Aula 4: Funções hiperbólicas

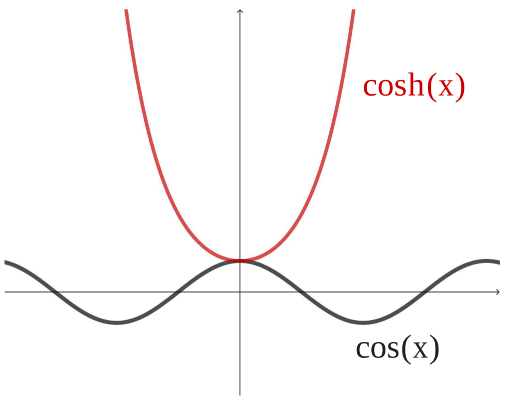
Fórmula de Euler: $\boxed{e^{ix} = \cos(x) + i\,\text{sen}\,(x)}$ $\qquad (i^2 = -1)$

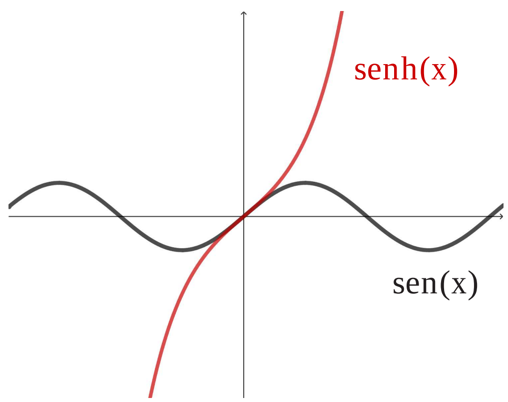
Daqui deduzimos:

$$\cos(x) = \frac{e^{ix} + e^{-ix}}{2} \quad , \quad \text{sen}\,(x) = \frac{e^{ix} - e^{-ix}}{2i} \tag{1}$$

Eliminando o i nas formulas anteriores obtemos o coseno e o seno hiperbólicos:

$$\cosh(x) = \frac{e^{x} + e^{-x}}{2} \quad , \quad \text{senh}\,(x) = \frac{e^{x} - e^{-x}}{2} \tag{2}$$





Aula 4: Fórmulas hiperbólicas e trigonométricas

$\cosh(x) = \cos(ix)$

$\operatorname{senh}(x) = -i \operatorname{sen}(ix)$

$e^{ix} = \cos(x) + i \operatorname{sen}(x)$

$\cosh^2(x) - \operatorname{senh}^2(x) = 1$

$\operatorname{tgh}(x) := \frac{\operatorname{senh}(x)}{\cosh(x)} = \frac{e^{2x}-1}{e^{2x}+1}$

$\operatorname{cotgh}(x) := \frac{\cosh(x)}{\operatorname{senh}(x)} = \frac{e^{2x}+1}{e^{2x}-1}$

$\cosh(x+y) = \cosh(x)\cosh(y) + \operatorname{senh}(x)\operatorname{sen}(y)$

$\cosh(x-y) = \cosh(x)\cosh(y) - \operatorname{senh}(x)\operatorname{sen}(y)$

$\operatorname{senh}(x+y) = \operatorname{senh}(x)\cosh(y) + \cosh(x)\operatorname{senh}(y)$

$\operatorname{senh}(x-y) = \operatorname{senh}(x)\cosh(y) - \cosh(x)\operatorname{senh}(y)$

$\cosh(2x) = \cosh^2(x) + \operatorname{senh}^2(x)$

$\operatorname{senh}(2x) = 2\operatorname{senh}(x)\cosh(x)$

$(\cosh(x))' = \operatorname{senh}(x)$

$(\operatorname{senh}(x))' = \cosh(x)$

$\cos(x) = \cosh(ix)$

$\operatorname{sen}(x) = -i \operatorname{senh}(ix)$

$e^{x} = e^{i(-ix)} = \cosh(x) + i \operatorname{senh}(x)$

$\cos^2(x) + \operatorname{sen}^2(x) = 1$

$\operatorname{tg} x = \frac{\operatorname{sen}(x)}{\cos(x)}$

$\operatorname{cotg} x = \frac{\cos(x)}{\operatorname{sen}(x)}$

$\cos(x+y) = \cos(x)\cos(y) - \operatorname{sen}(x)\operatorname{sen}(y)$

$\cos(x-y) = \cos(x)\cos(y) + \operatorname{sen}(x)\operatorname{sen}(y)$

$\operatorname{sen}(x+y) = \operatorname{sen}(x)\cos(y) + \cos(x)\operatorname{sen}(y)$

