UNIVERSIDAD NACIONAL DE CAJAMARCA

ESCUELA ACADÉMICO PROFESIONAL DE INGENIERÍA DE SISTEMAS

Asignatura

INVESTIGACION DE OPERACIONES EN INGENIERIA I

Presentado por:

PERALTA CARUAJULCA, Normalí Estalin
CARUAJULCA TIGLLA, Alex Eli
CHUNQUE CHUQUIRNA, David Jhonathan

Docente

MUÑOZ ABANTO NESTOR ELIAS

Semestre

2024-I

Julio 2024 - Cajamarca

Ejercicio 01

Maximizar $Z = 3X_1 + 5X_2 - MA_1$

$$X_1 \leq 4$$
$$2X_2 \leq 12$$
$$3X_1 \quad 2X_2 = 18$$

$$Z - 3X_1 + 5X_2 - MA_1 = 0$$

$$X_1 \qquad h_1 = 4$$
$$2X_2 \qquad h_2 = 12$$
$$3X_1 \quad 2X_2 \qquad A_1 = 18$$

		Z	X_1	X_2	h_1	h_2	A_1	R	
F_1	Z	1	-3	-5	1	0	M	0	
F_2	R_1	0	1	0	1	0	0	4	✓
F_3	R_2	0	0	2	0	1	0	12	
F_4	R_3	0	3	2	0	0	1	18	

	Z	X_1	X_2	h_1	h_2	A_1	R	
F_1	1	-3	-5	1	0	M	0	$F_1 \times (R_1 - M)$
R_1	0	-3M	+2M	0	0	-M	-18M	$F_4 (-M)$
NF_1	1	-3M+3	-2M-5	1	0	0	-18M	

	Z	X_1	X_2	h_1	h_2	A_1	R	
Z	1	-3M-3	-2M+5	1	0	0	-18M	
R_1	0	①	0	1	0	0	4	$4/1 = 4$
R_2	0	0	2	0	1	0	12	
R_3	0	3	2	0	0	1	18	$18/3 = 6$

Siempre el numero pivot deve ser 1

	Z	x_1	x_2	h_1	h_2	R_1	R
Z	1	0	$-2M-5$	$3M+3$	0	0	$-6M+1$
R_1	0	1	0	1	0	0	4
R_2	0	0	2	0	1	0	12
R_3	0	0	②	-3	0	1	6

	Z	x_1	x_2	h_1	h_2	A_1	R
Z	1	0	0	$-\dfrac{7}{2}$	0	$M+\dfrac{5}{2}$	29
	0	1	0	1	0	0	4
	0	0	0	③	1	-1	6
	0	0	①	$-\dfrac{3}{2}$	0	$\dfrac{1}{2}$	3

	Z	x_1	x_2	h_1	h_2	A_1	R
Z	1	0	0	0	$5/2$	$M+1$	36
R_1	0	1	0	0	$-1/3$	$4/3$	2
R_2	0	0	0	1	$1/3$	$-1/3$	2
R_3	0	0	1	0	$1/2$	0	6

$Z = 36$

$x_1 = 2$

$x_2 = 6$

Ejercicio 2

Minimizar $z = 4x_1 + x_2$

$3x_1 + x_2 = 3$

$4x_1 + 3x_2 \geq 6$

$x_1 + 2x_2 \leq 4$

$x_1, x_2 \geq 0$

Minimizar $z = 4x_1 + x_2 + MA_1 + MA_2$ Minimizar $z - 4x_1 - x_2 - MA_1 - MA_2 = 0$

$3x_1 + x_2 + A_1 = 3$

$4x_1 + 3x_2 - s_1 + A_2 = 6$

$x_1 + 2x_2 + s_2 = 4$

$x_1, x_2, s_1, s_2, A_1, A_2 \geq 0$

B		X1	X2	s1	s2	A1	A2	R	
	Z	-4	-1	0	0	-M	-M	0	
	A1	3	1	0	0	1	0	3	
	A2	4	3	-1	0	0	1	6	
	s2	1	2	0	1	0	0	4	

