press

The Forth

A hundred years ago, Einstein added to our 3 dimensional world, the 4th dimension of time, to give us the physics of spacetime.

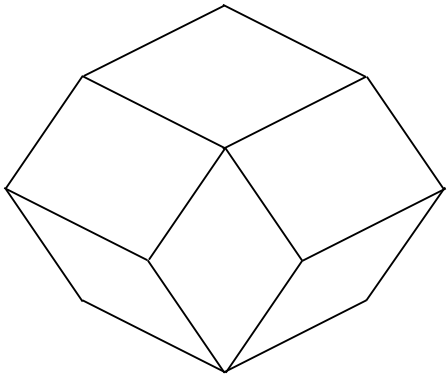
But, neither mathematicians nor physicists have shown us a simple way to draw a 4 dimensional hypercube.

So, I figured out a way by following Einstein's axiom, “Everything should be made as simple as possible, but no simpler.”

My steps are so easy, a grade schooler can follow them. I wonder who will be the youngest to draw a 4 dimensional Einstein hypercube.

Tiger Farm Press

Here is a 4D Hypercube:



It only has 6 front sides. How hard can that be to draw?

Hey, it's also called a tesseract. In the first Marvel *Avengers* movie, which has lots of superheros, the Tesseract stone was used to bridge the gap between worlds.

How cool is that?

Tiger Farm Press

Your Requirements

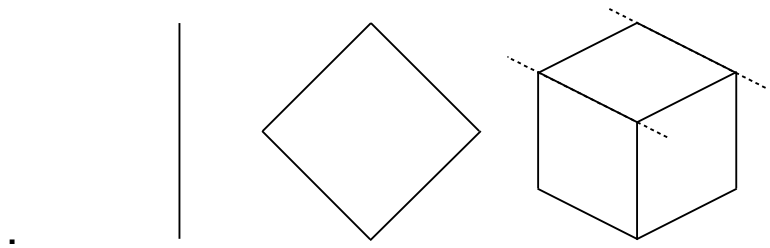
You must be familiar with these basic shapes:

+ a point to show position,

+ a line of 1 dimension,

+ a square of 2 dimensions, and

+ a cube of 3 dimensions:

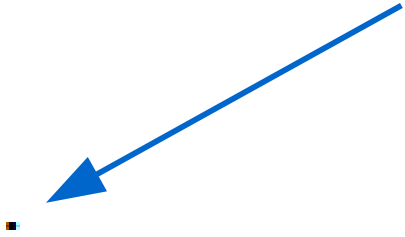


0D 1D 2D 3D

Tiger Farm Press

Drawing Shapes From 0 to 3 Dimensions

A 0 dimensional point: position

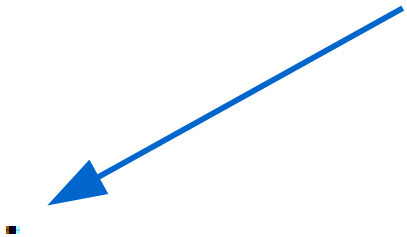


Tiger Farm Press

A 1 dimensional line segment: a measure of height

Tiger Farm Press

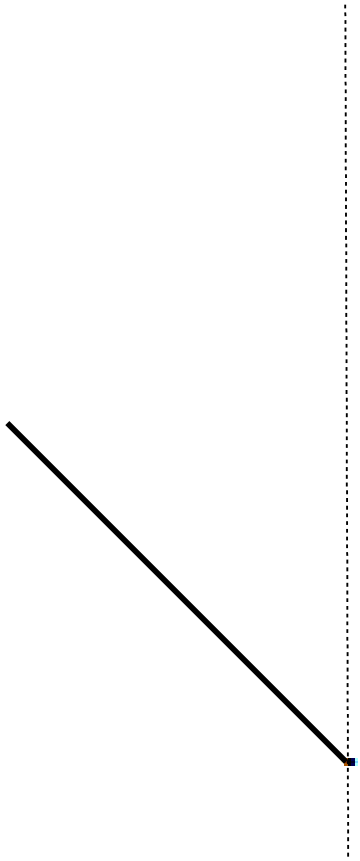
To draw a square, start with a point.



Tiger Farm Press

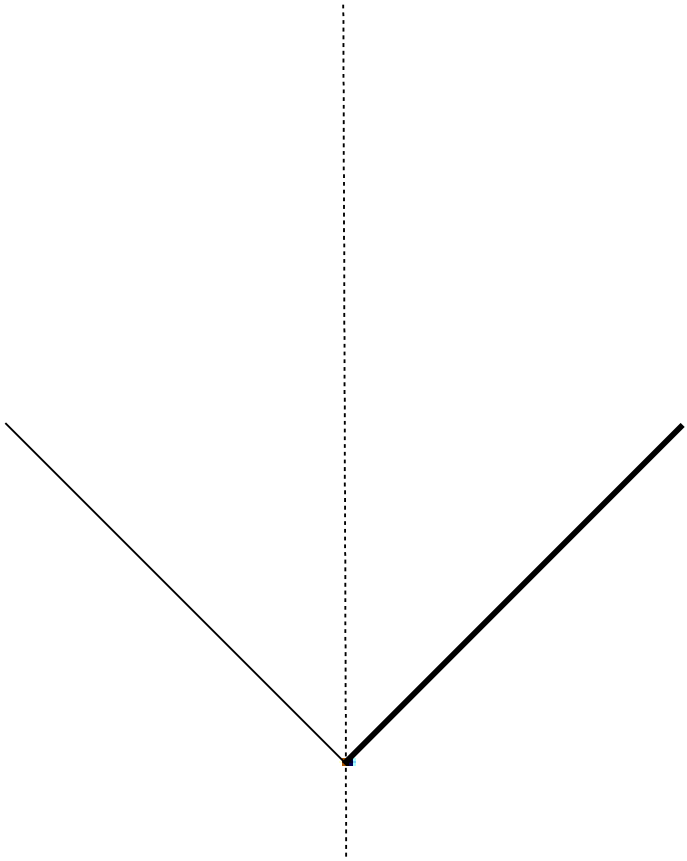
1st dimension: width

(dotted lines are for reference)



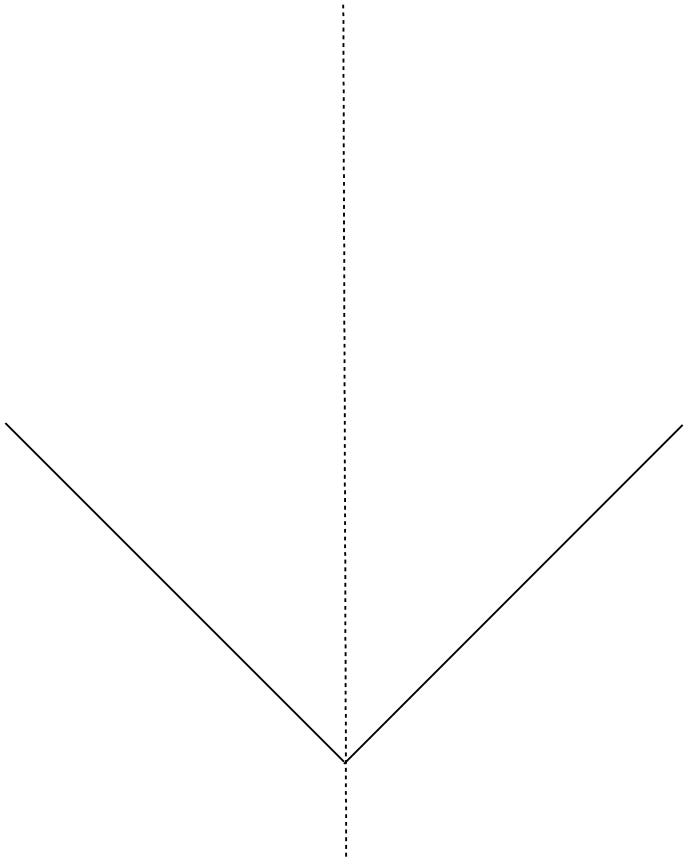
Tiger Farm Press

2nd dimension: length



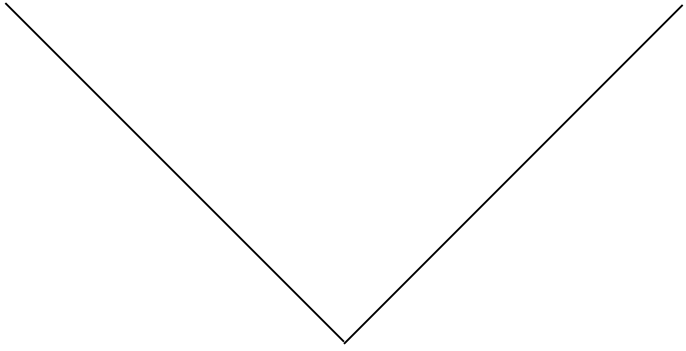
Tiger Farm Press

The segments are mirrored through the dotted line.

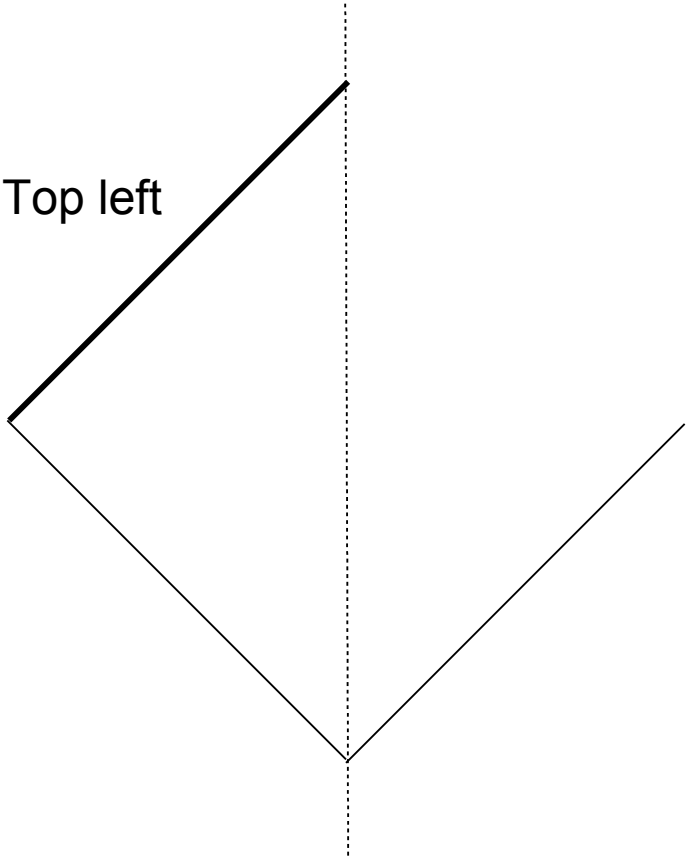


Tiger Farm Press

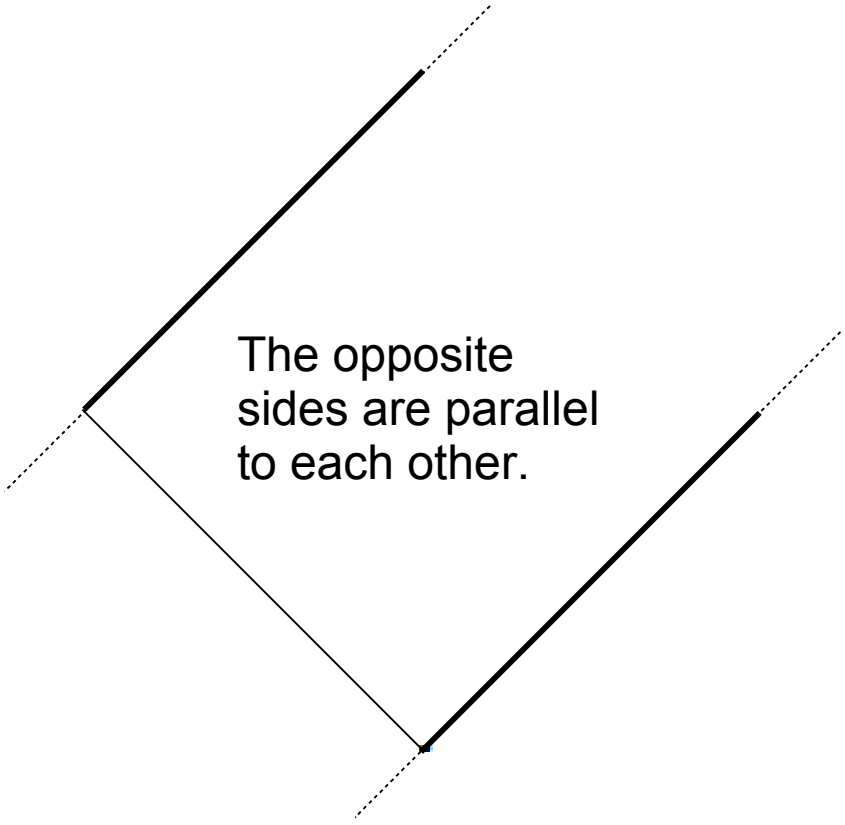
Width and length are the 2 dimensional base line segments of the square.



Tiger Farm Press

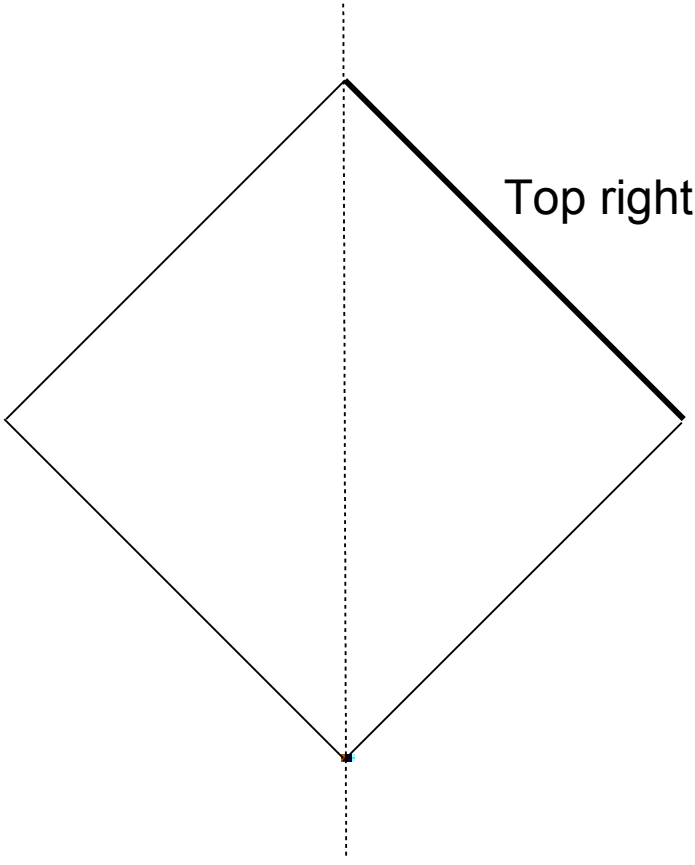


Tiger Farm Press



The opposite sides are parallel to each other.

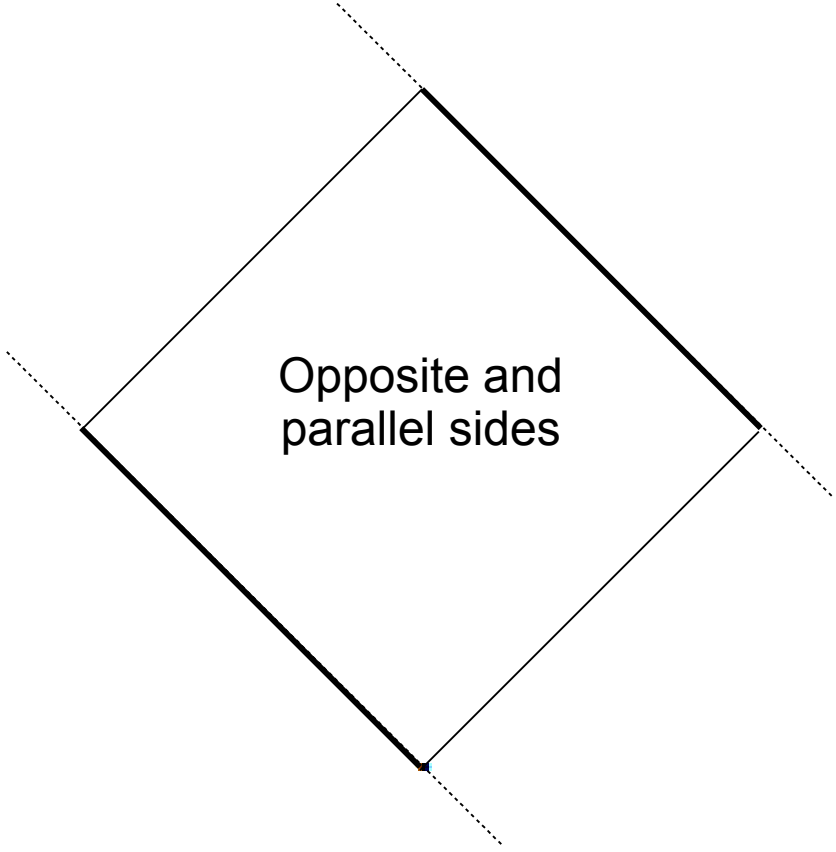
Tiger Farm Press



Tiger Farm Press

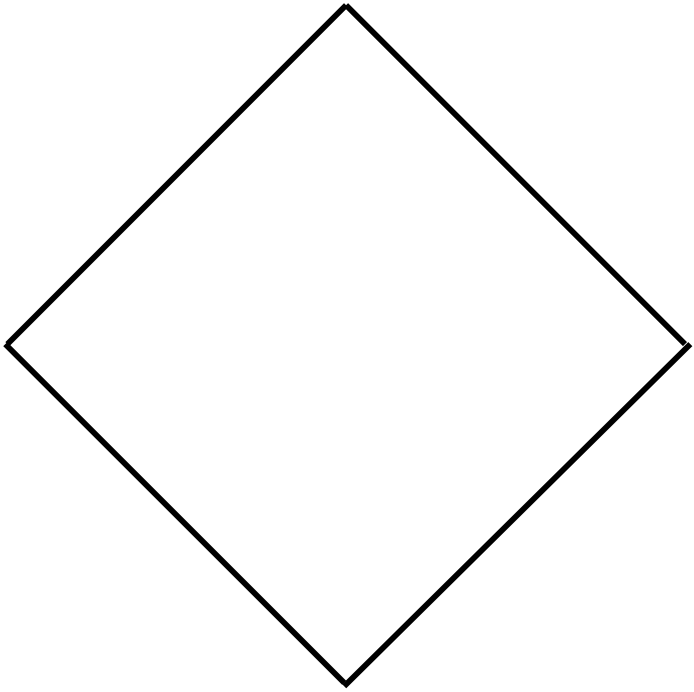
Key

Drawing tip: opposite sides of a square are parallel to each other.



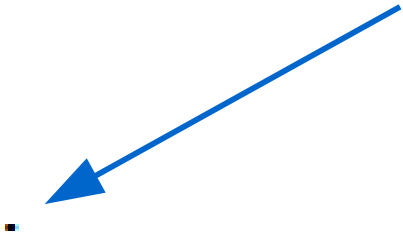
Tiger Farm Press

A Square



Tiger Farm Press

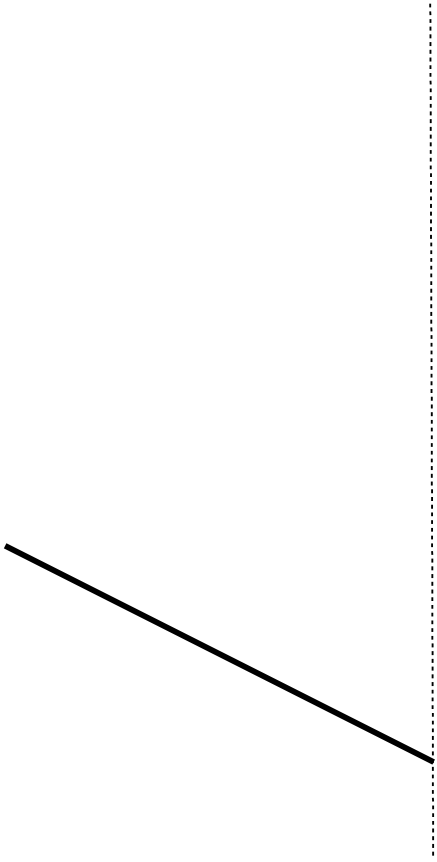
To draw a cube, start with a point.



Tiger Farm Press

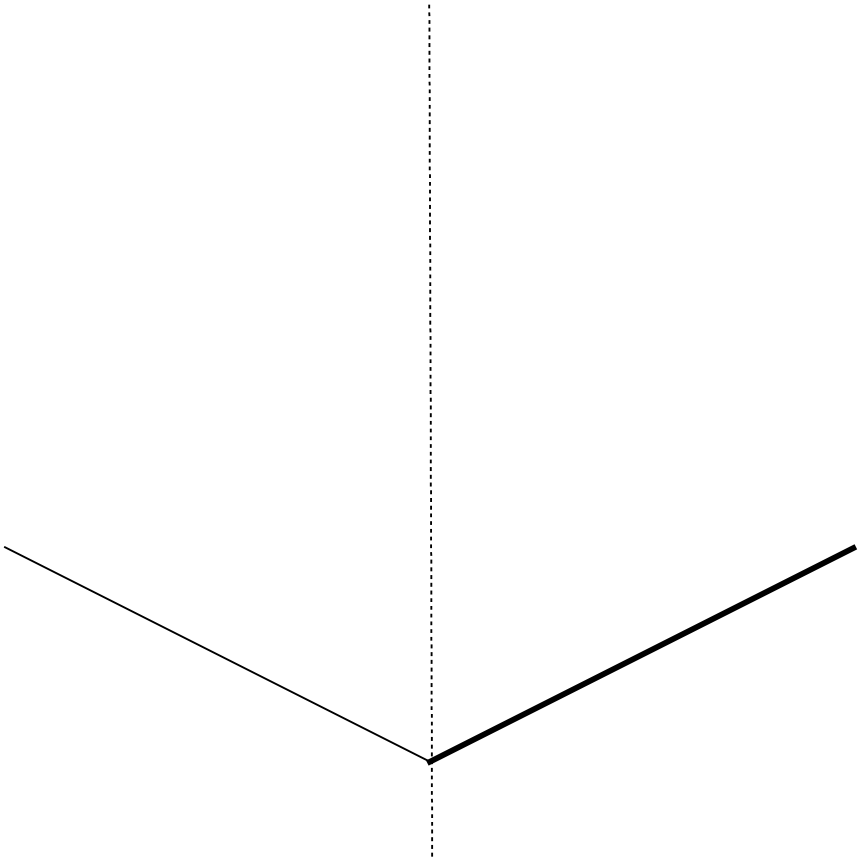
1st dimension: width

(dotted lines are for reference and to show perspective)



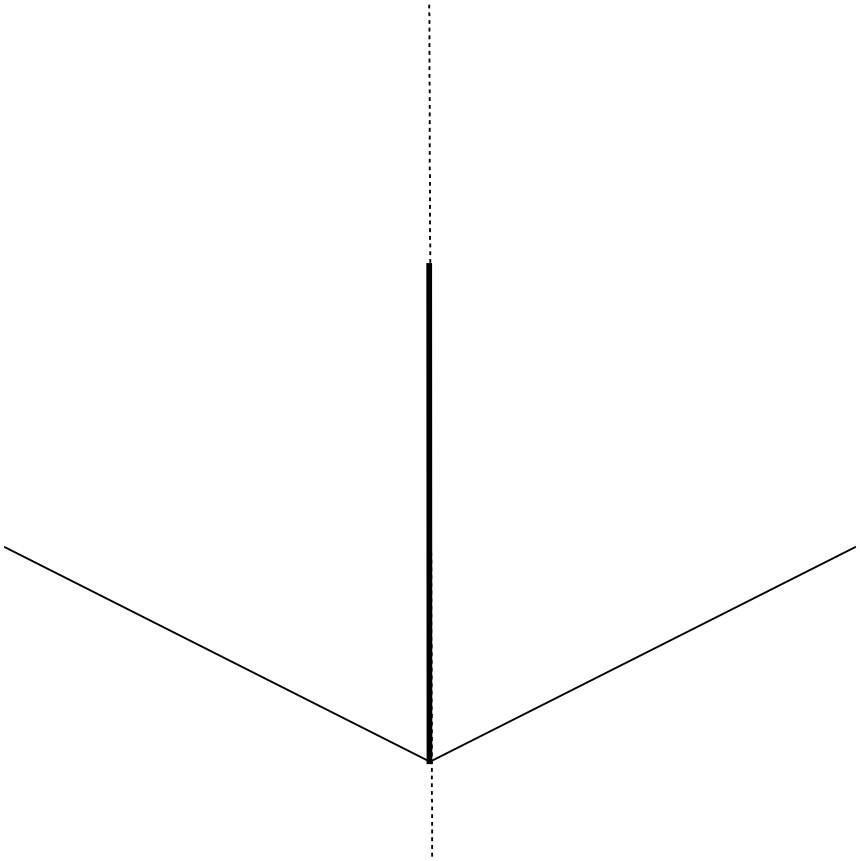
Tiger Farm Press

2nd dimension: length



Tiger Farm Press

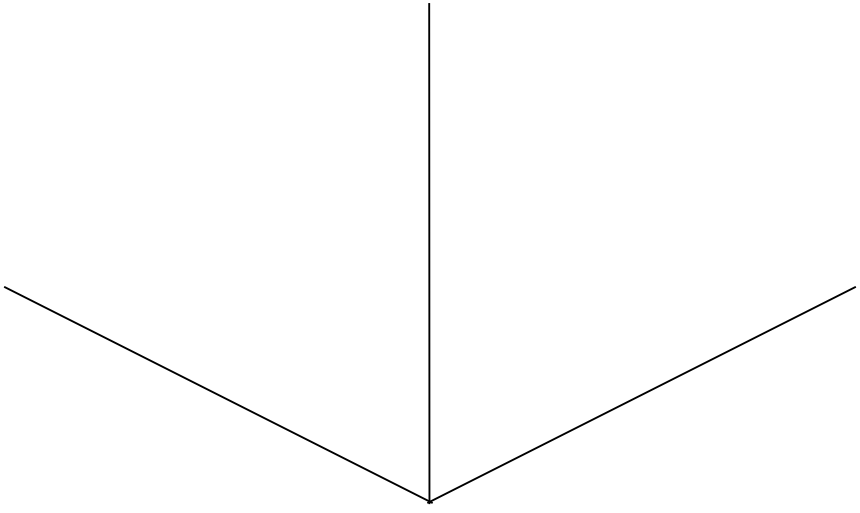
3rd dimension: height



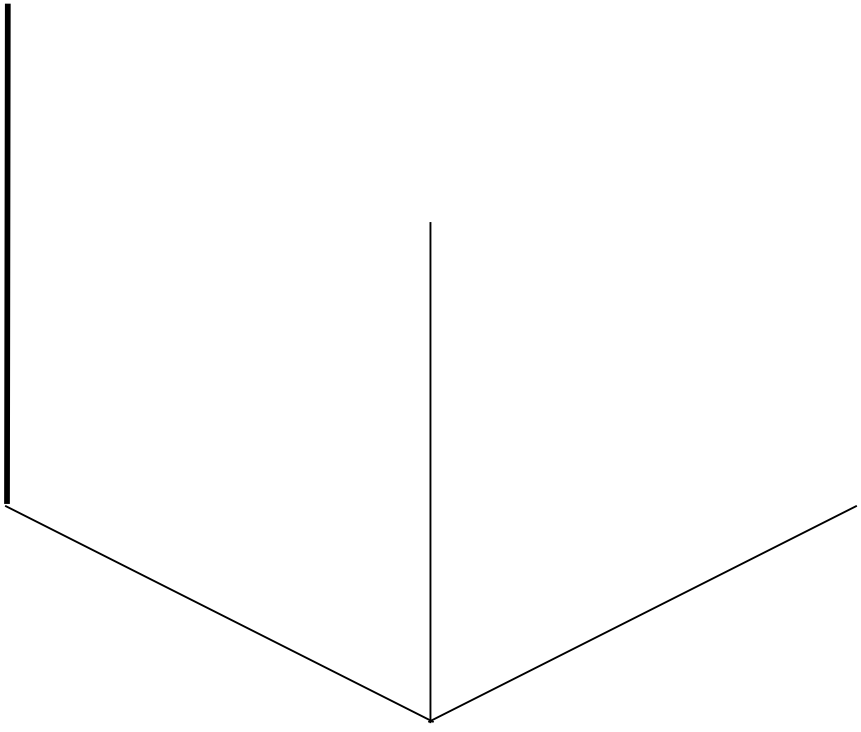
Tiger Farm Press

Key

When drawing cubes, start with the dimensional base lines.

