Tribhuvan University

Institute of Sciences and Technology

SCHOOL OF MATHEMATICAL SCIENCES

First Assessment 2079

Subject: **Multivariable Calculus for Data Science** *Full Marks*: 45

Course No: MSMT 554 *Pass Marks*: 22.5

Level: MDS. /I Year /II Semester *Time*: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Group A [5 ×3=15]

1. Define scalar triple product of three vectors and give its geometrical meaning. Using scalar triple product, verify that the vectors $\vec{u} = \vec{i} + 5\vec{j} - 2\vec{k}$, $\vec{v} = 3\vec{i} - \vec{j}$, and $\vec{w} = 5\vec{i} + 9\vec{j} - 4\vec{k}$ are coplanar. [1+2]

2. Find the parametric equations and symmetric equation for the lines through (2,1,0) and perpendicular to both the vectors $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$. [3]

3. Define curvature of a vector function $\vec{r} = \vec{r}(t)$. Find the curvature of the vector function $\vec{r}(t) = p\cos t\,\vec{i} + p\sin t\,\vec{j}$, where p is constant. [1+2]

4. Find the limit, if it exists, or show that the limit does not exist:

   a) $f(x, y) = \dfrac{5y^4\cos 2x}{x^4 + y^4}$ b) $f(x, y) = \dfrac{xy}{\sqrt{x^2 + y^2}}$ [1.5+1.5]

5. Explain why the function $f(x, y) = \sqrt{x + e^{4y}}$ is differentiable at the given point (3, 0). Find the linearization $L(x, y)$ of the function $f(x, y)$ at that point. [1+2]

Group B [5 ×6=30]

6. Prove the Parallelogram Law $|\vec{a} + \vec{b}|^2 + |\vec{a} - \vec{b}|^2 = 2|\vec{a}|^2 + 2|\vec{b}|^2$ for any two vectors $\vec{a}$ and $\vec{b}$. Give its geometric interpretation. Also if $\vec{a} + \vec{b}$ and $\vec{a} - \vec{b}$ are orthogonal, show that the vectors $\vec{a}$ and $\vec{b}$ must have the same length. [2+1+3]

   OR

   Derive a vector equation of a straight line

   a) through the given point $\vec{a}$ and parallel to the vector $\vec{b}$

   b) through two points $\vec{a}$ and $\vec{b}$. Also, find a vector equation for the line through the point (1, 0, 6) and perpendicular to the plane $x + 3y + z = 5$ [2+2+2]

7. Derive the expression for the derivative of scalar triple product of three vectors. Find the derivative of the scalar triple product of the vectors

   $t\vec{i} + t^2\vec{j} + t\vec{k}$, $(t+1)\vec{i} + (t+2)\vec{j} - 3t\vec{k}$ and $t^2\vec{i} + 2t\vec{j} + t\vec{k}$ at $t = 2$. [2+4]

8. Find the domain and range of the function $f(x, y) = \sqrt{16 - 4x^2 - y^2}$. Describe the graph of f. Sketch a contour map of this surface using level curves corresponding to $c = 1, 2, 3, 4, 5$ [1.5+1.5+1.5+1.5]

9. a) Let $f(x, y)$ be defined on an open disk D that contains the point (a, b). Prove that if the functions f_{xy} and f_{yx} are continuous on D, then $f_{xy}(a, b) = f_{yx}(a, b)$ [4]

   b) Show that $z = e^x \sin y$ satisfies the equation $\dfrac{\partial^2 z}{\partial x^2} + \dfrac{\partial^2 z}{\partial y^2} = 0$. [2]

10. a) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable functions of t and $z = f(x, y)$ is a differentiable function of x and y, then $z = f(x(t), y(t))$ is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt},$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y). [3]

b) Calculate $\partial z/\partial u$ and $\partial z/\partial v$ using the following functions:

$z = f(x, y) = 3x2 - 2xy + y2, x = x(u, v) = 3u + 2v, y = y(u, v) = 4u - v.$ [3]

OR

a) Prove that if f is a differentiable function of x and y, then f has a directional derivative in the direction of any unit vector $u = (a, b)$ and $D_u f(x, y) = f_x(x, y)a + f_y(x, y)b$. [3]

b) Find the direction for which the directional derivative of $f(x, y) = 3x^2 - 4xy + 2y^2$ at $(-2, 3)$ is a maximum. What is the maximum value? Find the maximum rate of change of $f(x, y) = \sqrt{x^2 + y^4}$ at $(-2, 3)$ and the direction in which this maximum rate of change occurs. [3]

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Tribhuvan University
Institute of Sciences and Technology
SCHOOL OF MATHEMATICAL SCIENCES
Second Assessment 2079

Subject: **Multivariable Calculus for Data Science**
Course No: **MDS 554**
Level: MDS/I **Year /II Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Attempt All Questions.

Group A [$5 \times 3 = 15$]

1. Find the normal vector and binormal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ where $x = t^2, y = t^2, z = t^3$ at point (1, 1, 1).

2. Evaluate $\iint_D (10x^2y^3 - 6)\, dA$, where D is the region bounded by $x = -2y^2$ and $x = y^3$.

3. Evaluate $\iiint_E (12y - 8x)\, dV$ where E is the region behind $y = 10 - 2z$ and in front of the region in the xz-plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

2• 4. If $\vec{F} = (2x + y)\vec{i} + (3y - x)\vec{j}$, evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where C is the curve in the xy-plane consisting of the straight lines from (0, 0) to (2, 0) and then to (3, 2).

5. If a closed surface S enclosed a volume V and $\vec{F} = x\vec{i} + 2y\vec{j} + 3z\vec{k}$, Using Gauss' theorem show that $\iint_S \vec{F} \cdot \vec{n}\, ds = 6V$.