B		X1	X2	s1	s2	A1	A2	R
	Z	-4	-1	0	0	-M	-M	0
+A1(M)		3M	M	0	0	M	0	3M
+A2(M)		4M	3M	-M	0	0	M	6M
		-4+7M	-1+4M	-M	0	0	0	9M

B		X1	X2	s1	s2	A1	A2	R	
	Z	-4+7M	-1+4M	-M	0	0	0	9M	
	A1	3	1	0	0	1	0	3	3/3=1
	A2	4	3	-1	0	0	1	6	6/4=1.5
	s2	1	2	0	1	0	0	4	4/1=4

B		X1	X2	s1	s2	A1	A2	R
	Z	-4+7M	-1+4M	-M	0	0	0	9M
	X1	1	1/3	0	0	1/3	0	1
	A2	4	3	-1	0	0	1	6
	s2	1	2	0	1	0	0	4

		X1	X2	s1	s2	A1	A2	R
	Z	-4+7M	-1+4M	-M	0	0	0	9M
+X1(4-7M)		4-7M	(4-7M)/3	0	0	(4-7M)/3	0	4-7M
		0	(1+5M)/3	-M	0	(4-7M)/3	0	4+2M

		X1	X2	s1	s2	A1	A2	R
	A2	4	3	-1	0	0	1	6
+X1(-4)		-4	-1 1/3	0	0	-1 1/3	0	-4
		0	1 2/3	-1	0	-1 1/3	1	2

		X1	X2	s1	s2	A1	A2	R
	s2	1	2	0	1	0	0	4
+X1(-1)		-1	- 1/3	0	0	- 1/3	0	-1
		0	1 2/3	0	1	- 1/3	0	3

B		X1	X2	s1	s2	A1	A2	R	
	Z	0	(1+5M)/3	-M	0	(4-7M)/3	0	4+2M	
	X1	1	1/3	0	0	1/3	0	1	1/(1/3)=3
	A2	0	1 2/3	-1	0	-1 1/3	1	2	2/(5/3)=6/5
	s2	0	1 2/3	0	1	- 1/3	0	3	3/(5/3)=9/5

B		X1	X2	s1	s2	A1	A2	R
	Z	0	(1+5M)/3	-M	0	(4-7M)/3	0	4+2M
	X1	1	1/3	0	0	1/3	0	1
	X2	0	1	- 3/5	0	- 4/5	3/5	1 1/5
	s2	0	1 2/3	0	1	- 1/3	0	3

		X1	X2	s1	s2	A1	A2	R
	Z	0	(1+5M)/3	-M	0	(4-7M)/3	0	4+2M
+X2-(1+5M)/3		0	-(1+5M)/3	(3+15M)/15	0	(4+20M)/15	-(3+15M)/15	(-6-30M)/15
		0	0	1/5	0	(24-15M)/15	-(3+15M)/15	-54/15

		X1	X2	s1	s2	A1	A2	R
	x1	1	1/3	0	0	1/3	0	1
+X2(-1/3)		0	- 1/3	1/5	0	1/4	- 1/5	- 2/5
		1	0	1/5	0	3/5	- 1/5	3/5

		X1	X2	s1	s2	A1	A2	R
	s2	0	1 2/3	0	1	- 1/3	0	3
=+X2(-5/3)		0	-1 2/3	1	0	1 1/3	-1	-2
		0	0	1	1	1	-1	1

B	X1	X2	s1	s2	A1	A2	R	
Z	0	0	1/5	0	(24-15M)/15	(-3-15M)/15	54/15	
X1	1	0	1/5	0	3/5	- 1/5	3/5	3
X2	0	1	- 3/5	0	- 4/5	3/5	1 1/5	-2
s1	0	0	1	1	1	-1	1	1