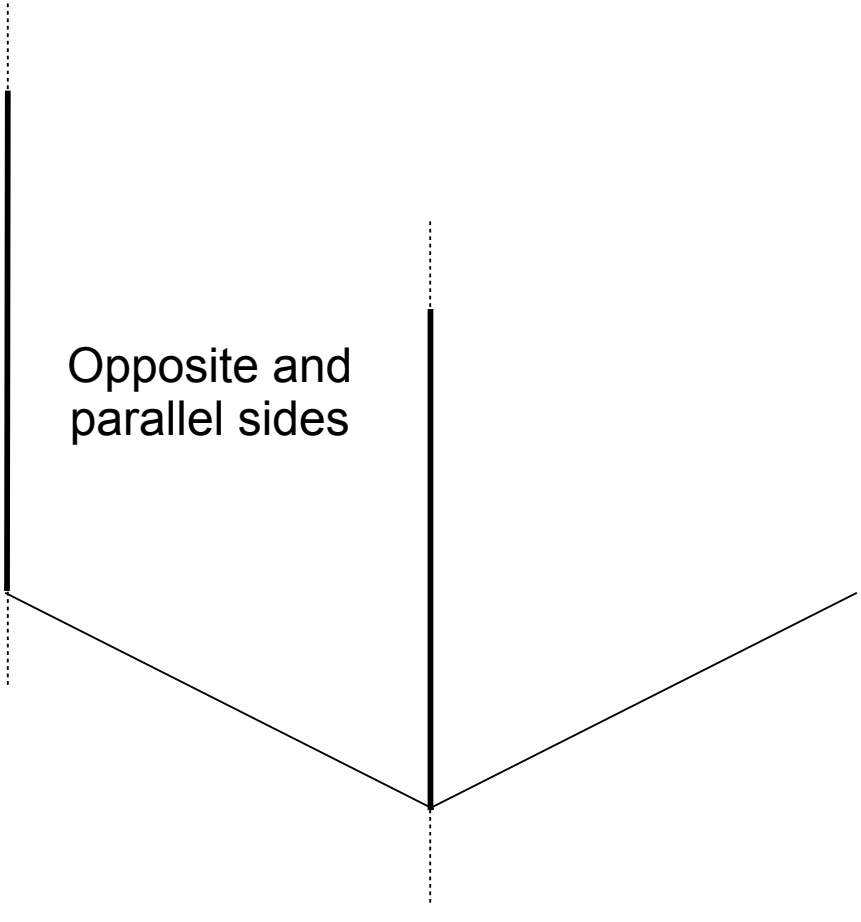


Tiger Farm Press

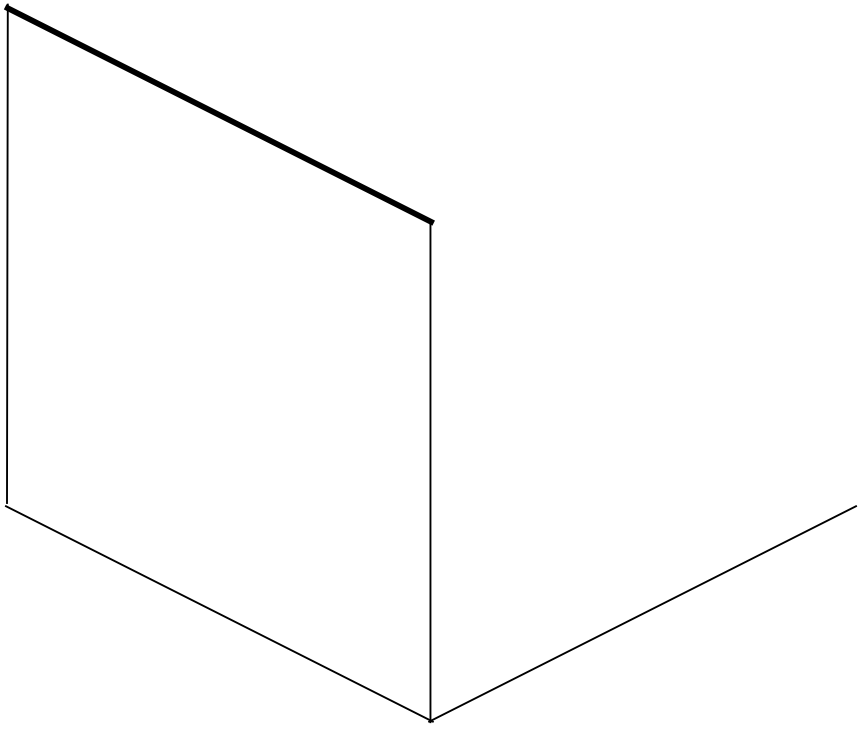


Tiger Farm Press

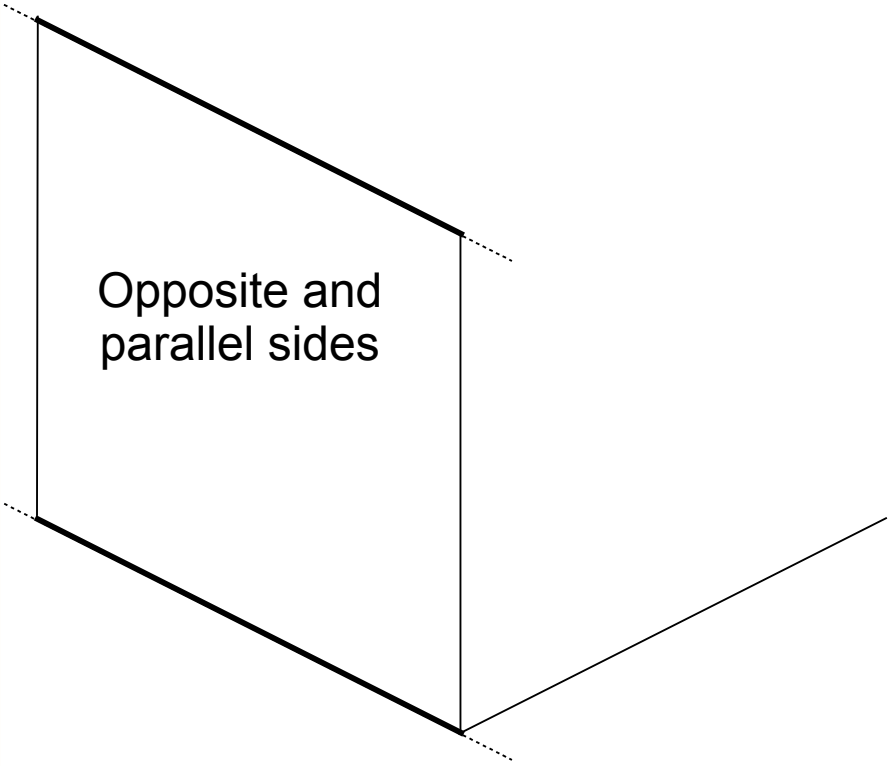
Opposite and parallel sides



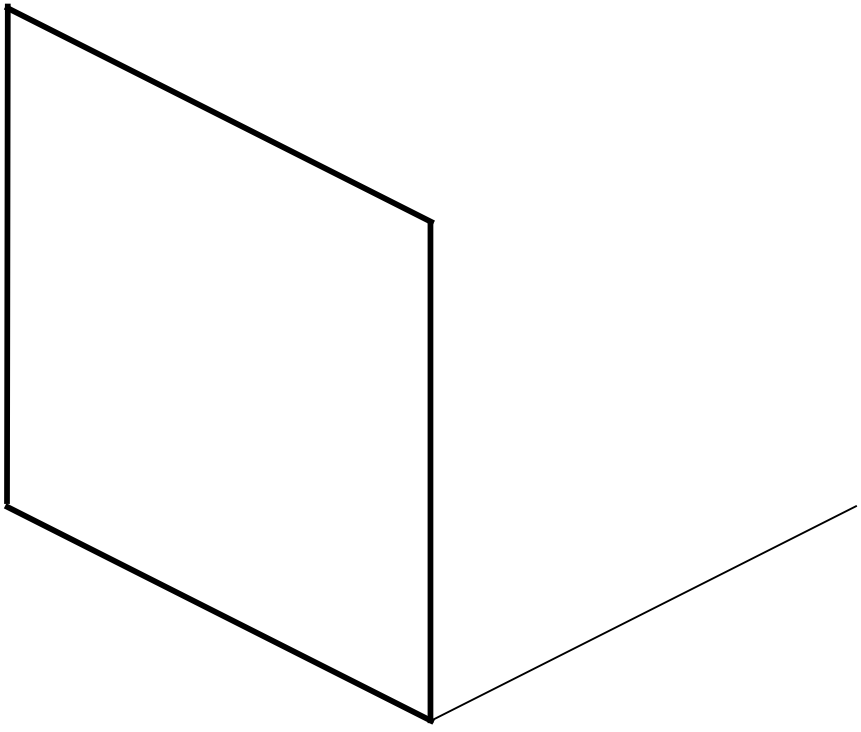
Tiger Farm Press



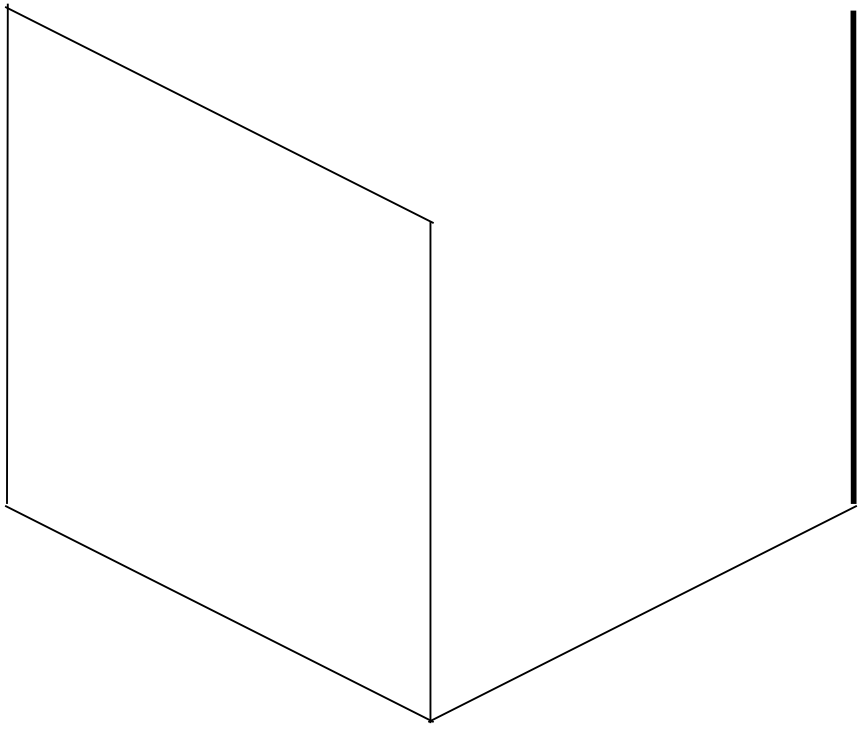
Tiger Farm Press



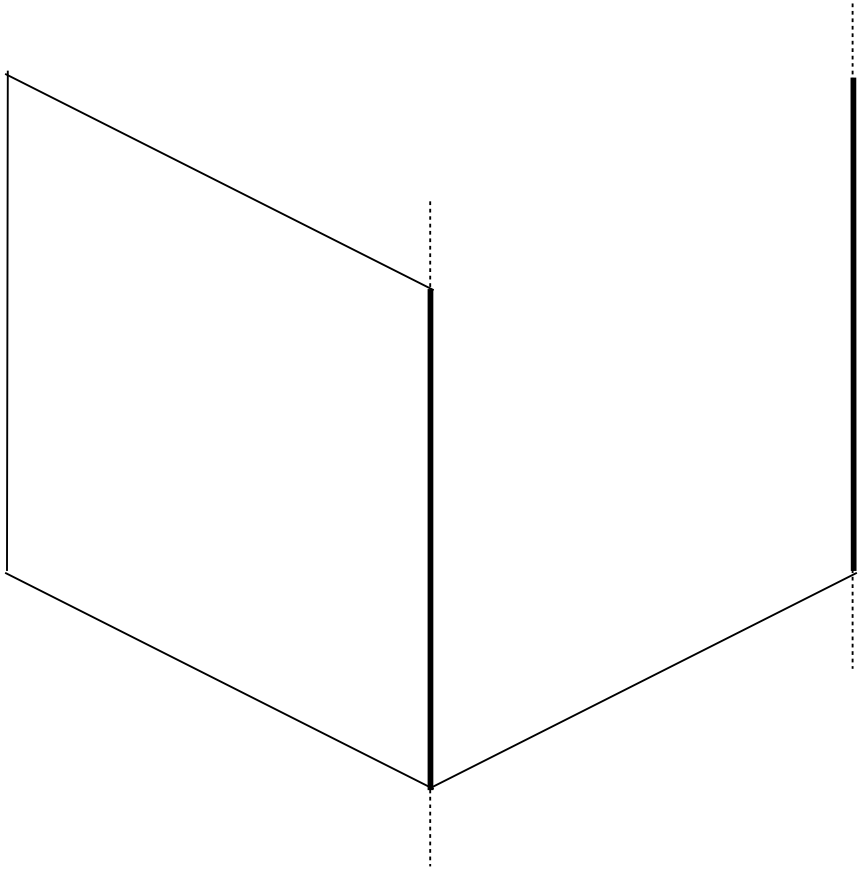
Tiger Farm Press



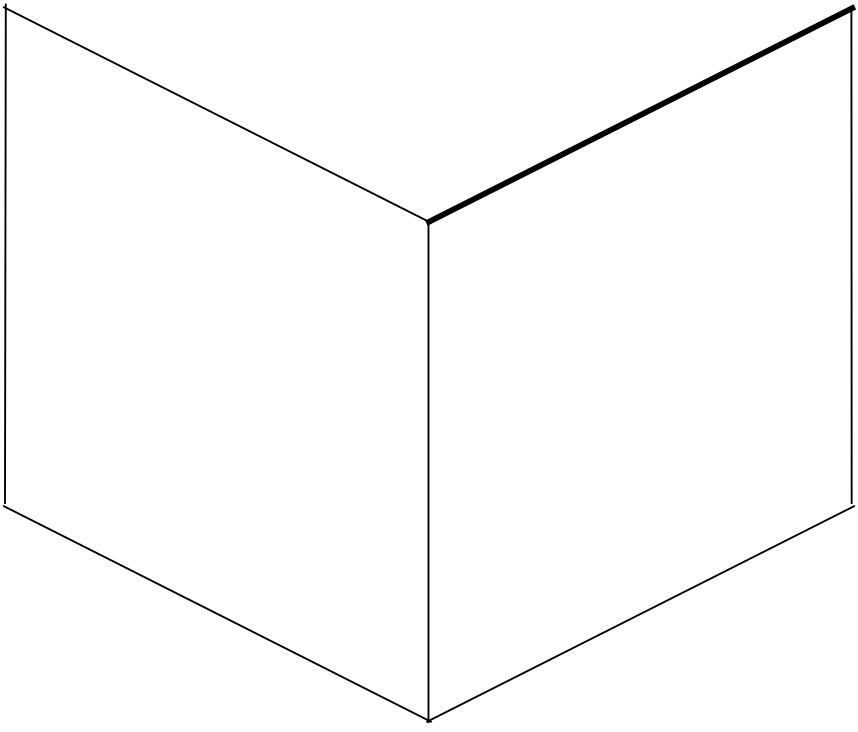
Tiger Farm Press



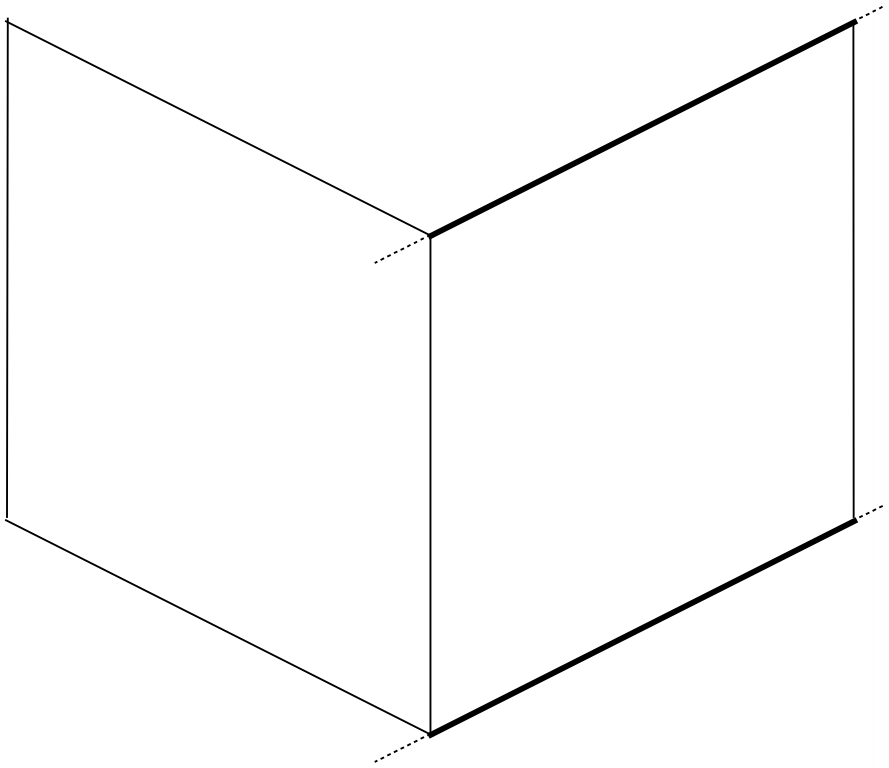
Tiger Farm Press



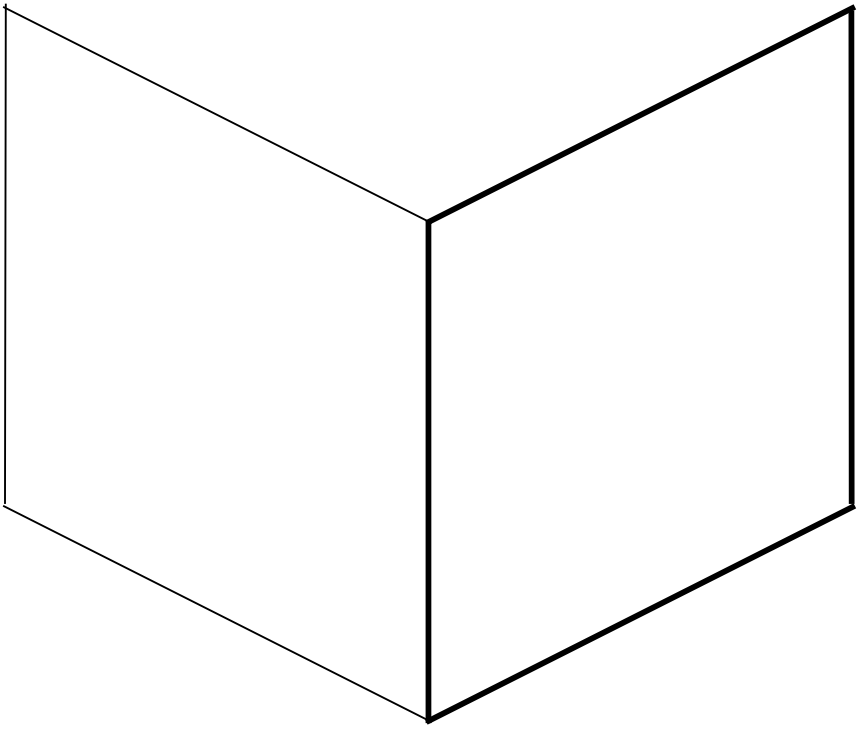
Tiger Farm Press



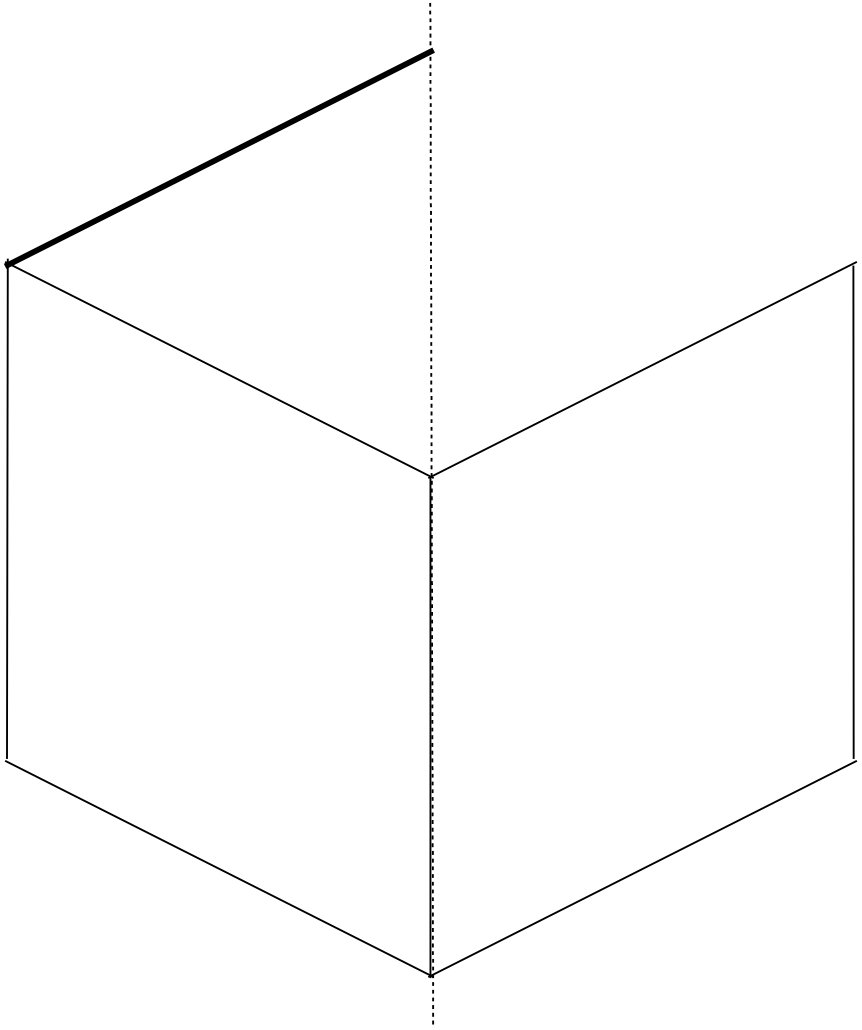
Tiger Farm Press



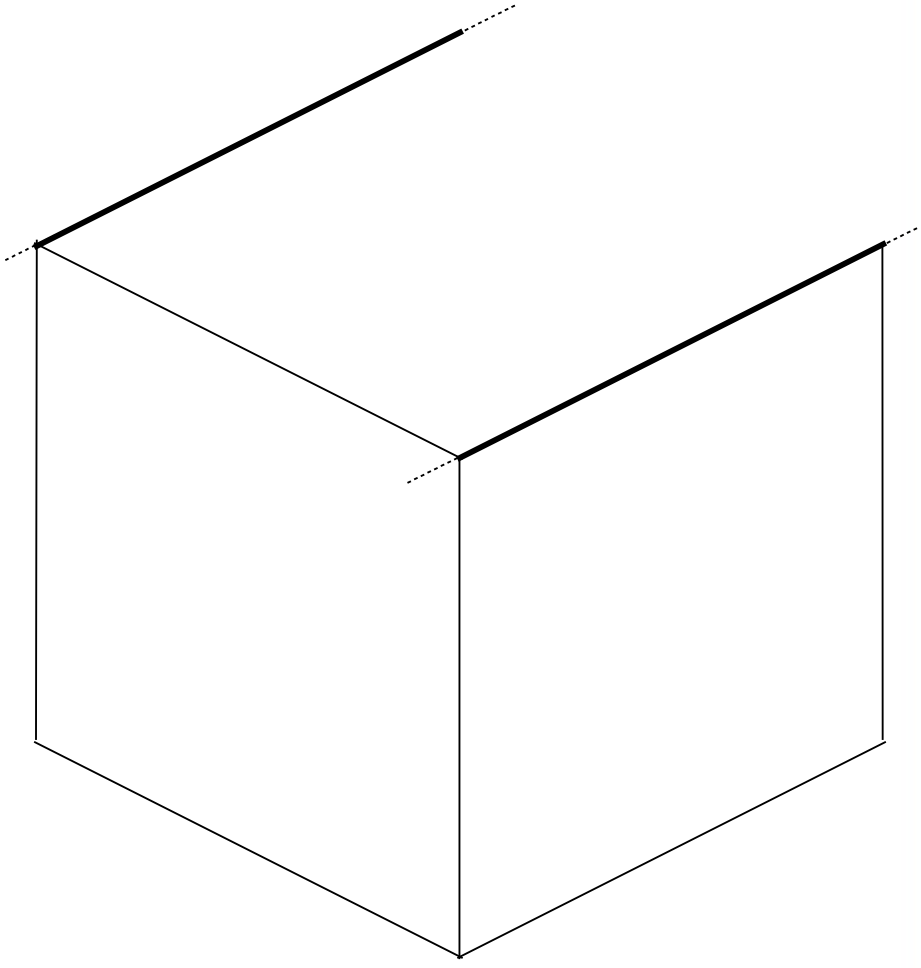
Tiger Farm Press



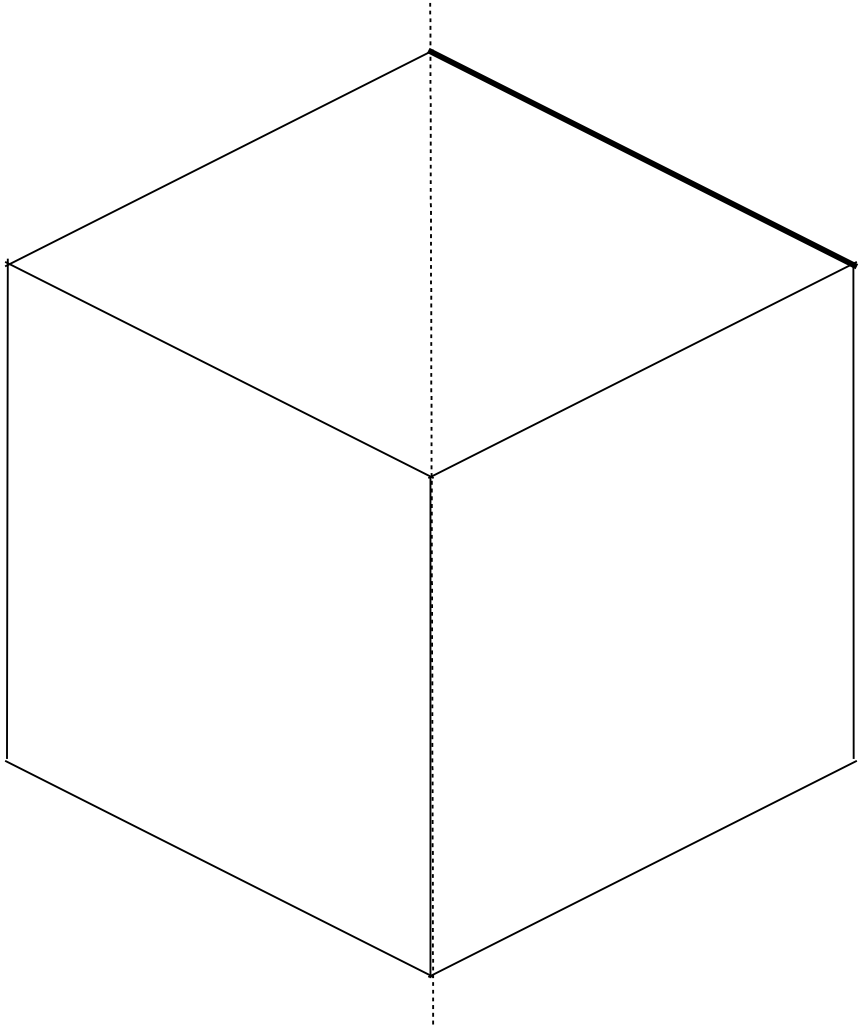
Tiger Farm Press



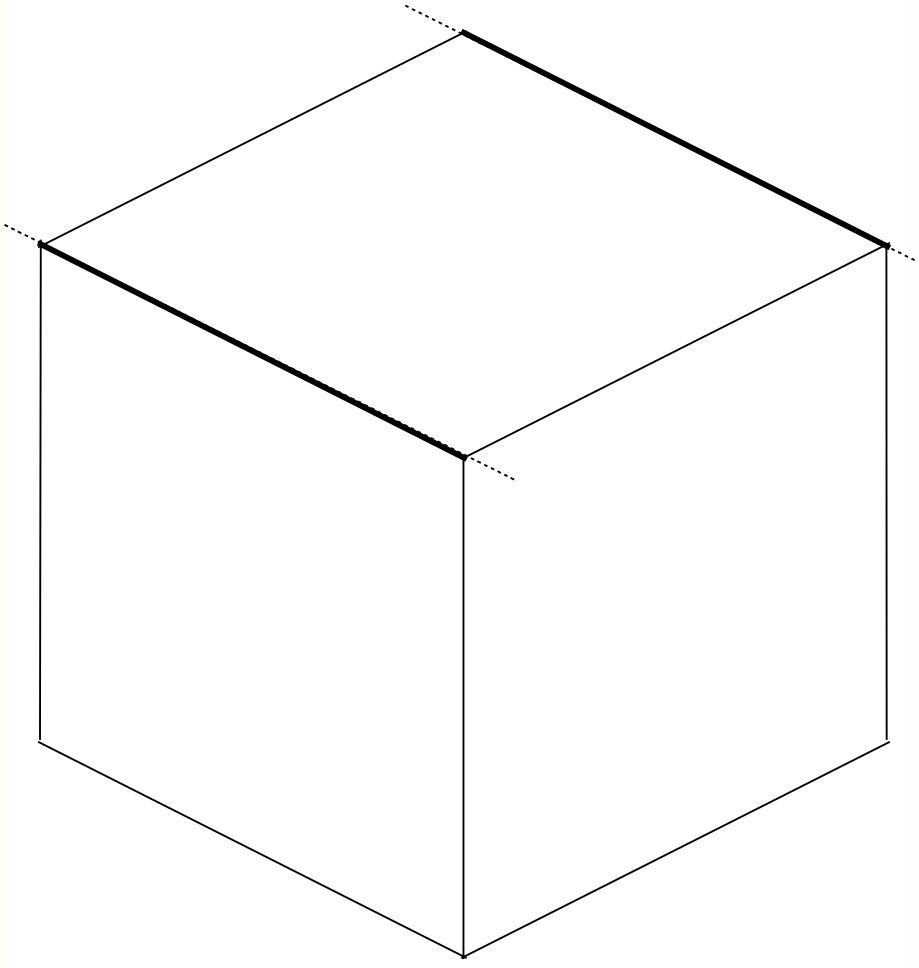
Tiger Farm Press



