Übung zur Vorlesung

Maschinelles Lernen 2

Sommersemester 2013

Abteilung Maschinelles Lernen
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Exercise Sheet 9

Due **June 17**, 9am local time on ISIS

19. **The Viterbi algorithm (70 + 30 + 15* + 15* P)**
(a) *Implement* the Viterbi algorithm which computes the most likely hidden state sequence for a sequence of observations. For this, write a MATLAB function `viterbi` with signature

```
x = viterbi(A, B, pi, y),
```

which takes the transistion matrix `A`, emission matrix `B`, initial distribution `pi`, and the observed sequence `y`. It outputs the MLE for the hidden state sequence `x`.

(Pseudocode of the algorithm can be found on page 2)

(b) Perform the following experiment: for the Hidden Markov Model of sheet 9, that is,

$$A = \begin{pmatrix} 0{,}1 & 0{,}9 \\ 0{,}5 & 0{,}5 \end{pmatrix} \quad B = \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}4 & 0{,}6 \end{pmatrix} \quad \pi = \begin{pmatrix} 1 & 0 \end{pmatrix},$$

generate sequences of length $\ell = 5, 10$ and 20; for each length generate a number of $N = 1000$ pairs of output sequences and hidden state sequences. On these sequences, for each length ℓ, compare (i) the Viterbi algorithm and (ii) the algorithm which randomly uniformly estimates a state sequence by (i) plotting for each length ℓ, and all integers $1 \leq k \leq \ell$, the relative frequency of the algorithm correctly estimating the hidden state at position k (this is three plots, one for each ℓ, and in each plot two curves), and (ii) for each length ℓ, computing the relative frequency of both algorithms succeeding in completely identifying the state sequence correctly (this is two numbers for each of the three ℓ).

Hint: if you want to do exercise (d*), and also generally, it is advised that you save the generated sequences for future use. For this, use the MATLAB command `save`.

(c*) In a Markov Model, there are often state sequences resp. distributions to which the model converges. *Prove* for the model from sheet 9 given in exercise (b) that the limit $\pi_\infty := \lim_{k\to\infty} \pi \cdot A^k$ exists, and it does not depend on the choice of π. That is, *prove* that π_∞ is the same, no matter what the initial distribution π is.

(d*) In your comparison in exercise (b), add the algorithm which does not estimate the outputs uniformly, but with independent probabilities $\pi_\infty \cdot B$.

Viterbi-algorithmus (cf. Rabiner, *A Tutorial on Hidden Markov Models and Selected Applications in Speech Recognition*, Proceedings of the IEEE, Vol. 77, No. 2, 1989, p. 264ff.)

Variables:		
	n	number of hidden states
	m	number of possible emissions
	ℓ	Length of sequence
	a_{ij}	transition probabilities
	b_{ij}	probability $P()$ of emitting symbol j in state i
	π_i	probability to start in state i
	$y(t)$	observation at time t
	$\widehat{x}(t)$	estimated state at time t
	P^*	probability for $y(t)$ given $\widehat{x}(t)$
	$\delta_i(t)$	"best" probability of state i at time t
	$\psi_i(t)$	"best" state i at time t

Initalization:

$$\begin{aligned}\delta_i(1) &= \pi_i \cdot b_{ix(1)}, \qquad 1 \leq i \leq n,\\ \psi_i(1) &= 0.\end{aligned}$$

Recursion:

$$\begin{aligned}\delta_j(t) &= \max_{1\leq i\leq n}[\delta_i(t-1)\cdot a_{ij}]\cdot b_{jx(t)}, \qquad 2 \leq t \leq \ell,\ 1 \leq j \leq n\\ \psi_j(t) &= \operatorname*{argmax}_{1\leq i\leq n}[\delta_i(t-1)\cdot a_{ij}], \qquad 2 \leq t \leq \ell,\ 1 \leq j \leq n.\end{aligned}$$

Termation:

$$\begin{aligned}P^* &= \max_{1\leq i\leq n}[\delta_i(\ell)],\\ \widehat{x}(T) &= \operatorname*{argmax}_{1\leq i\leq n}[\delta_i(\ell)].\end{aligned}$$

Backtracking:

$$\widehat{x}(t) = \psi_{\widehat{x}(t+1)}(t+1), \qquad t = \ell-1, \ell-2, \cdots, 1.$$

Please ask questions in the ISIS discussion forums for Machine Learning 2: `https://www.isis.tu-berlin.de/course/view.php?id=8005`