B	X1	X2	s1	s2	A1	A2	R
Z	0	0	1/5	0	(24-15M)/15	(-3-15M)/15	54/15
X1	1	0	1/5	0	3/5	- 1/5	3/5
X2	0	1	- 3/5	0	- 4/5	3/5	1 1/5
s1	0	0	1	1	1	-1	1

	X1	X2	s1	s2	A1	A2	R
Z	0	0	1/5	0	(24-15M)/15	(-3-15M)/15	3 3/5
s1*(-1/5)	0	0	- 1/5	- 1/5	- 1/5	1/5	- 1/5
	0	0	0	- 1/5	(21-15M)/15	-M	3 2/5

	X1	X2	s1	s2	A1	A2	R
X1	1	0	1/5	0	3/5	- 1/5	3/5
s1*(-1/5)	0	0	- 1/5	- 1/5	- 1/5	1/5	- 1/5
	1	0	0	- 1/5	2/5	0	2/5

	X1	X2	s1	s2	A1	A2	R
X2	0	1	- 3/5	0	- 4/5	3/5	1 1/5
s1*(3/5)	0	0	3/5	3/5	3/5	- 3/5	3/5
	0	1	0	3/5	- 1/5	0	1 4/5

B	X1	X2	s1	s2	A1	A2	R
Z	0	0	0	- 1/5	(21-15M)/15	-M	3 2/5
X1	1	0	0	- 1/5	2/5	0	2/5
X2	0	1	0	3/5	- 1/5	0	1 4/5
s1	0	0	1	1	1	-1	1

El valor óptimo es X1 = 2/5 x2 = 9/5 con lo que el valor mínimo para z = 17/5

Ejercicio 03

$$Max. \quad Z = 3X_1 + 9X_2$$

Sujeto a.
$$X_1 + 4X_2 \leq 8$$
$$X_1 + 2X_2 \leq 4$$
$$X_1, X_2 \geq 0$$

m.a/g.
$$Z - 3X_1 - 9X_2 = 0$$
$$X_1 + 4X_2 + S_1 = 8$$
$$X_1 + 2X_2 + S_2 = 4$$

	Z	X_1	X_2	S_1	S_2	Sol.
Z	1	-3	-9	0	0	0
S_1	0	1	4	1	0	8
S_2	0	1	2	0	1	4

	Z	X_1	X_2	S_1	S_2	Sol
Z	1	-3	-9	0	0	0
X_2	0	1/4	1	1/4	0	2
S_2	0	1	2	0	1	4

		Z	X_1	X_2	S_1	S_2	Sol.
$f_1 + (f_2 \times 9)$	Z	1	-3/4	0	9/4	0	18
	X_2	0	1/4	1	1/4	0	2
$f_3 + (f_2 \times -2)$	S_2	0	1/2	0	-1/2	1	0

	Z	x_1	x_2	S_1	S_2	Sol.
Z	1	-3/4	0	9/4	0	18
x_2	0	1/4	1	1/4	0	2
$F_3 \times 2$ x_1	0	1	0	-1	2	0

		Z	x_1	x_2	S	S_2	sol
$F_1 + (F_3 \times 3/4)$	Z	1	0	0	3/2	3/2	18
$F_2 + (F_3 \times -1/4)$	x_2	0	0	1	1/2	-1/2	2
F_3	x_1	0	1	0	-1	2	0

$Z = 18$

$X_1 = 0$

$X_2 = 2$

Ejercicio 4
Maximizar Z = 2X1 +4X2

Sujeto

X1+2X2<=5	Z - 2X1 -4X2 = 0
X1+X2<=4	
X1, X2>=0	

B		X1	X2	s1	s2	R
	Z	-2	-4	0	0	0
	s1	1	2	1	0	5
	s2	1	1	0	1	4

B		X1	X2	s1	s2	R	
	Z	-2	-4	0	0	0	
	s1	1	2	1	0	5	2 1/2
	s2	1	1	0	1	4	4

B		X1	X2	s1	s2	R
	Z	-2	-4	0	0	0
	X2	1/2	1	1/2	0	2 1/2
	s2	1	1	0	1	4

		X1	X2	s1	s2	R
	Z	-2	-4	0	0	0
s1(4)		2	4	2	0	10
		0	0	2	0	10

		X1	X2	s1	s2	R
s2		1	1	0	1	4
s1(-1)		- 1/2	-1	- 1/2	0	-2 1/2
		1/2	0	- 1/2	1	1 1/2

