

Holiday Homework \rightarrow Summations

25.01

A.1-1 as $\sum_{k=1}^n (ca_k + b_k) = c \sum_{k=1}^n a_k + \sum_{k=1}^n b_k$

$\rightarrow a_k = f_k(c), b = 0, c = 0$

$\therefore \sum_{k=1}^n O(f_k(c)) = O \sum_{k=1}^n f_k(c)$

A.1-2 $\sum_{k=1}^n (2k-1)$

$1 + 3 + 5 \dots + 2n-1$

~~$\sum_{k=1}^n (2k-1) = \Theta(n^2)$~~

$= 2n \times \frac{n}{2} = n^2$

$\sum_{k=0}^9 10^k = \frac{10^9 - 1}{9} = 111,111,111$

A.1-3 when the flip is interpreting a decimal in light of equation.

A.1-4

~~$\sum_{k=1}^n \frac{1}{2^k}$~~

~~$\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{2^k} = \frac{1}{3}$~~

$\sum_{k=0}^{\infty} \frac{1}{2^k} = \frac{2}{1} = 2$

A.1-5 ?

A.1-6. if $|x| < 1 \rightarrow \sum_{k=0}^{\infty} x^k = \frac{1}{1-x}, \sum_{k=0}^n k^2 = \frac{(n+1)(n+2)n}{6}$

$\frac{d}{dx} \sum_{k=0}^{\infty} x^k \times x = \sum_{k=0}^{\infty} kx^k$

$\frac{d}{dx} \left(\frac{d}{dx} \left(\frac{1}{1-x} \right) \right)$

$\frac{d}{dx} \sum_{k=0}^{\infty} k^2 x^k \times x = \sum_{k=0}^{\infty} k^3 x^k = \frac{-x(x+1)}{(x-1)^3}$

A.1-7 I don't know theta notation but I might come back to it.

A.1-8. sum everything $\rightarrow \sum_{k=1}^n k^c = \Theta(n^{c+1})$

$= 1^c + 2^c + 3^c \dots + n^c$

highest deg of $n = c$

$\therefore = \Theta(n^c)$

Sum =

$n^c \times n = n^{c+1}$

Arithmetic series: $(n-c)^k = \Theta(n^k)$

~~$\sum_{k=1}^n k^c$~~ avg = $\Theta(n^k)$ at const terms.