Group B [$5 \times 6 = 30$]

6. *Find the maximum and minimum values of $f(x, y, z) = y^2 - 10z$ subject to the constraint $x^2 + y^2 + z^2 = 36$.*

OR

The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farthest from the origin.

7. a) Use a double integral to determine the volume of the solid that is inside the cylinder $x^2 + y^2 = 16$, below $z = 2x^2 + 2y^2$ and above the xy-plane.

   b) Use a triple integral to determine the volume of the region below $z = 6 - x$, above $z = -\sqrt{4x^2 + 4y^2}$ inside the cylinder $x^2 + y^2 = 3$ with $x \leq 0$.

8. a) Evaluate $\iint_R (x + 2y)\, dA$ where R is the triangle with vertices (0, 3), (4, 1) and (2, 6) using the transformation $x = \frac{1}{2}(u - v)$, $y = \frac{1}{4}(3u + v + 12)$ to R.

   b) Determine the surface area of the portion of $y = 2x^2 + 2z^2 - 7$ that is inside the cylinder $x^2 + z^2 = 4$.

9. Define line integral. Is the the vector field $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - zx)\vec{j} + (z^2 - xy)\vec{k}$ a irrigational? Justify. Also find a scalar function ø such that $\vec{F} = \nabla ø$.

OR

State Green's theorem in the plane. Prove that the area enclosed by a simple closed curve C is given by $= \frac{1}{2}\int_C (x\,dy - y\,dx)$. Verify Green's theorem in the plane for $\int_C (2xy - x^2)\, dx + (x + y^2)\, dy$ where C is the closed curve given by $y = x^2$, $x^{3} = y^{2}$.

10. State Stokes' theorem in a surface S. Show that in a plane, Green's theorem is a particular case of Stokes' theorem. Verify Stokes' theorem for the vector function $\vec{F} = (x^2 + y^2)\,\vec{i} - 2xy\,\vec{j}$ taken round the rectangle in the xy-plane bounded by $x = 0, x = a, y = 0, y = b$.

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Tribhuvan University
Institute of Sciences and Technology
SCHOOL OF MATHEMATICAL SCIENCES

First Assessment 2080

Subject: **Multivariable Calculus for Data Science**
Course No: MDS 554
Level: MDS /I Year /II Semester

Full Marks: 45
Pass Marks: 22.5
Time: 2.00 hrs

Candidates are required to give their answer in their own words as far as practicable. All questions carry equal marks.

Group A [5×3=15]

1. State geometrical meaning of scalar triple product. Find the volume of the parallelepiped determined by the vectors $\vec{a} = \vec{i} + 2\vec{j} + 3\vec{k}$, $\vec{b} = -\vec{i} + \vec{j} + 2\vec{k}$, and $\vec{c} = 2\vec{i} + \vec{j} + 4\vec{k}$ [0.5+2.5]

2. Find the parametric and symmetric equation of the straight line through (2, 1,0) and perpendicular to both the vectors $\vec{i} + \vec{j}$ and $\vec{j} + \vec{k}$. [1.5+1.5]

3. Define normal and bi-normal vectors of a vector function of scalar variable t.

   Find the bi-normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, where $x = t^2, y = t^2, z = t^3$ at point (1, 1, 0). [1.5+1.5]

4. Find the domain and range of the function $f(x,y) = \sqrt{4 - x^2 - y^2}$. Sketching a Contour map, describe the level curves of the function for the values $c = 0, 1, 2, 3$.
   Find the limit, if it exists, or show that the limit does not exist at indicated point:

   (a) $f(x, y) = \dfrac{x^2 y}{x^4 + y^2}$ at (0, 0)

   (b) $f(x, y) = \dfrac{xy^3}{x + y}$ at (−1, 2) [1.5+1.5]

5. Show that the function $u = e^x \sin y + e^y \cos x$ satisfies Laplace's equation

   $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0.$$ [3]

Group B [5×6=30]

6. Find the vector equation of a straight line through the two vectors $\vec{a}$ and $\vec{b}$. Find an equation of the plane (by vector method) through the points whose position vectors are $3\vec{i} - \vec{j} + 2\vec{k}$, $8\vec{i} + 2\vec{j} + 4\vec{k}$ and $-\vec{i} - 2\vec{j} - 3\vec{k}$ [3+3]

7. Derive the formula for the derivative of vector triple product $\frac{d}{dt}[\vec{r_1} \times (\vec{r_2} \times \vec{r_3})]$ of three vectors $\vec{r_1}, \vec{r_2}$ and $\vec{r_3}$. If $\vec{r_1} = a\cos t\,\vec{i} + b\sin t\,\vec{j}$, $\vec{r_2} = -a\sin t\,\vec{i} + b\cos t\,\vec{j} + t\vec{k}$ and $\vec{r_3} = \vec{i} + 2\vec{j} + 3\vec{k}$, find $\frac{d}{dt}[\vec{r_1}\cdot(\vec{r_2} \times \vec{r_3})]$. [3+3]

OR

Define curvature of the vector function of scalar variable $\vec{r} = \vec{r}(t)$. Find the curvature of the curves

(a) $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$ at point $\theta = 0$. [3]

(b) $y = a \log \sec \frac{x}{a}$ at any point (x, y). [3]

8. Find the domain and range of the function $f(x, y) = \sqrt{4 - x^2 - y^2}$. Describe the graph of f. Sketch a contour map of this surface using level curves corresponding to $c = 0, 1, 2,$ 3,4. [1.5 + 1.5 + 1.5 + 1.5]

9. (a) Let $f(x, y)$ be defined on an open disk D that contains the point (a, b). Prove that if the functions f_{xy} and f_{yx} are continuous on D, then $f_{xy}(a, b) = f_{yx}(a, b)$. [4]

