Discrete Fourier Transform for Dummies

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1. Introduction

2. The Real DFT

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http://www.analog.com/media/en/technical-documentation/dsp-book/dsp_book_Ch31.pdf

$$ReX(k) = \frac{2}{N}\sum_{n=0}^{N-1} f(n)\cos(2\pi kn/N) \tag{1}$$

$$ImX(k) = \frac{-2}{N}\sum_{n=0}^{N-1} f(n)\sin(2\pi kn/N) \tag{2}$$

The N sample time domain signal $f(n)$ is decomposed into a set of N/2+1 cosine, and N/2+1 sine waves, with frequencies given by the index k. The amplitudes of the cosine waves are contained in $ReX(k)$, while the amplitudes of the sine waves are contained in $ImX(n)$. These equations operate by correlating the respective cosine or sine wave with the time domain signal. In spite of using the names: real part and imaginary part, there are no complex numbers in these equations[ANALOG 2016].

3. 1D DFT Definition

$$F(u) = \sum_{x=0}^{M-1} f(x)e^{-j2\pi ux/N} \tag{3}$$

where $u = 0, 1, 2, \dots, M-1$

$$f(x) = \frac{1}{M}\sum_{u=0}^{M-1} F(u)e^{j2\pi ux/N} \tag{4}$$

where $x = 0, 1, 2, \dots, M-1$

3.1. Implementation

4. 2D DFT Definition

$$F(k,l) = \frac{1}{MN}\sum_{m=0}^{M-1}\sum_{n=0}^{N-1} f(m,n)e^{-j2\pi(\frac{km}{M}+\frac{ln}{N})} \tag{5}$$

where $m = 0, 1, 2, \dots, M-1$ and $n = 0, 1, 2, \dots, N-1$

$$f(m,n) = \sum_{k=0}^{M-1} \sum_{l=0}^{N-1} F(k,l) e^{j2\pi(\frac{km}{M} + \frac{ln}{N})} \tag{6}$$

where $k = 0, 1, 2, \ldots, M-1$ and $l = 0, 1, 2, \ldots, N-1$

4.1. The spectrum

4.2. The magnitude

References

ANALOG (2016). The Complex Fourier Transform.