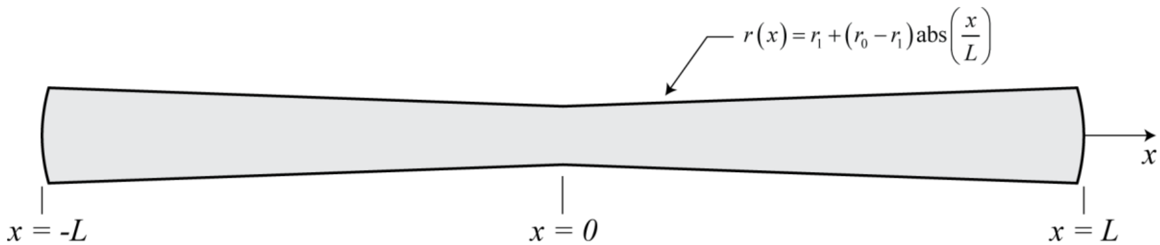
MAE 404/598 Finite Elements in Engineering
Project #1: **Due: March 5, 2020**

Current I is passed through the aluminum rod shown in the figure below. The relationship between the current and the amount of Joule heating per unit volume is $s = (I/A)^2 \rho$, where A is the cross-sectional area of the wire and $\rho = 2.82\times10^{-5}\ \Omega\cdot$mm is the electrical resistivity of aluminum.



The length of the rod is $2L$ = 20 mm and the radius varies linearly from the ends, where r_0 = 2 mm, to the midpoint, where r_1 = 1 mm. The temperature at the ends of the rods is held fixed at T = 20ºC. However due to the symmetry of the problem, you can consider only the domain $0 < x < L$, and apply a zero flux boundary condition at $x = 0$. Unless otherwise stated assume that the current load is 1000 A. The thermal conductivity of aluminum is k = 205 W/(m-K).

Write a finite element program in MATLAB to approximate the steady state temperature in the rod and compare with analytical solutions and an approximate solution from ABAQUS (or any other commercial finite element code).

Report format
Reports must be typed and submitted to Gradescope prior to the due date.

Grading scale:
Points will be awarded for correct completion of each of the tasks listed below with partial credit awarded for incomplete or incorrect attempts. Plots must contain axis labels and units as appropriate for full credit. You may attempt more points than required (however, your maximum score will be capped at 100%).

MAE 404: Grade calculated on 100 points.
MAE 598: Grade calculated on 200 points.

General points
- [10] Report is clearly written.
- [10] Plot temperature fields for analytical, ABAQUS and MATLAB solutions all on the same plot.
- [10] Plot heat flux fields for analytical, ABAQUS and MATLAB solutions all on the same plot.
- [15] Determine the maximum current that could be passed through the bar without melting it (the melting temperature of aluminum is 660 ºC). Explain how the assumptions in this model that might make this a bad prediction.

Analytical solution points
- [10] Analytical solution for the temperature field (show derivation – Mathematica is ok).
- [10] Analytical solution for the heat flux field (show derivation – Mathematica is ok).

MATLAB solution points
- [10] Plots of the temperature field for linear element solution with 4 and 20 elements.
- [10] Plots of the heat flux field for linear element solution with 4 and 20 elements.
- [15] Plots of the temperature field for quadratic element solution with 4 and 8 elements.
- [15] Plots of the heat flux field for quadratic element solution with 4 and 8 elements.
- [15] Plots of the temperature field for cubic element solution with 1 and 4 elements.
- [15] Plots of the heat flux field for cubic element solution with 1 and 4 elements.

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- [15] Comparison of times to (a) assemble and (b) solve the system of equations with 1000 elements for full storage and sparse storage of the conductivity matrix. (*Hint: Use tic() and toc() MATLAB functions for timing*)
- [20] Plot of L2 temperature error norm vs element size on a log-log plot and computed rate of convergence while using linear elements.
- [20] Plot of L2 temperature error norm vs element size on a log-log plot and computed rate of convergence while using quadratic elements.
- [20] Plot of flux error norm vs element size on a log-log plot and computed rate of convergence with linear elements.
- [20] Plot of flux error norm vs element size on a log-log plot and computed rate of convergence with quadratic elements.

ABAQUS solution points

- [10] Plot temperature and heat flux fields for linear elements.
- [20] Plot temperature and heat flux fields for quadratic elements.

Extra project (up to 100 points)

The resistivity of aluminum depends on its temperature. Assuming the change resistivity varies linearly with the change in temperature and given by $\rho(T) = 2.60\times10^{-5} + 1.10\times10^{-7}$ T, where T is in degrees Celsius, modify your MATLAB code to solve for the steady state temperature in the rod accounting for the change in the material's resistivity. *Hint – you will most likely need to use an iterative approach to solve the system.* For full credit, you must both have the correct solution and describe the method you used to solve.