Machine Intelligence II | **Exercise Sheet 1**
SS2012, Prof. Obermayer | due: 19.04.2012

Problem 1.1 (2 points)
Assume $\boldsymbol{W}$ to be an invertible square matrix, whose determinant is given by $\det \boldsymbol{W}$. Using the definitions of matrix gradient, determinant, inverse matrix and adjoint given in the mathematics tutorial, show that

$$\frac{\partial \log |\det \boldsymbol{W}|}{\partial \boldsymbol{W}} = (\boldsymbol{W}^T)^{-1},$$

where $|...|$ denotes the absolute value.

Problem 1.2 (2 points)
Let X be a random variable and $p : \mathbb{R} \to \mathbb{R}$ with:

$$p(x) = \begin{cases} c \cdot sin(x), & x \in [0, \pi] \\ 0, & elsewhere \end{cases}$$

a) (1 point)
Determine the parameter value $c \in \mathbb{R}$ so that $p(x)$ is a probability density.

b) (1 point)
Let $p(x)$ be the probability density of X. Determine the expected value $\langle X \rangle_p$ and the variance $\left\langle X^2 \right\rangle_p - \langle X \rangle_p^2$.

Problem 1.3 (2 points)
Assume that the probability density function of a two-dimensional random vector $\boldsymbol{z} = (x, y)^T$ is

$$p_{\boldsymbol{z}}(\boldsymbol{z}) = p_{x,y}(x, y) = \begin{cases} \frac{3}{7}(2 - x)(x + y), & x \in [0, 2], y \in [0, 1] \\ 0, & \text{elsewhere} \end{cases}$$

a) (1 point)
Write down the cumulative distribution function, $\boldsymbol{F}_{\boldsymbol{z}}(\boldsymbol{z}) = \boldsymbol{F}_{x,y}(x, y)$, by integrating over both x and y and taking into account the limits of the regions where the density is non-zero.

b) (1 point)
Write down the marginal densities $p_x(x)$ and $p_y(y)$ of the variables x and y and determine if the two variables are independent or uncorrelated.

Problem 1.4 (1 point)
Write down the Taylor series expansion of the function $\sqrt{1 + x}$ around $x = 0$ (up to 3rd order).

Problem 1.5 (1 point)
Consider the 3×3 matrix

$$A = \begin{pmatrix} 5 & 8 & 16 \\ 4 & 1 & 8 \\ -4 & -4 & -11 \end{pmatrix}$$

Calculate the determinant and the trace of A (directly, not via Eigenvalues).

Problem 1.6 (2 points)
Consider the two functions

$$f(x, y) := c + x^2 + y^2$$
$$g(x, y) := c + x^2 - y^2,$$

where c is a constant. Show that $\mathbf{a} = (0, 0)$ is a critical point of both functions. Check for f and for g whether $\mathbf{a}$ is a minimum, maximum or no extremum by calculating the Hessian matrix. Make use of the fact that a matrix is positive (negative) definite if and only if all its eigenvalues are positive (negative).

Total points: 10