Übung zur Vorlesung

Maschinelles Lernen 2

Sommersemester 2013

Abteilung Maschinelles Lernen
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Exercise Sheet 9

Due **June 17**, 9am local time on ISIS

1. **Hidden Markov Models (5 + 5 + 20 + 20 + 20 + 20 P)**
Consider the Hidden Markov Model with transition matrix

$$A = \begin{pmatrix} 0{,}1 & 0{,}9 \\ 0{,}5 & 0{,}5 \end{pmatrix}$$

and emission matrix

$$B = \begin{pmatrix} 0{,}2 & 0{,}8 \\ 0{,}4 & 0{,}6 \end{pmatrix}$$

The initial probability vector is

$$\pi = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

(a) *Draw* the graph of the model. As usual, use round shapes for hidden states and square shapes for visible states, and arrows for transitions/emissions; include transition/emission probabilities).

(b) We interpret the above as a model for an experiment with two hidden (unfair) coins and two visible coins. *Describe* such an experiment which can be modelled by the Markov model given above. Here, heads should correspond to the first indices in A, b, π, heads to the second.

(c) Using the Bayes formula for conditional probabilities, *compute* the probabilities

$$P\left(x|(\text{tails, tails})\right),$$

for all state sequences of length 2. That is, for each of the four sequences of states of length 2, compute the probability that an observation of "tails, tails" was caused by the hidden sequence of states x.

(d) *Write* a MATLAB function `hidden_coins.m`, in which you simulate the experiment. As input parameter, you should be able to specify a length n of the observation (in (c), this was fixed to 2). As output parameters, the function returns the n-vector of outputs, and the n-vector containing the state sequence.

(e) *Write* a MATLAB script `exp_hidden_coins.m`, in which you fix the observation length $n = 20$ and run `hidden_coins.m` for $N = 1000$ times. *Plot* the relative frequency of observing heads, plus standard deviation, against n.

(f) *Prove* in the scenario of (e) and the assumptions of the exercise: if $N \to \infty$, then the sample observation frequency of heads/tails at fixed position n converges, in probability and entry-wise, to the vector $\pi \cdot A^n \cdot B$.

Hint: Use the central limit theorem.

Please ask questions in the ISIS discussion forums for Machine Learning 2:
`https://www.isis.tu-berlin.de/course/view.php?id=8005`