(b) Let $f(x, y) = e^x \cos y$. Confirm that the mixed second-order partial derivatives of f are the same. [2]

10. (a) Prove that if a function $z = f(x, y)$ is differentiable at a point(a, b), then it is continuous at the point. [3]

(b) Show that $f(x, y) = xe^{xy}$ is differentiable at $(1, 0)$ and find its linearization there. Then use it to approximate $f(1.1, -0.1)$. [3]

OR

(a) Prove that if f is a differentiable function of x and y, then f has a directional derivative at (x_0, y_0) in the direction of any unit vector $u = (a, b)$ and

$$D_u f(x_0, y_0) = f_x(x_0, y_0)a + f_y(x_0, y_0)b.$$ [3]

(b) Find the closest points from the origin to the curve $x^2y = 16$. [3]

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Tribhuvan University
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SCHOOL OF MATHEMATICAL SCIENCES

First Re-Assessment 2080

Subject: **Multivariable Calculus for Data Science**
Course No: MDS 554
Level: **MDS /I Year /II Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2.00hrs

Candidates are required to give their answer in their own words as far as practicable. All questions carry equal marks.

Group A [5×3]

1. Determine whether the points $A(1, 3, 2)$, $B(3, -1, 6)$, $C(5, 2, 0)$, and $D(3, 6, -4)$ lie in the same plane. **[3]**
2. If $\vec{u} + \vec{v}$ and $\vec{u} - \vec{v}$ are orthogonal, show that the vectors $\vec{u}$ and $\vec{v}$ must have the same length. Also write a vector equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$. **[2+1]**
3. Define curvature of a vector function $\vec{r} = \vec{r}(t)$. Find the curvature of the vector function $\vec{r}(t) = a \cos t \,\vec{i} + a \sin t \,\vec{j}$, where a is constant. **[1+2]**
4. Find the domain and range of the function $f(x, y) = \frac{y}{\sqrt{x}}$. **[3]**
5. Show that the function $u = \ln(x + ct)$ satisfies the wave equation $\frac{\partial^2 u}{\partial t^2} = c^2\frac{\partial^2 u}{\partial x^2}$. **[3]**

Group B [5 ×6]

6. (a) Find a vector equation of a straight line through the given vector $\vec{a}$ and parallel to the vector $\vec{b}$. **[3]**
   (b) Find a vector equation and parametric equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$. **[3]**
7. (a) Write the formula for the derivative of scalar triple product of three vectors $\vec{r}_1, \vec{r}_2$ and $\vec{r}_3$. If $\vec{r}_1 = a \cos t \,\vec{i} + b \sin t \,\vec{j}$, and $\vec{r}_2 = -a \sin t \,\vec{i} + b \cos t \,\vec{j} + t\vec{k}$, find $\frac{d}{dt}(\vec{r}_2 \times \vec{r}_1)$. **[1+2]**
   (b) Evaluate the derivative of $\frac{\vec{r}}{r}$ w.r.t t. **[1]**
   (c) If $\frac{d\vec{a}}{dt} = \vec{c} \times \vec{a}$ and $\frac{d\vec{b}}{dt} = \vec{c} \times \vec{b}$, show that: $\frac{d}{dt}(\vec{a} \times \vec{b}) = \vec{c} \times (\vec{a} \times \vec{b})$. **[2]**

OR

7. (a) Find the tangent, normal and bi-normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ where $x = t, y = t^2, z = t^2$ at point $(1, 1, 1)$. **[3]**
   (b) Find the radius of curvature at any point ϕ for the parametric curve $x = a \cos \phi, y = b \sin \phi$. **[3]**

8. Find $\lim_{(x,\ y)\to(0,0)} \frac{3x^2y}{x^2+y^2}$ if it exists. [6]

9. (a) Prove that if $x = x(t)$, $y = y(t)$ and $z = f(x, y)$ are differentiable functions, then $z = f(x(t), y(t))$ is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial z}{\partial y}\frac{\partial y}{\partial t},$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y).

[4]

(b) If $z = f(x, y)$ and f has continuous second order partial derivatives and $x = r^2 + s^2$, $y = 2rs$, find $\frac{\partial z}{\partial r}$, $\frac{\partial^2 z}{\partial r^2}$. [2]

10. Find and classify all the critical points of $f(x, y) = 4 + x^3 + y^3 - 3xy$. [3]

OR

(a) Assume that $f(x, y)$ and $g(x, y)$ are differentiable functions. Prove that if $f(x, y)$ has a local minimum or a local maximum on the constraint curve $g(x, y) = 0$ at $P = (a, b)$, and if $\nabla g_P \neq 0$, then there is a scalar λ such that $\nabla f_P(x, y) = \lambda \nabla g_P(x, y)$.

[3]

(b) Find the point on the sphere $x^2 + y^2 + z^2 = 1$ farthest from $(1, 2, 3)$. [3]

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Second Assessment 2080

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Course No: **MDS 554**
Level: **MDS /I Year /II Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Group A [5×3=15]

1. Use a Riemann sum with $m = 2$, $n = 3$ to estimate the volume under $f(x,y) = 1-xy^2$ above the Rectangle $R= [0, 4] \times [-1, 2]$. Take the sample points to be (a) the lower right corners and (b) theupper left corners of the rectangles.

2. Evaluate $\int_2^3 \int_{-1}^4 \int_1^0 (4x^2y - z^3)dz\, dy\, dx$.

3. Define Gradient of a vector point function. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, then prove that $\nabla r^m = mr^{m-2}\vec{r}$.

4. Define line integral along a curve C. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2y^2\vec{i} + y\vec{j}$ and C is the curve from (0, 0) to (4, 4) along the parabola $y^2 = 4ax$.