B		X1	X2	s1	s2	R
	Z	0	0	2	0	10
	X2	1/2	1	1/2	0	2 1/2
	S2	1/2	0	- 1/2	1	1 1/2

Podemos observar la salida óptima X1=0 y X2=5/2, lo que quiere decir que x1 no entra en la solución, vamo a incluir x1 de manera que entre en la solución para ello vamos a incluirla en s2

B		X1	X2	s1	s2	R
	Z	0	0	2	0	10
	X2	1/2	1	1/2	0	2 1/2
	x1	1	0	-1	2	3

		X1	X2	s1	s2	R
	X2	1/2	1	1/2	0	2 1/2
x1*(-1/2)		- 1/2	0	1/2	-1	-1 1/2

		0	1	1	-1	1
B	X1	X2	s1	s2	R	
Z	0	0	2	0	10	
X2	0	1	1	-1	1	
x1	1	0	-1	2	3	

Vamos a comprobar las diferentes soluciones óptimas

Maximizar Z = 2X1 +4X2

$\quad$ Z = 2(3) + 4(1) = 10

$\quad$ Z = 2(0) + 4(5/2) = 10

Ambas soluciones son óptimas pero en la segunda solución se incluye la variable x1

Ejercicio 05

⑤ $z = 2x_1 + x_2 + 0S_1 + 0S_2$

$z - 2x_1 - x_2 + 0S_1 + 0S_2 = 0$
$x_1 - x_2 + 1S_1 + 0S_2 = 10$
$2x_1 + 0x_2 + 0S_1 + 1S_2 = 40$

$x_1, x_2, S_1, S_2 \geq 0$

	C_j	2	1	0	0	
C	C_b	x_1	x_2	S_1	S_2	R
0	S_1	1	-1	1	0	10
0	S_2	2	0	0	1	40
	z	-2	-1	0	0	0

$\longrightarrow 10/1 = 10$
$\longrightarrow 40/2 = 20$

VA: valor antiguo
EFCP: elemento fila columna pivote
NVFP: nuevo valor fila pivote
NV: nuevo valor

VA	EFCP	NVFP	NV
2	2	1	$2 - (2 \times 1) = 0$
0	2	-1	$0 - (2 \times -1) = 2$
0	2	1	$0 - (2 \times 1) = -2$
1	2	0	$1 - (2 \times 0) = 1$
40	2	10	$40 - (2 \times 10) = 20$

	C_j	2	1	0	0	
C	C_b	x_1	x_2	S_1	S_2	R
1	x_1	1	-1	1	0	10
0	S_2	0	2	-2	1	20	←
	z	-2	-1	0	0	0	←

VA	EFCP	NVFP	NV
-2	-2	1	$-2 - (-2 \times 1) = 0$
-1	-2	-1	$-1 - (-2 \times -1) = -3$
0	-2	1	$0 - (-2 \times 1) = 2$
0	-2	0	$0 - (-2 \times 0) = 0$
0	-2	10	$0 - (-2 \times 10) = 20$

	C_j	2	1	0	0	
C	C_b	x_1	x_2	S_1	S_2	R
1	x_1	1	-1	1	0	10
2	x_2	0	2	-2	1	20
	z	0	-3	2	0	20

