A.2 Holiday Homework

rounds down, rounds up to nearest int

$e^{\ln(3)} = 3$

A.2 → 2. $\sum_{k=0}^{\lfloor \lg n \rfloor} \lceil n/2^k \rceil$

$2^{\log_2(n)} = n$

$= \lceil \frac{n}{1} \rceil + \lceil \frac{n}{2} \rceil + \lceil \frac{n}{4} \rceil \ldots + \lceil 1 \rceil$

k=0 → every int below $\log_2(n)$

$\sum_{k=0}^{\lfloor \lg n \rfloor} \lceil n/2^k \rceil = 1$

$\frac{n}{1} + \frac{n}{4} + \frac{n}{8} + \frac{n}{16} \ldots + \frac{n}{2^{\log_2(n)}}$

$\therefore \sum_{k=0}^{\lfloor \lg n \rfloor} \lceil n / 2^k \rceil \leq \sum_{k=0}^{\lfloor \lg n \rfloor} \lceil n \lfloor \lg n \rfloor \rceil$

$= n \sum_{k=0}^{\lfloor \lg n \rfloor} \lfloor \lg n \rfloor$

$= O(\lg(n))$

X

$\sum_{k=0}^{\lfloor \lg n \rfloor} \lceil n / 2^k \rceil \leq \sum_{k=0}^{\lfloor \lg n \rfloor} (n / 2^k + 1) = \sum_{k=0}^{\lfloor \lg n \rfloor} \left(\frac{n}{2^k} + 1\right)$

$= \cancel{\sum_{k=0}^{\lfloor \lg n \rfloor} (n/2^k) + \sum_{k=0}^{\lfloor \lg n \rfloor} 1} = \sum_{k=0}^{\lfloor \lg n \rfloor} \frac{n}{2^k} + \sum_{k=0}^{\lfloor \lg n \rfloor} 1$

$\leq \sum_{k=0}^{\lfloor \lg n \rfloor} \frac{n}{2^k} + \lg n + 1$

$\leq \sum_{k=0}^{\infty} \left(\frac{n}{2^k}\right) + \lg n + 1$

$\leq \sum_{k=0}^{\infty} (2^{-k} n) + \lg n + 1$

$\leq n \sum_{k=0}^{\infty} (2^{-k}) + \lg n + 1$

$= \frac{n}{1 - \frac{1}{2}} + \lg n + 1$

$= O(n)$

A.2 → 3. $\sum_{k=1}^{n} \frac{1}{k}$

$= \sum_{k=\frac{n}{2}+1}^{n} \frac{1}{k} + \sum_{k=1}^{\frac{n}{2}} \frac{1}{k}$

$\geq \sum_{k=0}^{\frac{n}{2}} 0 + \sum_{k=\frac{n}{2}+1}^{n} \frac{n}{2k}$

$= \frac{n}{2} \sum_{k=\frac{n}{2}+1}^{n} \frac{1}{k}$

$\geq \frac{n}{2} \sum_{k=\frac{n}{2}+1}^{n} \frac{1}{(\frac{n}{2}+1)^2} = \frac{n}{2(\frac{n}{2}+1)^2} \sum_{k=\frac{n}{2}+1}^{n} 1$

$= \frac{n}{2(\frac{n}{2}+1)^2} \times \frac{n}{2} = \frac{n^2}{4(\frac{n^2}{4}+n+1)}$ → $= \frac{n^2}{n^2+4n+1}$