5. State Gauss's divergence theorem. If a closed surface S encloses a volume V and $\vec{F} = x\vec{i} + 2y\vec{j} + 3z\vec{k}$, Using Gauss's divergence theorem, find the value of $\iint_S \vec{F} \cdot \vec{n}\, ds$.

Group B [5 ×6=30]

6. a) Evaluate $\iint_D 5x^3 \cos y^3 dA$ where D is the region bounded by $y = 2$, $y = \frac{1}{4}x^2$ and the y-axis.

   b) A lamina occupies the part of the disk $x^2 + y^2 \leq 1$. Find its center of mass if the density at any point is proportional to the square of itsdistance from the origin.

OR

   a) Use a double integral to determine the volume of the solid that is bounded by $z = 8 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 4$.

   b) Evaluate $\iint_D xy^3 dA$ where D is the region bounded by $xy = 1, xy = 3, y = 2$ and $y= 6$ using the transformation $x = \frac{v}{6u}$, $y= 2u$.

7. **a)** Evaluate $\iiint\limits_E (12y - 8x)\, dV$ where E is the region behind $y = 10 - 2z$ and in front of the region in the xz- plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

b) Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2$, $0 \le y \le 1$.

8. **a)** Evaluate $\iiint\limits_E dV$, where E is the solid enclosed by the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$. Use the transformation $x = au$, $y = bv$, $z = cw$.

b) Determine the surface area region formed by the intersection of the two cylinders $x^2 + y^2 = 9$ and $x^2 + z^2 = 9$.

9. Define Surface integral. Let $\vec{G} = (x^2 - yz)\,\vec{i} + (y^2 - zx)\,\vec{j} + (z^2 - xy)\,\vec{k}$ be a vector field. Is $\vec{G}$ a irrotational? Justify. Also find a scalar function Ψ such that $\vec{G} = \nabla \Psi$. **[1+2+3]**

10. **a)** State Green's theorem in the XY- plane. Use Green's theorem to find the area bounded by the curve $4x^2 + 9y^2 = 36$. [1+2]

b) Verify Green's theorem in the plane for $\int_C [(3xy - 3x^2)\, dx + (2x + y^2)\, dy]$ where C is the closed curve given by the line $y = x$ and parabola $x = y^2$. [3]

OR

a) Find the equation of the tangent plane to the surface with parametric equation $x = u^2$, $y = v^2$, and $z = u + 2v$. [3]

b) State Stokes' theorem in a surface S. Verify Stokes' theorem for the vector function $\vec{F} = x\,\vec{i} + y\,\vec{j}$ around the square boundary $x = 0$, $y = 0$, $x = 2$, $y = 3$. [3]

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2079

Subject: **Multivariable Calculus for Data Science**
Course No: MDS 554
Level: **MDS /I Year /II Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Attempt **ALL** questions.

Group A [5×3 =15]

1. Find the normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$,where $x = t^2$, $y = t^2, z = t^3$ at point (1, 1, 1).

2. Find the equation of the tangent plane to $z = x^2 \cos(\pi y) - \frac{6}{xy^2}$ at point $(2,-1)$.

3. Find and classify all the critical points of the function: $f(x, y) = (y-2)x^2 - y^2$.

4. Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2, 0 \le y \le 1$.

5. With the help of Gauss's divergence theorem, show that

$$\iint_S \vec{F} \cdot \hat{n}\, ds = \frac{4}{3}\pi(a+b+c)$$

where $\vec{F} = ax\vec{i} + by\vec{j} + cz\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 1$.

Group B [5 ×6 =30]

6. Establish the vector equation of a straight line through two points $\vec{a}$ and $\vec{b}$. Find the vector equation of the line through the point (2, 1,0) and perpendicular to both the vectors $\vec{k} - 2\vec{j}$ and $\vec{j} + 2\vec{k}$. Also find the scalar and vector projections of $\vec{q} = \vec{i} - \vec{j} + \vec{k}$ onto $\vec{p} = \vec{i} + \vec{j} + \vec{k}$. [2+2+2]

7. Derive the expression for the derivative of vector triple product of three vectors. Find the derivative of the scalar triple product of the vectors $\vec{p} = (a\cos t, b\sin t, 0)$, $\vec{q} = (-a\sin t, b\cos t, t)$ and $\vec{r} = (1, 2, 3)$ at $t = 0$. [3+3]

8. Prove that if $x = x(t)$ and $y = y(t)$ are differentiable functions of t and $z = f(x, y)$ is a differentiable function of x and y, then $z = f(x(t), y(t))$ is a differentiable function of t and $\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$,
where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y). Also, find the maximum rate of change of $f(s, t) = te^{st}$ at point (0, 2) in the direction at which this maximum rate of change occurs. [3+3]

Calculate $\partial z/\partial s$ and $\partial z/\partial t$ using the following functions: $z = e^{x+2y}$, $x = s/t$, $y = t/s$.
Also Show that $u = e^{-x}\cos y - e^{-y}\cos x$ satisfies the Laplace equation $u_{xx} + u_{yy} = 0$.

[3+3]

9. Evaluate $\iint_R xy^3\, dA$ where R is the region bounded by $xy = 1, xy = 3, y = 2 , y = 6$ using the transformation $x = \frac{y}{6u}$, $y = 2u$. Using polar coordinates, find the area of the part of the surface $z = xy$ that lies within the cylinder $x^2 + y^2 = 1$. [3+3]

10. State Green's theorem in the plane and use it to find the area of the circle of radius 4 unit. Verify Green's theorem in the plane for $\int_C (2xy - x^2)\, dx + (x + y^2)\, dy$ where C is the closed curve given by the line $y = x$ and parabola $x = y^2$.

