Holiday Homework

A.1.5. entire right side is $\frac{n}{2} \times n^{c}$?

$= n^{c+1}$?

$\therefore \Theta(n^{c+1})$

~~A.1.6.~~ prove lower & upper?

$$\sum_{k=1}^{n} k^c = 1^c + 2^c \ldots + n^c$$

$$\leq (n)^c + n^c + n^c \ldots + n^c$$

$$\leq n^{c+1}$$

$$\therefore O(n^{c+1})$$

$$\sum_{k=1}^{n} k^c = 1^c + 2^c \ldots + n^c \geq \left(\frac{n}{2}\right)^c + \left(\frac{n}{2}+1\right)^c \ldots$$

$$\approx \Theta(n^c)$$

$$\rightarrow \Omega(n^{c+1})$$

A.1.7. $\sum_{k=1}^{n} \sqrt{k \lg k} = \cancel{\sqrt{\lg 1}}\ 0, \cancel{\lg 2}\ \sqrt{2}, \sqrt{3 \lg 3} \ldots + \sqrt{n \lg n}$

$$\leq n \times \sqrt{n \lg n}$$

$$\leq n (n \lg n)^{\frac{1}{2}}$$

$$\leq n^{\frac{3}{2}} \lg^{\frac{1}{2}} n$$

$$\therefore O(n^{\frac{3}{2}} \lg^{\frac{1}{2}} n)$$

$$\sum_{k=1}^{n} \sqrt{k \lg k} = 0 + \sqrt{2} \ldots + \sqrt{n \lg n}$$

$$\cancel{\sqrt{n \lg n}} \leq \sum_{k=1}^{n} \sqrt{k \lg k} \geq \sum_{k=\frac{n}{2}}^{n} \sqrt{k \lg k} \geq \sqrt{k \lg k} \sum_{k=\frac{n}{2}} 1 \geq \sqrt{k \lg n} \left(\frac{n}{2}\right)$$

$$\geq \frac{1}{2} (n^{\frac{3}{2}} \lg^{\frac{1}{2}} n)$$

$$\rightarrow \Omega(n^{\frac{3}{2}} \lg^{\frac{1}{2}} n)$$

$$\therefore \Theta(n^{\frac{3}{2}} \lg^{\frac{1}{2}} n)$$

A.2 → 1 $\sum_{k=1}^{n} \frac{1}{k^2} = 1 + \frac{1}{4} \ldots + \frac{1}{k^2}$

$$\cancel{\leq \frac{1}{k^2}} \leq 1 \times n$$

$$\cancel{\leq \frac{1}{n}} \quad (\leq n)$$