$\operatorname{sen}(x-y) = \operatorname{sen}(x)\cos(y) - \cos(x)\operatorname{sen}(y)$

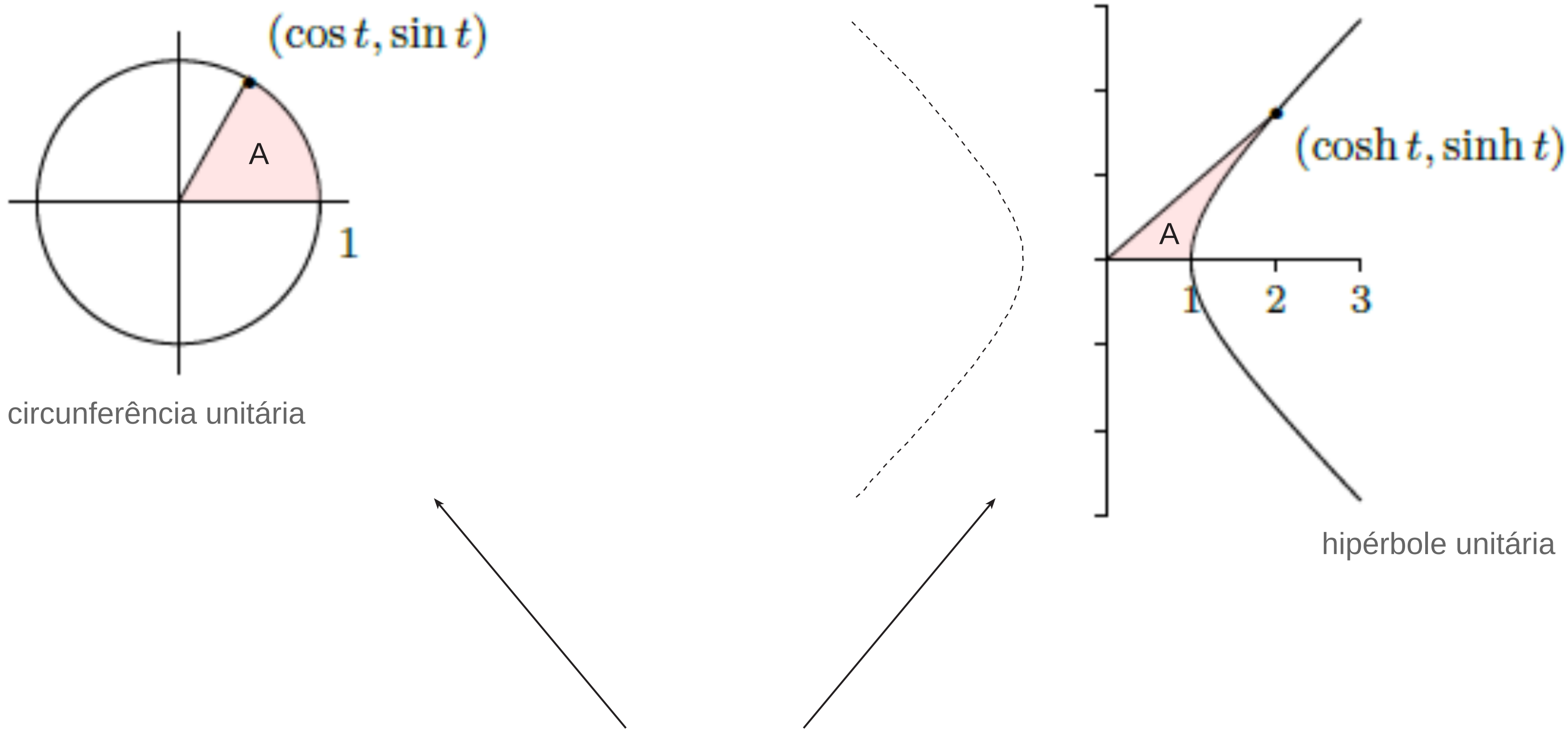
$\cos(2x) = \cos^2(x) - \operatorname{sen}^2(x)$