[1+2+3]

OR

Find the equation of the tangent plane to the surface with parametric equation $x = u^2, y = v^2$, and $z = u + 2v$. Verify Stokes' theorem for the vector function $\vec{F} = x\vec{i} + y\vec{j}$ around the square boundary $x = 0, y = 0, x = a, y = a$. [3+ 3]

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Tribhuvan University
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First Assessment 2081

Subject: **Multivariable Calculus for Data Science** *Full Marks*: 45
Course No: MDS 554 *Pass Marks*: 22.5
Level: **MDS /I Year /II Semester** *Time*: 2 hrs

Candidates are required to give their answers in their own words as far as practicable.

Attempt ALL questions.

Group A [5×3= 15]

1. Define vector projections of a vector $\vec{a}$ to another vector $\vec{b}$. Find the vector projection of $\vec{b} = (1,-1,2)$ onto $\vec{a} = 7\vec{i} + \vec{j} + 2\vec{k}$ [1+2]
2. Find an equation of the plane (by vector method) through the points whose position vectors are $3\vec{i} - \vec{j} + 2\vec{k}$, $8\vec{i} + 2\vec{j} + 4\vec{k}$ and $-\vec{i} - 2\vec{j} - 3\vec{k}$
3. Prove that the limit $\lim_{(x,y)\to(0,0)} \frac{xy - y^2}{y^2 + x}$ does not exist.
4. Show that the function $u(x, y) = e^x \cos y$ satisfy the Laplace equation $u_{xx} + u_{yy} = 0$.
5. Find the directional derivative of the function $f(x, y) = e^x \sin y$ at the point $P(1, \pi/2)$ in the direction of $v = -\vec{i}$.

Group B [5 ×6 = 30]

6. Derive a vector equation of a straight line (a) through the given point $\vec{a}$ and parallel to the vector $\vec{b}$ (b) through two points $\vec{a}$ and $\vec{b}$.
Also find a vector equation for the line through the point (1, 0, 6) and perpendicular to the plane $x + 3y + z = 5$. [(2+2)+2]

7. a) Define scalar triple product of three vectors and state its geometrical meaning. Are the following four points (2, 3, −1), (1, −2, 3), (3, 4, −2) and (1, − 6, 4) coplanar? [1+2]
b) If $\vec{v}$ is a unit vector, prove that $\left|\vec{v} \times \frac{d\vec{v}}{dt}\right| = \left|\frac{d\vec{v}}{dt}\right|$.

8. a) If $\vec{r_1} = t\vec{i} + \sin t\vec{j}$, $\vec{r_2} = -\sin t\vec{i} + t\vec{j} + \vec{k}$ and $\vec{r_3} = \vec{i} + 2\vec{j} + t\vec{k}$, find $\frac{d}{dt}[\vec{r_1}\ (\vec{r_2} \times \vec{r_3})]$.
b) Define curvature of the vector function of scalar variable $\vec{r} = \vec{r}(t)$. Find the curvature of the curve $y = a \log \sec \frac{x}{a}$ at any point (x, y). [1+2]

OR

Define the divergence and curl of a vector point function. If $f(x,y,z) = x^2y - y^3z^2$, find ∇f at the point. Evaluate: ∇r^3 where $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$. [2+2+2]

9.a) If a function $z = f(x, y)$ is differentiable at a point, then it is continuous at the point

b) Use the Chain Rule to calculate $\frac{d}{dt} f(c(t))$.

$$f(x, y) = x^2 - 3xy, \qquad c(t) = (\cos t, \sin t), \qquad t = 0$$

OR

a) If f is a differentiable function of x and y, then f has a directional derivative at (x_0, y_0) in the direction of a unit vector $u = (a, b)$ and

$$D_u f(x_0, y_0) = f_x(x_0, y_0)a + f_y(x_0, y_0)b.$$

b) Find the gradient of the function $f(x, y, z) = 3x^2 - 5y^2 + 2z^2$ at the point $P(1, 1, -2)$.

10. Find the extreme values of $f(x, y) = x^2 + 2y^2$ subject to the constraint $g(x, y) = 4x - 6y = 25$.

a) Show that the Lagrange equations yield $2x = 4\lambda$, $4y = -6\lambda$.

b) Show that if $x = 0$ or $y = 0$, then the Lagrange equations give $x = y = 0$. Since $(0, 0)$ does not satisfy the constraint, you may assume that x and y are nonzero.

c) Use the Lagrange equations to show that $y = -\frac{3}{4}x$.

d) Substitute in the constraint equation to show that there is a unique critical point P.

e) Does P correspond to a minimum or maximum value of f?

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Tribhuvan University

Institute of Science and Technology

2081

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MDS / I Year / Second Semester/ Science
Data Science (MDS 554)
(Multivariable Calculus for Data Science)

Full Marks: 45
Pass Marks: 22.5
Time: 2 hours.

Candidates are required to give their answers in their own words as for as practicable.
The figures in the margin indicate full marks. The symbols have their usual meanings.
Attempt all questions.

Group A [5 × 3 = 15]

1. Define curvature. Find the radius of curvature at any point ϕ for the parametric curve $x = a\cos\phi$, $y = b\sin\phi$. [3]

2. Prove that the limit $\lim_{(x, y)\to(0, 0)} \dfrac{xy - y^2}{y^2 + x}$ does not exist. [3]

3. Find the gradient of the function $z = \ln(x^2 - y)$ at the point (2, 3). [3]

4. Evaluate the iterated integral $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1}(x^2 + y^2 + z^2)\,dx\,dy\,dz.$ [3]

5. State Gauss divergence theorem. Find the value of $\iint_S \vec{F}\cdot\hat{n}ds$, where $\vec{F} = px\vec{i} + qy\vec{j} + rz\vec{k}$ and S is the surface of the sphere $x^2 + y^2 + z^2 = 9$. [3]

Group B [5 × 6 = 30]