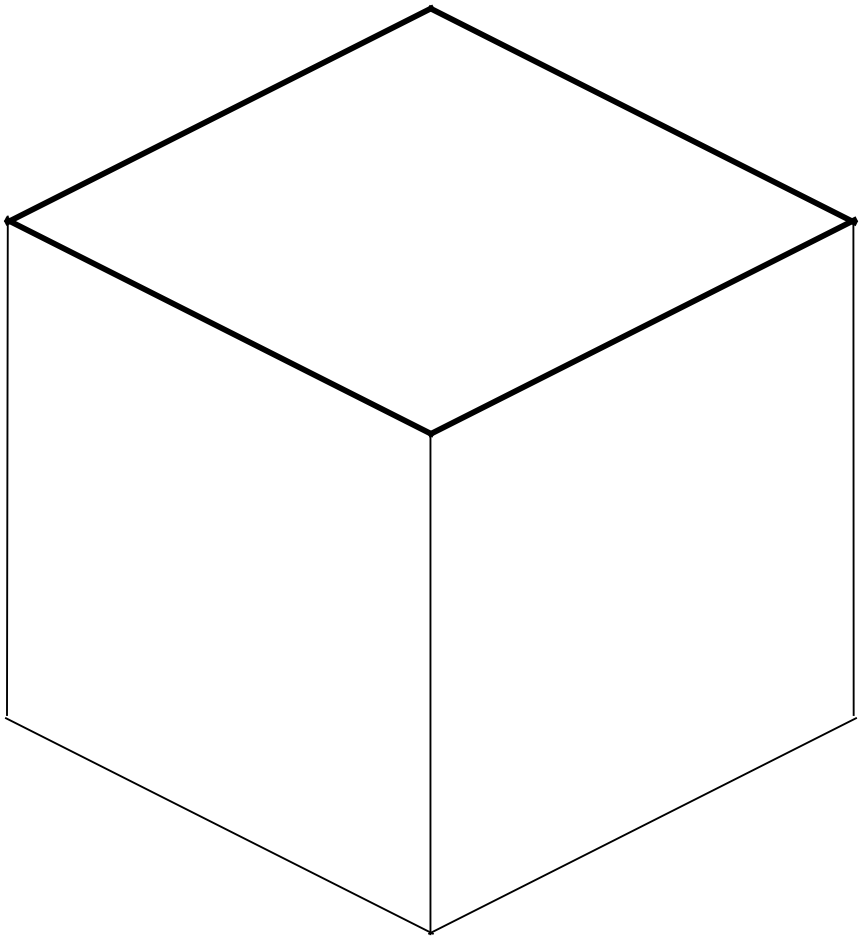
Tiger Farm Press



Tiger Farm Press

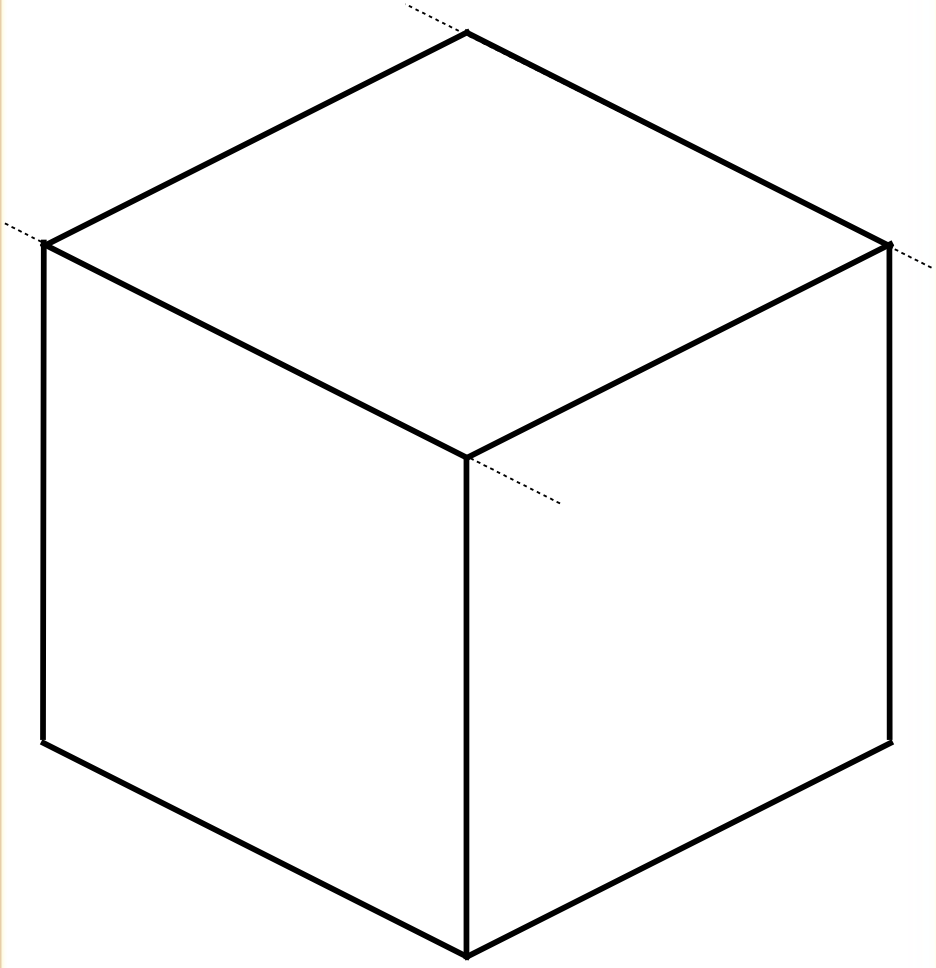


Tiger Farm Press



Tiger Farm Press

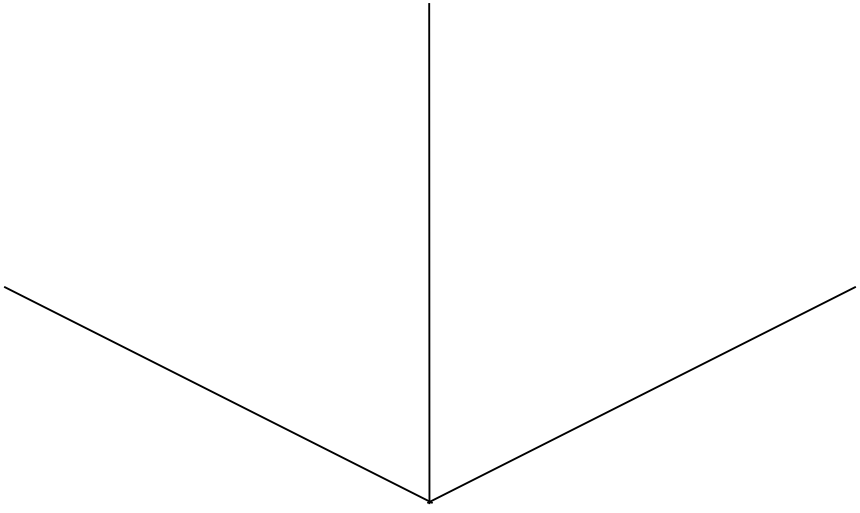
3D Cube



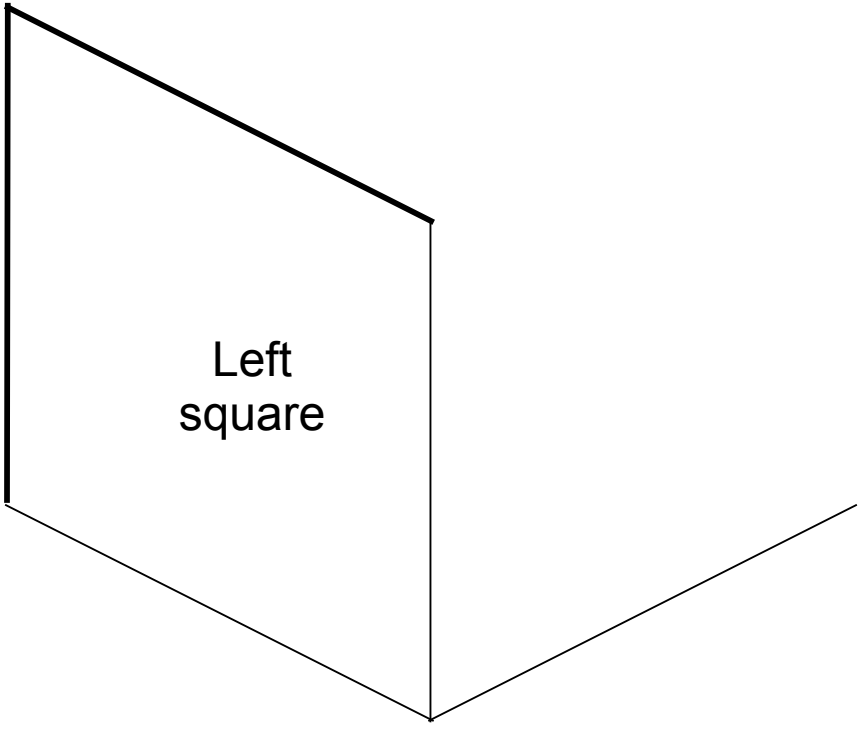
Tiger Farm Press

In the following sequence, the 3D cube is redrawn one square side at a time.

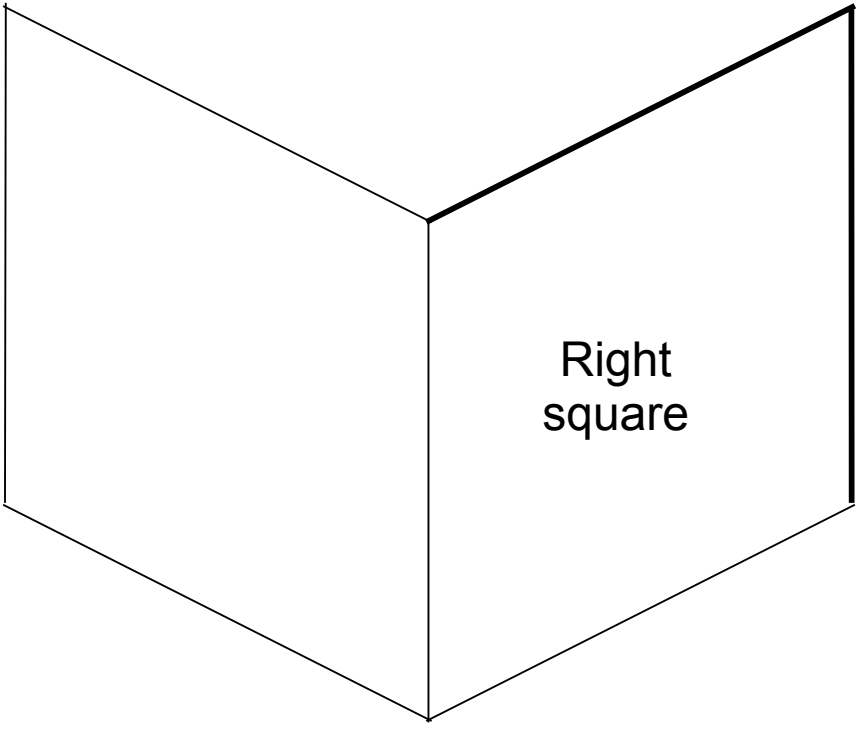
How many dimensional base lines for a 3 dimensional cube?



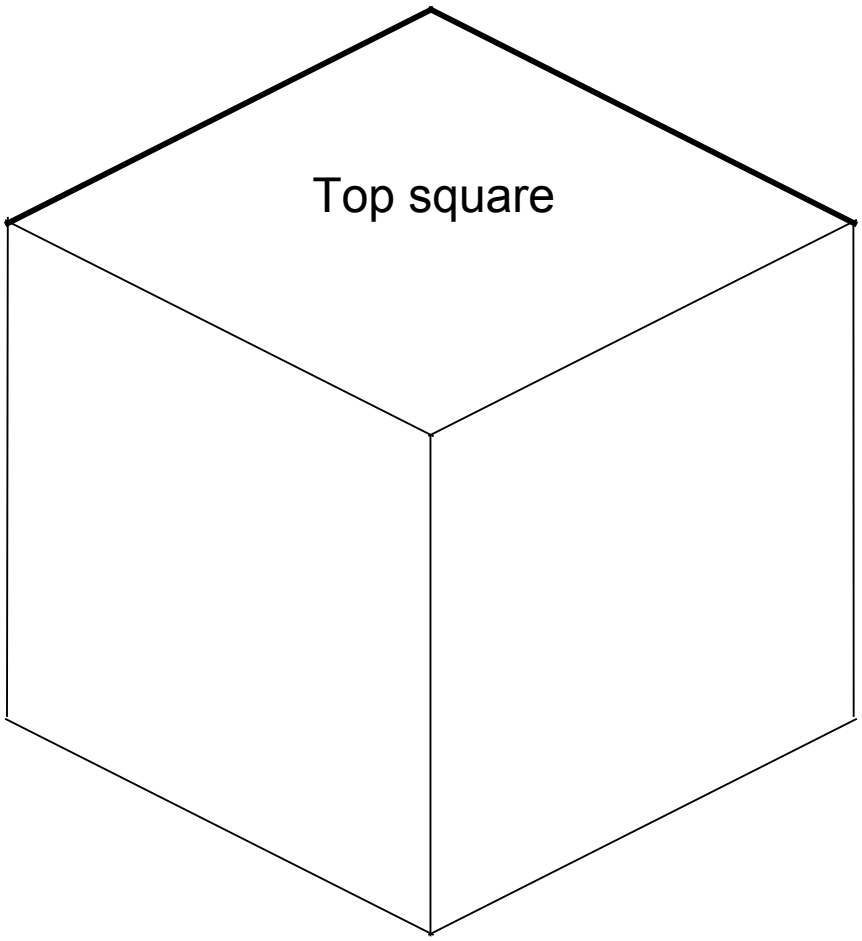
Tiger Farm Press



Tiger Farm Press



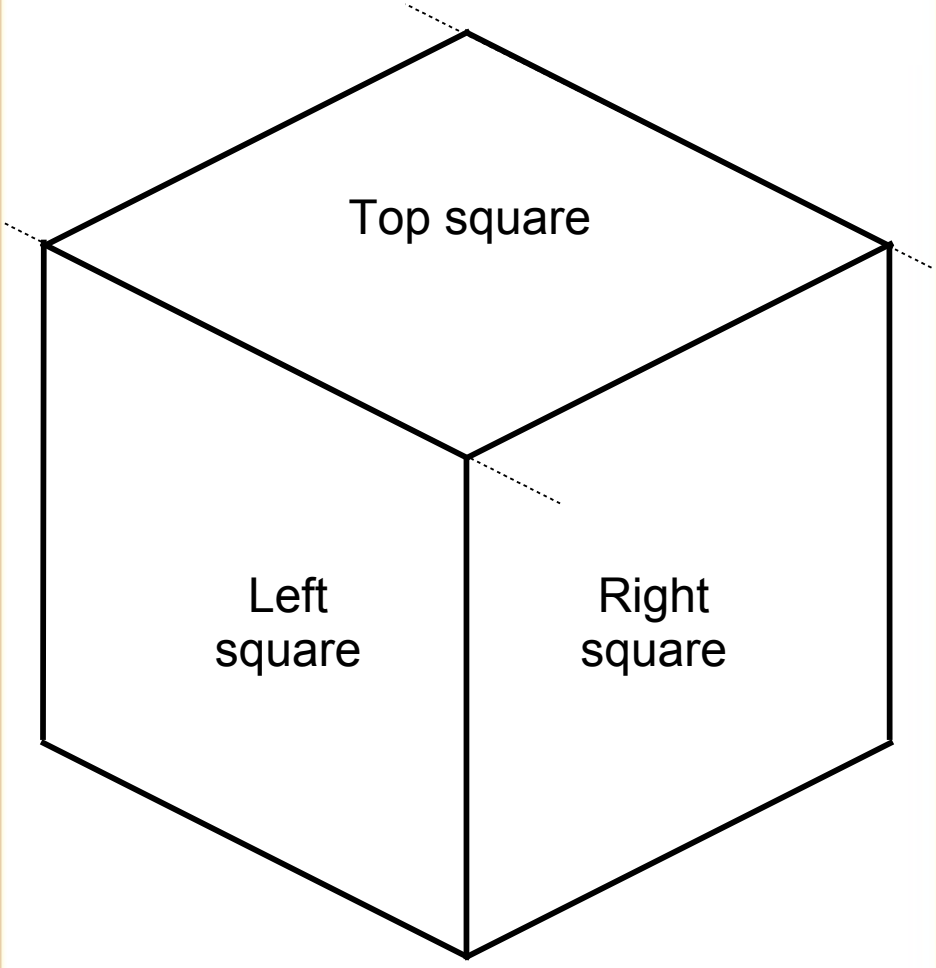
Tiger Farm Press



Tiger Farm Press

Key

To draw a cube, draw the square sides.



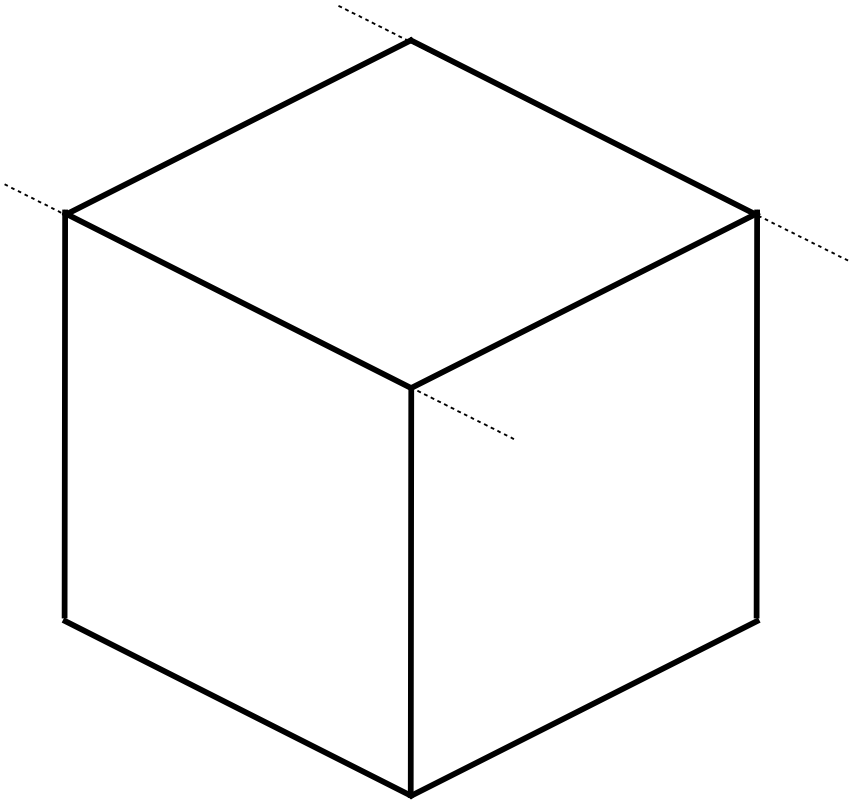
Tiger Farm Press

Hidden Lines

Key

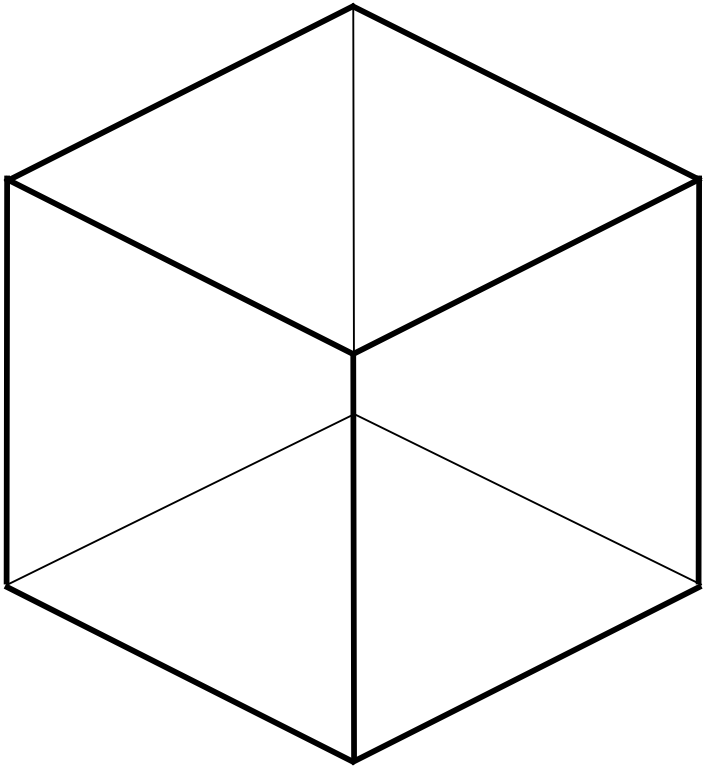
For simplicity, draw cubes without hidden lines.

