Textbook: **Multivariable Calculus: Concepts and Contexts**, Fourth Edition
by James Stewart

3 Home Work III: Partial derivatives

Due Date: **Bhadra 20, 2080**

Exercises 11.3

11, 15, 17, 18, 39, 43, 45, 46, 49, 51, 53, 57, 59, 71, 72, 78

1. Show that the function satisfies Laplace's equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2} = 0$.

   (a) $u = x^2 - y^2 + 2xy$

   (b) $u = e^x \sin y + e^y \cos x$

   (c) $u = \ln(x^2 + y^2) + 2\tan^{-1}(y/x)$

2. Show that the function satisfies Wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$

   (a) $u = \cos(4x + 4ct)$

   (b) $u = \ln(x + ct)$

   (c) $u = \sin c\omega t \sin \omega x$ for all real values of ω.

3. Show that the function satisfies the Heat (diffusion) equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$.

   (a) $u = e^{-t}\sin(x/c)$

   (b) $u = e^{-t}\cos(x/c)$

(c) $u = e^{-t/2}e^{-x^2/(4t)}$

4. (a) Verify for the Cobb-Douglas production function $P(L, K) = 1.01L^{0.75}K^{0.25}$ that the production will be doubled if both the amount of labor and the amount of capital are doubled.

   Determine whether this is also true for the general production function $P(L, K) = cL^{\alpha}K^{\beta}$. If not, under what condition this will be true.

5. Let's consider a small printing business where N is the number of workers, V is the value of the equipment (in units of \$25,000), and P is the production, measured in thousands of pages per day. Suppose the production function for this company is given by

$$P = f(N, V) = 2N^{0.6}V^{0.4}.$$

   (a) If this company has a labor force of 100 workers and 200 units' worth of equipment, what is the production output of the company?

   (b) Find $f_N(100, 200)$ and $f_V(100, 200)$. Interpret your answers in terms of production.

6. A model for the surface area of a human body is given by the function

$$S = f(w, h) = 0.1091w^{0.425}h^{0.725}$$

   where w is the weight (in pounds), h is the height (in inches), and S is measured in square feet.

   (i) Find and interpret it.

   (ii) What is your own surface area?