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# Numerische Mathematik für Ingenieure II
## Homework 12

**Programming exercise 15:** (12 points)
In this programming exercise serveral iterative solvers for linear algebraic systems are compared. Here we consider matrices and right hand sides arising from a finite difference discretization with uniform grid size $h = 1/(201)$ of the following PDEs on the domain $\Omega = (0, 1)$ with zero Dirichlet boundary conditions and $f(x) = \sin(2\pi x)\exp(10x)$:

$$-u'' + 50u = f, \tag{1}$$
$$-u'' + 10^{-4}u' + 50u = f, \tag{2}$$
$$-u'' + 10^{-2}u' + 50u = f, \tag{3}$$
$$-u'' + u' + 50u = f, \tag{4}$$
$$-u'' + 10^{2}u' + 50u = f, \tag{5}$$
$$-u'' + 10^{4}u' + 50u = f. \tag{6}$$

Discretize the second order term $u''$ with the 3-point stencil and the first order term $u'$ with backward differences to obtain the corresponding linear algebraic systems $L_h^{(j)} x^{(j)} = f_h^{(j)}$ with $L_h^{(j)} \in \mathbb{R}^{200,200}$ and $f_h^{(j)} \in \mathbb{R}^{200}$ ($j \in \{1, \ldots, 6\}$). Test the following methods without preconditioning:

| | |
|---|---|
| (1) `pcg` | (2) `minres` |
| (3) `bicg` | (4) `qmr` |
| (5) `gmres` (without restart!) | |

Familiarize yourself with the interface of these functions in MATLAB's documentation (`help` and `doc`). Use all methods with `x0=zeros(200,1)`, `tol=1e-6` and `maxit=250`. Write a `function flags = p15compare()` that carries out the following computations:

- Create a figure window for each linear algebraic system $j \in \{1, \ldots, 6\}$ and plot *all* residual norms relative to the initial residual (use `semilogy`). Note that some of MATLAB's implementations return the last residual seperately.
- Return the `flag` values for all linear algebraic systems and methods in the matrix `flags` $\in \mathbb{R}^{6,5}$, that is `flags(`$j$`,`$i$`)` is the flag obtained for the $j$-th linear algebraic system with the $i$-th method.

Annotate your plots!

**Exercise 20:** (6 points)

(a) Let $\|x\|_\infty = \max_{i\in\{1,\dots,n\}} |x_i|$ be the maximum norm for $x \in \mathbb{R}^n$. Show that for the induced matrix norm (that is $\|A\|_\infty = \max_{\|x\|_\infty=1} \|Ax\|_\infty$ for $A \in \mathbb{R}^{n,n}$) the following holds:

$$\|A\|_\infty = \max_{i\in\{1,\dots,n\}} \sum_{j=1}^{n} |a_{ij}|$$

(Note: therefore $\|A\|_\infty$ is also called *row sum norm.*)

(b) Let $A \in \mathbb{R}^{n,n}$ be strictly diagonally dominant, that is

$$|a_{ii}| > \sum_{\substack{j=1\\ j\neq i}}^{n} |a_{ij}| \qquad \forall i \in \{1,\dots,n\}.$$

Show: for each $b \in \mathbb{R}^n$ the Jacobi method for the solution of $Ax = b$ converges.

Hint: Show that for the iteration matrix $M^{-1}N = -D^{-1}(L+R)$ (see lecture) under the assumption of strict diagonal dominance $\|M^{-1}N\|_\infty < 1$ holds. Use the result of (a).