This is a cube without hidden lines:



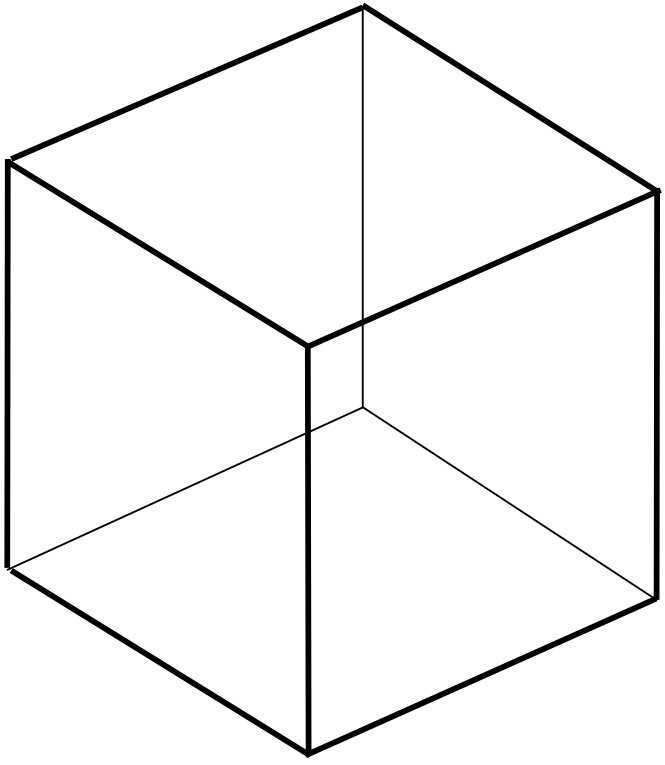
Tiger Farm Press

A cube with hidden lines:



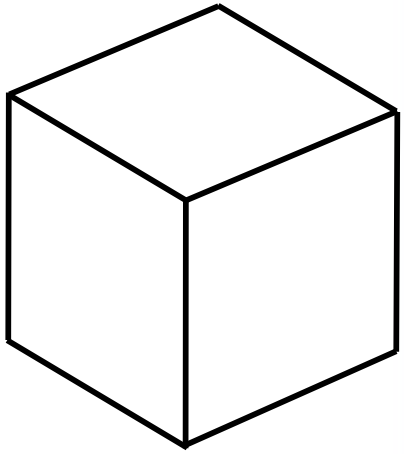
Tiger Farm Press

A cube with hidden lines, rotated right:

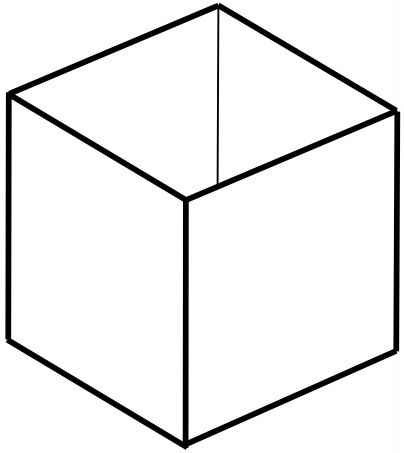


Tiger Farm Press

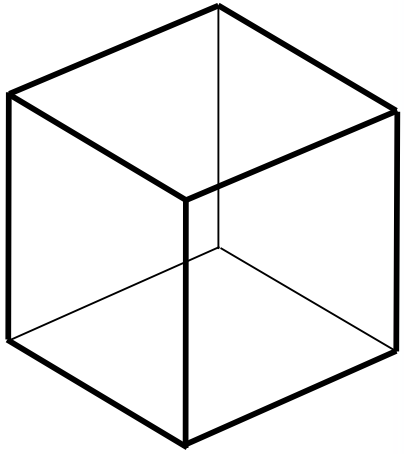
A cube without hidden lines looks like a closed box.



This is a cube box without a top.



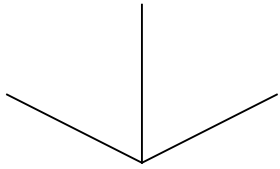
A cube with hidden lines is the frame of a box.



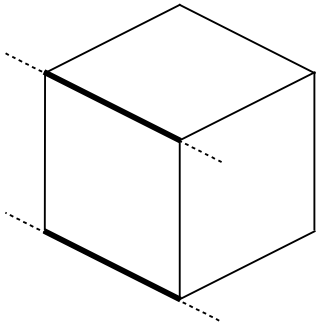
Tiger Farm Press

Key Drawing Points

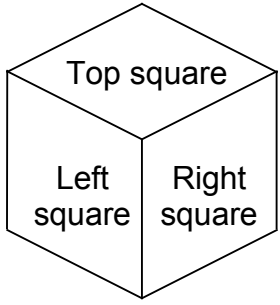
+ Start with the base dimensional lines.



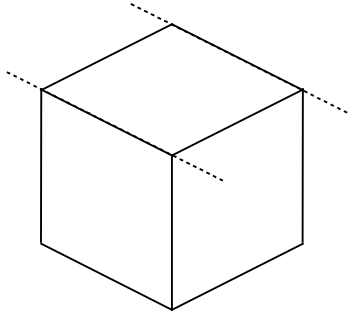
+ Each of a cube's square sides, have opposite and parallel sides.



+ All sides of a cube are square.



+ Draw a cube without hidden lines.

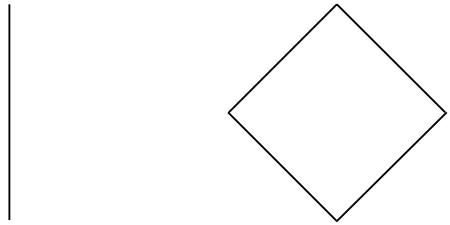


Tiger Farm Press

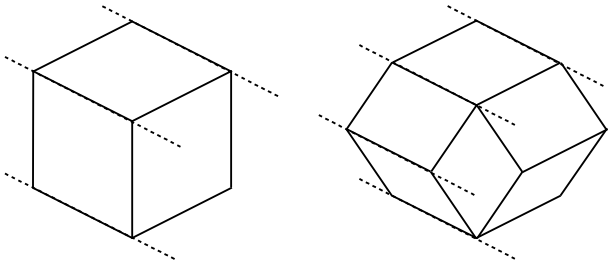
Drawing 4D Hypercubes

A note before beginning:

1 and 2 dimensional shapes can be drawn on your 2 dimensional screen in proper perspective.



However, 3 and 4 dimensional cubes have to be drawn to fit on a 2 dimensional screen. Examples with dotted perspective lines:

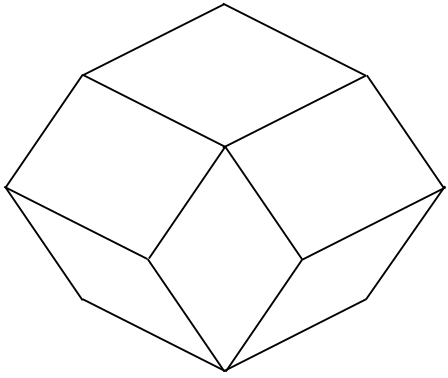


Tiger Farm Press

I will draw 2 hypercubes.

The first drawing includes detail explanations.

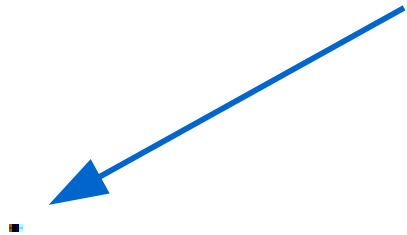
Then you will be ready for the quick draw of the 6 front sides of a 4D cube.



Tiger Farm Press

(1)
Drawing a 4D Cube with Detail Explanations

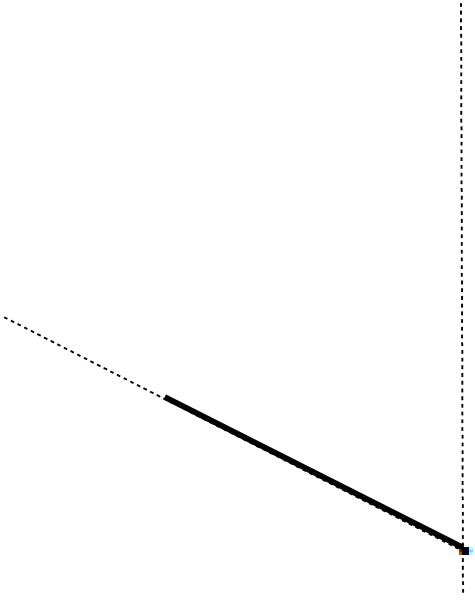
To draw a 4D cube, start with a point.



Tiger Farm Press

1st dimension: width

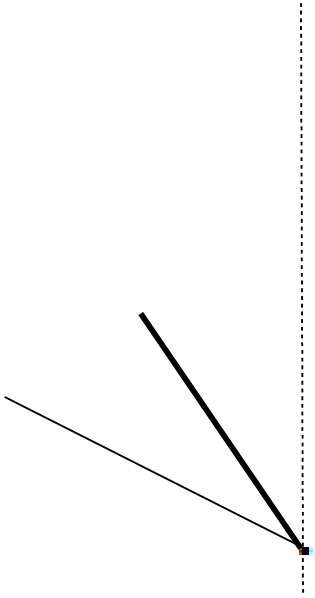
(a measure of distance)



Tiger Farm Press

2nd dimension: height

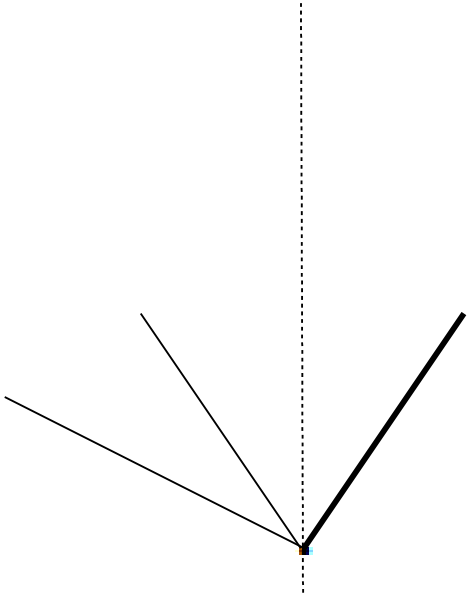
(a measure of distance)



Tiger Farm Press

3rd dimension: length

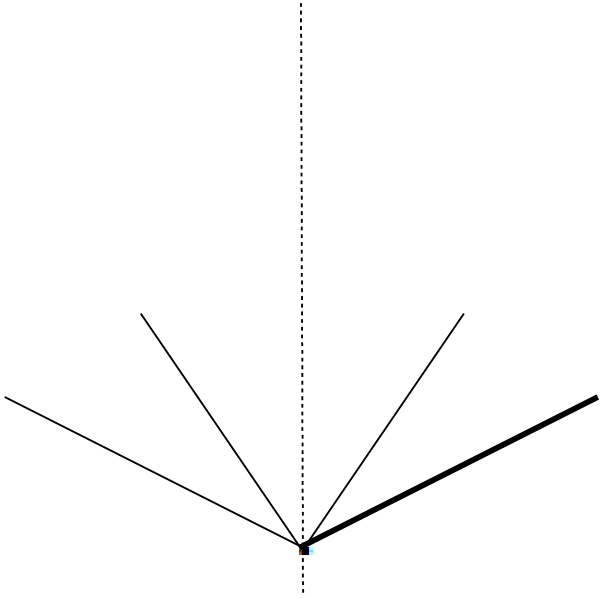
(a measure of distance)



Tiger Farm Press

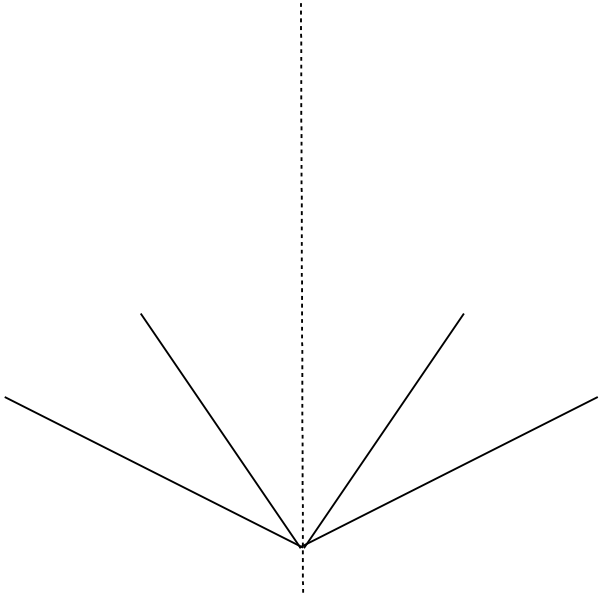
4th dimension: time

(a duration of time)



Tiger Farm Press

The left segments are mirrors of the right segments.



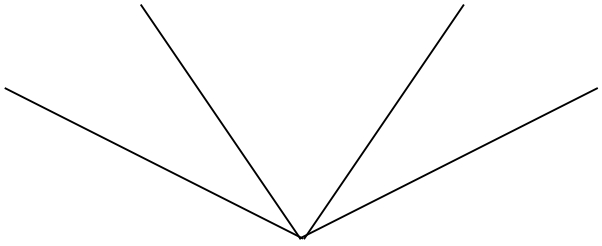
Tiger Farm Press

Key

When drawing a 4 dimensional cube, start with 4 dimensional base lines.

How many dimensional base lines for a 2D square and a 3D cube?

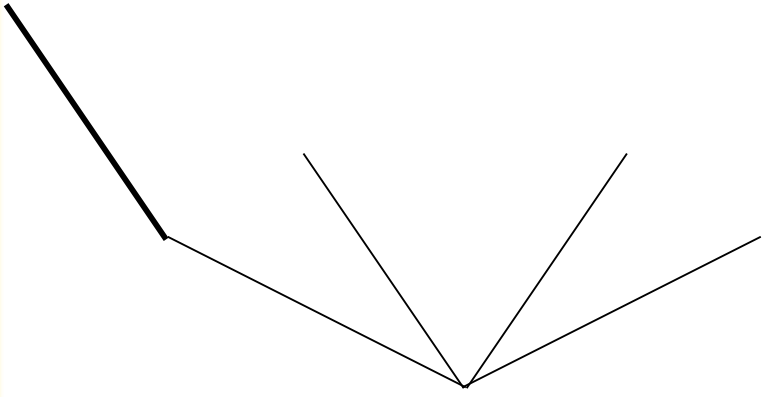
If you don't know, delete this book because you're too stupid to continue. Just messing with ya. I'm sure you're bloody brilliant, so continue.



Tiger Farm Press

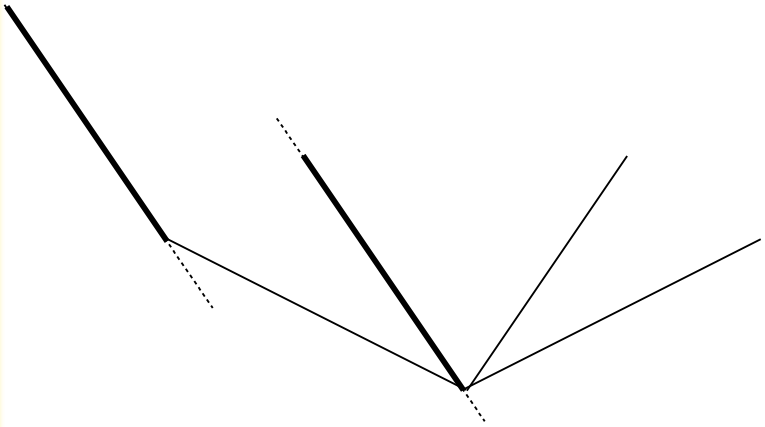
Use the base lines as guide lines.

Draw the first line, opposite and parallel to the 2nd dimensional base line.



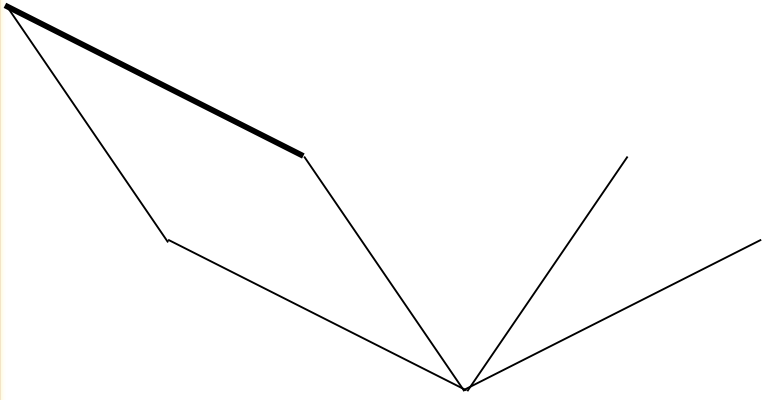
Tiger Farm Press

Opposite and
parallel sides



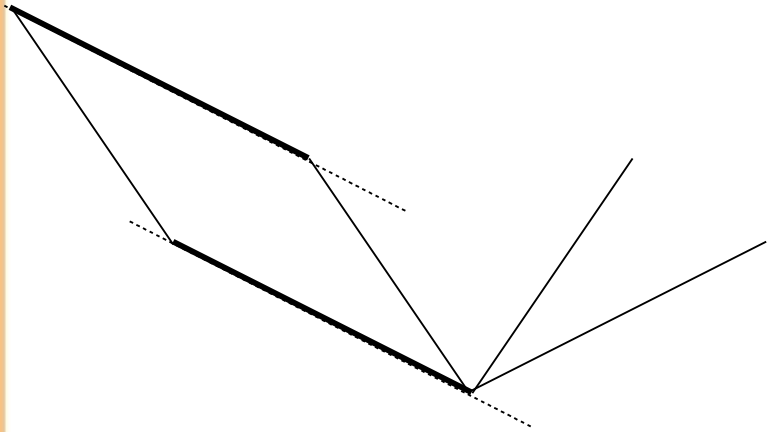
Tiger Farm Press

Draw the second line opposite and parallel to the 1st dimensional base line.



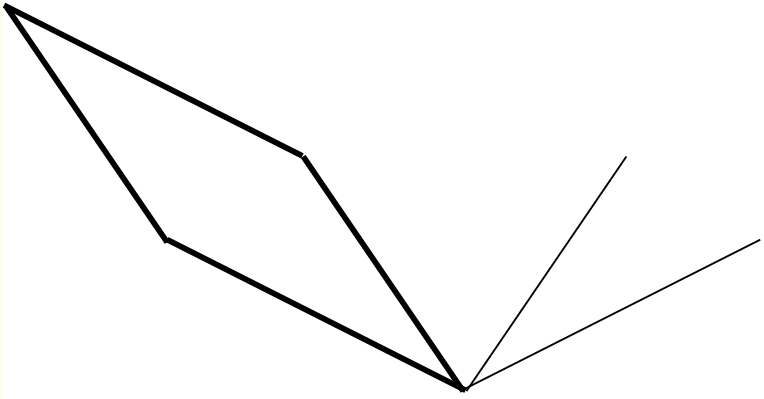
Tiger Farm Press

Opposite and
parallel sides

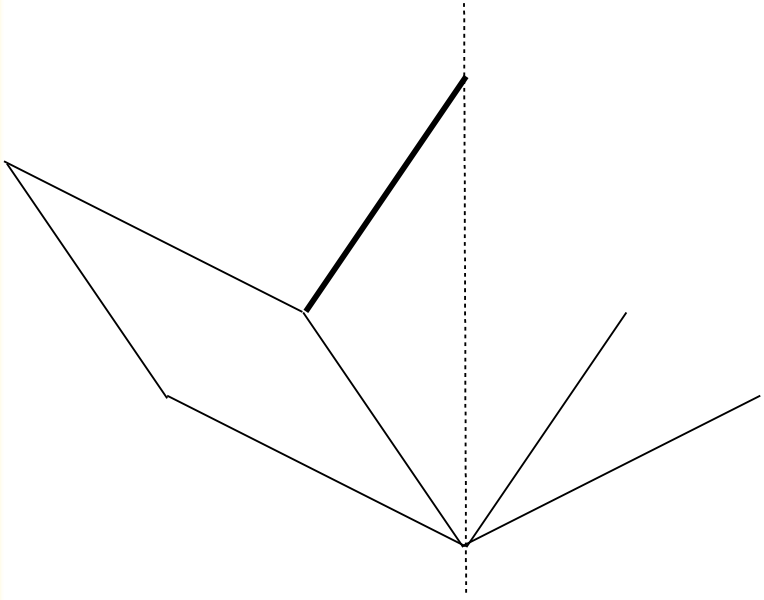


Tiger Farm Press

Bottom
left
square

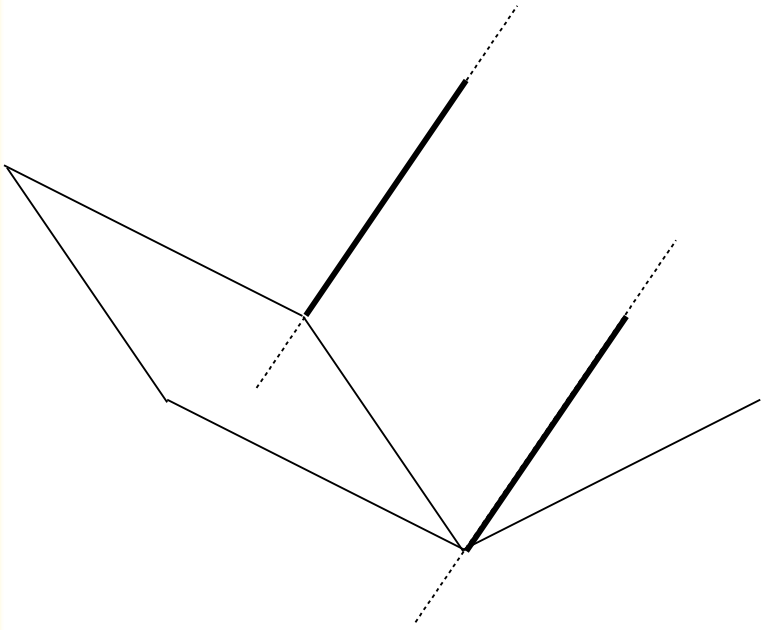


Tiger Farm Press

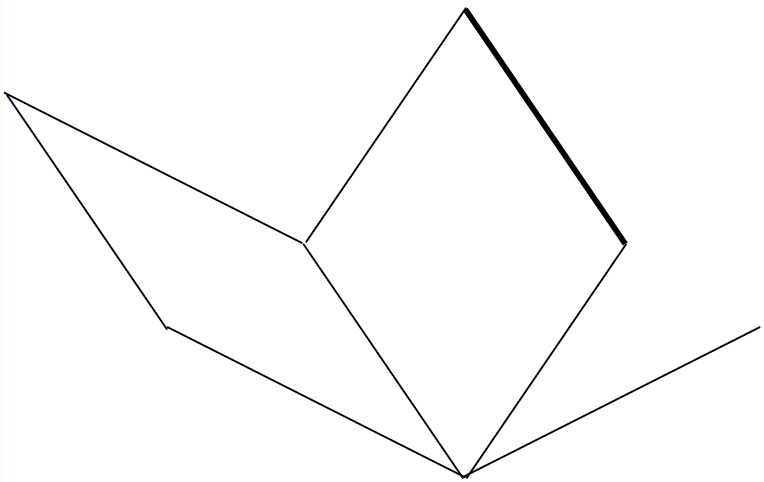


Tiger Farm Press

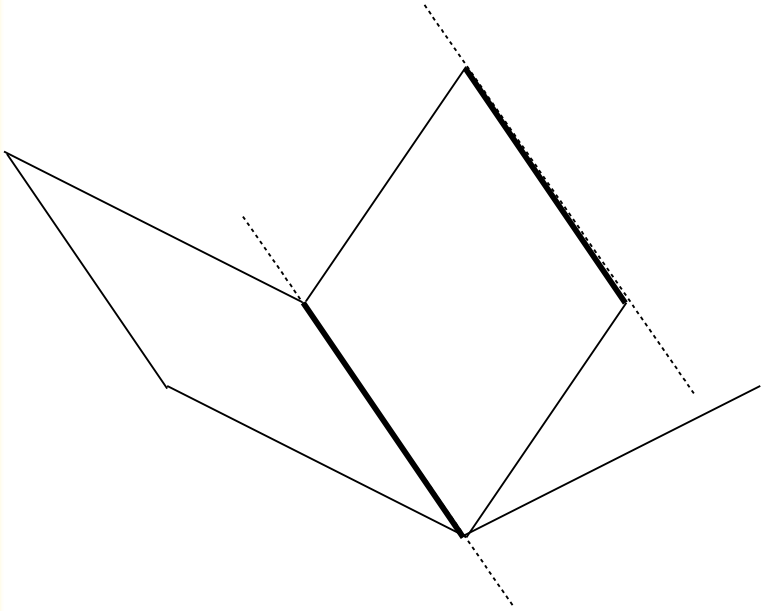
By now, you know that the highlighted segments are opposite and parallel to each other. So, I'll shut the fuck up about it.



