Tutorial-4

Ques 1 → $T(n) = 3T(n/2) + n^2$

$a = 3$
$b = 2$
$f(n) = n^2$
$\therefore n^{\log_b a} = n^{\log_2 3}$
$f(n) < n^{\log_b a}$ CASE I
$T(n) = \Theta(n^{\log_2 3})$
$T(n) = O(n)$

Ques 2 - $T(n) = 4T(n/2) + n^2$

$a = 2$
$b = 2$
$f(n) = n^2$
$\therefore n^{\log_b a} = n^{\log_2 4} = n^2$
$f(n) = n^{\log_2 4}$
$T(n) = O(n^{\log_b a} \log n)$
$= O(n^2 \log n)$

Ques 3 → $T(n) = T(n/2) + 2^n$

$a = 1$
$b = 2$
$f(n) = 2^n$
$n^{\log_b a} = n^{\log_2 1} = 1$
$f(n) \geq n^{\log_b a}$
$T(n) = O(2^n)$

Ques 4 → $T(n) = 2^n T(n/2) + n^n$

$a = 2$
$b = 2$
$f(n) = n^n$
$n^{\log_2 2} = n^1$
$f(n) > n^{\log_b a}$
$T(n) = O(n^n)$

Ques 5 → $T(n) = 16\, T(n/4) + n$

$a = 16$
$b = 4$
$fn = n$
$n^{\log_4 16}$
$n^{\log_4 4^2}$
n^2
$f(n) < n^{\log_b a}$
$T(n) = O(n^{\log_2 4^2})$
$T(n)\ O(n^2)$

Ques 6 → $T(n) = 2T(n/2) + n \log n$

$a = 2$
$b = 2$
$f(n) = n \log n$
$n^{\log_b a} = n^{\log_2 2} = n$
$f(n) > n^{\log_2 2}$
$[T(n) = n \log n]$

Ques 7 → $T(n) = 2T(n/2) + \frac{n}{\log n}$

$a = 2$
$b = 2$
$f(n) \leq \frac{n}{\log n}$
$n^{\log_b a} = n^{\log_2 2} = n$
$f(n) < n^{\log_b a}$
$O(n)$

Ques 8 → $T(n) = 2T(n/4) + n^{0.51}$

$a = 2$
$b = 4$
$fn = n^{0.51}$
$n^{\log_4 a}$
$n^{0.5}$
$f(n) = n^{\log_b a}$
$T(n) = (n^{0.5} \log n)$

Ques 9 → $T(n) = 0.5\,T(n/2) + 1/n$

$a = 0.5$, $b = 2$, $f(n) = 1/n$

$n^{\log_b a} = n^{\log_2 0.5}$

$n \log_2 0.5$

n^{-1}

$f(n) = n^{\log_b a}$

$T(n) = O(\frac{1}{n} \log n)$

Ques 10 → $T(n) = 16T(n/4) + n!$

$a = 16$, $b = 4$, $f(n) = n!$

$n^{\log_b a} = n^{\log_4 16}$

$\Rightarrow n^2$

$f(n) > n^{\log_b a}$

$T(n) = O(n!)$

Ques 11 → $T(n) = 4T(n/2) + \log n$

$a = 4$, $b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 4}$

n^2

$f(n) < n^{\log_b a}$

$T(n) \Rightarrow O(n^2)$

Ques 12 → $T(n) = \sqrt{n}\,T(n/2) + \log n$

$a = \sqrt{n}$, $b = 2$, $f(n) = \log n$

$n^{\log_b a} = n^{\log_2 \sqrt{n}}$

$n^{\log_2 n^{1/2}}$

$f(n) < n^{\log_2 n^{1/2}}$

$O(n^{\log_2 \sqrt{n}})$

Ques 13 → $T(n) = 3T(n/2) + n$

$a = 3$, $b = 2$, $f(n) = n$

$n^{\log_2 3}$

$n^{1.5}$

$T(n) = O(n^{1.5})$

Ques 14 → $T(n) = 3T(n/3) + \sqrt{n}$

$a = 3$, $b = 3$, $f(n) = \sqrt{n}$

$n^{\log_b a} = n$

$n^{1/2} < n$

$T(n) = O(n)$

Ques 15 → $T(n) = 4T(n/2) + cn$

$a = 4$, $b = 2$, $f(n) = cn$

$n^{\log_2 4} = n^2$

$f(n) < n^{\log_2 2^2}$

$T(n) = O(n^2)$

Ques 16 → $3T(n/4) + n \log n$

$a = 3$, $b = 4$, $f(n) = n \log n$

$n^{\log_4 3}$

$= n$

$f(n) > n^{\log_b a}$

$T(n) = O(n \log n)$

Ques 17 → $T(n) = 3T(n/3) + n/2$

$a = 3$, $b = 3$, $f(n) = n/2$

$n^{\log_b a}$

$n^{\log_3 3} = n$

$f(n) < n^{\log_b a}$

$T(n) = n$

Ques 18 → $T(n) = 6T(n/2) + n^2 \log n$

$a = 6$, $b = 2$

$n^{\log_2 6}$

$f(n) = n^2 \log n > n^{\log_b a}$

$T(n) = n^2 \log n$

19 → $T(n) = 4T(n/2) + n\log n$

$a = 4$, $b = 2$ $\quad n^{\log_2 2^2} = n^2$

$f(n) = n\log n$

$f(n) < n^2$

$T(n) = O(n^2)$

20 → $T(n) = 64T(n/8) - n^2\log n$

$a = 64$, $b = 8$ $\quad n^{\log_b a} = n^{\log_8 8^2}$

n^2

$f(n) = n^2\log n$

$f(n) > n^{\log_b a}$

$T(n) = O(n^2\log n)$

21 → $T(n) = 7T(n/3) + n^2$

$a = 7$, $b = 3$ $\quad n^{\log_3 7}$

$n^{1.7}$

$f(n) = n^2$

$f(n) > n^{\log_b a}$

$T(n) = O(n^2)$

22 → $T(n/2) + n(2 - \cos n)$

$a = 1$, $b = 2$ $\quad n^{\log_2 1} = 1$

$f(n) = n(2 - \cos n)$

$f(n) > n^{\log_b a}$

$T(n) = O(n(2 - \cos n))$