Ex 3

Tutoriums-Anmerkungen in blau (oder rosa oder so)

$$H_\Delta = \hbar\omega_{eg}|e\rangle\langle e| + \hbar\omega_{sg}|s\rangle\langle s| + \frac{\hbar\Omega_1}{2}\left(e^{-i\omega_1 t}|e\rangle\langle g| + e^{i\omega_1 t}|g\rangle\langle e|\right) + \frac{\hbar\Omega_2}{2}\left(e^{-i\omega_2 t}|e\rangle\langle s| + e^{i\omega_2 t}|s\rangle\langle e|\right)$$

$$\downarrow U(t)$$

$$\tilde{H}_\Delta = -\hbar\Delta|e\rangle\langle e| - \hbar\delta|s\rangle\langle s| + \frac{\hbar}{2}\Omega_1\left(|e\rangle\langle g| + |g\rangle\langle e|\right) + \frac{\hbar\Omega_2}{2}\left(|e\rangle\langle s| + |s\rangle\langle e|\right)$$

$\Delta = \omega_1 - \omega_{eg}$

$\delta = \omega_1 - \omega_2 - \omega_{sg}$

Step 1 $U_1 = e^{-i\omega_1 t|e\rangle\langle e|}$ $\rightarrow$ $UH(t)U^\dagger$ step by step.

$$\underbrace{\left[UH(t)U^\dagger + i\hbar\dot{U}U^\dagger\right]}_{\tilde{H}(t)}$$

- $U|e\rangle\langle e|U^\dagger = |e\rangle\langle e|$, $U|e\rangle\langle g|U^\dagger = e^{i\omega_L t}|e\rangle\langle g|$
- $i\hbar\dot{U}U^\dagger = -\hbar\omega_L|e\rangle\langle e|$

$U\hbar\omega_{eg}|e\rangle\langle e|U^\dagger = |e\rangle\langle e|\hbar\omega_{eg}$

$U\hbar\omega_{sg}|s\rangle\langle s|U^\dagger = \hbar\omega_{sg}|s\rangle\langle s|$

$U\frac{\hbar\Omega_1}{2}e^{-i\omega_1 t}|e\rangle\langle g|U^\dagger = \frac{\hbar\Omega_1}{2}e^{-i\omega_1 t}e^{-i\omega_1 t}|e\rangle\langle g|$

$U\frac{\hbar\Omega_1}{2}e^{i\omega_1 t}|g\rangle\langle e|U^\dagger = \frac{\hbar\Omega_1}{2}e^{i\omega_1 t}e^{i\omega_1 t}|g\rangle\langle e|$

$U\frac{\hbar\Omega_2}{2}e^{-i\omega_2 t}|e\rangle\langle s|U^\dagger = \frac{\hbar\Omega_2}{2}e^{-i\omega_2 t}e^{-i\omega_1 t}|e\rangle\langle s|$

$U\frac{\hbar\Omega_2}{2}e^{i\omega_2 t}|s\rangle\langle e|U^\dagger = \frac{\hbar\Omega_2}{2}e^{i\omega_2 t}e^{i\omega_1 t}|s\rangle\langle e|$

$$U|e\rangle = e^{-i\omega_1 t|e\rangle\langle e|}|e\rangle = \begin{pmatrix}1 & 0\\ 0 & e^{-i\omega_1 t}\end{pmatrix}\begin{pmatrix}0\\1\end{pmatrix} = \begin{pmatrix}0\\ e^{-i\omega_1 t}\end{pmatrix} = e^{-i\omega_1 t}|e\rangle$$

$$U|g\rangle = e^{-i\omega_1 t|e\rangle\langle e|}|g\rangle = \begin{pmatrix}1 & 0\\ 0 & e^{-i\omega_1 t}\end{pmatrix}\begin{pmatrix}1\\0\end{pmatrix} = \begin{pmatrix}1\\0\end{pmatrix} \quad |g\rangle$$

$$\langle e|U^\dagger = (U|e\rangle)^\dagger = (e^{-i\omega_1 t}|e\rangle)^\dagger = \langle e|e^{i\omega_1 t}$$