Tiger Farm Press



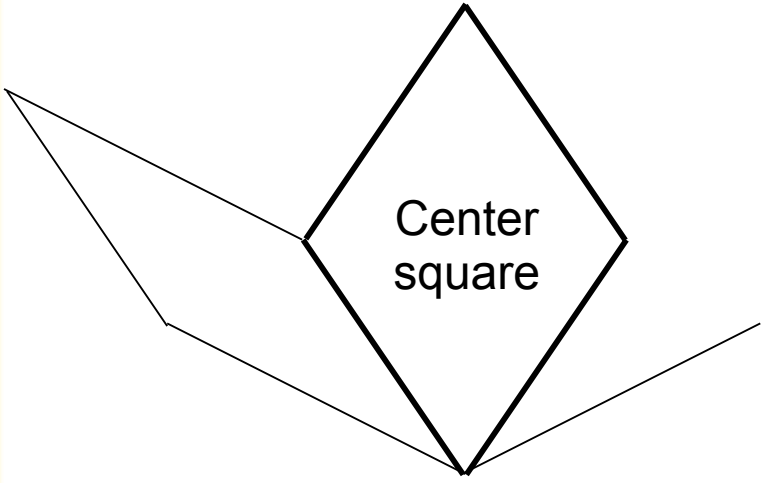
Tiger Farm Press



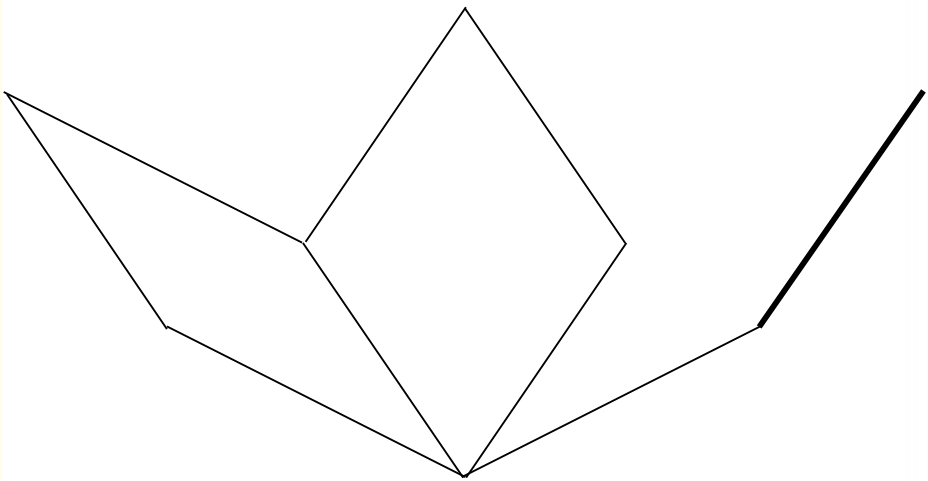
Tiger Farm Press

Important

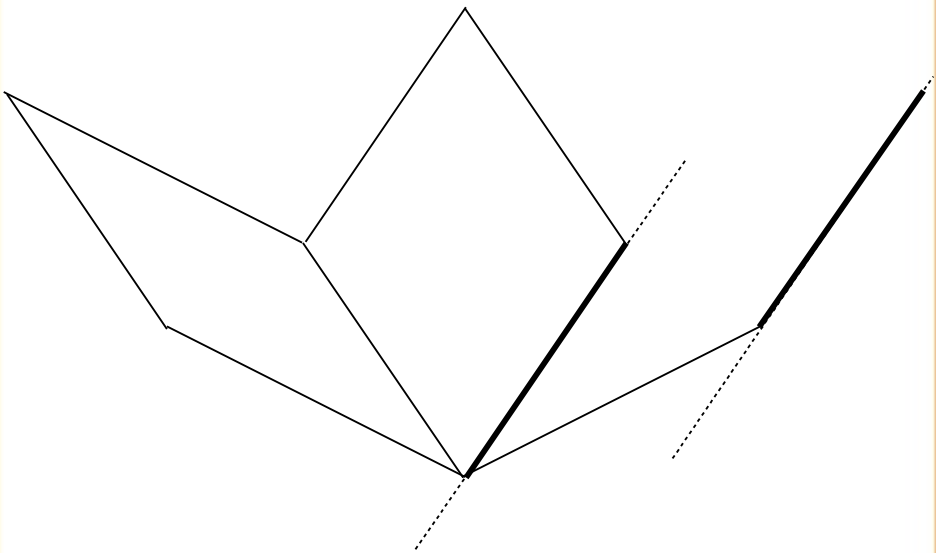
It may not be important to put the dishes in the dishwasher right away. But it is important to know that all sides of a cube are square.



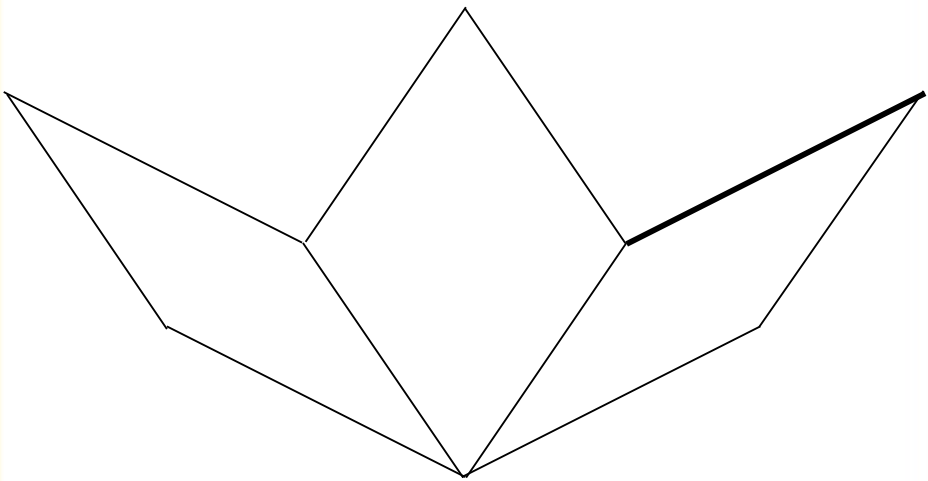
Tiger Farm Press



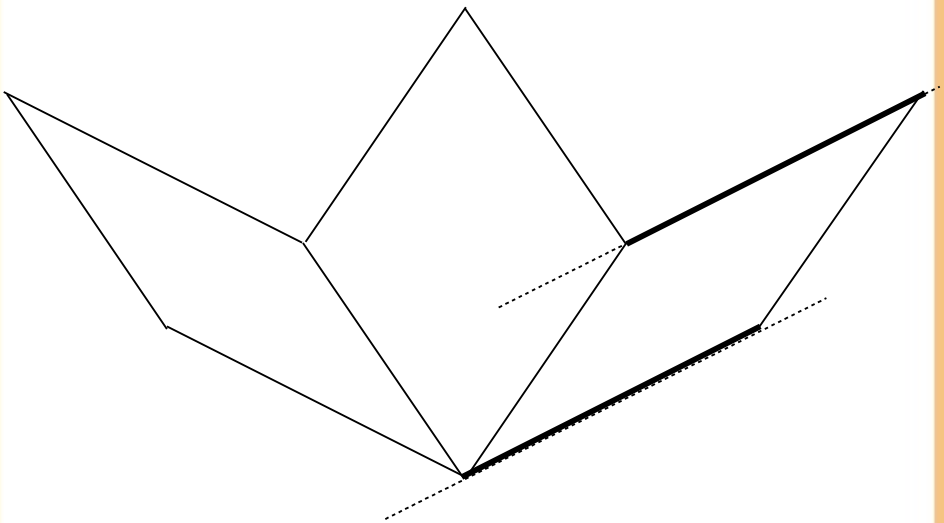
Tiger Farm Press



Tiger Farm Press

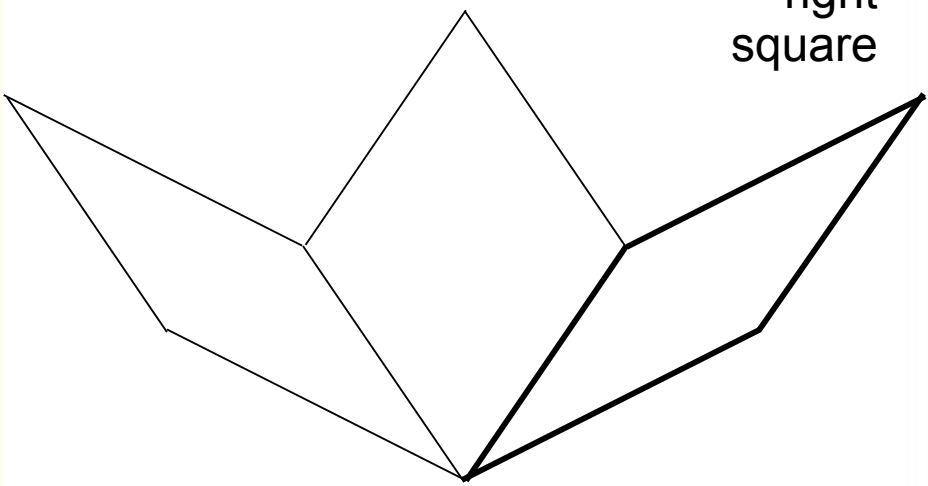


Tiger Farm Press

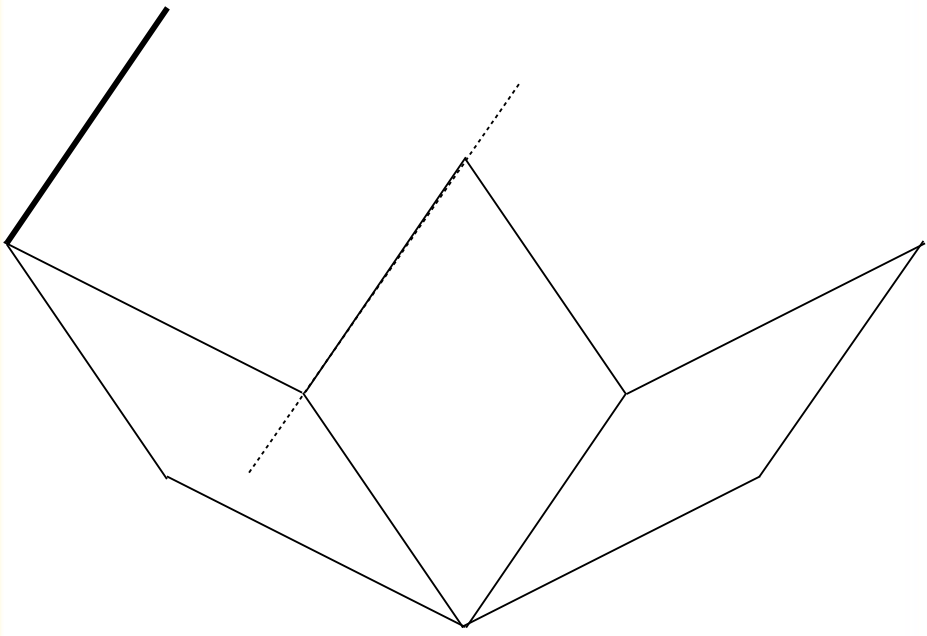


Tiger Farm Press

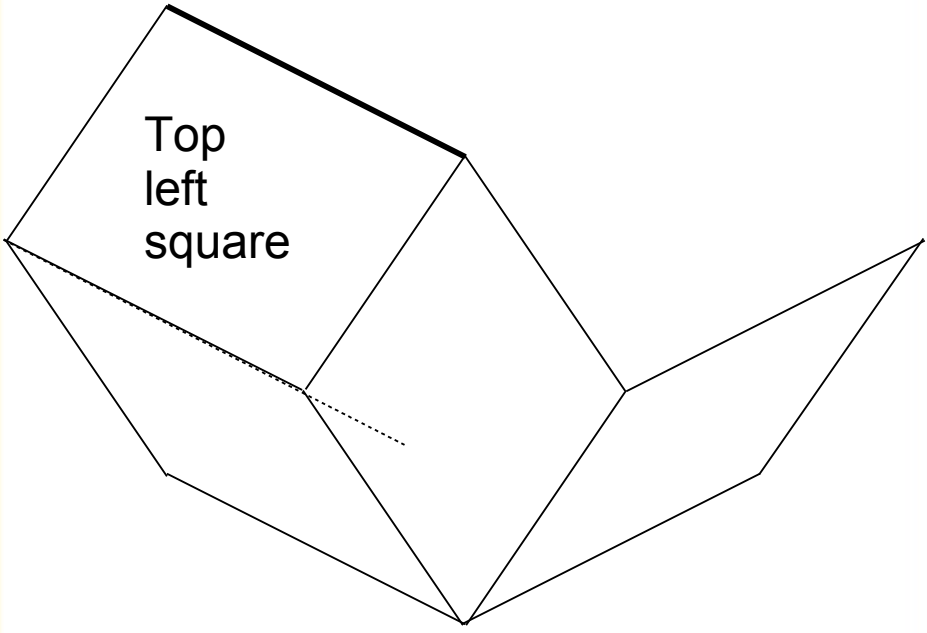
Bottom
right
square



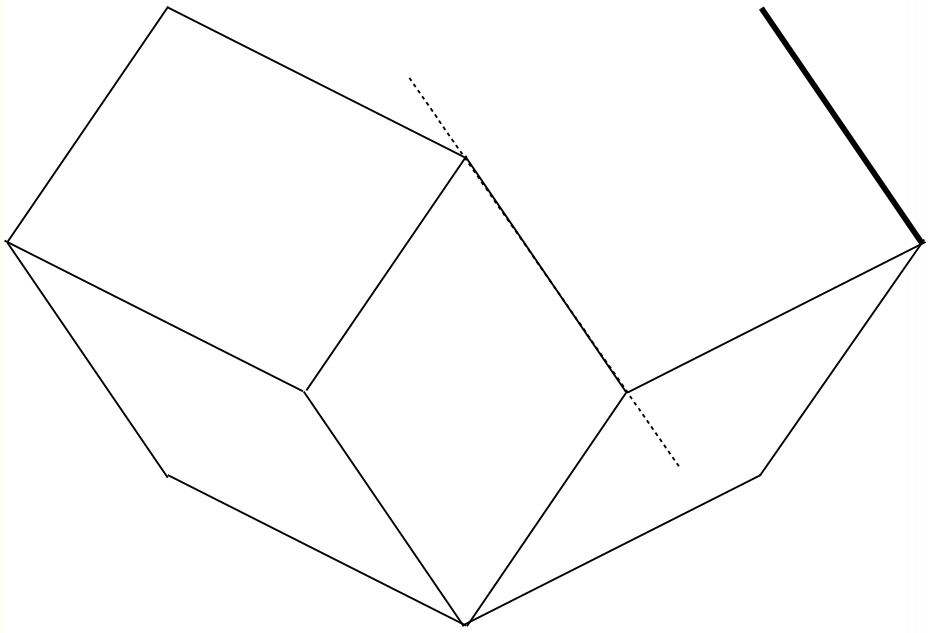
Tiger Farm Press



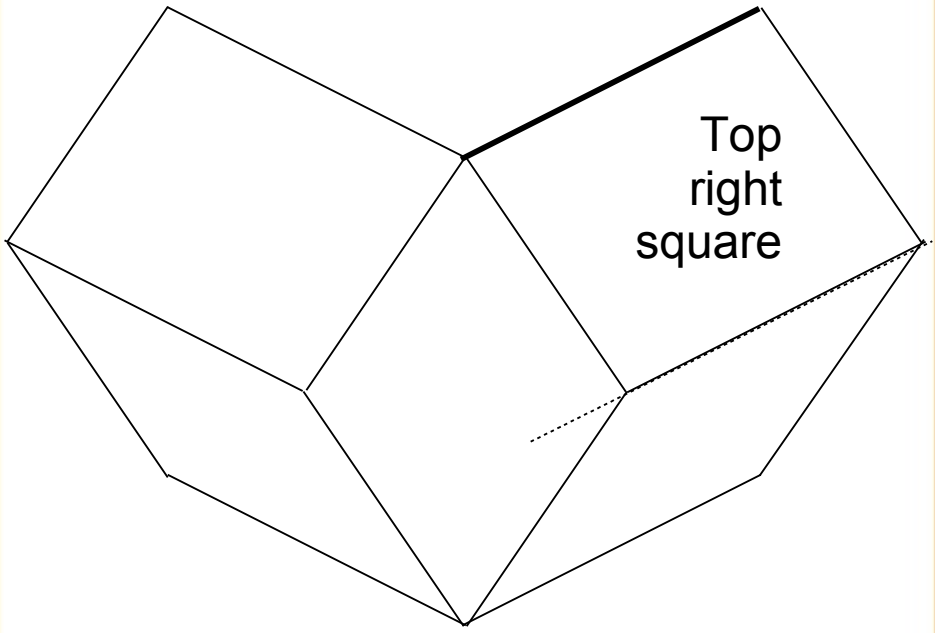
Tiger Farm Press



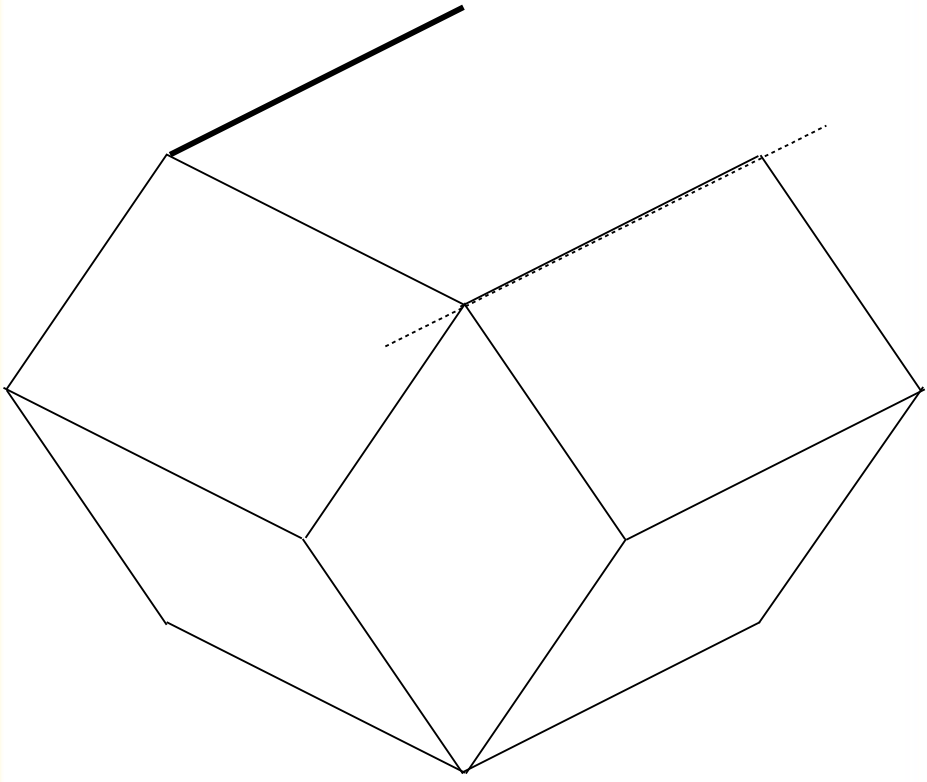
Tiger Farm Press



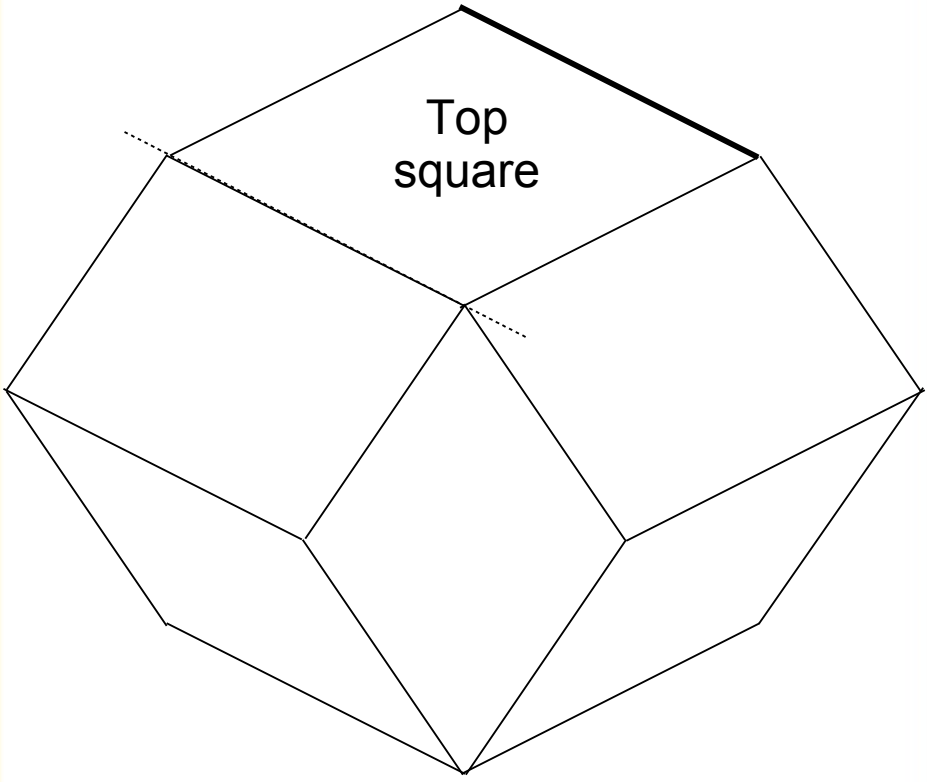
Tiger Farm Press



Tiger Farm Press

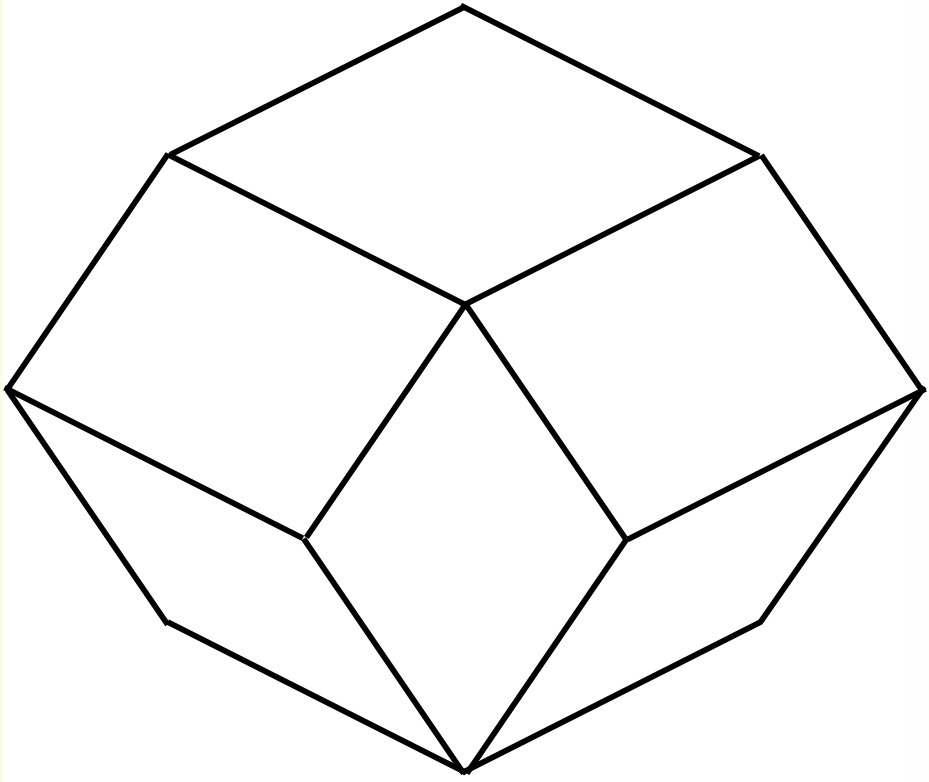


Tiger Farm Press



Tiger Farm Press

4D Cube, a Hypercube

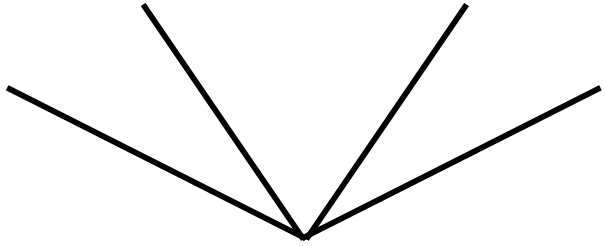


Tiger Farm Press

(2)
Drawing a 4D Cube One Square at a Time

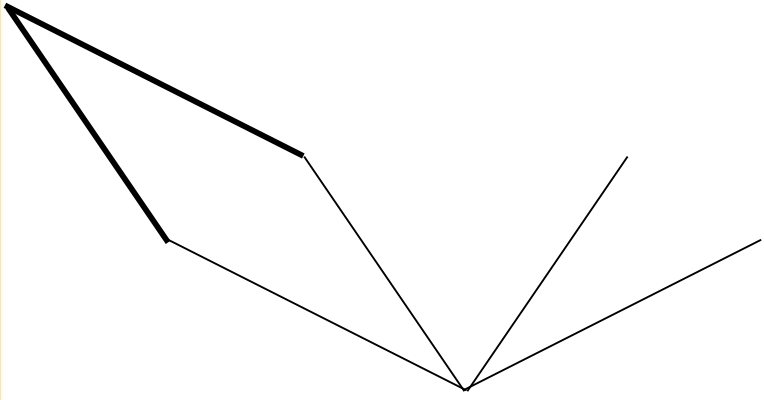
Start with the base lines.

Draw the cube's square sides,
using the base lines,
as guide lines.