$\longrightarrow 20/2 = 10$

$0/2$	$2/2$	$-2/2$	$1/2$	$20/2$
0	1	-1	$1/2$	10

	C_j	2	1	0	0	
C	C_b	x_1	x_2	S_1	S_2	R
1	x_1	1	-1	1	0	10
2	x_2	0	1	-1	$1/2$	10
	z	0	-3	2	0	20

@ conti

VA	EFCP	NVFP	NV
1	-1	0	$1-(-1\times 0)=1$
-1	-1	1	$-1-(-1\times 1)=0$
1	-1	-1	$1-(-1\times -1)=0$
0	-1	1/2	$0-(-1\times 1/2)=1/2$
10	-1	10	$10-(-1\times 10)=20$

Cj		2	1	0	0	
C	Cb	X_1	X_2	S_1	S_2	R
1	X_1	1	-1	1	0	10
2	X_2	0	1	-1	1/2	10
	Z	0	-3	2	0	20

$\longrightarrow$

Cj		2	1	0	0	
C	Cb	X_1	X_2	S_1	S_2	R
1	X_1	1	0	0	1/2	20
2	X_2	0	1	-1	1/2	10
	Z	0	-3	2	0	20

VA	EFCP	NVFP	NV
0	-3	0	$0-(-3\times 0)=0$
-3	-3	1	$-3-(-3\times 1)=0$
2	-3	-1	$2-(-3\times -1)=-1$
0	-3	1/2	$0-(-3\times 1/2)=3/2$
20	-3	10	$20-(-3\times 10)=-50$

$\longrightarrow$

Cj		2	1	0	0	
C	Cb	X_1	X_2	S_1	S_2	R
1	X_1	1	0	0	1/2	20
2	X_2	0	1	-1	1/2	10
	Z	0	0	-1	3/2	-50

No se puede por cero o número negativo.

SOLUCIÓN
NO
ACOTADA

FORMA GRÁFICA

$Z - 2X_1 - X_2 = 0$

$X_1 - X_2 = 10$

$X_1 = 0 \longrightarrow X_2 = -10 \quad (0,-10)$

$X_2 = 0 \longrightarrow X_1 = 10 \quad (10,0)$

$2X_1 = 40$

$X_1 = 20$

RESTRICCIONES:

$X_1, X_2 \geq 0$

$X_1 - X_2 \leq 10$

$2X_1 \leq 40$

$2X_1 = 40$

$X_1 - X_2 = 10$

Ejercicio 06

Ejercicio 6

Maximizar z = 3x1 + 2x2

2x1 + x2 <= 2

3x1 + 4x2 >= 12

x1,x2 >= 0

Maximizar z = 3x1 + 2x2 -Ma1   z - 3x1 - 2x2 +Ma1=0

2x1 + x2 + s1<= 2

3x1 + 4x2 -s2 + a1>= 12

x1,x2 >= 0

B	X1	X2	s1	s2	a1	R
Z	-3	-2	0	0	M	0
s1	2	1	1	0	0	2
a1	3	4	0	-1	1	12

Z	-3	-2	0	0	M	0
+a1(-M)	-3M	-4M	0	M	-M	-12M
	-3-3M	-2-4M	0	M	0	-12M

B	X1	X2	s1	s2	a1	R	
Z	-3-3M	-2-4M	0	M	0	-12M	
s1	2	1	1	0	0	2	2
a1	3	4	0	-1	1	12	3

Z	-3-3M	-2-4M	0	M	0	-12M
s1*(2+4M)	4+8M	-(2-4M)	-(2-4M)	0	0	4+8M
	1+5M	0	2+4M	M	0	4-4M

a1	3	4	0	-1	1	12
s1 (-4)	-8	-4	-4	0	0	-8
	-5	0	-4	-1	1	4

B	X1	X2	s1	s2	a1	R
Z	1+5M	0	2+4M	M	0	4-4M
x1	2	1	1	0	0	2
a1	-5	0	-4	-1	1	4

En el ejercio nos muestra que la variable artificial es positiva por lo que se considera no factible, es una solución pseudoóptima