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WS 11/12
To be submitted in the lecture on: 30.11.2011

Numerische Mathematik für Ingenieure II
Homework 6

Exercise 12: (8 points)
Let $\Omega \subset \mathbb{R}^2$ be a set for which the Gaussian integral theorem can be applied. For the following problems determine their variational formulations of the form:

$$\text{Find } u \in V \text{ such that } a(u,v) = g(v) \text{ for all } v \in V.$$

Please specify the test space V, the bilinear form $a : V \times V \to \mathbb{R}$ and the linear form $g : V \to \mathbb{R}$. When setting up the bilinear form make sure that the order of the occurring derivatives is reduced to a minimum.

(a)

$$\begin{aligned} -\Delta u + cu &= f_1 && \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} &= f_2 && \text{on } \partial\Omega, \end{aligned}$$

with $c \in \mathbb{R}$, $c > 0$ and $f_1, f_2 \in C^0(\overline{\Omega})$.

(b)

$$\begin{aligned} -\Delta u + g_1 u_x + g_2 u_y &= f && \text{in } \Omega, \\ \frac{\partial u}{\partial \nu} + g_3 u &= 0 && \text{on } \partial\Omega, \end{aligned}$$

with $g_1, g_2, g_3, f \in C^0(\overline{\Omega})$.

(c)

$$\begin{aligned} \Delta^2 u &= f && \text{in } \Omega, \\ u = \frac{\partial u}{\partial \nu} &= 0 && \text{on } \partial\Omega, \end{aligned}$$

with $f \in C^0(\overline{\Omega})$. (Hint: Use Green's identity twice).

Exercise 13: (8 points)
Consider the following Dirichlet boundary value problem

$$\begin{aligned} -u''(x) &= \pi^2 \sin(\pi x), && \text{for all } x \in \Omega := (0,1), \\ u(0) = u(1) &= 0, \end{aligned}$$

with the exact solution

$$u(x) = \sin(\pi x).$$

(a) Write the variational formulation of this problem (see Exercise 12), with

$$V := C_0^1(\overline{\Omega}).$$

(b) Let the following two spaces be given

$$\begin{aligned} V_h^{(1)} &:= \operatorname{span}\{b_1(x) = \frac{1}{3}x^2 - \frac{1}{3}x, \quad b_2(x) = x^4 - 2x^3 + x^2\}, \\ V_h^{(2)} &:= \operatorname{span}\{b_1(x) = \frac{1}{2}x, \ b_2(x) = x^3 - \frac{3}{2}x^2 + \frac{5}{8}x\}. \end{aligned}$$

Which of these two spaces is a proper ansatz space for the Galerkin approximation? Explain why one choice of ansatz space is better suited for the Galerkin approximation.

(c) Calculate the Galerkin approximation u_h with respect to the previously chosen ansatz space.