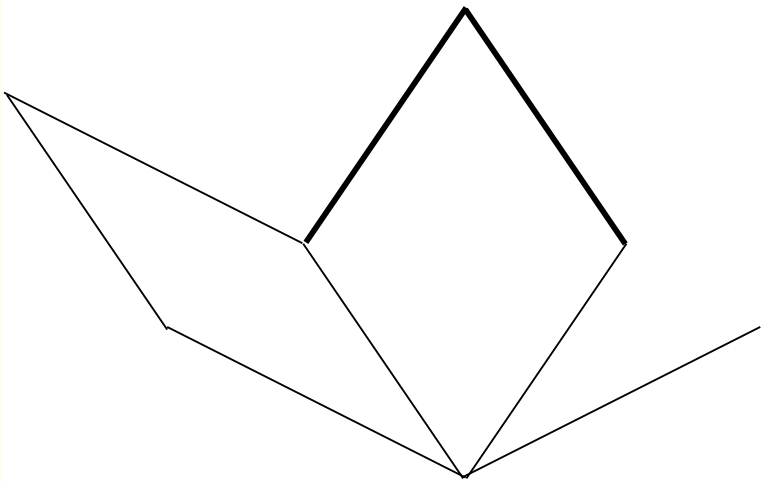
Tiger Farm Press

Side 1



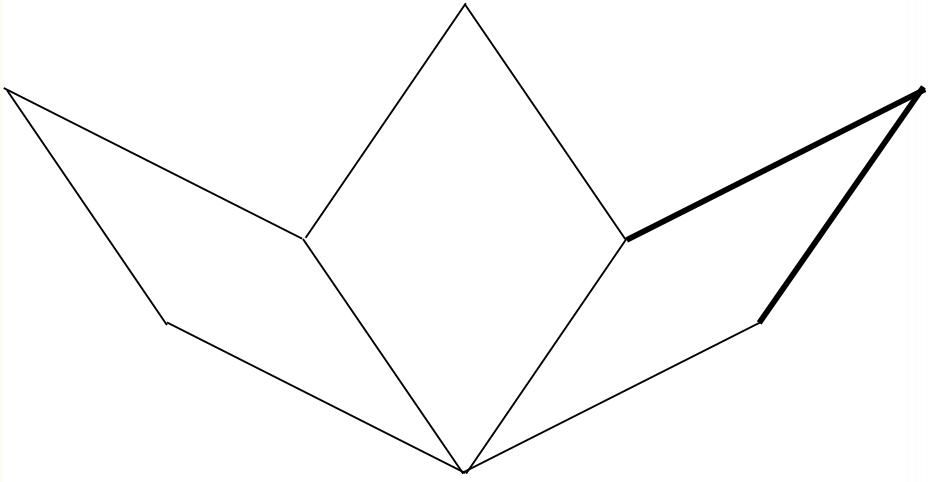
Tiger Farm Press

Side 2



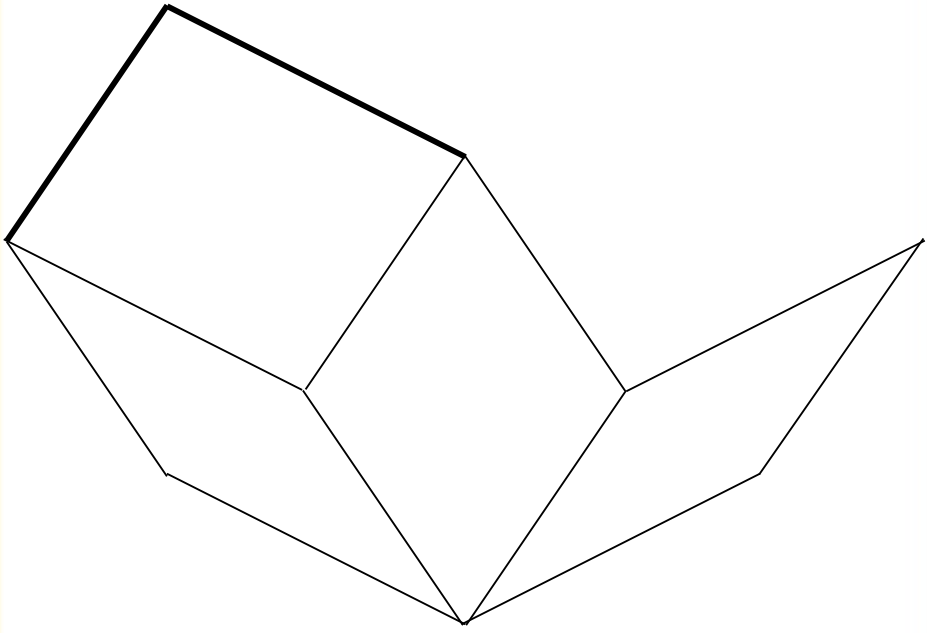
Tiger Farm Press

Side 3



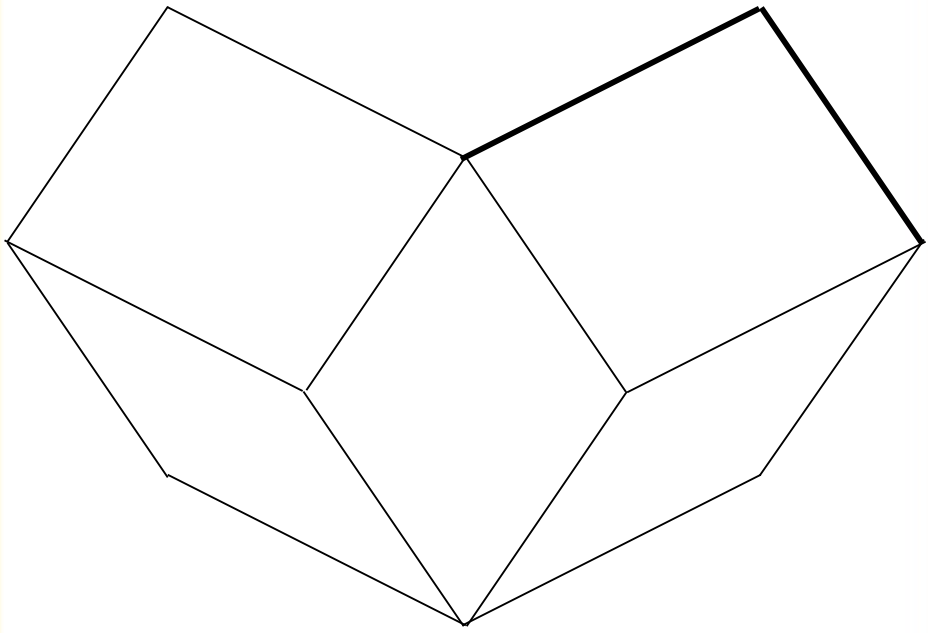
Tiger Farm Press

Side 4



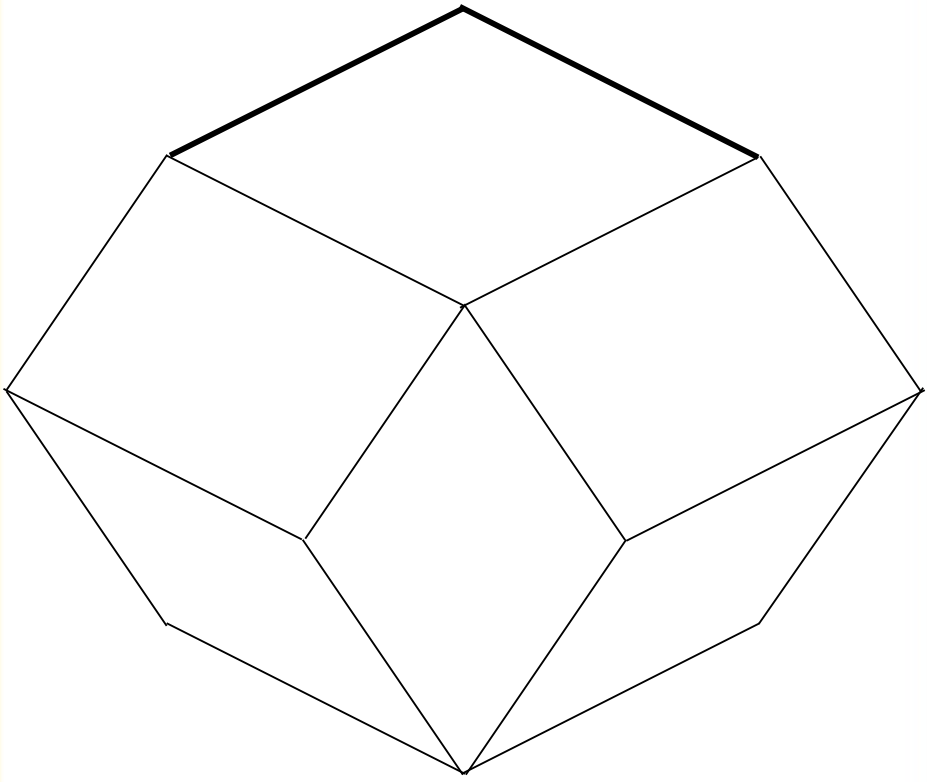
Tiger Farm Press

Side 5



Tiger Farm Press

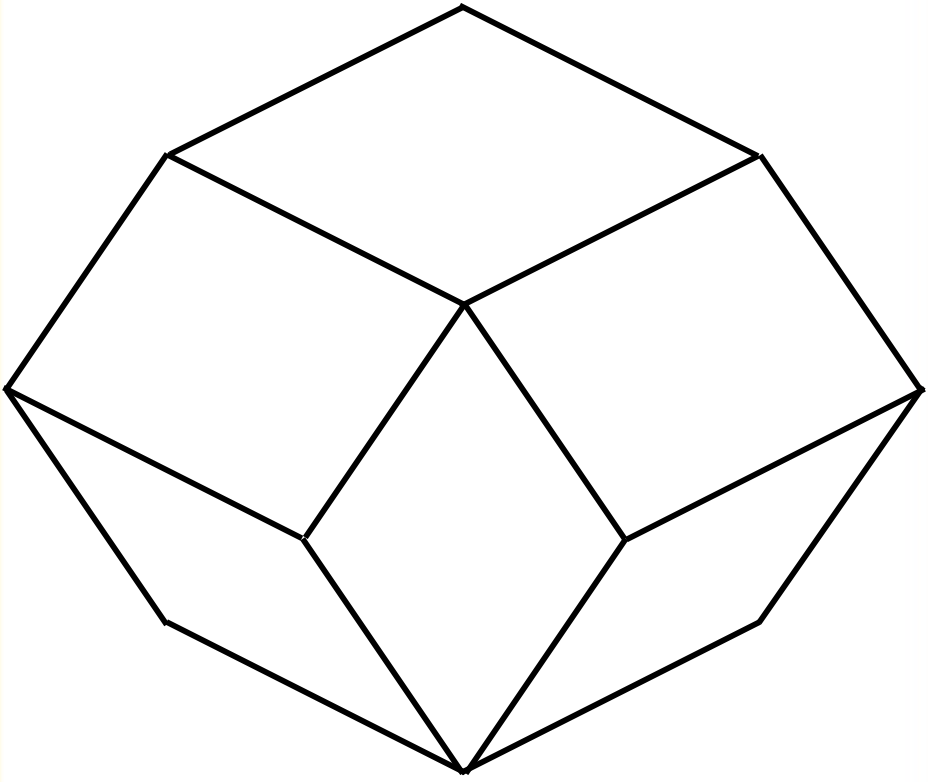
Side 6



Tiger Farm Press

4D Cube, an Einstein Cube

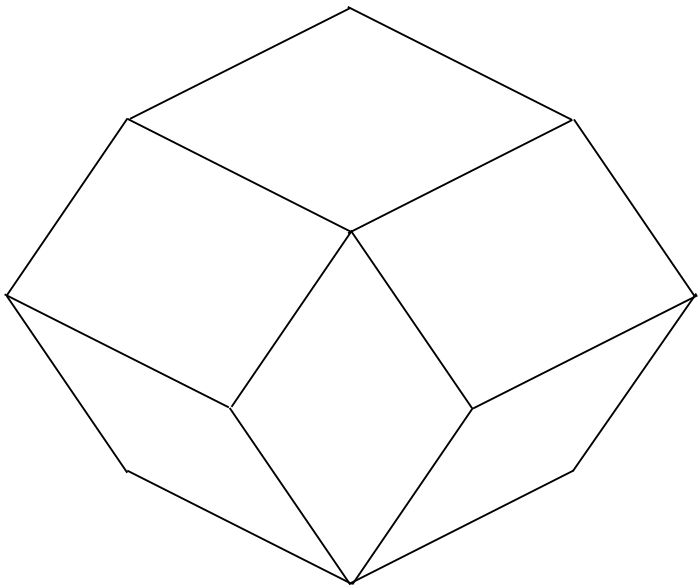
“As simple as possible, but no simpler,” Einstein's axiom.



Tiger Farm Press

What name do you like best?

1. Hypercube
2. Tesseract
3. 4D Cube
4. Einstein cube
5. Einstein spacetime hypercube

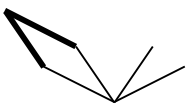


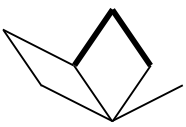
Tiger Farm Press

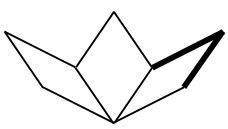
Now, use a doodle program on your phone, or find a pen and paper, and be the first person at your company or school to draw a hypercube.

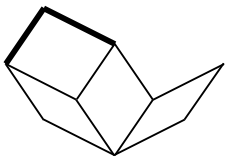
1. Start with the 4 base line segments:

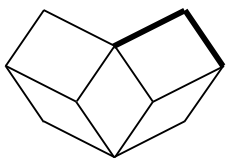
2. Draw each square side:

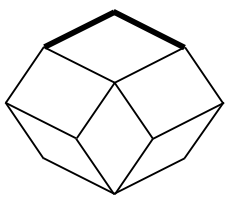
Side:1 

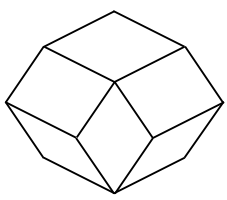
Side:2 

Side:3 

Side:4 

Side:5 

Side:6 

Boom!
a hypercube: 

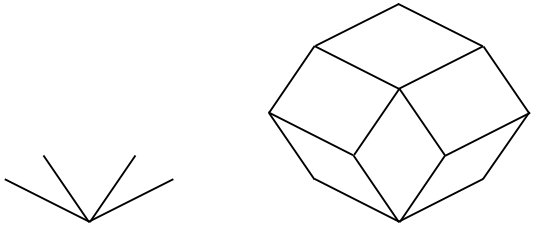
Tiger Farm Press

Easy eh?

After all, this isn't rocket science.
It's only Einstein science.

As Einstein said, “Everything should be made as simple as possible, but no simpler.”

Only 2 instructions:
1. Draw the base lines.
2. Draw the square sides.



It can't get any simpler than that.

Tiger Farm Press

Seeing in 4D

Riley said, “It would be interesting to see in 4 dimensions.”

“You do all the time,” I said.

With a ruler, you can measure the 3 dimensions of an object such as a smartphone. You see 3D objects.

With a smartphone, you can record the 4^{th} dimension of an object by taking a video of it. You watch 3D objects for a duration of time (1D).

A video is a recording of 4 dimensions: 3 dimensional objects in 1 dimension of time—spacetime.

“Okay, that make sense,” he agreed.

Tiger Farm Press

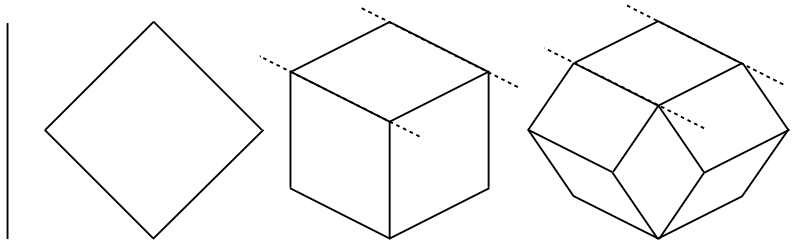
Beyond 4D Spacetime

Can You?

If you can't draw a 4 dimensional cube, don't bother continuing, you hit your geometric limit—no need to read on.

If you can, then with one more skill, you can draw a String Theory 10 dimensional hypercube.

1D 2D 3D 4D … 10D



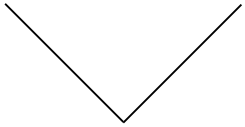
Tiger Farm Press

IQ Challenge (Branden's idea)

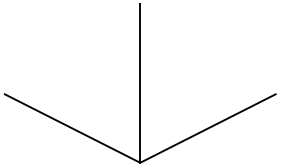
The Pattern to higher dimensions can be seen in the base line sequence.

1D

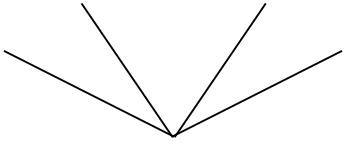
2D



3D



4D



5D Draw the 5D base lines before looking at the next page.

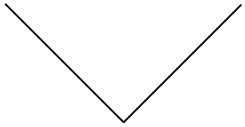
Tiger Farm Press

IQ Challenge (Branden's idea)

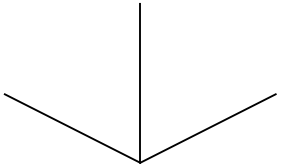
The Pattern to higher dimensions can be seen in the base line sequence.

1D

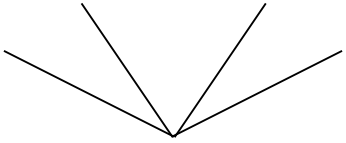
2D



3D



4D



5D Draw the 5D base lines before looking at the next page.

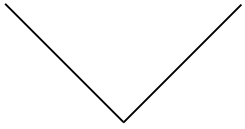
Tiger Farm Press

Key

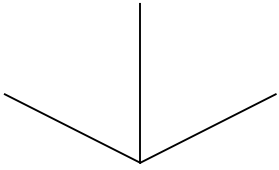
To draw higher dimensional cubes, figure out the pattern.

1D

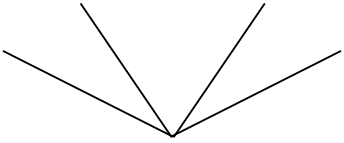
2D



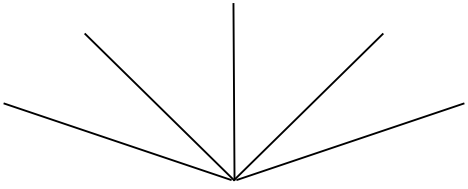
3D



4D



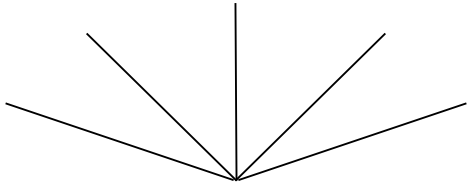
5D



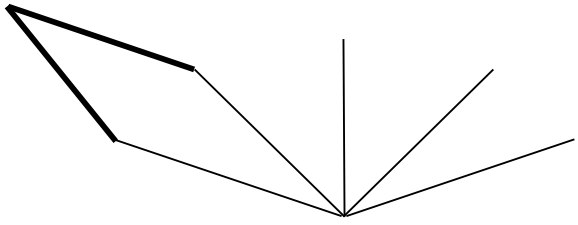
Tiger Farm Press

IQ Challenge

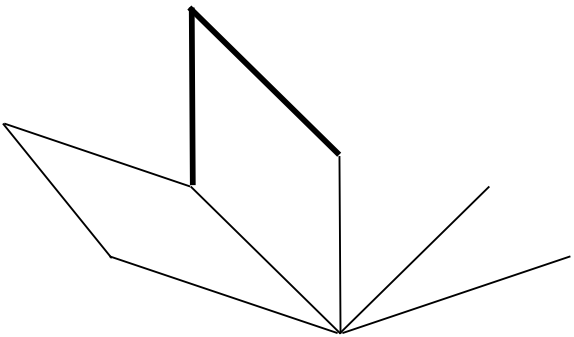
Draw a 5D cube on top of the base lines. All you gotta do is draw squares using the base lines as guide lines.



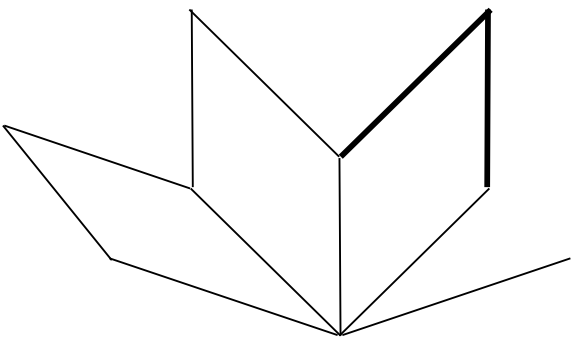
Tiger Farm Press



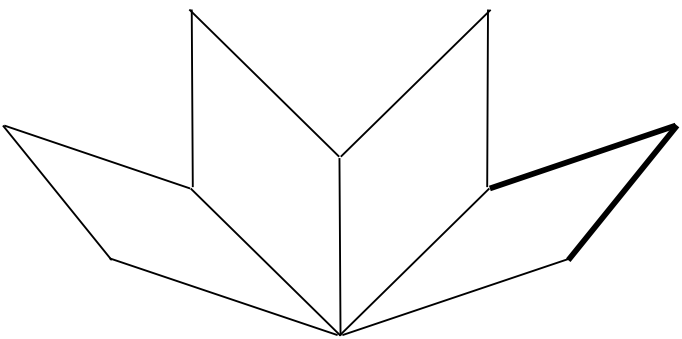
Tiger Farm Press



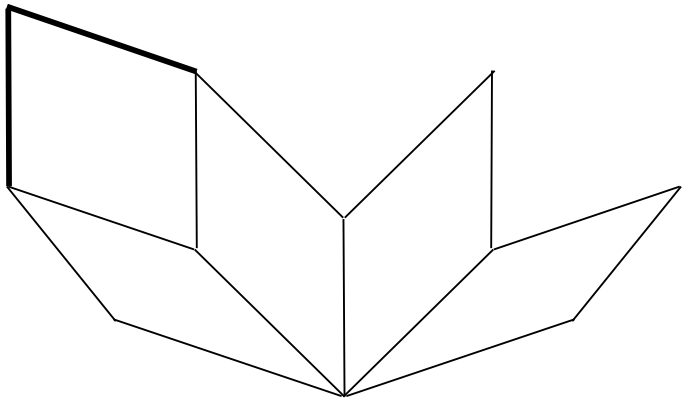
Tiger Farm Press



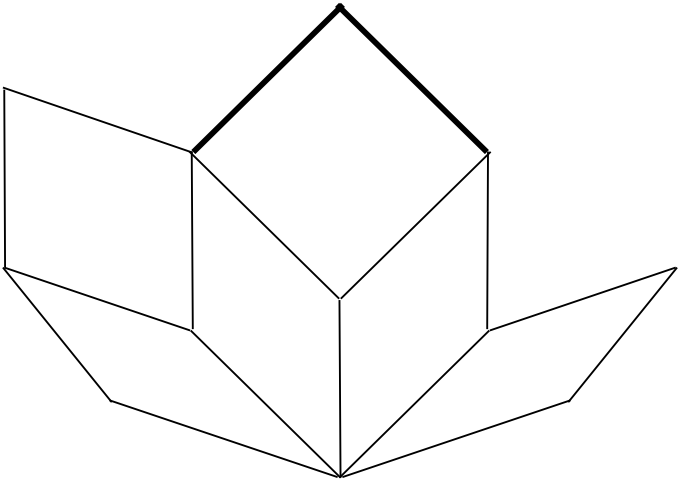
Tiger Farm Press



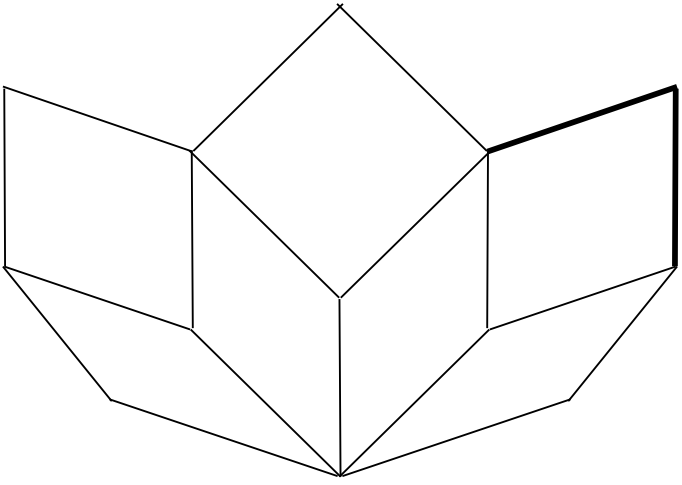
Tiger Farm Press



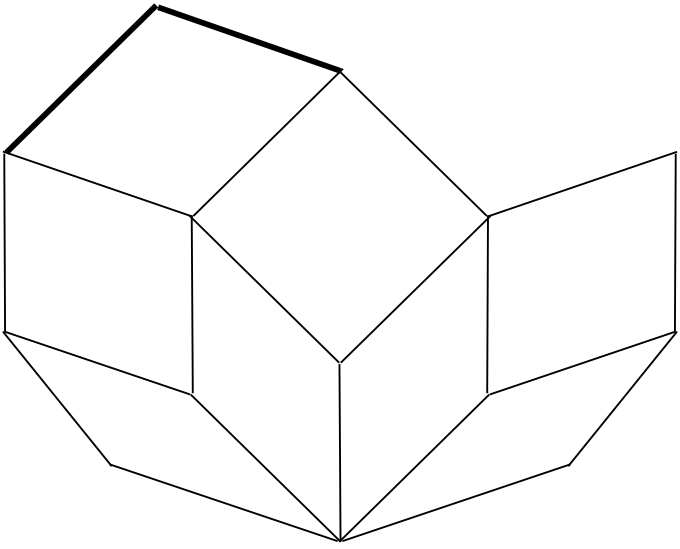
Tiger Farm Press



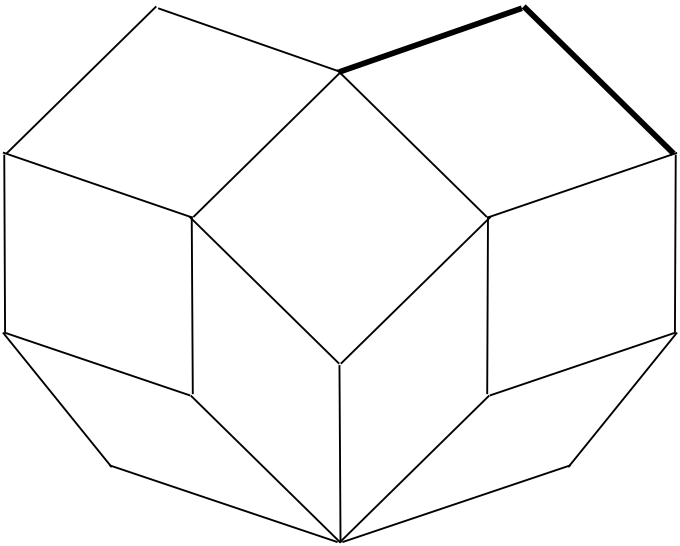
Tiger Farm Press



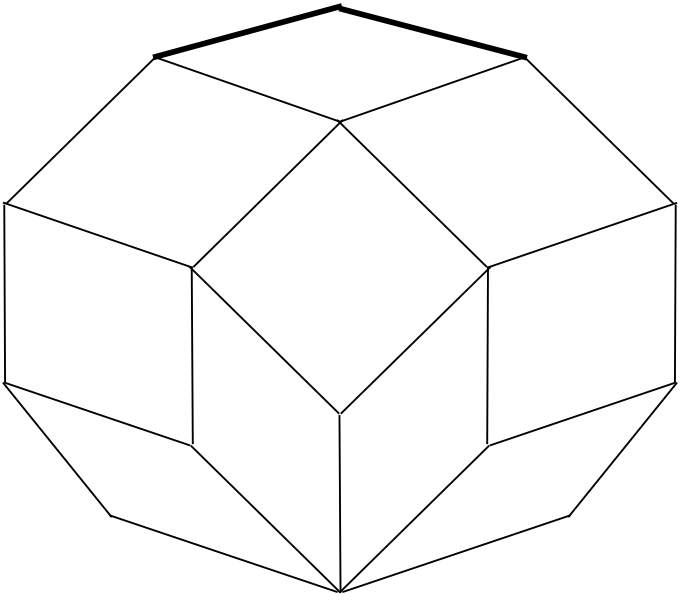
Tiger Farm Press



Tiger Farm Press

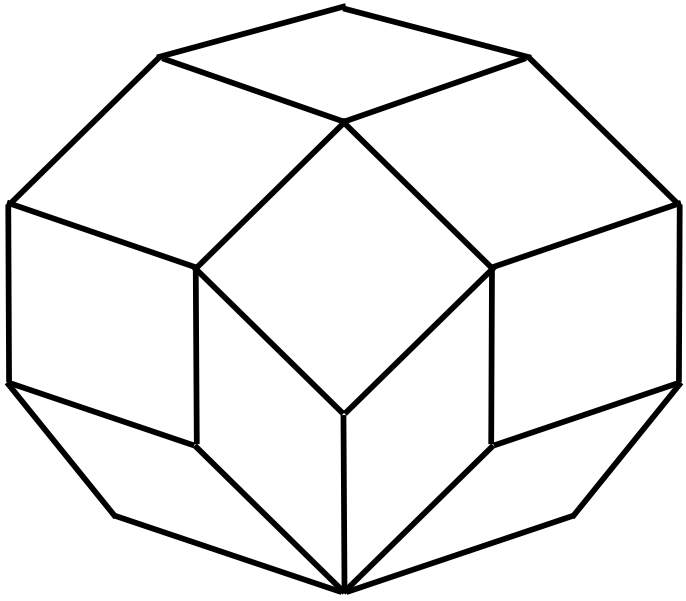


Tiger Farm Press



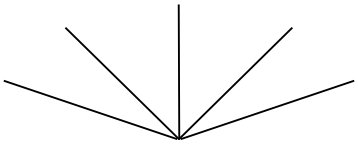
Tiger Farm Press

5D Cube



Tiger Farm Press

5D base lines:



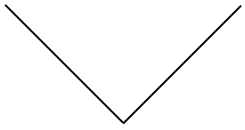
The middle base line:

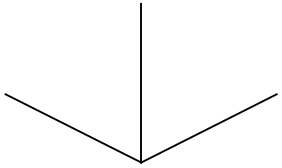
Riley, you get it?