6. Using vector method, prove the projection law c = a cosB + b cosA in any triangle. Write the vector equation of a straight line through two points. Find the vector equation of the straight line through the point (1, 2,0) and perpendicular to both the vectors $\vec{j} + 2\vec{k} + \vec{i}$ and $\vec{i} + \vec{k} - 2\vec{j}$. [2+1+3]

7. a) Write the formula for the derivative of scalar triple product of three vectors $\vec{r_1}$, $\vec{r_2}$ and $\vec{r_3}$. If $\vec{r_1} = a\cos t\,\vec{i} + b\sin t\,\vec{j}$, and $\vec{r_2} = -a\sin t\,\vec{i} + t\vec{k}$, find $\dfrac{d}{dt}(\vec{r_2}\times\vec{r_1})$. [1+2]

   b) Find the bi-normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, where $x = t^2, y = t, z = t$ at point (1, 1, 1). [3]

8. a) Find the linearization $L(x, y)$ of $f(x, y) = x^2y^3$ at $(a, b) = (2, 1)$. Use it to estimate $f(2.01, 1.02)$ and $f(1.97, 1.01)$ and compare with actual values. [3+3]

b) Use the Chain Rule to calculate $\frac{d}{dt}f(\mathbf{c}(t))$, if $\mathbf{f}(x, y) = 3x - 7y$, $\mathbf{c}(t) = (t^2, t^3)$, $t = 2$.

[3+3]

OR

Find the minimum and maximum values of the function subject to the given constraint if $f(x, y) = x^2y^4$, $x^2 + 2y^2 = 6$ [6]

9. a) Use double integration to find the area of the plane region enclosed by the curves $y^2 = 9 - x$ and $y^2 = 9 - 9x$. [6]

b) Find the center of mass of a thin plate of density $\rho = 3$ bounded by the lines $x = 0$, $y = x$, and the parabola $y = 2 - x^2$ in the first quadrant. [3+3]

10. Use Green's theorem to find the area of hypocycloid $x^{2/3} + y^{2/3} = a^{2/3}$.

Verify Green's theorem in the plane for $\int_C (2xy - x^2)\,dx + (x + y^2)\,dy$ where C is the closed curve given by the line $y = x$ and parabola $x = y^2$. [3+3]

OR

Define divergence and curl of a vector function. State Stokes' theorem. Verify Stokes' theorem for the vector function $\vec{F} = x\vec{i} + y\vec{j}$ around the square boundary $x = 0, y = 0, x = p, y = p$. [2+ 4]

Tribhuvan University
Institute of Sciences and Technology

SCHOOL OF MATHEMATICAL SCIENCES

First Re-Assessment 2082

Subject: **Multivariable Calculus for Data Science** | *Full Marks*: 45
Course No: MDS 554 | *Pass Marks*: 22.5
Level: **MDS /I Year /II Semester** | *Time*: 2.00 hrs

Candidates are required to give their answer in their own words as far as practicable.

Group A [5×3]

1. Determine whether the points $P(3, 6, -4)$, $Q(1, 3, 2)$, $R(3, -1, 6)$, $S(5, 2, 0)$ are coplanar or not? [3]

2. Write a vector equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + 2z = 5$. If $\vec{w} + \vec{u}$ and $\vec{w} - \vec{u}$ are perpendicular, show that the vectors $\vec{w}$ and $\vec{u}$ must have the same length. [1+2]

3. Define curvature of a vector function $\vec{r} = \vec{r}(t)$. Find the curvature of the vector function $\vec{r}(t) = a\cos t\,\vec{i} + a\sin t\,\vec{j}$, where a is constant. [1+2]

4. Show that the function $z = e^{-t}\sin(x/c)$ satisfies the **heat equation** $u_t = u_{xx}$ for $t > 0$. [3]

5. Find the linearization $L(x, y)$ of $f(x, y) = xe^{y}$ at $(a, b) = (0.9, 0.2)$. Compare the value of the approximation $L(0.9, 0.2)$ with the exact value $f(0.9, 0.2)$. [3]

Group B [5 ×6]

6. (a) Find a vector equation of a straight line through the given vector $\vec{v}$ and parallel to the vector $\vec{u}$. [3]

   (b) Find a vector equation and parametric equation for the line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$. [3]

7. (a) Write the formula for the derivative of scalar triple product of three vectors $\vec{r_1}, \vec{r_2}$ and $\vec{r_3}$. If $\vec{r_1} = a\cos t\,\vec{i} + b\sin t\,\vec{j}$, and $\vec{r_2} = -a\sin t\,\vec{i} + b\cos t\,\vec{j} + t\vec{k}$, find $\frac{d}{dt}(\vec{r_2} \times \vec{r_1})$. [1+2]

   (b) Evaluate the derivative of $\frac{\vec{r}}{r}$ w.r.t t. [1]

   (c) If $\frac{d\vec{p}}{dt} = \vec{r} \times \vec{p}$ and $\frac{d\vec{q}}{dt} = \vec{r} \times \vec{q}$, show that: $\frac{d}{dt}(\vec{p} \times \vec{q}) = \vec{r} \times (\vec{p} \times \vec{q})$. [2]

OR

7. (a) Find the tangent and normal vector of the space curve $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ where $x = t, y = t^2, z = t^2$ at point $(1, 1, 1)$. [3]

   (b) Find the radius of curvature at any point ϕ for the parametric curve $x = a\cos\phi$, $y = b\sin\phi$. [3]

8. Find the domain and range of the function $f(x, y) = \sqrt{x^2 + 4y^2}$. Describe the graph of f. Sketch the level curves for the function corresponding to $c = 1, 2, 3, 4, 5$. [1.5+1.5+1.5+1.5]

