The traveling salesman problem & minimum spanning tree algorithm

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The traveling salesman problem (TSP) is a classic example of a simple NP-complete optimization problem[1]. It can be stated as follows: if a salesman wishes to visit exactly once each city on a given list and then return home, what route should he choose?

The underlaying decision problem is the NP-complete hamiltonian cycle problem[1]. Hamiltonian cycle of an undirected graph is a cycle that contains each vertex of the graph. The solution of TSP is a hamiltonian cycle of minimum length.

There are various approximation algorithms for solving TSP. I implement the minimum spanning tree algorithm (MST) which in the worst case gives a solution two times the optimal solution. The time and space comlexity of the MST-algorithm is $\Theta(n^2)$.[1]

For the MST to work as stated the graph for which traveling salesman problem is computed must satisfy the triangle inequality $c(u, v) \geq c(u, v) + c(v, w)$ for all vertices u, v, w where function c gives the cost of moving from one vertex to another.

The MST for complete undirected graph $G = (V, E)$ with nonnegative cost $c(u, v)$ for each $u, v \in V$ works as follows:

1. select vertex $r \in V$ to be the "root" vertex
2. compute a minimum spanning tree T for G from r using Prim's algorithm
3. do a depth-first search on T starting from r and order vertices according to when they are first visited
4. return the obtained hamiltonian cycle

[1] Cormen et al., 2009

Prim's algorithm will be implemented with binary heap for good performance. The input for the MST must be in the form of adjacency matrix.

MST algorithm is not the best known polynomial approximation algorithm for TSP. Christofides algorithm has the upper bound of 1.5 times the optimal solution. It is a bit more complicated to implement, and its time complexity is $O(n^3)$.[2]

References

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Lawler, E.L., J.K. Lenstra, Kan A.H.G Rinooy, and Shmoys D.B. 1985. *The traveling salesman problem.* John Wiley & Sons.

[2] Lawler et al., 1985