Backprop from Scratch

June 3, 2022

[1]:
```python
import numpy as np
import matplotlib.pyplot as plt
```

[2]:
```python
# Hyperparameters
stepsize = 1e-4
```

[3]:
```python
def sigmoid(x):
    return 1 / (1 + np.exp(-x))
```

[4]:
```python
def loss(y, out):
    L = 0
    for i in range(y.size):
        L += (1/2) * (y[i]-out[i])**2
    return L
```

[5]:
```python
# Network:
# input x: R^(3*1)
# hidden layer h: R^(4*1)
# output layer out: R^(2*1)

# Graph:
# x -> W1*x = z -> sigmoid(z) = h -> W2*h = out

# weights
def init_weights():
    # init (W1, W2) ~ Normal Gaussian
    W1 = np.random.rand(4,3)
    W2 = np.random.rand(2,4)
    return W1, W2

def forward(x, W1, W2):
    z = np.dot(W1, x) # h*1
    h = sigmoid(z) #h*1

    out = np.dot(W2, h) #o*1
    return out
```

[6]:
```
# init model
W1, W2 = init_weights()
x = np.random.rand(3,1)

out = forward(x, W1, W2)
y = np.array([10,-2])[:,None]

print(x)
print(y)
print(out)
L = loss(y, out)
L
```

```
[[0.64939334]
 [0.7704799 ]
 [0.66021291]]
[[10]
 [-2]]
[[1.56664067]
 [1.00139225]]
```

[6]:
```
array([40.06495249])
```

[7]:
```
# calculate dL_dW2
def W2_grad(x, y, out, W1):
    h = sigmoid(np.dot(W1, x))
    dL_dW2 = np.dot((-y + out), h.T)
    return dL_dW2

W2_grad(x, y, out, W1)
```

[7]:
```
array([[-5.82016268, -4.9195419 , -5.69856975, -5.99530692],
       [ 2.07136806,  1.75084143,  2.02809372,  2.13370106]])
```

[8]:
```
# calculate dL_dW1
def W1_grad(x, y, out, W1, W2):
    z = np.dot(W1, x)

    a = (-y + out).T
    b = np.dot(a, W2)

    c = sigmoid(z) * (1 - sigmoid(z))

    d = b.T * c

    dL_dW1 = d * x.T
    return dL_dW1
```

```
W1_grad(x, y, out, W1, W2)
```

[8]:
```
array([[ 0.0231634 ,  0.02748247,  0.02354933],
       [-0.92055436, -1.09220188, -0.93589175],
       [-0.8262928 , -0.98036421, -0.84005969],
       [-0.5623849 , -0.66724777, -0.57175481]])
```

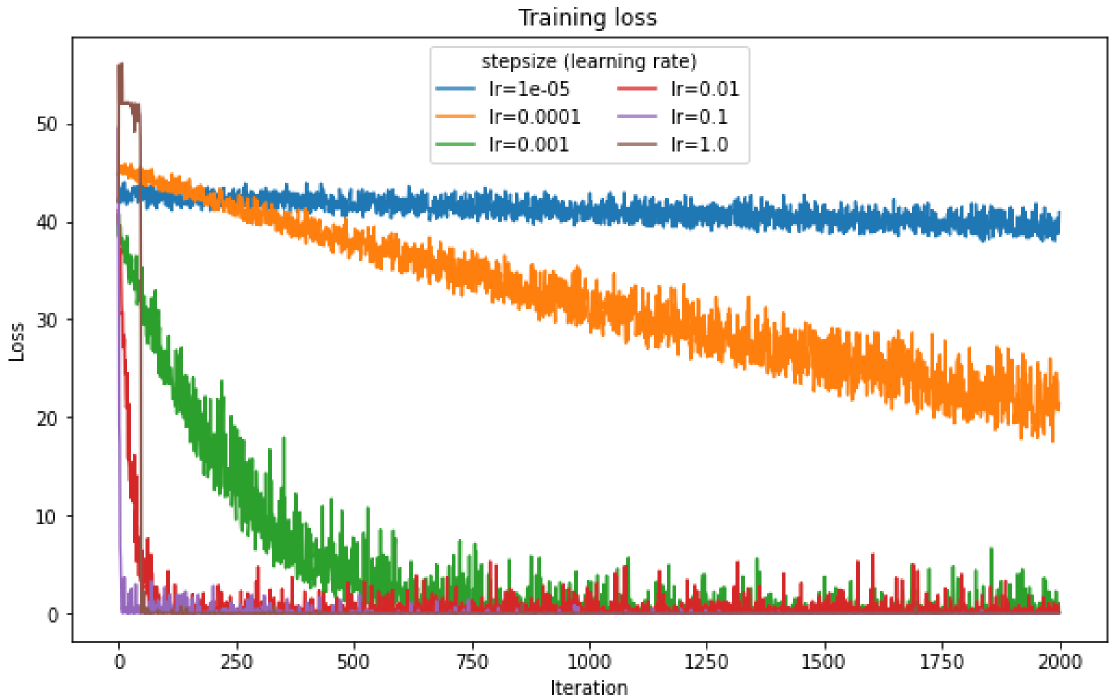
[9]:
```
def update_weights(x, y, out, W1, W2, stepsize):
    new_W1 = W1 - stepsize * W1_grad(x, y, out, W1, W2)
    new_W2 = W2 - stepsize * W2_grad(x, y, out, W1)
    return new_W1, new_W2

W1, W2 = update_weights(x, y, out, W1, W2, stepsize)
W1, W2
```

[9]:
```
(array([[0.39648036, 0.37602858, 0.38404979],
        [0.18591119, 0.04167148, 0.2785186 ],
        [0.19024909, 0.06115962, 0.8537801 ],
        [0.0545512 , 0.43444166, 0.80239174]]),
 array([[0.15106017, 0.73649631, 0.82716617, 0.66862462],
        [0.47818199, 0.12466736, 0.38767533, 0.47286998]]))
```

Fitting to a fixed label $y = [10, -2]$:

[10]:
```
# Training loop

plt.figure(figsize=(10, 6))
plt.title('Training loss')

learning_rates = [1e-5, 1e-4, 1e-3, 1e-2, 1e-1, 1e+0]
for stepsize in learning_rates:
    W1, W2 = init_weights()
    Losses = []

    for i in range(2_000):
        x = np.random.rand(3,1)
        y = np.array([10,-2])[:,None]
        out = forward(x, W1, W2)
        Losses.append(loss(y, out).item())
        W1, W2 = update_weights(x, y, out, W1, W2, stepsize)

    plt.plot(range(0, len(Losses)), Losses, label=f'lr={stepsize}')

plt.xlabel('Iteration')
plt.ylabel('Loss')
plt.legend(loc='upper center', title='stepsize (learning rate)',ncol=2)
```

```
plt.show()
```



[11]:

```
# inference
x = np.random.rand(3,1)
print("input x:\n", x, "\n")
y = np.array([10,-2])[:,None]
print("label y:\n", y, "\n")
pred = forward(x, W1, W2)
print("prediction:\n", pred, "\n")
print("Loss:", loss(y, pred))
```

```
input x:
 [[0.42963737]
 [0.79965055]
 [0.36776805]]

label y:
 [[10]
 [-2]]

prediction:
 [[ 9.88078792]
 [-1.97326016]]
```

```
Loss: [0.00746327]
```