$$\langle g|U^\dagger = \langle g|$$

$$U|e\rangle\langle g|U^\dagger = e^{-i\omega_1 t}|e\rangle\langle g|,\quad U|g\rangle\langle e|U^\dagger = |g\rangle\langle e|e^{i\omega_1 t}$$

$$\Rightarrow UH(t)U^\dagger = \hbar\omega_{eg}|e\rangle\langle e| + \hbar\omega_{sg}|s\rangle\langle s| + \frac{\hbar\Omega_1}{2}\left(e^{-2i\omega_1 t}|e\rangle\langle g| + e^{2i\omega_1 t}|g\rangle\langle e|\right) + \frac{\hbar\Omega_2}{2}\left(e^{-i(\omega_1+\omega_2)t}|e\rangle\langle s| + e^{i(\omega_1+\omega_2)t}|s\rangle\langle e|\right)$$

$i\hbar\dot{U}U^\dagger$:

$\dot{U} = i\omega_1 e^{i\omega_1 t|e\rangle\langle e|}$

$U = e^{i\omega_1 t|e\rangle\langle e|}$

$$\rightarrow \dot{U}U^\dagger = i\omega_1 e^{i\omega_1 t|e\rangle\langle e|}e^{-i\omega_1 t|e\rangle\langle e|} = i\omega_1|e\rangle\langle e|$$

$$\rightarrow i\hbar\dot{U}U^\dagger = -\hbar\omega_1|e\rangle\langle e| \rightarrow \hbar\omega_{eg}|e\rangle\langle e| - \hbar\omega_1|e\rangle\langle e| = -\hbar\Delta|e\rangle\langle e|$$

$$\Rightarrow \tilde{H}'_\Delta(t) = -\hbar\Delta|e\rangle\langle e| + \hbar\omega_{sg}|s\rangle\langle s| + \frac{\hbar\Omega_1}{2}\left(e^{-2i\omega_1 t}|e\rangle\langle g| + e^{2i\omega_1 t}|g\rangle\langle e|\right) + \frac{\hbar\Omega_2}{2}\left(e^{-i(\omega_1+\omega_2)t}|e\rangle\langle s| + e^{i(\omega_1+\omega_2)t}|s\rangle\langle e|\right)$$

Now step 2: $U_2 = e^{i(\omega_1-\omega_2)t\,|s\rangle\langle s|}$

$\Rightarrow -\hbar\Delta U |e\rangle\langle e| U^\dagger = -\hbar\Delta |e\rangle\langle e|$

$\hbar\omega_{sg} U |s\rangle\langle s| U^\dagger = \hbar\omega_{sg} |s\rangle\langle s|$

$\frac{\hbar\Omega_1}{2} e^{-2i\omega_1 t} U |e\rangle\langle g| U^\dagger = \frac{\hbar\Omega_1}{2} \cancel{e^{-2i\omega_1 t}} |e\rangle\langle g|$ fast oscillating

$\frac{\hbar\Omega_1}{2} e^{2i\omega_1 t} U |g\rangle\langle e| U^\dagger = \frac{\hbar\Omega_1}{2} \cancel{e^{2i\omega_1 t}} |g\rangle\langle e|$

$\frac{\hbar\Omega_2}{2} e^{-i(\omega_1+\omega_2 t)} U |e\rangle\langle s| U^\dagger = \frac{\hbar\Omega_2}{2} e^{-i(\omega_1+\omega_2)t} e^{-i(\omega_1-\omega_2)t} |e\rangle\langle s| = \frac{\hbar\Omega_2}{2} \cancel{e^{-2i\omega_1 t}} |e\rangle\langle s|$ fast osc.

$\frac{\hbar\Omega_2}{2} e^{i(\omega_1+\omega_2 t)} U |s\rangle\langle e| U^\dagger = \frac{\hbar\Omega_2}{2} e^{i(\omega_1+\omega_2)t} e^{i(\omega_1-\omega_2)t} |s\rangle\langle e|$

$= \frac{\hbar\Omega_2}{2} \cancel{e^{2i\omega_1 t}} |s\rangle\langle e|$ fast oscillating terms

and $i\dot{U}U^\dagger = i\,i(\omega_1-\omega_2) e^{i(\omega_1-\omega_2)t|s\rangle\langle s|} e^{-i(\omega_1-\omega_2)t|s\rangle\langle s|}) = -(\omega_1-\omega_2)|s\rangle\langle s|$

