Topics

- Canonical Forms
- Minimization of Boolean functions using the Karnaugh map method

Preliminary Definitions

- A **literal** is a variable or the complement of a variable. Examples: x, y, \bar{x} .
- A **product term** is a single literal or a logical product of two or more literals. Examples: \bar{z} , $x \cdot y$, $x \cdot \bar{y} \cdot z$.
- A sum term is a single literal or a logical sum of two or more literals. Examples: \overline{z} , x + y, $x + \overline{y} + z$.
- A **normal term** is a product or sum term in which no variable appears more than once.
- An *n*-variable **minterm** is a normal product term with *n* literals. There are 2^{*n*} such product terms.
 - A minterm m_i corresponds to row i of the truth table.
 - In minterm m_i, a particular variable appears complemented if the corresponding bit in the binary representation
 of i is 0; otherwise, it is uncomplemented.
- An *n*-variable **maxterm** is a normal sum term with *n* literals. There are 2ⁿ such sum terms.
 - A maxterm M_i corresponds to row i of the truth table.
 - In maxterm M_{i} , a particular variable appears complemented if the corresponding bit in the binary representation of i is 1; otherwise, it is uncomplemented.

Canonical Forms:

canonical form:
$$f(x_0,x_1,...,x_{n-1}) = \sum_{i=0}^{2^n-1} m_i \cdot f_i$$
 sum of products, SC:

recanonical form:
$$f(x_0,x_1,...,x_{n-1}) = \prod_{i=0}^{2^n-1} (f_i + M_i)$$
 Produce of sums, PC :

3rd canonical form:
$$f(x_0, x_1, ..., x_{n-1}) = \prod_{i=0}^{2^n - 1} \overline{f_i \cdot m_i}$$

4th canonical form :
$$f(x_0, x_1, ..., x_{n-1}) = \sum_{i=0}^{2^n - 1} \overline{f_i + M_i}$$

Problems

- 1. Consider the following Boolean function $f(x, y, z) = x' \cdot y + z' + x \cdot y' \cdot z$.
 - a. Draw the logic circuit
 - b. Construct the truth table for the function f(x,y,z).
 - c. From the truth table write all the canonical forms.
- 2. Write all the canonical forms for the Boolean functions f,g,h,w of (x,y,z) defined in the following truth table:

X	У	Z	f	g	h	W
0	0	0	0	1	0	1
0	0	1	1	0	1	0
0	1	0	1	1	1	0
0	1	1	0	0	1	0
1	0	0	1	1	1	1
1	0	1	0	0	1	1
1	1	0	0	1	1	0
1	1	1	1	0	0	1

3. Find the minimal SOP and POS algebraic expressions for the functions defined by the following Karnaugh maps:

K1

ab cd	00	01	11	10
00	1	1		
01				
11		1	1	
10		1	1	

K2

ab cd	00	01	11	10
00	1	1	1	1
01				
11				
10				

K3

ab cd	00	01	11	10
00	1			1
01				
11				
10	1			1

 K^2

C	ab d	00	01	11	10
(00		1	1	
(01			1	
	11		1	1	
	10		1	1	

- 4. Propose a 4'variable Boolean function, that has more than one minimal expression (in either POS or SOP form). Locate the essential prime implicants (or essential prime implicates) if they exist.
- 5. Let $f(a, b, c, d) = a' \cdot c' + b' \cdot c' + a \cdot c \cdot d + a' \cdot b \cdot c'$.
 - a. Write the Karnaugh map directly from the expression
 - b. Find the minimal SOP expression for f(a, b, c, d).
- 6. Let f(a, b, c, d) = (a + b').(c'+d).(b'+d')
 - a. Write the Karnaugh map directly from the expression
 - b. Find the minimal SOP expression for f(a, b, c, d).

7. Find the minimal SOP expressions for the two following Boolean functions. Compare the results.

a.
$$f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (0,1,4,5,12,13)$$

b. $g(x_3, x_2, x_1, x_0) = \prod m_{x_3, x_2, x_1, x_0} (2,3,6,7,8,9,10,11,14,15)$

8. Find the minimal SOP and POS expressions for the following Boolean functions

a.
$$(w, x, y, z) = \sum m_{w,x,y,z} (0,1,2,4,6,9,11)$$

b.
$$f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (0,1,4,5,8,9,10,11,12,14)$$

c.
$$f(x_4, x_3, x_2, x_1, x_0) = \sum m_{x_4, x_3, x_2, x_1, x_0} (0,2,4,5,6,8,9,10,12,14,17,18,20,24,25,28,30)$$

9. Sometimes the output for a given combination of the input variables is not possible to define and/or is irrelevant. The existence of these don't care conditions may facilitate the minimization process. Find the minimal SOP expressions for the following Karnaugh maps:

K1

1					
	ab c	00	01	11	10
	0		х	х	1
	1	1	х	1	

K2

ab cd	00	01	11	10
00		1	1	
01	1	х		х
11	х		х	1
10		1	1	

10. Obtain the minimal SOP and POS expressions for the following functions with don't care conditions:

a.
$$f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (4,5,6,8,9,10,13) + \sum d_{x_3, x_2, x_1, x_0} (0,7,15)$$

b.
$$f(x_3, x_2, x_1, x_0) = \sum m_{x_3, x_2, x_1, x_0} (1,3,5,7,9) + \sum d_{x_3, x_2, x_1, x_0} (6,12,13)$$