9. (a) Define the directional derivative of a function f at (x_0, y_0). Prove that if f is a differentiable function of x and y, then f has a directional derivative at (x_0, y_0) in the direction of a **unit vector** $u = (a, b)$ and $D_u f(x_0, y_0) = f_x(x_0, y_0)a + f_y(x_0, y_0)b$. [3]

(b) Find the directional derivative of $f(x, y, z) = xy + z^3$ at $P = (3, -2, -1)$ in the direction pointing to the origin. [3]

OR

(a) Prove that if $x = x(t)$ and $y = y(t)$ are differentiable functions of t and $z = f(x, y)$ is a differentiable function of x and y, then $z = f(x(t), y(t))$ is a differentiable function of t and

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y}\frac{dy}{dt},$$

where the ordinary derivatives are evaluated at t and the partial derivatives are evaluated at (x, y). [3]

(b) Calculate $\partial z/\partial s$ and $\partial z/\partial t$ at the indicated s and t values if [3]

$$z = x^2 y,\ x = s - t,\ y = 2s + 4t; \quad s = 1, t = 0.$$

10. Find the minimum and maximum values of the function $f(x, y) = 2x + 3y$ subject to the constraint $x^2 + y^2 = 4$. [6]

$x = 1$

$y = \sqrt{3}$

$1 + 3 = 4.$

$2 \times 1 + 3 \times \sqrt{3} = 7.19$

Tribhuvan University

Institute of Sciences and Technology

SCHOOL OF MATHEMATICAL SCIENCES

Second Assessment 2082

Subject: **Multivariable Calculus for Data Science** — *Full Marks:* 45

Course No: **MDS 554** — *Pass Marks:* 22.5

Level: **MDS /I Year /II Semester** — *Time:* 2 hrs

Candidates are required to give their answer in their own words as far as practicable.

Attempt all questions

Group A [5×3=15]

1. Use a Riemann sum with $m = 2$, $n = 3$ to estimate the volume under $f(x, y) = 1 + x^2 + 3y$ above the Rectangle $R = [1, 2] \times [0, 3]$. (a) Take the sample points to be the lower right corners. (b) Use the Midpoint Rule to estimate the volume in part (a).

2. Evaluate $\int_0^2 \int_{-1}^{y^2} \int_{-1}^{z} yz\, dx\, dz\, dy$.

3. State Stokes' theorem in a surface S. Use this theorem to prove that $\int_C \vec{r} \cdot d\vec{r} = 0$.

4. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = y^2\,\vec{i} + x^2\,\vec{j}$ and C is the straight line from $(0, 0)$ to $(1, 0)$ and then from $(1, 0)$ to $(1, 1)$.

5. State Gauss's divergence theorem. If a closed surface S encloses a volume V. Using Gauss's divergence theorem, find the value of $\iint_S \vec{F} \cdot \vec{n}\, ds$, where $\vec{F} = 2x\,\vec{i} + 2y\,\vec{j} + 3z\,\vec{k}$

Group B [5 ×6=30]

6. a) Evaluate $\iint_D 5x^3 \cos y^3\, dA$ where D is the region bounded by $y = 2$, $y = \frac{1}{4}x^2$ and the y-axis.

   b) A lamina occupies the part of the disk $x^2 + y^2 \le 1$. Find its center of mass if the density at any point is proportional to the square of its distance from the origin.

7. a) Use a double integral to determine the volume of the solid that is bounded by $z = 8 - x^2 - y^2$ and $z = 3x^2 + 3y^2 - 4$.

   b) Evaluate $\iint_D xy^3\, dA$ where D is the region bounded by $xy = 1$, $xy = 3$, $y = 2$ and $y = 6$ using the transformation $x = \frac{v}{6u}$, $y = 2u$.

8. a) Evaluate $\iiint_E (12y - 8x)\, dV$ where E is the region behind $y = 10 - 2z$ and in front of the region in the xz-plane bounded by $z = 2x$, $z = 5$ and $x = 0$.

$x^2 = 4y$

x 0 ±2

y 0 1

(0,0)

$x^2 = 4y$

$\vec{F}\cdot d\vec{r} = \int$

$z = \frac{1}{4}x^2$

$x^2 = 8$

$x = \pm 2\sqrt{2}$

64 × 2 = 128 × 2 = 256

$\int \vec{F} \cdot \vec{n}\, dr = \iiint_V dV$

$dy\,dx$

$dx\,dz$

2x

$(\nabla \cdot \vec{F})\, dv$

$x^2 = 4y$

$x^2 = 8$ $x = \pm 2\sqrt{2}$

b) Use a triple integral to determine the volume of the region below $z = 4 - xy$ and above the region in the xy-plane defined by $0 \le x \le 2$, $0 \le y \le 1$.

OR

a) Solve for x and y in terms of u and v, and then find the Jacobian $\partial(x, y)/\partial(u, v)$, if $u = 2x - 5y$, $v = x + 2y$

b) Use the transformation $u = x - 2y$, $v = 2x + y$ to find $\iint_R \frac{x - 2y}{2x + y}\, dA$ where R is the rectangular region enclosed by the lines $x - 2y = 1$, $x - 2y = 4$, $2x + y = 1$, $2x + y = 3$.