$\operatorname{sen}(2x) = 2\operatorname{sen}(x)\cos(x)$

$(\cos(x))' = -\operatorname{sen}(x)$

$(\operatorname{sen}(x))' = \cos(x)$

Aula 4: Semelhanças

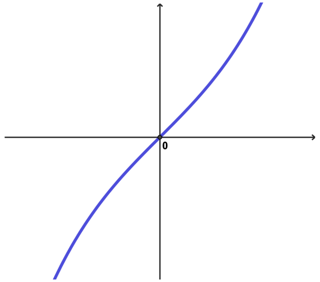


Área $A = \frac{t}{2}$

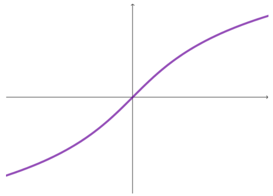
Aula 4: Funções hiperbólicas inversas

pf: parte de f ao subdominio (f restrita ao subdominio)

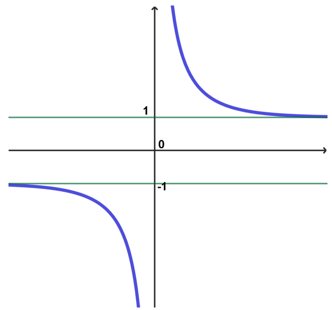
$\text{senh}(x)$



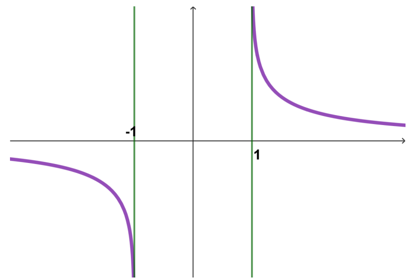
inversa $\longrightarrow$ $\overset{-1}{\text{senh}}(x)=\ln(x+\sqrt{x^2+1})$



$\text{cotgh}(x)$

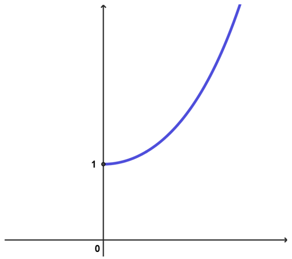


inversa $\longrightarrow$ $\overset{-1}{\text{cotgh}}(x)=\frac{1}{2}\ln(\frac{x+1}{x-1})$

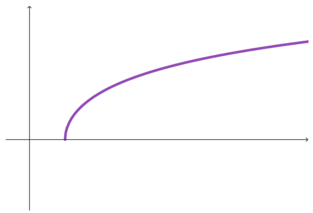


$\text{pcosh}(x)$

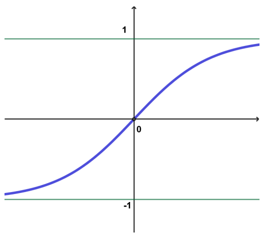
$x \geqslant 0$



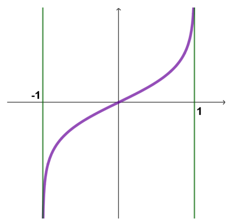
inversa $\longrightarrow$ $\overset{-1}{\text{pcosh}}(x)=\ln(x+\sqrt{x^2-1})$



$\text{tgh}(x)$



inversa $\longrightarrow$ $\overset{-1}{\text{tgh}}(x)=\frac{1}{2}\ln(\frac{1+x}{1-x})$



Nota: (1) À semelhança das funções trigonométricas “inversas”, também aqui é habitual designar-se por arcf à inversa de f (f=senh, tgh, cotgh) ou pf (f=cosh).

Neste último caso, $\text{arccosh} = \overset{-1}{\text{pcosh}}(x)$.

(2) $\cosh(\overset{-1}{\text{pcosh}}(x)) = x$ mas $\cancel{\overset{-1}{\text{pcosh}}(\cosh(x))} = x$ $(=|x|)$.