$\Rightarrow \tilde{H} = -\hbar\Delta |e\rangle\langle e| - \hbar\delta |s\rangle\langle s| + \frac{\hbar\Omega_1}{2}(|e\rangle\langle g| + |g\rangle\langle e|) + \frac{\hbar\Omega_2}{2}(|e\rangle\langle s| + |s\rangle\langle e|)$

wrong 6l

use $U_1 = e^{+i\omega_1 t\,|e\rangle\langle e|}$

$U_2 = e^{+i(\omega_1-\omega_2)t\,|s\rangle\langle s|}$

$U_1 = e^{-i\omega_1 t\,|g\rangle\langle g|}$ works well but then we have to energy shift

$U_2 U_1 H U_1^\dagger U_2^\dagger = \hbar\omega_{eg}|e\rangle\langle e| + \hbar\omega_{sg}|s\rangle\langle s| + \Omega_1(|e\rangle\langle g| + h.c.) + \Omega_2(|e\rangle\langle s| + h.c.),$

$i\dot{U}_1 U_1^\dagger + i\dot{U}_2 U_2^\dagger = \omega_1 |g\rangle\langle g| + \omega_2 |s\rangle\langle s|$

$\tilde{H} = \omega_{eg}|e\rangle\langle e| + \omega_1 |g\rangle\langle g| + \omega_{sg}$

$H \to H - \omega_1 \mathbb{1} = H - \omega_1(|e\rangle\langle e| + |g\rangle\langle g| + |s\rangle\langle s|)$

$\to \tilde{H} - \omega_1 \mathbb{1} = |e\rangle\langle e|(\omega_{eg}-\omega_1) + |g\rangle\langle g|(\omega_1-\omega_1) + |s\rangle\langle s|$

→ what we did here

$U_1 = e^{-i\omega_1 |g\rangle\langle g| t}$

$U_2 = e^{-i\omega_2 t |S\rangle\langle S|}$

$H \to H - \omega_1 \mathbb{1}$

$U = e^{i\omega_1 t \mathbb{1}}$

$H - i\dot{U}U^\dagger$

$\begin{pmatrix} UHU^\dagger = H \\ -i\dot{U}U^\dagger = -\omega_1 \mathbb{1} \end{pmatrix}$

$U_1' = e^{i\omega_1 t |e\rangle\langle e|}$

$U_2' = e^{i(\omega_1 - \omega_2) t |S\rangle\langle S|}$

$\to UU_2U_1 = e^{i\omega_1 t \mathbb{1}} e^{-i\omega_1 t |g\rangle\langle g|} e^{-i\omega_2 t |S\rangle\langle S|} = U_2' U_1'$

$\hat{H} = \omega_{eg} |e\rangle\langle e| + \Omega(|e\rangle\langle g| + |g\rangle\langle e|)$

$\Omega \ll \omega_{eg}$ → use perturbation theory

$\hat{H} = \omega_{eg} |e\rangle\langle e| + \Omega(e^{-i\omega_{eg} t}|e\rangle\langle g| + e^{i\omega_{eg} t}|g\rangle\langle e|)$ can we still apply perturbation theory?

$\Omega \ll \omega_{eg}$

$U_1 = e^{i\omega_{eg} t |e\rangle\langle e|}$

$H = \Omega(|e\rangle\langle g| + |g\rangle\langle e|) + (\omega_{eg} \cancelto{0}{-\omega_{eg}})|e\rangle\langle e|$

→ can't just do the same perturbation theory now since there's no ω_{eg}-term to compare it to

$\Delta \neq 0$

$H = \Omega(|e\rangle\langle g| + |g\rangle\langle e|) + \Delta |e\rangle\langle e|$

→ control Δ to adjust the magnitude of perturbation

best way to see if a term is big/negligible or not is to switch to a ref. frame where it is time-independent and compare it to other terms as well the question is not only "is it fast oscillating" but rather "is it fast oscillating in a frequency that is away from the systems frequency?" (I think)

faster option to solve_ivp: often: just take $U = e^{-iHt/\hbar}$, especially if you're just interested in one specific zeitpunkt