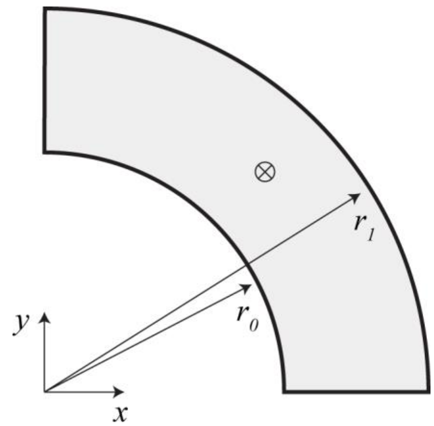
MAE 404/598 Finite Elements in Engineering

Project #2: **Due: See Gradescope Deadline**

This project is to be completed in teams of two students. Each team will submit a <u>single</u> PDF report to Gradescope.



Write a finite element code to analyze the temperature field for a thin plate subjected to a flame impinging on it at a location of (x_c, y_c). The inner radius of the plate r_0 = 10 mm and the outer radius is r_1 = 20 mm. The governing equation acting on the plate is:

$$k\Delta_z \nabla^2 T + s = \sigma\left(T^4 - T_\infty^4\right)$$

where k = 17 W/(m-K) is the thermal conductivity of steel, $\sigma = 5.670\times10^{-8}$ $\mathrm{Wm^{-2}K^{-4}}$ is the Stefan-Boltzmann constant, Δ_z = 1 mm is the plate thickness, and T_∞ = 300 K is the ambient temperature. Note that with radiative heat transport, an absolute temperature scale must be used. The flame is modeled as a heat source *s(x,y)*, given as:

$$s(x,y) = \exp\left(-\frac{(x-x_c)^2 + (y-y_c)^2}{R^2}\right) \quad \mathrm{W/mm^2},$$

where $x_c = \frac{1}{2}(r_1 + r_0)\cos(\pi/4)$ and $y_c = \frac{1}{2}(r_1 + r_0)\sin(\pi/4)$,

and where R = 3 mm is the radius of the flame.

Radiative heat transfer cools the plate, governed by the Stefan-Boltzmann law. The radiation heat flux can be linearized using a temperature-dependent heat transfer coefficient, h_{eff}

$$\sigma\left(T^4 - T_\infty^4\right) = h_{eff}\left(T - T_\infty\right), \text{ where } h_{eff} = \sigma\left(T^2 + T_\infty^2\right)\left(T + T_\infty\right).$$

Assume that there is negligible heat flux at the boundary of the domain.

Task 1:

Develop the weak form and discretized finite element equations for the governing equation. For the radiation term, use the effective heat transfer coefficient h_{eff} to avoid having fourth order terms. The weak form must also account for both natural and essential boundary conditions.

Task 2:

Solve for the equilibrium temperature field of the plate given the heat loads of the flame and radiative heat transfer described above. Note that the radiation term makes this a nonlinear problem. You can solve this using one of several approaches:

1) Approximate the average temperature of the domain and use it to compute an average effective heat transfer value. The average temperature can be determined by solving the following:

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$$\int_{\Omega} s\, d\Omega = \int_{\Omega} \sigma\left(\overline{T}^4 - T_0^4\right) d\Omega = A_{\Omega}\, \sigma\left(\overline{T}^4 - T_0^4\right)$$

2) Solve the problem iteratively by assuming an initial temperature field (a good initial guess is to use $\overline{T}$ from calculated by the approach above).

Write a finite element program in MATLAB to approximate the steady state temperature in the rod and compare with analytical solutions and an approximate solution from ABAQUS (or any other commercial finite element code).

Grading scale:

Points will be awarded for correct completion of each of the tasks listed below with partial credit awarded for incomplete or incorrect attempts. Plots must contain axis labels and units as appropriate for full credit. You may attempt more points than required (however, your maximum score will be capped at 100%).

The total number of points needed for a full 100% depends on how many people are in your group:

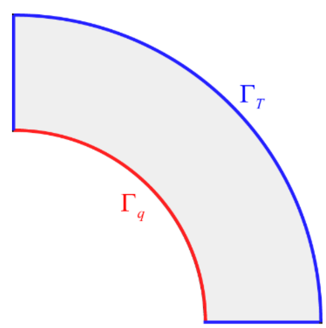
- For each MAE 404 student on a team, add **60 points**.
- For each MAE 598 student on a team, add **100 points**.

General points

- [10] Report is clearly written. This means that all plots are correctly labelled, all results have a few sentences explaining the observed trends, the problem you are solving is clearly stated.
- [20] Derivation of the weak form and discretized equations (Task 1). Show how the strong form is developed into a weak form, and then how the test and trial functions are discretized to arrive at the discrete (matrix) equations.
- [10] Clearly describe the method used to solve the problem in your MATLAB code. You should describe how you deal with the nonlinear nature of the problem, whether you solved a linearized version of the problem, or you used iteration to solve the problem, and what other techniques you used (e.g. how you stabilized the iterations).

Convergence testing – method of manufactured solutions

- [10] Use the method of manufactured solutions to verify that linear elements (either QUAD4 or TRI3) reproduce your manufactured solution at the ideal convergence rate. Test both essential and natural boundary conditions (using the boundaries defined in the figure).
- [20] Repeat with quadratic elements.
- [20] Describe your implementation of the method of manufactured solutions, be sure to describe:
  - the chosen solution for the temperature field
  - heat source term that produces the solution
  - boundary conditions (must use both essential and natural BCs)



MATLAB solution points

- [10] Plot the temperature field for linear TRI3 elements.
- [10] Plot the magnitude of the heat flux field for linear TRI3 elements.
- [10] Plot the temperature field for linear QUAD4 elements.
- [10] Plot the magnitude of the heat flux field for linear QUAD4 elements.
- [15] Plot the temperature field for quadratic TRI6 elements.
- [15] Plot the magnitude of the heat flux field for quadratic TRI6 elements.
- [20] Plot the temperature field for quadratic QUAD8 elements.
- [20] Plot the magnitude of the heat flux field for quadratic QUAD8 elements.

- [20] For any of the above mesh types, demonstrate that your finite element mesh is sufficiently refined. Generate a table where each row shows the approximate element size, the maximum temperature, and the maximum internal heat flux. The element sizes should follow a geometric sequence: h, h/2, h/4, h/8 …

ABAQUS solution points

- [30] Plot temperature and heat flux fields for linear elements.
- [30] Plot temperature and heat flux fields for quadratic elements.
- [20] Given the node limitations you may have with ABAQUS or other commercial FEA software, explain whether you believe your ABAQUS solution is consistent with your MATLAB solution.