Implementierung der Cut & Count-Technik am Beispiel Steiner tree

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27. Oktober 2016

Überblick

1. Cut & Count
   - Allgemeines
2. Cut & Count mit Steiner Tree
   - Cut
   - Count
   - (Nice) Tree Decomposition
3. Implementierung

Cut & Count-Technik

- Technik um connectivity-type Probleme mithilfe von Randomisierung zu lösen(Marek Cygan, Jesper Nederlof, Marcin Pilipczuk, Michał Pilipczuk, Johan van Rooij, Jakub Onufry Wojtaszczyk)
- angewendet auf viele verschiedene Probleme (z.B. Longest Path, Steiner Tree, Feedback Vertex Set uvm)
- Randomisierung durch Isolation-Lemma
- als Ergebnis ein einseitiger Monte-Carlo-Algorithmus mit Laufzeit $c^{tw(G)}|V|^{\mathcal{O}(1)}$

Theorem

There exists a randomized algorithm, which given a graph G with n vertices, a tree decomposition of G of width t and a number k in $3^t n^{\mathcal{O}(1)}$ time either states that there exists a connected vertex cover of size at most k in G, or that it could not verify this hypothesis. If there indeed exists such a cover, the algorithm will return “unable to verify” with probability at most $1/2$.

Steiner Tree

Problem

Input: An undirected graph $G = (V, E)$, a set of terminals $T \subseteq V$ and an integer k.
Question: Is there a set $X \subseteq V$ of cardinality k such that $T \subseteq X$ and $G[X]$ is connected?

Cut (1)

- definiere Gewichtsfunktion $\omega : V \to \{1, \dots, N\}$
- sei $\mathcal{R}_W$ die Menge aller Teilmengen von X aus V mit $T \subseteq X$, $\omega(X) = W$ und $|X| = k$
- sei $\mathcal{S}_W = \{X \in \mathcal{S}_W | G[X] \text{ ist zusammenhängend}\}$
- $\cup_W \mathcal{S}_W$ ist die Menge der Lösungen
- gibt es ein W für das die Menge nichtleer ist, so gibt der Algorithmus eine positive Antwort

Cut (2)

- einen beliebigen Terminalknoten $v \in T$ als v_1 festlegen
- sei $\mathcal{C}_W$ die Menge aller Subgraphen, die einen konsistenten Cut $(X, (X_1, X_2))$ bilden, wobei $X \in \mathcal{R}_W$ und $v_1 \in X_1$

Lemma 3.3

Let $G = (V, E)$ be a graph and let X be a subset of vertices such that $v_1 \in X \subseteq V$. The number of consistently cut subgraphs $(X, (X_1, X_2))$ such that $v_1 \in X_1$ is equal to $2^{cc(G[X])-1}$.

Count (1)

- aus Lemma 3.3 ist bekannt: $|\mathcal{C}| = \sum_{X \in \mathcal{R}} 2^{cc(G[X])-1}$
- wir legen W fest und ignorieren die Indices: $|\mathcal{C}| \equiv |X \in \mathcal{R}| cc(G[X]) = 1| = |\mathcal{S}|$

Lemma 3.4

Let G, ω, $\mathcal{C}_W$ and $\mathcal{S}_W$ be as defined above. Then for every W, $|\mathcal{S}_W| \equiv |\mathcal{C}_W|$.

Einschub: (Nice) Tree Decomposition (1)

Tree Decomposition

A tree decomposition of a graph G is a tree $\mathbb{T}$ in which each vertex $x \in \mathbb{T}$ has an assigned set of vertices $B_x \subseteq V$ (called a bag) such that $\cup_{x \in \mathbb{T}} B_x = V$ with the following properties:

- for any $uv \in E$, there exists an $x \in \mathbb{T}$ such that $u, v \in B_x$
- if $v \in B_x$ and $v \in B_y$, then $v \in B_z$ for all z on the path from x to y in $\mathbb{T}$

Einschub: (Nice) Tree Decomposition (2)

Nice Tree Decomposition

...

- **Leaf bag**: a leaf x of $\mathbb{T}$ with $B_x = \emptyset$
- **Introduce vertex bag**: an internal vertex x of $\mathbb{T}$ with one child vertex y for which $B_x = B_y \cup \{v\}$ for some $v \notin B_y$. This bag is said to introduce v
- **Introduce edge bag**: an internal vertex x of $\mathbb{T}$ labeled with an edge $uv \in E$ with one child bag y for which $u, v \in B_x = B_y$. This bag is said to introduce uv
- **Forget bag**: an internal vertex x of $\mathbb{T}$ with one child bag y for which $B_x = B_y\{v\}$ for some $v \in B_y$. This bag is said to forget v
- **Join bag**: an internal vertex x with two children vertices l and r with $B_r = B_l$

Count (2)

- Kardinalität von $\mathcal{C}_W$ modulo 2 kann mit dynamischen Programm berechnet werden
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Bullet Points

- Lorem ipsum dolor sit amet, consectetur adipiscing elit
- Aliquam blandit faucibus nisi, sit amet dapibus enim tempus eu
- Nulla commodo, erat quis gravida posuere, elit lacus lobortis est, quis porttitor odio mauris at libero
- Nam cursus est eget velit posuere pellentesque
- Vestibulum faucibus velit a augue condimentum quis convallis nulla gravida

Blocks of Highlighted Text

Block 1

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Block 2

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Block 3

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Multiple Columns

Heading

1. Statement
2. Explanation
3. Example

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Table

Treatments	Response 1	Response 2
Treatment 1	0.0003262	0.562
Treatment 2	0.0015681	0.910
Treatment 3	0.0009271	0.296

Tabelle : Table caption

Theorem

Theorem (Mass–energy equivalence)

$E = mc^2$

Verbatim

Example (Theorem Slide Code)

```
\begin{frame}
\frametitle{Theorem}
\begin{theorem}[Mass--energy equivalence]
$E = mc^2$
\end{theorem}
\end{frame}
```

Figure

Uncomment the code on this slide to include your own image from the same directory as the template .TeX file.

Citation

An example of the `\cite` command to cite within the presentation:

This statement requires citation [Smith, 2012].

References

John Smith (2012)
Title of the publication
Journal Name 12(3), 45 – 678.

The End