9. a) Let $\vec{G} = (2xz^3 + 6y)\,\vec{i} + (6x - 2yz)\,\vec{j} + (3x^2z^2 - y^2)\,\vec{k}$

be a vector field. Is $\vec{G}$ a conservative? Justify. Also find its scalar potential. [1+2]

b) Evaluate $\iint_S \vec{F} \cdot \vec{n}\, ds$ (the flux) for the vector field $\vec{F} = yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k}$ through that of the sphere $x^2 + y^2 + z^2 = 1$ in the first octant

10. a) State Green's theorem in the XY- plane. Use Green's theorem to find the area bounded by the astroid $x^{2/3} + y^{2/3} = a^{2/3}$. [1+2]

b) Verify Green's theorem in the plane for $\int_C [(3xy - 3x^2)\, dx + (2x + y^2)\, dy]$ where C is the closed curve given by the line $y = x$ and parabola $x = y^2$. [3]

OR

a) Find the equation of the tangent plane to the surface with parametric equation $x = u + v$, $y = u^2$, and $z = v^2$. [3]

b) Verify Stokes' theorem for the vector function $\vec{F} = x^2\,\vec{i} + y^2\,\vec{j}$ around the square boundary $x = 0$, $y = 0$, $x = 6$, $y = 3$. [3]

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x 0 -1 2 -2

y 0 1 4 4

$\pi ab,\ \pi r^2$

$\frac{3}{8} a^2 \cdot 2\pi = \frac{4}{8}\pi a^3$

$2 \cdot \left(\frac{\cos 2\theta}{2}\right)$

~~cos~~ 3

Tribhuvan University
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SCHOOL OF MATHEMATICAL SCIENCES
Second Reassessment 2082

Subject: **Multivariable Calculus for Data Science**
Course No: **MDS 554**
Level: **MDS. /I Year /II Semester**

Full Marks: 45
Pass Marks: 22.5
Time: 2 hrs

Candidates are required to give their answers in their own words as far as practicable. All questions carry equal marks.

Attempt ALL questions.

Group A [5×3=15]

1. Use a Riemann sum with $m = 2, n = 3$ to estimate the volume under $f(x, y) = 1 + x^2 + 3y$ above the Rectangle $R = [1, 2] \times [0, 3]$ (a) Take the sample points to be the lower right corners. (b) Use the Midpoint Rule to estimate the volume in part (a)
2. Evaluate $\int_1^3 \int_x^{x^2} \int_0^{\ln z} xe^y \, dy \, dz \, dx$.

3. Let $\vec{G} = yz\vec{i} + zx\vec{j} + xy\vec{k}$ be a vector field. Is $\vec{G}$ a irrotational? Justify. Also find its scalar potential.
[1+2]

4. Define line integral along a curve C. Evaluate the line integral $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F} = x^2y^2\vec{i} - y\vec{j}$ and C is the curve from (0, 0) to (4, 4) along the parabola $y^2 = 4ax$.
[1+2]

5. State the formula for Gauss Divergence Theorem. Applying Gauss' theorem, evaluate $\iint_S \vec{r} \cdot \vec{n} \, dS$, where S is a closed surface enclosing a volume V and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$.
[1+2]

Group B [5×6=30]

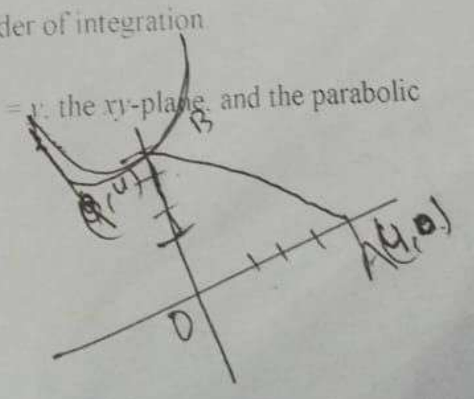
a) By converting to polar coordinates, evaluate the iterated integral $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2) \, dy \, dx$.

b) Find the mass of the rectangular region $0 \le x \le 2$, $0 \le y \le 1$ with density function $\rho(x, y) = 1 - y$

a) Use a double integral to determine the volume of the solid bounded by the cylinder $x^2 + y^2 = 9$ and the planes $z = 0$ and $z = 3 - x$.

b) Evaluate the integral $\int_0^1 \int_{4x}^4 e^{-y^2} \, dy \, dx$ by first reversing the order of integration

8. a) Evaluate $\iiint_E y \, dV$, where E is the solid enclosed by the plane $z = y$, the xy-plane, and the parabolic cylinder $y = 1 - x^2$



$M_x = \iint p(x,y) \, dx \, dy$

$M_y = \iint p(x,y) \, dx \, dy$

b) Use a triple integral to determine the volume of the solid in the first octant bounded by the coordinate planes and the plane $3x + 6y + 4z = 12$

OR

a) Find the Jacobian $\partial(x, y, z)/\partial(u, v, w)$, if $x = 3u + v$, $y = u - 2w$, $z = v + w$

b) Evaluate $\iint_R (6x - 3y)\, dA$ where R is the parallelogram with vertices $(2, 0)$, $(5, 3)$, $(6, 7)$ and $(3, 4)$ using the transformation $x = (1/3)(v - u)$, $y = (1/3)(4v - u)$ to R

9. What is meant by a parametric space? Find the equation of the tangent plane to the surface with parametric equation $x = u^2$, $y = v^2$, and $z = u + 2v$. Prove that the area enclosed by a simple closed curve C is given by $= \frac{1}{2} \int_C (x\, dy - y\, dx)$ [1+3+2]

OR

State Green's theorem in the plane and use Green's theorem to find the area bounded by the ellipse $x^2 + 4y^2 = 4$. Verify Green's theorem in the plane for $\int_C (2y - 4x^2)\, dx + (3x + y^2)\, dy$ where C is the closed curve given by the line $y = 2x$ and parabola $x = y^2$ [1+3+2]

10. State Stokes' theorem in a surface S. Show that in a plane, Green's theorem is a particular case of Stokes' theorem. Verify Stokes' theorem for the vector function $\vec{F} = x\vec{i} + y\vec{j}$ around the square boundary $x = 0$, $y = 0$, $x = 3$, $y = 3$ [1+2+3]

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$(\vec{r} - \vec{a}) \cdot \vec{n} = 0$

$\vec{n} = \vec{r}_u \times \vec{r}_v$