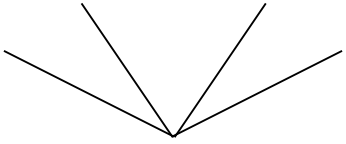
Tiger Farm Press

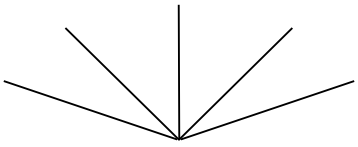
Are you feeling brilliant?

Draw the 6 dimensional base lines before looking at the next page.

2D 

3D 

4D 

5D 

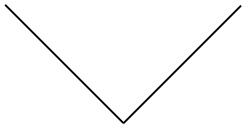
6D ?

Tiger Farm Press

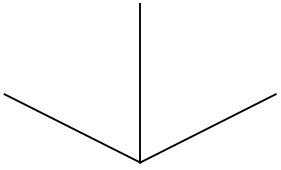
Next, draw a 6D cube.

Step: 1, draw the base lines. Done.
Step: 2, draw the square sides. Can do.

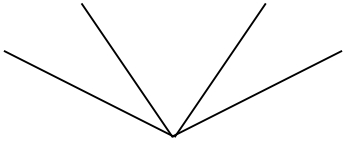
2D



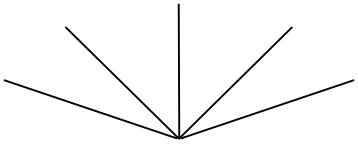
3D



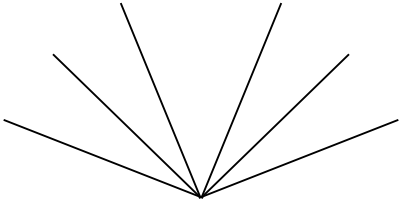
4D



5D



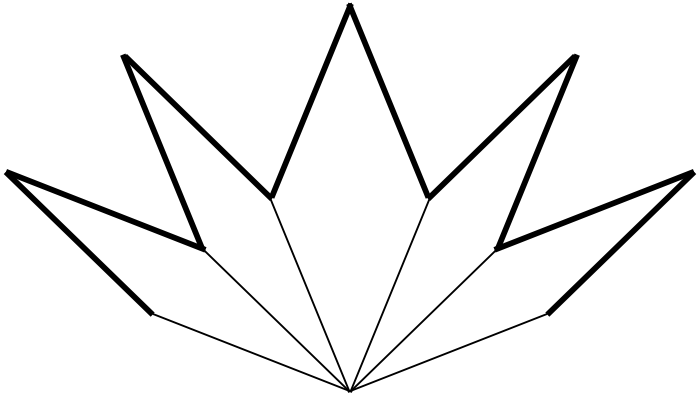
6D



Tiger Farm Press

Step: 2

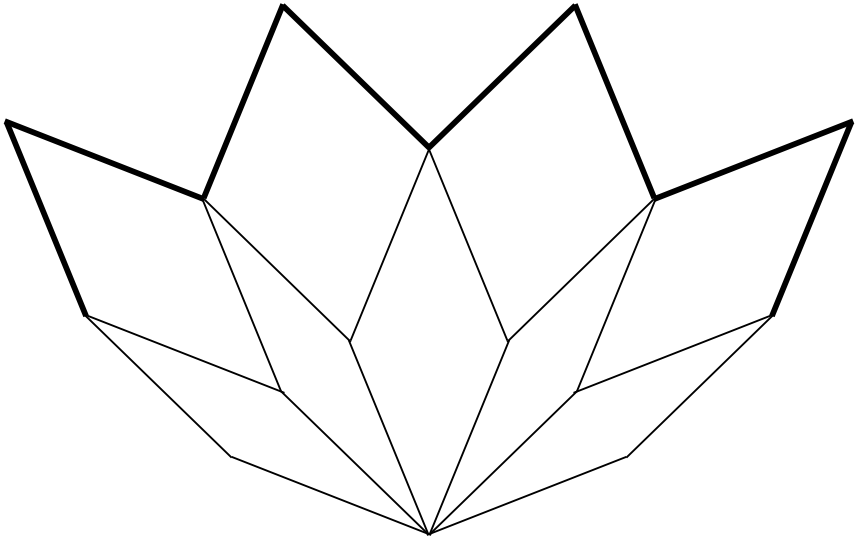
The 1st row of squares of the 6D cube.



Tiger Farm Press

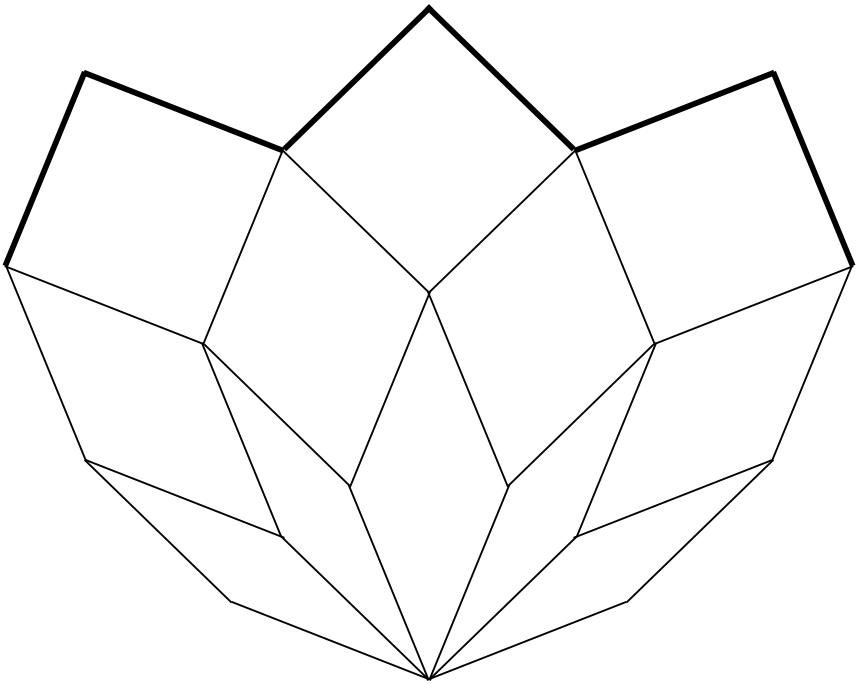
Step: 3

“It's like drawing a crystal flower,” Riley commented.



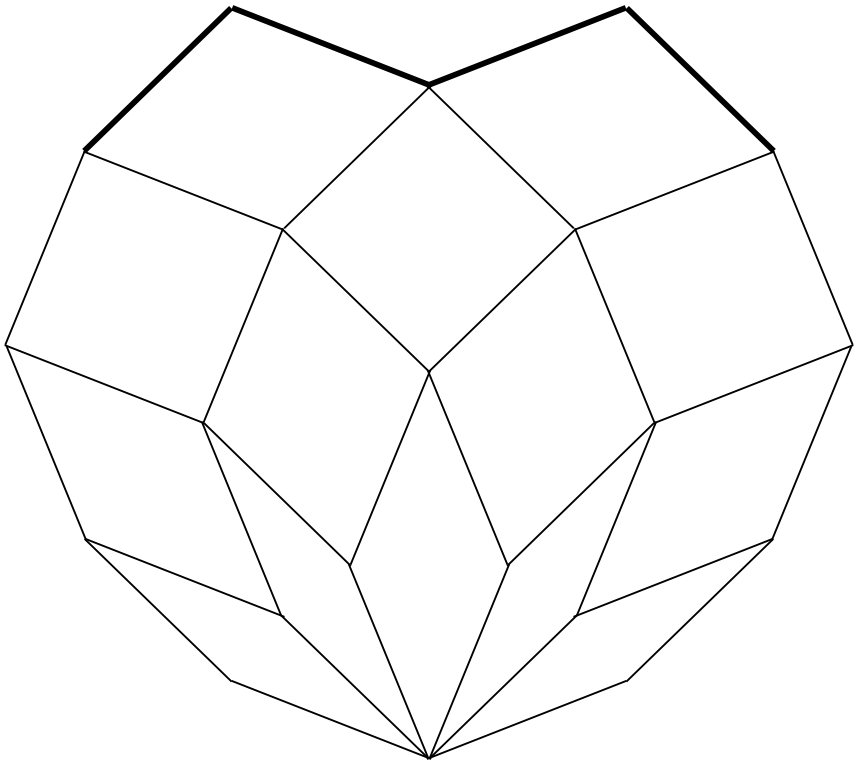
Tiger Farm Press

Step: 4



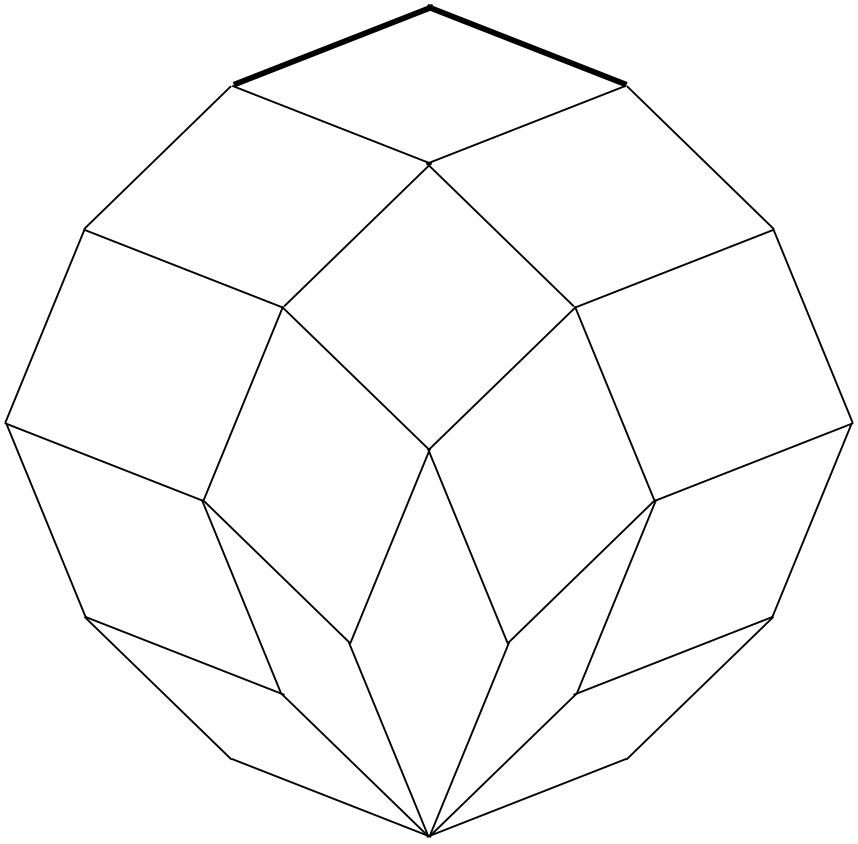
Tiger Farm Press

Step: 5



Tiger Farm Press

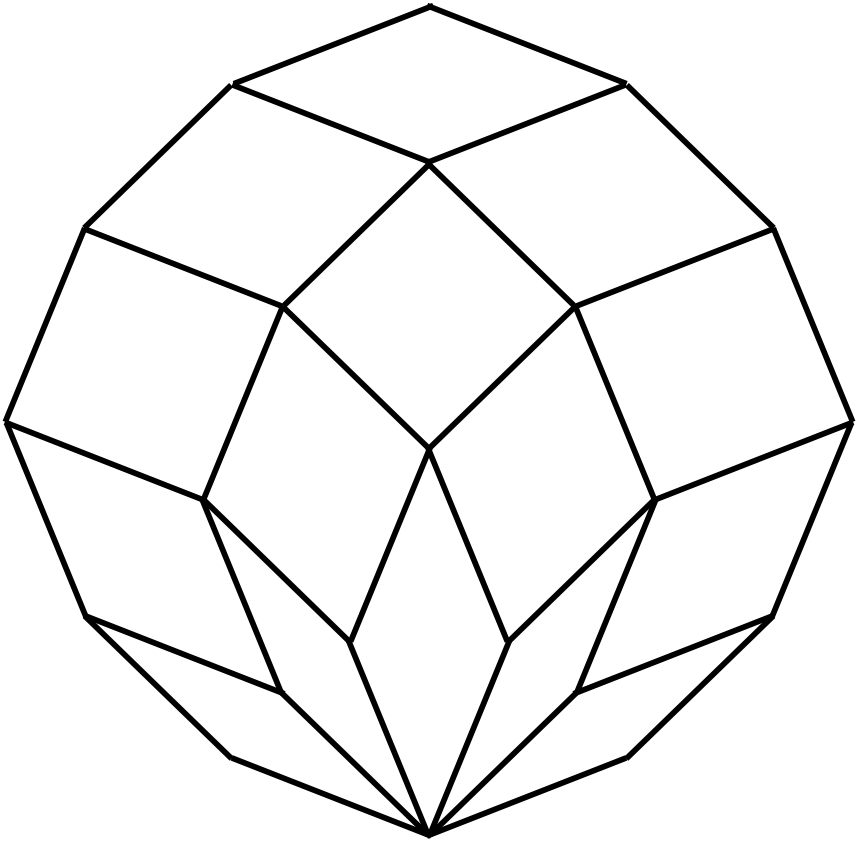
Step: 6



Tiger Farm Press

6D Cube

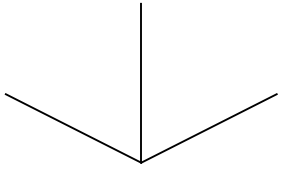
A cool design for a Swarovski crystal.



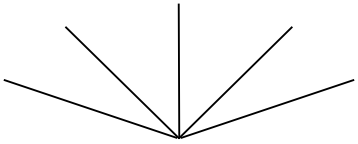
Tiger Farm Press

The odd dimensional base line pattern.

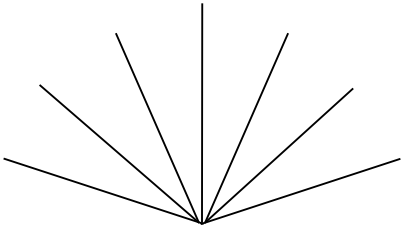
3D



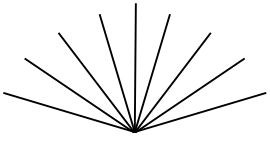
5D



7D



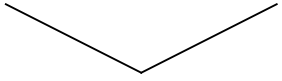
9D



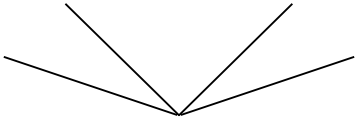
Tiger Farm Press

The even dimensional base line pattern. The difference between odd and even, is one: | .

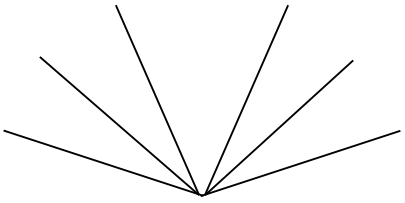
2D



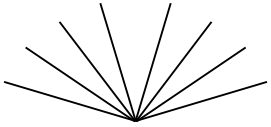
4D



6D

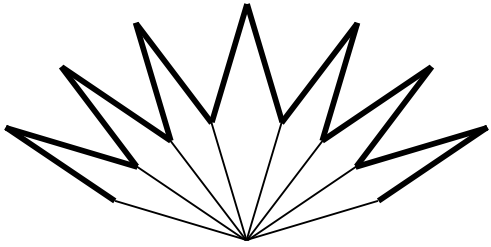


8D

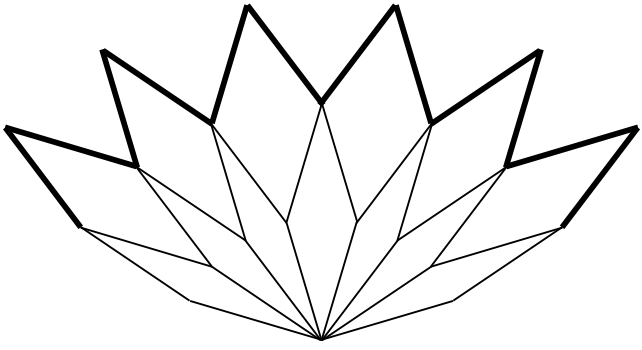


Tiger Farm Press

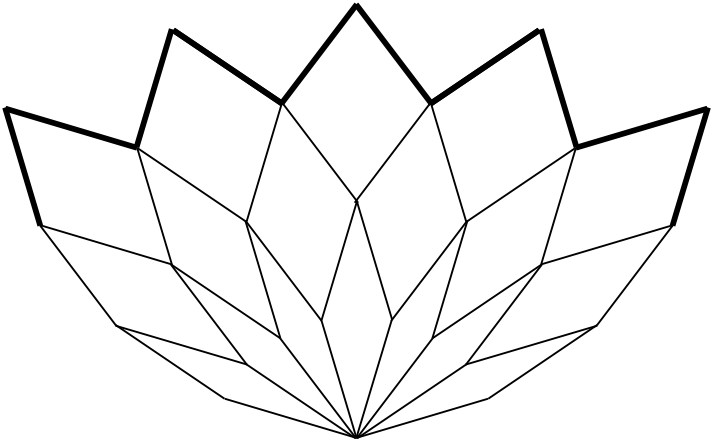
1st row of square sides for an 8D cube:



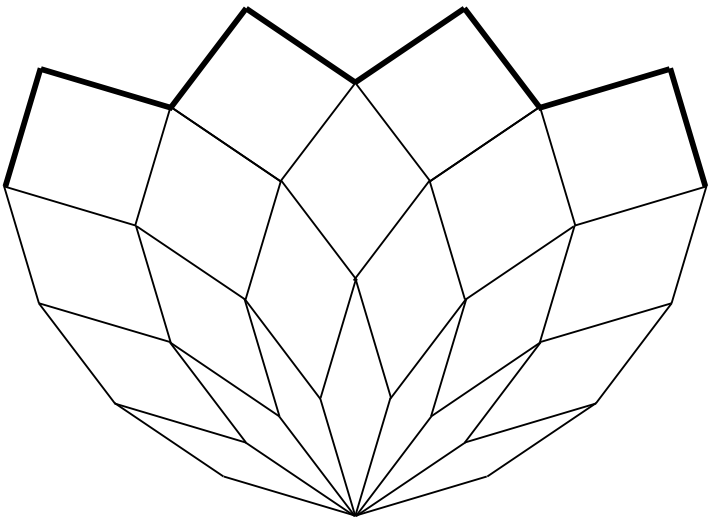
Tiger Farm Press



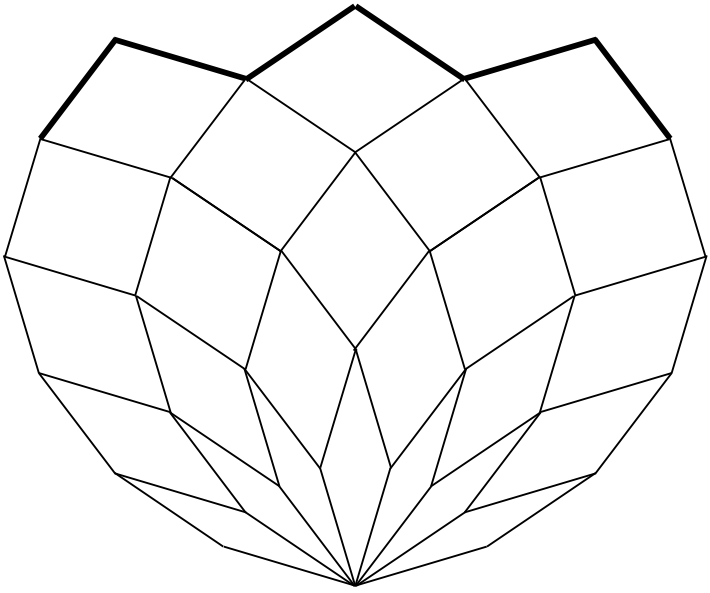
Tiger Farm Press



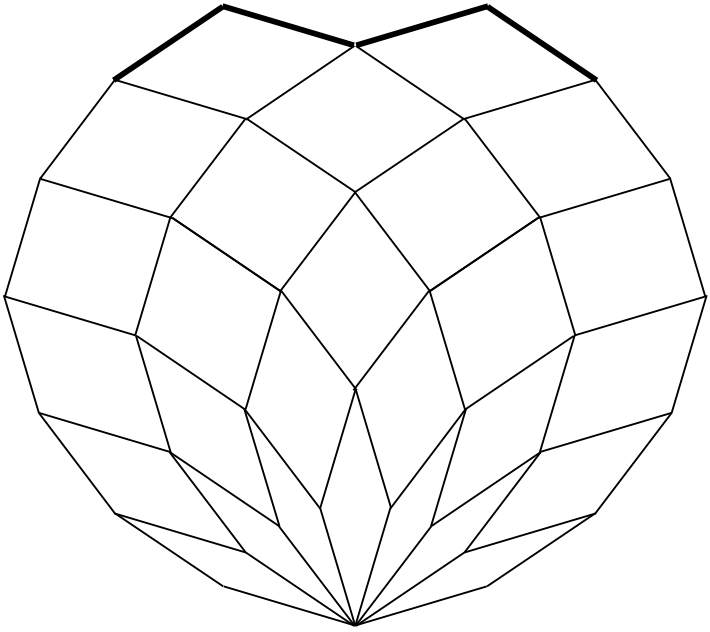
Tiger Farm Press



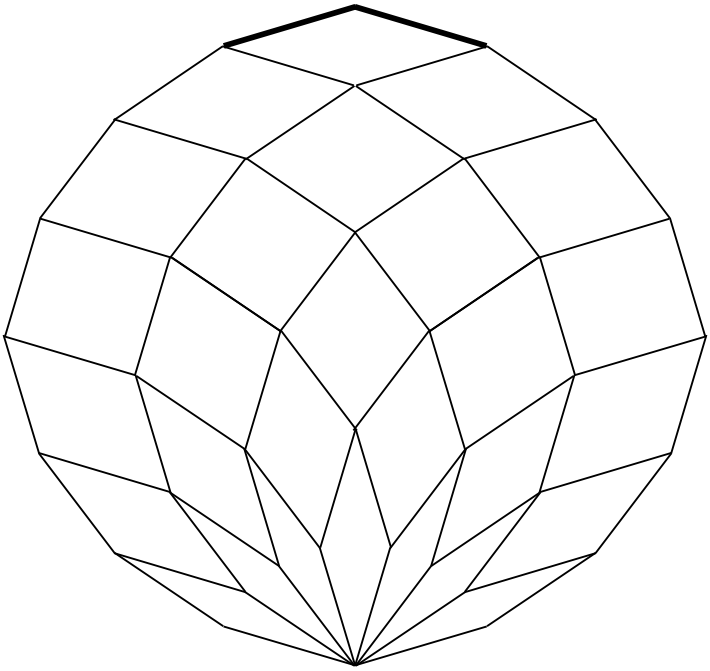
Tiger Farm Press



Tiger Farm Press



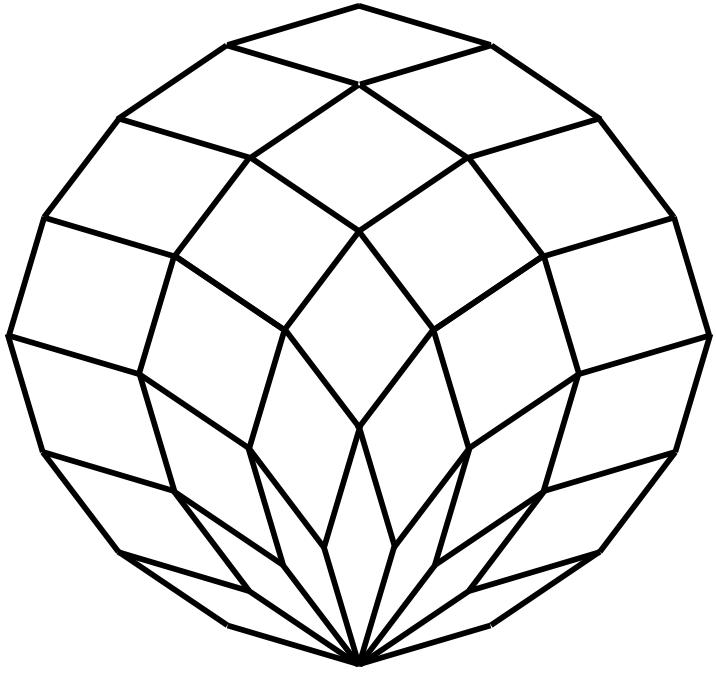
Tiger Farm Press



Tiger Farm Press

8D Cube

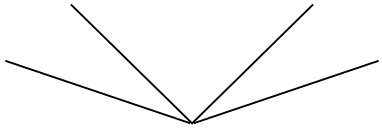
Only 2 more dimensions to String Theory's 10 dimensions.



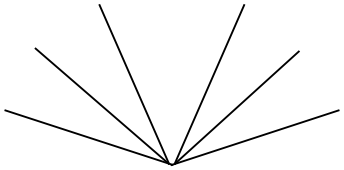
Tiger Farm Press

Here is the sequence of dimensional base line segments up to String Theory's 10D spacetime world.

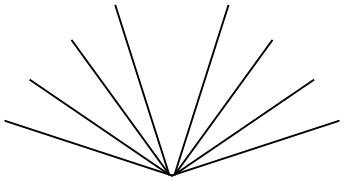
4D



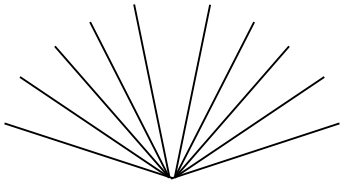
6D



8D



10D



You can do it, draw a String Theory 10 dimensional cube.

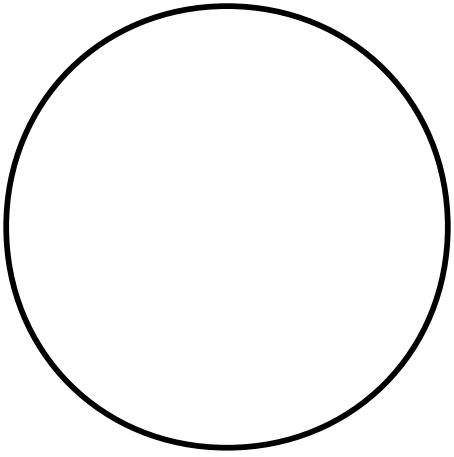
Tiger Farm Press

Advanced IQ Question

Now that you've graduated to the dimensions of String Theory, can you describe the diagram below?

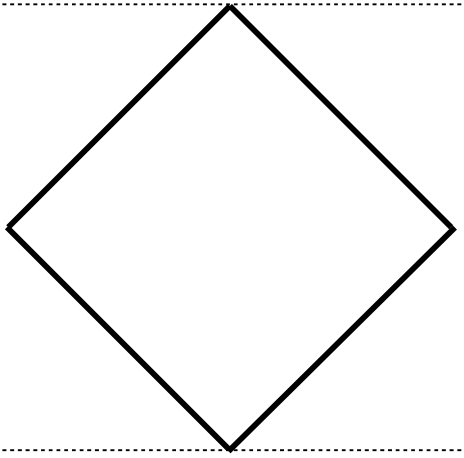
Actually, you would have to be a fucking genius. Then again, maybe you are. After all, you do know how to draw an Einstein cube.

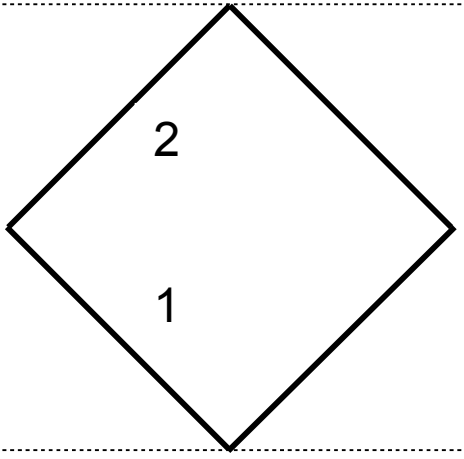
In the following pages is the geometric inductive proof.



Tiger Farm Press

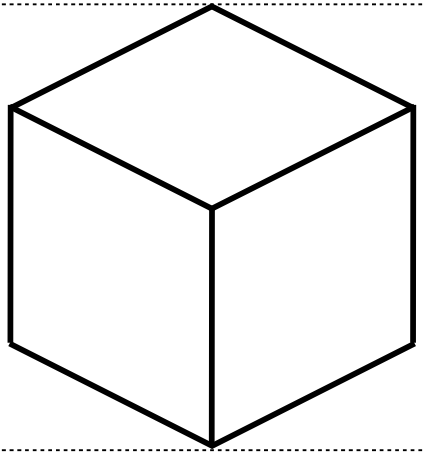
Here is a square, which is a 2D cube.

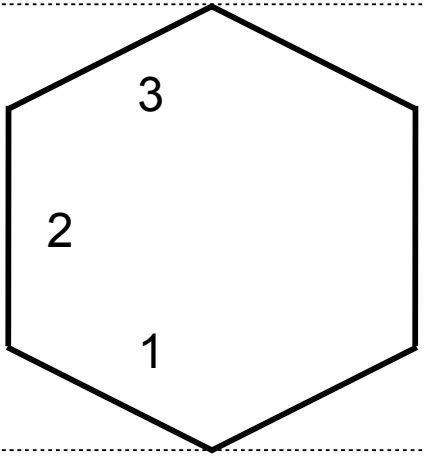




Tiger Farm Press

A 3D cube and the outline of the 3D cube.

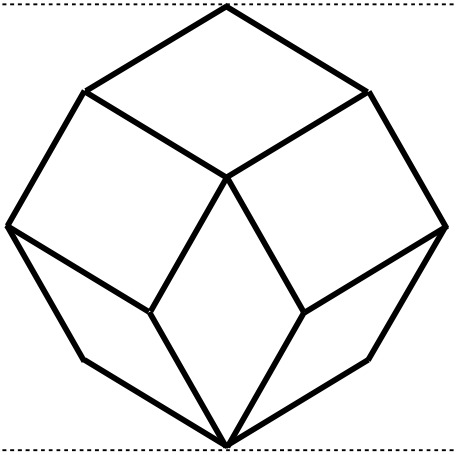


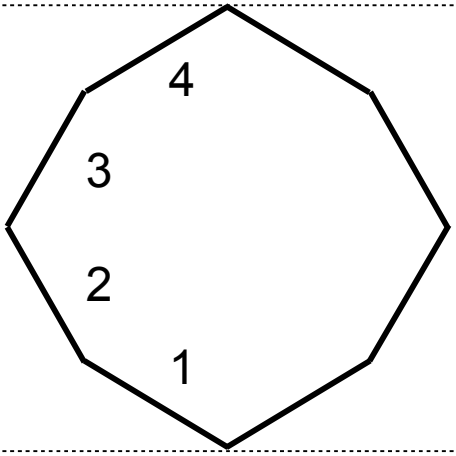


Tiger Farm Press

A 4D cube and its outline.

Note, all the diagrams in this sequence have the same height.

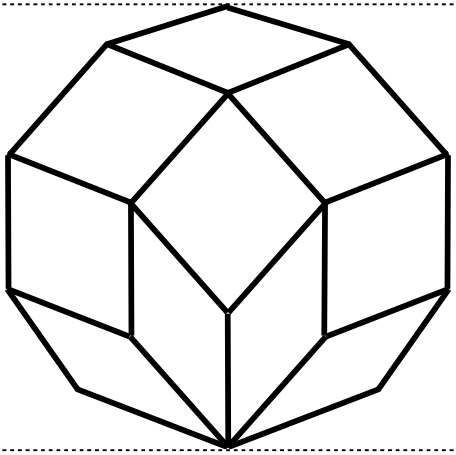


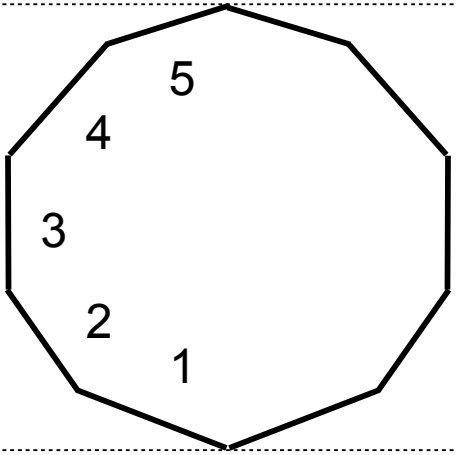


Tiger Farm Press

A 5D cube and its outline.

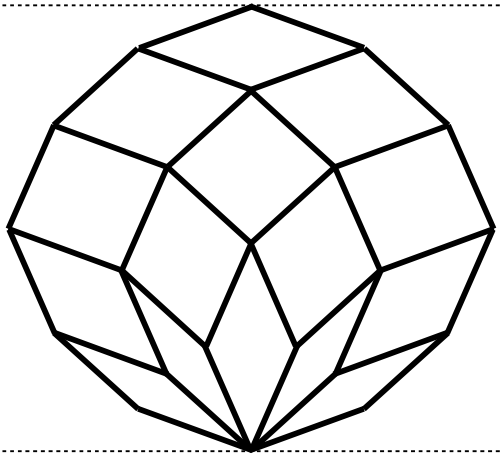
Do you see the pattern?

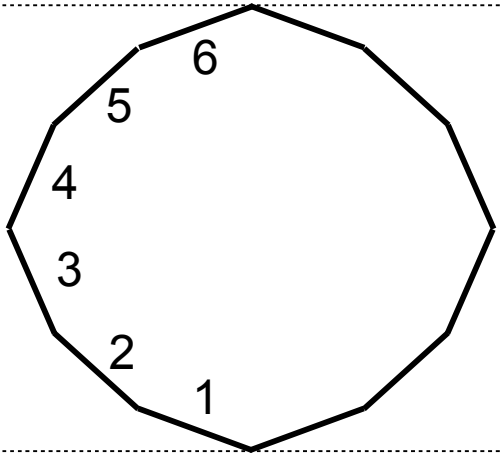




Tiger Farm Press

A 6D cube and its outline.

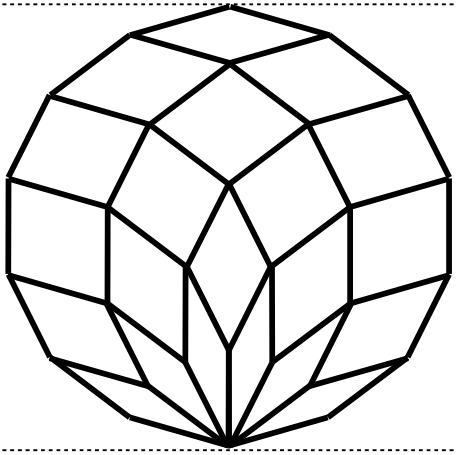


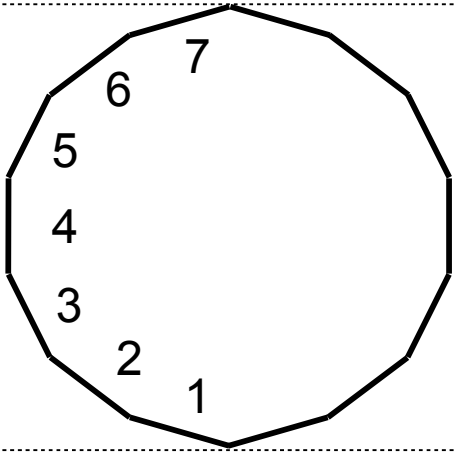


Tiger Farm Press

A 7D cube and its outline.

As dimensions increase,
the line segments get shorter.

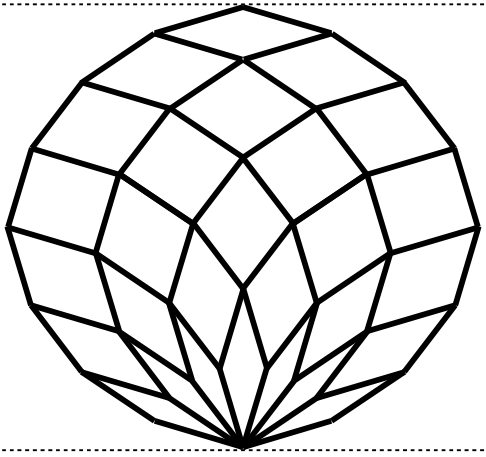


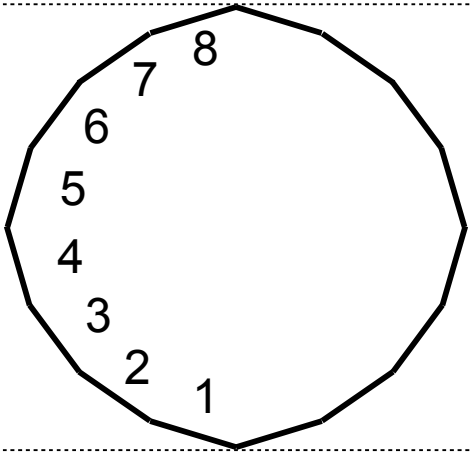


Tiger Farm Press

A 8D cube and its outline.

The number of connection points and lines within the outline, are increasing.

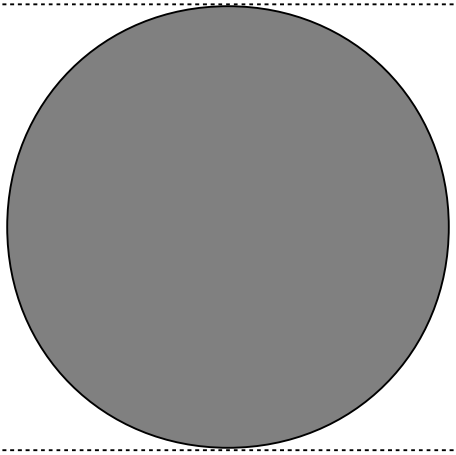


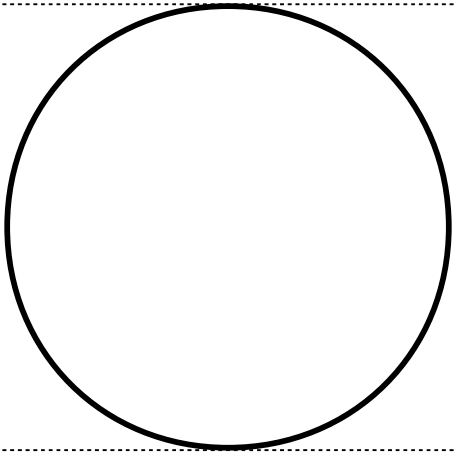


Tiger Farm Press

A many-D cube and its outline.

When the dimensional line length is less than a pixel, the cube looks like a sphere and its outline, a circle.





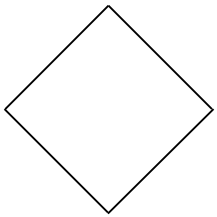
Tiger Farm Press

In Summary

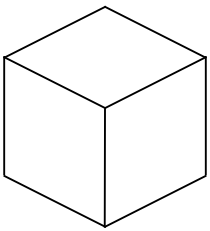
From my 3D world, I looked for a pattern to higher dimensions.

1D

2D

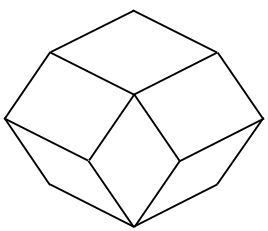


3D



Einstein added another dimension for me to use, time.

4D



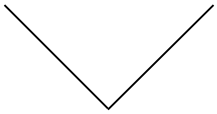
3D space + 1D time = 4D spacetime

Tiger Farm Press

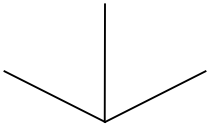
The pattern can be found in the dimensional base lines.

1D

2D

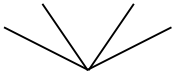


3D



It takes imagination to jump from 3 to 4. From odd to even, split the 1 (|) into 2 (∨). (Riley's idea)

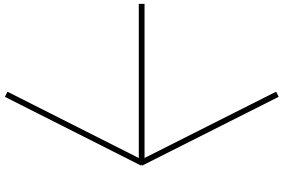
4D



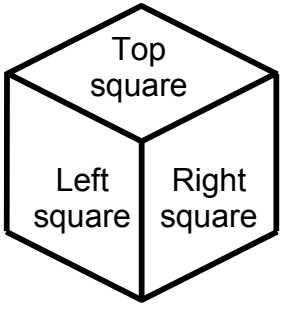
Tiger Farm Press

When drawing cubes of higher dimensions, the properties of a cube should be maintained.

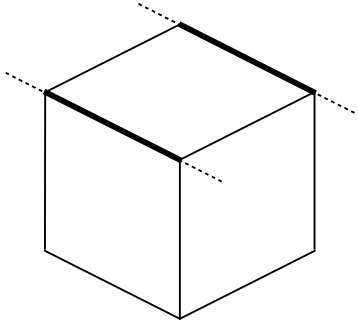
+ The number of base dimensional lines, equals the number of dimensions.



+ All sides of a cube are square.



+ Each of a cube's square sides, have opposite and parallel sides.

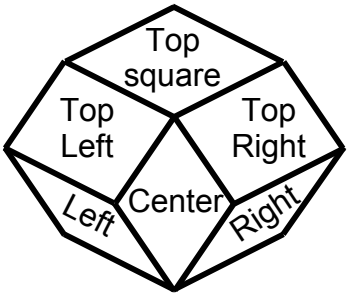


Tiger Farm Press

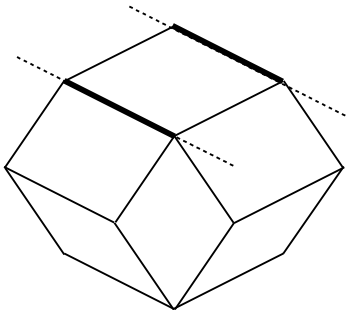
The 3D cube properties are retained in the 4D cube.

+ The number of base dimensional lines, equals the number of dimensions.

+ All sides of a cube are square.



+ Each of a cube's square sides, have opposite and parallel sides.

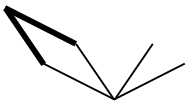


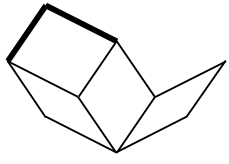
Tiger Farm Press

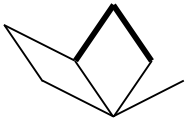
To Draw Hypercubes:

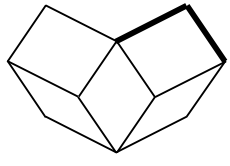
1. Draw the dimensional base lines.

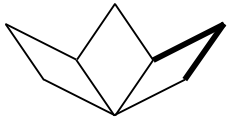
2. Draw the square sides on top.

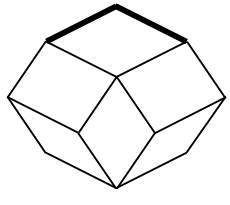


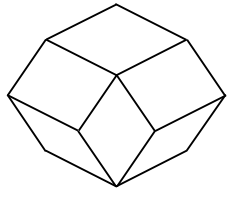










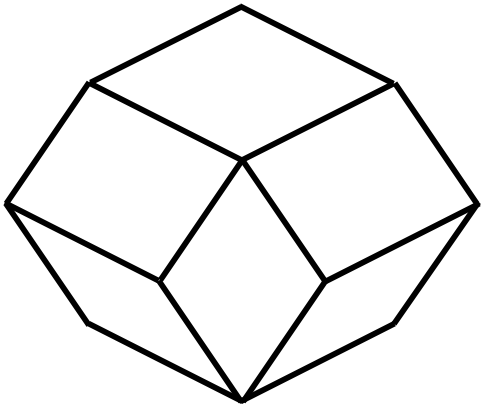


Tiger Farm Press

Einstein's Hypercube

It has been 100 years since Einstein presented the world with his vision of 4 dimensional spacetime.

And now you know how to draw an Einstein hypercube.



It cannot be simpler.
It's as simple as can be,
And still be a hypercube.
It's an Einstein hypercube.

Tiger Farm Press

However, the Earth is Not Flat.

Thinking that 3D space is all there is, is like thinking the earth is flat. We know better than that.

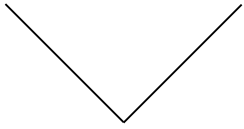
Thinking that 4D spacetime is all there is, is like thinking the planets and the sun rotate around the earth. We know better than that, too.

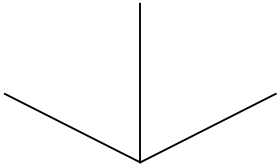
Knowing better, I needed a pattern to higher dimensions.

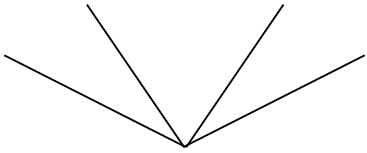
Tiger Farm Press

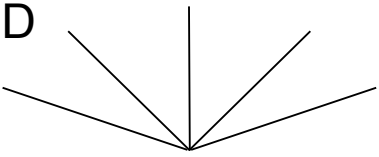
And so, I found the sequential pattern of dimensional base line segments to the higher dimensions.

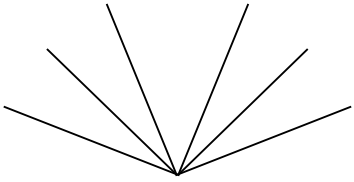
1D

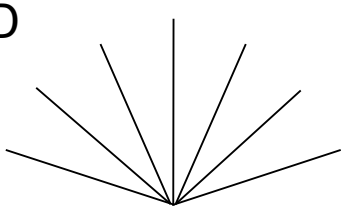
2D 

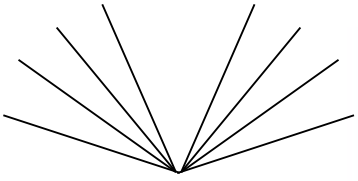
3D 

4D 

5D 

6D 

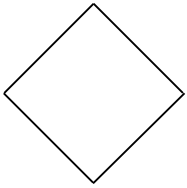
7D 

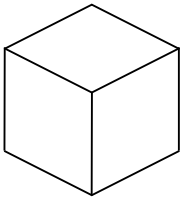
8D 

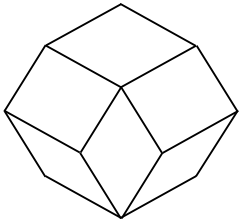
Tiger Farm Press

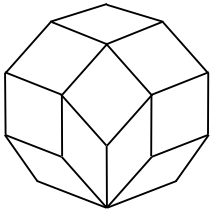
Given the base lines, it's simply a matter of drawing the square sides to draw cubes of higher dimensions.

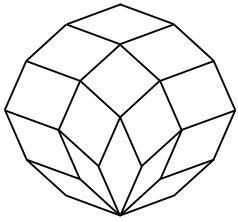
1D

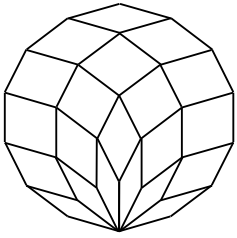
2D 

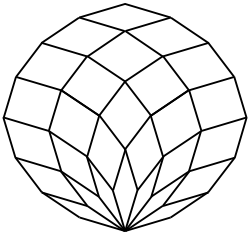
3D 

4D 

5D 

6D 

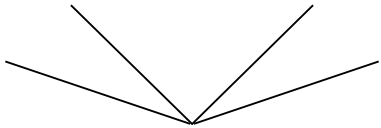
7D 

8D 

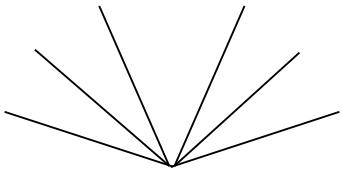
Tiger Farm Press

It was elementary to extend the sequence to String Theory's 10 dimensional spacetime world.

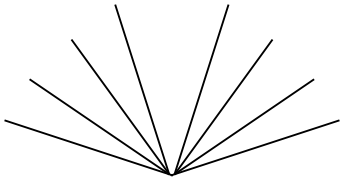
4D



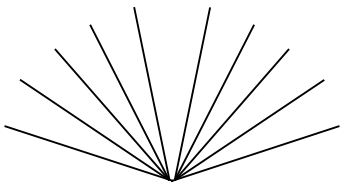
6D



8D



10D



Tiger Farm Press

These sequences, from squares to sphere and circle, summaries the sequential pattern from our world to the universe of many dimensions.

2D

3D

4D

5D

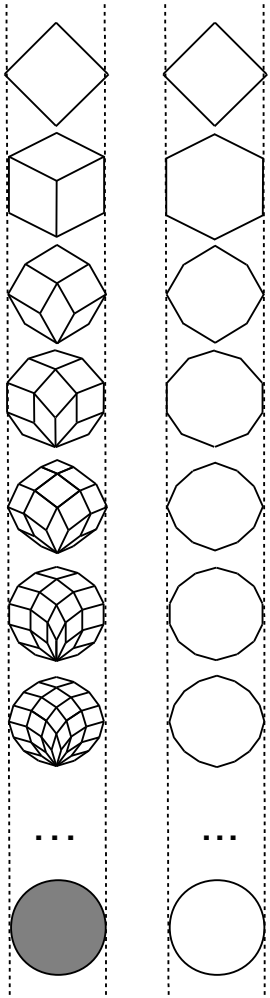
6D

7D

8D

…

Many-D



Tiger Farm Press

Our Many Dimensional Universe

Thinking the earth is the center of our solar system, is like thinking our 3D world is the center of our multiple dimensional universe.

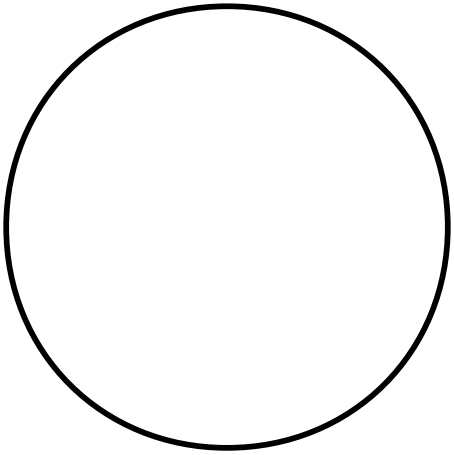
People were amazed, and shocked, when they realized our earth was not the center of our solar system.

People should be just as amazed to realize that our 3D space is not the center of our multi-dimensional universe.

Stick that in your hobbit pipe and smoke it.

Tiger Farm Press

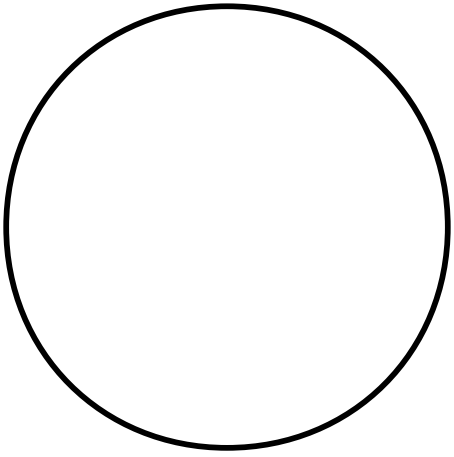
And so, what's this?



Tiger Farm Press

Did you say, *“A hypercube outline?”*

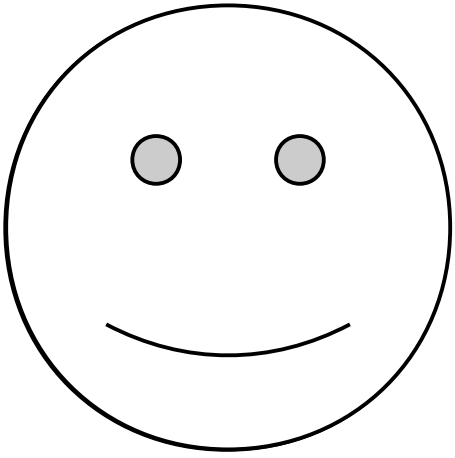
You got it, Sherlock.



Tiger Farm Press

And remember, our dimensions are not the center of our universe.

We just like to think they are :-)



Tiger Farm Press